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ASSIGNMENT 3 - THEORETICAL

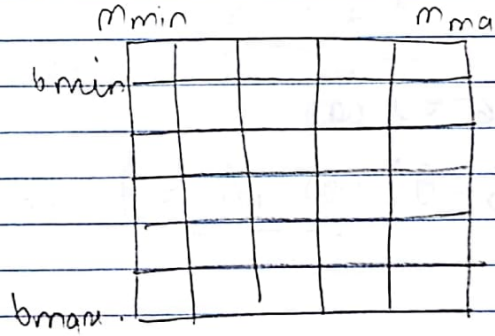
A1) Hough Transform: Parametrization.

⇒ Explain why this option did not become a popular choice.

⇒ The reason this option did not become a popular choice is, when I have perpendicular, vertical lines, m_0 becomes ∞.

So it gets impossible to define it.

⇒ Since $m_0 \to \infty$, we can no longer divide the hough space in equal intervals (grid) as $m = \infty$.



⇒ Also, you can have infinite combinations of m_0 & b_0 which satisfy the equation :-

$y = m_0 x + b_0$

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A2) Hough Transform : Polar Representation I.

Show that the polar transformation of a line $x\cos\theta + y\sin\theta = \rho$, represents a cosine function in parameter space with axes θ & ρ.

$\Rightarrow$ Given a point $(x_i, y_i) \Rightarrow (\rho, \theta)$ are parameters.

$$\rho = x_i\cos\theta + y_i\sin\theta$$

$$\frac{\rho}{A_i} = \frac{x_i}{A_i}\cos\theta + \frac{y_i}{A_i}\sin\theta$$

$$\frac{\rho}{A_i} = \underbrace{\cos\delta_i\cos\theta + \sin\delta_i\sin\theta}_{\cos(\theta-\delta_i)}$$

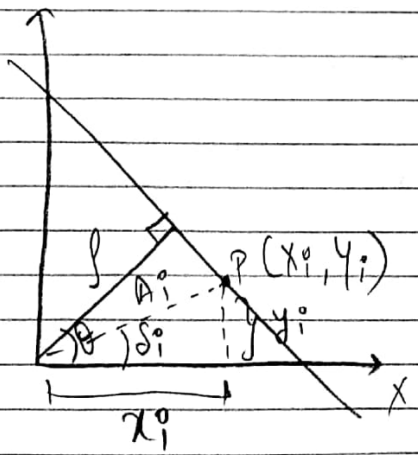
$$\Rightarrow \rho = A_i\cos(\theta-\delta_i)$$

$$A_i = \sqrt{x_i^2+y_i^2}$$

$$\delta_i = \tan^{-1}\left(\frac{y_i}{x_i}\right)$$

Given $(x_i, y_i) \rightarrow (A_i, \delta_i) \Rightarrow$ cos-function in ρ, θ space.

where:-



$$\cos\delta_i = \frac{x_i}{A_i} \qquad A_i = \sqrt{x_i^2+y_i^2}$$

$$\sin\delta_i = \frac{y_i}{A_i} \qquad \delta_i = \tan^{-1}\left(\frac{y_i}{x_i}\right)$$

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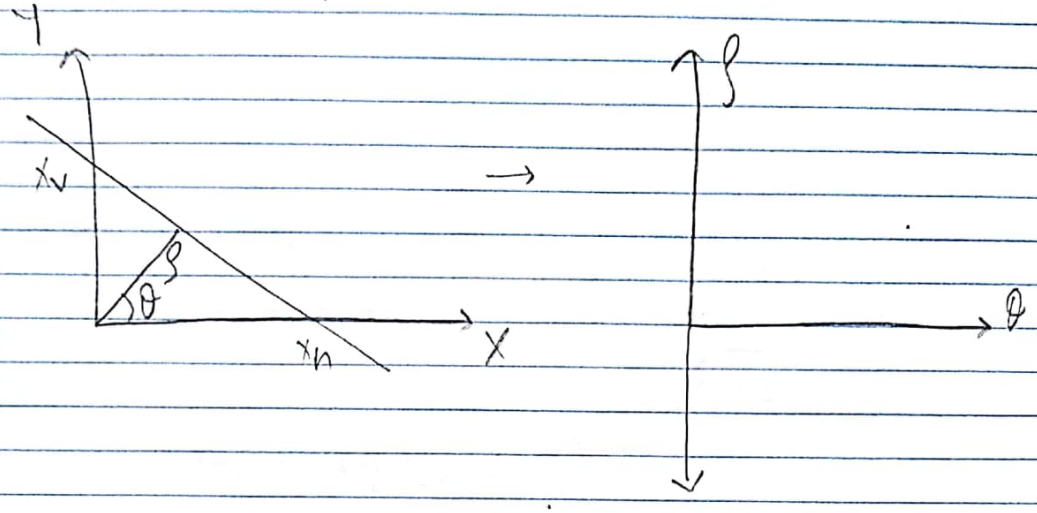
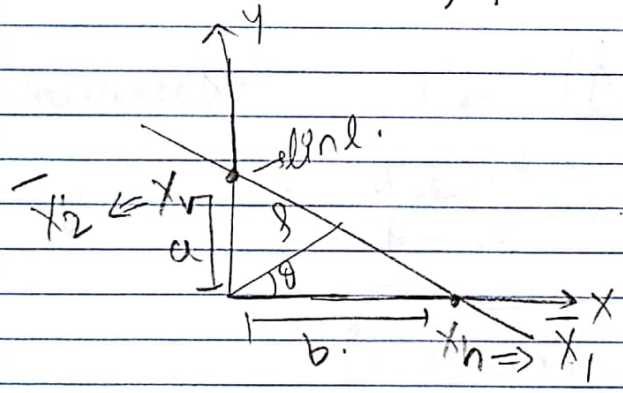
A3) Hough Transform: Polar Representation II.



To be able to plot the corresponding cosine curves in (θ, ρ) parameters space, we use:-



Assuming:- x_v & x_2 < a & b.

~~$\bar{x}_1 : \rho_v = x$~~

$\bar{x}_1 : \rho_1 = a \cos(\theta - \pi/2)$ -(1)

$\bar{x}_2 : \rho_2 = b \cos(\theta - 0)$ -(2)

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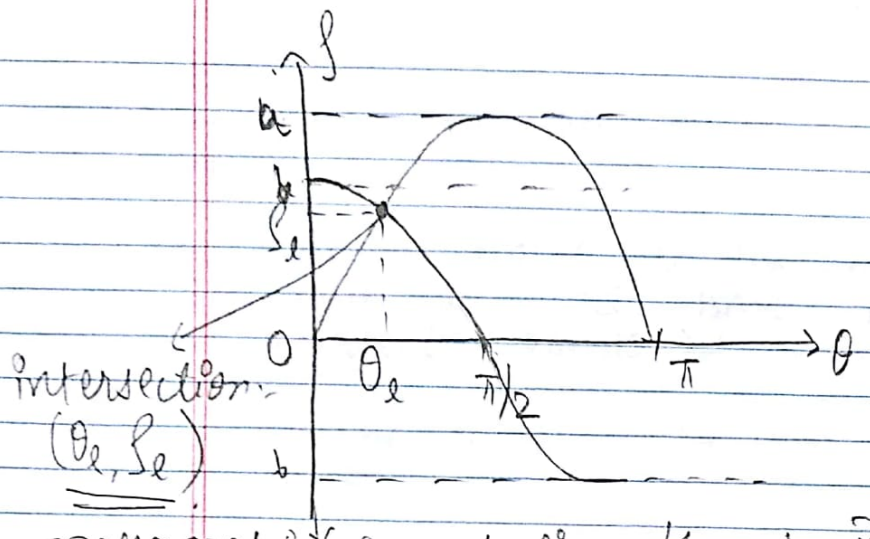

Intersection (θ_ℓ, ρ_ℓ)

= parametrization of line through $\bar{x}_1$ & $\bar{x}_2$

From eq ① & ②.

$$\rho_1 = a \sin\theta$$

$$\rho_2 = b \cos\theta$$

Intersection point is (θ_ℓ, ρ_ℓ).

$$\therefore a \sin\theta = b \cos\theta$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\rho = a \sin\left(\tan^{-1}\left(\frac{b}{a}\right)\right) = a \sin\left(\frac{b/a}{\sqrt{b^2/a^2+1}}\right)$$

$$= a \sin\left(\frac{b/a}{\frac{\sqrt{a^2+b^2}}{a}}\right) = \frac{ab}{\sqrt{a^2+b^2}}$$

∴ Intersection point is :- $\left(\tan^{-1}\left(\frac{b}{a}\right), \frac{ab}{\sqrt{a^2+b^2}}\right)$.

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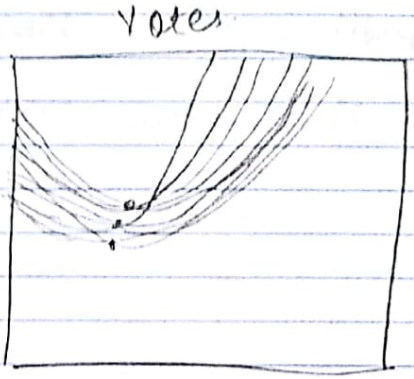
A4) Noisy line structures.

⇒ Due to noise in the image in our hough space we will have multiple curve intersection points.

Thereby making the peak fuzzy & hard to locate.



Like in this image, we can see that there are multiple points highlighted, which are close to each other but are there because of noise.

As the level of uniform noise increases, the maximum number of votes increases too.

In order to be able to deal with noise :-

- Choose a good grid / discretization
  - If we choose too coarse then large votes will be obtained when too many different lines correspond to a single bucket.
  - If you choose too fine then you will miss lines because some points that are not exactly collinear cast votes for different buckets.
- Increment neighbouring bins (smoothing).

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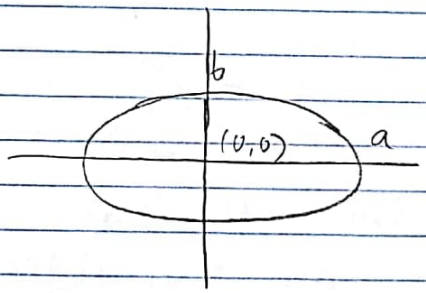
- Use only relevant features by getting rid of irrelevant ones.

eg:- By taking only edge points with significant gradient magnitude.

A5) Hough Transform for Ellipses.

To find ellipses using Hough Transform.

Standard form means its centered at the origin.



Horizontal major axis.

Equation:-

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow$$

without orientation, it will need to two parameters ~~to~~ (a, b) to be able to define it in a Hough Space.

If the ellipses is not centered at O, then:-

$$\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{b^2} = 1$$

It will need 4 parameters:- $x_c, y_c, a, b,$.

With Orientation it will need :-> x_c, y_c, a, b, θ.

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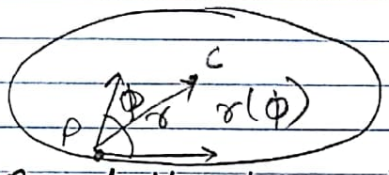
GHT :- Generalized Hough Transform.

GHT can be used to find a given arbitrary shape without having its mathematical equation.

Since ellipses is also another 2D arbitrary shape, GHT can be used to find ellipses using R-tables.

R-tables contain the edge orientation info along with the vectors. → Hence storing the angle along with magnitude.



R-table.

edge orientation	vectors.
ϕ_1	$\vec{r}_1^1, \vec{r}_2^1, \dots$
ϕ_2	$\vec{r}_1^2, \vec{r}_2^2 \dots$
:	— —

we record the shape into R-table.
Using this R-table, we can find the shape in a given image.

So in this case its the center c of the ellipses.

We take a reference point (c) & then find the radius from point P to c. & find the orientation ϕ.

Orientation

⇒ For rotation by θ of the shape :- we rotate all the vectors in the R-table by the same angle θ.