ECSE 493: Lab #1 Report

Due on Friday, February 20, 2015

Musallam 10:35am

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Problem 1

(a)

The torque for the DC motor is

$$T = K_m K_g i_m,$$

where K_m is the back emf constant and K_g is the gear ratio. The back emf V_b is given by

$$V_b = K_g K_m \omega.$$

Doing KVL around the equivalent circuit gives us the following equation for the applied voltage V

$$V = i_m R_m + V_b,$$

Where R_m is the motor resistance. Solving for i_m yields,

$$i_m = \frac{V}{R_m} - \frac{K_g K_m}{R_m}\omega.$$

Plugging this into the torque equation, we get

$$T = \frac{K_g K_m V}{R_m} - \frac{K_g^2 K_m^2}{R_m}\omega.$$

Taking into account that the force F of the motor applied to the cart is given by T/r and the angular velocity ω can be expressed by $\omega = \dot{x}/r$, we can express F as

$$F = \frac{K_g K_m V}{r R_m} - \frac{K_g^2 K_m^2}{R_m}\frac{\dot{x}}{r^2}.$$

Summing the forces in the free body diagram and assuming that the effect of friction is negligible, we can get the following second order differential equation for the variables V and x.

$$m\ddot{x} = \frac{K_g K_m}{r R_m}V - \frac{K_g^2 K_m^2}{r^2 R_m}\dot{x}$$

Taking the laplace transform of this function with respect to the position of the cart x yeilds

$$ms^2 X(s) + \frac{K_g^2 K_m^2}{r^2 R_m} s X(s) = \frac{K_g K_m}{r R_m} V(s)$$

Therefore, the transfer function of the cart with voltage as the input and position as the output, is given by

$$\frac{X(s)}{V(s)} = \frac{\frac{K_g K_m}{r R_m}}{ms^2 + \frac{K_g^2 K_m^2}{r^2 R_m} s}$$

Problem 2

(a)