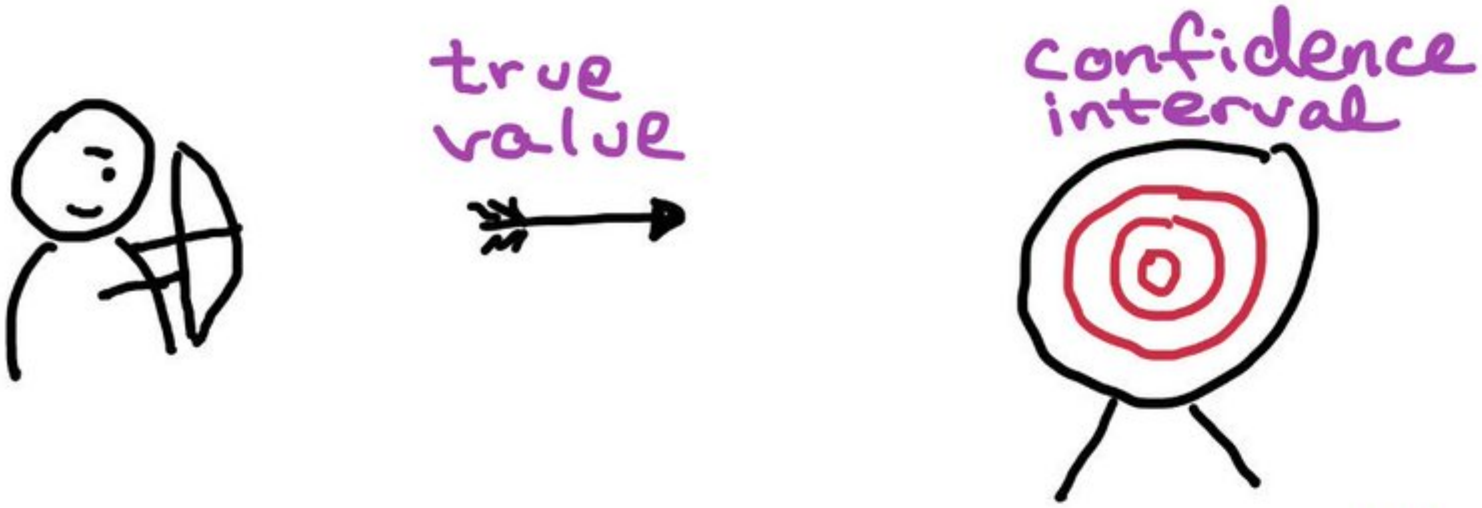
Confidence Intervals

We call a confidence interval a q% confidence interval if it is constructed such that it contains the true parameter at least q% of the time if we repeat the experiment a large number of times.

People think confidence intervals are like archery:

- the target is fixed & the true value might end up in the interval



@epiellie

But really confidence intervals are more like ring toss:

- the true value is fixed & the interval might end up around it.

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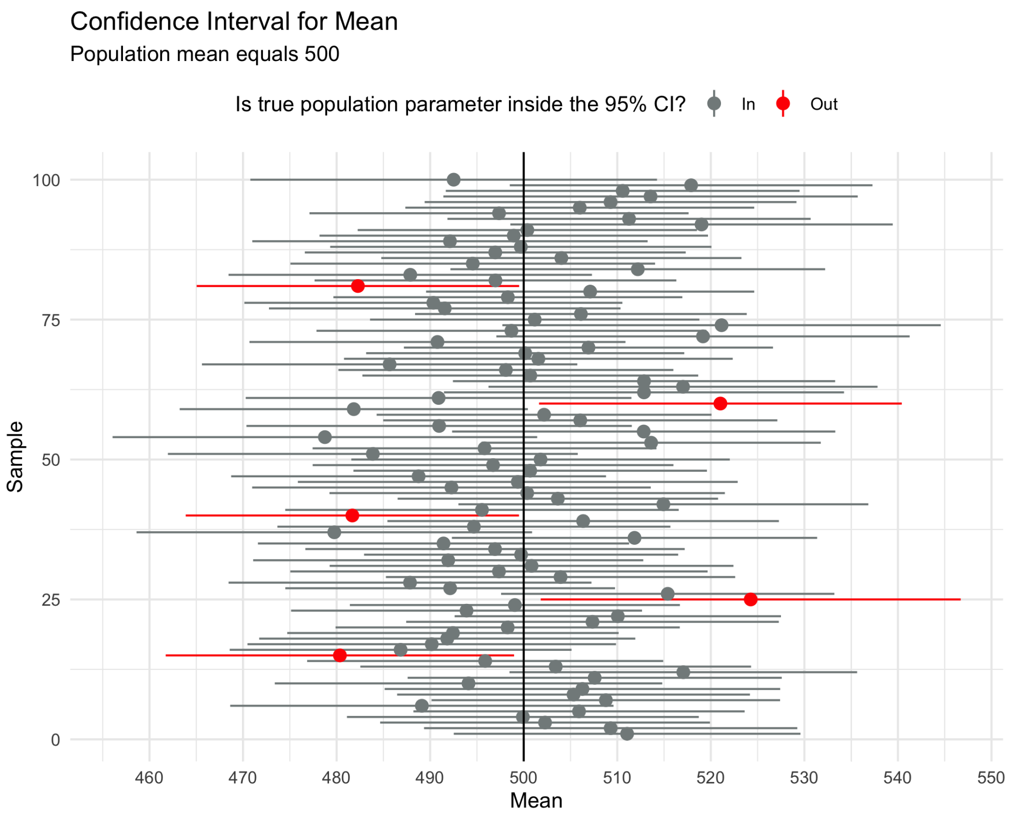
Confidence Intervals



- *interval estimate* = plausible range of values of the population parameter given our data

$$CI_{95} = \bar{x} \pm 1.96SE$$

- 95% CI is constructed so that:
  - when we draw *repeated samples*
  - 95% of CIs calculated with this formula *on these samples*
  - cover the true mean (not the means from other samples!)

- one single calculated CI tells us *nothing* about:
  - means of other samples
  - individual observations in our sample and/or population

- better say *"CIs cover/contain the true value 95% of times"*: this implies that it is the CIs that are changing over samples, not the true value, which is fixed

Probabilistic Interpretation of CIs

$$CI_{95} : [\bar{x} - 1.96SE, \bar{x} + 1.96SE]$$

- Randomness comes from the stage of drawing a sample (we only have one, but hypothetically, we draw them repeatedly)

- **After we draw the random sample**, calculating the CI is a matter of procedure:
  - there is no more randomness, it's just applying the formula
  - hence, we can think of this as a realized experiment
  - if CI bounds are just numbers, the true fixed value is either inside (1) or not (0)
  - we say: a single *calculated* CI contains the true parameter or not (and in real life, we don't know if it does, so we hope it is one of the ones that cover the true value)

- **Before the random sample is drawn**, we can apply probabilistic interpretation to CI:
  - for a 95% confidence interval (*before* the sample for calculating that CI is drawn!), there is a 95% chance that a CI will contain μ
  - the **random** interval $[\bar{x} - 1.96SE, \bar{x} + 1.96SE]$ contains μ with probability 0.95. It is a random interval, since the endpoints change with different samples (i.e., we don't have not drawn the sample yet)

A single CI is a range of plausible values for the true population parameter.

And by the construction of it, we would expect it that 95% of such intervals in the long run would cover the true mean.

And all we can do is hope that the CI we have is the one that contains the true population parameter.