(*) If its a maximization problem then coeff. of var. in obj. f^n are considered -ve.

SIMPLEX METHOD

As per the graphical method, optimum sol^n is obtined from the corner points of the sol^n space. This result forms the basis for algebraic simplex method used for solving LP model. Corner points are determined algebraically: ~~this~~

1. Convert the constraint inequality into equality.
2. Simplex method solves LP in iterations, each iteration moves the sol^n to a new corner point that has the potential to improve the value of objective f^n. ~~The process~~
3. The process ends once the optimum sol^n is reached

2 assumptions we make in this method:

1. All constraints (with the exception of non-ve restrictions) are equations with non-ve RHS.
2. All variables are non-ve.

Converting inequalities to equations:

In $\leq$ constraint, RHS represents the limit on the availability of resources and LHS represents the usage of these limited resources by the activities of the model. The difference is the unused or slack amount of the resources.

$$4x_1 + 3x_2 \leq 240$$

$$4x_1 + 3x_2 + \underbrace{s_1}_{\text{slack var.}} = 240$$

$$2x_1 + x_2 \geq 40$$

$$2x_1 + x_2 - \underbrace{x_3}_{\text{surplus var.}} = 40$$

continue of Flair furniture Q^n

$$\text{Max } Z - 70x_1 - 50x_2 + 0s_1 + 0s_2 = 0$$

$$4x_1 + 3x_2 + s_1 = 240 \quad \text{— (1.)}$$

$$2x_1 + x_2 + s_2 = 100 \quad \text{— (2.)}$$

$$s_1, s_2, x_1, x_2 \geq 0$$

Basic	x_1	x_2	s_1	s_2	Sol^n.
Z	-70	-50	0	0	0
s_1	4	3	1	0	240
s_2	2	1	0	1	100

continue

Leaving var. $\rightarrow$ $s_1 : \frac{240}{4} = 60$ X

$s_2 : \frac{100}{2} = 50$ ✓ ← Least

Pivot element $\rightarrow$ 2

Z					
s_1					
x_1	1	$1/2$	0	$1/2$	50

* The entering var. incase of maximiz. prob is the one var. with the most -ve coeff. in the obj. f^n.

* The leaving var. is the one with the least ratio. which is

$\hookrightarrow \dfrac{sol^n}{\text{constraint coeff. under entering var.}} \; (\geq 0)$

* Pivot element is intersection of entering and leaving var.

* we find Pivot row by dividing the orig. leaving var. row with pivot element
$\dfrac{\text{leaving var. row}}{\text{Pivot row}}$

* New z row $\Rightarrow$ z = orig. z row $-$ (it pivitol column coeff.)(the new pivot row)

$$z = z - (\;)(\;)$$
$$= (-70\ -50\ 0\ 0\ 0) - (-70)(1\ \tfrac{1}{2}\ 0\ \tfrac{1}{2}\ 50)$$
$$= (-70\ -50\ 0\ 0\ 0) - (-70\ -35\ 0\ -35\ -3500)$$

$$z = (0\ -15\ 0\ 35\ 3500)$$

$$s_1 = (4\ 3\ 1\ 0\ 240) - (4)(1\ \tfrac{1}{2}\ 0\ \tfrac{1}{2}\ 50)$$

$$= (4\ 3\ 1\ 0\ 240) - (4\ 2\ 0\ 2\ 200)$$

$$= (0\ 1\ 1\ -2\ 40)$$

* Since corresponding to z, none of the coeff. are -ve for the non-basic var. s_1 & s_2 it implies optimality where x_1 is 30, x_2 is 40 and z is 4100.

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Redoing iterations

Basic	x_1 ↓	x_2	s_1	s_2	Sol^n
Z	-70	-50	0	0	0
s_1	4	3	1	0	240
← s_2	$\boxed{2}$	1	0	1	100
Z	0	-15 ↓	0	35	3500
← s_1	0	$\boxed{1}$	1	-2	40
x_1	1	$1/2$	0	$1/2$	50
Z	0	0	$+15$	5	4100
x_2	0	1	1	-2	40
x_1	1	0	$-1/2$	$3/2$	30

$z = z - (-15)(0\ 1\ 1\ -2\ 40)$

$= z - (0/-15/-15/30/\ldots$

$x_1 = x_1 - \left(\frac{1}{2}\right)(0\ 1\ 1\ -2\ 40)$

$x_1 - (0\ \frac{1}{2}\ \frac{1}{2}\ -1\ 20)$

(Q.) Min $Z = x_1 - x_2 + 2x_3$

subj. to $x_1 + 2x_2 + x_3 \leq 20$

$x_1 - 2x_2 + x_3 \geq -30 \rightarrow \cancel{-ve}$

$2x_1 + 3x_2 - x_3 \leq 50$

$x_1, x_2, x_3 \geq 0$

Ans.)

$Z - x_1 + x_2 - 2x_3 = 0$

$x_1 + 2x_2 + x_3 + S_1 = 20$

$-x_1 + 2x_2 - x_3 + S_2 = 30$

$2x_1 + 3x_2 - x_3 + S_3 = 50$

~~(1 | 2 | 1 | 1 | 0 | 0 | 20)~~

$S_3 = S_3 - (3/2 \mid 3 \mid 3/2 \mid 3/2 \mid 0 \mid 0 \mid 30)$

Basic	x_1	$x_2 \downarrow$	x_3	s_1	s_2	s_3	Solⁿ
Z	-1	1	-2	0	0	0	0
← S_1	1	$\boxed{2}$	1	1	0	0	20
S_2	-1	2	-1	0	1	0	30
S_3	2	3	-1	0	0	1	50
Z	-3/2	0	-5/2	-1/2	0	0	-10
x_2	1/2	1	1/2	1/2	0	0	10
S_2	-2	0	-2	-1	1	0	10
S_3	1/2	0	-5/2	-3/2	0	1	20

$Z = Z -$ (1)(1/2 1 1/2 1/2 0 0 ...

$S_2 = S_2 -$ (2)(1/2 1 1/2 1/2 0 0 ...

Solⁿ: $x_1, x_3, s_1 = 0$

$x_2 = 10, \; s_2 = 10, \; s_3 = 20, \; z = -10$

Artificial starting Sol^n

$R_1, R_2 \rightarrow$ Artifi... var.

Constraint $\rightarrow x_1 + x_2 = 7$

$\rightarrow x_1 + x_2 + R_1 = 7$

$\rightarrow x_1 + 3x_2 \geq 10$

$x_1 + 3x_2 \underbrace{-x_3}_{\text{surplus var.}} + R_2 = 10$

~~Since~~ LP problems with constraints which are eq^n or $\geq$, artificial var. are intro. that act as slack var. in the first iteration then they are slowly disposed at later iterations. Final sol^n will be such that artificial var. does not exist. Artificial var. is used as slack to get the initial basic feasible sol^n so that using the simplex method, optimality is reached!

2 methods are used:

1. Big M - Method
2. Two-phase method

Max Prob. ⊛ Coeff. of artificial var. in objective f^n is $(-M)$

Min Prob. " " " " " " " " " " " " " " " " " is (M).

3.4 A 5 (a) $\text{Max } z = 2x_1 + 3x_2 - 5x_3$

s.t $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$

$\boxed{x_1, x_2, x_3 \geq 0}$

First using Big M-Method

Ans.

$x_1 + x_2 + x_3 + R_1 = 7$

$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10$

$(x_4) \to$ surplus, $(R_1, R_2) \to$ Artificial var.

$\text{Max } z = 2x_1 + 3x_2 - 5x_3 + 0 \cdot x_4 - MR_1 - MR_2$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol^n
Z	-2	-3	5	0	M	M	0
R_1	1	1	1	0	1	0	7
R_2	2	-5	1	-1	0	1	10
$Z - MR_1 - MR_2 = Z$	$(-2-3M)$ ↓	$(-3+4M)$	$(5-2M)$	M	0	0	$-17M$
R_1	1	1	1	0	1	0	7
← R_2	2	-5	1	-1	0	1	10
Z	0	$\left(-8-\frac{7M}{2}\right)$ ↓	$6-\frac{M}{2}$	$-1-\frac{M}{2}$	0	$1+\frac{3M}{2}$	$10-2M$
← R_1	0	$7/2$	$1/2$	$1/2$	1	$-1/2$	2
x_1	1	$-5/2$	$1/2$	$-1/2$	0	$1/2$	5
Z	0	0	$50/7$	$1/7$	$\frac{16}{7}+M$	$-\frac{1}{7}+M$	$102/7$
x_2	0	1	$1/7$	$1/7$	$2/7$	$-1/7$	$4/7$
x_1	1	0	$6/7$	$-1/7$	$5/7$	$1/7$	$45/7$

$Z = z - (-2-3M)(x_1)$

$\boxed{x_1 = R_2/2}$

$\boxed{x_2 = \frac{R_1 \times 2}{7}}$

$$\boxed{Z = \frac{102}{7}, \quad x_1 = \frac{45}{7}, \quad x_2 = \frac{4}{7}}$$

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(Q.) $\text{Min } Z = 4x_1 - 8x_2 + 3x_3$

s.t $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$, $x_1, x_2, x_3 \geq 0$

Ans.) $x_1 + x_2 + x_3 + R_1 = 7$

$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10$

$\text{Min } z = 4x_1 - 8x_2 + 3x_3 + MR_1 + MR_2$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol^n
Z	-4	8	-3	0	$-M$	$-M$	0
R_1	1	1	1	0	1	0	7
R_2	2	-5	1	-1	0	1	10
$(+MR_1, +MR_2)$ Z	$(-4+3M)\downarrow$	$(8-4M)$	$(-3+2M)$	$-M$	0	0	$17M$
R_1	1	1	1	0	1	0	7
$\leftarrow R_2$	2	-5	1	-1	0	1	10
Z	0	$-2+\frac{7M}{2}\downarrow$	$-1+\frac{M}{2}$	$-2+\frac{M}{2}$	0	$2-\frac{3M}{2}$	$20+2M$
$\leftarrow R_1$	0	$7/2$	$1/2$	$1/2$	1	$-1/2$	2
x_1	1	$-5/2$	$1/2$	$-1/2$	0	$1/2$	5
Z	0	0	$-5/7$	$-12/7$	$\frac{4}{7}-M$	$\frac{12}{7}-M$	$148/7$
x_2	0	1	$1/7$	$1/7$	$2/7$	$-1/7$	$4/7$
x_1	1	0	$6/7$	$-1/7$	$5/7$	$1/7$	$45/7$

$$\therefore\ Z = \frac{148}{7}, \quad x_1 = \frac{45}{7}, \quad x_2 = \frac{4}{7}$$

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$$\text{Max } z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

s.t. $x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

(*) x_3 and x_4 can form basic variables but their coeff. in the obj. f^n should be zero, to do that we perform a row operation

Basic	x_1	x_2	x_3	x_4	sol^n
z	-2	-4	-4	3	0
x_3	1	1	1	0	4
x_4	1	4	0	1	8
$z + 4x_3 - 3x_4 = z$	-1	$-12 \downarrow$	0	0	-8
x_3	1	1	1	0	4
$\leftarrow x_4$	1	$\boxed{4}$	0	1	8
z	2	0	0	3	16
x_3	$3/4$	0	1	$-1/4$	2
x_2	$1/4$	1	0	$1/4$	2

$\therefore x_1 = 0, x_2 = 2, x_3 = 2, x_4 = 0, z = 16$

(1) Big-M ✓ Done

(2) Two-phase Method

We find basic feasible solⁿ to the original L.P by solving the phase-1.

In phase-1, the obj. f^n is to minimize the sum of all artificial var. irrespective of the fact that the orig. prob. is a maximization or minimization prob.

Solving phase-1 will force artificial variables to be 0. At the completion of phase-1, we reintroduce the original L.P's obj. f^n and determine the optimal sol^n to the original L.P.

(*) Remarks

Since $R_i \geq 0$, solving phase-1 L.P will result in one of the following 3 cases:

(i) Case I: The optimal value of R is > 0, it implies that the original L.P has no feasible sol^n.

(ii) Case II: The optimal value of R is $= 0$ ~~it implies~~ and no artific. var. are in the optimal phase-1 basis. In this case, we draw all columns in the optimal phase-1 table that corresponds to the artificial var.. We now combine the original obj. f^n with the constraints from the optimal phase-1 table. This gives the phase-2 L.P. The optimal sol^n to the phase-2 L.P is also the optimal sol^n for the original L.P.

(iii) <u>Case III</u> : The optimal value of R is = 0 and
atleast 1 artificial var. is in the optimal phase-1
basis. In this case, we find the optimal solⁿ to
the orig. L.P if at the end of phase-1 we
draw from the optimum phase-1 table all
non-basic artificial var. and any var. from
the orig. problem that has a −ve coeff. in
row of the optimal phase-1 table.

For question on next page

* Since tie is b/w slack and artifi. var for ratios ~~of Sa~~ to be considered as a leaving var. , artificial var. → preffered.

2phase * <u>Phase-I</u> → <u>minimization</u>

<u>Phase-II</u> → we use orig. obj. f^n

* If both constrainsts ≤ then simplex.
 if ≤ and ≥ then <u>big M</u> or <u>2phase</u>.

22/02/22

Q's on 2-phase method

(1.) $\text{Max } z = 2x_1 + 2x_2 + 4x_3$

s.t. $2x_1 + x_2 + x_3 \le 2$

$3x_1 + 4x_2 + 2x_3 \ge 8$

$x_1, x_2, x_3 \ge 0$

Ans.) $2x_1 + x_2 + x_3 + s_1 = 2$

$3x_1 + 4x_2 + 2x_3 - x_4 + R_1 = 8$

Phase I : $\min r = R_1$

Basic	x_1	x_2	x_3	x_4	s_1	R_1	Sol^n
r	0	0	0	0	0	−1	0
s_1	2	1	1	0	1	0	2
R_1	3	4	2	−1	0	1	8
$r+R_1$	3	4 ↓	2	−1	0	0	8
s_1	2	1	1	0	1	0	2
← R_1	3	$\boxed{4}$	2	−1	0	1	8
r	0	0	0	0	0	−1	0
s_1	5/4	0	1/2	1/4	1	−1/4	0
x_2	3/4	1	1/2	−1/4	0	1/4	2

Since optimality is reached in Phase - I, it implies the artificial var. is minimized.
For phase-II, we consider the original obj. f^n.

Phase - II

Basic	x_1	x_2	x_3	x_4	s_1	Sol^n
Z	-2	-2	-4	0	0	0
s_1	$5/4$	0	$1/2$	$1/4$	1	0
x_2	$3/4$	1	$1/2$	$-1/4$	0	2
$Z : Z+2x_2$	$-1/2$	0	$-3 \downarrow$	$-1/2$	0	4
$\leftarrow s_1$	$5/4$	0	$\boxed{1/2}$	$1/4$	1	0
x_2	$3/4$	1	$1/2$	$-1/4$	0	2
Z	7	0	0	1	6	4
x_3	$5/2$	0	1	$1/2$	2	0
x_2	$-1/2$	1	0	$-1/2$	-1	2

$$\boxed{\therefore Z = 4, \; x_1 = 0, \; x_2 = 2, \; x_3 = 0, \; x_4 = 0}$$

2-phase

(Q.) Max $z = 2x_1 + 5x_2$

s.t

$3x_1 + 2x_2 \geq 6$

$2x_1 + x_2 \leq 2$

$\boxed{x_1, x_2 \geq 0}$

Ans:) $3x_1 + 2x_2 - x_3 + R_1 = 6$

$2x_1 + x_2 + s_1 = 2$

Phase - I: $\boxed{\text{Min } r = R_1}$

Basic	x_1	x_2	x_3	R_1	s_1	Soln
r	0	0	0	-1	0	0
R_1	3	2	-1	1	0	6
s_1	2	1	0	0	1	2
$r: r+R_1$	3 ↓	2	-1	0	0	6
R_1	3	2	-1	1	0	6
← s_1	$\boxed{2}$	1	0	0	1	2
r	0	$1/2$ ↓	-1	0	$-3/2$	3
R_1	0	$1/2$	-1	1	$-3/2$	3
← x_1	1	$\boxed{1/2}$	0	0	$1/2$	1
r	-1	0	-1	0	-2	2
R_1	-1	0	-1	1	-2	2
x_2	2	1	0	0	1	2

~~Since~~ Phase - I is at optimum level but the artificial var. R_1 continues to be in the basis, this implies infeasible solⁿ.

2-3/0...

Special cases in Simplex Method

(1.) Degeneracy / Degenerate Sol^n

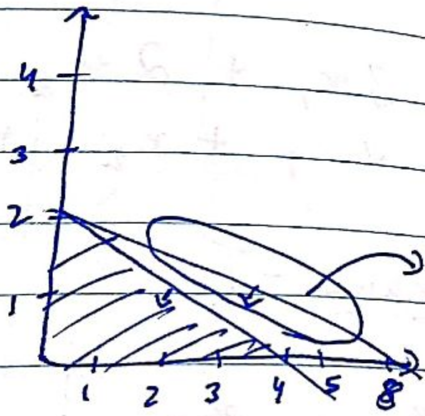
It implies that atleast one of the constraints is redundant.

Eg: $Max\ Z = 3x_1 + 9x_2$

s.t $x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$



For Degeneracy in Simplex method:

If in a basic feasible sol^n, one of the var. is present at 0 level then it is a degenerate sol^n. This usually happens when there is a tie for the leaving var..

There's nothing alarming in having a degen. sol^n except that in some cases it leads to cycling.

Cycling is a phenomenon in which the leaving var. again enters the basis and leaves repeatedly w/o optimality being breached.

Degeneracy can be temporary or permanent.

$x_1 + 4x_2 \le 8$

$x_1 + 2x_2 \le 4 \qquad x_1, x_2 \ge 0$

Eg: Max $z = 3x_1 + 9x_2 + 0s_1 + 0s_2$

$x_1 + 4x_2 + s_1 = 8$

$x_1 + 2x_2 + s_2 = 4 \qquad x_1, x_2, s_1, s_2 \ge 0$

Basic	x_1	x_2 ↓	s_1	s_2	sol^n
z	-3	-9	0	0	0
← s_1	1	$\boxed{4}$	1	0	8
s_2	1	2	0	1	4
z	$-3/4$ ↓	0	$9/4$	0	18
x_2	$1/4$	1	$1/4$	0	2
← s_2	$\boxed{1/2}$	0	$-1/2$	1	$\boxed{0}$ → Degen.
z	0	0	$3/2$	$3/2$	18
x_2	0	1	$1/2$	$-1/2$	2
x_1	1	0	-1	2	$\boxed{0}$

∴ optimal. $z = 18,\ x_1 = 0,\ x_2 = 2$

but a degen. sol^n (meaning one of constraints is redundant)

(2·) Alternative Optima

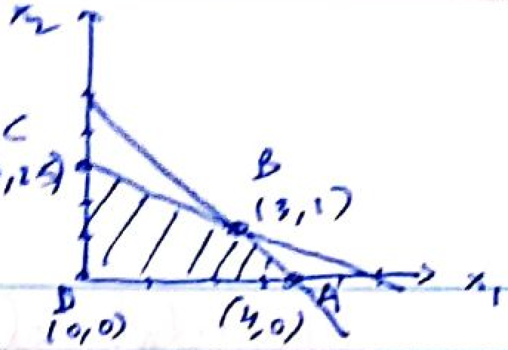
when the obj. f^n is $\parallel^{el}$ to a binding constraint then the obj. f^n will assume the same optimal value at morethan one corner point, this implies that the problem has alternative optimal sol^n.

If in the optimal table, the coeff. of non-basic var. is zero (0) then that var. can enter the basis and the obj. f^n will not change.

$(4,0)$ A: $z = 8$

$(3,1)$ B: $\underline{z = 10}$ ✓

$(0,2.5)$ C: $\underline{z = 10}$ ✓



Q) $\text{Max } z = 2x_1 + 4x_2$

s.t $x_1 + 2x_2 \le 5$

$x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$

Ans.) $x_1 + 2x_2 + s_1 = 5$

$x_1 + x_2 + s_2 = 4$

Basic	x_1	x_2 ↓	s_1	s_2	sol^n
z	-2	-4	0	0	0
← s_1	1	$\boxed{2}$	1	0	5
s_2	1	1	0	1	4
z	0 ↓	0	2	0	$\boxed{10}$
x_2	1/2	1	1/2	0	5/2
← s_2	$\boxed{1/2}$	0	$-1/2$	1	3/2
z	0	0	2	0	$\boxed{10}$
x_2	0	1	1	-1	1
x_1	1	0	-1	2	3

3. Unbounded solⁿ

In some of the L.P.P's, the values of some decision var. can be increased indefinitely w/o violating any of the constraints meaning the solⁿ space is unbounded in atleast one direction.

The solⁿ space is unbounded if in a simplex iteration or entries corresponding to the non-basic var. are -ve or 0 (zero) in all the constraint rows and in the obj. fⁿ row the entry is -ve or 0 (zero) incase of maximization prob. and +ve or 0 (zero) incase of minimization prob.

(Q.) Max $z = 2x_1 + x_2$

$x_1 - x_2 \leq 10$
$2x_1 \leq 40$, $(x_1, x_2 \geq 0)$

Ans.) $x_1 - x_2 + s_1 = 10$
$2x_1 + s_2 = 40$

basic	x_1	x_2	s_1	s_2	Sol^n
z	-2	-1	0	0	0
s_1	1	-1	1	0	10
s_2	2	0	0	1	40
z					
x	x	x	x	x	x
y					

$\therefore$ unbounded

4. Infeasible Sol^n

L.P models with inconsistent constraints have no feasible sol^n. If unable to drive all the artificial var. from the basis by the time optimality condition is reached then there is no feasible sol^n.

(Q.)

$\text{Max } z = 2x_1 + 4x_2$

$x_1 + x_2 \leq 10$

$x_1 + x_2 \geq 20 \qquad x_1, x_2 \geq 0$

Ans.)

$\text{Max } z = 2x_1 + 4x_2 - MR_1$

$x_1 + x_2 + S_1 = 10$

$x_1 + x_2 - x_3 + R_1 = 20$

Basic	x_1	x_2	x_3	S_1	R_1	Sol^n
Z	-2	-4	0	0	M	0
S_1	1	1	0	1	0	10
R_1	1	1	-1	0	1	20
$Z: Z-MR_1$	$-2-M$	$-4-M$	M	0	0	$-20M$
$\leftarrow S_1$	1	$\boxed{1}$	0	1	0	10
R_1	1	1	-1	0	1	20
Z	2	0	M	$4+M$	0	$40-10M$
x_2	1	1	0	1	0	10
R_1	0	0	-1	-1	1	10

Optimal already but artificial still remains in basis.

$\therefore$ infeasible sol^n