Introduktion

Sample space $\Omega = \{e_1, \ldots, e_n\}$

Event $E \subseteq \Omega$

Probability function $P(E) = x$ $\quad 0 \leq x \leq 1$

$$\forall E : |E| = i > 1 : P(E) = \sum_{i}^{n} P(\{e_i\}), e_i \in E$$

Probability space (Ω, P)

Bayes Theorem $P(S|T) = \frac{P(T|S)P(S)}{P(T)}$

Total Probability $\frac{P(B|A)P(A)}{\sum_r P(B|r)}$

Conditional Probability $\frac{P(B|A)P(A)}{\sum_r P(B|r)P(r)}$

Axiom 1 $P(\Omega) = 1$

Axiom 2 $0 \leq P(E) \leq 1, \forall E \subseteq \Omega$

Axiom 3 $E, F \subseteq \Omega : E \cap F = \emptyset \Rightarrow P(E \cup F) = P(E) + P(F)$

In general $P(E \cup F) = P(E) + P(F) - P(E \cap F) \quad (P(\emptyset) = 0)$

Conditional Probability $P(E|F) = \frac{P(E \cap F)}{P(F)}$ Probability of E occurring when F occurred

Independence

Product Rule $P(x, y) = P(x|y)P(y)$

Chain Rule $P(x_1, x_2) = P(x_2|x_1) \cdot P(x_1) = P(x_1|x_2) \cdot P(x_2)$

Independence $I_p(A, B) \Leftrightarrow P(a|b) = P(a) \vee \Big(P(a) = 0 \vee P(b) = 0\Big)$

Conditional Independence $I_p(A, B|C) \Leftrightarrow P(a|b, c) = P(a|b)$

Symmetry $I_p(X, Y|Z) \Rightarrow I_p(Y, X|Z)$

Decomposition $I_p(X, \{Y, W\}|Z) \Rightarrow I_p(X, Y|Z)$

Weak Union $I_p(X, \{Y, W\}|Z) \Rightarrow I_p(X, Y|\{Z, W\})$

Contraction $I_p(X, W|Z, Y)$ **und** $I_p(X, Y|Z) \Rightarrow I_p(X, \{Y, W\}|Z)$

Intersection $I_p(X, Y|\{Z, W\})$ **und** $I_p(X, W|Z, Y) \Rightarrow I_p(X, \{Y, W\}|Z)$

For positive distributions and for mutually disjoint sets X, Y, Z, W

Markov

DAG	$G = (V, E)$
Nodes	$V := \{A, B, C, ...\}$
Edge	$A \to B := (A, B) \in E \wedge (B, A) \notin E$
Parent node	$PA_X :=$ nodes with a direct path to X
Non-Descendants	$ND_X :=$ nodes that have no path from X
Markov Condition	$I_p(X, ND_X \mid PA_X)$
G satisfies MC	$\forall X \in V : I_p(X, ND_X \mid PA_X)$
Chain	$\rho = [X, ..., Y] := X - ... - Y \qquad X, ..., Y \in A \subseteq V$
Directed Chain	$X(\leftarrow \vee \to)...(\leftarrow \vee \to)Y$
Blockade by A	$\exists Z \in A : \leftarrow Z \leftarrow$
	$\exists Z \in A : \leftarrow Z \to$
	$\exists Z \in A : \forall z \in desc_Z : z \notin A \wedge z \notin \rho \wedge \to Z \leftarrow$
d-Separation	$I_G(X, Y \mid A) :=$ every chain between X and Y is blocked by A
Active Chain	ρ is not blocked by A given A

Subjective / Bayesian Probabilities

Prior Probability	probability of some event prior to updating its probability using new information.
Posteriori Probability	probability of some event after updating its probability based on new information.
Join Prob. Distribution	