

# A First-Order Logic Formalization of the Unified Foundational Ontology

Daniele Porelo, Claudenir M. Fonseca, João Paulo A. Almeida,  
Giancarlo Guizzardi, Tiago Prince Sales

December 3, 2021

## Abstract

This document presents a formalization of the Unified Foundational Ontology (UFO) in first-order logic. This formalization is documented by means of three complementary representations: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation. The presented formalization is supported by consistency and satisfiability checks performed through automated proofing tools.

## 1 Introduction

This document presents a formalization of the Unified Foundational Ontology (UFO) in first-order logic. This formalization is documented by means of three complementary representations: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation. The presented formalization is supported by consistency and satisfiability checks performed through automated proofing tools.

The remainder of this document is organized as a single formalization section (Section 2), which contains subsections for each submodule of the ontology.

## 2 Formalization

This section contains the formalization of the Unified Foundational Ontology (UFO) in first-order logics. This formalization is organized in several subsections where each presents the formalization of a portion of the whole ontology. The formalization is presented through different equivalent representations, designed to support the understanding of its contents: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation.

The UML-based diagrammatic representation serves as a visual representation certain predicates and axioms, being each element in Figure 1 being translated as follows:

- Rectangle shape (Figure 1a): visual representation of unary predicates associated to types in the ontology; the associated predicate is shown in lower camel case with no spaces.

*classA(x)*

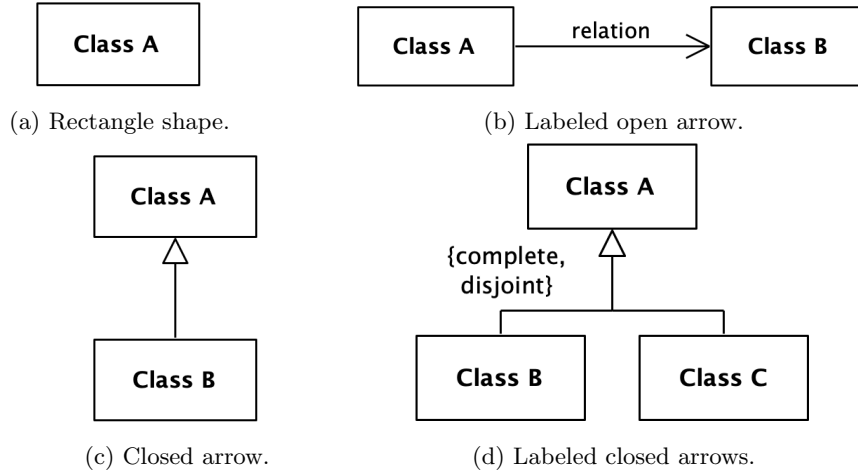


Figure 1: UML-based representation of first-order logic axioms.

- Open arrow (Figure 1b): visual representation of binary predicates; the predicate associated to the arrows' label is shown in lower camel case with no spaces; the predicate can only be true for any  $x$  and  $y$  if it is also true predicates associated to the types of each end (keeping the order of the arrow in the binary predicate's positions); this representation may also be associated to ternary predicates iff its third position represents a time-index.

$$\forall x, y (relation(x, y) \rightarrow (classA(x) \wedge classB(y)))$$

$$\forall x, y, w (relation(x, y, w) \rightarrow (classA(x) \wedge classB(y) \wedge world(w)))$$

- Closed arrow (Figure 1c): visual representation of specializations between ontology's types, where the type in the tail of the arrow is a subtype of the type in the head of the arrow.

$$\forall x (classB(x) \rightarrow classA(x))$$

- Labeled closed arrow (Figure 1d): visual representation of disjoint and/or complete constraints over sets specializations between ontology's types.

$$\forall x (classB(x) \rightarrow classA(x))$$

$$\forall x (classC(x) \rightarrow classA(x))$$

$$\forall x (classA(x) \rightarrow (classB(x) \vee classC(x)))$$

$$\neg \exists x (classB(x) \wedge classC(x))$$

{complete}  
{disjoint}

## 2.1 Partial Taxonomy of UFO: Thing

This subsection presents most general types of UFO's taxonomy specializing the type **Thing** (Figure 2).

- a1** For every  $x$ ,  $x$  is a **Thing** iff  $x$  is either a **Type** or an **Individual**.

$$\forall x (\text{type\_}(x) \vee \text{individual}(x) \leftrightarrow \text{thing}(x))$$

```
1 (cl-text ax_thing_taxonomy
2 (forall (x)
3 (iff (or (type_ x) (individual x))
```

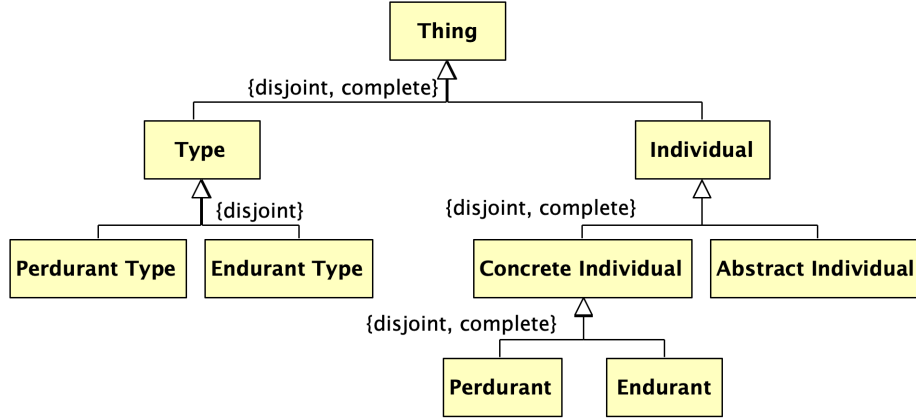


Figure 2: Visual representation of UFO's taxonomy of Thing.

```

4  (thing x))
5  )
6  )

```

- a2** There is no  $x$  such that it is a Type and an Individual.

$$\neg \exists x (\text{type\_}(x) \wedge \text{individual}(x))$$

```

7  (cl-text ax_thing_partition
8  (not (exists (x)
9  (and (type_ x) (individual x)))
10 )
11 )

```

- a3** For every  $x$ ,  $x$  is an Individual iff  $x$  is either a Concrete Individual or an Abstract Individual.

$$\forall x (\text{concreteIndividual}(x) \vee \text{abstractIndividual}(x) \leftrightarrow \text{individual}(x))$$

```

12 (cl-text ax_individual_taxonomy
13 (forall (x)
14 (iff (or (concreteIndividual x) (abstractIndividual x))
15 (individual x))
16 )
17 )

```

- a4** There is no  $x$  such that it is a Concrete Individual and an Abstract Individual.

$$\neg \exists x (\text{concreteIndividual}(x) \wedge \text{abstractIndividual}(x))$$

```

18 (cl-text ax_individual_partition
19 (not (exists (x)
20 (and (concreteIndividual x) (abstractIndividual x)))
21 )
22 )

```

- a5** For every  $x$ ,  $x$  is a Concrete Individual iff  $x$  is either a Perdurant or an Endurant.

$$\forall x (\text{endurant}(x) \vee \text{perdurant}(x) \leftrightarrow \text{concreteIndividual}(x))$$

```

23 (cl-text ax_concreteIndividual_taxonomy
24 (forall (x)
25   (iff (or (endurant x) (perdurant x))
26         (concreteIndividual x))
27   )
28 )

```

**a6** There is no  $x$  such that it is a Perdurant and an Endurant.

$\neg \exists x(\text{endurant}(x) \wedge \text{perdurant}(x))$

```

29 (cl-text ax_concreteIndividual_partition
30 (not (exists (x)
31   (and (endurant x) (perdurant x)))
32 )
33 )

```

**a7** For every  $x$ ,  $x$  is a Concrete Individual iff  $x$  is either a Perdurant or an Endurant.

$\forall x(\text{endurantType}(x) \vee \text{perdurantType}(x) \rightarrow \text{type\_}(x))$

```

34 (cl-text ax_type_taxonomy
35 (forall (x)
36   (if (or (endurantType x) (perdurantType x))
37       (type_ x))
38   )
39 )

```

**a8** There is no  $x$  such that it is a Perdurant Type and an Endurant Type.

$\neg \exists x(\text{endurantType}(x) \wedge \text{perdurantType}(x))$

```

40 (cl-text ax_type_partition
41 (not (exists (x)
42   (and (endurantType x) (perdurantType x)))
43 )
44 )

```

## 2.2 Partial Taxonomy of UFO: Abstract Individual

This subsection presents a portion of UFO's taxonomy specializing the type Abstract Individual (Figure 3).

**a9** Every  $x$  that is a Quale is also an Abstract Individual.

$\forall x(\text{quale}(x) \rightarrow \text{abstractIndividual}(x))$

```

1 (cl-text ax_abstractIndividual_taxonomy_quale
2 (forall (x)
3   (if (quale x)
4       (abstractIndividual x))
5   )
6 )

```

**a10** Every  $x$  that is a Set is also an Abstract Individual.

$\forall x(\text{set\_}(x) \rightarrow \text{abstractIndividual}(x))$

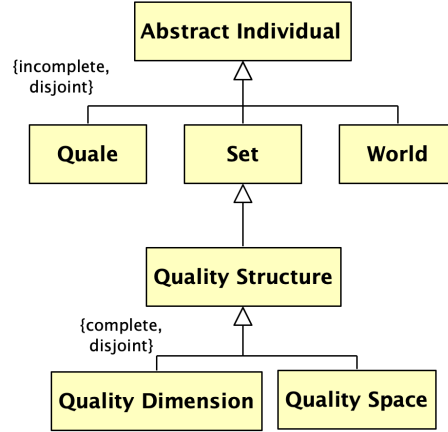


Figure 3: Visual representation of UFO's taxonomy of Abstract Individual.

```

7 (cl-text ax_abstractIndividual_taxonomy_set
8 (forall (x)
9   (if (set_ x)
10     (abstractIndividual x))
11   )
12 )

```

**a11** Every  $x$  that is a World is also an Abstract Individual.

$\forall x(\text{world}(x) \rightarrow \text{abstractIndividual}(x))$

```

13 (cl-text ax_abstractIndividual_taxonomy_world
14 (forall (x)
15   (if (world x)
16     (abstractIndividual x))
17   )
18 )

```

**a12** There is no  $x$  such that it is a Quale, a Set, and a World (pairwise disjoint).

$\neg \exists x((\text{quale}(x) \wedge \text{set}_-(x)) \vee (\text{quale}(x) \wedge \text{world}(x)) \vee (\text{set}_-(x) \wedge \text{world}(x)))$

```

19 (cl-text ax_abstractIndividual_pairwiseDisjoint
20 (not (exists (x)
21   (or (and (quale x) (set_ x)) (and (quale x) (world x)) (and (set_ x) (world x)
22   )))
23 )

```

**a13** Every  $x$  that is a Quality Structure is also a Set.

$\forall x(\text{qualityStructure}(x) \rightarrow \text{set}_-(x))$

```

24 (cl-text ax_set_taxonomy_qualityStructure
25 (forall (x)
26   (if (qualityStructure x)
27     (set_ x))
28   )
29 )

```

**a14** For every  $x$ ,  $x$  is a Quality Structure iff  $x$  is either a Quality Dimension or a Quality Space.

$$\forall x(\text{qualityDimension}(x) \vee \text{qualitySpace}(x) \leftrightarrow \text{qualityStructure}(x))$$

```

30 (cl-text ax_qualityStructure_taxonomy
31 (forall (x)
32   (iff (or (qualityDimension x) (qualitySpace x))
33         (qualityStructure x))
34 )
35 )

```

**a15** There is no  $x$  such that it is a Quality Dimension and a Quality Space.

$$\neg \exists x(\text{qualityDimension}(x) \wedge \text{qualitySpace}(x))$$

```

36 (cl-text ax_qualityStructure_partition
37 (not (exists (x)
38   (and (qualityDimension x) (qualitySpace x)))
39 )
40 )

```

## 2.3 Partial Taxonomy of UFO: Endurant

This subsection presents a portion of UFO's taxonomy specializing the type Endurant (Figure 4).

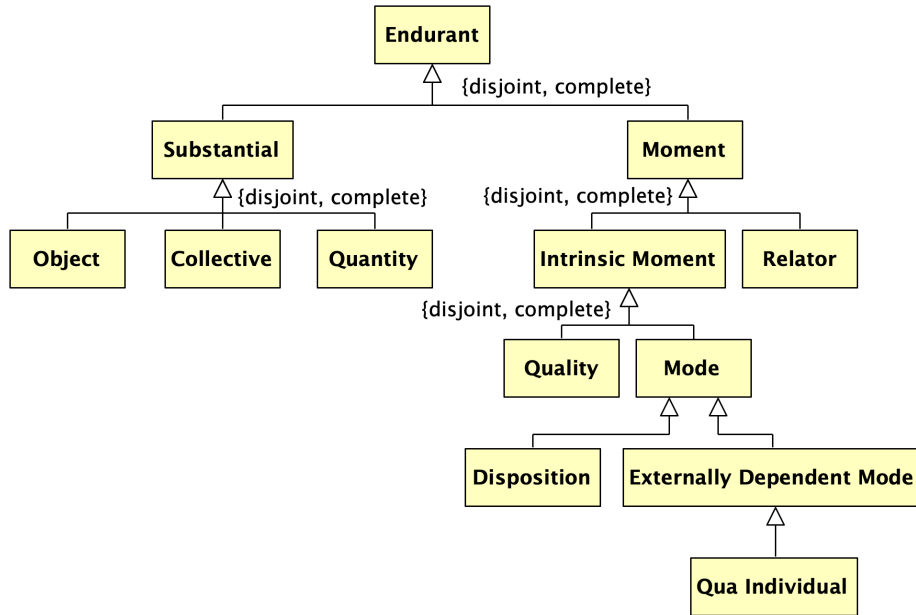


Figure 4: Visual representation of UFO's taxonomy of Endurant.

**a16** For every  $x$ ,  $x$  is an Endurant iff  $x$  is either a Substantial or a Moment.

$$\forall x(\text{substantial}(x) \vee \text{moment}(x) \leftrightarrow \text{endurant}(x))$$

```

1 (cl-text ax_endurant_taxonomy
2 (forall (x)
3   (iff (or (substantial x) (moment x))
4     (endurant x))
5   )
6 )

```

**a17** There is no  $x$  such that it is a Substantial and a Moment.

$\neg \exists x(\text{substantial}(x) \wedge \text{moment}(x))$

```

7 (cl-text ax_endurant_partition
8 (not (exists (x)
9   (and (substantial x) (moment x)))
10 )
11 )

```

**a18** For every  $x$ ,  $x$  is a Substantial iff  $x$  is either an Object, a Collective, or a Quantity.

$\forall x(\text{object}(x) \vee \text{collective}(x) \vee \text{quantity}(x) \leftrightarrow \text{substantial}(x))$

```

12 (cl-text ax_substantial_taxonomy
13 (forall (x)
14   (iff (or (object x) (collective x) (quantity x))
15     (substantial x))
16   )
17 )

```

**a19** There is no  $x$  such that it is an Object, a Collective, and a Quantity (pairwise disjoint).

$\neg \exists x((\text{object}(x) \wedge \text{collective}(x)) \vee (\text{object}(x) \wedge \text{quantity}(x)) \vee (\text{collective}(x) \wedge \text{quantity}(x)))$

```

18 (cl-text ax_substantial_partition
19 (not (exists (x)
20   (or (and (object x) (collective x)) (and (object x) (quantity x)) (and (
21     collective x) (quantity x))))
22 )

```

**a20** For every  $x$ ,  $x$  is a Moment iff  $x$  is either an Intrinsic Moment or a Relator.

$\forall x(\text{intrinsicMoment}(x) \vee \text{relator}(x) \leftrightarrow \text{moment}(x))$

```

23 (cl-text ax_moment_taxonomy
24 (forall (x)
25   (iff (or (intrinsicMoment x) (relator x))
26     (moment x))
27   )
28 )

```

**a21** There is no  $x$  such that it is an Intrinsic Moment and a Relator.

$\neg \exists x(\text{intrinsicMoment}(x) \wedge \text{relator}(x))$

```

29 (cl-text ax_moment_partition
30 (not (exists (x)
31   (and (intrinsicMoment x) (relator x)))
32 )
33 )

```

**a22** For every  $x$ ,  $x$  is an Intrinsic Moment iff  $x$  is either a Quality or a Mode.

$\forall x(\text{quality}(x) \vee \text{mode}(x) \leftrightarrow \text{intrinsicMoment}(x))$

```
34 (cl-text ax_intrinsicMoment_taxonomy
35 (forall (x)
36   (iff (or (quality x) (mode x))
37     (intrinsicMoment x))
38   )
39 )
```

**a23** There is no  $x$  such that it is an Intrinsic Moment and a Relator.

$\neg \exists x(\text{quality}(x) \wedge \text{mode}(x))$

```
40 (cl-text ax_intrinsicMoment_partition
41 (not (exists (x)
42   (and (quality x) (mode x)))
43   )
44 )
```

**a24** Every  $x$  that is a Disposition is also a Mode.

$\forall x(\text{disposition}(x) \rightarrow \text{mode}(x))$

```
45 (cl-text ax_mode_taxonomy_disposition
46 (forall (x)
47   (if (disposition x)
48     (mode x))
49   )
50 )
```

**a25** Every  $x$  that is an Externally Dependent Mode is also a Mode.

$\forall x(\text{externallyDependentMode}(x) \rightarrow \text{mode}(x))$

```
51 (cl-text ax_mode_taxonomy_externallyDependentMode
52 (forall (x)
53   (if (externallyDependentMode x)
54     (mode x))
55   )
56 )
```

**a26** Every  $x$  that is an Qua Individual is also an Externally Dependent Mode.

$\forall x(\text{quaIndividual}(x) \rightarrow \text{externallyDependentMode}(x))$

```
57 (cl-text ax_externallyDependentMode_taxonomy_quaIndividual
58 (forall (x)
59   (if (quaIndividual x)
60     (externallyDependentMode x))
61   )
62 )
```

## 2.4 Partial Taxonomy of UFO: Endurant Type by Ontological Natures

This subsection presents a portion of UFO's taxonomy specializing the type Endurant Type classified by ontological natures (Figure 5).



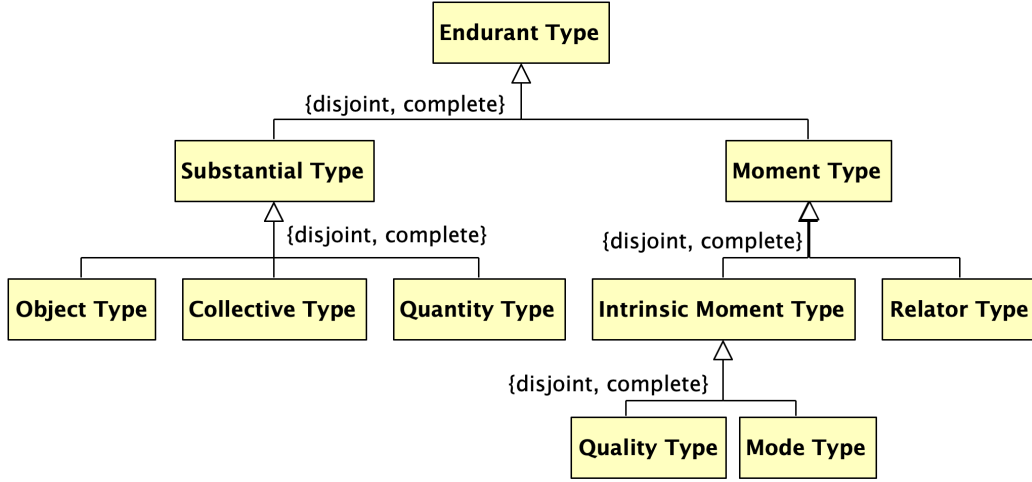


Figure 5: Visual representation of UFO’s taxonomy of Endurant Type classified by ontological natures.

**a27** For every  $x$ ,  $x$  is an Endurant Type iff  $x$  is either a Substantial Type or a Moment Type.

$$\forall x(\text{substantialType}(x) \vee \text{momentType}(x) \leftrightarrow \text{endurantType}(x))$$

```

1 (cl-text ax_endurantType_taxonomy_nature
2 (forall (x)
3   (iff (or (substantialType x) (momentType x))
4     (endurantType x))
5   )
6 )

```

**a28** There is no  $x$  such that it is a Substantial Type and a Moment Type.

$$\neg \exists x(\text{substantialType}(x) \wedge \text{momentType}(x))$$

```

7 (cl-text ax_endurantType_partition_nature
8 (not (exists (x)
9   (and (substantialType x) (momentType x)))
10 )
11 )

```

**a29** For every  $x$ ,  $x$  is a Substantial Type iff  $x$  is either an Object Type, a Collective Type, or a Quantity Type.

$$\forall x(\text{objectType}(x) \vee \text{collectiveType}(x) \vee \text{quantityType}(x) \leftrightarrow \text{substantialType}(x))$$

```

12 (cl-text ax_substantialType_taxonomy
13 (forall (x)
14   (iff (or (objectType x) (collectiveType x) (quantityType x))
15     (substantialType x))
16   )
17 )

```

**a30** There is no  $x$  such that it is an Object Type, a Collective Type, and a Quantity Type (pairwise disjoint).

$$\neg\exists x((\text{objectType}(x)\wedge\text{collectiveType}(x))\vee(\text{objectType}(x)\wedge\text{quantityType}(x))\vee(\text{collectiveType}(x)\wedge\text{quantityType}(x)))$$

```

18 (cl-text ax_substantialType_partition
19 (not (exists (x)
20   (or (and (objectType x) (collectiveType x)) (and (objectType x) (quantityType
21     x)) (and (collectiveType x) (quantityType x))))
22 )

```

**a31** For every  $x$ ,  $x$  is a Moment Type iff  $x$  is either an Intrinsic Moment Type or a Relator Type.

$$\forall x(\text{intrinsicMomentType}(x) \vee \text{relatorType}(x) \leftrightarrow \text{momentType}(x))$$

```

23 (cl-text ax_momentType_taxonomy
24 (forall (x)
25   (iff (or (intrinsicMomentType x) (relatorType x))
26     (momentType x))
27 )
28 )

```

**a32** There is no  $x$  such that it is an Intrinsic Moment Type and a Relator Type.

$$\neg\exists x(\text{intrinsicMomentType}(x) \wedge \text{relatorType}(x))$$

```

29 (cl-text ax_momentType_partition
30 (not (exists (x)
31   (and (intrinsicMomentType x) (relatorType x)))
32 )
33 )

```

**a33** For every  $x$ ,  $x$  is an Intrinsic Moment Type iff  $x$  is either a Quality Type or a Mode Type.

$$\forall x(\text{qualityType}(x) \vee \text{modeType}(x) \leftrightarrow \text{intrinsicMomentType}(x))$$

```

34 (cl-text ax_intrinsicMomentType_taxonomy
35 (forall (x)
36   (iff (or (qualityType x) (modeType x))
37     (intrinsicMomentType x))
38 )
39 )

```

**a34** There is no  $x$  such that it is an Intrinsic Moment Type and a Relator Type.

$$\neg\exists x(\text{qualityType}(x) \wedge \text{modeType}(x))$$

```

40 (cl-text ax_intrinsicMomentType_partition
41 (not (exists (x)
42   (and (qualityType x) (modeType x)))
43 )
44 )

```

## 2.5 Partial Taxonomy of UFO: Endurant Type by Modal Properties of Types

This subsection presents a portion of UFO's taxonomy specializing the type Endurant Type classified by the modal properties of types (Figure 6).

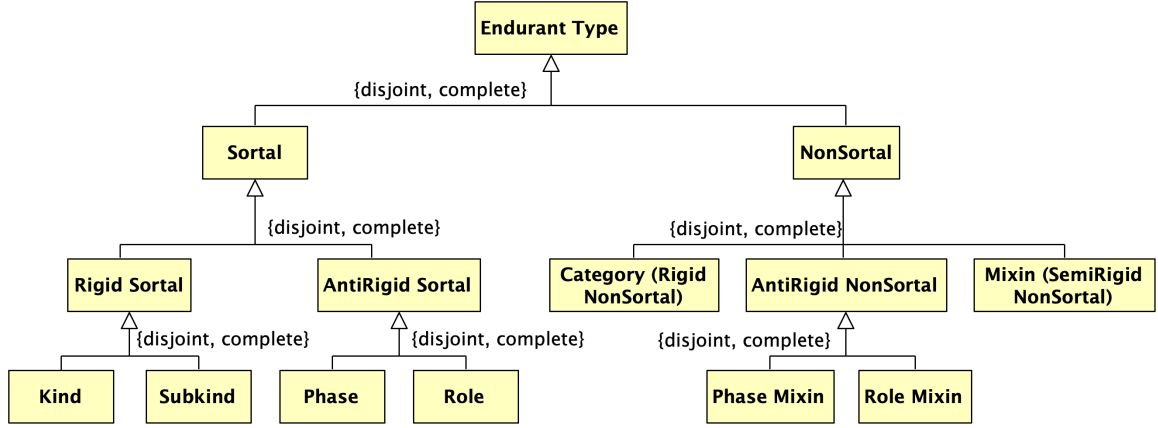


Figure 6: Visual representation of UFO's taxonomy of Endurant Type classified by the modal properties of types.

**a35** For every  $x$ ,  $x$  is an Endurant Type iff  $x$  is either a Sortal or a NonSortal.

$$\forall x(\text{sortal}(x) \vee \text{nonSortal}(x) \leftrightarrow \text{endurantType}(x))$$

```

1 (cl-text ax_endurantType_taxonomy_properties
2 (forall (x)
3   (iff (or (sortal x) (nonSortal x))
4     (endurantType x))
5   )
6 )

```

**a36** There is no  $x$  such that it is a Sortal and a NonSortal.

$$\neg \exists x(\text{sortal}(x) \wedge \text{nonSortal}(x))$$

```

7 (cl-text ax_endurantType_partition_properties
8 (not (exists (x)
9   (and (sortal x) (nonSortal x)))
10 )
11 )

```

**a37** For every  $x$ ,  $x$  is a Sortal iff  $x$  is either a Rigid Sortal or an AntiRigid Sortal.

$$\forall x(\text{rigidSortal}(x) \vee \text{antiRigidSortal}(x) \leftrightarrow \text{sortal}(x))$$

```

12 (cl-text ax_sortal_taxonomy
13 (forall (x)
14   (iff (or (rigidSortal x) (antiRigidSortal x))
15     (sortal x))
16   )
17 )

```

**a38** There is no  $x$  such that it is a Rigid Sortal and an AntiRigid Sortal.

$$\neg \exists x(\text{rigidSortal}(x) \wedge \text{antiRigidSortal}(x))$$

```

18 (cl-text ax_sortal_partition
19 (not (exists (x)
20   (and (rigidSortal x) (antiRigidSortal x)))

```

```

21 )
22 )

```

**a39** For every  $x$ ,  $x$  is a Rigid Sortal iff  $x$  is either a Kind or a Subkind.

$$\forall x(\text{kind}(x) \vee \text{subkind}(x) \leftrightarrow \text{rigidSortal}(x))$$

```

23 (cl-text ax_rigidSortal_taxonomy
24 (forall (x)
25   (iff (or (kind x) (subkind x))
26         (rigidSortal x))
27   )
28 )

```

**a40** There is no  $x$  such that it is a Kind and a Subkind.

$$\neg \exists x(\text{kind}(x) \wedge \text{subkind}(x))$$

```

29 (cl-text ax_rigidSortal_partition
30 (not (exists (x)
31   (and (kind x) (subkind x)))
32 )
33 )

```

**a41** For every  $x$ ,  $x$  is an AntiRigid Sortal iff  $x$  is either a Phase or a Role.

$$\forall x(\text{phase}(x) \vee \text{role}(x) \leftrightarrow \text{antiRigidSortal}(x))$$

```

34 (cl-text ax_antiRigidSortal_taxonomy
35 (forall (x)
36   (iff (or (phase x) (role x))
37         (antiRigidSortal x))
38   )
39 )

```

**a42** There is no  $x$  such that it is a Phase and a Role.

$$\neg \exists x(\text{phase}(x) \wedge \text{role}(x))$$

```

40 (cl-text ax_antiRigidSortal_partition
41 (not (exists (x)
42   (and (phase x) (role x)))
43 )
44 )

```

**a43** For every  $x$ ,  $x$  is a NonSortal iff  $x$  is either a Rigid NonSortal, an AntiRigid NonSortal, or a SemmiRigid NonSortal.

$$\forall x(\text{rigidNonSortal}(x) \vee \text{semiRigidNonSortal}(x) \vee \text{antiRigidNonSortal}(x) \leftrightarrow \text{nonSortal}(x))$$

```

45 (cl-text ax_nonSortal_taxonomy
46 (forall (x)
47   (iff (or (rigidNonSortal x) (semiRigidNonSortal x) (antiRigidNonSortal x))
48         (nonSortal x))
49   )
50 )

```

- a44** There is no  $x$  such that it is an Rigid NonSortal, an AntiRigid NonSortal, and a SemiRigid NonSortal (pairwise disjoint).

$$\neg \exists x ((\text{rigidNonSortal}(x) \wedge \text{semiRigidNonSortal}(x)) \vee (\text{rigidNonSortal}(x) \wedge \text{antiRigidNonSortal}(x)) \vee (\text{semiRigidNonSortal}(x) \wedge \text{antiRigidNonSortal}(x)))$$

```

51 (cl-text ax_nonSortal_partition
52 (not (exists (x)
53   (or (and (rigidNonSortal x) (semiRigidNonSortal x)) (and (rigidNonSortal x) (
54     antiRigidNonSortal x)) (and (semiRigidNonSortal x) (antiRigidNonSortal x))))
55 )

```

- a45** Every  $x$  that is a Rigid NonSortal is also a Category.

$$\forall x (\text{rigidNonSortal}(x) \leftrightarrow \text{category}(x))$$

```

56 (cl-text ax_rigidNonSortal_taxonomy
57 (forall (x)
58   (iff (rigidNonSortal x)
59     (category x))
60   )
61 )

```

- a46** Every  $x$  that is a SemiRigid NonSortal is also a Mixin.

$$\forall x (\text{semiRigidNonSortal}(x) \leftrightarrow \text{mixin}(x))$$

```

62 (cl-text ax_semiRigidNonSortal_taxonomy
63 (forall (x)
64   (iff (semiRigidNonSortal x)
65     (mixin x))
66   )
67 )

```

- a47** For every  $x$ ,  $x$  is an AntiRigid NonSortal iff  $x$  is either a Phase Mixin or a Role Mixin.

$$\forall x (\text{phaseMixin}(x) \vee \text{roleMixin}(x) \leftrightarrow \text{antiRigidNonSortal}(x))$$

```

68 (cl-text ax_antiRigidNonSortal_taxonomy
69 (forall (x)
70   (iff (or (phaseMixin x) (roleMixin x))
71     (antiRigidNonSortal x))
72   )
73 )

```

- a48** There is no  $x$  such that it is a Phase Mixin and a Role Mixin.

$$\neg \exists x (\text{phaseMixin}(x) \wedge \text{roleMixin}(x))$$

```

74 (cl-text ax_antiRigidNonSortal_partition
75 (not (exists (x)
76   (and (phaseMixin x) (roleMixin x))))
77 )
78 )

```

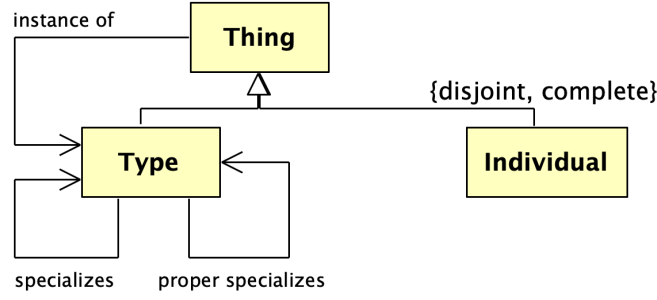


Figure 7: Types, individuals, instantiation, and specialization.

## 2.6 On the Definitions of Type, Individual, Instantiation, and Specialization

This subsection introduces the UFO's definitions of Type and Individual as well as the definitions for the relations of instance of, specializes, and proper specializes.

- a49** The instance of relation can only hold between  $x$ ,  $y$ , and  $w$ , where  $x$  is an instance of a type  $y$  in  $w$ , and  $w$  is the world where this relation holds.

$$\forall x, y, w (\text{iof}(x, y, w) \rightarrow \text{type\_}(y) \wedge \text{world}(w))$$

```

1 (cl-text ax_dIof
2 (forall (x y w)
3   (if (iof x y w)
4     (and (type_ y) (world w)))
5   )
6 )

```

- a50** For every  $x$ ,  $x$  is a type iff there exists some  $y$  and  $w$  such that  $y$  is instance of  $x$  in the world  $w$ .

$$\forall x (\text{type\_}(x) \leftrightarrow \exists y, w (\text{iof}(y, x, w)))$$

```

7 (cl-text ax_dType
8 (forall (x)
9   (iff (type_ x)
10     (exists (y w)
11       (iof y x w))
12     )
13   )
14 )

```

- a51** For every  $x$ ,  $x$  is an individual iff there is no pair  $y$  and  $w$  such that  $y$  is instance of  $x$  in  $w$ .

$$\forall x (\text{individual}(x) \leftrightarrow \neg \exists y, w (\text{iof}(y, x, w)))$$

```

15 (cl-text ax_dIndividual
16 (forall (x)
17   (iff (individual x)
18     (not (exists (y w)
19       (iof y x w))
20     ))
21   )
22 )

```

**a52** The instance  $x$  in an instance of relation must be either a type or an individual.

$\forall x, y, w(\text{iof}(x, y, w) \rightarrow \text{type\_}(x) \vee \text{individual}(x))$

```
23 (cl-text ax_multiLevel
24 (forall (x y w)
25   (if (iof x y w)
26     (or (type_ x) (individual x)))
27   )
28 )
```

**a53** Chains of instance of relations are limited to a depth of two.

$\neg \exists x, y, z, w(\text{type\_}(x) \wedge \text{iof}(x, y, w) \wedge \text{iof}(y, z, w))$

```
29 (cl-text ax_twoLevelConstrained
30 (not (exists (x y z w)
31   (and (type_ x) (iof x y w) (iof y z w)))
32 )
33 )
```