A First-Order Logic Formalization of the Unified Foundational Ontology

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Abstract

This document presents a formalization of the Unified Foundational Ontology (UFO) in first-order logic. This formalization is documented by means of three complementary representations: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation. The presented formalization is supported by consistency and satisfiability checks performed through automated proofing tools.

1 Introduction

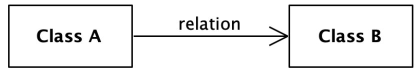
This document presents a formalization of the Unified Foundational Ontology (UFO) in first-order logic. This formalization is documented by means of three complementary representations: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation. The presented formalization is supported by consistency and satisfiability checks performed through automated proofing tools.

The remainder of this document is organized as a single formalization section (Section 2), which contains subsections for each submodule of the ontology.

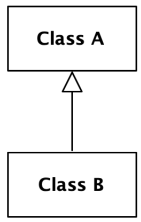
2 Formalization

This section contains the formalization of the Unified Foundational Ontology (UFO) in first-order logics. This formalization is organized in several subsections where each presents the formalization of a portion of the whole ontology. The formalization is presented through different equivalent representations, designed to support the understanding of its contents: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation.

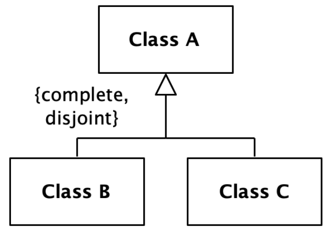
(a) Rectangle shape.



(b) Labeled open arrow.



(c) Closed arrow.



(d) Labeled closed arrows.

Figure 1: UML-based representation of first-order logic axioms.

The UML-based diagrammatic representation serves as a visual representation certain predicates and axioms, being each element in Figure 1 being translated as follows:

- Rectangle shape (Figure 1a): visual representation of unary predicates associated to types in the ontology; the associated predicate is shown in lower camel case with no spaces.

  $classA(x)$

- Open arrow (Figure 1b): visual representation of binary predicates; the predicate associated to the arrows' label is shown in lower camel case with no spaces; the predicate can only be true for any x and y if it is also true predicates associated to the types of each end (keeping the order of the arrow in the binary predicate's positions); this representation may also be associated to ternary predicates ifif its third position represents a time-index.

  $\forall x, y(relation(x, y) \rightarrow (classA(x) \wedge classB(y)))$

  $\forall x, y, w(relation(x, y, w) \rightarrow (classA(x) \wedge classB(y) \wedge world(w)))$

- Closed arrow (Figure 1c): visual representation of specializations between ontology's types, where the type in the tail of the arrow is a subtype of the type in the head of the arrow.

  $\forall x(classB(x) \rightarrow classA(x))$

- Labeled closed arrow (Figure 1d): visual representation of disjoint and/or complete constraints over sets specializations between ontology's types.

  $\forall x(classB(x) \rightarrow classA(x))$
  $\forall x(classC(x) \rightarrow classA(x))$
  $\forall x(classA(x) \rightarrow (classB(x) \vee classC(x)))$ {complete}
  $\neg\exists x(classB(x) \wedge classC(x))$ {disjoint}

2.1 Partial Taxonomy of UFO: Thing

This subsection presents most general types of UFO's taxonomy specializing the type Thing (Figure 2).

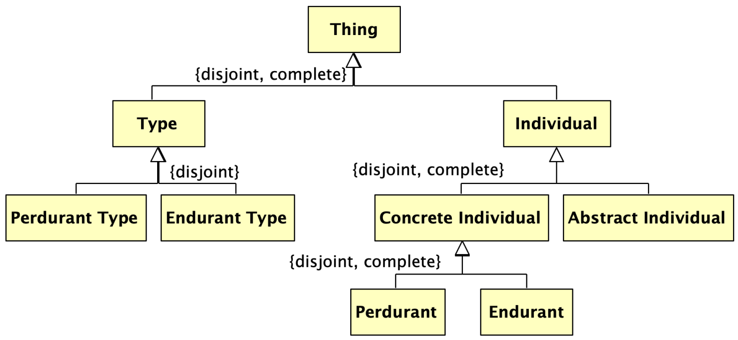


Figure 2: Visual representation of UFO's taxonomy of Thing.

a1 For every x, x is a Thing iff x is either a Type or an Individual.

$\forall x(\mathsf{type\_}(x) \lor \mathsf{individual}(x) \leftrightarrow \mathsf{thing}(x))$

```
(cl-text  ax_thing_taxonomy
(forall (x)
  (iff (or (type_ x) (individual x))
    (thing x))
  )
)
```

a2 There is no x such that it is a Type and an Individual.

$\neg\exists x(\mathsf{type\_}(x) \land \mathsf{individual}(x))$

```
(cl-text  ax_thing_partition
(not (exists (x)
  (and (type_ x) (individual x)))
)
)
```

a3 For every x, x is an Individual iff x is either a Concrete Individual or an Abstract Individual.

$\forall x(\mathsf{concreteIndividual}(x) \lor \mathsf{abstractIndividual}(x) \leftrightarrow \mathsf{individual}(x))$

```
(cl-text  ax_individual_taxonomy
(forall (x)
  (iff (or (concreteIndividual x) (abstractIndividual x))
    (individual x))
  )
)
```

a4 There is no x such that it is a Concrete Individual and an Abstract Individual.

$\neg\exists x(\mathsf{concreteIndividual}(x) \wedge \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_individual_partition
(not (exists (x)
  (and (concreteIndividual x) (abstractIndividual x)))
)
)
```

a5 For every x, x is a Concrete Individual iff x is either a Perdurant or an Endurant.

$\forall x(\mathsf{endurant}(x) \vee \mathsf{perdurant}(x) \leftrightarrow \mathsf{concreteIndividual}(x))$

```
(cl-text  ax_concreteIndividual_taxonomy
(forall (x)
  (iff (or (endurant x) (perdurant x))
    (concreteIndividual x))
  )
)
```

a6 There is no x such that it is a Perdurant and an Endurant.

$\neg\exists x(\mathsf{endurant}(x) \wedge \mathsf{perdurant}(x))$

```
(cl-text  ax_concreteIndividual_partition
(not (exists (x)
  (and (endurant x) (perdurant x)))
)
)
```

a7 For every x, x is a Concrete Individual iff x is either a Perdurant or an Endurant.

$\forall x(\mathsf{endurantType}(x) \vee \mathsf{perdurantType}(x) \rightarrow \mathsf{type\_}(x))$

```
(cl-text  ax_type_taxonomy
(forall (x)
  (if (or (endurantType x) (perdurantType x))
    (type_ x))
  )
)
```

a8 There is no x such that it is a Perdurant Type and an Endurant Type.

$\neg\exists x(\mathsf{endurantType}(x) \wedge \mathsf{perdurantType}(x))$

```
(cl-text  ax_type_partition
(not (exists (x)
  (and (endurantType x) (perdurantType x)))
)
)
```

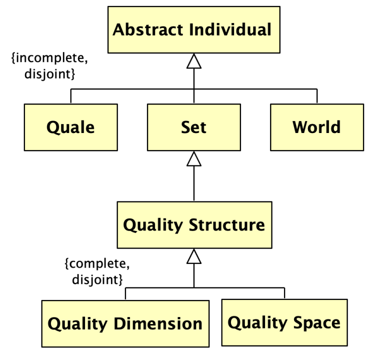


Figure 3: Visual representation of UFO's taxonomy of Abstract Individual.

2.2 Partial Taxonomy of UFO: Abstract Individual

This subsection presents a portion of UFO's taxonomy specializing the type Abstract Individual (Figure 3).

a9 Every x that is a Quale is also an Abstract Individual.

$\forall x(\mathsf{quale}(x) \rightarrow \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_abstractIndividual_taxonomy_quale
(forall (x)
  (if (quale x)
    (abstractIndividual x))
  )
)
```

a10 Every x that is a Set is also an Abstract Individual.

$\forall x(\mathsf{set\_}(x) \rightarrow \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_abstractIndividual_taxonomy_set
(forall (x)
  (if (set_ x)
    (abstractIndividual x))
  )
)
```

a11 Every x that is a World is also an Abstract Individual.

$\forall x(\mathsf{world}(x) \rightarrow \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_abstractIndividual_taxonomy_world
(forall (x)
  (if (world x)
    (abstractIndividual x))
  )
)
```

a12 There is no x such that it is a Quale, a Set, and a World (pairwise disjoint).

$\neg\exists x((\mathsf{quale}(x) \wedge \mathsf{set\_}(x)) \vee (\mathsf{quale}(x) \wedge \mathsf{world}(x)) \vee (\mathsf{set\_}(x) \wedge \mathsf{world}(x)))$

```
(cl-text  ax_abstractIndividual_pairwiseDisjoint
(not (exists (x)
  (or (and (quale x) (set_ x)) (and (quale x) (world x)) (and
    (set_ x) (world x))))
)
)
```

a13 Every x that is a Quality Structure is also a Set.

$\forall x(\mathsf{qualityStructure}(x) \rightarrow \mathsf{set\_}(x))$

```
(cl-text  ax_set_taxonomy_qualityStructure
(forall (x)
  (if (qualityStructure x)
    (set_ x))
  )
)
```

a14 For every x, x is a Quality Structure iff x is either a Quality Dimension or a Quality Space.

$\forall x(\mathsf{qualityDimension}(x) \vee \mathsf{qualitySpace}(x) \leftrightarrow \mathsf{qualityStructure}(x))$

```
(cl-text  ax_qualityStructure_taxonomy
(forall (x)
  (iff (or (qualityDimension x) (qualitySpace x))
    (qualityStructure x))
  )
)
```

a15 There is no x such that it is a Quality Dimension and a Quality Space.

$\neg\exists x(\mathsf{qualityDimension}(x) \wedge \mathsf{qualitySpace}(x))$

```
(cl-text  ax_qualityStructure_partition
(not (exists (x)
  (and (qualityDimension x) (qualitySpace x)))
)
)
```

2.3 Partial Taxonomy of UFO: Endurant

This subsection presents a portion of UFO's taxonomy specializing the type Endurant (Figure 4).

a16 For every x, x is an Endurant iff x is either a Substantial or a Moment.

$\forall x(\mathsf{substantial}(x) \vee \mathsf{moment}(x) \leftrightarrow \mathsf{endurant}(x))$

```
(cl-text  ax_endurant_taxonomy
(forall (x)
  (iff (or (substantial x) (moment x))
    (endurant x))
  )
)
```

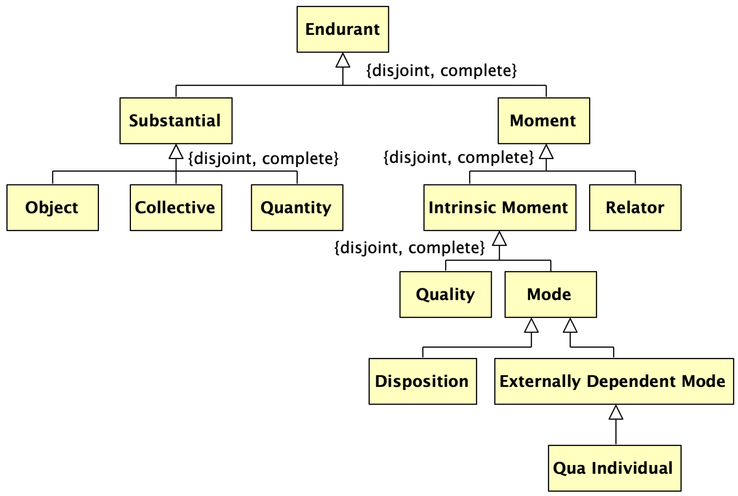


Figure 4: Visual representation of UFO's taxonomy of Endurant.

a17 There is no x such that it is a Substantial and a Moment.

$\neg\exists x(\mathsf{substantial}(x) \wedge \mathsf{moment}(x))$

```
(cl-text  ax_endurant_partition
(not (exists (x)
  (and (substantial x) (moment x)))
)
)
```

a18 For every x, x is a Substantial ifif x is either an Object, a Collective, or a Quantity.

$\forall x(\mathsf{object}(x) \vee \mathsf{collective}(x) \vee \mathsf{quantity}(x) \leftrightarrow \mathsf{substantial}(x))$

```
(cl-text  ax_substantial_taxonomy
(forall (x)
  (iff (or (object x) (collective x) (quantity x))
    (substantial x))
  )
)
```

a19 There is no x such that it is an Object, a Collective, and a Quantity (pairwise disjoint).

$\neg\exists x((\mathsf{object}(x) \wedge \mathsf{collective}(x)) \vee (\mathsf{object}(x) \wedge \mathsf{quantity}(x)) \vee (\mathsf{collective}(x) \wedge \mathsf{quantity}(x)))$

```
(cl-text  ax_substantial_partition
(not (exists (x)
  (or (and (object x) (collective x)) (and (object x) (
    quantity x)) (and (collective x) (quantity x))))
)
)
```

a20 For every x, x is a Moment ifif x is either an Intrinsic Moment or a Relator.

$\forall x(\mathsf{intrinsicMoment}(x) \lor \mathsf{relator}(x) \leftrightarrow \mathsf{moment}(x))$

```
(cl-text  ax_moment_taxonomy
(forall (x)
  (iff (or (intrinsicMoment x) (relator x))
    (moment x))
  )
)
```

a21 There is no x such that it is an Intrinsic Moment and a Relator.

$\neg\exists x(\mathsf{intrinsicMoment}(x) \land \mathsf{relator}(x))$

```
(cl-text  ax_moment_partition
(not (exists (x)
  (and (intrinsicMoment x) (relator x)))
)
)
```

a22 For every x, x is an Intrinsic Moment ifif x is either a Quality or a Mode.

$\forall x(\mathsf{quality}(x) \lor \mathsf{mode}(x) \leftrightarrow \mathsf{intrinsicMoment}(x))$

```
(cl-text  ax_intrinsicMoment_taxonomy
(forall (x)
  (iff (or (quality x) (mode x))
    (intrinsicMoment x))
  )
)
```

a23 There is no x such that it is an Intrinsic Moment and a Relator.

$\neg\exists x(\mathsf{quality}(x) \land \mathsf{mode}(x))$

```
(cl-text  ax_intrinsicMoment_partition
(not (exists (x)
  (and (quality x) (mode x)))
)
)
```

a24 Every x that is a Disposition is also a Mode.

$\forall x(\mathsf{disposition}(x) \rightarrow \mathsf{mode}(x))$

```
(cl-text  ax_mode_taxonomy_disposition
(forall (x)
  (if (disposition x)
    (mode x))
  )
)
```

a25 Every x that is an Externally Dependent Mode is also a Mode.

$\forall x(\mathsf{externallyDependentMode}(x) \rightarrow \mathsf{mode}(x))$

```
(cl-text  ax_mode_taxonomy_externallyDependentMode
(forall (x)
  (if (externallyDependentMode x)
    (mode x))
  )
)
```

a26 Every x that is an Qua Individual is also an Externally Dependent Mode.

$\forall x(\mathsf{quaIndividual}(x) \rightarrow \mathsf{externallyDependentMode}(x))$

```
(cl-text  ax_externallyDependentMode_taxonomy_quaIndividual
(forall (x)
  (if (quaIndividual x)
    (externallyDependentMode x))
  )
)
```

2.4 UFO Taxonomy

2.4.1 Partial Taxonomy of Endurant

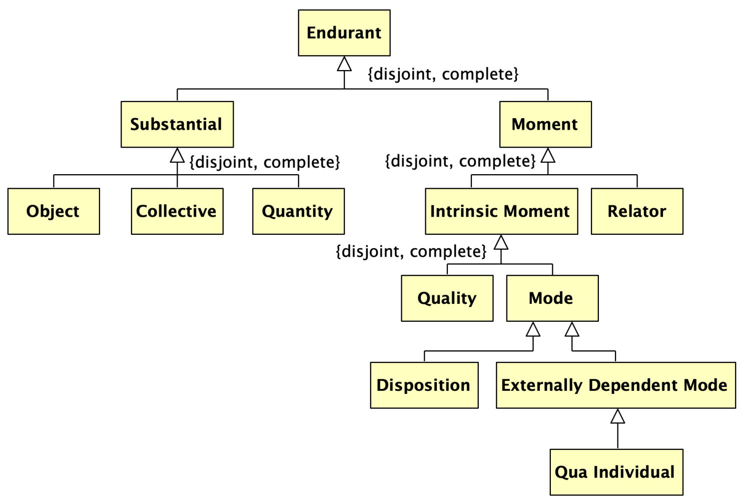


Figure 5: Partial Taxonomy of UFO – Endurant.

```
% fof(ax_abstractIndividual_instances, axiom, (
%   set_(set1) & quale(quale1) & qualityStructure(qualityStructure1
    ) & qualityDimension(qualityDimension1) & qualitySpace(
    qualitySpace1) & world(world1)
% )).

% Endurant

fof(ax_endurant_taxonomy, axiom, (
  ![X]: ((substantial(X) | moment(X)) <=> (endurant(X)))
```

```
)).

fof(ax_endurant_partition, axiom, (
  ~?[X]: (substantial(X) & moment(X))
)).

% Substantial

fof(ax_substantial_taxonomy, axiom, (
  ![X]: ((object(X) | collective(X) | quantity(X)) <=> (substantial
    (X)))
)).

fof(ax_substantial_partition, axiom, (
  ~?[X]: ((object(X) & collective(X)) | (object(X) & quantity(X)) |
     (collective(X) & quantity(X)))
)).

% Moment

fof(ax_moment_taxonomy, axiom, (
  ![X]: ((intrinsicMoment(X) | relator(X)) <=> (moment(X)))
)).

fof(ax_moment_partition, axiom, (
  ~?[X]: (intrinsicMoment(X) & relator(X))
)).

% Intrinsic Moment

fof(ax_intrinsicMoment_taxonomy, axiom, (
  ![X]: ((quality(X) | mode(X)) <=> (intrinsicMoment(X)))
)).

fof(ax_intrinsicMoment_partition, axiom, (
  ~?[X]: (quality(X) & mode(X))
)).

% Mode

fof(ax_mode_taxonomy_externallyDependentMode, axiom, (
  ![X]: (externallyDependentMode(X) => (mode(X)))
)).

% Externally Dependent Mode

fof(ax_externallyDependentMode_taxonomy_quaIndividual, axiom, (
  ![X]: (quaIndividual(X) => (externallyDependentMode(X)))
)).

% Endurant partial taxonomy instances
% (tested to rule out trivial models)
```

2.4.2 Partial Taxonomy of Endurant Type (on ontological natures)

```
% fof(ax_endurant_instances, axiom, (
```

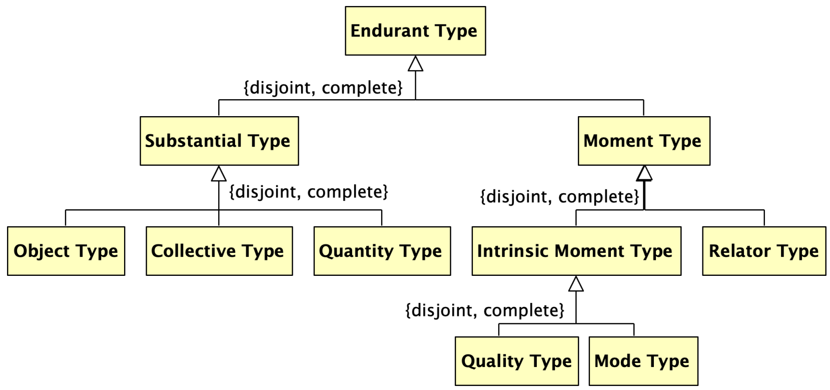


Figure 6: Partial Taxonomy of UFO – Endurant Types (by ontological nature).

```
%   substantial(substantial1) & moment(moment1) & object(object1) & collective(collective1) & quantity(quantity1) & intrinsicMoment(intrinsicMoment1) & relator(relator1) & quality(quality1) & mode(mode1) & disposition(disposition1) & externallyDependentMode(externallyDependentMode1) & quaIndividual(quaIndividual1)
% )).

% Endurant Type (by ontological nature)

fof(ax_endurantType_taxonomy_nature, axiom, (
  ![X]: ((substantialType(X) | momentType(X)) <=> (endurantType(X)))
)).

fof(ax_endurantType_partition_nature, axiom, (
  ~?[X]: (substantialType(X) & momentType(X))
)).

% Substantial Type

fof(ax_substantialType_taxonomy, axiom, (
  ![X]: ((objectType(X) | collectiveType(X) | quantityType(X)) <=> (substantialType(X)))
)).

fof(ax_substantialType_partition, axiom, (
  ~?[X]: ((objectType(X) & collectiveType(X)) | (objectType(X) & quantityType(X)) | (collectiveType(X) & quantityType(X)))
)).

% Moment Type

fof(ax_momentType_taxonomy, axiom, (
  ![X]: ((intrinsicMomentType(X) | relatorType(X)) <=> (momentType(X)))
)).

```

```
fof(ax_momentType_partition, axiom, (
  ~?[X]: (intrinsicMomentType(X) & relatorType(X))
)).

% Intrinsic Moment Type

fof(ax_intrinsicMomentType_taxonomy, axiom, (
  ![X]: ((qualityType(X) | modeType(X)) <=> (intrinsicMomentType(X)
    ))
)).

fof(ax_intrinsicMomentType_partition, axiom, (
  ~?[X]: (qualityType(X) & modeType(X))
)).

% Endurant Type (by ontological nature) partial taxonomy instances
% (tested to rule out trivial models)
```

2.4.3 Partial Taxonomy of Endurant Type (on modal properties of types)

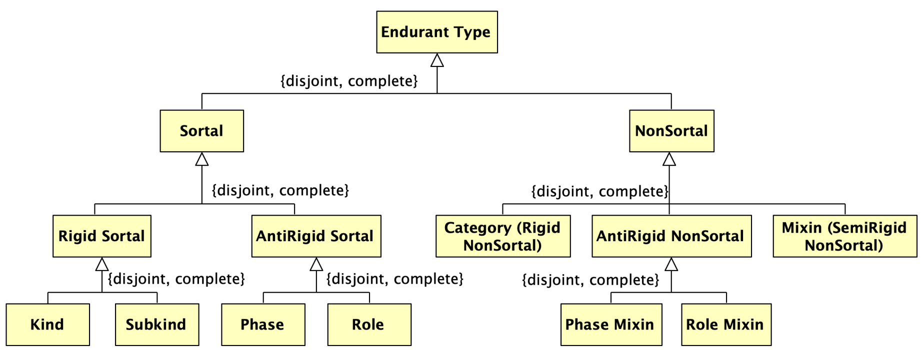


Figure 7: Partial Taxonomy of UFO – Endurant Types (by modal properties of types).

```
% fof(ax_endurantType_instances_natures, axiom, (
%   substantialType(substantialType1) & momentType(momentType1) &
    objectType(objectType1) & collectiveType(collectiveType1) &
    quantityType(quantityType1) & intrinsicMomentType(
    intrinsicMomentType1) & relatorType(relatorType1) & qualityType
    (qualityType1) & modeType(modeType1)
% )).

% Endurant Type (by modal properties of types)

fof(ax_endurantType_taxonomy_properties, axiom, (
  ![X]: ((sortal(X) | nonSortal(X)) <=> (endurantType(X)))
)).

fof(ax_endurantType_partition_properties, axiom, (
  ~?[X]: (sortal(X) & nonSortal(X))
)).
```

```
% Sortal

fof(ax_sortal_taxonomy, axiom, (
  ![X]: ((rigidSortal(X) | antiRigidSortal(X)) <=> (sortal(X)))
)).

fof(ax_sortal_partition, axiom, (
  ~?[X]: (rigidSortal(X) & antiRigidSortal(X))
)).

% Rigid Sortal

fof(ax_rigidSortal_taxonomy, axiom, (
  ![X]: ((kind(X) | subkind(X)) <=> (rigidSortal(X)))
)).

fof(ax_rigidSortal_partition, axiom, (
  ~?[X]: (kind(X) & subkind(X))
)).

% Anti-Rigid Sortal

fof(ax_antiRigidSortal_taxonomy, axiom, (
  ![X]: ((phase(X) | role(X)) <=> (antiRigidSortal(X)))
)).

fof(ax_antiRigidSortal_partition, axiom, (
  ~?[X]: (phase(X) & role(X))
)).

% Non-Sortal

fof(ax_nonSortal_taxonomy, axiom, (
  ![X]: ((rigidNonSortal(X) | semiRigidNonSortal(X) |
    antiRigidNonSortal(X)) <=> (nonSortal(X)))
)).

fof(ax_nonSortal_partition, axiom, (
  ~?[X]: ((rigidNonSortal(X) & semiRigidNonSortal(X)) | (
    rigidNonSortal(X) & antiRigidNonSortal(X)) | (
    semiRigidNonSortal(X) & antiRigidNonSortal(X)))
)).

% Category

fof(ax_rigidNonSortal_taxonomy, axiom, (
  ![X]: (rigidNonSortal(X) <=> (category(X)))
)).

% Mixin

fof(ax_semiRigidNonSortal_taxonomy, axiom, (
  ![X]: (semiRigidNonSortal(X) <=> (mixin(X)))
)).

% Anti-Rigid Non-Sortal
```

```

fof(ax_antiRigidNonSortal_taxonomy, axiom, (
  ![X]: ((phaseMixin(X) | roleMixin(X)) <=> (antiRigidNonSortal(X))
    )
)).

fof(ax_antiRigidNonSortal_partition, axiom, (
  ~?[X]: (phaseMixin(X) & roleMixin(X))
)).

% Endurant Type (by modal properties of types) partial taxonomy
    instances
% (tested to rule out trivial models)
```

2.4.4 Defining Types, Individuals, and Specialization

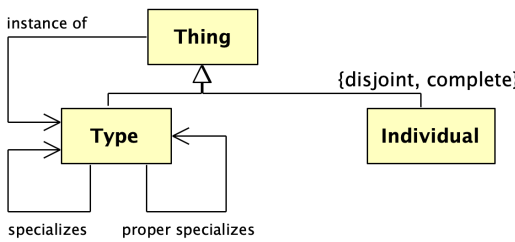


Figure 8: Types, individuals, instantiation, and specialization.

```
% fof(ax_endurantType_instances_properties, axiom, (
%   sortal(sortal1) & nonSortal(nonSortal1) & rigidSortal(
    rigidSortal1) & antiRigidSortal(antiRigidSortal1) & kind(kind1)
     & subkind(subkind1) & phase(phase1) & role(role1) & category(
    category1) & mixin(mixin1) & antiRigidNonSortal(
    antiRigidNonSortal1) & phaseMixin(phaseMixin1) & roleMixin(
    roleMixin1)
% )).

%%%%%%%%%%% Instance of, Types, and Individuals %%%%%%%%%%

fof(ax_dIof, axiom, (
  ![X,Y,W]: (iof(X,Y,W) => (type_(Y) & world(W)))
)).

fof(ax_dType_a1, axiom, (
  ![X]: (type_(X) <=> (?[Y,W]: iof(Y,X,W)))
)).

fof(ax_dIndividual_a2, axiom, (
  ![X]: (individual(X) <=> (~?[Y,W]: iof(Y,X,W)))
)).

fof(ax_multiLevel_a3, axiom, (
  ![X,Y,W]: (iof(X,Y,W) => (type_(X) | individual(X)))
)).
```

```
fof(ax_twoLevelConstrained_a4, axiom, (
  ~?[X,Y,Z,W]: (type_(X) & iof(X,Y,W) & iof(Y,Z,W))
)).

% Instantiation relations
% (tested to rule out trivial models)

% fof(ax_iofInUse, axiom, (
%   type_(t2) & individual(i2) & world(w2) & iof(i2,t2,w2)
% )).

% Ax |= "th_everythingIsAThing_t1"; conjecture commented for
    convenience

% fof(th_everythingIsAThing_t1, conjecture, (
%   ![X]: (type_(X) | individual(X))
% )).

% Ax |= "th_thingPartition_t2"; conjecture commented for
    convenience

% fof(th_thingPartition_t2, conjecture, (
%   ~?[X]: (type_(X) & individual(X))
% )).

%%%%%%%% Specialization and Proper Specialization %%%%%%%%

fof(ax_dSpecializes, axiom, (
  ![X,Y]: (specializes(X,Y) => (type_(X) & type_(Y)))
)).

fof(ax_specialization_a5, axiom, (
  ![T1,T2]: (specializes(T1,T2) <=> (
    type_(T1) & type_(T2) & ![W]: (world(W) => ![E]: (iof(E,T1,W)
    => iof(E,T2,W)))
  ))
)).

fof(ax_properSpecializes_d1, axiom, (
  ![X,Y]: (properSpecializes(X,Y) <=> (specializes(X,Y) & ~
    specializes(Y,X)))
)).

% Ax |= "th_cyclicSpecializations_t3"; conjecture commented for
    convenience

% fof(th_cyclicSpecializations_t3, conjecture, (
%   ![X,Y]: (specializes(X,Y) => (specializes(X,X) & specializes(Y,
    Y)))
% )).

% Ax |= "th_transitiveSpecializations_t4"; conjecture commented for
     convenience

% fof(th_transitiveSpecializations_t4, conjecture, (
%   ![X,Y,Z]: ((specializes(X,Y) & specializes(Y,Z)) => (
```

```
    specializes(X,Z)))
% )).

fof(ax_sharedSpecializations_a6, axiom, (
  ![T1,T2]: (![X,W]: ((iof(X,T1,W) & iof(X,T2,W) & ~specializes(T1,T2) & ~specializes(T2,T1)) => (
      (?[T3]: (specializes(T1,T3) & specializes(T2,T3) & iof(X,T3,W)))
      | (?[T3]: (specializes(T3,T1) & specializes(T3,T2) & iof(X,T3,W)))
  )))
)).

% Specialization relations
% (tested to rule out trivial models)

% fof(ax_specializesInUse, axiom, (
```

2.4.5 Defining Rigidity and Sortality

```
%%%%%%%%%%%%%%%%% Sortality and Rigidity %%%%%%%%%%%%%%%%%

% Rigidity

fof(ax_dRigid_a18, axiom, (
  ![T]: (rigid(T) <=> (endurantType(T) & (
    ![X]: ((?[W1]: (world(W1) & iof(X,T,W1))) => (![W2]: (world(W2) => iof(X,T,W2))))
  )))
)).

fof(ax_dAntiRigid_a19, axiom, (
  ![T]: (antiRigid(T) <=> (endurantType(T) & (
    ![X]: ((?[W1]: (world(W1) & iof(X,T,W1))) => (?[W2]: (world(W2) & ~iof(X,T,W2)))
  ))))
)).

fof(ax_dSemiRigid_a20, axiom, (
  ![T]: (semiRigid(T) <=> (endurantType(T) & ~rigid(T) & ~antiRigid(T)))
)).

% Ax |= "th_thEndurantTypeHaveRigidity_t5"; conjecture commented for convenience

% fof(th_thEndurantTypeHaveRigidity_t5, conjecture, (
%   ![T]: (endurantType(T) <=> (rigid(T) | semiRigid(T) | antiRigid(T)))
% )).

% Ax |= "th_pairwiseDisjointRigidities_t6"; conjecture commented for convenience

% fof(th_pairwiseDisjointRigidities_t6, conjecture, (
```

```
%   ~![T]: ((rigid(T) & semiRigid(T)) | (semiRigid(T) & antiRigid(T)) | (rigid(T) & antiRigid(T)))
% )).

% Ax |= "th_rigidAntiRigidSpecializationConstraint_t7"; conjecture commented for convenience

% fof(th_rigidAntiRigidSpecializationConstraint_t7, conjecture, (
%   ~![T1,T2]: (rigid(T1) & antiRigid(T2) & specializes(T1,T2))
% )).

% Ax |= "th_semiRigidAntiRigidSpecializationConstraint_t8"; conjecture commented for convenience

% fof(th_semiRigidAntiRigidSpecializationConstraint_t8, conjecture, (
%   ~![T1,T2]: (semiRigid(T1) & antiRigid(T2) & specializes(T1,T2))
% )).

% Rigidity properties
% (tested to rule out trivial models)

% fof(ax_rigidityInUse, axiom, (
%   endurantType(t4_1) & endurantType(t4_2) & endurantType(t4_3) & rigid(t4_1) & semiRigid(t4_2) & antiRigid(t4_3) & properSpecializes(t4_1,t4_2) & properSpecializes(t4_3,t4_1)
% )).

% Sortality

fof(ax_endurantsKind_a21, axiom, (
  ![E]: (endurant(E) => (
    ?[U]: (kind(U) & (![W]: (world(W) => iof(E,U,W))))
  ))
)).

fof(ax_uniqueKind_a22, axiom, (
  ![E,U,W]: ((world(W) & kind(U) & iof(E,U,W)) => (
    ~?[U2,W2]: (kind(U2) & iof(E,U2,W2) & ~(U = U2))
  ))
)).

% Changing "ax_dSortal_a23" from the form it was defined in the paper to "sortals are endurant types that specialize some ultimate sortal" seem to express the same concept while speeding up the execution of SPASS considerably

% fof(ax_dSortal_a23, axiom, (
%   ![S]: (sortal(S) <=> (endurantType(S) & (?[U]: (kind(U) & (![E,W]: (iof(E,S,W) => iof(E,U,W)))))))
% )).

fof(ax_dSortal_a23, axiom, (
  ![S]: ((sortal(S)) <=> (endurantType(S) & (?[U]: (kind(U) & specializes(S,U)))))
)).

```

```
% If we have the taxonomy's axiomatization, then a24 becomes a theorem
% Ax |= "th_nonSortalsAreEndurantsThatAreNotSortals_a24"; conjecture commented for convenience

% fof(th_nonSortalsAreEndurantsThatAreNotSortals_a24, conjecture, (
%   ![NS]: ((nonSortal(NS)) <=> (endurantType(NS) & ~sortal(NS)))
% )).

% Ax |= "th_kindsAreRigid_t9"; conjecture commented for convenience

% fof(th_kindsAreRigid_t9, conjecture, (
%   ![U]: ((kind(U)) => (rigid(U)))
% )).

% Ax |= "th_kindsHaveDisjointExtensions_t10"; conjecture commented for convenience

% fof(th_kindsHaveDisjointExtensions_t10, conjecture, (
%   ![K1,K2]: ((kind(K1) & kind(K2) & ~(K1=K2)) => (
%     ~?[X,W1,W2]: (world(W1) & world(W2) & iof(X,K1,W1) & iof(X,K2,W2)))
%   )
% )).

% Ax |= "th_kindsHaveDisjointTaxonomies_t11"; conjecture commented for convenience

% fof(th_kindsHaveDisjointTaxonomies_t11, conjecture, (
%   ![K1,K2]: ((kind(K1) & kind(K2) & ~(K1=K2)) => (
%     ~?[T]: (specializes(T,K1) & specializes(T,K2)))
%   )
% )).

% Ax |= "th_kindsAreSortal_t12"; conjecture commented for convenience

% fof(th_kindsAreSortal_t12, conjecture, (
%   ![K]: ((kind(K)) => (sortal(K)))
% )).

% Ax |= "th_sortalSpecializeKinds_t13"; conjecture commented for convenience

% fof(th_sortalSpecializeKinds_t13, conjecture, (
%   ![S]: ((sortal(S)) => (?[K]: (kind(K) & specializes(S,K))))
% )).

% Ax |= "th_sortalsSpecializeAUniqueKind_t14"; conjecture commented for convenience

% fof(th_sortalsSpecializeAUniqueKind_t14, conjecture, (
%   ![S]: ((sortal(S)) => (~?[U,U2]: (kind(U) & kind(U2) & specializes(S,U) & specializes(S,U2) & ~(U=U2))))
% )).

% Sortality properties
```

```
% (tested to rule out trivial models)

% fof(ax_sortalityInUse, axiom, (
%   endurant(e5_1) & endurant(e5_2) & world(w5) & kind(k5_1) & kind
    (k5_2) & iof(e5_1,k5_1,w5) & iof(e5_1,k5_1,w5) & ~(k5_1=k5_2)
% )).

% Sortality + Rigidity

fof(ax_rigidSortalsAreRigidAndSortal_xx, axiom, (
  ![T]: ((rigidSortal(T)) <=> (rigid(T) & sortal(T)))
)).

fof(ax_antiRigidSortalsAreAntiRigidAndSortal_xx, axiom, (
  ![T]: ((antiRigidSortal(T)) <=> (antiRigid(T) & sortal(T)))
)).

fof(ax_rigidNonSortalsAreRigidAndNonSortal_xx, axiom, (
  ![T]: ((rigidNonSortal(T)) <=> (rigid(T) & nonSortal(T)))
)).

fof(ax_antiRigidNonSortalsAreAntiRigidAndNonSortal_xx, axiom, (
  ![T]: ((antiRigidNonSortal(T)) <=> (antiRigid(T) & nonSortal(T)))
)).

fof(ax_semiRigidNonSortalsAreSemiRigidAndNonSortal_xx, axiom, (
  ![T]: ((semiRigidNonSortal(T)) <=> (semiRigid(T) & nonSortal(T)))
)).

% If we have the taxonomy's axiomatization, then a25 becomes a
    theorem
% Ax |= "th_kindAndSubkindAreDisjoint_a25"; conjecture commented
    for convenience

% fof(th_kindAndSubkindAreDisjoint_a25, conjecture, (
%   ~?[T]: (kind(T) & subkind(T))
% )).

% If we have the taxonomy's axiomatization, then a26 becomes a
    theorem
% Ax |= "th_kindAndSubkindAreRigidSortals_a26"; conjecture
    commented for convenience

% fof(th_kindAndSubkindAreRigidSortals_a26, conjecture, (
%   ![T]: ((kind(T) | subkind(T)) <=> (rigid(T) & sortal(T)))
% )).

% If we have the taxonomy's axiomatization, then a27 becomes a
    theorem
% Ax |= "th_phaseAndRoleAreDisjoint_a27"; conjecture commented for
    convenience

% fof(th_phaseAndRoleAreDisjoint_a27, conjecture, (
%   ~?[T]: (phase(T) & role(T))
% )).

% If we have the taxonomy's axiomatization, then a28 becomes a
```

```
    theorem
% Ax |= "th_phaseAndRoleAreAntiRigidSortals_a28"; conjecture commented for convenience

% fof(th_phaseAndRoleAreAntiRigidSortals_a28, conjecture, (
%   ![T]: ((phase(T) | role(T)) <=> (antiRigid(T) & sortal(T)))
% )).

% Skipping (a29) because we leave the concept of semi-rigid sortals out of this ontology.

% If we have the taxonomy's axiomatization, then a30 becomes a theorem
% Ax |= "th_categoriesAreRigidNonSortals_a30"; conjecture commented for convenience

% fof(th_categoriesAreRigidNonSortals_a30, conjecture, (
%   ![T]: ((category(T)) <=> (rigid(T) & nonSortal(T)))
% )).

% If we have the taxonomy's axiomatization, then a31 becomes a theorem
% Ax |= "th_mixinsAreSemiRigidNonSortals_a31"; conjecture commented for convenience

% fof(th_mixinsAreSemiRigidNonSortals_a31, conjecture, (
%   ![T]: ((mixin(T)) <=> (semiRigid(T) & nonSortal(T)))
% )).

% If we have the taxonomy's axiomatization, then a32 becomes a theorem
% Ax |= "th_phaseMixinAndRoleMixinAreDisjoint_a32"; conjecture commented for convenience

% fof(th_phaseMixinAndRoleMixinAreDisjoint_a32, conjecture, (
%   ~?[T]: (phaseMixin(T) & roleMixin(T))
% )).

% If we have the taxonomy's axiomatization, then a33 becomes a theorem
% Ax |= "ax_phaseMixinAndRoleMixinAreAntiRigidSortals_a33"; conjecture commented for convenience

% fof(th_phaseMixinAndRoleMixinAreAntiRigidSortals_a33, conjecture, (
%   ![T]: ((phaseMixin(T) | roleMixin(T)) <=> (antiRigid(T) & nonSortal(T)))
% )).

% Ax |= "th_leafCategoriesArePairwiseDisjoint_t18"; conjecture commented for convenience

% fof(th_leafCategoriesArePairwiseDisjoint_t18, conjecture, (
%   ~?[T]: (endurantType(T) & (
%     (
%       (kind(T) & subkind(T))
%       | (kind(T) & phase(T))
```

```
%       | (kind(T) & role(T))
%       | (kind(T) & category(T))
%       | (kind(T) & mixin(T))
%       | (kind(T) & phaseMixin(T))
%       | (kind(T) & roleMixin(T))
%     ) | (
%       (subkind(T) & phase(T))
%       | (subkind(T) & role(T))
%       | (subkind(T) & category(T))
%       | (subkind(T) & mixin(T))
%       | (subkind(T) & phaseMixin(T))
%       | (subkind(T) & roleMixin(T))
%     ) | (
%       (phase(T) & role(T))
%       | (phase(T) & category(T))
%       | (phase(T) & mixin(T))
%       | (phase(T) & phaseMixin(T))
%       | (phase(T) & roleMixin(T))
%     ) | (
%       (role(T) & category(T))
%       | (role(T) & mixin(T))
%       | (role(T) & phaseMixin(T))
%       | (role(T) & roleMixin(T))
%     ) | (
%       (category(T) & mixin(T))
%       | (category(T) & phaseMixin(T))
%       | (category(T) & roleMixin(T))
%     ) | (
%       (mixin(T) & phaseMixin(T))
%       | (mixin(T) & roleMixin(T))
%     ) | (
%       (phaseMixin(T) & roleMixin(T))
%     )
%   ))
% )).

% Ax |= "th_leafCategoriesCompletelyCategorizeAllEndurantTypes_t19"; conjecture commented for convenience

% fof(th_leafCategoriesCompletelyCategorizeAllEndurantTypes_t19, conjecture, (
%   ![T]: (endurantType(T) => (
%     kind(T) | subkind(T) | phase(T) | role(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T)
%   ))
% )).

% Sortality and rigidity properties combined
% (tested to rule out trivial models)

% fof(ax_sortalityAndRigidityInUse, axiom, (
%   endurant(e6_1) & endurant(e6_2) & world(w6) & kind(k6_1) & kind(k6_2) & iof(e6_1,k6_1,w6) & iof(e6_1,k6_1,w6) & ~(k6_1=k6_2)
% )).
```

2.4.6 Defining Endurant Types

```

```

```
% Defining the taxonomy of types of ontological natures through the categorization of the taxonomy of concrete individuals

fof(ax_perdurantTypeDefinition_a44, axiom, (
  ![T]: (perdurantType(T) <=> (
    type_(T) & (![P,W]: ((world(W) & iof(P,T,W)) => (perdurant(P))))
  ))
)).

fof(ax_endurantTypeDefinition_a44, axiom, (
  ![T]: (endurantType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (endurant(E))))
  ))
)).

fof(ax_substantialTypeDefinition_a44, axiom, (
  ![T]: (substantialType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (substantial(E))))
  ))
)).

fof(ax_momentTypeDefinition_a44, axiom, (
  ![T]: (momentType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (moment(E))))
  ))
)).

fof(ax_objectTypeDefinition_a44, axiom, (
  ![T]: (objectType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (object(E))))
  ))
)).

fof(ax_collectiveTypeDefinition_a44, axiom, (
  ![T]: (collectiveType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (collective(E))))
  ))
)).

fof(ax_quantityTypeDefinition_a44, axiom, (
  ![T]: (quantityType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (quantity(E))))
  ))
)).

fof(ax_intrinsicMomentTypeDefinition_a44, axiom, (
  ![T]: (intrinsicMomentType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (intrinsicMoment(E))))
  ))
)).

fof(ax_relatorTypeDefinition_a44, axiom, (
  ![T]: (relatorType(T) <=> (
```

```
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (relator(E))))
  ))
)).

fof(ax_qualityTypeDefinition_a44, axiom, (
  ![T]: (qualityType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (quality(E))))
  ))
)).

fof(ax_modeTypeDefinition_a44, axiom, (
  ![T]: (modeType(T) <=> (
    type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (mode(E))))
  ))
)).

% Types Definition
% (tested to rule out trivial models)
% TODO: investigate why we cannot list four different endurant types (it may have something to do with "intrinsicMoment" and "intrinsicMomentType")

% fof(ax_typesDefinitionsInstances, axiom, (
%   objectType(ot7) & collectiveType(ct7) & modeType(mt7)
% )).

% Ax |= "th_leafCategoriesArePairwiseDisjoint_t21"; conjecture commented for convenience
% Having the previously defined taxonomy, this should be quite trivial

% fof(th_leafCategoriesArePairwiseDisjoint_t21, conjecture, (
%   ~?[T]: (type_(T) & (
%     (
%       (objectType(T) & collectiveType(T)) | (objectType(T) & quantityType(T)) | (objectType(T) & modeType(T)) | (objectType(T) & qualityType(T)) | (objectType(T) & relatorType(T)) | (objectType(T) & perdurantType(T))
%     ) | (
%       (collectiveType(T) & quantityType(T)) | (collectiveType(T) & modeType(T)) | (collectiveType(T) & qualityType(T)) | (collectiveType(T) & relatorType(T)) | (collectiveType(T) & perdurantType(T))
%     ) | (
%       (quantityType(T) & modeType(T)) | (quantityType(T) & qualityType(T)) | (quantityType(T) & relatorType(T)) | (quantityType(T) & perdurantType(T))
%     ) | (
%       (modeType(T) & qualityType(T)) | (modeType(T) & relatorType(T)) | (modeType(T) & perdurantType(T))
%     ) | (
%       (qualityType(T) & relatorType(T)) | (qualityType(T) & perdurantType(T))
%     ) | (
%       relatorType(T) & perdurantType(T)
%     )
%   ))
```

```
% )).

% Ultimate Sortals Definitions (by ontological nature)

fof(ax_objectKindDefinition_a45, axiom, (
  ![T]: (objectKind(T) <=> (objectType(T) & kind(T)))
)).

fof(ax_collectiveKindDefinition_a45, axiom, (
  ![T]: (collectiveKind(T) <=> (collectiveType(T) & kind(T)))
)).

fof(ax_quantityKindDefinition_a45, axiom, (
  ![T]: (quantityKind(T) <=> (quantityType(T) & kind(T)))
)).

fof(ax_modeKindDefinition_a45, axiom, (
  ![T]: (modeKind(T) <=> (modeType(T) & kind(T)))
)).

fof(ax_qualityKindDefinition_a45, axiom, (
  ![T]: (qualityKind(T) <=> (qualityType(T) & kind(T)))
)).

fof(ax_relatorKindDefinition_a45, axiom, (
  ![T]: (relatorKind(T) <=> (relatorType(T) & kind(T)))
)).

% Ultimate sortals (by ontological nature) instances
% (tested to rule out trivial models)
% TODO: investigate why we cannot list all different types of
    ultimate sortals at once

% fof(ax_typesDefinitionsInstances, axiom, (
%   objectKind(ok9) & collectiveKind(ck9) & quantityKind(quank9) &
    relatorKind(rk9) & modeKind(mk9) & qualityKind(qualk9)
% )).

% Skipping (t22) because (a21) makes it trivial

% Ax |= "th_endurantsInstantiateEndurantKindsOfSomeNature_a46";
    conjecture commented for convenience
% This axiom is actually a theorem in this version of the
    axiomatization

% fof(th_endurantsInstantiateEndurantKindsOfSomeNature_a46,
    conjecture, (
%   ![E]: (endurant(E) => (
%     ?[K,W]: ((objectKind(K) | collectiveKind(K) | quantityKind(K)
     | modeKind(K) | qualityKind(K) | relatorKind(K))
%     & iof(E,K,W))
%   ))
% )).

% Ax |= "th_endurantSortalsCompleteness_t23"; conjecture commented
    for convenience
% Thanks to the taxonomy, we already have "sortal(T) =>
```

```
endurantType(T)", but I leave it like this to be consistent with the paper

% fof(th_endurantSortalsCompleteness_t23, conjecture, (
%   ![T]: ((endurantType(T) & sortal(T)) => (objectKind(T) | collectiveKind(T) | quantityKind(T) | qualityKind(T) | modeKind(T) | relatorKind(T) | phase(T) | role(T)))
% )).

% Ax |= "th_objectTypesSpecializeAKindOfSameNature_t24"; conjecture commented for convenience

% fof(th_objectTypesSpecializeAKindOfSameNature_t24, conjecture, (
%   ![T]: ((objectType(T) & sortal(T)) <=> (?[K]: (objectKind(K) & specializes(T,K))))
% )).

% Ax |= "th_collectiveTypesSpecializeAKindOfSameNature_t24"; conjecture commented for convenience

% fof(th_collectiveTypesSpecializeAKindOfSameNature_t24, conjecture, (
%   ![T]: ((collectiveType(T) & sortal(T)) <=> (?[K]: (collectiveKind(K) & specializes(T,K))))
% )).

% Ax |= "th_quantityTypesSpecializeAKindOfSameNature_t24"; conjecture commented for convenience

% fof(th_quantityTypesSpecializeAKindOfSameNature_t24, conjecture, (
%   ![T]: ((quantityType(T) & sortal(T)) <=> (?[K]: (quantityKind(K) & specializes(T,K))))
% )).

% Ax |= "th_modeTypesSpecializeAKindOfSameNature_t24"; conjecture commented for convenience

% fof(th_modeTypesSpecializeAKindOfSameNature_t24, conjecture, (
%   ![T]: ((modeType(T) & sortal(T)) <=> (?[K]: (modeKind(K) & specializes(T,K))))
% )).

% Ax |= "th_qualityTypesSpecializeAKindOfSameNature_t24"; conjecture commented for convenience

% fof(th_qualityTypesSpecializeAKindOfSameNature_t24, conjecture, (
%   ![T]: ((qualityType(T) & sortal(T)) <=> (?[K]: (qualityKind(K) & specializes(T,K))))
% )).

% Ax |= "th_relatorTypesSpecializeAKindOfSameNature_t24"; conjecture commented for convenience

% fof(th_relatorTypesSpecializeAKindOfSameNature_t24, conjecture, (
%   ![T]: ((relatorType(T) & sortal(T)) <=> (?[K]: (relatorKind(K) & specializes(T,K))))
```

```
% )).

% Ax |= "th_sortalLeafCategoriesAreDisjoint_t25"; conjecture commented for convenience

% fof(th_sortalLeafCategoriesAreDisjoint_t25, conjecture, (
%   ![T]: (objectKind(T) => (~(collectiveKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (collectiveKind(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (quantityKind(T) => (~(objectKind(T) | collectiveKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (modeKind(T) => (~(objectKind(T) | quantityKind(T) | collectiveKind(T) | qualityKind(T) | relatorKind(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (qualityKind(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | collectiveKind(T) | relatorKind(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (relatorKind(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | collectiveKind(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (category(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | collectiveKind(T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (mixin(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) | collectiveKind(T) | phaseMixin(T) | roleMixin(T))))
%   & ![T]: (phaseMixin(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) | mixin(T) | collectiveKind(T) | roleMixin(T))))
%   & ![T]: (roleMixin(T) => (~(objectKind(T) | quantityKind(T) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) | mixin(T) | phaseMixin(T) | collectiveKind(T))))
% )).

% Ax |= "th_sortalLeafCategoriesAreComplete_t26"; conjecture commented for convenience

% fof(th_sortalLeafCategoriesAreComplete_t26, conjecture, (
%   ![T]: ((endurantType(T)) => (objectKind(T) | collectiveKind(T) | quantityKind(T) | qualityKind(T) | modeKind(T) | relatorKind(T) | phase(T) | role(T) | category(T) | mixin(T) | phaseMixin(T) | roleMixin(T)))
% )).
```

2.4.7 Mereology

```

% TODO: review whether it is necessary to reduce mereology to concrete individuals; I am leaving this axiom out for the moment

% fof(ax_partArguments, axiom, (
%   ![X,Y]: (part(X,Y) => (concreteIndividual(X) & concreteIndividual(Y)))
```

```
% )).

fof(ax_reflexiveParthood, axiom, (
  ![X]: (partOf(X,X))
)).

fof(ax_antiSymmetricParthood_a47, axiom, (
  ![X,Y]: ((partOf(X,Y) & partOf(Y,X)) => (X=Y))
)).

fof(ax_antiSymmetricParthood_a48, axiom, (
  ![X,Y]: ((partOf(X,Y) & partOf(Y,X)) => (X=Y))
)).

fof(ax_transitiveParthood_a49, axiom, (
  ![X,Y,Z]: ((partOf(X,Y) & partOf(Y,Z)) => (partOf(X,Z)))
)).

fof(ax_overlappingWholes_a50, axiom, (
  ![X,Y]: ((overlap(X,Y)) <=> (?[Z]: (partOf(Z,X) & partOf(Z,Y))))
)).

fof(ax_strongSupplementation_a51, axiom, (
  ![X,Y]: (~partOf(X,Y) <=> ?[Z]: (partOf(Z,X) & ~overlap(Z,Y)))
)).

fof(ax_properPart_a52, axiom, (
  ![X,Y]: (properPartOf(X,Y) <=> (partOf(X,Y) & ~partOf(Y,X)))
)).

fof(ax_binarySum_a53, axiom, (
  ![X,Y,Z]: (sum(Z,X,Y) <=> ![W]: (overlap(W,Z) <=> (overlap(W,X) | overlap(W,Y))))
)).

% Mereology in use
% (tested to rule out trivial models)

% fof(ax_mereologyInUse, axiom, (
%   concreteIndividual(ci10_1) & concreteIndividual(ci10_2) & concreteIndividual(ci10_3) & concreteIndividual(ci10_4) & concreteIndividual(ci10_5) & ~(ci10_1=ci10_2) & ~(ci10_2=ci10_3) & ~(ci10_3=ci10_4) & ~(ci10_4=ci10_5) & properPart(ci10_1,ci10_2) & properPart(ci10_3,ci10_4) & sum(ci10_5,ci10_3,ci10_4)
% )).

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Composition %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fof(ax_function, axiom,  (
  ![X,Y]: (functionsAs(X,Y) => (endurant(X) & type_(Y)))
)).
```

2.4.8 Composition

```
  ![X,Y]: (functionsAs(X,Y) => (endurant(X) & type_(Y)))
)).

fof(ax_genericFunctionalDependence_a55, axiom, (
```

```
  ![T1,T2,W]: (gfd(T1,T2,W) <=>
    ![E1]: ((iof(T1,E1,W) & functionsAs(T1,E1)) => ?[E2]: (~(E1=E2) & iof(T2,E2,W) & functionsAs(T2,E2))))
)).

fof(ax_individualFunctionalDependence_a56, axiom, (
  ![E1,T1,E2,T2,W]: (ifd(E1,T1,E2,T2,W) <=> (
    gfd(T1,T2,W) & iof(E1,T1,W) & iof(E2,T2,W) & (functionsAs(E1,T1) => functionsAs(E2,T2))
  ))
)).

fof(ax_componentOf_a57, axiom, (
  ![E1,T1,E2,T2,W]: (componentOf(E1,T1,E2,T2,W) <=> (properPartOf(E1,E2) & ifd(E1,T1,E2,T2,W)))
)).

% Composition in use
% (tested to rule out trivial models)

% fof(ax_compositionInUse, axiom, (
%   componentOf(e11_1,t11_1,e11_2,t11_2,w11) & ~(e11_1=e11_2) & ~(e11_1=t11_1) & ~(e11_2=t11_2) & ~(e11_1=t11_2) & ~(e11_2=t11_1) & ~(t11_1=t11_2)
% )).

%%%%%%%%%%%%%%%%%%%%%% Constitution %%%%%%%%%%%%%%%%%%%%%%

fof(ax_constitutedByInvolvedNatures_a58, axiom, (
  ![X,Y,W]: (constitutedBy(X,Y,W) => ((endurant(X) <=> endurant(Y)) & (perdurant(X) <=> perdurant(Y)) & world(W)))
```

2.4.9 Constitution

```

fof(ax_constitutedByDifferentKinds_a59, axiom, (
  ![E1,E2,T1,T2,W]: ((constitutedBy(E1,E2,W) & iof(E1,T1,W) & iof(E2,T2,W) & kind(T1) & kind(T2)) => (~(T1=T2)))
)).

% Ax |= "th_noSelfConstitution_t27"; conjecture commented for convenience

% fof(th_noSelfConstitution_t27, conjecture, (
%   ~?[X,W]: (endurant(X) & constitutedBy(X,X,W))
% )).

fof(ax_genericConstitutionalDependence_a60, axiom, (
  ![T1,T2]: (genericConstitutionalDependence(T1,T2) <=> (
    type_(T1) & type_(T2) & ![E1,W]: (iof(E1,T1,W) => (
      ?[E2]: (constitutedBy(E1,E2,W) & iof(E2,T2,W)
    )))
  ))
)).

fof(ax_constitution_a61, axiom, (
  ![E1,T1,E2,T2,W]: (constitution(E1,T1,E2,T2,W) <=> (
```

```
    iof(E1,T1,W) & iof(E2,T2,W) & genericConstitutionalDependence(
    T1,T2) & constitutedBy(E1,E2,W)
  ))
)).

fof(
    ax_wheneverAConstitutedPerdurantExistsTheConstitutedByRelationHolds_a62
    , axiom, (
  ![P1,P2,W1]: ((constitutedBy(P1,P2,W1) & perdurant(P1)) => (![W2
    ]: (exists(P1,W2) => constitutedBy(P1,P2,W2))))
)).

fof(ax_constitutedByIsAsymmetric_a63, axiom, (
  ![E1,E2,W]: (constitutedBy(E1,E2,W) => ~constitutedBy(E2,E1,W))
)).

% Constitution in use
% (tested to rule out trivial models)

% fof(ax_constitutionInUse, axiom, (
%   object(e12_1) & object(e12_2) & objectKind(k12_1) & objectKind(
    k12_2) & world(w12) & ~(k12_1=k12_2) & iof(e12_1,k12_1,w12) &
    iof(e12_2,k12_2,w12) & constitutedBy(e12_1,e12_2,w12) &
    genericConstitutionalDependence(k12_1,k12_2) & constitution(
    e12_1,k12_1,e12_2,k12_2,w12)
% )).

%%%%%%%%%%%%%%%%% Existential Dependence %%%%%%%%%%%%%%%%%

fof(ax_exists_a64, axiom, (
  ![X,W]: (exists(X,W) => (thing(X) & world(W)))
```

2.4.10 Existential Dependence

```

fof(ax_existentiallyDependsOn_a65, axiom, (
  ![X,Y]: (existentiallyDependsOn(X,Y) <=> (![W]: (exists(X,W) =>
    exists(Y,W))))
)).

fof(ax_existentiallyIndependentOf_a66, axiom, (
  ![X,Y]: (existentiallyIndependentOf(X,Y) <=> (~
    existentiallyDependsOn(X,Y) & ~existentiallyDependsOn(Y,X)))
)).

% Existential dependence in use
% (tested to rule out trivial models)

% fof(ax_constitutionInUse, axiom, (
%   object(e13_1) & object(e13_2) & object(e13_3) & ~(e13_1=e13_2)
    & ~(e13_1=e13_3) & ~(e13_2=e13_3) & existentiallyDependsOn(
    e13_2,e13_1) & existentiallyIndependentOf(e13_3,e13_1)
% )).

% TODO: introduce transitivity and anti-symmetry of existential
    dependence
% TODO: introduce continuity of existence with perdurants never
    ceasing to exist
```

```

%%%%%%%%%%%%%%%%%% Moments and Inherence %%%%%%%%%%%%%%%%%

%  Inherence
```

2.4.11 Moments and Inherence

```
  ![M,X]: (inheresIn(M,X) => existentiallyDependsOn(M,X))
)).

fof(ax_thingsInvolvedInInherence_a68, axiom, (
  ![M,X]: (inheresIn(M,X) => (moment(M) & (type_(X) | endurant(X))))
)).

% TODO: add definition (d5) for the "bearer" axiom

fof(ax_irreflexiveInherence, axiom, (
  ![X]: (~inheresIn(X,X))
)).

fof(ax_asymmetricInherence, axiom, (
  ![X,Y]: (inheresIn(X,Y) => ~inheresIn(Y,X))
)).

fof(ax_intransitiveInherence, axiom, (
  ![X,Y,Z]: ((inheresIn(X,Y) & inheresIn(Y,Z)) => ~inheresIn(X,Z))
)).

fof(ax_uniqueInherence_a69, axiom, (
  ![X,Y,Z]: ((inheresIn(X,Y) & inheresIn(X,Z)) => (Y=Z))
)).

% Moments

fof(ax_dMomentOf_d6, axiom, (
  ![M,X]: (momentOf(M,X) <=> (inheresIn(M,X) | (
    ?[M2]: (inheresIn(M,M2) & momentOf(M2,X))
  )))
)).

fof(ax_dUltimateBearerOf_d7, axiom, (
  ![B,M]: (ultimateBearerOf(B,M) <=> (~moment(B) & momentOf(M,B)))
)).

fof(ax_everyMomentHasUniqueAUltimateBearer_a70, axiom, (
  ![M]: (moment(M) => (?[B]: (ultimateBearerOf(B,M) & (
    ![B2]: (ultimateBearerOf(B2,M) <=> (B=B2))
  ))))
)).

fof(ax_noMomentOfCycles, axiom, (
  ~?[M]: momentOf(M,M)
)).

% Ax |= "th_irreflexiveInherence_t28"; conjecture commented for convenience

```

```
% fof(th_irreflexiveInherence_t28, conjecture, (
%   ~?[X]: (inheresIn(X,X))
% )).

% Ax |= "th_asymmetricInherence_t29"; conjecture commented for convenience

% fof(th_asymmetricInherence_t29, conjecture, (
%   ~?[X,Y]: (inheresIn(X,Y) & inheresIn(Y,X))
% )).

% Ax |= "th_antiTransitiveInherence_t30"; conjecture commented for convenience

% fof(th_antiTransitiveInherence_t30, conjecture, (
%   ![X,Y,Z]: ((inheresIn(X,Y) & inheresIn(Y,Z)) => (~inheresIn(X,Z)))
% )).

% TODO: add instances

%%%%%%%%%%%%%%%%%%%%%%%% Relators %%%%%%%%%%%%%%%%%%%%%%%%

% External Dependence and Externally Dependent Modes
```

2.4.12 Relators

```
  ~?[M,X]: (externallyDependsOn(M,X) <=> (existentiallyDependsOn(M,X) & (![Y]: (inheresIn(M,Y) => existentiallyIndependentOf(X,Y)))))
)).

fof(ax_dExternallyDependentMode_a72, axiom, (
  ![M]: (externallyDependentMode(M) <=> (mode(M) & (?[X]: (externallyDependsOn(M,X)))))
)).

% Founded by

fof(ax_foundedByInvolvedThings_a73, axiom, (
  ![M,P]: (foundedBy(M,P) <=> ((externallyDependentMode(M) | relator(M)) & perdurant(P)))
)).

fof(ax_relationalModesHaveAFoundationEvent_a74, axiom, (
  ![M]: ((externallyDependentMode(M) | relator(M)) => (?[P]: (foundedBy(M,P))))
)).

fof(ax_uniqueFoundationEvents_a74, axiom, (
  ![M,P1,P2]: ((foundedBy(M,P1) & foundedBy(M,P2)) => (P1=P2))
)).

% TODO: add definition (d8) for the "foundationOf" axiom

% Qua Individual

fof(ax_dQuaIndividualOf_a75, axiom, (
```

```
  ![X,Y]: (quaIndividualOf(X,Y) <=> (![Z]: (overlap(Z,X) <=> (
    externallyDependentMode(Z) & inheresIn(Z,Y) & (![P]: (foundedBy
    (X,P) => foundedBy(Z,P)))
  ))))
)).

% Ax |= "
    th_thePartsOfAQuaIndividualShareTheFoundationOfTheWhole_t31";
    conjecture commented for convenience

% fof(th_thePartsOfAQuaIndividualShareTheFoundationOfTheWhole_t31,
    conjecture, (
%   ![X,Y,Z]: ((quaIndividual(X) & partOf(Z,X)) => (![P]: (
    foundedBy(Z,P) => foundedBy(X,P))))
% )).

fof(ax_dQuaIndividual_a76, axiom, (
  ![X]: (quaIndividual(X) <=> ?[Y]: (quaIndividualOf(X,Y)))
)).

% Qua Individual is already defined as a subtype of Externally
    Dependent Mode in the taxonomy; skipping (a78)

% Skipping (a79); already defined in (a74)

fof(ax_thePartsOfARelatorShareTheFoundationOfTheWhole_a80, axiom, (
  ![X,Y,Z]: ((relator(X) & partOf(Z,X)) => (![P]: (foundedBy(Z,P)
    => foundedBy(X,P))))
)).

fof(ax_dRelator_a81, axiom, (
  ![R]: (relator(R) <=> (
    (?[X]: (properPartOf(X,R))
    & (![Y,Z]: ((properPartOf(Y,R) & properPartOf(Z,R)) => (
    quaIndividual(Y) & quaIndividual(Z) & existentiallyDependsOn(Y,
    Z) & existentiallyDependsOn(Z,Y) & (![P]: (foundedBy(Y,P) <=>
    foundedBy(Z,P))))))
    & (![Y2,Z2]: ((properPartOf(Y2,R) & quaIndividual(Z2) &
    existentiallyDependsOn(Y2,Z2) & existentiallyDependsOn(Z2,Y2) &
     (![P2]: (foundedBy(Y2,P2) <=> foundedBy(Z2,P2)))) => (
    properPartOf(Z2,R))))
  )))
)).

% Ax |= "th_relatorsImplyTheExistenceOfAtLeastTwoQuaIndividuals_t32
    "; conjecture commented for convenience

% fof(th_relatorsImplyTheExistenceOfAtLeastTwoQuaIndividuals_t32,
    conjecture, (
%   ![R]: (relator(R) => (?[Q1,X,Q2,Y]: (quaIndividualOf(Q1,X) &
    quaIndividualOf(Q2,Y) & ~(Q1=Q2))))
% )).

fof(ax_dMediates_a82, axiom, (
  ![R,E]: (mediates(R,E) <=> (relator(R) & endurant(E) & (?[Q]: (
    quaIndividualOf(Q,E) & partOf(Q,R)))))
)).
```

```
% Ax |= "th_relatorsMediateAtLeastTwoThings_t33"; conjecture commented for convenience

% fof(th_relatorsMediateAtLeastTwoThings_t33, conjecture, (
%   ![R]: (relator(R) => (?[E1,E2]: (~(E1=E2) & mediates(R,E1) & mediates(R,E2))))
% )).

% TODO: add definition (d9) for the "relator bearer" axiom

% TODO: add instances

%%%%%%%%%%%%%%%%%%% Characterization %%%%%%%%%%%%%%%%%%%

fof(ax_endurantTypeCharacterizationByMomentTypes_a83, axiom, (
  ![ET,MT]: (characterizes(MT,ET) => (
```

2.4.13 Characterization

```
    & momentType(M)
    & (![E,W]: (iof(E,ET,W) => (?[M]: (iof(M,MT,W) & inheresIn(M,E)))))
    & (![M2,W2]: (iof(M2,MT,W2) => (?[E2]: (iof(E2,ET,W2) & inheresIn(M2,E2)))))
  ))
)).

% Ax |= "th_qualitiesInheresInAUniqueEndurantConnectThroughCharacteization_a84"; conjecture commented for convenience

% fof(th_qualitiesInheresInAUniqueEndurantConnectThroughCharacteization_a84, conjecture, (
%   ![QT,ET]: ((characterizes(QT,ET) & qualityType(QT)) => (![Q,W]: (iof(Q,QT,W) => (?[E]: (iof(E,ET,W) & inheresIn(Q,E) & (![E2]: (inheresIn(Q,E2) <=> (E=E2)))))))))
% )).

% TODO: add instances

%%%%%%%%%%% Qualities and Quality Structures %%%%%%%%%%%

% Skipping (a85); previously introduced in the taxonomy
% Skipping (a86); previously introduced in the taxonomy
```

2.4.14 Qualities and Quality Structures

```

% Quality Structures

fof(ax_dQualityStructure_d10, axiom, (
  ![QS]: (qualityStructure(QS) <=> (?[QT]: (qualityType(QT) & associatedWith(QS,QT))))
)).

```

```
fof(ax_dQualityStructure_d10, axiom, (
  ![QS]: (qualityStructure(QS) <=> (?[QT]: (qualityType(QT) & associatedWith(QS,QT))))
)).

%%%%%%%%%%%%%%%% Endurants and Perdurants %%%%%%%%%%%%%%%%

fof(ax_manifestsInvolvedThings_a104, axiom, (
  ![E,P]: (manifests(E,P) => (endurant(E) & perdurant(P)))
)).

fof(ax_lifeOfInvolvedThings_a105, axiom, (
  ![E,P]: (lifeOf(P,E) => (
```

2.4.15 Endurants and Perdurants

```
    & (![P2]: (overlap(P2,P) <=> (perdurant(P2) & manifests(E,P2))))
  ))
)).

% TODO: review ax_lifeOfInvolvedThings_a105 and its translation of the small sigma predicate schema in (a105)

fof(ax_meetsInvolvedThings_a106, axiom, (
  ![P1,P2]: (meets(P1,P2) => (perdurant(P1) & perdurant(P2)))
)).

% TODO: add instances
```