A First-Order Logic Formalization of the Unified Foundational Ontology

Daniele Porelo, Claudenir M. Fonseca, João Paulo A. Almeida,
Giancarlo Guizzardi, Tiago Prince Sales

December 3, 2021

Abstract

This document presents a formalization of the Unified Foundational Ontology (UFO) in first-order logic. This formalization is documented by means of three complementary representations: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation. The presented formalization is supported by consistency and satisfiability checks performed through automated proofing tools.

1 Introduction

This document presents a formalization of the Unified Foundational Ontology (UFO) in first-order logic. This formalization is documented by means of three complementary representations: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation. The presented formalization is supported by consistency and satisfiability checks performed through automated proofing tools.

The remainder of this document is organized as a single formalization section (Section 2), which contains subsections for each submodule of the ontology.

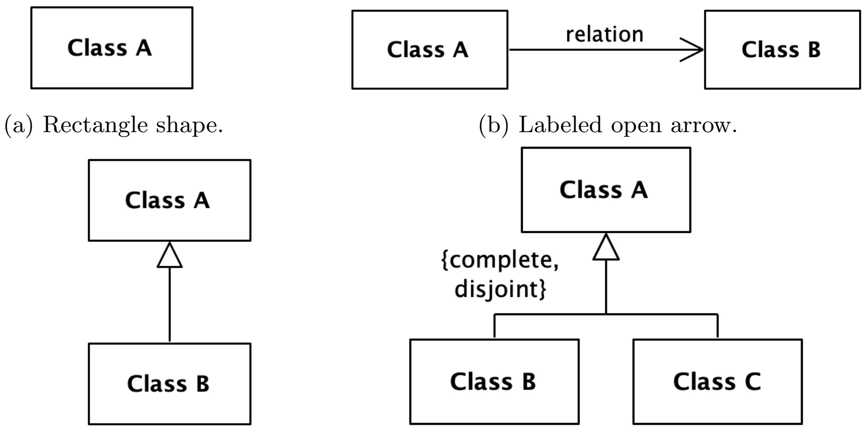
2 Formalization

This section contains the formalization of the Unified Foundational Ontology (UFO) in first-order logics. This formalization is organized in several subsections where each presents the formalization of a portion of the whole ontology. The formalization is presented through different equivalent representations, designed to support the understanding of its contents: (i) a representation in standard Common Logic using the CLIF syntax; (ii) a representation in natural language; and, when applicable, (iii) a UML-based diagrammatic representation.

The UML-based diagrammatic representation serves as a visual representation certain predicates and axioms, being each element in Figure 1 being translated as follows:

- Rectangle shape (Figure 1a): visual representation of unary predicates associated to types in the ontology; the associated predicate is shown in lower camel case with no spaces.

  $classA(x)$



(a) Rectangle shape.

(b) Labeled open arrow.

(c) Closed arrow.

(d) Labeled closed arrows.

Figure 1: UML-based representation of first-order logic axioms.

- Open arrow (Figure 1b): visual representation of binary predicates; the predicate associated to the arrows' label is shown in lower camel case with no spaces; the predicate can only be true for any x and y if it is also true predicates associated to the types of each end (keeping the order of the arrow in the binary predicate's positions); this representation may also be associated to ternary predicates iff its third position represents a time-index.

  $\forall x, y(relation(x, y) \rightarrow (classA(x) \wedge classB(y)))$

  $\forall x, y, w(relation(x, y, w) \rightarrow (classA(x) \wedge classB(y) \wedge world(w)))$

- Closed arrow (Figure 1c): visual representation of specializations between ontology's types, where the type in the tail of the arrow is a subtype of the type in the head of the arrow.

  $\forall x(classB(x) \rightarrow classA(x))$

- Labeled closed arrow (Figure 1d): visual representation of disjoint and/or complete constraints over sets specializations between ontology's types.

  $\forall x(classB(x) \rightarrow classA(x))$
  $\forall x(classC(x) \rightarrow classA(x))$
  $\forall x(classA(x) \rightarrow (classB(x) \vee classC(x)))$ {complete}
  $\neg\exists x(classB(x) \wedge classC(x))$ {disjoint}

2.1 Partial Taxonomy of UFO: Thing

This subsection presents most general types of UFO's taxonomy specializing the type Thing (Figure 2).

a1 For every x, x is a Thing iff x is either a Type or an Individual.

$\forall x(\mathsf{type\_}(x) \vee \mathsf{individual}(x) \leftrightarrow \mathsf{thing}(x))$

```
(cl-text  ax_thing_taxonomy
(forall (x)
  (iff (or (type_ x) (individual x))
```

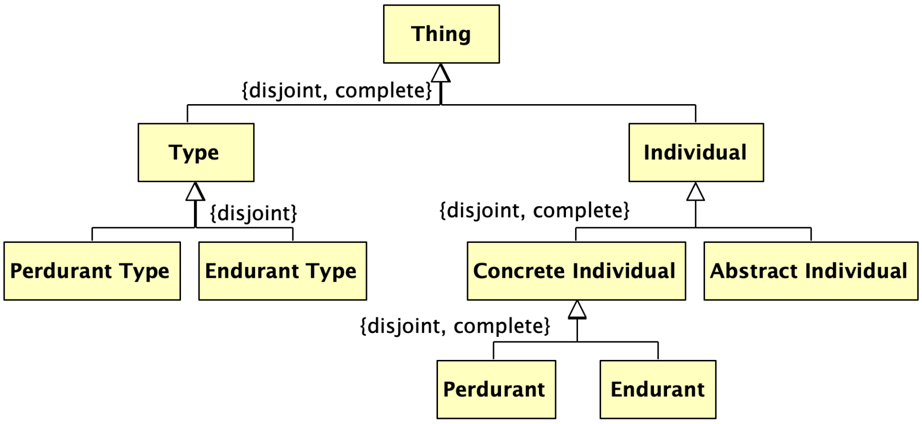


Figure 2: Visual representation of UFO's taxonomy of Thing.

```
    (thing x))
  )
)
```

a2 There is no x such that it is a Type and an Individual.

$\neg\exists x(\mathsf{type\_}(x) \wedge \mathsf{individual}(x))$

```
(cl-text  ax_thing_partition
(not (exists (x)
  (and (type_ x) (individual x)))
)
)
```

a3 For every x, x is an Individual iff x is either a Concrete Individual or an Abstract Individual.

$\forall x(\mathsf{concreteIndividual}(x) \vee \mathsf{abstractIndividual}(x) \leftrightarrow \mathsf{individual}(x))$

```
(cl-text  ax_individual_taxonomy
(forall (x)
  (iff (or (concreteIndividual x) (abstractIndividual x))
    (individual x))
  )
)
```

a4 There is no x such that it is a Concrete Individual and an Abstract Individual.

$\neg\exists x(\mathsf{concreteIndividual}(x) \wedge \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_individual_partition
(not (exists (x)
  (and (concreteIndividual x) (abstractIndividual x)))
)
)
```

a5 For every x, x is a Concrete Individual iff x is either a Perdurant or an Endurant.

$\forall x(\mathsf{endurant}(x) \vee \mathsf{perdurant}(x) \leftrightarrow \mathsf{concreteIndividual}(x))$

```
(cl-text  ax_concreteIndividual_taxonomy
(forall (x)
  (iff (or (endurant x) (perdurant x))
    (concreteIndividual x))
  )
)
```

a6 There is no x such that it is a Perdurant and an Endurant.

$\neg\exists x(\mathsf{endurant}(x) \wedge \mathsf{perdurant}(x))$

```
(cl-text  ax_concreteIndividual_partition
(not (exists (x)
  (and (endurant x) (perdurant x)))
)
)
```

a7 For every x, x is a Concrete Individual ifif x is either a Perdurant or an Endurant.

$\forall x(\mathsf{endurantType}(x) \vee \mathsf{perdurantType}(x) \rightarrow \mathsf{type\_}(x))$

```
(cl-text  ax_type_taxonomy
(forall (x)
  (if (or (endurantType x) (perdurantType x))
    (type_ x))
  )
)
```

a8 There is no x such that it is a Perdurant Type and an Endurant Type.

$\neg\exists x(\mathsf{endurantType}(x) \wedge \mathsf{perdurantType}(x))$

```
(cl-text  ax_type_partition
(not (exists (x)
  (and (endurantType x) (perdurantType x)))
)
)
```

2.2 Partial Taxonomy of UFO: **Abstract Individual**

This subsection presents a portion of UFO's taxonomy specializing the type Abstract Individual (Figure 3).

a9 Every x that is a Quale is also an Abstract Individual.

$\forall x(\mathsf{quale}(x) \rightarrow \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_abstractIndividual_taxonomy_quale
(forall (x)
  (if (quale x)
    (abstractIndividual x))
  )
)
```

a10 Every x that is a Set is also an Abstract Individual.

$\forall x(\mathsf{set\_}(x) \rightarrow \mathsf{abstractIndividual}(x))$

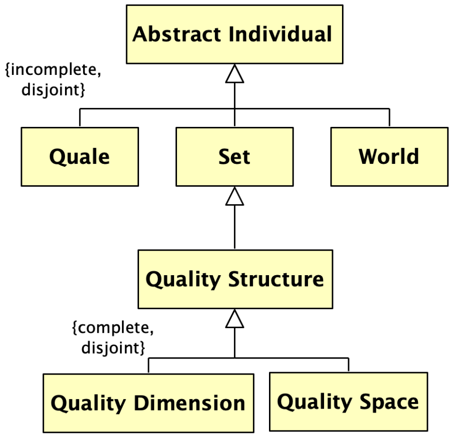


Figure 3: Visual representation of UFO's taxonomy of Abstract Individual.

```
(cl-text  ax_abstractIndividual_taxonomy_set
(forall (x)
  (if (set_ x)
    (abstractIndividual x))
  )
)
```

a11 Every x that is a World is also an Abstract Individual.

$\forall x(\mathsf{world}(x) \rightarrow \mathsf{abstractIndividual}(x))$

```
(cl-text  ax_abstractIndividual_taxonomy_world
(forall (x)
  (if (world x)
    (abstractIndividual x))
  )
)
```

a12 There is no x such that it is a Quale, a Set, and a World (pairwise disjoint).

$\neg\exists x((\mathsf{quale}(x) \wedge \mathsf{set\_}(x)) \vee (\mathsf{quale}(x) \wedge \mathsf{world}(x)) \vee (\mathsf{set\_}(x) \wedge \mathsf{world}(x)))$

```
(cl-text  ax_abstractIndividual_pairwiseDisjoint
(not (exists (x)
  (or (and (quale x) (set_ x)) (and (quale x) (world x)) (and (set_ x) (world x)
    )))
)
)
```

a13 Every x that is a Quality Structure is also a Set.

$\forall x(\mathsf{qualityStructure}(x) \rightarrow \mathsf{set\_}(x))$

```
(cl-text  ax_set_taxonomy_qualityStructure
(forall (x)
  (if (qualityStructure x)
    (set_ x))
  )
)
```

a14 For every x, x is a Quality Structure iff x is either a Quality Dimension or a Quality Space.

$\forall x(\mathsf{qualityDimension}(x) \lor \mathsf{qualitySpace}(x) \leftrightarrow \mathsf{qualityStructure}(x))$

```
(cl-text  ax_qualityStructure_taxonomy
(forall (x)
  (iff (or (qualityDimension x) (qualitySpace x))
    (qualityStructure x))
  )
)
```

a15 There is no x such that it is a Quality Dimension and a Quality Space.

$\neg\exists x(\mathsf{qualityDimension}(x) \land \mathsf{qualitySpace}(x))$

```
(cl-text  ax_qualityStructure_partition
(not (exists (x)
  (and (qualityDimension x) (qualitySpace x)))
)
)
```

2.3 Partial Taxonomy of UFO: Endurant

This subsection presents a portion of UFO's taxonomy specializing the type Endurant (Figure 4).

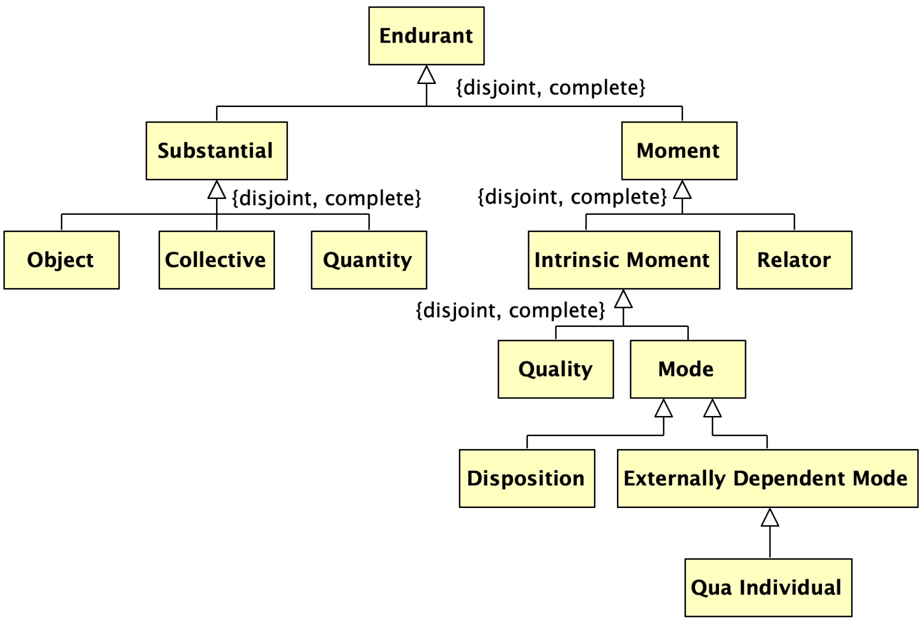


Figure 4: Visual representation of UFO's taxonomy of Endurant.

a16 For every x, x is an Endurant iff x is either a Substantial or a Moment.

$\forall x(\mathsf{substantial}(x) \lor \mathsf{moment}(x) \leftrightarrow \mathsf{endurant}(x))$

```
(cl-text  ax_endurant_taxonomy
(forall (x)
  (iff (or (substantial x) (moment x))
    (endurant x))
  )
)
```

a17 There is no x such that it is a Substantial and a Moment.

$\neg\exists x(\mathsf{substantial}(x) \wedge \mathsf{moment}(x))$

```
(cl-text  ax_endurant_partition
(not (exists (x)
  (and (substantial x) (moment x)))
)
)
```

a18 For every x, x is a Substantial iff x is either an Object, a Collective, or a Quantity.

$\forall x(\mathsf{object}(x) \vee \mathsf{collective}(x) \vee \mathsf{quantity}(x) \leftrightarrow \mathsf{substantial}(x))$

```
(cl-text  ax_substantial_taxonomy
(forall (x)
  (iff (or (object x) (collective x) (quantity x))
    (substantial x))
  )
)
```

a19 There is no x such that it is an Object, a Collective, and a Quantity (pairwise disjoint).

$\neg\exists x((\mathsf{object}(x) \wedge \mathsf{collective}(x)) \vee (\mathsf{object}(x) \wedge \mathsf{quantity}(x)) \vee (\mathsf{collective}(x) \wedge \mathsf{quantity}(x)))$

```
(cl-text  ax_substantial_partition
(not (exists (x)
  (or (and (object x) (collective x)) (and (object x) (quantity x)) (and (
    collective x) (quantity x))))
)
)
```

a20 For every x, x is a Moment iff x is either an Intrinsic Moment or a Relator.

$\forall x(\mathsf{intrinsicMoment}(x) \vee \mathsf{relator}(x) \leftrightarrow \mathsf{moment}(x))$

```
(cl-text  ax_moment_taxonomy
(forall (x)
  (iff (or (intrinsicMoment x) (relator x))
    (moment x))
  )
)
```

a21 There is no x such that it is an Intrinsic Moment and a Relator.

$\neg\exists x(\mathsf{intrinsicMoment}(x) \wedge \mathsf{relator}(x))$

```
(cl-text  ax_moment_partition
(not (exists (x)
  (and (intrinsicMoment x) (relator x)))
)
)
```

a22 For every x, x is an Intrinsic Moment iiff x is either a Quality or a Mode.

$\forall x(\mathsf{quality}(x) \vee \mathsf{mode}(x) \leftrightarrow \mathsf{intrinsicMoment}(x))$

```
(cl-text  ax_intrinsicMoment_taxonomy
(forall (x)
  (iff (or (quality x) (mode x))
    (intrinsicMoment x))
  )
)
```

a23 There is no x such that it is an Intrinsic Moment and a Relator.

$\neg\exists x(\mathsf{quality}(x) \wedge \mathsf{mode}(x))$

```
(cl-text  ax_intrinsicMoment_partition
(not (exists (x)
  (and (quality x) (mode x)))
)
)
```

a24 Every x that is a Disposition is also a Mode.

$\forall x(\mathsf{disposition}(x) \rightarrow \mathsf{mode}(x))$

```
(cl-text  ax_mode_taxonomy_disposition
(forall (x)
  (if (disposition x)
    (mode x))
  )
)
```

a25 Every x that is an Externally Dependent Mode is also a Mode.

$\forall x(\mathsf{externallyDependentMode}(x) \rightarrow \mathsf{mode}(x))$

```
(cl-text  ax_mode_taxonomy_externallyDependentMode
(forall (x)
  (if (externallyDependentMode x)
    (mode x))
  )
)
```

a26 Every x that is an Qua Individual is also an Externally Dependent Mode.

$\forall x(\mathsf{quaIndividual}(x) \rightarrow \mathsf{externallyDependentMode}(x))$

```
(cl-text  ax_externallyDependentMode_taxonomy_quaIndividual
(forall (x)
  (if (quaIndividual x)
    (externallyDependentMode x))
  )
)
```

2.4 Partial Taxonomy of UFO: Endurant Type by Ontological Natures

This subsection presents a portion of UFO's taxonomy specializing the type Endurant Type classified by ontological natures (Figure 5).

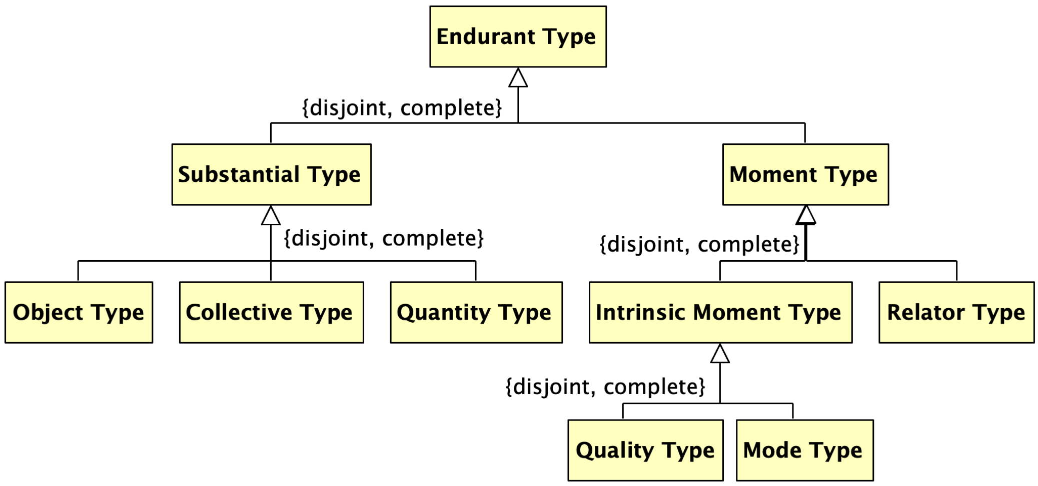


Figure 5: Visual representation of UFO's taxonomy of Endurant Type classified by ontological natures.

a27 For every x, x is an Endurant Type iff x is either a Substantial Type or a Moment Type.

$\forall x(\mathsf{substantialType}(x) \lor \mathsf{momentType}(x) \leftrightarrow \mathsf{endurantType}(x))$

```
(cl-text  ax_endurantType_taxonomy_nature
(forall (x)
  (iff (or (substantialType x) (momentType x))
    (endurantType x))
  )
)
```

a28 There is no x such that it is a Substantial Type and a Moment Type.

$\neg\exists x(\mathsf{substantialType}(x) \land \mathsf{momentType}(x))$

```
(cl-text  ax_endurantType_partition_nature
(not (exists (x)
  (and (substantialType x) (momentType x)))
)
)
```

a29 For every x, x is a Substantial Type iff x is either an Object Type, a Collective Type, or a Quantity Type.

$\forall x(\mathsf{objectType}(x) \lor \mathsf{collectiveType}(x) \lor \mathsf{quantityType}(x) \leftrightarrow \mathsf{substantialType}(x))$

```
(cl-text  ax_substantialType_taxonomy
(forall (x)
  (iff (or (objectType x) (collectiveType x) (quantityType x))
    (substantialType x))
  )
)
```

a30 There is no x such that it is an Object Type, a Collective Type, and a Quantity Type (pairwise disjoint).

$\neg\exists x((\mathsf{objectType}(x) \wedge \mathsf{collectiveType}(x)) \vee (\mathsf{objectType}(x) \wedge \mathsf{quantityType}(x)) \vee (\mathsf{collectiveType}(x) \wedge \mathsf{quantityType}(x)))$

```
(cl-text  ax_substantialType_partition
(not (exists (x)
  (or (and (objectType x) (collectiveType x)) (and (objectType x) (quantityType
    x)) (and (collectiveType x) (quantityType x))))
)
)
```

a31 For every x, x is a Moment Type ifif x is either an Intrinsic Moment Type or a Relator Type.

$\forall x(\mathsf{intrinsicMomentType}(x) \vee \mathsf{relatorType}(x) \leftrightarrow \mathsf{momentType}(x))$

```
(cl-text  ax_momentType_taxonomy
(forall (x)
  (iff (or (intrinsicMomentType x) (relatorType x))
    (momentType x))
  )
)
```

a32 There is no x such that it is an Intrinsic Moment Type and a Relator Type.

$\neg\exists x(\mathsf{intrinsicMomentType}(x) \wedge \mathsf{relatorType}(x))$

```
(cl-text  ax_momentType_partition
(not (exists (x)
  (and (intrinsicMomentType x) (relatorType x)))
)
)
```

a33 For every x, x is an Intrinsic Moment Type ifif x is either a Quality Type or a Mode Type.

$\forall x(\mathsf{qualityType}(x) \vee \mathsf{modeType}(x) \leftrightarrow \mathsf{intrinsicMomentType}(x))$

```
(cl-text  ax_intrinsicMomentType_taxonomy
(forall (x)
  (iff (or (qualityType x) (modeType x))
    (intrinsicMomentType x))
  )
)
```

a34 There is no x such that it is an Intrinsic Moment Type and a Relator Type.

$\neg\exists x(\mathsf{qualityType}(x) \wedge \mathsf{modeType}(x))$

```
(cl-text  ax_intrinsicMomentType_partition
(not (exists (x)
  (and (qualityType x) (modeType x)))
)
)
```

2.5 Partial Taxonomy of UFO: **Endurant Type** by Modal Properties of Types

This subsection presents a portion of UFO's taxonomy specializing the type Endurant Type classified by the modal properties of types (Figure 6).

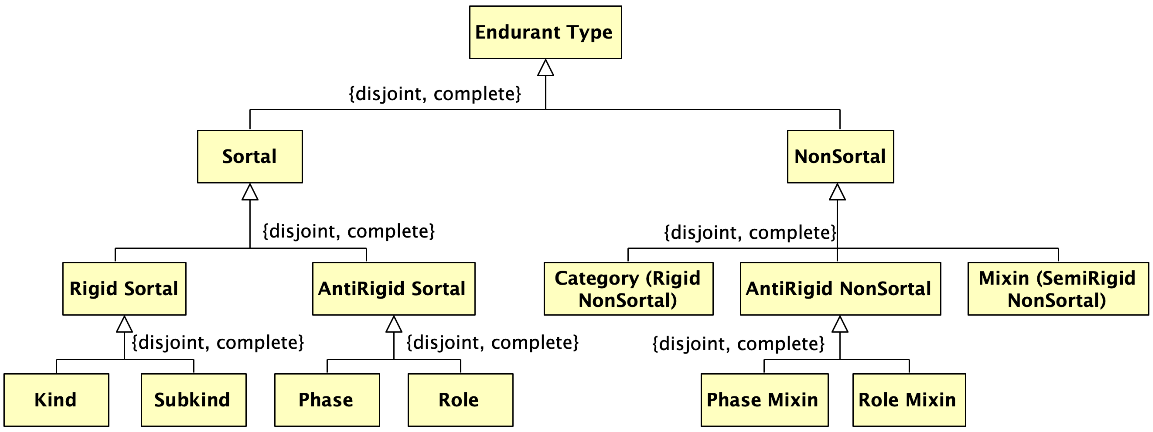


Figure 6: Visual representation of UFO's taxonomy of Endurant Type classified by the modal properties of types.

a35 For every x, x is an Endurant Type iiff x is either a Sortal or a NonSortal.

$\forall x(\mathsf{sortal}(x) \vee \mathsf{nonSortal}(x) \leftrightarrow \mathsf{endurantType}(x))$

```
(cl-text  ax_endurantType_taxonomy_properties
(forall (x)
  (iff (or (sortal x) (nonSortal x))
    (endurantType x))
  )
)
```

a36 There is no x such that it is a Sortal and a NonSortal.

$\neg\exists x(\mathsf{sortal}(x) \wedge \mathsf{nonSortal}(x))$

```
(cl-text  ax_endurantType_partition_properties
(not (exists (x)
  (and (sortal x) (nonSortal x)))
)
)
```

a37 For every x, x is a Sortal iiff x is either a Rigid Sortal or an AntiRigid Sortal.

$\forall x(\mathsf{rigidSortal}(x) \vee \mathsf{antiRigidSortal}(x) \leftrightarrow \mathsf{sortal}(x))$

```
(cl-text  ax_sortal_taxonomy
(forall (x)
  (iff (or (rigidSortal x) (antiRigidSortal x))
    (sortal x))
  )
)
```

a38 There is no x such that it is a Rigid Sortal and an AntiRigid Sortal.

$\neg\exists x(\mathsf{rigidSortal}(x) \wedge \mathsf{antiRigidSortal}(x))$

```
(cl-text  ax_sortal_partition
(not (exists (x)
  (and (rigidSortal x) (antiRigidSortal x)))
```

```
)
)
```

a39 For every x, x is a Rigid Sortal ifif x is either a Kind or a Subkind.

$\forall x(\mathsf{kind}(x) \vee \mathsf{subkind}(x) \leftrightarrow \mathsf{rigidSortal}(x))$

```
(cl-text  ax_rigidSortal_taxonomy
(forall (x)
  (iff (or (kind x) (subkind x))
    (rigidSortal x))
  )
)
```

a40 There is no x such that it is a Kind and a Subkind.

$\neg\exists x(\mathsf{kind}(x) \wedge \mathsf{subkind}(x))$

```
(cl-text  ax_rigidSortal_partition
(not (exists (x)
  (and (kind x) (subkind x)))
)
)
```

a41 For every x, x is an AntiRigid Sortal ifif x is either a Phase or a Role.

$\forall x(\mathsf{phase}(x) \vee \mathsf{role}(x) \leftrightarrow \mathsf{antiRigidSortal}(x))$

```
(cl-text  ax_antiRigidSortal_taxonomy
(forall (x)
  (iff (or (phase x) (role x))
    (antiRigidSortal x))
  )
)
```

a42 There is no x such that it is a Phase and a Role.

$\neg\exists x(\mathsf{phase}(x) \wedge \mathsf{role}(x))$

```
(cl-text  ax_antiRigidSortal_partition
(not (exists (x)
  (and (phase x) (role x)))
)
)
```

a43 For every x, x is a NonSortal ifif x is either a Rigid NonSortal, an AntiRigid NonSortal, or a SemmiRigid NonSortal.

$\forall x(\mathsf{rigidNonSortal}(x) \vee \mathsf{semiRigidNonSortal}(x) \vee \mathsf{antiRigidNonSortal}(x) \leftrightarrow \mathsf{nonSortal}(x))$

```
(cl-text  ax_nonSortal_taxonomy
(forall (x)
  (iff (or (rigidNonSortal x) (semiRigidNonSortal x) (antiRigidNonSortal x))
    (nonSortal x))
  )
)
```

a44 There is no x such that it is an Rigid NonSortal, an AntiRigid NonSortal, and a SemiRigid NonSortal (pairwise disjoint).

$\neg\exists x((\mathsf{rigidNonSortal}(x) \land \mathsf{semiRigidNonSortal}(x)) \lor (\mathsf{rigidNonSortal}(x) \land \mathsf{antiRigidNonSortal}(x)) \lor (\mathsf{semiRigidNonSortal}(x) \land \mathsf{antiRigidNonSortal}(x)))$

```
(cl-text  ax_nonSortal_partition
(not (exists (x)
  (or (and (rigidNonSortal x) (semiRigidNonSortal x)) (and (rigidNonSortal x) (
    antiRigidNonSortal x)) (and (semiRigidNonSortal x) (antiRigidNonSortal x))))
)
)
```

a45 Every x that is a Rigid NonSortal is also a Category.

$\forall x(\mathsf{rigidNonSortal}(x) \leftrightarrow \mathsf{category}(x))$

```
(cl-text  ax_rigidNonSortal_taxonomy
(forall (x)
  (iff (rigidNonSortal x)
    (category x))
  )
)
```

a46 Every x that is a SemiRigid NonSortal is also a Mixin.

$\forall x(\mathsf{semiRigidNonSortal}(x) \leftrightarrow \mathsf{mixin}(x))$

```
(cl-text  ax_semiRigidNonSortal_taxonomy
(forall (x)
  (iff (semiRigidNonSortal x)
    (mixin x))
  )
)
```

a47 For every x, x is an AntiRigid NonSortal ifif x is either a Phase Mixin or a Role Mixin.

$\forall x(\mathsf{phaseMixin}(x) \lor \mathsf{roleMixin}(x) \leftrightarrow \mathsf{antiRigidNonSortal}(x))$

```
(cl-text  ax_antiRigidNonSortal_taxonomy
(forall (x)
  (iff (or (phaseMixin x) (roleMixin x))
    (antiRigidNonSortal x))
  )
)
```

a48 There is no x such that it is a Phase Mixin and a Role Mixin.

$\neg\exists x(\mathsf{phaseMixin}(x) \land \mathsf{roleMixin}(x))$

```
(cl-text  ax_antiRigidNonSortal_partition
(not (exists (x)
  (and (phaseMixin x) (roleMixin x)))
)
)
```