A TPTP Formalization of the Unified Foundational Ontology

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October 12, 2021

Abstract

This document presents a formalization of the Unified Foundation Ontology (UFO) expressed in first-order logics through the TPTP syntax. This formalization is intended to support verification of UFO's theory through automated provers and consistency checkers.

1 Introduction

This document presents a formalization of the Unified Foundation Ontology (UFO) expressed in first-order logics through the TPTP syntax. This formalization is intended to support verification of UFO's theory through automated provers and consistency checkers.

2 UFO's TPTP Specification

2.1 UFO Taxonomy

2.1.1 Partial Taxonomy of Thing

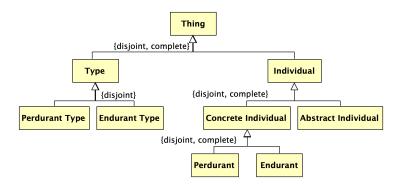


Figure 1: Partial Taxonomy of UFO - Thing.

```
18 )).
19
fof(ax_individual_partition, axiom, (
  ~?[X]: (concreteIndividual(X) & abstractIndividual(X))
22 )).
23
24 % Concrete Individual
25
fof(ax_concreteIndividual_taxonomy, axiom, (
    ![X]: ((endurant(X) | perdurant(X)) <=> (concreteIndividual(X)))
28 )).
29
30 fof(ax_concreteIndividual_partition, axiom, (
"?[X]: (endurant(X) & perdurant(X))
32 )).
33
34 % Type
fof(ax_type_taxonomy, axiom, (
   ![X]: ((endurantType(X) | perdurantType(X)) => (type_(X)))
37
38 )).
39
40 fof(ax_type_partition, axiom, (
  ~?[X]: (endurantType(X) & perdurantType(X))
41
42 )).
43
44 % Thing partial taxonomy instances
45 % (tested to rule out trivial models)
47 % fof(ax_thing_instances, axiom, (
      type_(type1) & individual(individual1) & concreteIndividual(
      concreteIndividual1) & abstractIndividual(abstractIndividual1)
      & endurant(endurant1) & perdurant(perdurant1) & endurantType(
       endurantType1) & perdurantType(perdurantType1)
49 % )).
```

2.1.2 Partial Taxonomy of Abstract Individual

```
51 % Abstract Individual 52
```

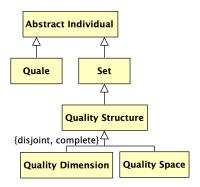


Figure 2: Partial Taxonomy of UFO – Abstract Individual.

```
53 fof(ax_abstractIndividual_taxonomy_quale, axiom, (
![X]: (quale(X) => (abstractIndividual(X)))
55 )).
fof(ax_abstractIndividual_taxonomy_set, axiom, (
![X]: (set_(X) => (abstractIndividual(X)))
59 )).
60
_{\rm 61} fof(ax_abstractIndividual_taxonomy_world, axiom, (
  ![X]: (world(X) => (abstractIndividual(X)))
64
65 % Set
fof(ax_set_taxonomy_qualityStructure, axiom, (
![X]: (qualityStructure(X) => (set_(X)))
69 )).
70
_{71} % Quality Structure
73 fof(ax_qualityStructure_taxonomy, axiom, (
    ![X]: ((qualityDimension(X) | qualitySpace(X)) <=> (
      qualityStructure(X)))
75 )).
fof(ax_qualityStructure_partition, axiom, (
  ~?[X]: (qualityDimension(X) & qualitySpace(X))
78
79 )).
80
81 % Abstract Individual partial taxonomy instances
82 % (tested to rule out trivial models)
84 % fof(ax_abstractIndividual_instances, axiom, (
      set_(set1) & quale(quale1) & qualityStructure(qualityStructure1
      ) & qualityDimension(qualityDimension1) & qualitySpace(
      qualitySpace1) & world(world1)
86 % )).
```

2.1.3 Partial Taxonomy of Endurant

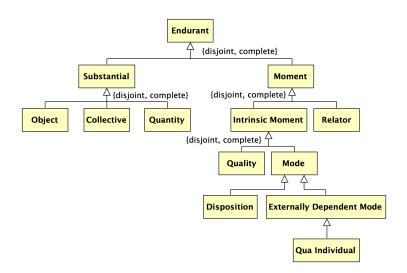


Figure 3: Partial Taxonomy of UFO - Endurant.

```
88 % Endurant
89
90 fof(ax_endurant_taxonomy, axiom, (
   ![X]: ((substantial(X) | moment(X)) <=> (endurant(X)))
91
93
_{\rm 94} fof(ax_endurant_partition, axiom, (
   "?[X]: (substantial(X) & moment(X))
95
96 )).
97
98 % Substantial
99
fof(ax_substantial_taxonomy, axiom, (
    ![X]: ((object(X) | collective(X) | quantity(X)) <=> (substantial
       (X)))
102 )).
103
fof(ax_substantial_partition, axiom, (
     ~?[X]: ((object(X) & collective(X)) | (object(X) & quantity(X)) |
105
        (collective(X) & quantity(X)))
106 )).
107
108 % Moment
109
110 fof(ax_moment_taxonomy, axiom, (
    ![X]: ((intrinsicMoment(X) | relator(X)) <=> (moment(X)))
111
112 )).
113
fof(ax_moment_partition, axiom, (
"?[X]: (intrinsicMoment(X) & relator(X))
116 )).
117
118 % Intrinsic Moment
```

```
119
120 fof(ax_intrinsicMoment_taxonomy, axiom, (
    ![X]: ((quality(X) | mode(X)) <=> (intrinsicMoment(X)))
121
122 )).
123
   fof(ax_intrinsicMoment_partition, axiom, (
124
     \tilde{R} [X]: (quality(X) & mode(X))
126
127
128 % Mode
129
   \verb| fof(ax_mode_taxonomy_externallyDependentMode, axiom, (\\
130
     ![X]: (externallyDependentMode(X) => (mode(X)))
131
132
  )).
   % Externally Dependent Mode
134
   fof(ax_externallyDependentMode_taxonomy_quaIndividual, axiom, (
136
     ![X]: (quaIndividual(X) => (externallyDependentMode(X)))
138 )).
139
   % Endurant partial taxonomy instances
140
   % (tested to rule out trivial models)
141
142
143 % fof(ax_endurant_instances, axiom, (
       substantial(substantial1) & moment(moment1) & object(object1) &
144
        collective(collective1) & quantity(quantity1) &
       intrinsicMoment(intrinsicMoment1) & relator(relator1) & quality
       (quality1) & mode(mode1) & disposition(disposition1) &
       externallyDependentMode(externallyDependentMode1) &
       quaIndividual (quaIndividual1)
145 % )).
```

2.1.4 Partial Taxonomy of Endurant Type (on ontological natures)

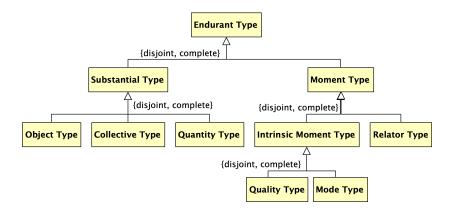


Figure 4: Partial Taxonomy of UFO – Endurant Types (by ontological nature).

```
147 % Endurant Type (by ontological nature)
148
```

```
149 fof(ax_endurantType_taxonomy_nature, axiom, (
     ![X]: ((substantialType(X) | momentType(X)) <=> (endurantType(X))
       )
  )).
fof(ax_endurantType_partition_nature, axiom, (
154
     ~?[X]: (substantialType(X) & momentType(X))
155 )).
157 % Substantial Type
158
159
  fof(ax_substantialType_taxonomy, axiom, (
     ![X]: ((objectType(X) | collectiveType(X) | quantityType(X)) <=>
       (substantialType(X)))
161 )).
162
163
   fof(ax_substantialType_partition, axiom, (
     ~?[X]: ((objectType(X) & collectiveType(X)) | (objectType(X) &
164
       quantityType(X)) | (collectiveType(X) & quantityType(X)))
165 )).
  % Moment Type
167
168
169 fof(ax_momentType_taxonomy, axiom, (
     ![X]: ((intrinsicMomentType(X) | relatorType(X)) <=> (momentType(
170
       X)))
171 )).
173 fof(ax_momentType_partition, axiom, (
     ~?[X]: (intrinsicMomentType(X) & relatorType(X))
174
175 )).
176
177 % Intrinsic Moment Type
178
fof(ax_intrinsicMomentType_taxonomy, axiom, (
    ![X]: ((qualityType(X) | modeType(X)) <=> (intrinsicMomentType(X)
      ))
181 )).
182
183 fof(ax_intrinsicMomentType_partition, axiom, (
    ~?[X]: (qualityType(X) & modeType(X))
184
185 )).
186
187 % Endurant Type (by ontological nature) partial taxonomy instances
  % (tested to rule out trivial models)
189
190 % fof(ax_endurantType_instances_natures, axiom, (
       substantialType(substantialType1) & momentType(momentType1) &
       objectType(objectType1) & collectiveType(collectiveType1) &
       quantityType(quantityType1) & intrinsicMomentType(
       intrinsicMomentType1) & relatorType(relatorType1) & qualityType
       (qualityType1) & modeType(modeType1)
192 % )).
```

$2.1.5 \quad \text{Partial Taxonomy of Endurant Type (on modal properties of types)}$

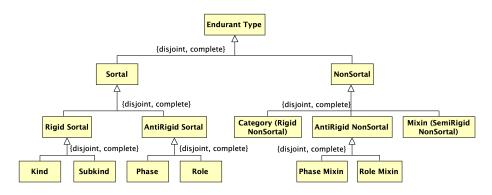


Figure 5: Partial Taxonomy of UFO – Endurant Types (by modal properties of types).

```
194 % Endurant Type (by modal properties of types)
195
{\tt 196} fof(ax_endurantType_taxonomy_properties, axiom, (
     ![X]: ((sortal(X) | nonSortal(X)) <=> (endurantType(X)))
198 )).
199
   fof(ax_endurantType_partition_properties, axiom, (
200
     ~?[X]: (sortal(X) & nonSortal(X))
201
203
204
   % Sortal
205
206 fof(ax_sortal_taxonomy, axiom, (
     ![X]: ((rigidSortal(X) | antiRigidSortal(X)) <=> (sortal(X)))
208 )).
209
fof(ax_sortal_partition, axiom, (
     ~?[X]: (rigidSortal(X) & antiRigidSortal(X))
211
212 )).
213
214
   % Rigid Sortal
215
fof(ax_rigidSortal_taxonomy, axiom, (
    ![X]: ((kind(X) | subkind(X)) <=> (rigidSortal(X)))
217
218 )).
219
220 fof(ax_rigidSortal_partition, axiom, (
     ~?[X]: (kind(X) & subkind(X))
222 )).
223
224 % Anti-Rigid Sortal
225
fof(ax_antiRigidSortal_taxonomy, axiom, (
![X]: ((phase(X) | role(X)) <=> (antiRigidSortal(X)))
```

```
228 )).
229
fof(ax_antiRigidSortal_partition, axiom, (
     ~?[X]: (phase(X) & role(X))
232 )).
233
234 % Non-Sortal
235
fof(ax_nonSortal_taxonomy, axiom, (
     ![X]: ((rigidNonSortal(X) | semiRigidNonSortal(X) |
       antiRigidNonSortal(X)) <=> (nonSortal(X)))
238 )).
239
240 fof(ax_nonSortal_partition, axiom, (
      ~?[X]: ((rigidNonSortal(X) & semiRigidNonSortal(X)) | (
       rigidNonSortal(X) & antiRigidNonSortal(X)) | (
       semiRigidNonSortal(X) & antiRigidNonSortal(X)))
242 )).
243
244 % Category
246 fof(ax_rigidNonSortal_taxonomy, axiom, (
    ![X]: (rigidNonSortal(X) <=> (category(X)))
247
248 )).
249
250 % Mixin
251
fof(ax_semiRigidNonSortal_taxonomy, axiom, (
    ![X]: (semiRigidNonSortal(X) <=> (mixin(X)))
253
254 )).
255
256 % Anti-Rigid Non-Sortal
fof(ax_antiRigidNonSortal_taxonomy, axiom, (
     ![X]: ((phaseMixin(X) | roleMixin(X)) <=> (antiRigidNonSortal(X))
260 )).
fof(ax_antiRigidNonSortal_partition, axiom, (
263
     ~?[X]: (phaseMixin(X) & roleMixin(X))
264 )).
265
266 % Endurant Type (by modal properties of types) partial taxonomy
       instances
267 % (tested to rule out trivial models)
268
269 % fof(ax_endurantType_instances_properties, axiom, (
       sortal(sortal1) & nonSortal(nonSortal1) & rigidSortal(
       rigidSortal1) & antiRigidSortal(antiRigidSortal1) & kind(kind1)
        & subkind(subkind1) & phase(phase1) & role(role1) & category(
       category1) & mixin(mixin1) & antiRigidNonSortal(
       antiRigidNonSortal1) & phaseMixin(phaseMixin1) & roleMixin(
       roleMixin1)
271 % )).
```

2.1.6 Defining Types, Individuals, and Specialization

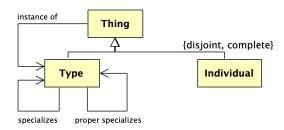


Figure 6: Types, individuals, instantiation, and specialization.

```
273 %%%%%%%%%% Instance of, Types, and Individuals %%%%%%%%%%%%
274
fof(ax_dIof, axiom, (
    ![X,Y,W]: (iof(X,Y,W) => (type_(Y) & world(W)))
276
277 )).
278
279
  fof(ax_dType_a1, axiom, (
    ![X]: (type_(X) <=> (?[Y,W]: iof(Y,X,W)))
280
282
285 )).
^{287} % TODO: confirm whether we are including second-order types in this
       formalization
fof(ax_multiLevel_a3, axiom, (
    ![X,Y,W]: (iof(X,Y,W) => (type_(X) | individual(X)))
291 )).
292
293 fof(ax_twoLevelConstrained_a4, axiom, (
     ~?[X,Y,Z,W]: (type_(X) & iof(X,Y,W) & iof(Y,Z,W))
294
295 )).
296
297 % Instantiation relations
298 % (tested to rule out trivial models)
299
300 % fof(ax_iofInUse, axiom, (
      type_(t2) & individual(i2) & world(w2) & iof(i2,t2,w2)
301 %
302 % )).
303
304 % Ax |= "th_everythingIsAThing_t1"; conjecture commented for
       convenience
305
306 % fof(th_everythingIsAThing_t1, conjecture, (
307 % ![X]: (type_(X) | individual(X))
309
310 % Ax |= "th_thingPartition_t2"; conjecture commented for
   convenience
```

```
311
312 % fof(th_thingPartition_t2, conjecture, (
     ~?[X]: (type_(X) & individual(X))
313 %
314 % )).
315
316 %%%%%%% Specialization and Proper Specialization %%%%%%%%
317
fof(ax_dSpecializes, axiom, (
     ![X,Y]: (specializes(X,Y) => (type_(X) & type_(Y)))
320 )).
321
322 fof(ax_specialization_a5, axiom, (
     ![T1,T2]: (specializes(T1,T2) <=> (
323
       type_{T1} & type_{T2} & ![W]: (world(W) => ![E]: (iof(E,T1,W))
       => iof(E,T2,W)))
     ))
325
326 )).
327
328 fof(ax_properSpecializes_d1, axiom, (
     ![X,Y]: (properSpecializes(X,Y) <=> (specializes(X,Y) & ~
329
       specializes(Y,X)))
330 )).
331
332 % Ax |= "th_cyclicSpecializations_t3"; conjecture commented for
       convenience
334 % fof(th_cyclicSpecializations_t3, conjecture, (
       ![X,Y]: (specializes(X,Y) => (specializes(X,X) & specializes(Y,
335 %
       Y)))
336 % )).
337
338 % Ax |= "th_transitiveSpecializations_t4"; conjecture commented for
        convenience
339
340 % fof(th_transitiveSpecializations_t4, conjecture, (
341
      ![X,Y,Z]: ((specializes(X,Y) & specializes(Y,Z)) => (
       specializes(X,Z)))
342 % )).
343
344 fof(ax_sharedSpecializations_a6, axiom, (
     ![T1,T2]: (?[X,W]: ((iof(X,T1,W) & iof(X,T2,W) & ~specializes(T1,
       T2) & ^{\circ} specializes(T2,T1)) => (
          (?[T3]: (specializes(T1,T3) \& specializes(T2,T3) \& iof(X,T3,W) ) \\
346
       )))|
         (?[T3]: (specializes(T3,T1) & specializes(T3,T2) & iof(X,T3,W
       )))
     )))
348
349 )).
350
   % Specialization relations
352 % (tested to rule out trivial models)
353
354 % fof(ax_specializesInUse, axiom, (
       endurantType(t3_1) & endurantType(t3_2) & specializes(t3_1,t3_2
       ) & properSpecializes(t3_1,t3_2) & specializes(t3_1,t3_1) &
       endurant(e3) & world(w3) & iof(e3,t3_1,w3)
356 % )).
```

2.1.7 Defining Rigidity and Sortality

```
360 % Rigidity
361
362 % TODO: I don't find we need to attach the "rigid(T)" predicate to
       the "endurant(T)" predicate like the paper does, so let's
       review this idea.
_{363} % TODO: verify whether it is a problem not to introduce predicates
       "world(W1) &" and "world(W2) &" before each instantiation
364
   fof(ax_dRigid_a18, axiom, (
365
366
     ![T]: (rigid(T) <=> (endurantType(T) & (
       ![X]: ((?[W1]: (world(W1) & iof(X,T,W1))) => (![W2]: (world(W2)
367
        => iof(X,T,W2))))
    )))
368
369 )).
370
371 fof(ax_dAntiRigid_a19, axiom, (
     ![T]: (antiRigid(T) <=> (endurantType(T) & (
       ![X]: ((?[W1]: (world(W1) & iof(X,T,W1))) => (?[W2]: (world(W2)
373
        & ~iof(X,T,W2)))
     ))))
374
375 )).
fof(ax_dSemiRigid_a20, axiom, (
     ![T]: (semiRigid(T) <=> (endurantType(T) & ~rigid(T) & ~antiRigid
       (T)))
379 )).
380
381 % Ax |= "th_thEndurantTypeHaveRigidity_t5"; conjecture commented
       for convenience
382
383 % fof(th_thEndurantTypeHaveRigidity_t5, conjecture, (
       ![T]: (endurantType(T) <=> (rigid(T) | semiRigid(T) | antiRigid
384
       (T)))
385 % )).
386
   % Ax |= "th_pairwiseDisjointRigidities_t6"; conjecture commented
       for convenience
388
389 % fof(th_pairwiseDisjointRigidities_t6, conjecture, (
        \tilde{T} [T]: ((rigid(T) & semiRigid(T)) | (semiRigid(T) & antiRigid(T)
390 %
       )) | (rigid(T) & antiRigid(T)))
391 % )).
392
393 % Ax |= "th_rigidAntiRigidSpecializationConstraint_t7"; conjecture
       commented for convenience
395 % fof(th_rigidAntiRigidSpecializationConstraint_t7, conjecture, (
396 %
       ~![T1,T2]: (rigid(T1) & antiRigid(T2) & specializes(T1,T2))
397 % )).
398
399 % Ax |= "th_semiRigidAntiRigidSpecializationConstraint_t8";
       conjecture commented for convenience
401 % fof(th_semiRigidAntiRigidSpecializationConstraint_t8, conjecture,
```

```
402 % ~ [[T1,T2]: (semiRigid(T1) & antiRigid(T2) & specializes(T1,T2))
403 % )).
404
405 % Rigidity properties
406 % (tested to rule out trivial models)
407
408 % fof(ax_rigidityInUse, axiom, (
      endurantType(t4_1) & endurantType(t4_2) & endurantType(t4_3) &
409 %
       rigid(t4_1) & semiRigid(t4_2) & antiRigid(t4_3) &
       properSpecializes(t4_1,t4_2) & properSpecializes(t4_3,t4_1)
410 % )).
411
412 % Sortality
413
414 fof(ax_endurantsKind_a21, axiom, (
     ![E]: (endurant(E) => (
416
       ?[U]: (kind(U) & (![W]: (world(W) => iof(E,U,W))))
417
418 )).
419
420 fof(ax_uniqueKind_a22, axiom, (
    ![E,U,W]: ((world(W) & kind(U) & iof(E,U,W)) => (
421
       "?[U2,W2]: (kind(U2) & iof(E,U2,W2) & "(U = U2))"
422
    ))
423
424 )).
425
% Changing "ax_dSortal_a23" from the form it was defined in the
       paper to "sortals are endurant types that specialize some
       ultimate sortal" seem to express the same concept while
       speeding up the execution of SPASS considerably
428 % fof(ax_dSortal_a23, axiom, (
       ![S]: (sortal(S) <=> (endurantType(S) & (?[U]: (kind(U) & (![E,
       W]: (iof(E,S,W) \Rightarrow iof(E,U,W))))))
430 % )).
431
fof(ax_dSortal_a23, axiom, (
    ![S]: ((sortal(S)) <=> (endurantType(S) & (?[U]: (kind(U) &
       specializes(S,U)))))
434 )).
435
^{436} % If we have the taxonomy's axiomatization, then a24 becomes a
       theorem
437 % Ax |= "th_nonSortalsAreEndurantsThatAreNotSortals_a24";
       conjecture commented for convenience
438
439 % fof(th_nonSortalsAreEndurantsThatAreNotSortals_a24, conjecture, (
440 % ![NS]: ((nonSortal(NS)) <=> (endurantType(NS) & ~sortal(NS)))
441 % )).
442
443 % Ax |= "th_kindsAreRigid_t9"; conjecture commented for convenience
445 % fof(th_kindsAreRigid_t9, conjecture, (
446 % ![U]: ((kind(U)) => (rigid(U)))
447 % )).
449 % Ax |= "th_kindsHaveDisjointExtensions_t10"; conjecture commented
```

```
for convenience
451 % fof(th_kindsHaveDisjointExtensions_t10, conjecture, (
       ![K1,K2]: ((kind(K1) & kind(K2) & ~(K1=K2)) => (
452 %
         ~?[X,W1,W2]: (world(W1) & world(W2) & iof(X,K1,W1) & iof(X,K2
453 %
       ,W2)))
454 %
455 % )).
456
457 % Ax |= "th_kindsHaveDisjointTaxonomies_t11"; conjecture commented
       for convenience
458
459 % fof(th_kindsHaveDisjointTaxonomies_t11, conjecture, (
      ![K1,K2]: ((kind(K1) & kind(K2) & ~(K1=K2)) => (
461 %
          ~?[T]: (specializes(T,K1) & specializes(T,K2)))
462
463 % )).
464
465 % Ax |= "th_kindsAreSortal_t12"; conjecture commented for
       convenience
_{467} % fof(th_kindsAreSortal_t12, conjecture, (
468 % ![K]: ((kind(K)) => (sortal(K)))
469 % )).
470
471 % Ax |= "th_sortalSpecializeKinds_t13"; conjecture commented for
       convenience
473 % fof(th_sortalSpecializeKinds_t13, conjecture, (
      ![S]: ((sortal(S)) => (?[K]: (kind(K) & specializes(S,K))))
475 % )).
476
477 % Ax |= "th_sortalsSpecializeAUniqueKind_t14"; conjecture commented
        for convenience
479 % fof(th_sortalsSpecializeAUniqueKind_t14, conjecture, (
     ![S]: ((sortal(S)) => (~?[U,U2]: (kind(U) & kind(U2) &
480 %
       specializes(S,U) & specializes(S,U2) & ~(U=U2))))
481 % )).
482
483 % Sortality properties
484 % (tested to rule out trivial models)
486 % fof(ax_sortalityInUse, axiom, (
       endurant(e5_1) & endurant(e5_2) & world(w5) & kind(k5_1) & kind
        (k5_2) \& iof(e5_1,k5_1,w5) \& iof(e5_1,k5_1,w5) \& ~(k5_1=k5_2) \\
489
490 % Sortality + Rigidity
492 fof(ax_rigidSortalsAreRigidAndSortal_xx, axiom, (
   ![T]: ((rigidSortal(T)) <=> (rigid(T) & sortal(T)))
493
494 )).
495
496 fof(ax_antiRigidSortalsAreAntiRigidAndSortal_xx, axiom, (
497 ![T]: ((antiRigidSortal(T)) <=> (antiRigid(T) & sortal(T)))
```

```
499
fof(ax_rigidNonSortalsAreRigidAndNonSortal_xx, axiom, (
   ![T]: ((rigidNonSortal(T)) <=> (rigid(T) & nonSortal(T)))
501
503
fof(ax_antiRigidNonSortalsAreAntiRigidAndNonSortal_xx, axiom, (
505
   ![T]: ((antiRigidNonSortal(T)) <=> (antiRigid(T) & nonSortal(T)))
506 )).
508 fof(ax_semiRigidNonSortalsAreSemiRigidAndNonSortal_xx, axiom, (
    ![T]: ((semiRigidNonSortal(T)) <=> (semiRigid(T) & nonSortal(T)))
509
510 )).
511
512 % If we have the taxonomy's axiomatization, then a25 becomes a
       theorem
513 % Ax |= "th_kindAndSubkindAreDisjoint_a25"; conjecture commented
      for convenience
515 % fof(th_kindAndSubkindAreDisjoint_a25, conjecture, (
       ~?[T]: (kind(T) & subkind(T))
516 %
517 % )).
518
519 % If we have the taxonomy's axiomatization, then a26 becomes a
       theorem
520 % Ax |= "th_kindAndSubkindAreRigidSortals_a26"; conjecture
       commented for convenience
522 % fof(th_kindAndSubkindAreRigidSortals_a26, conjecture, (
     ![T]: ((kind(T) | subkind(T)) <=> (rigid(T) & sortal(T)))
523 %
524 % )).
526 % If we have the taxonomy's axiomatization, then a27 becomes a
527 % Ax |= "th_phaseAndRoleAreDisjoint_a27"; conjecture commented for
      convenience
529 % fof(th_phaseAndRoleAreDisjoint_a27, conjecture, (
530 %
       ~?[T]: (phase(T) & role(T))
531 % )).
532
533 % If we have the taxonomy's axiomatization, then a28 becomes a
       theorem
534 % Ax |= "th_phaseAndRoleAreAntiRigidSortals_a28"; conjecture
      commented for convenience
536 % fof(th_phaseAndRoleAreAntiRigidSortals_a28, conjecture, (
     ![T]: ((phase(T) | role(T)) <=> (antiRigid(T) & sortal(T)))
537 %
538 % )).
539
540 % Skipping (a29) because we leave the concept of semi-rigid sortals
        out of this ontology.
541
542 % If we have the taxonomy's axiomatization, then a 30 becomes a
      theorem
543 % Ax |= "th_categoriesAreRigidNonSortals_a30"; conjecture commented
       for convenience
```

```
545 % fof(th_categoriesAreRigidNonSortals_a30, conjecture, (
546 % ![T]: ((category(T)) <=> (rigid(T) & nonSortal(T)))
547 % )).
548
549 % If we have the taxonomy's axiomatization, then a31 becomes a
       theorem
550 % Ax |= "th_mixinsAreSemiRigidNonSortals_a31"; conjecture commented
       for convenience
552 % fof(th_mixinsAreSemiRigidNonSortals_a31, conjecture, (
553 %
      ![T]: ((mixin(T)) <=> (semiRigid(T) & nonSortal(T)))
554 % )).
555
556 % If we have the taxonomy's axiomatization, then a32 becomes a
       theorem
557 % Ax |= "th_phaseMixinAndRoleMixinAreDisjoint_a32"; conjecture
       commented for convenience
559 % fof(th_phaseMixinAndRoleMixinAreDisjoint_a32, conjecture, (
       ~?[T]: (phaseMixin(T) & roleMixin(T))
560 %
  % )).
561
562
_{563} % If we have the taxonomy's axiomatization, then a33 becomes a
       theorem
564 % Ax |= "ax_phaseMixinAndRoleMixinAreAntiRigidSortals_a33";
       conjecture commented for convenience
565
566 % fof(th_phaseMixinAndRoleMixinAreAntiRigidSortals_a33, conjecture,
       ![T]: ((phaseMixin(T) | roleMixin(T)) <=> (antiRigid(T) &
567
       nonSortal(T)))
568 % )).
569
570 % Ax |= "th_leafCategoriesArePairwiseDisjoint_t18"; conjecture
       commented for convenience
572 % fof(th_leafCategoriesArePairwiseDisjoint_t18, conjecture, (
573 %
       ~?[T]: (endurantType(T) & (
574 %
575 %
            (kind(T) & subkind(T))
            | (kind(T) & phase(T))
576 %
577 %
            | (kind(T) & role(T))
578 %
            | (kind(T) & category(T))
579 %
            | (kind(T) & mixin(T))
580 %
            | (kind(T) & phaseMixin(T))
581 %
           | (kind(T) & roleMixin(T))
582 %
         ) | (
583 %
           (subkind(T) & phase(T))
            | (subkind(T) & role(T))
584 %
              (subkind(T) & category(T))
585
586 %
            | (subkind(T) & mixin(T))
587 %
            | (subkind(T) & phaseMixin(T))
588 %
            | (subkind(T) & roleMixin(T))
589 %
         ) | (
590 %
            (phase(T) & role(T))
591 %
            | (phase(T) & category(T))
592 %
           | (phase(T) & mixin(T))
```

```
| (phase(T) & phaseMixin(T))
593 %
594
   %
              (phase(T) & roleMixin(T))
         ) | (
595 %
            (role(T) & category(T))
596
            | (role(T) & mixin(T))
597 %
598
   %
            | (role(T) & phaseMixin(T))
599
   %
            | (role(T) & roleMixin(T))
600 %
         ) | (
601 %
            (category(T) & mixin(T))
            | (category(T) & phaseMixin(T))
602 %
            | (category(T) & roleMixin(T))
603
604
   %
         ) | (
   %
            (mixin(T) & phaseMixin(T))
605
606 %
            | (mixin(T) & roleMixin(T))
607 %
         ) | (
            (phaseMixin(T) & roleMixin(T))
608
609 %
       ))
610 %
611 % )).
612
   % Ax |= "th_leafCategoriesCompletelyCategorizeAllEndurantTypes_t19
        "; conjecture commented for convenience
614
_{615} % fof(th_leafCategoriesCompletelyCategorizeAllEndurantTypes_t19,
       conjecture, (
       ![T]: (endurantType(T) => (
616
         kind(T) | subkind(T) | phase(T) | role(T) | category(T) |
617 %
       mixin(T) | phaseMixin(T) | roleMixin(T)
618 %
       ))
619 % )).
620
621 % Sortality and rigidity properties combined
622 % (tested to rule out trivial models)
623
   % fof(ax_sortalityAndRigidityInUse, axiom, (
624
625
       endurant(e6_1) & endurant(e6_2) & world(w6) & kind(k6_1) & kind
        (k6_2) \& iof(e6_1,k6_1,w6) \& iof(e6_1,k6_1,w6) \& ~(k6_1=k6_2) \\
626 % )).
```

2.1.8 Defining Endurant Types

```
630 % Defining the taxonomy of types of ontological natures through the
         categorization of the taxonomy of concrete individuals
632 fof(ax_perdurantTypeDefinition_a44, axiom, (
      ![T]: (perdurantType(T) <=> (
633
        \label{eq:type_T} \mbox{type_(T) \& (![P,W]: ((world(W) \& iof(P,T,W)) => (perdurant(P)))}
634
     ))
635
636 )).
638 fof(ax_endurantTypeDefinition_a44, axiom, (
     ![T]: (endurantType(T) <=> (
639
        \label{eq:type_T} \mbox{type_(T) \& (![E,W]: ((world(W) \& iof(E,T,W)) => (endurant(E))))}
640
641
642 )).
643
644 fof(ax_substantialTypeDefinition_a44, axiom, (
```

```
![T]: (substantialType(T) <=> (
645
       type_{T}(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (substantial(E)))
       )))
     ))
648 )).
649
650
   fof(ax_momentTypeDefinition_a44, axiom, (
     ![T]: (momentType(T) <=> (
651
       type_{-}(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (moment(E))))
     ))
653
654 )).
655
656 fof(ax_objectTypeDefinition_a44, axiom, (
     ![T]: (objectType(T) <=> (
       type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (object(E))))
658
659
660 )).
661
   fof(ax_collectiveTypeDefinition_a44, axiom, (
     ![T]: (collectiveType(T) <=> (
663
       type_{T}(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (collective(E)))
       ))
     ))
665
666 )).
667
   fof(ax_quantityTypeDefinition_a44, axiom, (
     ![T]: (quantityType(T) <=> (
669
       type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (quantity(E))))
670
671
672 )).
673
fof(ax_intrinsicMomentTypeDefinition_a44, axiom, (
     ![T]: (intrinsicMomentType(T) <=> (
       type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (
676
       intrinsicMoment(E))))
     ))
678 )).
680 fof(ax_relatorTypeDefinition_a44, axiom, (
681
     ![T]: (relatorType(T) <=> (
       type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (relator(E))))
682
683
684 )).
685
686 fof(ax_qualityTypeDefinition_a44, axiom, (
687
     ![T]: (qualityType(T) <=> (
       type_(T) & (![E,W]: ((world(W) & iof(E,T,W)) => (quality(E))))
688
     ))
689
690 )).
fof(ax_modeTypeDefinition_a44, axiom, (
     ![T]: (modeType(T) <=> (
693
       type_{-}(T) & (![E,W]: ((world(W) & iof(E,T,W)) \Rightarrow (mode(E))))
694
695
696 )).
697
698 % Types Definition
```

```
699 % (tested to rule out trivial models)
700 % TODO: investigate why we cannot list four different endurant
       types (it may have something to do with "intrinsicMoment" and "
       intrinsicMomentType")
701
702 % fof(ax_typesDefinitionsInstances, axiom, (
703 %
     objectType(ot7) & collectiveType(ct7) & modeType(mt7)
704 % )).
705
706 % Ax |= "th_leafCategoriesArePairwiseDisjoint_t21"; conjecture
       commented for convenience
   \% Having the previously defined taxonomy, this should be quite
       trivial
708
709 % fof(th_leafCategoriesArePairwiseDisjoint_t21, conjecture, (
       ~?[T]: (type_(T) & (
710 %
711 %
         (
           (objectType(T) & collectiveType(T)) | (objectType(T) &
712 %
       quantityType(T)) | (objectType(T) & modeType(T)) | (objectType(
       T) & qualityType(T)) | (objectType(T) & relatorType(T)) | (
       objectType(T) & perdurantType(T))
         ) | (
713 %
           (collectiveType(T) & quantityType(T)) | (collectiveType(T)
714 %
       & modeType(T)) | (collectiveType(T) & qualityType(T)) | (
       collectiveType(T) & relatorType(T)) | (collectiveType(T) &
       perdurantType(T))
715 %
         ) | (
           (quantityType(T) & modeType(T)) | (quantityType(T) &
716 %
       qualityType(T)) | (quantityType(T) & relatorType(T)) | (
       quantityType(T) & perdurantType(T))
717 %
         ) | (
           (modeType(T) & qualityType(T)) | (modeType(T) & relatorType
718 %
       (T)) | (modeType(T) & perdurantType(T))
719 %
         ) | (
           (qualityType(T) & relatorType(T)) | (qualityType(T) &
720 %
       perdurantType(T))
721 %
         ) | (
722 %
           relatorType(T) & perdurantType(T)
723 %
724 %
       ))
725 % )).
726
727 % Ultimate Sortals Definitions (by ontological nature)
728
fof(ax_objectKindDefinition_a45, axiom, (
   ![T]: (objectKind(T) <=> (objectType(T) & kind(T)))
730
731 )).
732
733 fof(ax_collectiveKindDefinition_a45, axiom, (
    ![T]: (collectiveKind(T) <=> (collectiveType(T) & kind(T)))
735 )).
736
737 fof(ax_quantityKindDefinition_a45, axiom, (
    ![T]: (quantityKind(T) <=> (quantityType(T) & kind(T)))
738
739 )).
740
741 fof(ax_modeKindDefinition_a45, axiom, (
```

```
742 ![T]: (modeKind(T) <=> (modeType(T) & kind(T)))
743 )).
744
745 fof(ax_qualityKindDefinition_a45, axiom, (
   ![T]: (qualityKind(T) <=> (qualityType(T) & kind(T)))
746
747 )).
fof(ax_relatorKindDefinition_a45, axiom, (
    ![T]: (relatorKind(T) <=> (relatorType(T) & kind(T)))
751 )).
752
753 % Ultimate sortals (by ontological nature) instances
754 % (tested to rule out trivial models)
_{755} % TODO: investigate why we cannot list all different types of
       ultimate sortals at once
757 % fof(ax_typesDefinitionsInstances, axiom, (
      objectKind(ok9) & collectiveKind(ck9) & quantityKind(quank9) &
758 %
       relatorKind(rk9) & modeKind(mk9) & qualityKind(qualk9)
759 % )).
760
_{761} % Skipping (t22) because (a21) makes it trivial
762
763 fof(ax_typesDefinitionsInstances_a46, axiom, (
    ![E]: (endurant(E) => (
764
       ?[K]: ((objectKind(K) | collectiveKind(K) | quantityKind(K) |
765
       modeKind(K) | qualityKind(K) | relatorKind(K))
       & (![W]: (iof(E,K,W))))
766
    ))
767
768 )).
769
770 % Ax |= "th_endurantSortalsCompleteness_t23"; conjecture commented
       for convenience
_{771} % Thanks to the taxonomy, we already have "sortal(T) =>
       endurantType(T)", but I leave it like this to be consistent
       with the paper
772
_{773} % fof(th_endurantSortalsCompleteness_t23, conjecture, (
       ![T]: ((endurantType(T) & sortal(T)) => (objectKind(T) |
       collectiveKind(T) | quantityKind(T) | qualityKind(T) | modeKind
       (T) | relatorKind(T) | phase(T) | role(T)))
777 % Ax |= "th_objectTypesSpecializeAKindOfSameNature_t24"; conjecture
        commented for convenience
779 % fof(th_objectTypesSpecializeAKindOfSameNature_t24, conjecture, (
      ![T]: ((objectType(T) & sortal(T)) <=> (?[K]: (objectKind(K) &
       specializes(T,K))))
781 % )).
782
783 % Ax |= "th_collectiveTypesSpecializeAKindOfSameNature_t24";
       conjecture commented for convenience
784
785 % fof(th_collectiveTypesSpecializeAKindOfSameNature_t24, conjecture
       ![T]: ((collectiveType(T) \& sortal(T)) \iff (?[K]: (
```

```
collectiveKind(K) & specializes(T,K))))
787 % )).
788
  % Ax |= "th_quantityTypesSpecializeAKindOfSameNature_t24";
       conjecture commented for convenience
790
   % fof(th_quantityTypesSpecializeAKindOfSameNature_t24, conjecture,
       ![T]: ((quantityType(T) & sortal(T)) <=> (?[K]: (quantityKind(K
      ) & specializes(T,K))))
793
794
795 % Ax |= "th_modeTypesSpecializeAKindOfSameNature_t24"; conjecture
       commented for convenience
796
   % fof(th_modeTypesSpecializeAKindOfSameNature_t24, conjecture, (
798
      ![T]: ((modeType(T) & sortal(T)) <=> (?[K]: (modeKind(K) &
       specializes(T,K))))
799 % )).
800
   % Ax |= "th_qualityTypesSpecializeAKindOfSameNature_t24";
       conjecture commented for convenience
802
803 % fof(th_qualityTypesSpecializeAKindOfSameNature_t24, conjecture, (
      ![T]: ((qualityType(T) & sortal(T)) <=> (?[K]: (qualityKind(K)
804
       & specializes(T,K))))
805 % )).
806
807 % Ax |= "th_relatorTypesSpecializeAKindOfSameNature_t24";
       conjecture commented for convenience
809 % fof(th_relatorTypesSpecializeAKindOfSameNature_t24, conjecture, (
       ![T]: ((relatorType(T) & sortal(T)) <=> (?[K]: (relatorKind(K)
810 %
       & specializes(T,K))))
811
812
813 % Ax |= "th_sortalLeafCategoriesAreDisjoint_t25"; conjecture
       commented for convenience
814
815 % fof(th_sortalLeafCategoriesAreDisjoint_t25, conjecture, (
      ![T]: (objectKind(T) => (~(collectiveKind(T) | quantityKind(T)
       | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) |
       mixin(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (collectiveKind(T) => (~(objectKind(T) | quantityKind(T)))
817 %
       ) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T)
        | mixin(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (quantityKind(T) => (~(objectKind(T) | collectiveKind(T
818 %
       ) | modeKind(T) | qualityKind(T) | relatorKind(T) | category(T)
        | mixin(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (modeKind(T) => (~(objectKind(T) | quantityKind(T) |
       collectiveKind(T) | qualityKind(T) | relatorKind(T) | category(
       T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
820 %
       & ![T]: (qualityKind(T) => (~(objectKind(T) | quantityKind(T) |
        modeKind(T) | collectiveKind(T) | relatorKind(T) | category(T)
        | mixin(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (relatorKind(T) => (~(objectKind(T) | quantityKind(T) |
821 %
      modeKind(T) | qualityKind(T) | collectiveKind(T) | category(T)
```

```
| mixin(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (category(T) => (~(objectKind(T) | quantityKind(T) |
       modeKind(T) | qualityKind(T) | relatorKind(T) | collectiveKind(
       T) | mixin(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (mixin(T) => (~(objectKind(T) | quantityKind(T) |
823 %
       modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) |
       collectiveKind(T) | phaseMixin(T) | roleMixin(T))))
       & ![T]: (phaseMixin(T) => (~(objectKind(T) | quantityKind(T) |
824 %
       modeKind(T) | qualityKind(T) | relatorKind(T) | category(T) |
       mixin(T) | collectiveKind(T) | roleMixin(T))))
       & ![T]: (roleMixin(T) => (~(objectKind(T) | quantityKind(T) |
825 %
       \verb|modeKind(T)| | qualityKind(T)| | relatorKind(T)| | category(T)|
       mixin(T) | phaseMixin(T) | collectiveKind(T))))
826 % )).
827
828 % Ax |= "th_sortalLeafCategoriesAreComplete_t26"; conjecture
       commented for convenience
830 % fof(th_sortalLeafCategoriesAreComplete_t26, conjecture, (
      ![T]: ((endurantType(T)) => (objectKind(T) | collectiveKind(T)
831 %
       | quantityKind(T) | qualityKind(T) | modeKind(T) | relatorKind(
      T) | phase(T) | role(T) | category(T) | mixin(T) | phaseMixin(T
      ) | roleMixin(T)))
832 % )).
```