University of Cagliari

MSc programs in Computer Engineering, Cybersecurity and Artificial Intelligence, and in Electronic Engineering

Artificial Intelligence

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Exercises on logical languages for knowledge representation

1. Give a representation in predicate logic of the following propositions related to search trees, starting from the definition of a *common* domain of discourse and of the predicate, function and constant symbols.

   (a) The root node of a search tree has zero depth

   (b) A node that has not yet been expanded has no successors

   (c) A node that has already been expanded has at least one successor

   (d) Each node has exactly one parent node, except for the root node

2. Give a representation in predicate logic of the following groups of propositions. First of all, separately for each group, *clearly* define the domain of discourse and the predicate, function and constant symbols.

   (a) Not all students attend both the History and the Biology courses. The best mark in History is higher than the best mark in Biology. One only student did not pass the exam in History. One only student did not pass both the exam in History and the exam in Biology.

   (b) Politicians always fool someone, and sometimes fool everyone, but they do not always fool everyone. (*From a phrase attributed to Abraham Lincoln: "You can fool some of the people all of the time, and all of the people some of the time, but you can not fool all of the people all of the time."*)

   (c) Every person who does not like any vegetarian is intelligent. Nobody likes a vegetarian who is intelligent. There is a woman who likes every man who is not a vegetarian.

   (d) Some barber shaves every man who does not shave himself.

   (e) Nobody likes a professor, unless he is an intelligent professor.

   (f) All Germans speak the same languages.

3. Give a representation in predicate logic of the following propositions, starting from the definition of a *common* domain of discourse and of the predicate, function and constant symbols.

   (a) Cows, pigs and horses are mammals

   (b) The child of a horse is a horse

   (c) Bluebeard is a horse

   (d) Bluebeard is Charlie's father

   (e) Child and father are inverse relations

   (f) Every mammal has a father

4. Give a representation in predicate logic of the following propositions, including the definition of a *common* domain of discourse and of the predicate, function and constant symbols.
   (a) Tony, Mike and John are members of the Alpine Club
   (b) Every member of the Alpine Club is a skier or a climber
   (c) No climber likes rain
   (d) Every skier likes snow
   (e) Mike does not like everything that Tony likes
   (f) Mike likes everything Tony does not like
   (g) Tony likes both rain and snow
5. Give a representation in predicate logic of the following propositions related to the *block's world* domain, including the defintion of a *common* domain of discourse and of the predicate, function and constant symbols:
   (a) If all heavy objects are blue, then all non-heavy objects are green
   (b) Every object is either blue or green, but not both
   (c) If there is some non-heavy object, then all heavy objects are blue

Solution

The following ones are *possible* solutions to each exercise. Different solutions can be devised by making different choices on the domain, constants, predicates and functions.

1. Domain: a set made up of nodes, and of the set of natural numbers (denoting the depth of nodes).

   Predicate symbols: Node(·) (being a node), Root(·) (being a root node), Expanded(·) (being a node which has already been expanded), Successor(⟨child node⟩,⟨parent node⟩) (to be the successor node of), Equal(·,·) (being the same entity). Note that a predicate representing the relationship "being the parent node of" is not necessary, since the predicate "Successor" can be used to this aim.

   Function symbols: Depth(·) (the depth of a node).

   Constant symbols: Z (the number zero).

   (a) $\forall x\ Root(x) \Rightarrow Equal(Depth(x), Z)$
   (b) $\forall x\ \neg Expanded(x) \Rightarrow \neg\exists y\ Successor(y, x)$
   (c) $\forall x\ Expanded(x) \Rightarrow \exists y\ Successor(y, x)$
   (d) $\forall x\ Node(x) \wedge \neg Root(x) \Rightarrow (\exists y\ Successor(x, y) \wedge (\forall z\ Successor(x, z) \Rightarrow Equal(z, y)))$
2. A distinct definition of domain, constants, predicates and functions is given for each group of propositions.
   (a) Domain: a set including students, courses and marks. Constant symbols: `Biology` and `History` (denoting two courses). Predicate symbols: Student(·), Attends(⟨student⟩,⟨course⟩), Passes(⟨student⟩,⟨course⟩), Equal(·, ·), GreaterThan(⟨mark_1⟩,⟨mark_2⟩) (which means: $\text{mark}_1 > \text{mark}_2$), GreaterOrEqual(⟨$\text{mark}_1$⟩,⟨$\text{mark}_2$⟩) (which means: $\text{mark}_1 \geq \text{mark}_2$).
   Function symbols: Mark(⟨student⟩,⟨course⟩) (denotes the mark got by a student in an exam).

       "Not all students attend both the History and the Biology courses:"
       $\neg(\forall x\ Student(x) \Rightarrow Attends(x, History) \wedge Attends(x, Biology))$
       Equivalently, "some student did not attend either History or Biology:"
       $\exists x\ Student(x) \wedge (\neg Attends(x, History) \vee \neg Attends(x, Biology))$

"The best mark in History is higher than the best mark in Biology" can be restated as: "some student got a mark in History which is higher than the marks got in Biology by all students, and is also higher than or equal to all the marks got in History (including itself):"

$$\exists y\ Student(y) \wedge\ (\forall x\ Student(x) \Rightarrow Greater(Mark(y, History), Mark(x, Biology))) \wedge$$
$$(\forall z\ Student(z) \Rightarrow GreaterOrEqual(Mark(y, History), Mark(z, History)))$$

"One only student did not pass the exam in History." One may think that the sentence:

$$\exists x\ Student(x) \wedge \neg Passes(x, History)$$

is the correct representation of the above proposition. However it is not, since the existential quantifier $\exists$ means "there is *at least* one domain element that...," not "there is *exactly* one domain element that..." To correctly translate the above proposition, it can be restated as: "some student did not pass the exam in History, and every student who did not pass the exam in History is equal to the former:"

$$\exists x\ Student(x) \wedge \neg Passes(x, History) \wedge (\forall y\ Student(y) \wedge \neg Passes(y, History) \Rightarrow Equal(x, y))$$

"One only student did not pass both the exam in History and the exam in Biology:"

$$\exists\ x\ Student(x) \wedge \neg Passes(x, History) \wedge \neg Passes(x, Biology) \wedge$$
$$(\forall y\ (Student(y) \wedge \neg Passes(y, History) \wedge \neg Passes(y, Biology) \Rightarrow Equal(x, y))$$

(b) Domain: a set including people, some of which are politicians, and discrete time instants (the meaning of "always" and "sometimes" is expressed with reference to discrete time instants).

Predicate symbols: Person($\cdot$), Politician($\cdot$), Instant($\cdot$), Fools($\langle$who$\rangle$,$\langle$whom$\rangle$,$\langle$when (time instant)$\rangle$).

The above proposition can be subdivided into three distinct ones:

"Politicians always fool someone" (in other words: "every politician in every instant fools someone"):

$$\forall x, t\ Politician(x) \wedge Instant(t) \Rightarrow (\exists y\ Person(y) \wedge Fools(x, y, t))$$

"Politicians sometimes fool everyone" (i.e., "for each politician there is some time instant in which he fools everyone"):

$$\forall x\ Politician(x) \Rightarrow (\exists t\ Instant(t) \wedge (\forall y\ Person(y) \Rightarrow Fools(x, y, t)))$$

"Politicians do not always fool everyone" (i.e., "it is false that politicians always fool everyone"):

$$\neg(\forall x, y, t\ Politician(x) \wedge Person(y) \wedge Instant(t) \Rightarrow Fools(x, y, t))$$

(c) Domain: a set of people (men and women), some of which is vegetarian.

Predicate symbols: Likes($\langle$who$\rangle$,$\langle$whom$\rangle$), Man($\cdot$), Woman($\cdot$), Vergetarian($\cdot$), Intelligent($\cdot$).

"Every person who does not like any vegetarian is intelligent:"

$$\forall x\ (\forall y\ Vegetarian(y) \Rightarrow \neg Likes(x, y)) \Rightarrow Intelligent(x)$$

"Nobody likes a vegetarian who is intelligent:"

$$\forall x, y\ (Intelligent(y) \wedge Vegetarian(y)) \Rightarrow \neg Likes(x, y)$$

"There is a woman who likes every man who is not a vegetarian:"

$$\exists x\ Woman(x) \wedge (\forall y\ Man(y) \wedge \neg Vegetarian(y) \Rightarrow Likes(x, y))$$

(d) Domain: a set of people, including men and women, some of which are barbers.

Predicate symbols: Man($\cdot$), Barber($\cdot$), Shaves($\langle$who$\rangle$,$\langle$whom$\rangle$).

$$\exists x\ Barber(x) \wedge (\forall y\ Man(y) \wedge \neg Shaves(y, y) \Rightarrow Shaves(x, y))$$

(e) Domain: a set of people, some of which are professors.

Predicate symbols: Likes($\langle$who$\rangle$,$\langle$whom$\rangle$), Intelligent($\cdot$), Professor($\cdot$).

This proposition can be restated as: "nobody likes professors who are not intelligent:"

$$\forall x, y\ Professor(y) \wedge \neg Intelligent(y) \Rightarrow \neg Likes(x, y)$$

(f) Domain: a set including people, some of which are Germans, and languages.

Predicate symbols: Language($\cdot$), German($\cdot$), Speaks($\langle$who$\rangle$,$\langle$language$\rangle$).

$$\forall x, y, z\ German(x) \wedge German(y) \wedge Language(z) \wedge Speaks(x, z) \Rightarrow Speaks(y, z)$$

An equivalent sentence:
$\forall x, z\ German(x) \land Language(z) \land Speaks(x, z) \Rightarrow (\forall y\ German(y) \Rightarrow Speaks(y, z))$

3. Domain: a set of animals including cows, horses and pigs.
   Predicate symbols: Mammal(·), Horse(·), Cow(·), Pig(·), Child(⟨who⟩,⟨of whom⟩), Father(⟨who⟩,⟨of whom⟩).
   Constant symbols: Bluebeard, Charlie.
   (a) $\forall x\ Cow(x) \lor Pig(x) \lor Horse(x) \Rightarrow Mammal(x)$
   (b) $\forall x, y\ Horse(x) \land Child(y, x) \Rightarrow Horse(y)$
   (c) $Horse(Bluebeard)$
   (d) $Father(Bluebeard, Charlie)$
   (e) $\forall x, y\ Child(x, y) \Leftrightarrow Father(y, x)$
   (f) $\forall x\ Mammal(x) \Rightarrow \exists y\ Father(y, x)$
4. Domain: a set made up of people, clubs, and entities including snow and rain.
   Constant symbols: Tony, John, Mike, AlpineClub, Rain, Snow.
   Predicate symbols: Member(⟨who⟩,⟨club⟩), Skier(·), Climber(·), Likes(⟨who⟩,⟨what⟩).
   (a) $MAC(Tony),\ MAC(John),\ MAC(Mike)$ (three distinct sentences)
   (b) $\forall x\ Member(x, AlpineClub) \Rightarrow Skier(x) \lor Climber(x)$
   (c) $\forall x\ Climber(x) \Rightarrow \neg Likes(x, Rain)$
   (d) $\forall x\ Skier(x) \Rightarrow Likes(x, Snow)$
   (e) $\forall x\ Likes(Tony, x) \Rightarrow \neg Likes(Mike, x)$
   (f) $\forall x\ \neg Likes(Tony, x) \Rightarrow Likes(Mike, x)$
   (g) $Likes(Tony, Rain),\ Likes(Tony, Snow)$ (two distinct sentences)
5. Domain: a set of objects. Predicate symbols: Heavy(·), Blue(·), Green(·).
   (a) $(\forall x\ Heavy(x) \Rightarrow Blue(x)) \Rightarrow (\forall y\ \neg Heavy(y) \Rightarrow Green(y))$
   (b) $\forall x\ \ (Blue(x) \land \neg Green(x)) \lor (\neg Blue(x) \land Green(x))$
   (c) $(\exists x\ \neg Heavy(x)) \Rightarrow (\forall y\ Heavy(y) \Rightarrow Blue(y))$