Python Practical Session on the Recognition of MNIST Handwritten Digits

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MNIST Handwritten Digit Dataset

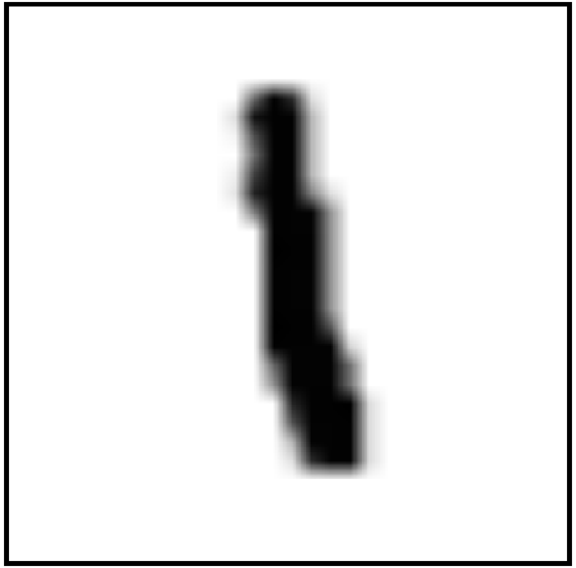
- Popular benchmark dataset for image classification
  - Y. LeCun, L. Jackel, L. Bottou, A. Brunot, C. Cortes, J. Denker, H. Drucker, I. Guyon, U. Muller, E. Sackinger, P. Simard, and V. Vapnik. Comparison of learning algorithms for handwritten digit recognition. In Int'l Conf. on Artificial Neural Networks, pp. 53–60, 1995.

- Handwritten digit images (centered and normalized as 28x28 images) belonging to 10 different classes (digits from 0 to 9)

- Each image **x** is stored as a flat vector of 28x28 = 784 values (gray-level value of each pixel)

- The class label of each digit is also provided

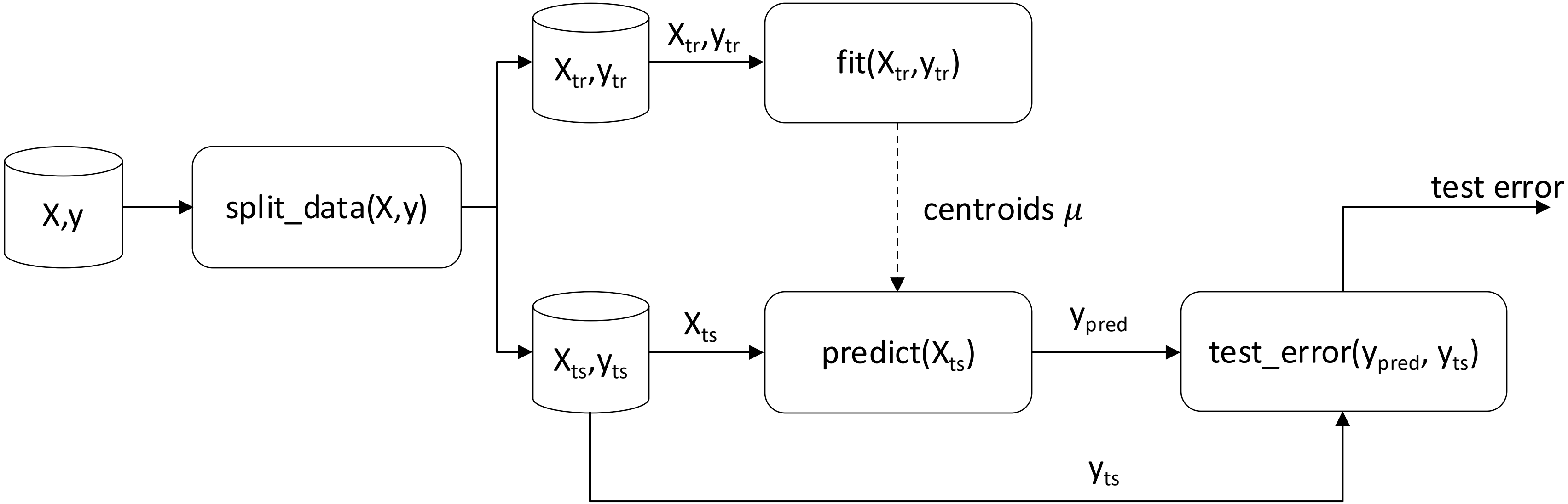
MNIST Handwritten Digit Dataset



$$\simeq \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .6 & .8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .5 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & .7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .9 & 1 & .1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .3 & 1 & .1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Coding Exercise

- We have to build a simple system that recognizes the class of digit images
- Its architecture is depicted below



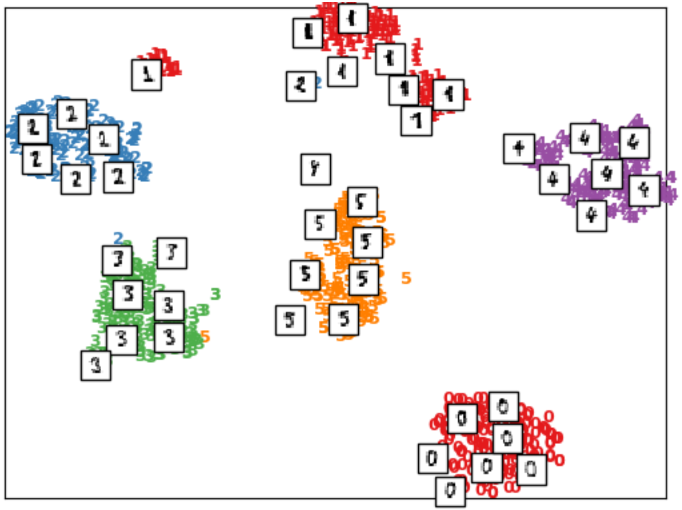
Coding Exercise

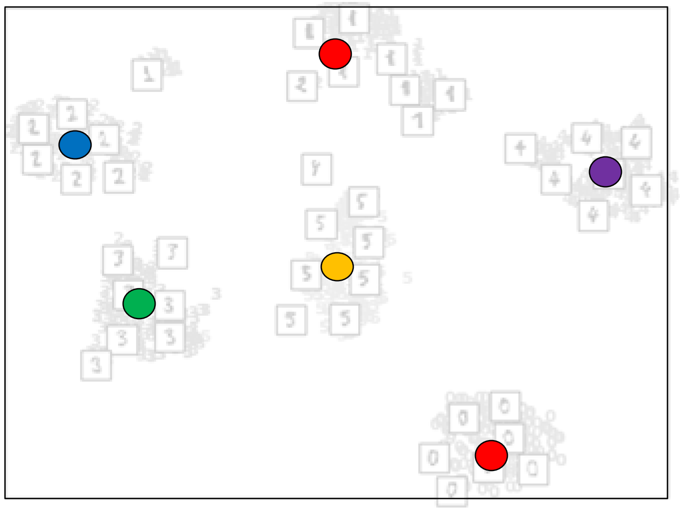
- The system should work as follows. Given the digit images as rows of X (namely, a matrix with *num_samples* rows and *num_dimensions* = 784 columns), and their labels as a vector y of *num_samples* elements:
    1. Split the input data X,y into a training (Xtr, ytr) and a test set (Xts, yts) by randomly selecting a fraction of images from X and labels from y to be part of the training set, while assigning the remaining ones to the test set;
    2. Compute the average image (centroid) for each class from the training data (fit), and display the centroids (average digits) in a plot;
    3. Compute the distance of each test image in Xts against all centroids, and predict the label of the current sample ypred as that of the closest centroid (predict);
    4. Evaluate the test error, namely, the fraction of misclassified test images (for which ypred != yts).

- This is your very first, simple implementation of the machine-learning algorithm known as Nearest Mean Centroid (NMC) classifier.

NMC Classifier: «*fit*»

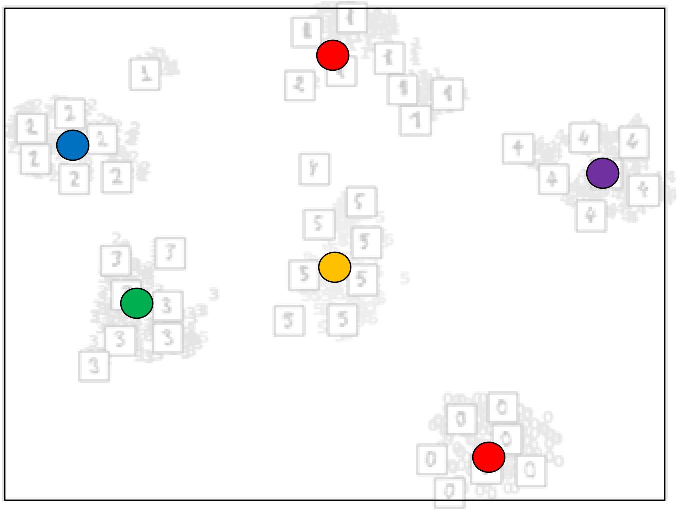
- Each digit image can be represented as a point in a vector space where each dimension corresponds to the value of each pixel
  - we can plot training digits of different classes with different colors / markers
  - *fit* estimates the average image (centroid) for each class

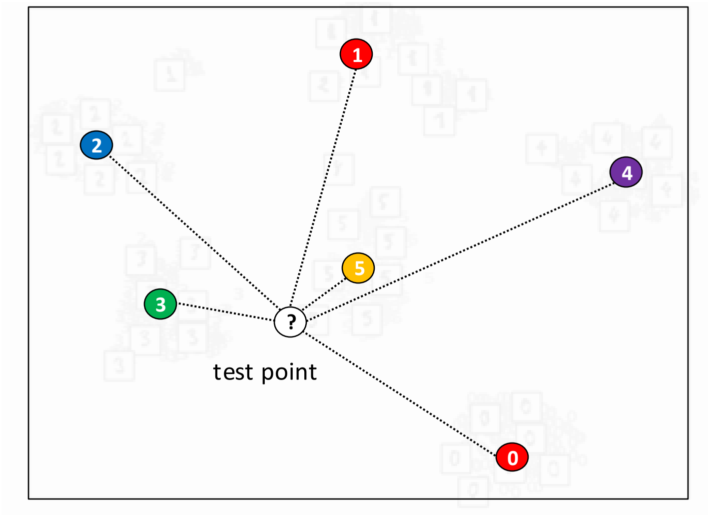




NMC Classifier: «*predict*»

- *Predict* computes the Euclidean distance of a given test point against all centroids, and assigns it to the class of the closest centroid
  - The test point ('?') below is classified as a '5', as it is closer to the centroid of class '5'
  - The length of each dashed line is the distance between the test point and the given centroid





Why Training and Test Data?

- The idea of using separate training and test data is motivated by the need of testing the performance (*test error*) of the learning algorithm on never-before-seen data, namely, data which has not been used to learn its parameters (i.e., the centroids, in our case).

- Measuring the performance of the classifier on the same data used for training is wrong and too optimistic: the learning algorithm has already been shown such images while estimating the centroids!

- *We will see more in the Machine Learning course*

Solution

The solution code (implementation) is provided separately
Here only the key algorithmic details are explained

Load Data

- load_data (filename) simply uses read_csv from *pandas* to load data from a file

- The file contains the class labels in the first column, and the digit images as rows
  - In practice, each row contains the class label and then 784 gray-level pixel values (the image)

- We thus extract the class labels y from the first column, and the digit images X as the remaining data
  - y is simply a numpy array of (*num_samples,)*
  - X is a numpy array of (*num_samples, num_pixels*)

```python
def load_data(filename):
    """
    Load data from a csv file

    Parameters
    ----------
    filename : string
        Filename to be loaded.

    Returns
    -------
    X : ndarray
        the data matrix.

    y : ndarray
        the labels of each sample.
    """
    data = read_csv(filename)
    z = np.array(data)
    y = z[:, 0]
    X = z[:, 1:]
    return X, y
```

Split Data

- This function has to split X,y into a training and a test set
- The training set has to include *tr_fraction* elements (in %)
- We first compute *n_tr*: how many samples should go in the training set
- Then, we create a vector of indices *idx,* and shuffle it. The first *n_tr* indices will be the indices of the training samples, while the remaining indices will be used to select the test samples.

```python
def split_data(X, y, tr_fraction=0.6):
    """
    Split the data X,y into two random subsets

    """
    num_samples = y.size
    n_tr = int(num_samples * tr_fraction)

    idx = np.array(range(0, num_samples))
    np.random.shuffle(idx)  # shuffle indices

    tr_idx = idx[0:n_tr]
    ts_idx = idx[n_tr:]

    Xtr = X[tr_idx, :]
    ytr = y[tr_idx]

    Xts = X[ts_idx, :]
    yts = y[ts_idx]

    return Xtr, ytr, Xts, yts
```

Fit: Compute the Centroids

- Computes the average centroid image for each class from the training data
  - *centroids* is a matrix of 10 rows x 784 columns, as it needs to store 1 centroid (784 values) per class (and we have 10 classes)
- First, we initialize *centroids* (a matrix of zeros)
- Then, we iterate over the classes, and extract the samples *xk* belonging to class *k*
  - *ytr==classes[k]* returns a boolean vector (true if the label in ytr is equal to the label of class k) which can be used to select the samples of class k from Xtr
- These samples are then averaged using np.mean along the correct axis to obtain the corresponding centroid, which is stored as row *k* in centroids

```
def fit(Xtr, ytr):
    """
    Compute the average centroid
    for each class
    """
    classes = np.unique(ytr)
    num_classes = classes.size

    centroids = np.zeros(
            shape=(num_classes, Xtr.shape[1]))

    for k in range(num_classes):
        xk = Xtr[ytr == classes[k], :]
        centroids[k, :] = np.mean(xk, axis=0)

    return centroids
```

Predict Classes of Unseen Test Data

- *Predict* computes a matrix *dist_euclidean* of size n_ts x 10, being n_ts the number of test samples in Xts, which contains the values of the Euclidean distance computed between each test sample and the 10 class centroids
- Then, the index of the closest centroid for each test sample is retrieved with np.argmin
  - if the class labels are contiguous (0,1,2,...9), then these indices are already the class labels *ypred* and can be returned (if classes is None, we assume that this is the case)
  - otherwise, we use them to index the class labels (passed as the input parameter *classes*) and return them

```python
def predict(Xts, centroids, classes=None):
    """
    Predicts the label of each sample in Xts based on
    the closest centroid.
    """
    dist_euclidean = euclidean_distances(Xts, centroids)
    ypred = np.argmin(dist_euclidean, axis=1)
    if classes is not None:
        ypred = classes[ypred]
    return ypred
```

Have a look at the code too. There is a more complex implementation of this function where the distance matrix is computed explicitly – not using a library function!

Compute the Test Error

- This function just counts the fraction of different elements between the predicted labels ypred and the true labels yts
  - Cast to float is required to avoid integer division (which would return 0 or 1 in this case)

```
def compute_ts_error(ypred, yts):
    """
    Compute the fraction of elements that
    are different in ypred and yts
    (classification errors)

    Parameters
    ----------
    ypred: the set of predicted class labels
    yts: the true labels of test samples

    Returns
    -------
    test_error: the classification error
    """
    test_error = np.sum(ypred != yts) / float(ypred.size)
    return test_error
```

From functions to classes: the NMC classifier class

Implementation of the NMC classifier as a Python class

The NMC Classifier as a Python Class

- Finally, the previous functions can be put together in a class, representing our learning algorithm.

- _centroids is a protected attribute that will store the centroids (the corresponding property allows reading their values from the outside)

- fit and predict slightly change to use the internal variable _centroids (rather than returning the centroids and taking them as input)

```python
class NMC(object):
    """Nearest Mean Centroid (NMC) classifier."""

    def __init__(self):
        self._centroids = None
        self._classes = None  # class labels may not be contiguous

    @property
    def centroids(self):
        return self._centroids

    def fit(self, Xtr, ytr):
        self._classes = np.unique(ytr)
        num_classes = self._classes.size
        self._centroids = np.zeros(shape=(num_classes, Xtr.shape[1]))
        for k in xrange(num_classes):
            xk = Xtr[ytr == self._classes[k], :]
            self._centroids[k, :] = np.mean(xk, axis=0)

    def predict(self, Xts):

        if self._centroids is None:
            raise ValueError("The classifier is not trained. Call fit!")

        dist_euclidean = euclidean_distances(Xts, self._centroids)
        idx_min = np.argmin(dist_euclidean, axis=1)
        yc = self._classes[idx_min]
        return yc
```

Extras

Additional Programming Exercises

1. Create an abstract class for data loaders, with an abstract method `load_data()`
   - Inherit the MNIST loader from the abstract class
   - Create a data loader for the LFW dataset

2. Create a package of classes that generate random perturbations
   - From the Uniform distribution
   - From the Gaussian distribution

Write a main program to show how the test error of the NMC classifier increases as the level of perturbation applied to the MNIST digit images increases.