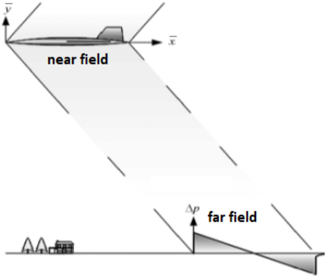
Sonic boom prediction in far field for supersonic vehicles via open-source software.

Kirill Vatutin, Andrey Epikhin, Matvey Kraposhin

11 December 2020.

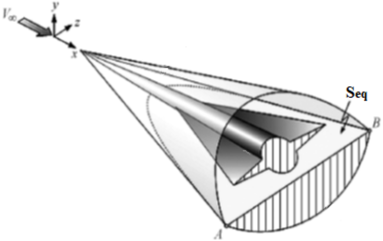
Purpose of the work



stages of work:

- Improvement of the implementation of quasi-gasdynamic(QGD) equations obtained on the basis of the Navier-Stokes equations to determine the pressure drop in the near field.
- Development of an implementation for forecasting a sound wave in the far field using Whitham functions based on OpenFOAM.

- **1. Definition of an equivalent body of revolution**



a) determination of the optimal distance to a reference surface.
Reference surface must be perpendicular to the direction of gravity.
b) interpolating values to a reference surface;
c) computation of $dS_{eq}(x)/dx$:

$$\frac{dS_{eq}(x)}{dx} = -\frac{1}{V_{inf}} \int_A^B (\beta u + v)\, dz = -\frac{2\beta}{V_{inf}} \int_A^B u\, dz = -\frac{2}{V_{inf}} \int_A^B v\, dz$$

Algorithm.

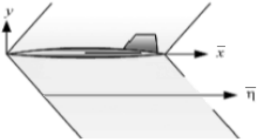
- **2. Computation of Whitham function $\bar{F}(\bar{\eta})$:**

$$\bar{S}'_{eq}(\bar{x}) = \frac{\gamma M^2_{inf}}{2\pi L\sqrt{2\beta}} S'_{eq}(x) \quad \Rightarrow \quad \bar{\Phi}(\bar{\eta}) = \int_0^{\bar{\eta}} \frac{\bar{S}'_{eq}(\bar{x})}{\sqrt{\bar{\eta}-\bar{x}}}\, d\bar{x} \quad \Rightarrow \quad \bar{F}(\bar{\eta}) = \frac{\partial}{\partial\bar{\eta}} \bar{\Phi}(\bar{\eta})$$

- **3. Determination of damping factors k, k_1, k_2:**

$$k = \frac{\gamma+1}{2\pi\sqrt{2}} \frac{M^{3/2} \left| \int_0^H \frac{dy}{\sqrt{\rho|(h-y)(h-|h-y|\gamma)|}} \right|}{a^2_{inf}} \sqrt{\frac{l(0)}{\rho_{inf}}} \frac{C_y}{l^2\sqrt{l}}$$

$$k_1 = \frac{1}{\pi} \sqrt{\frac{M}{2l(0)|1-\gamma|}} \frac{C_y}{l\sqrt{l}}, \quad k_2 = \frac{l}{M_{inf} a}$$



- **4. Final plot of pressure drop on the ground:**

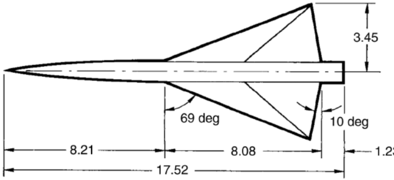
$$\Delta p = k_1 \bar{F}(\bar{\eta}), \; t = k_2[\bar{\eta} - k\bar{F}(\bar{\eta})]$$

Statement of the problem for near field.

Physical parameters:

Mach number	$M = 1.68,$
Angle of attack	$AoA = 4.74,$
Unperturbed air pressure at the height of the aircraft flight	$p_{inf} = 101325\,[Pa],$
Acceleration of gravity	$g = 9.8\,[m/s^2\,].$
Adiabatic exponent	$\gamma = 1.4$

Geometrical parameters: $L = 0.1752[m]$



Boundary conditions on inlet and bottom:

$$U_x = M \cdot a \cdot cos(AoA); U_y = M \cdot a \cdot sin(AoA);$$

$$U_z = 0;\ p = 101325[Pa];\ T = 300[K];$$

Conditions on body:

$$\vec{U} = 0;\ \frac{\partial p}{\partial \vec{n}} = 0;\ \frac{\partial T}{\partial \vec{n}} = 0;$$

Condition on the other boundaries:

$$\frac{\partial \vec{U}}{\partial \vec{n}} = 0;\ \frac{\partial p}{\partial \vec{n}} = 0;\ \frac{\partial T}{\partial \vec{n}} = 0;$$

Numerical parameters:

$$\alpha_{QGD} = 0.5,\ \ Sc_{QGD} = 1,\ \ Pr_{QGD} = 0.72.$$

ISPRAS

The system of quasi-gasdynamic equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \left(\vec{U} - \vec{W} \right) = 0, \tag{1}$$

$$\frac{\partial \rho \vec{U}}{\partial t} + \nabla \cdot \left(\rho(\vec{U} - \vec{W}) \otimes \vec{U} \right) + \nabla p - \vec{F} = \nabla \cdot \hat{\Pi}, \tag{2}$$

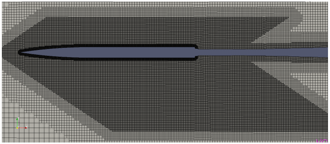
$$\frac{\partial \rho e}{\partial t} + \nabla \cdot \left(\rho(\vec{U} - \vec{W})(e + \frac{p}{\rho}) \right) + \nabla \cdot \vec{q} = \nabla \cdot (\hat{\Pi} \cdot \vec{U}). \tag{3}$$

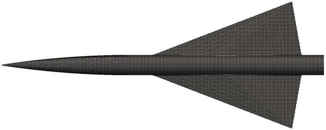
Here, viscous stress tensor: $\hat{\Pi}$;
extra speed: $\vec{W} = \frac{\tau}{\rho} \left(\nabla \cdot (\rho \vec{U} \otimes \vec{U}) + \nabla p - \vec{F} \right)$;

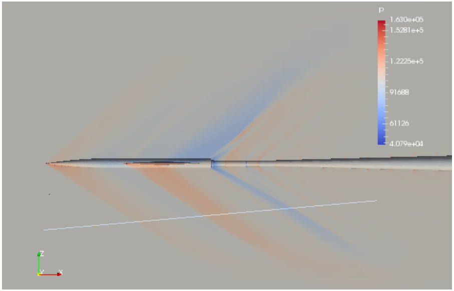
heat flux: $\vec{q} = -\kappa \nabla T - \tau \vec{U} \rho (\vec{U} \cdot \nabla u + p \vec{U} \cdot \nabla \frac{1}{\rho}$;
internal energy per unit mass: $u = \frac{p}{\rho(\gamma - 1)}$;
thermal conductivity: $\kappa = \frac{\mu C_p}{Pr}$;
regularization parameter: $\tau = \frac{\mu}{p Sc} + \alpha_{QGD} \frac{\Delta_h}{a}$;

gravity force: $\vec{F} = \rho \vec{g}$.

Results of calculations of sonic boom in the near field.

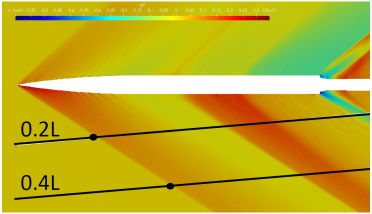


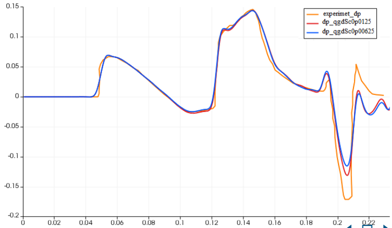




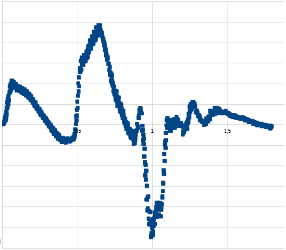
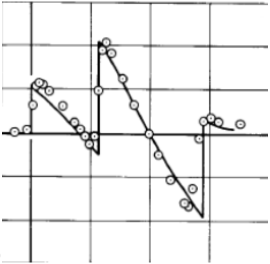
Results of calculations of sonic boom in the near field.

Comparison of the accuracy of the results of modeling the pressure drop in the near field at a distance of 0.2 and 0.4 of the model length with the experiment





Results of calculations of sonic boom in far field.

a)  b) 

Qualitative comparison of a) the obtained normalized pressure drop in the far field at a distance of 3.6 from the aircraft length with b) the experimental normalized pressure drop.

Further directions of work to improve the results:

- Study of the criterion for the selection of QGD parameters when modeling various layouts and flight modes of supersonic aircraft.
- Formulation of selection criteria reference surface in the near field for increased simulation accuracy.
- Estimation of the error in modeling the pressure drop on the ground with experiment.

Sirius. Summer: start your project.

“Sirius. Summer: Start Your Project ” is a part of the concept of healthy recreation for children, designed to attract talented young people to work on urgent tasks of Russian science and priority technologies for the development of their regions.

Thanks for attention.

QGDsolver for OpenFOAM: https://github.com/unicfdlab/QGDsolver

This work was supported by the Russian Foundation for Basic Research (grant No. 19-11-00169)