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Spatial decomposition of covariance functions in a process of data assimilation by the Generalized Kalman filter method

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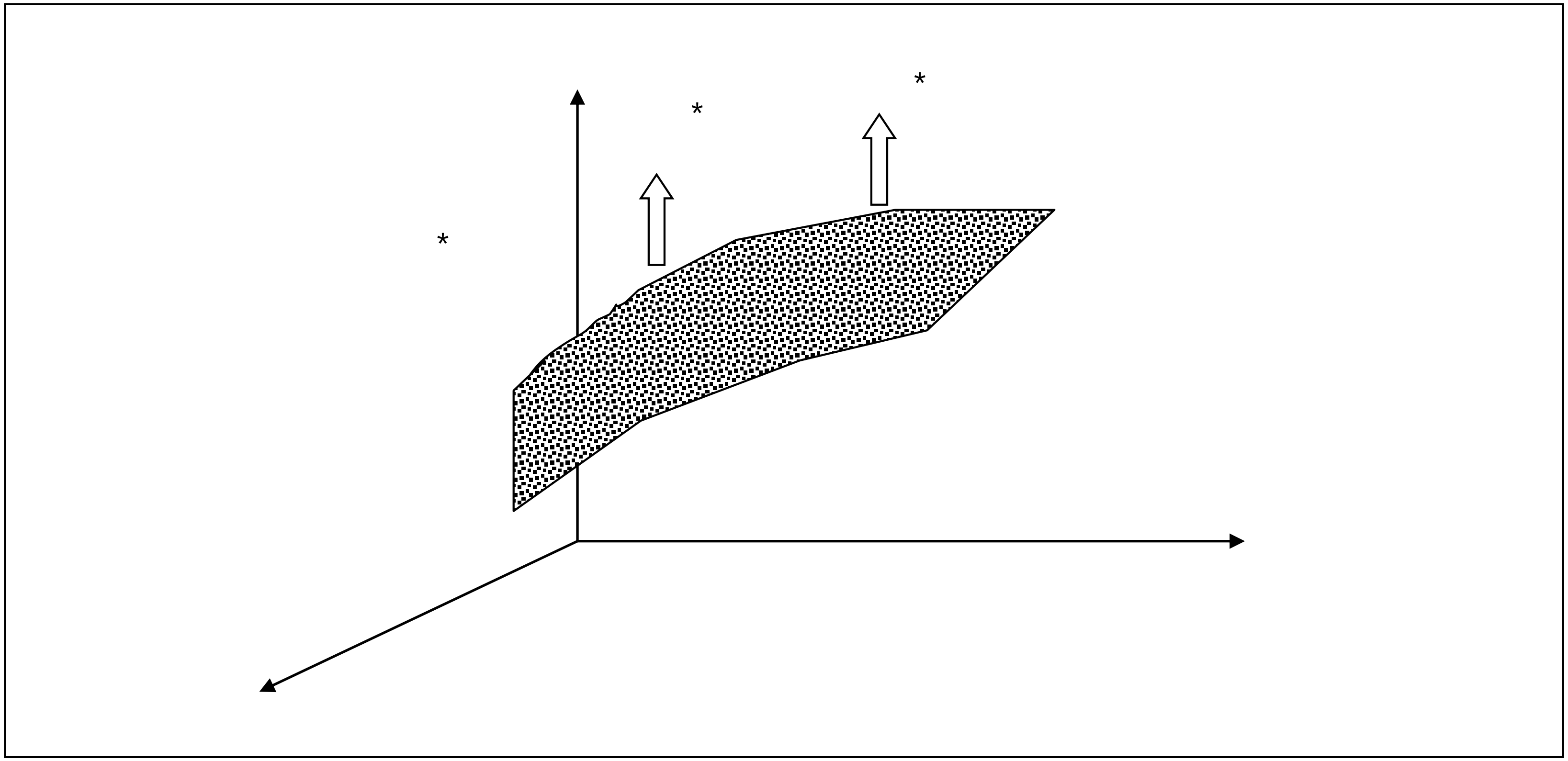
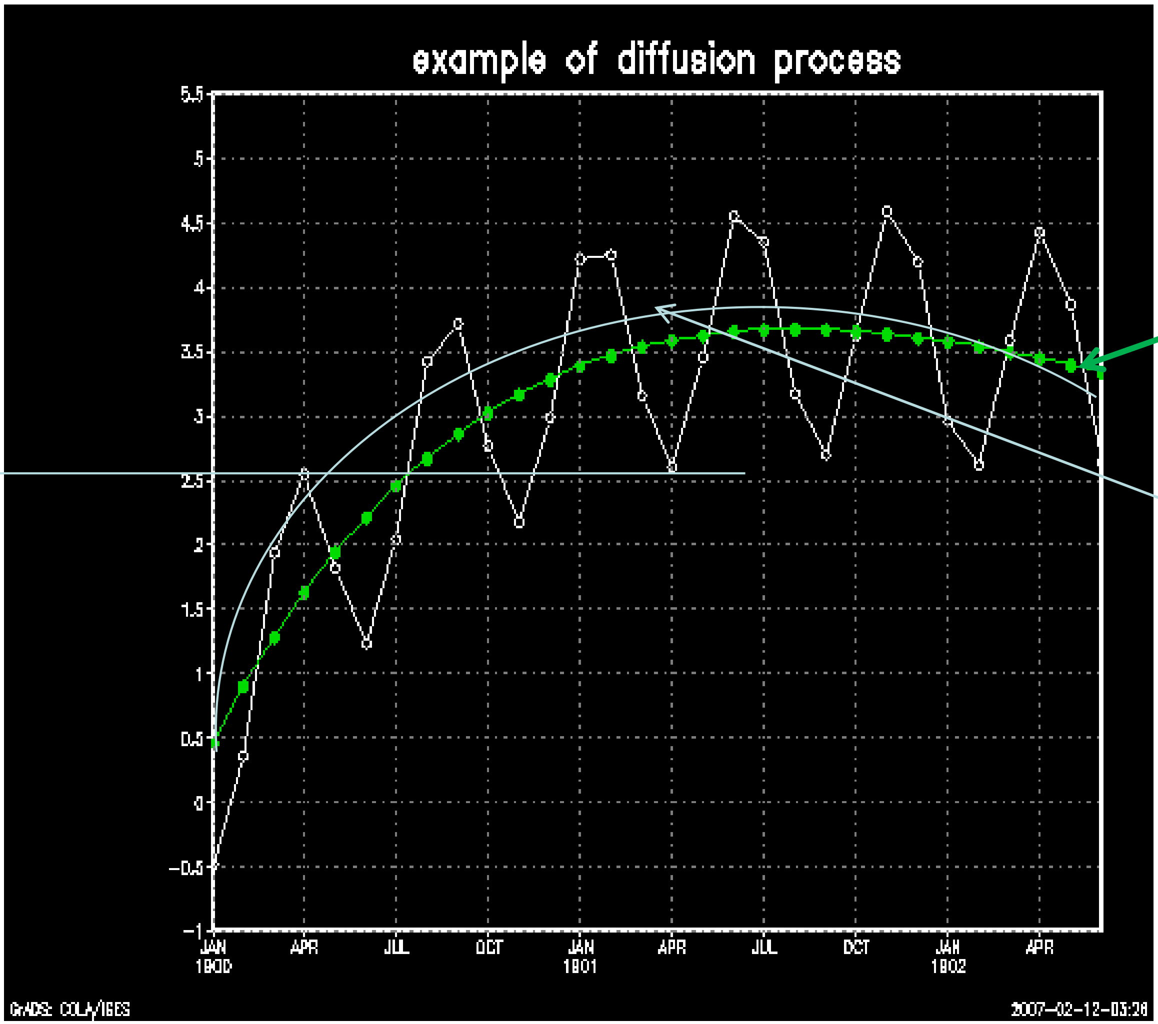
The goal of investigation:
Modeling of the ocean circulation in the North Atlantic using the NEMO model with the data assimilation method GKFDA and comparison of various numerical calculation schemes.

Geometric interpretation

The main idea of all data assimilation methods is the best approximation of the model fields to observed data maintaining the model conservation laws.





$$a(t) = (I - KH)\Lambda$$

$$a(t) = \Lambda(t)$$

Ocean circulation model NEMO

NEMO – Nucleus for European Modelling of the Ocean

- created in Pierre Simon Laplace Institute (Paris)
- consists of parallel executed several modules

NEMO structure

- NEMO-OCE solves the basic Navier-Stokes equations and models ocean thermodynamics.

- NEMO-ICE simulates the dynamics of sea ice, including changes in ice thickness and salinity.

- NEMO-TOP models transport routes and biogeochemical processes

- Interface block XIOS

Ocean Model NEMO-OCE

Navier-Stokes equations with Boussinesq assumptions made

$$\frac{\partial \mathbf{U_h}}{\partial t} = \left[(\nabla \times \mathbf{U}) \times \mathbf{U} + \frac{1}{2} \nabla (\mathbf{U}^2) \right]_{\mathbf{h}} - f\, \mathbf{k} \times \mathbf{U_h} - \frac{1}{\rho_0} \nabla_{\mathbf{h}} p + \mathbf{D^U} + \mathbf{F^U}$$

$$\frac{\partial p}{\partial z} = -\rho g, \quad \nabla \cdot \mathbf{U} = 0, \quad \rho = \rho(T, S, p)$$

$\mathbf{U_h}$ is the horizontal velocity, $\mathbf{U} = \mathbf{U_h} + w\mathbf{k}$, w is the vertical component of velocity
ρ is water density, ρ_0 is horizontal average density,
g is gravity parameter f is Coriolis acceleration,

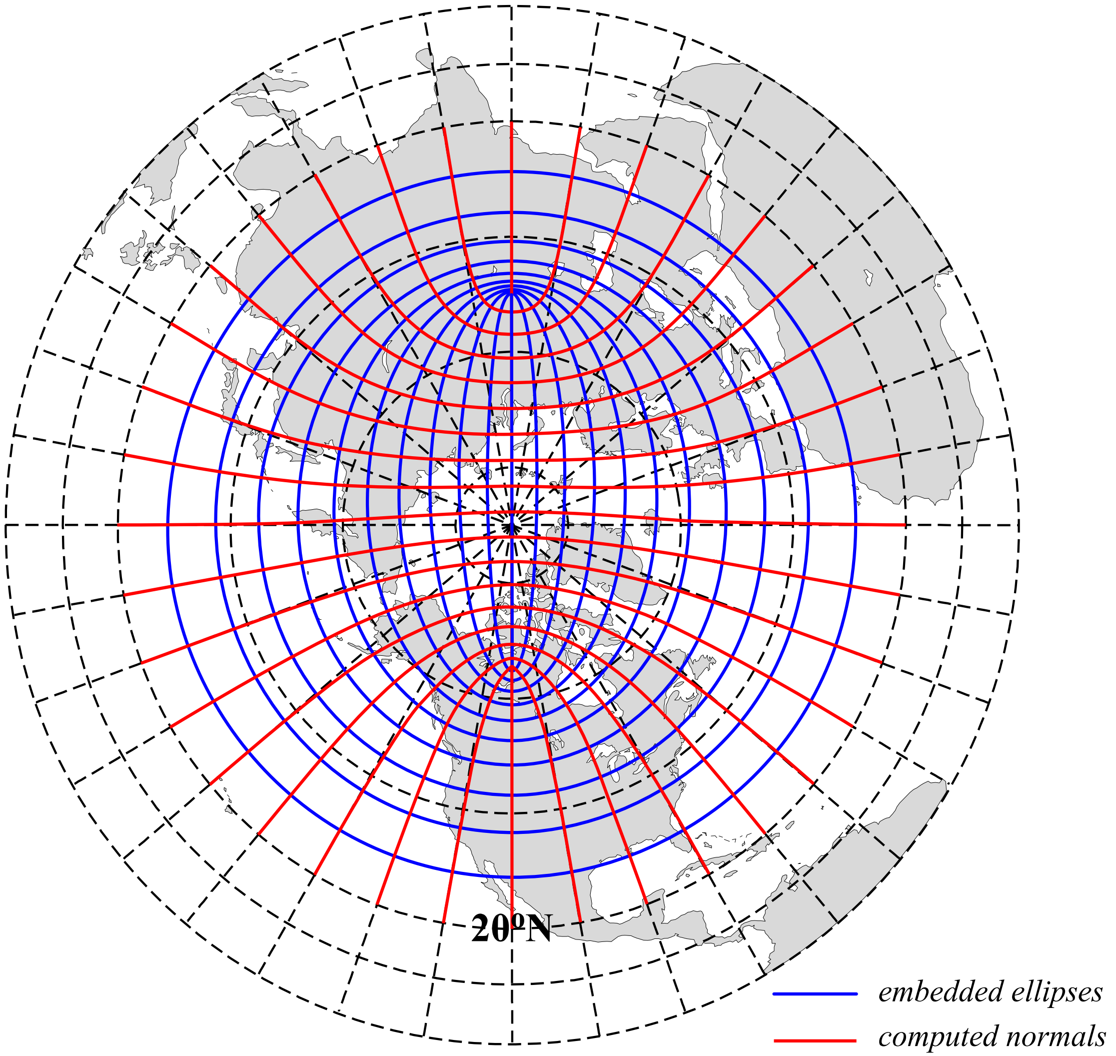
The equation of heat transfer

$$\frac{\partial T}{\partial t} = -\nabla \left(T\, \mathbf{U} \right) + D^T + F^T$$

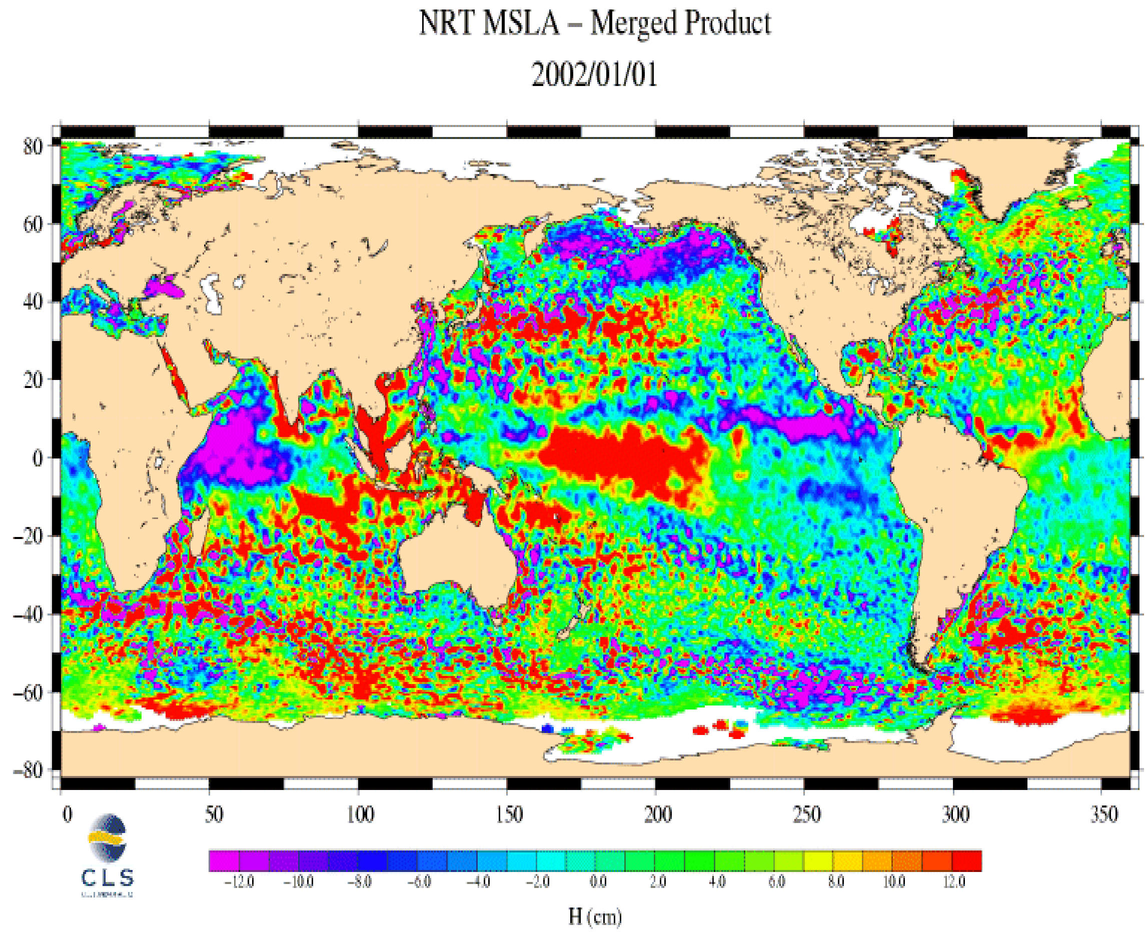
The equation for salinity

$$\frac{\partial S}{\partial t} = -\nabla \left(S\, \mathbf{U} \right) + D^S + F^S$$

ORCA Three Pole Ocean Mesh for NEMO



OBSERVATION DATA
by satellites Jason-2

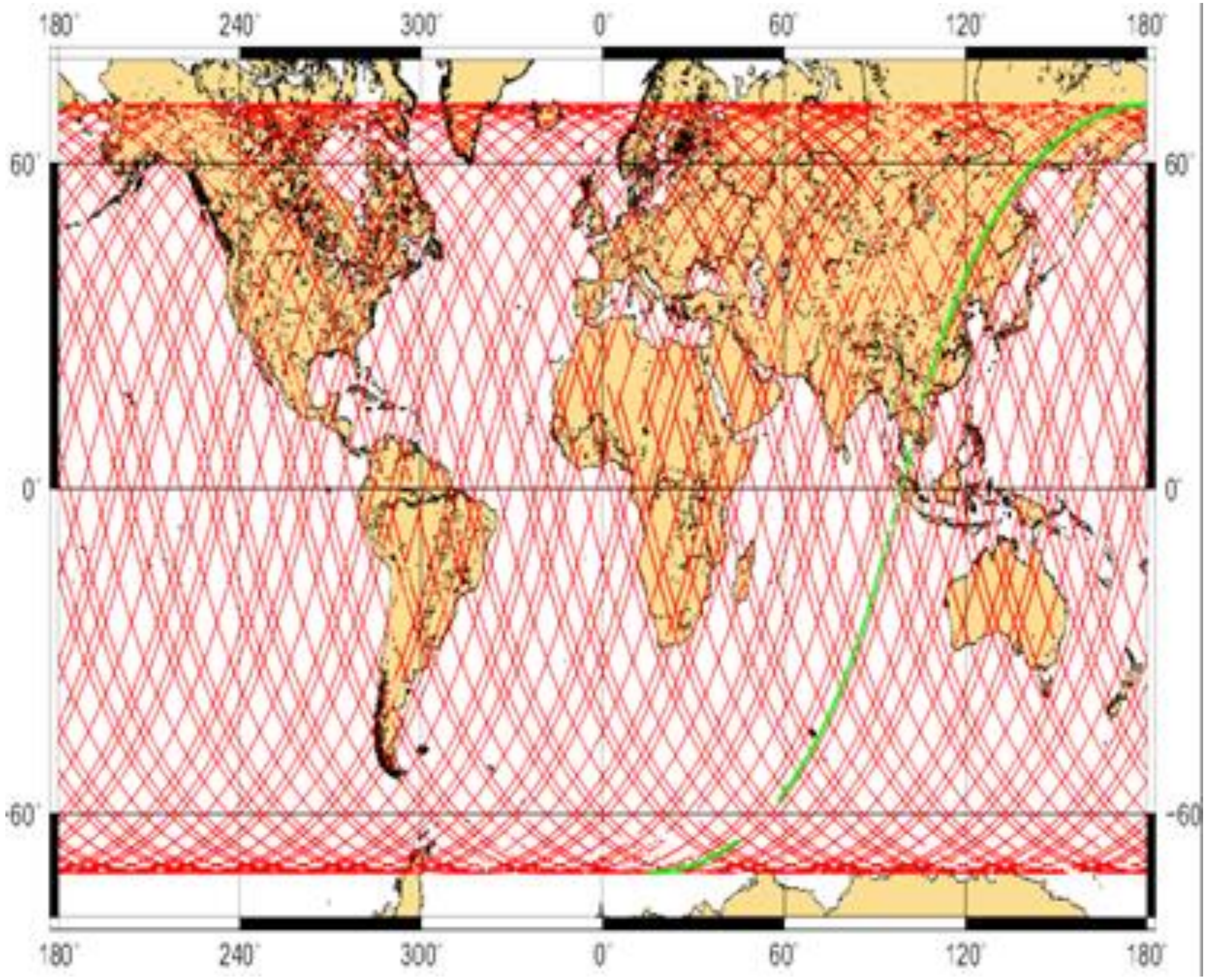


Sea surface height (SSH)

www.aviso.oceanobs.com

OBSERVATION DATA

Global scheme of satellite tracks Jason-2



Main assimilation model

$$X_a = X_b + K(Y - HX_b) \qquad (1)$$

X is the state vector of the ocean parameters, which includes the temperature (θ), the salinity (S), the sea surface height (SSH), the sea surface height anomaly (SSHA) (is the difference between the SSH and the long-term average at the same points), at all

X_b is the background state vector

X_a is the analysis state vector – the ocean parameters after assimilation

Y is the vector of the observable parameters $\dim Y < \dim X$

H is the projection operator matrix

K is Kalman gain matrix (Kalman filter)

A method for constructing the gain matrix K,
which provides better data assimilation was developed.

Generalized Kalman filter for data assimilation — GKFDA method

$$\frac{\partial X}{\partial t} = \Lambda(X, t) \qquad (1)$$

$$X(0) = X_0$$

$$0 \le t \le T \qquad 0 = t_0 < t_1 < \ldots. < t_N = T, \quad t_{n+1} = t_n + \Delta t$$

$$X_{a,n} = X_{b,n} + K_n(Y_n - HX_{b,n}) \qquad (2)$$

$$K_n = (\sigma_n^2)^{-1}(\Lambda_n - C_n)(H\Lambda_n)^{\mathrm{T}} Q_n^{-1} \qquad (3)$$

$$\sigma_n^2 = (H\Lambda_n)^{\mathrm{T}} Q_n^{-1}(H\Lambda_n) \qquad (4)$$

Decomposition of the covariance matrix into the set of eigenvalues and eigenvectors

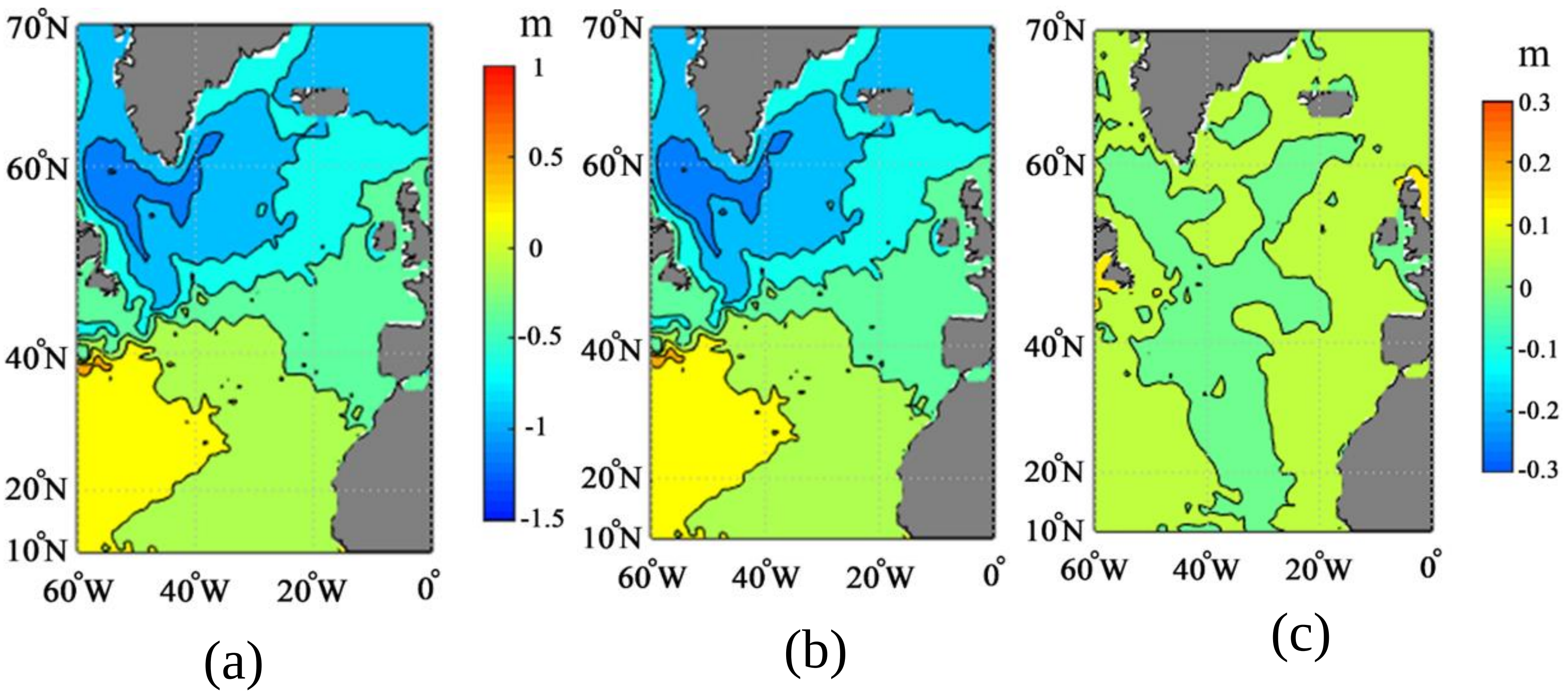
$$Q_n^{-1}(Y_n - HX_{b,n}) = \sum_{i=1}^{N} \alpha_i^n Q_n^{-1} e_i^n = \sum_{i=1}^{N} \alpha_i^n (\lambda_i^n)^{-1} e_i^n$$

$$X_{a,n} = X_{b,n} + (\sigma_n^{-1})^2 (\Lambda_n - C_n) \sum_{i=1}^{N} (\lambda_i^n)^{-1} \alpha_i^n \kappa_i^n$$

where $\kappa_i^n = (H\Lambda_n)^T e_i^n$

RESULTS OF THE NUMERICAL SIMULATION
using observation data by satellites from archive AVISO

Results of calculations of the SSH fields with the GKFDA method:
(a) by using inversion of the covariance matrix;
(b) by using decomposition of the covariance matrix into the set of eigenvalues and eigenvectors;
(c) difference between these fields

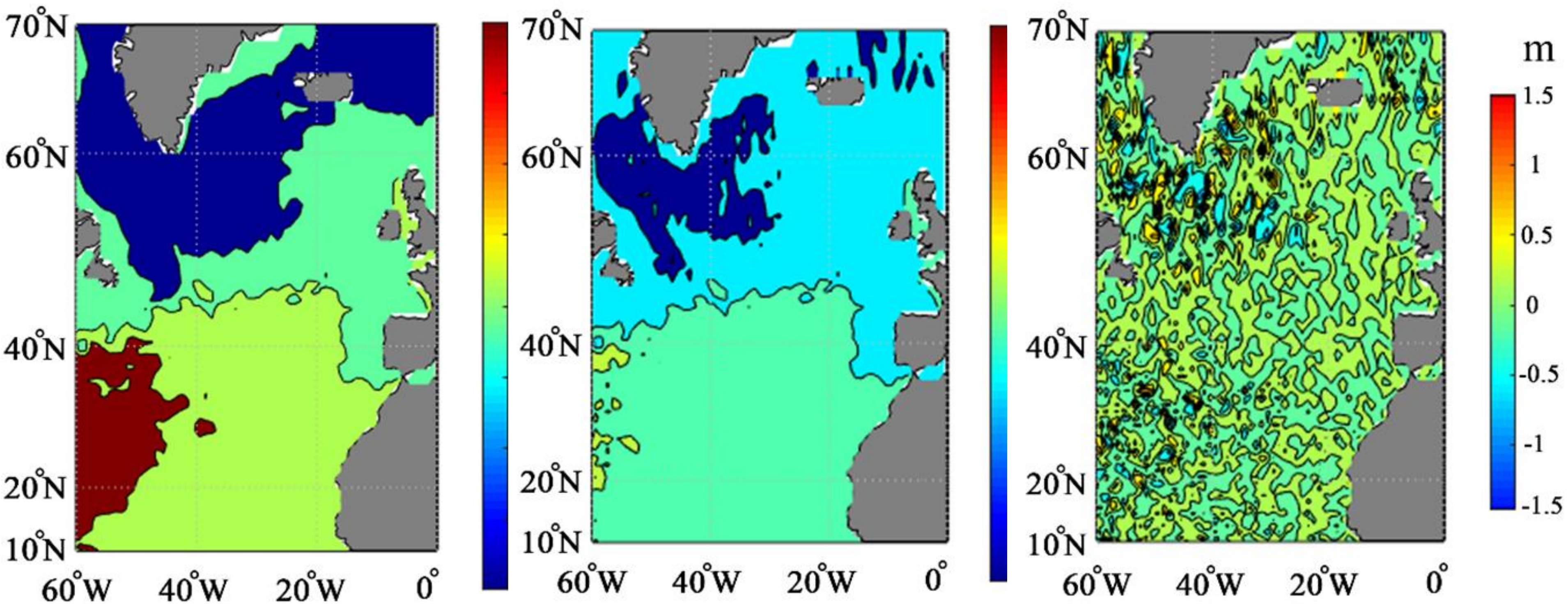


Eigenvalues of the covariance matrix decomposition

Date	λ_1^1	λ_2^1	λ_3^1
July 1, 2013	0.9	0.03	0.002

Sea Surface Height

The values of the first (a) second (b) and third (c) eigenvectors in the covariance matrix decomposition



(a) (b) (c)

Conclusion

The decomposition of the matrix into the sum of eigenvectors allows us to study in detail the structure of the correcting term in formula for matrix decomposition. In particular, it determes in which regions the observational data will have the largest impact and in what way they make their contribution to the corrected filed.