On Implementation of Discontinuous Galerkin Scheme for Gas Dynamics Problems Using Open-Source Software

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Introduction

Gas dynamics specifics

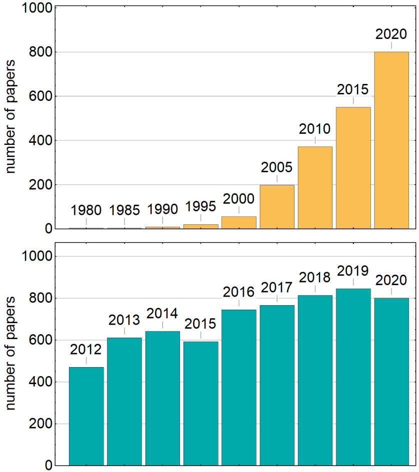
- Discontinuities in solution
- Hydro- and gas dynamic instabilities (Rayleigh — Taylor, Kelvin — Helmholtz, *etc.*)
- Directions of disturbancies propagation in subsonic and supersonic flows

Methods

- **FDM** — only structured meshes, simple geometry
- **FEM** — unstructured meshes, continuous solution, high-order
- **FVM** — unstructured meshes, low-order

Brief DG review

Publication statistic at Scopus



Discontinuous Galerkin method

FEM + FVM = DG

Advantages

- Compact stencil
- Easy to increase the accuracy order

Main difficulties

- Limitation of solution in case of strong discontinuities
- Implementation complexity

Euler equations

$$\left.\begin{aligned} &\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0, \\ &\frac{\partial \rho \mathbf{v}}{\partial t} + \operatorname{div}\left(\rho \mathbf{v} \otimes \mathbf{v} + p\hat{\mathcal{I}}\right) = \mathbf{0}, \\ &\frac{\partial e}{\partial t} + \operatorname{div}\left[(e+p)\,\mathbf{v}\right] = 0 \end{aligned}\right\} \quad \begin{aligned} &\frac{\partial \mathbf{U}}{\partial t} + \operatorname{div}\mathcal{F}(\mathbf{U}) = \mathbf{0}, \\ &\mathcal{F}(\mathbf{U}) = \operatorname{diag}\{\mathbf{F}, \mathbf{G}, \mathbf{H}\} \end{aligned} \quad \begin{aligned} \mathbf{U} &= [\rho,\ \rho u,\ \rho v,\ \rho w,\ e]^T, \\ \mathbf{F} &= [\rho u,\ \rho u^2 + p,\ \rho uv,\ \rho uw,\ (e+p)u]^T, \\ \mathbf{G} &= [\rho v,\ \rho vu,\ \rho v^2 + p,\ \rho vw,\ (e+p)v]^T, \\ \mathbf{H} &= [\rho w,\ \rho wu,\ \rho wv,\ \rho w^2 + p,\ (e+p)w]^T. \end{aligned}$$

ρ – density, $\mathbf{v} = (u,\ v,\ w)^T$ – vector of velocity, p – pressure, $e = \rho\varepsilon + \rho\frac{\mathbf{v}^2}{2}$ – volumetric total energy

EoS for perfect gas

$$p = (\gamma - 1)\rho\varepsilon, \quad \gamma > 1$$

General approach

Solution approximation on cell

$$\mathbf{U}_h(\vec{x}, t) = \sum_{j=1}^{N} \sum_{s=0}^{N_f} \mathbf{U}_j^{(s)}(t)\varphi_j^{(s)}(\vec{x}), \qquad \varphi_j^{(s)}(\vec{x}) \in \left\{ f(\vec{x}) : f|_{I_k} \in P^m(I_k),\ k = \overline{1,\ N} \right\}$$

Spatial discretization

$$\frac{d}{dt}\left(\sum_{s=0}^{N_f} \mathbf{U}_j^{(s)}(t) \int\limits_{I_j} \varphi_j^{(s)} \varphi_j^{(r)} d\Omega \right) - \int\limits_{I_j} \mathcal{F}_j \cdot \nabla \varphi_j^{(r)} d\Omega + \int\limits_{\partial I_j} (\mathbf{n} \cdot \mathcal{F}_j)\varphi_j^{(r)} d\Gamma = 0,$$

Time discretization: Runge–Kutta method

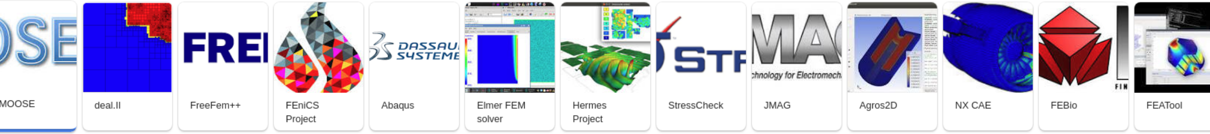
$$\mathbf{U}^* = \mathbf{U}^n + \tau \mathbf{L}_h(\mathbf{U}^n)$$

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \frac{1}{2}\mathbf{U}^* + \frac{1}{2}\tau \mathbf{L}_h\left(\mathbf{U}^*\right)$$

What about big codes?

”Is there any software or source code of Discontinuous Galerkin method?”[1]

- Last answer: 09.11.2020
- **13** codes are proposed
- **10** codes are alive now



[1]ResearchGate, 2014

What is offered in large libraries?

Diversity of features

- DG as one of the FEM-based approaches
- different solvers for various problems
- unstructured meshes and adaptive mesh refinement
- massive parallelism

- Common structure of codes: FEM libraries + DG support + some addons
- First steps in DG: solvers for advection equation
- Most of packages: **DG only for problems with continuous solution**
- Compressible flow solvers are rare

How to choose the code?

Code “at a glance”

- contains needed features;
- documentation and set of tutorials;
- community (workshops, feedbacks...);
- compatibility with other formats.

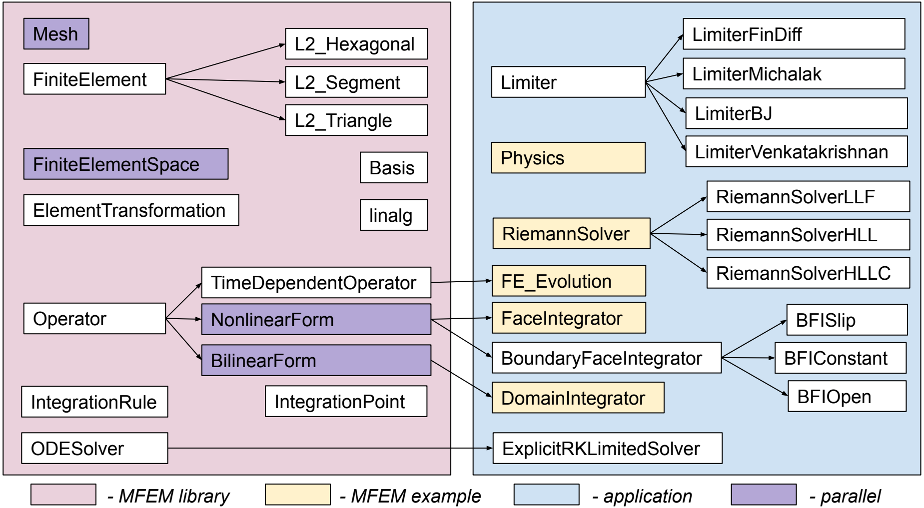
First experience

- fast and comfortable installation;
- running of tutorials;
- verification with own tests;
- readability and flexibility of code.

MFEM

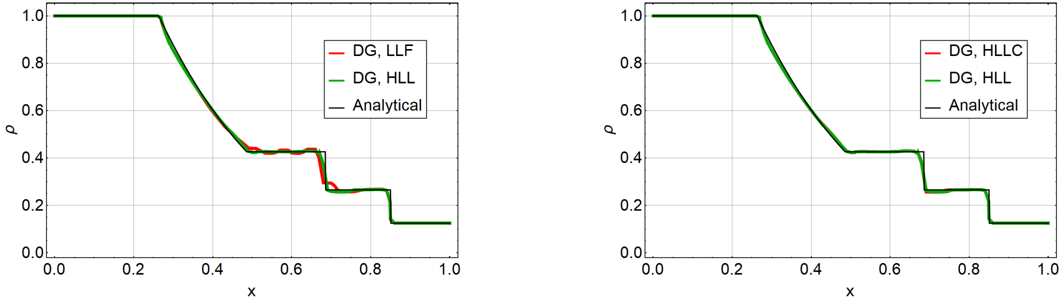
- C++
- Lawrence Livermore National Laboratory, 2019
- FEM and DG for continuous solutions
- Lagrange polynomials
- MPI parallelism
- Many features for meshes
- Supports VTK, gmsh and other formats

RKDG application based on MFEM library



Sod problem

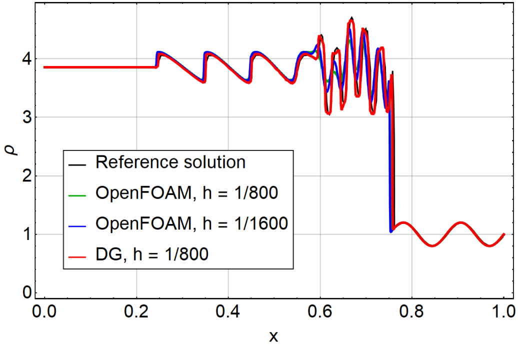
$$(\rho, u, v, w, p) = \begin{cases} (1, 0, 0, 0, 1), & x \leq 0.5; \\ (0.125, 0, 0, 0, 0.1), & x > 0.5. \end{cases}$$



Barth – Jespersen, Co = 0.1, $t^* = 0.2$, $h = 0.01$

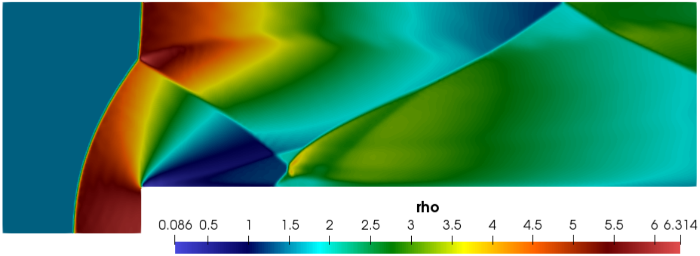
Shu – Osher problem, M = 3

$$(\rho, u, v, w, p) = \begin{cases} (3.857143, 2.629369, 0, 0, 10.3333), & x \leq 0.125; \\ (1 + 0.2\sin 8x, 0, 0, 0, 0.1), & x > 0.125. \end{cases}$$

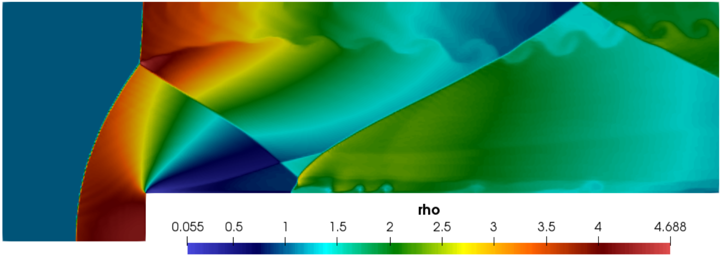


Barth – Jespersen, HLL, Co = 0.1, t = 0.178*

Wind Tunnel with a Forward Step, M = 3

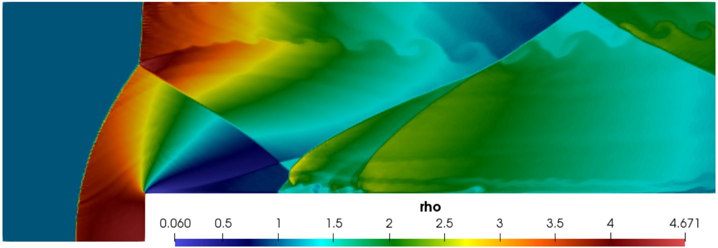


OpenFOAM
32 cells per step height
$Co = 0.1$

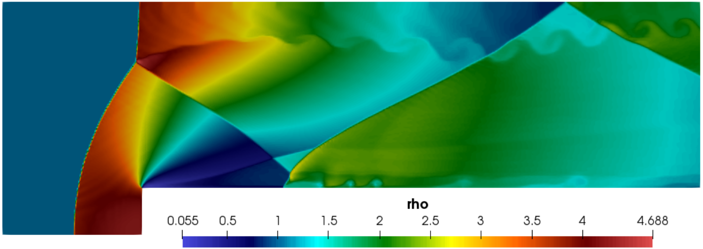


DG
32 cells per step height
$Co = 0.1$

Wind Tunnel with a Forward Step, M = 3

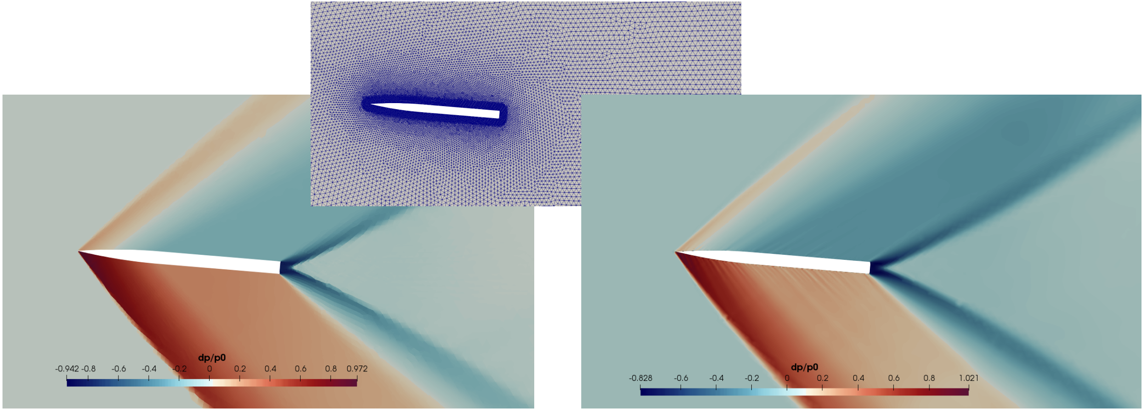


DG
32 cells per step height
$\mathit{Co} = 0.1$
no linearization



DG
32 cells per step height
$\mathit{Co} = 0.1$
additional linearization
in troubled cells

2D Wing, M = 1.66

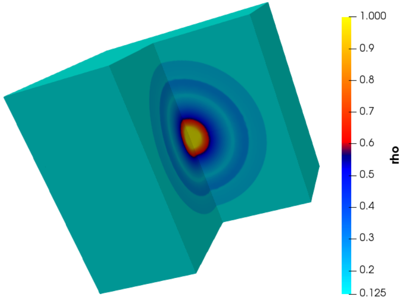


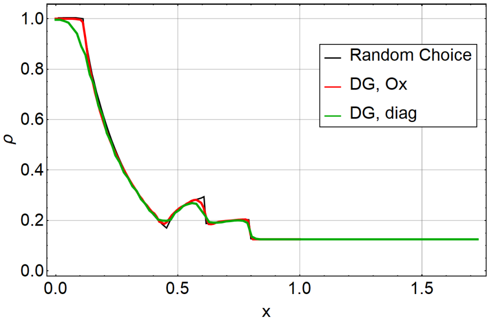
DG
Co = 0.1

OpenFOAM
Co = 0.1

Sod-like spherical explosion

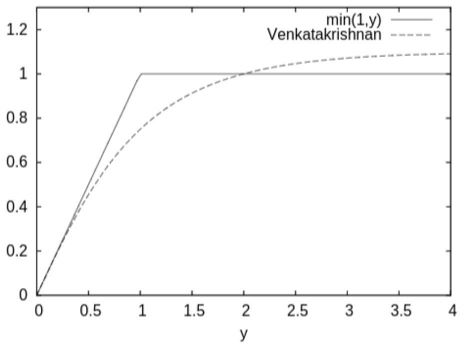
$$(\rho, u, v, w, p) = \begin{cases} (1, 0, 0, 0, 1), & r \leq 0.4; \\ (0.125, 0, 0, 0, 0.1), & r > 0.4. \end{cases}$$

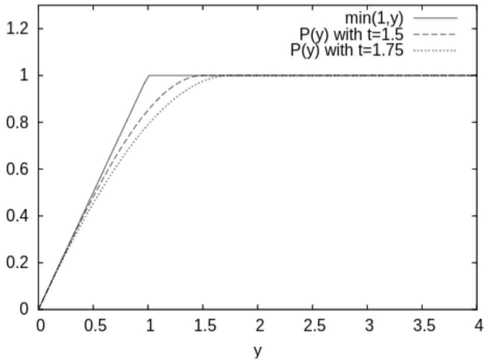




BJ, HLL, Co = 0.1, t^* = 0.25

LMP-based limiters[2]

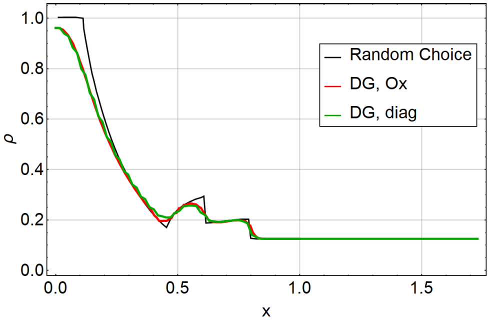




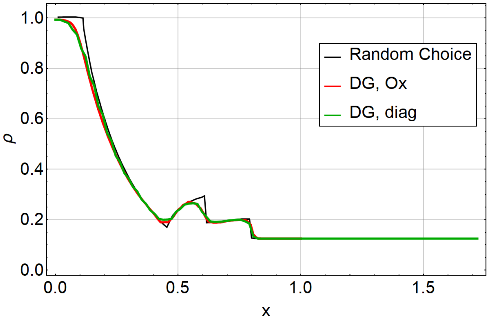
[2]Michalak K., Ollivier-Gooch C. Limiters for unstructured higher-order accurate solutions of the Euler equations // 46th AIAA AerospaceSciences Meeting and Exhibit, 2008

Sod-like spherical explosion

$$(\rho, u, v, w, p) = \begin{cases} (1, 0, 0, 0, 1), & r \le 0.4; \\ (0.125, 0, 0, 0, 0.1), & r > 0.4. \end{cases}$$



Venkatakrishnan, HLL, Co = 0.1, t^* = 0.25



Michalak, HLL, Co = 0.1, t^* = 0.25

Conclusions and Perspectives

1. The RKDG application for compressible gas flows simulation based on free finite element library MFEM has been implemented.
2. First results for MFEM-based application were demonstrated. Four test cases (1D and 3D Sod problem, Shu – Osher problem, flow past a forward step) were considered.
3. The DG advantage over the finite volume method is presented. An influence of numerical flux type and different LPM limiter modifications was discussed.
4. The following perspectives could be considered: testing of other limitation techniques; functionality enhancement for Navier – Stokes equations, real gases or gas mixtures; applicability for adaptive meshes.

Thank you for your attention!