Research on the wake of the ducted propeller with POD and DMD

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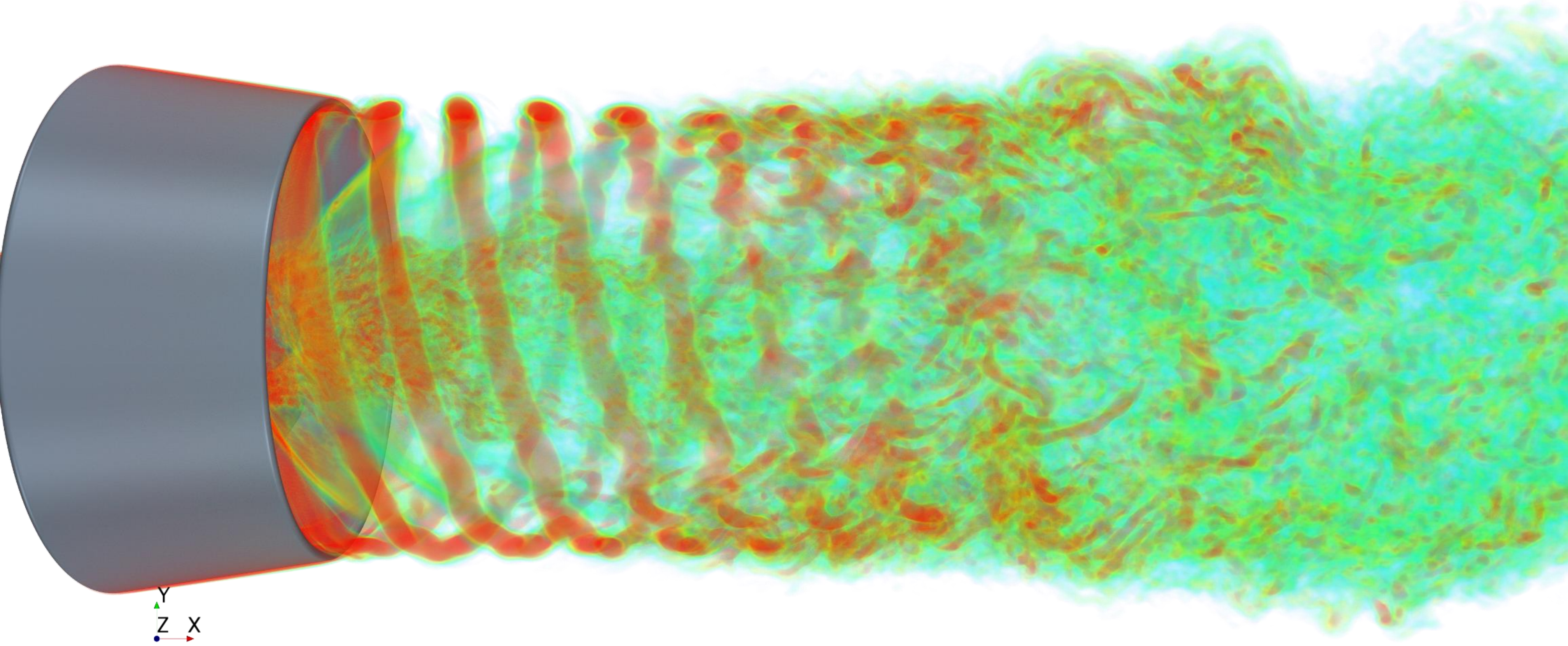
The wake of the ducted propeller



Layout

- Modal decomposition methods
- Overview of numerical simulation
- Data preprocessing
- Modes and reconstruction results display

Proper Orthogonal Decomposition (POD)

The snapshot matrix

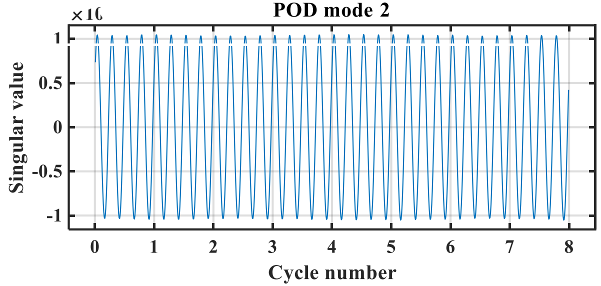
Singular value matrix

$$X = U\Sigma V^T$$

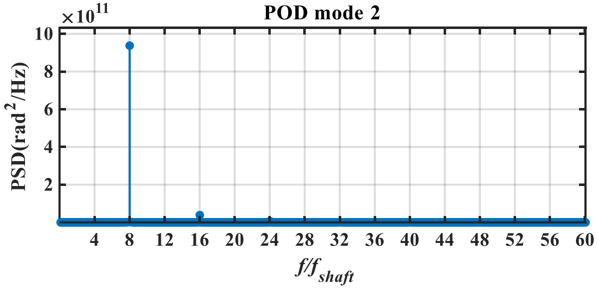
Right singular matrix

POD mode matrix

$$a_i(t) = \sigma_i v_i^T$$



FFT



Power spectral density (PSD)

Dynamic mode decomposition(DMD)

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X}$$

$$\Rightarrow \quad \boldsymbol{A} = \boldsymbol{Y}\boldsymbol{X}^{\dagger}$$

SVD: $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{T}$

$$\Rightarrow \quad \mathbf{A}' = \mathbf{U}_{\mathrm{r}}^{\mathrm{T}}\mathbf{A}\mathbf{U}_{\mathrm{r}}$$

Dimensionality reduction projection

EVD

$$\Rightarrow \quad \mathbf{A}'\mathbf{w}_{\mathrm{i}} = \mathbf{w}_{\mathrm{i}}\lambda_{\mathrm{i}}$$

$$g_i = real\left(ln(\lambda_i)\right)/\Delta t$$

$$\boldsymbol{\phi}_i = \boldsymbol{U}_r\boldsymbol{w}_i \qquad f_i = imag\left(ln(\lambda_i)\right)/2\pi\Delta t$$

$$\alpha_i = \Phi_i^{\dagger}\boldsymbol{x}_0$$

Creat two matrices:

```
X=matrix(:,1:end-1);
Y=matrix(:,2:end);
```

Compute the economy SVD of X:

```
[U,S,V] = svd(X,'econ')
```

Compute the approximate matrix:

```
Atilde = U'*Y*V/S
```

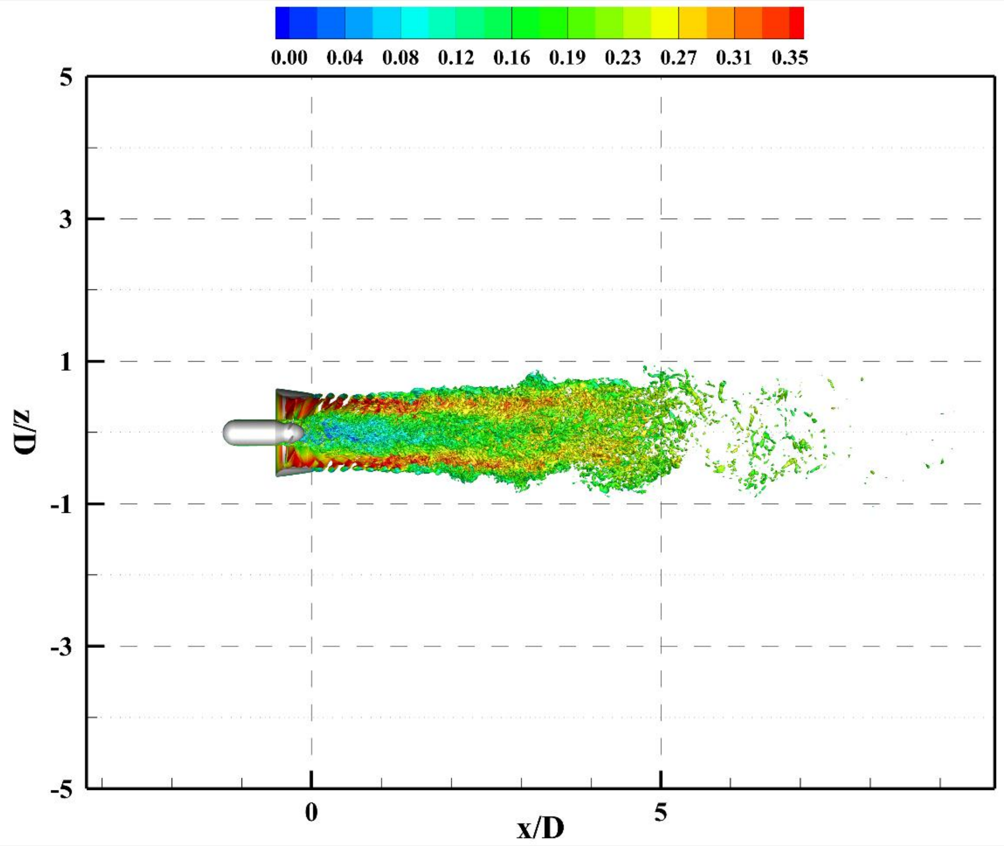
Decompose:

```
[W, D] = eig(Atilde)
```

DMD mode:

```
Phi = U*W
```

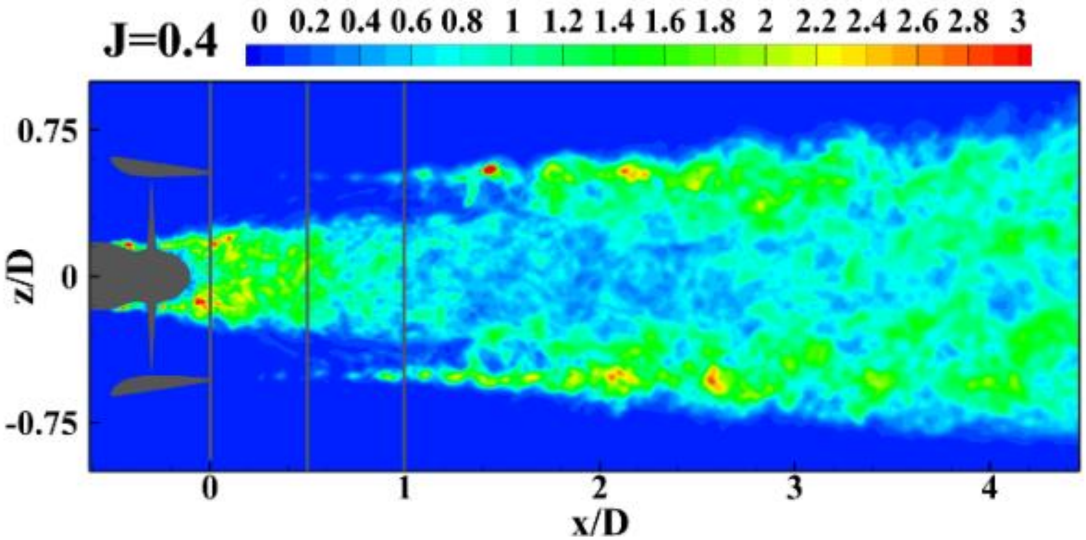
Numerical set-up



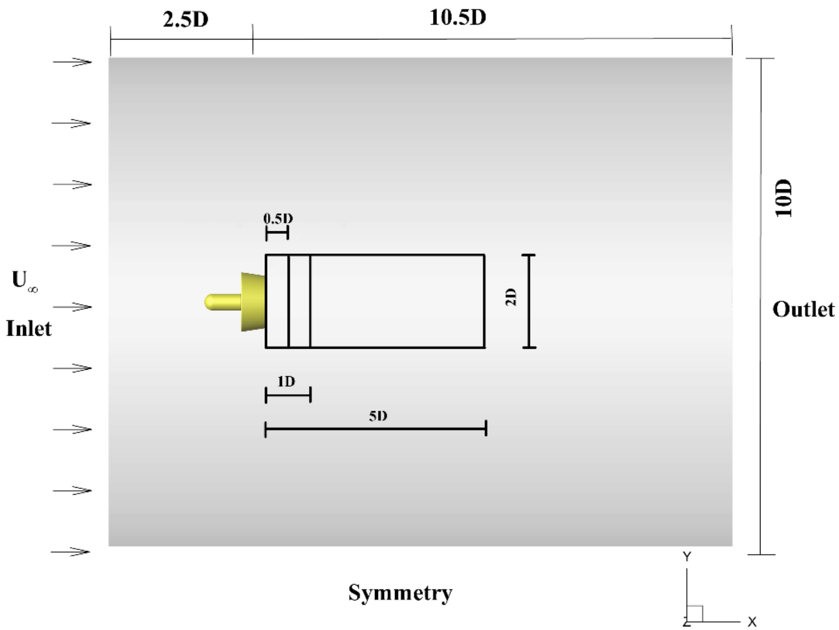
Parameters	Unit	Value
Propeller diameter D	m	0.10
Number of blades Z	-	4
Pitch P/D	-	1.00
Rotation rate n	rps	17.65
Advance coefficient J	-	0.4
Length	m	13D
Diameter	m	10D
Total computational cells	-	33×10^6
Re_n	-	1.765×10^5
Output interval dt	s	0.0001575

Analysis variable: **magnitude vorticity** calculated by the **vorticity** function object of **OpenFOAM-v3.0+**

Computational subdomain



Phase-averaged turbulence kinetic energy field

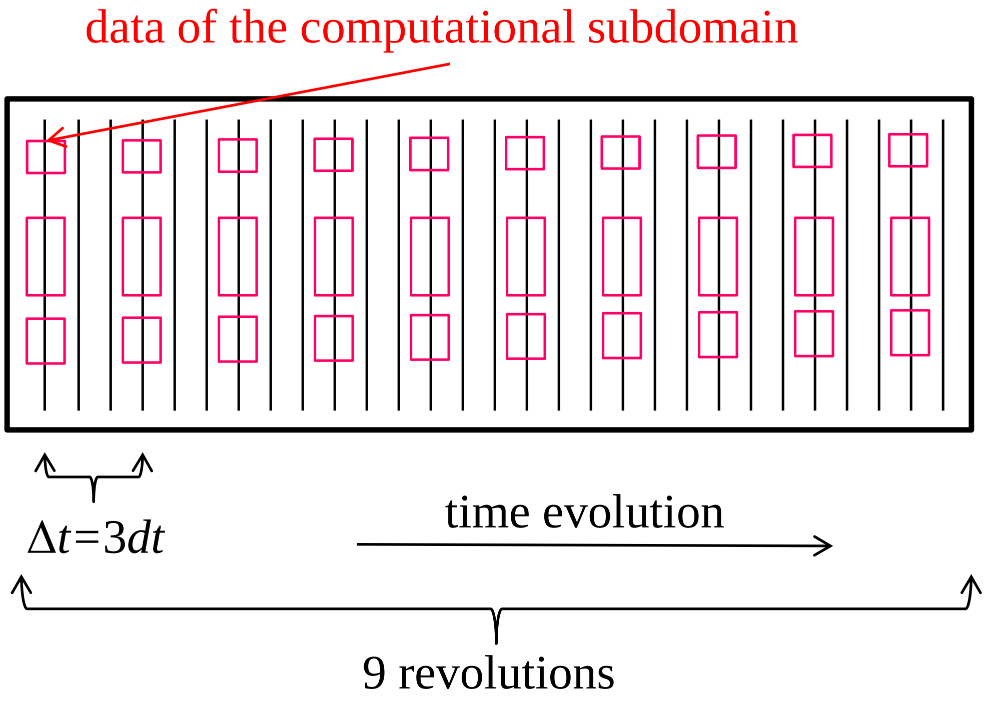


Computational subdomain of the ducted propeller

The bird order of the cylindrical computational subdomain are obtained through the **topoSet** function in OpenFOAM.

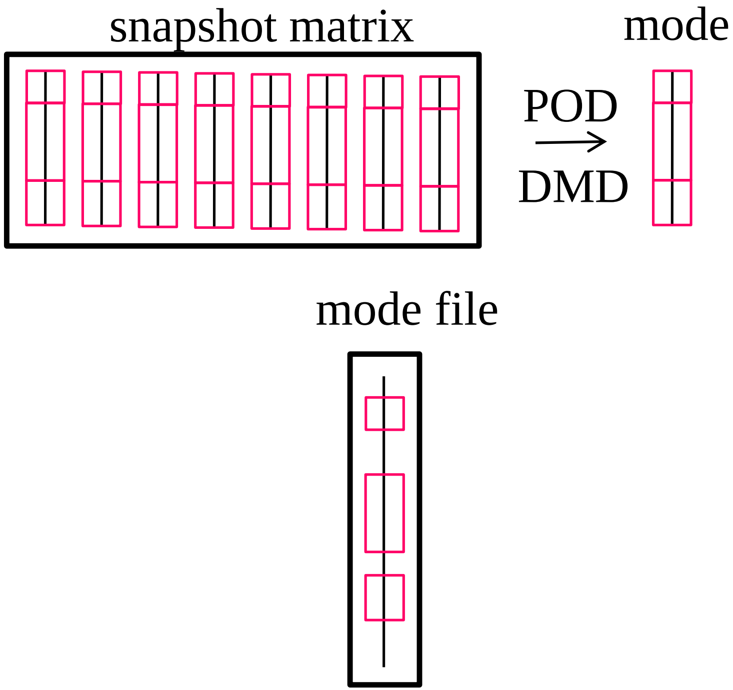
1.9×10^6, 3.55×10^6, 1.17×10^7

Data preprocessing



Speed up reading data:

```
Memmapfile('file path')
```



The snapshot sampling interval

According to the Nyquist-Shannon criterion, the minimum sampling frequency f_{samp} should be two times of the blade passage frequency (BPF) f_{blade}:

$$f_{samp} = 1/\Delta t > 2f_{blade}$$

Parameters	Unit	Value
original output frequency f_{max}	Hz	6349
BPF f_{blade}	Hz	141

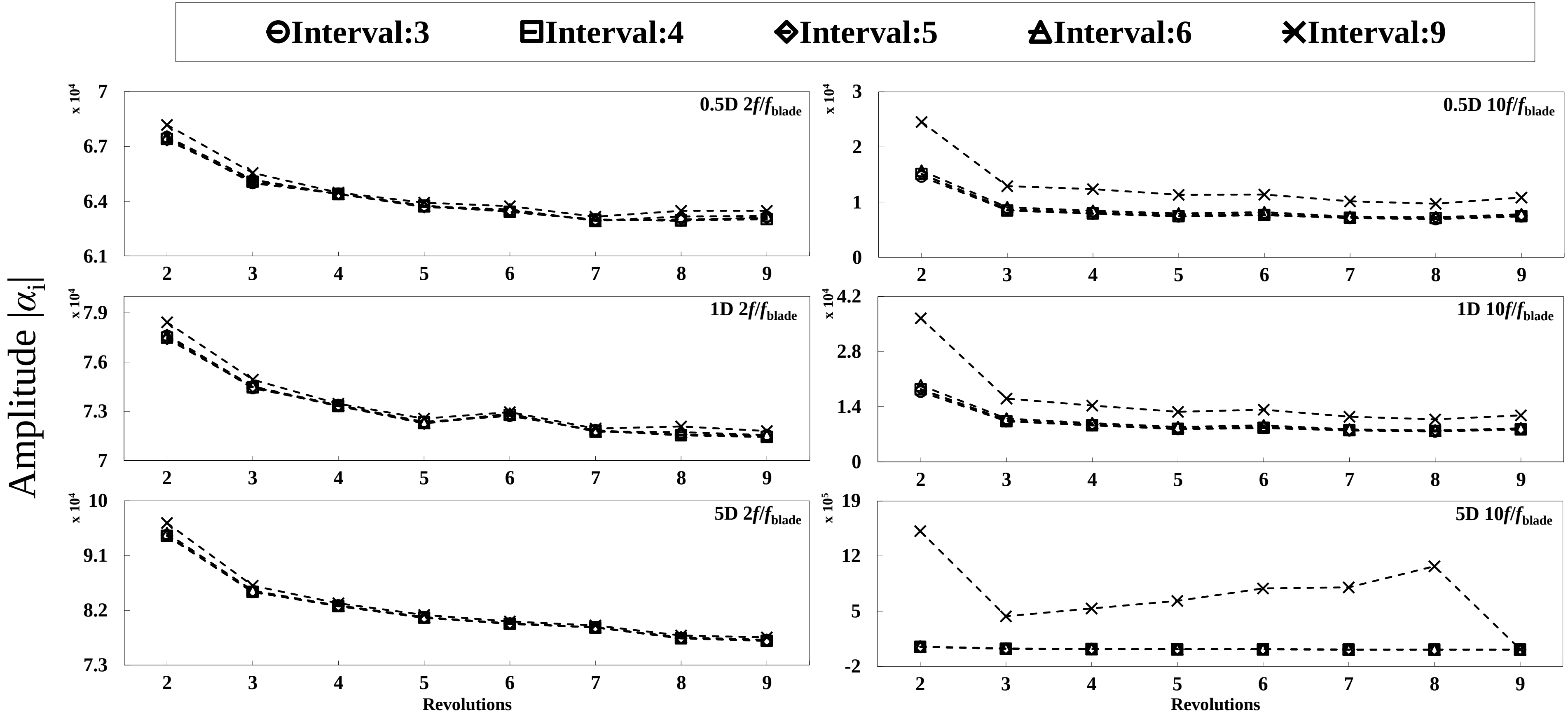
The sampling interval $\Delta t = 3dt, 4dt, 5dt, 6dt, 9dt$

The data coverage revolutions

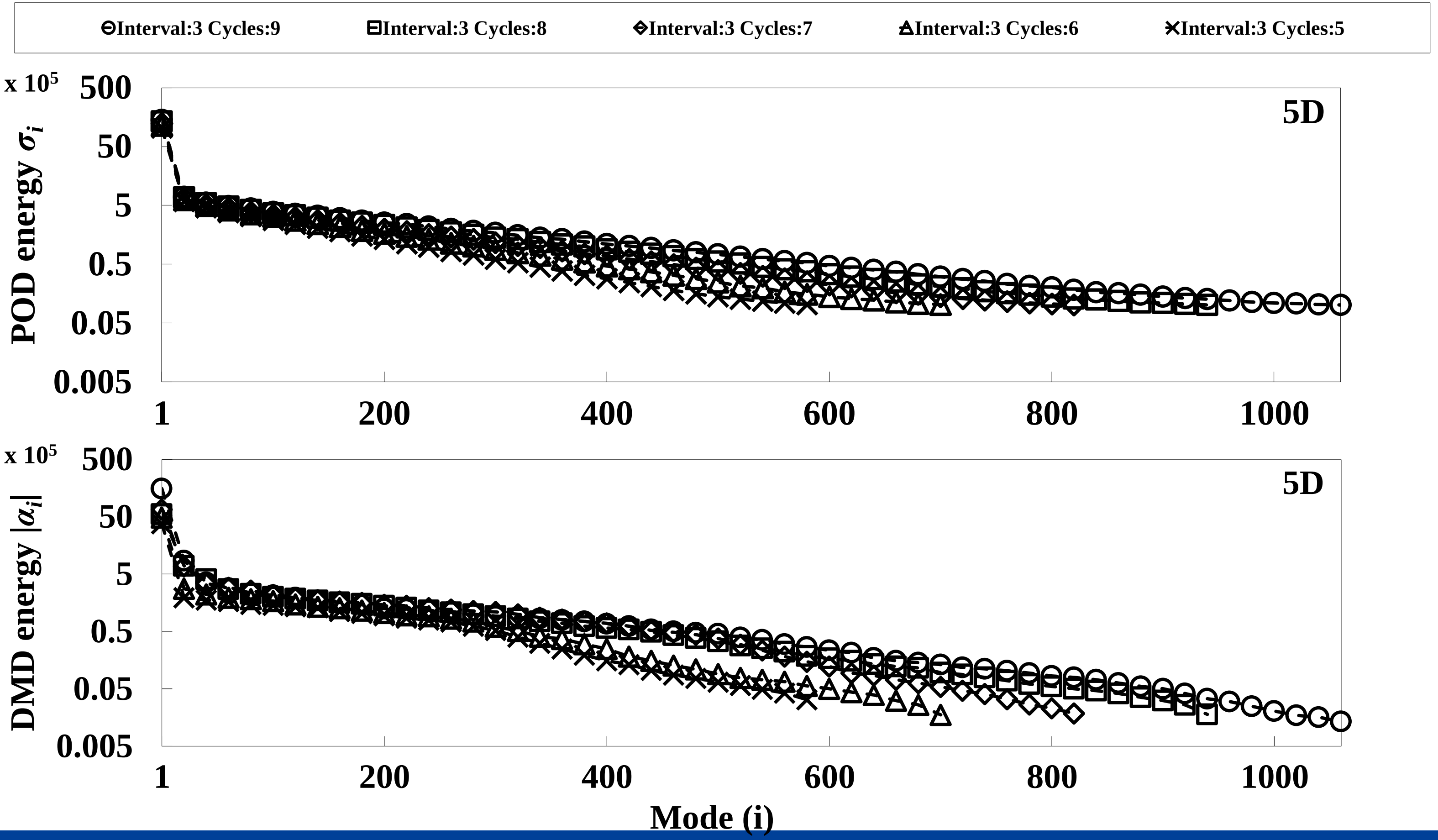
According to the research of K. K. Chen, J. H. Tu, and C. W. Rowley (2012), DMD is not sensitive to whether the analysis data cover integer number of periods, while POD may generate unexpected results when analyzing the data with non-integer multiple periods.

Parameters	Unit	Value
Diameter	m	2D
Length	m	0.5D, 1D, 5D (1.9×10^6, 3.55×10^6, 1.17×10^7)
The snapshot sampling intervals	s	$3dt, 4dt, 5dt, 6dt, 9dt$ (3°, 4°, 5°, 6°, 9°)
The data coverage revolutions	-	2-9 (0.1134-0.6237)

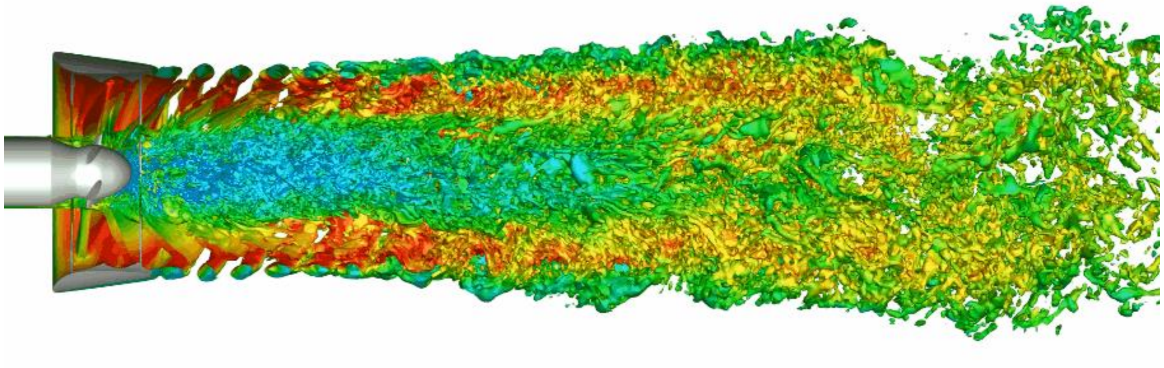
Results


Interval:3
Interval:4
Interval:5
Interval:6
Interval:9
Amplitude $|\alpha_i|$
0.5D $2f/f_{blade}$
0.5D $10f/f_{blade}$
1D $2f/f_{blade}$
1D $10f/f_{blade}$
5D $2f/f_{blade}$
5D $10f/f_{blade}$
x 10^4
x 10^5
Revolutions

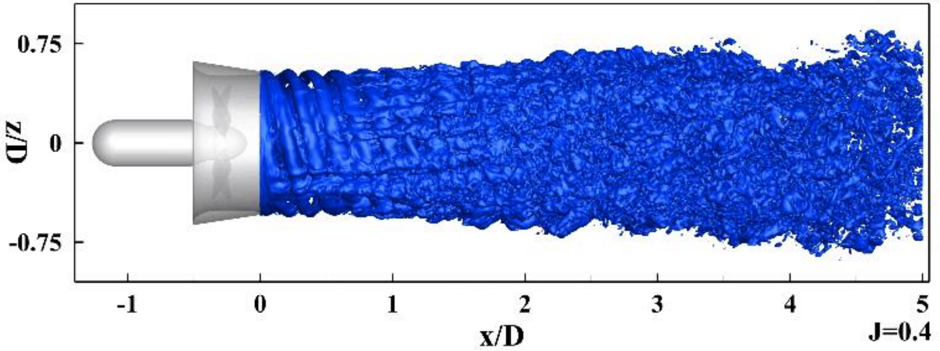
Results


Interval:3 Cycles:9
Interval:3 Cycles:8
Interval:3 Cycles:7
Interval:3 Cycles:6
Interval:3 Cycles:5
x 10^5
POD energy σ_i
500
50
5
0.5
0.05
0.005
5D
1
200
400
600
800
1000
x 10^5
DMD energy $|\alpha_i|$
500
50
5
0.5
0.05
0.005
5D
1
200
400
600
800
1000
Mode (i)

Reconstruction

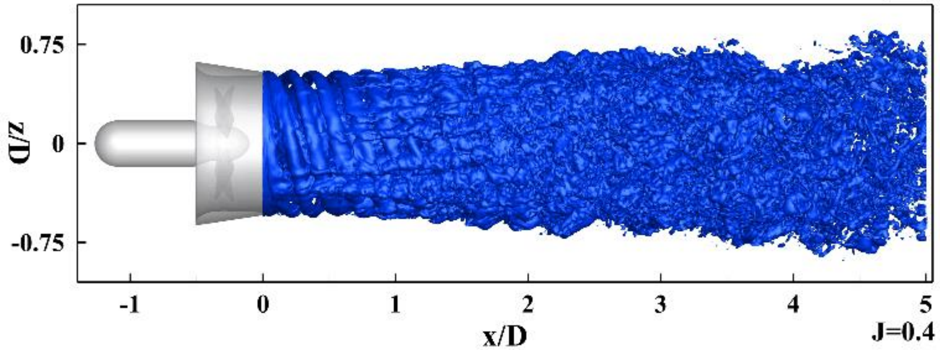


DMD Reconstruction



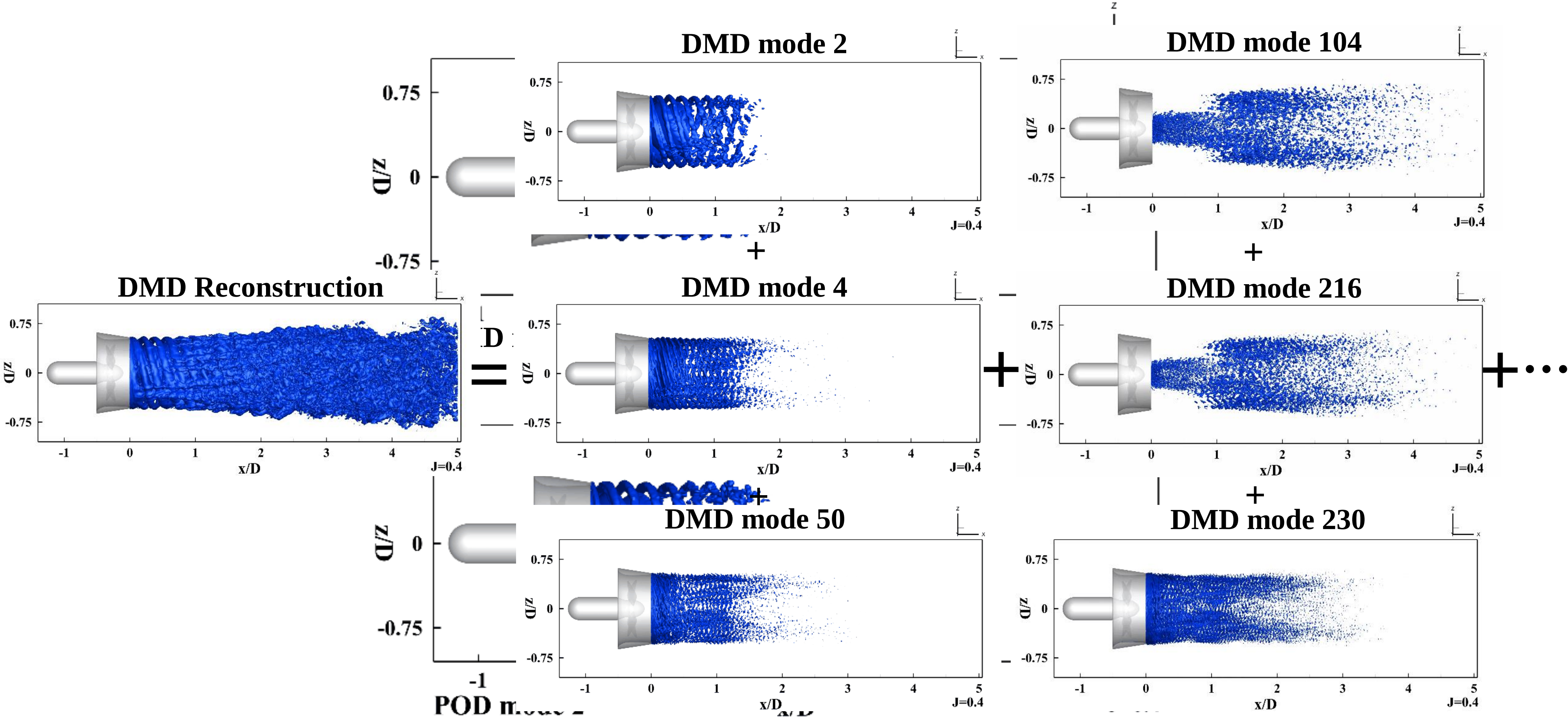
DMD reconstruction error: $\|\boldsymbol{X}' - \boldsymbol{A}\boldsymbol{X}\|_F = 6.39 \times 10^{-5}$

POD Reconstruction



POD reconstruction error: $\left\|\boldsymbol{X} - \sum_{i=1}^{960} \boldsymbol{u}_i a_i(t)\right\|_F = 5.67 \times 10^{-8}$

Modes


DMD Reconstruction
=
DMD mode 2
+
DMD mode 4
+
DMD mode 50
DMD mode 104
+
DMD mode 216
+…
+
DMD mode 230
z/D
x/D
0.75
0
-0.75
-1
0
1
2
3
4
5
J=0.4

Summary

- POD modes are independent in space, while DMD modes are in frequency.
- When the sampling interval reaches a certain level, it only affects the frequency range of the modes captured by POD and DMD, and has little influence on the modal convergence.
- The key factor affecting the modal convergence is the data coverage revolutions, which determines the frequency interval between two captured modes. A larger subdomain requires more revolutions.

Future work

- Future work will focus on the relationship between modes and physical phenomena under different operating conditions

- **Thanks!**