The multifractal analysis in Geophysical Flows.
In memory of
Prof. Jose Manuel Redondo Apraiz

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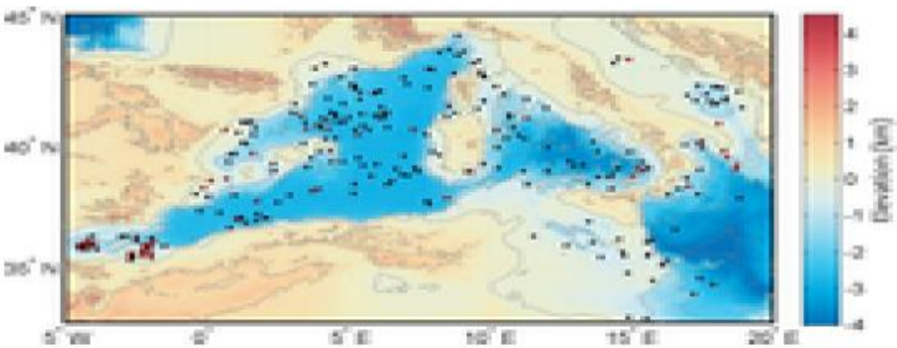
ISP RAS OPEN 2020
10.12.2020-11.12.2020

Content

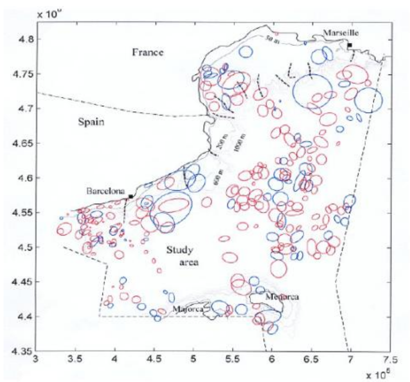
- Geophysical Flows
- Motivation
- Multifractal analysis
- Prof. Jose Manuel Redondo Apraiz
- Mathematical modeling
- Some photos for memory
- Conclusions

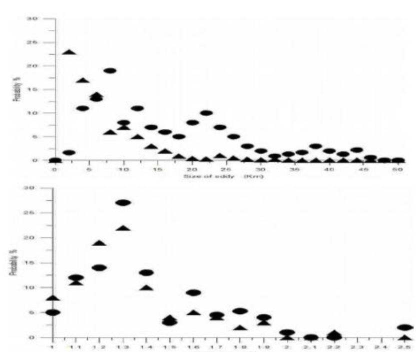
Geophysical Flows

Positions of detected eddies in the Western Mediterranean sea



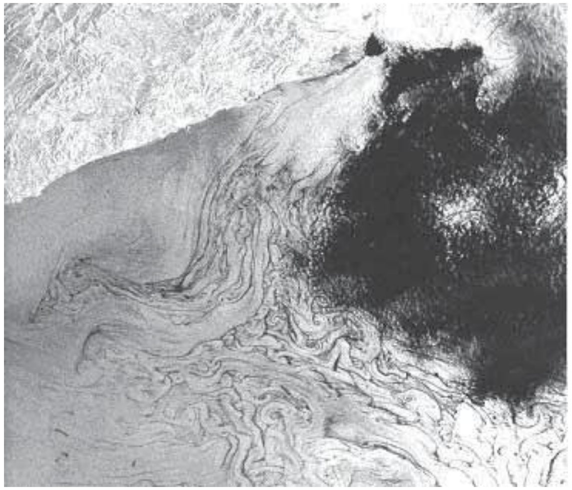
Position, anysotropy and PDF of spiral eddies in the Gulf of Lyons





J. M. Redondo, A. Matulka, A. Platonov E. Sekula, P. Fraunie. Eddy measurements, coastal turbulence and statistics in the gulf of Lions. Ocean Sci. Discuss., 10, 55–81, 2013

SAR image of the ocean surface



Oil spills and vortical features

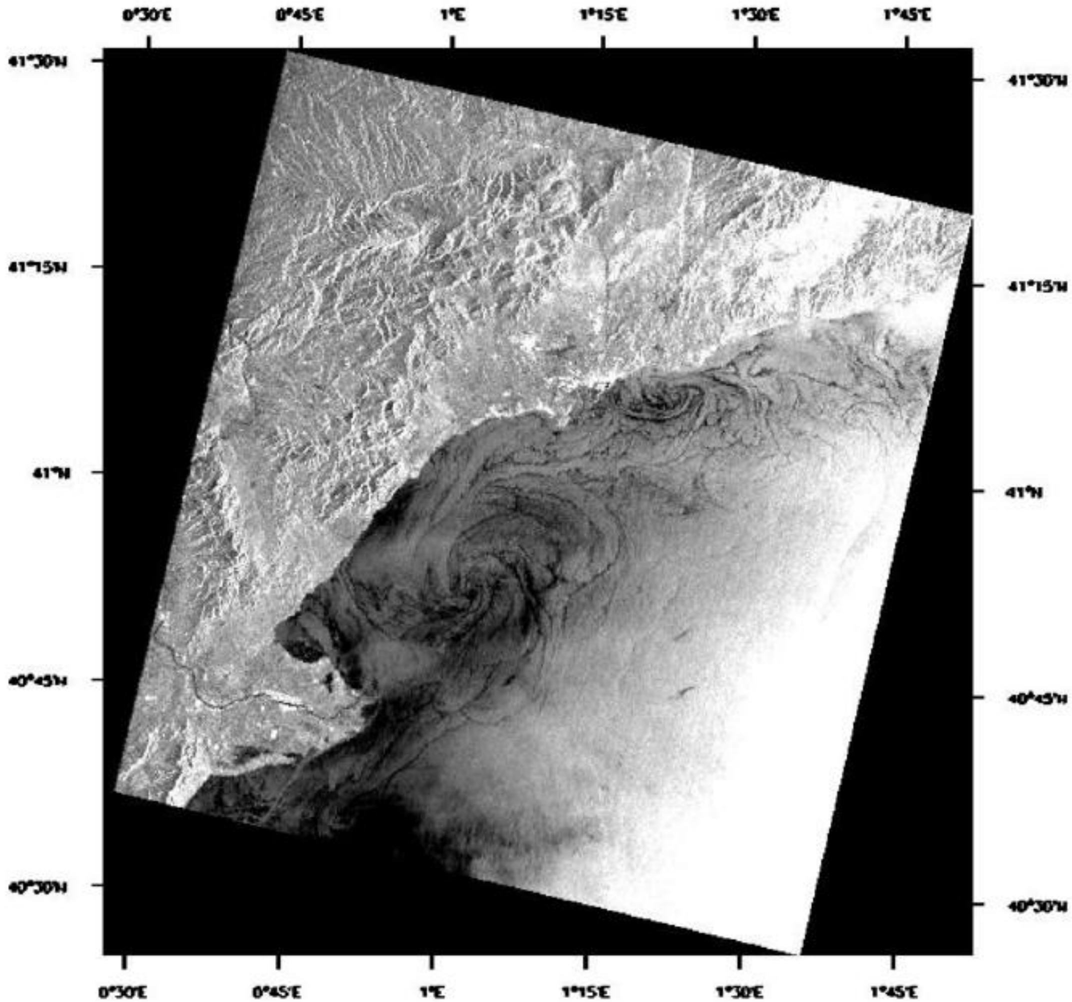
$$R_D = \frac{N}{f}h.$$ Rossby deformation number

$$N = \left(\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{1/2}$$ Brunt-Vaisalla frequency

$$Rig = \frac{g}{\rho}\frac{\partial\rho/\partial z}{(\partial u/\partial z)^2},$$ Gradient Richardson number

Geophysical Flows

Detection of eddies in the NW Mediterranean by SAR (ERS-1/2) Dynamic features on sea surface near **the Ebro delta river**



Motivations

1) Many applications in environmental (flows through trees) and geophysical flows (plant canopies, breaking ocean waves, plumes) as well as in industry (combustion applications, mixing devices, spoilers on wing) **of fractal-forced** or **fractal-generated turbulence**

2) How to create **ideal turbulence experiments** with

- (i) a very wide range of outer-to-inner scales
- (ii) fully controlled conditions in the laboratory
- (iii) the possibility to accurately measure down to the smallest scales

3) Better: how to tamper with **the turbulence in the laboratory**?

4) Effects on drag properties?

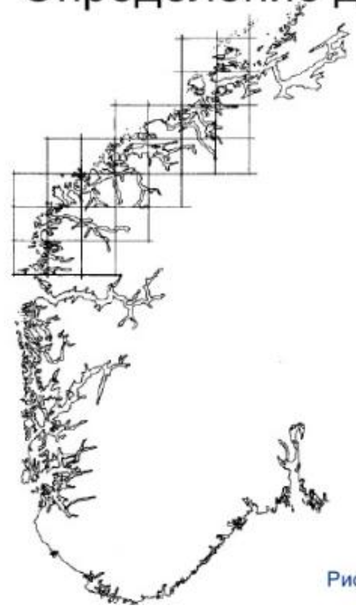
5) How does **a turbulence decay** when it is generated by creating many eddies of many different sizes at once?

6) How does a turbulent flow scale when it is generated **by a fractal** which has its own intrinsic scaling?

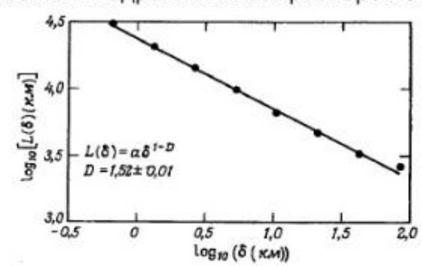
7) How to organize a **Multiscale flow control**?

Fractals in nature

Определение длины береговой линии



Карта побережья Норвегии покрывается сеткой квадратных ячеек размером $\delta\times\delta$.



Измеренная длина береговой линии, как функция шага δ (измеряется в км). Прямая на графике в двойном логарифмическом масштабе соответствует зависимости

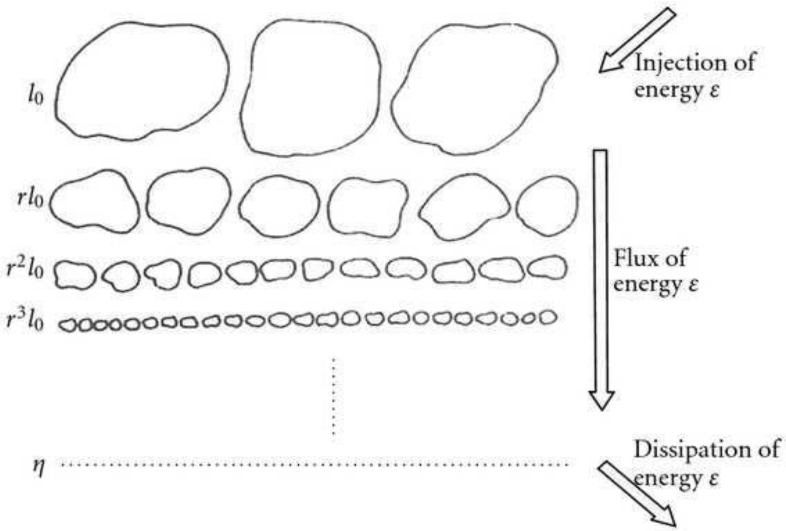
$$L_\delta = a\delta^{1-D}, \quad \text{где } D \approx 1.52$$

Рисунки взяты из книги Федер Е. Фракталы. М.: Мир. 1991

Turbulence studying and modeling

RANS, LES, DNS

The Kolmogorov (1941) -5/3 Law



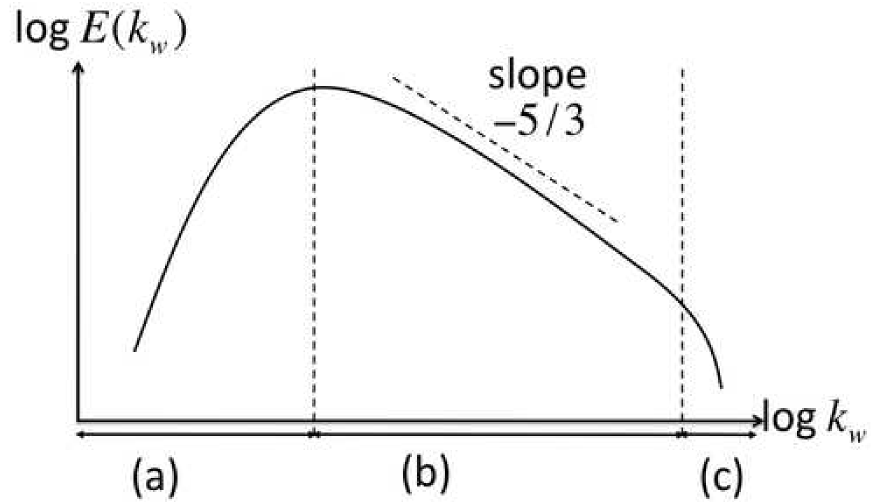


Fig. : Energy cascade and spectrum

Goal of research: To study influence of stratification on Energy cascade and spectrum using data from geophysical flows

Wilcox, David C. Turbulence Modeling for CFD. Third Edition. 2006. 536 p.

The theory of turbulence: structure function, the intermittency exponent, the Hausdroff dimension

$$B(R)^P = \left\langle \left(\left(u(x+R) - u(x) \right) \right)^P \right\rangle = \left\langle \delta_R (u)^P \right\rangle$$ **The structure function**

$$B(R)^P \sim \langle \varepsilon \rangle^{P/3} R^{P/3}$$

$$B(R)^P \sim \langle \varepsilon_R \rangle^{P/3} R^{P/3} \sim R^{\xi_p}$$

$$\xi_p = \frac{p}{3} + \frac{1}{18} \mu p (3 - p)$$ **the scaling exponent**

$$\mu = 2 - \xi_6$$

μ **is the intermittency exponent**

$$\xi_p = \frac{p}{3} + \frac{1}{3} (3 - D)(3 - p)$$

D **is the Hausdroff dimension**

$$E(k) \propto k^{-\left(\frac{5}{3} + \frac{3-D}{3}\right)}$$ **Energy spectrum**

Calculation of Fractal Dimensions

The Moment method uses mainly three functions: $\tau(q)$ - mass exponent function, α - coarse Hölder exponent, and $f(\alpha)$ - multifractal spectrum

$$n(\delta) \propto \delta^{-D_0}$$

number n of features of a certain size δ

$$D_0 = \lim_{\delta \to 0} \frac{\log n(\delta)}{\log \frac{1}{\delta}}$$

the fractal dimension D_0

$$\chi(q,\delta) = \sum_{i=1}^{n(\delta)} \mu_i^q(\delta) = \sum_{i=1}^{n(\delta)} m_i^q$$

the partition function $\chi(q, \delta)$

$$<\tau(q)>= \lim_{\delta \to 0} \frac{\log <\chi(q,\delta)>}{\log(\delta)} = \lim_{\delta \to 0} \frac{\log <\sum_{i=1}^{n(\delta)} m_i^q>}{\log(\delta)}$$

the mass exponent function $\tau(q)$

$$D_q = \lim_{\delta \to 0} \frac{1}{1-q} \frac{\log \sum_{i=1}^{n(\delta)} m_i^q}{\log \delta}$$

generalized dimensions D_q,

$$<\alpha(q)>= \frac{d<\tau(q)>}{dq}$$

The singularity index (α)

$$<f(\alpha)>= q<\alpha(q)> - <\tau(q)>$$

$f(\alpha)$ is a scaling exponent of the cells with common α

FRACTAL ANALYSIS

- **Fractal studies provide a natural method for analysing turbulent fields like ABL, wakes, plumes and their turbulent cascade processes.**

- **If there is a subrange where production and dissipation are at equilibrium, it is possible a functional relation between the exponent β of the spectral density function and the fractal dimension *D* of the scalar field represented in the images:**

$$\beta = 2E_U + 1 - 2D \quad \Rightarrow \quad D = E_U + \frac{1-\beta}{2}, \qquad E_U : Euclidian\ \text{dim}\,ension$$

- **The last aim is to investigate the intermittency of the mixing flows**

Measuring the maximum fractal dimension and using results of another researchers relating to the sixth and third order structure function scaling exponents.

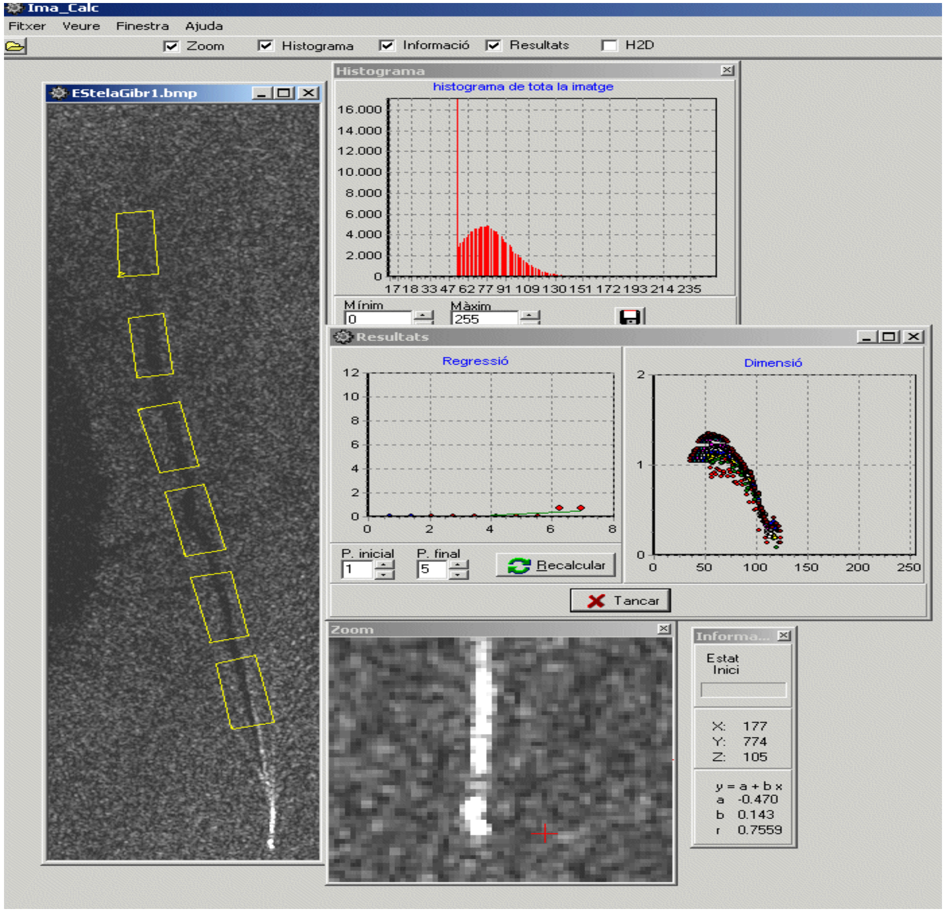
Fractal Dimension and The ImaCalc software

Oil spills on SAR image

The multifractal analysis allows to quantify the level of **geometric complexity** and in some cases it can be predicted some dynamic aspects of the fluid.

We applied the multifractal analysis using the ImaCalc software developed in 2005 by Joan Grau during the PhD Thesis in the Technical University of Catalonia, Spain for geophysical flows during the last 5 years.

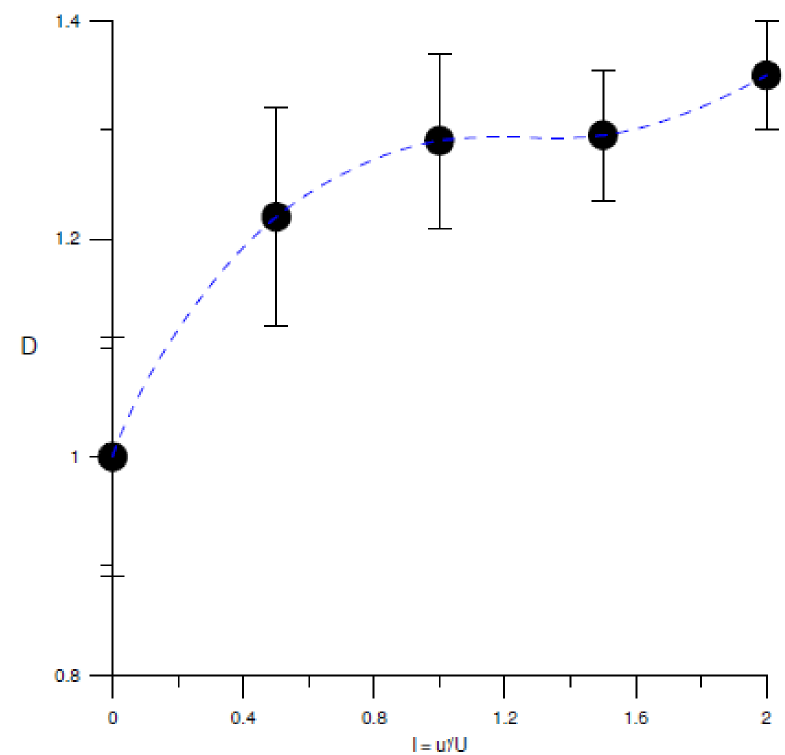
The ImaCalc software uses **the box counting method** to calculate the fractal dimension through t**he observation of flow's turbulent images,** where the method evaluates the **different gray scales**.



Mulifractal analysis:

The basic theory and method of box-counting was followed and we used self-similarity to identify different dynamic processes that might influence the radar back-scattering from the ocean surface. The image analysis algorithms are able to detect the self-similar characteristics for different SAR intensity levels, *i*.

The fractal dimension *D (i)* is then a function of intensity and may be calculated using



where *N*(*i*) is the number of boxes of size ***e*** needed to cover the SAR contour of intensity ***i***. The algorithm operates dividing the surface into smaller and smaller square boxes and counting the number of them which have values close to the SAR radiation level under study

$$n(\delta) \propto \delta^{-D_0} \quad , \qquad D_0 = \lim_{\delta \to 0} \frac{\log n(\delta)}{\log \frac{1}{\delta}}$$

OBITUARY: JOSE MANUEL REDONDO APRAIZ

(04.04.1956-16.07.2020)

Jose Manuel studied different complex processes in geophysical hydrodynamics with highly intermittent spiky measures and no uniformities. For example, the distribution of turbulent kinetic energy dissipation rate. These intermittent processes cannot be described totally by typical moment methods. Therefore, a multifractal method was required.

Jose Manuel studied such flows as vertical plumes, turbulent wakes and jets, the effect of rotation and stratification on diffusion of particles, vortices in ABL with PhD students and co-workers .

Jose Manuel completed his PhD in 1989 in Cambridge, DAMTP, UK. He was a professor position in the department of Física Aplicada at Universitat Politècnica de Catalunya – UPC Barcelona.

Jose Manuel had publications in many famous journals: Journal of Fluid Mechanics, Physics of Fluids, Journal of applied meteorology and climatology, Flow, turbulence and combustion, Experiments in fluids, Nonlinear Processes in Geophysics, Applied scientific research, Continental Shelf Research, Physica Scripta, etc.

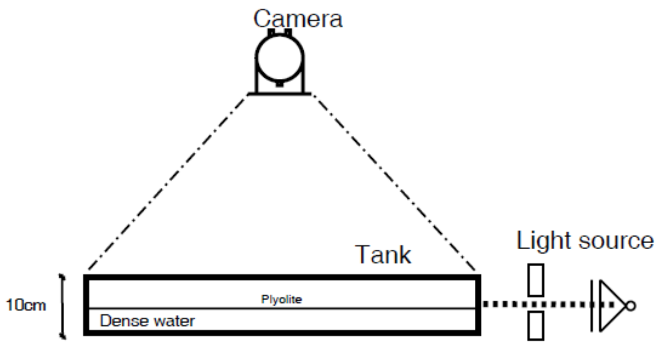
José Manuel, as the son of a sailor, loved the sea and mostly applied his research activity to marine science. He represented Spain, swimming butterfly as a young man

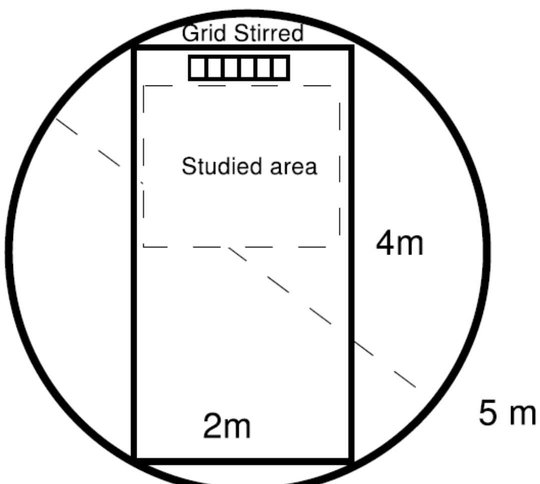
Jose Manuel was a leader of SIG 14 “Stably Stratified and Rotating Turbulence” of Ercoftac

15 Languages of Jose Manuel from researchgate.net: Basque, Catalan , Valencian, Spanish Castilian, Chinese, Danish, English, French, German, Greek, Modern Italian, Japanese, Portuguese, Russian, Slovak,

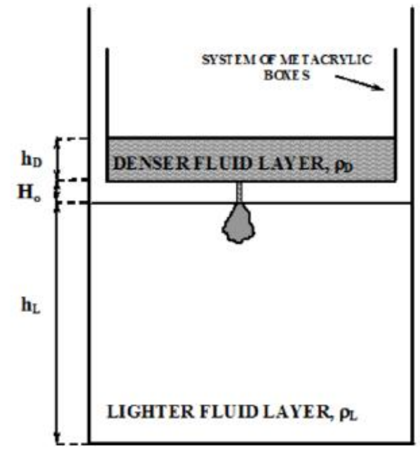
EXPERIMENTS OF PROF. JOSE MANUEL REDONDO APRAIZ

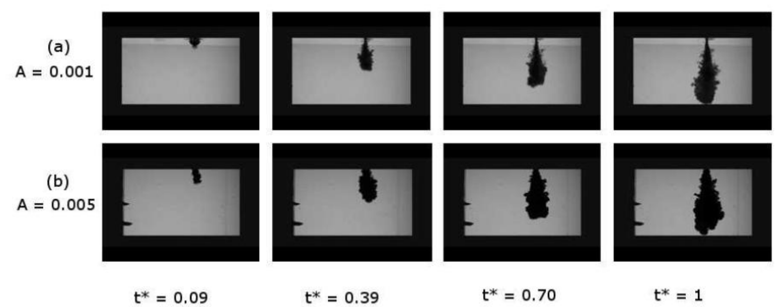
Experiments in stratified and rotating decaying 2D flows





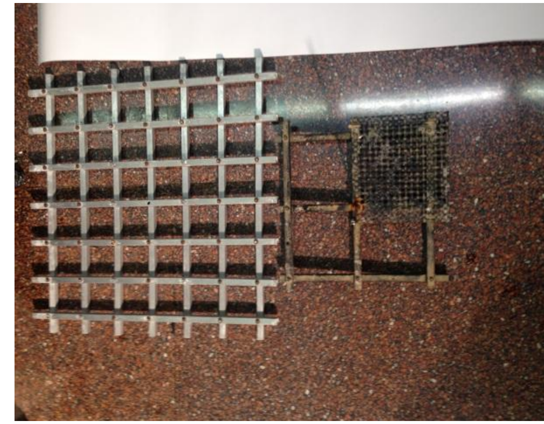
Experimental configuration that shows the initial state of the fluid system with a starting forced plume





A. Matulka, P. López, J. M. Redondo, A. Tarquis. On the entrainment coefficient in a forced plume: quantitative effects of source parameters. Nonlin. Processes Geophys., 21, 269–278, 2014

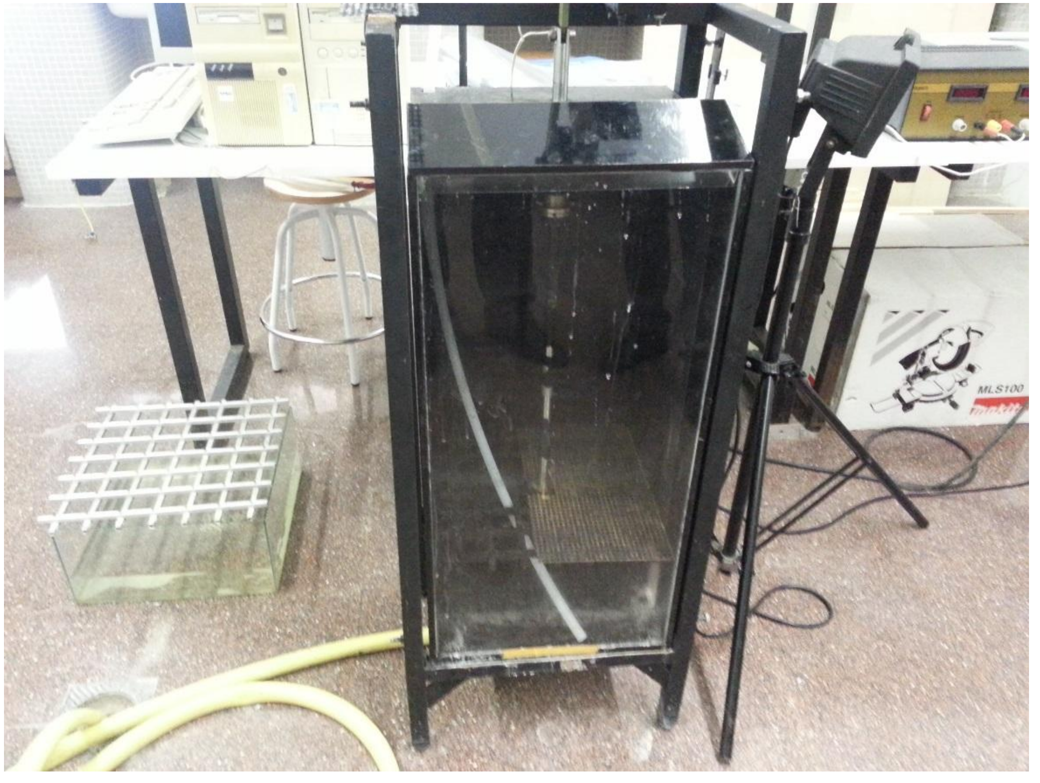
Experimentas of fractal grates and oscillation grates
(UPC, Barcelona, Prof. J.M. Redondo)

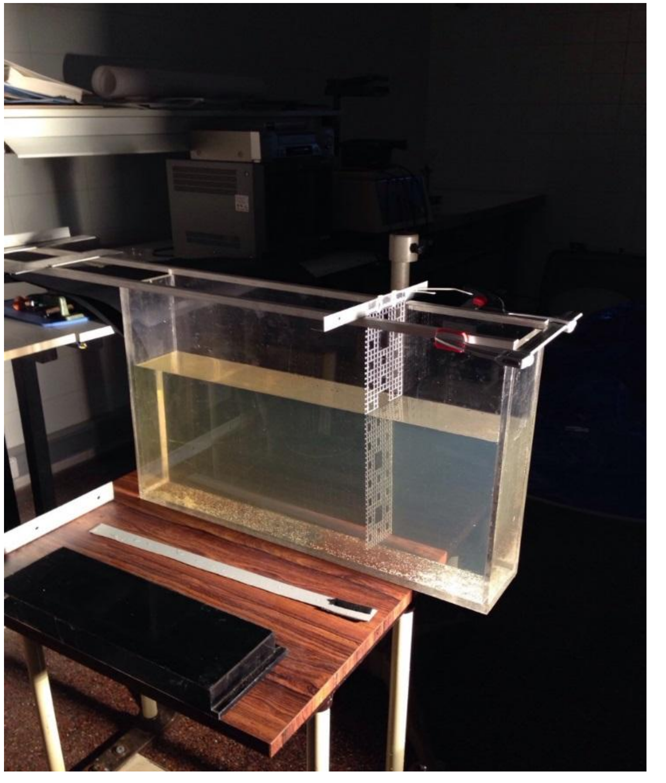


Two scale grate

Sierpinski grate

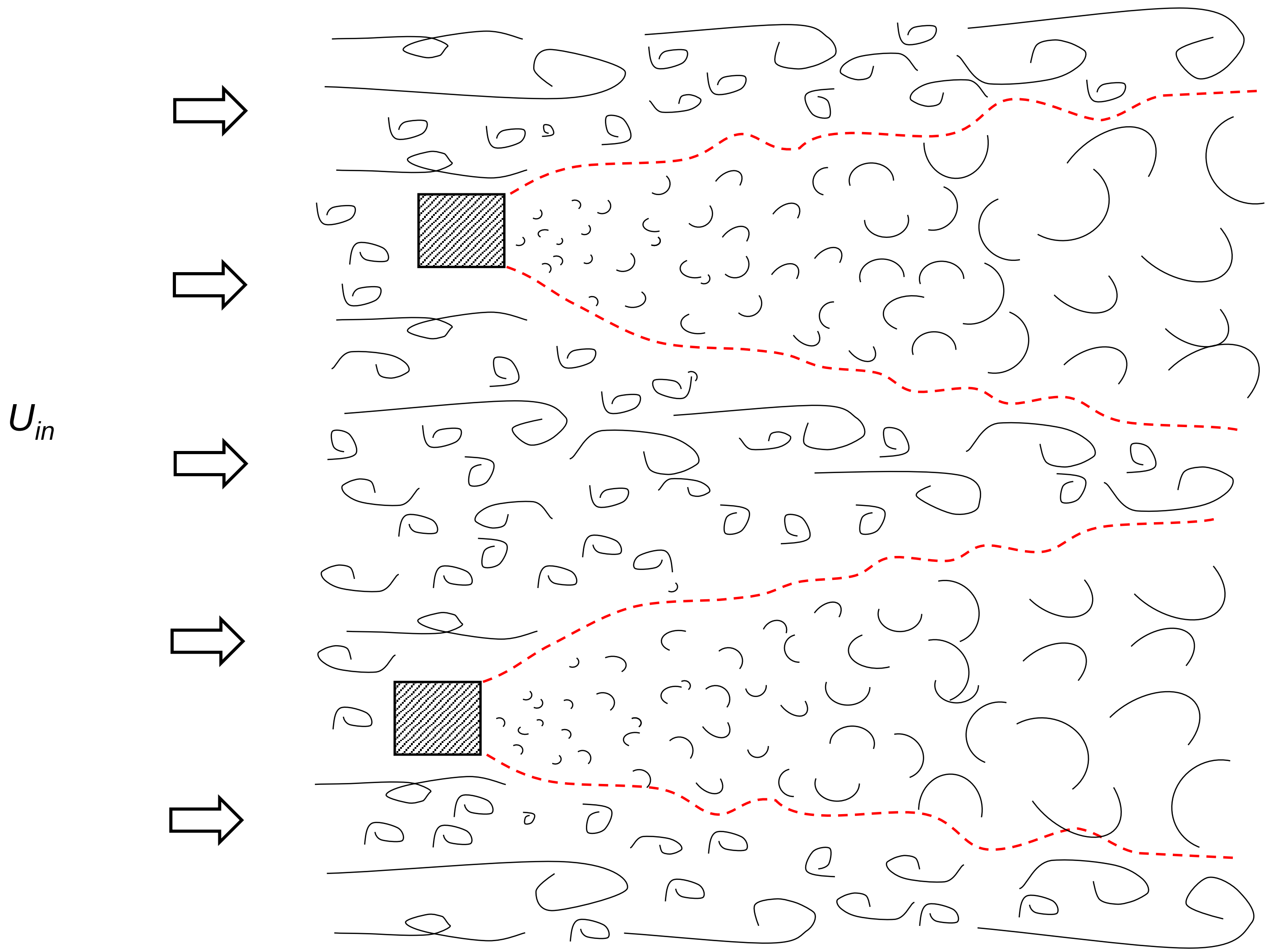
Vertical tank and horizontal water channel



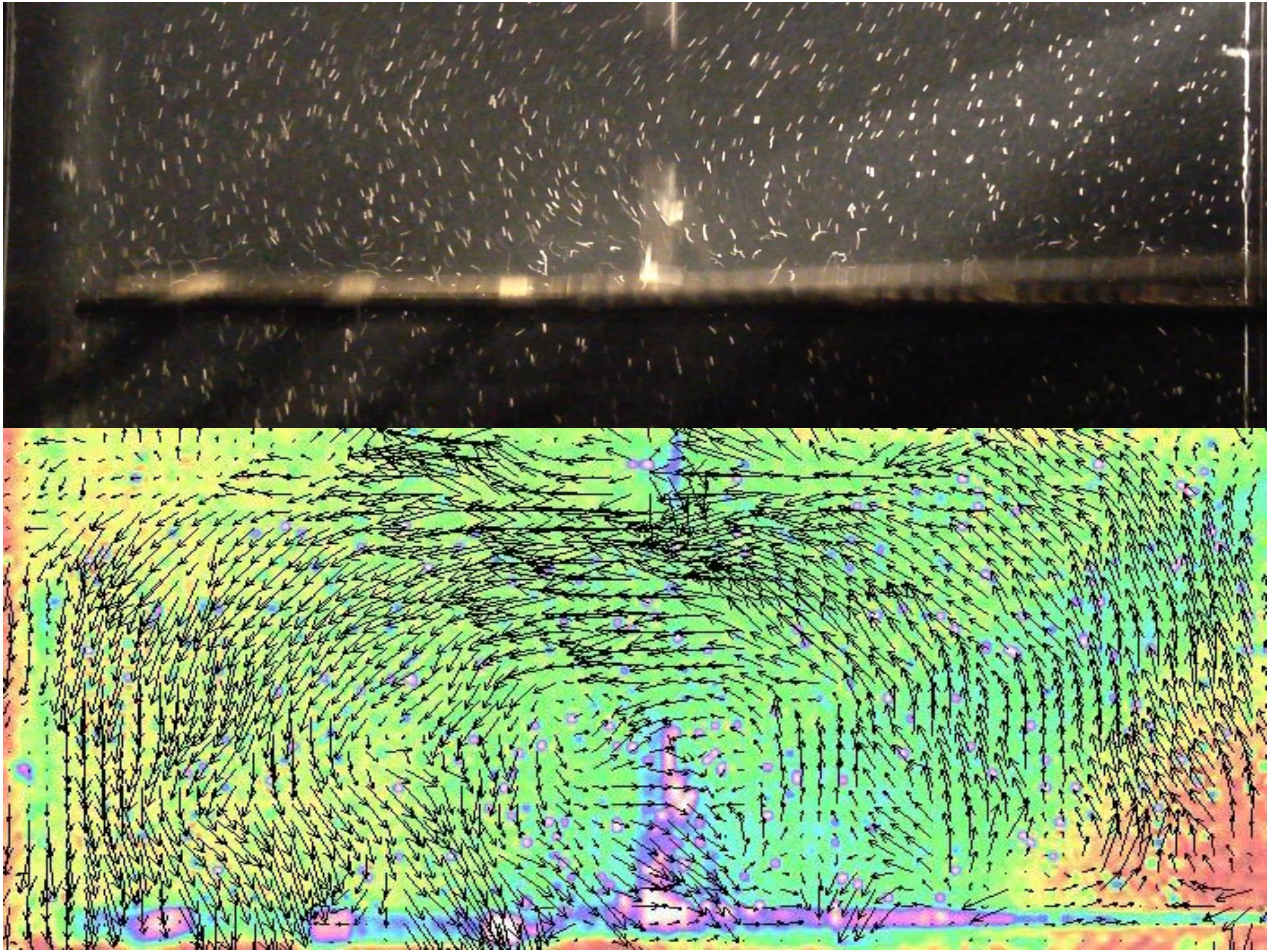


Sierpinsky multi-fractal grid
in water channel

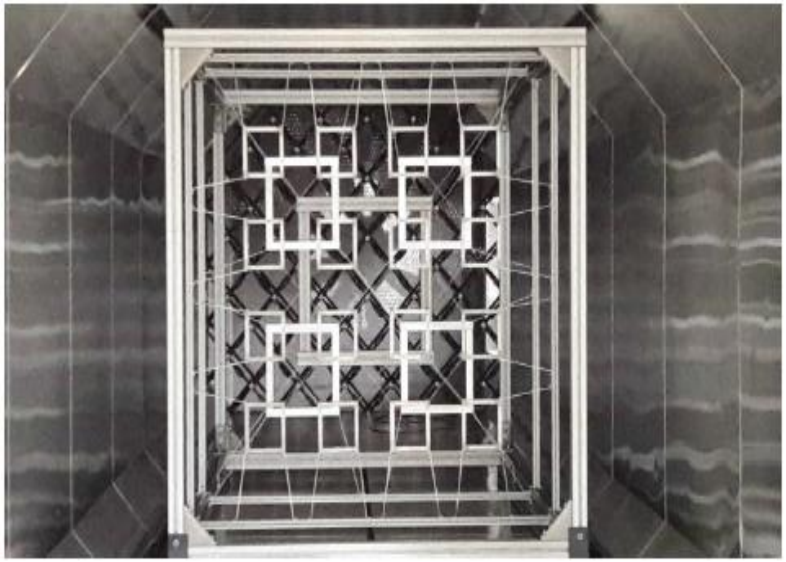
General schematic of wake interactions behind a fractal square grid



Pattern of Field velocities of oscillation grates



3D SPARSE GRID TURBULENCE GENERATORS (3DSGT)



$M = (1-\sigma)F(Re;\rho,U,\mu,L)$ - mass flux rate from the grid

The goal: to generate the turbulence behind non-regular grids to reduces the effective blockage ratio

A three-dimensional sparse grid turbulence generators (3DSGT) as defined in Figure was built using the technical facilities at the Max Plank Institute in Gottingen.

The new type of grid turbulence generator, the 3D sparse grid (3DS), is a co-planar arrangement of co-frames each containing a different length scale of grid elements.

Multi-scale instabilities leading to mixing due to Rayleigh-Taylor and Richtmeyer-Meshkov instabilities

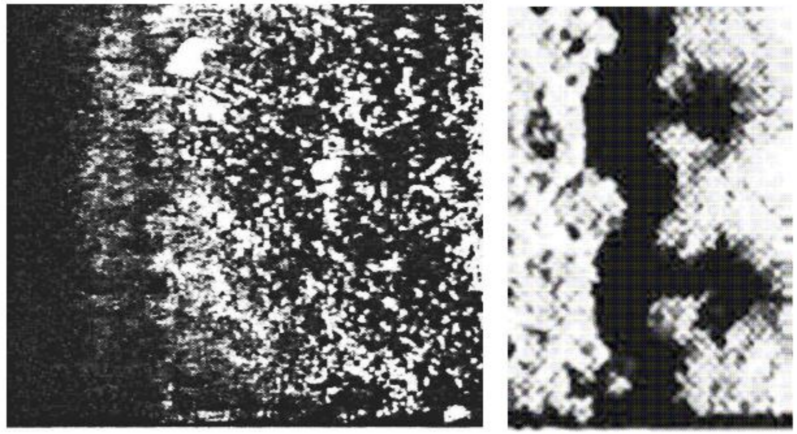


Figure 1: Detail of the mixing front in a Shock tube RM.
Marseille large facility DEA–UPC. (Redondo 1997)

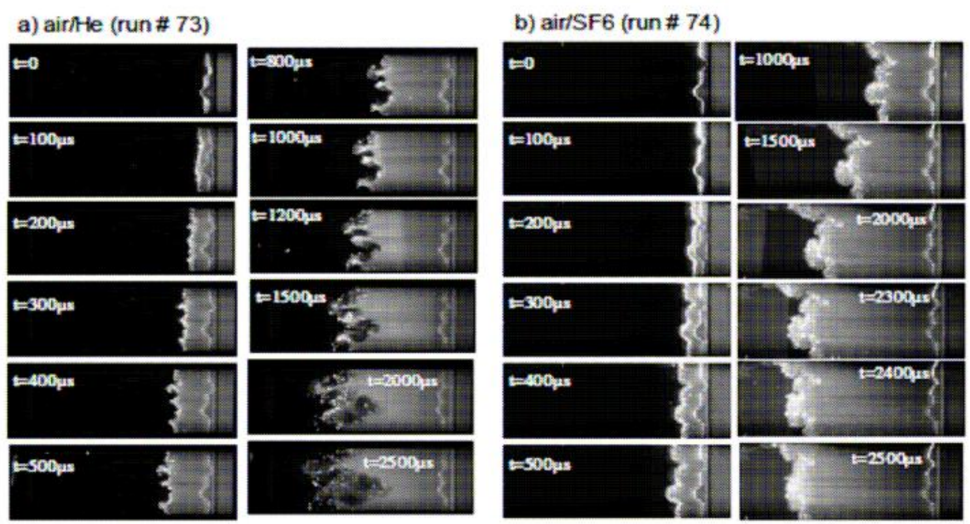


Figure 2: LIF image of the details of the evolution of two RM Shock front experiments

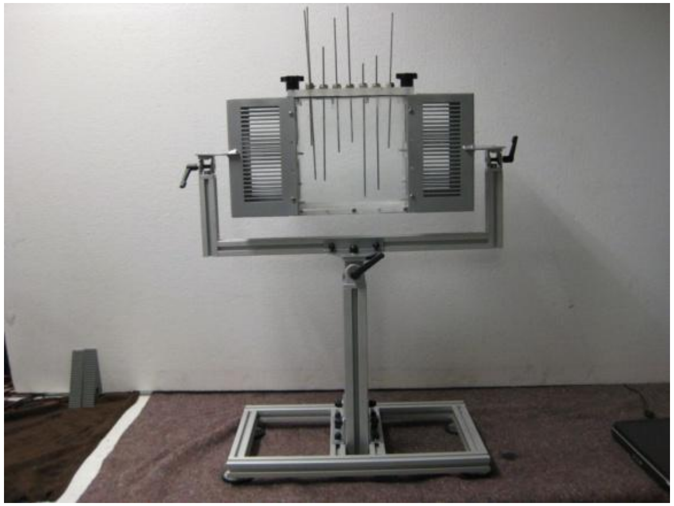
DESCRIPTION OF PROTOTYPE

Description of the apparatus:

The heat sources or sinks can be regulated both in power and sign. The thermal convective driven flows are generated by Peltier effect in 4 wall extended positions of 0.05 x 0.05 cm each, these can be regulated independently. Another advantage of the full control of the angle at which convection takes place. Because the fluid enclosure may become totally isolated, the range of zenital and azimutal angles span almost 180 and 360 degrees.

Working conditions:

The thermoelectric driven heating and cooling experimental device may be used in order to map the different transitions between twodimensional convection in an enclosure and the 3 D complex flows. The size of the enclosure is of 0.2 x 0.2 x 0.1 m



Mathematical modeling

SOWFA* - an open source library for numerical simulation of flow parameters in wind farms

- The library is written in National Renewable Energy Laboratory (NREL) in 2010.
- Is supported by National Center for Atmospheric Research (WRF code)
- Simulation of flow parameters in Atmospheric Boundary Layer - **ABLsolver**
- Wind turbine simulation - **pisoTurbineFoam**
- Wind plant simulation - **windPlantSolver**
- Wind plant simulation with FSI (FAST solver) – **windPlantSolverFast**
- Large Eddy Simulation (LES) model
- Different SGS models for LES
- Special utilities for boundary conditions
- Scales from **10e-3 m till 10e5** m
- **Re ~ 10e7**

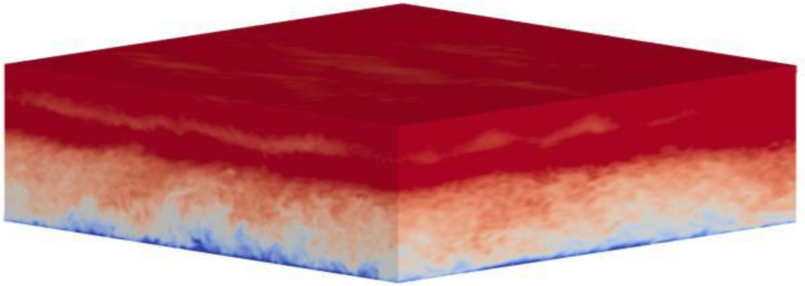


Fig. Velocity magnitude

Domain: 3 km x 3 km x 1 km V=8 м/с at H=90 m.
Solver ABLSolver.

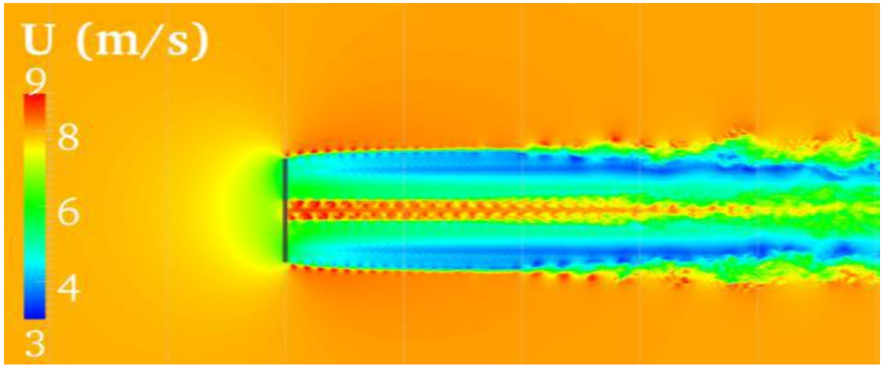


Fig. Velocity field. Solver pisoturbineFoam

* SOWFA, The Simulator for Wind Farm Applications, created by The National Renewable Energy, distributed online at https://wind.nrel.gov/SOWFA, 2016.

Mathematical model in SOWFA: Finite Volume Method and Large Eddy Simulations

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0$$ - mass conservation equation

$$\bar{u}_j = u_j - u'_j$$ - velocity after procedure of filtration

Coriolis force:

$$-2\varepsilon_{ijk}\Omega_j \bar{u}_k$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_j \bar{u}_i\right) = -2\varepsilon_{ijk}\Omega_j \bar{u}_k - \frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\left(R_{ij}^D\right) + \left(\frac{\rho_b}{\rho_0} - 1\right) g_i - \left\langle \frac{\partial p}{\partial x_i} \right\rangle + f_i$$ - momentum equation

the Boussinesq approximation

Where ε_{ijk} - the alternating tensor,

Ω_j - Rotation Rate Vector for Earth,

$\tilde{p}$ - Modified pressure variable,

R_{ij}^D - Fluid stress tensor.

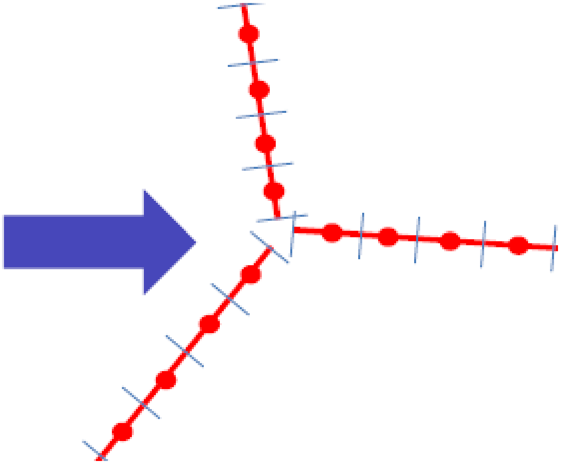
$$\Omega_j = \omega \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \end{bmatrix}$$

ω is the planetary rotation rate (rad/s), ϕ - latitude

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial u_j \bar{\theta}}{\partial x_j} = -\frac{\partial \tau_{\theta_i}}{\partial x_j}$$ - a potential temperature transport equation

Where $\bar{\theta}_j$ - the resolved-scale potential temperature, τ_j - is the SGS temperature flux

Actuator Line Model for wind turbine



$$f_i^{turbine}(r) = \frac{F_i^{actuator}}{\varepsilon^3 \pi^{3/2}} \exp\left[-\left(\frac{r}{\varepsilon} \right)^2 \right]$$

Total Aerodynamic Force

Aerodynamics coefficients $Cx(\alpha)$ $C_y(\alpha)$ are known for different profiles

Angle of Attack from -180 till 180. The Simple bodies for wind turbine are: "Cylinder1, "Cylinder2", airfoil profiles "DU40_A17", "DU35_A17", "DU30_A17", "DU25_A17", "DU21_A17", "NACA64_A17"

The Surface Shear Stress Model

$$u_*^2 = \sqrt{\langle \tau_{13S}(x,y) \rangle^2 + \langle \tau_{23S}(x,y) \rangle^2}$$

friction velocity

$$\left| \langle \overline{U}(z_1) \rangle \right| = \frac{u_*}{k} \left[\log\left(\frac{z_1}{z_0} \right) - \psi_m \left(\frac{z_1}{L} \right) \right]$$

Monin-Obukhov ABL similarity laws (angle brackets denote planar average)

$$L = -u_* \frac{\theta_0}{kgq_s}$$

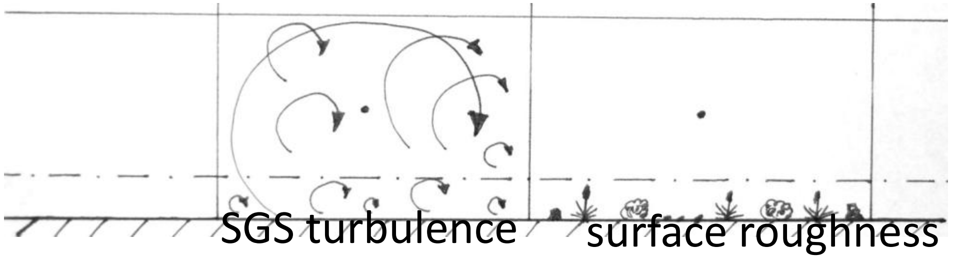
The Obukhov length

$$\tau_{i3S}(x,y) = -u_*^2 \frac{\overline{U}_i(x,y,z_1)}{\left| \langle \overline{U}(z_1) \rangle \right|}$$

The surface shear stress model of Schumann

Constraints

- Relies on planar averages (angle brackets)
- Mathematically valid only for flow over flat terrain



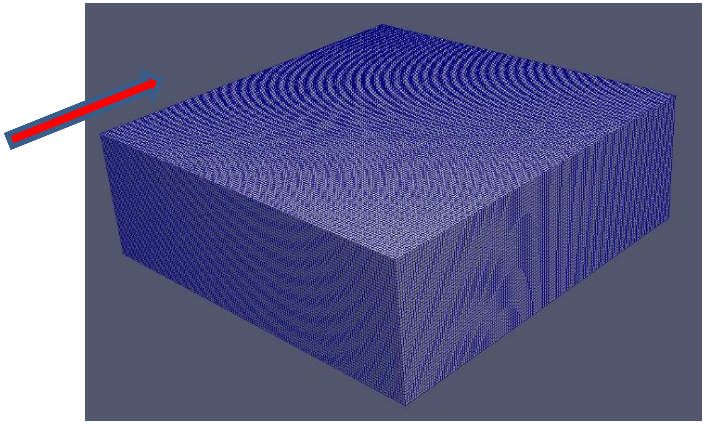
Neutral/Stable Stratification ABL test case

- **Global Energy and Water Cycle Experiment Atmospheric Boundary Layer Study (GABLS) model intercomparison case**
- **Flat terrain**
- **3000 m** x **3000 m** x **1020 m**
- **150x150x51 grid (20 m) and 300x300x102 grid (10 m)**
- **Surface cooling rate 1.38889 K/s**
- **Periodic BCs**
- **Geostrophic wind U=8 m/s**
- **41.3 N latitude**
- ***zo* =0.15 m**
- **SGS models:**

  Standard Smagorinsky

  Dynamic Smagorinsky

U_g=(8.0, 0.0, 0.0)



Numerical domain and grid

U magnitude field. t = 20000 s. top - “neutral", bottom – “stable".

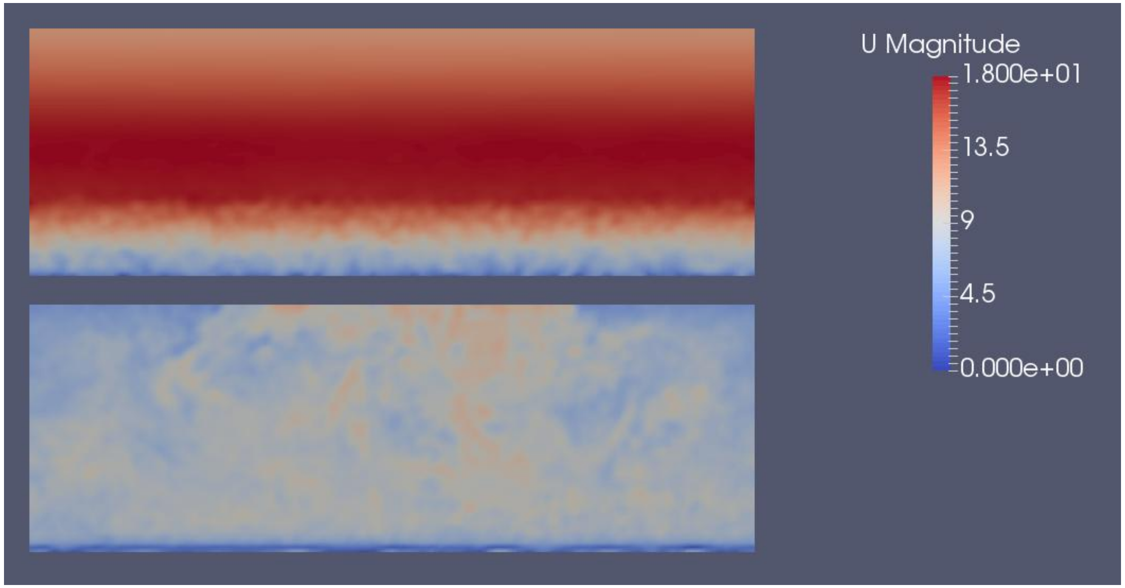
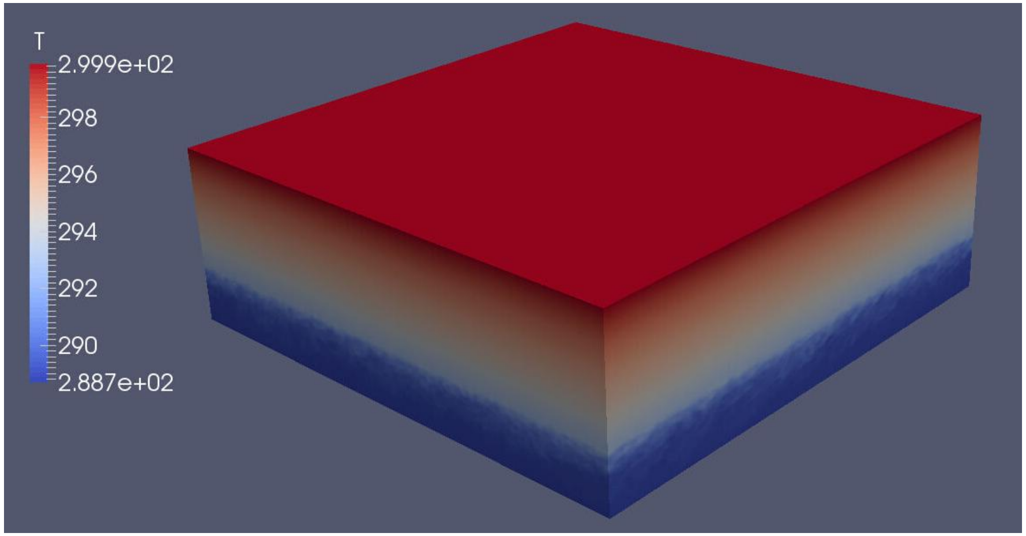
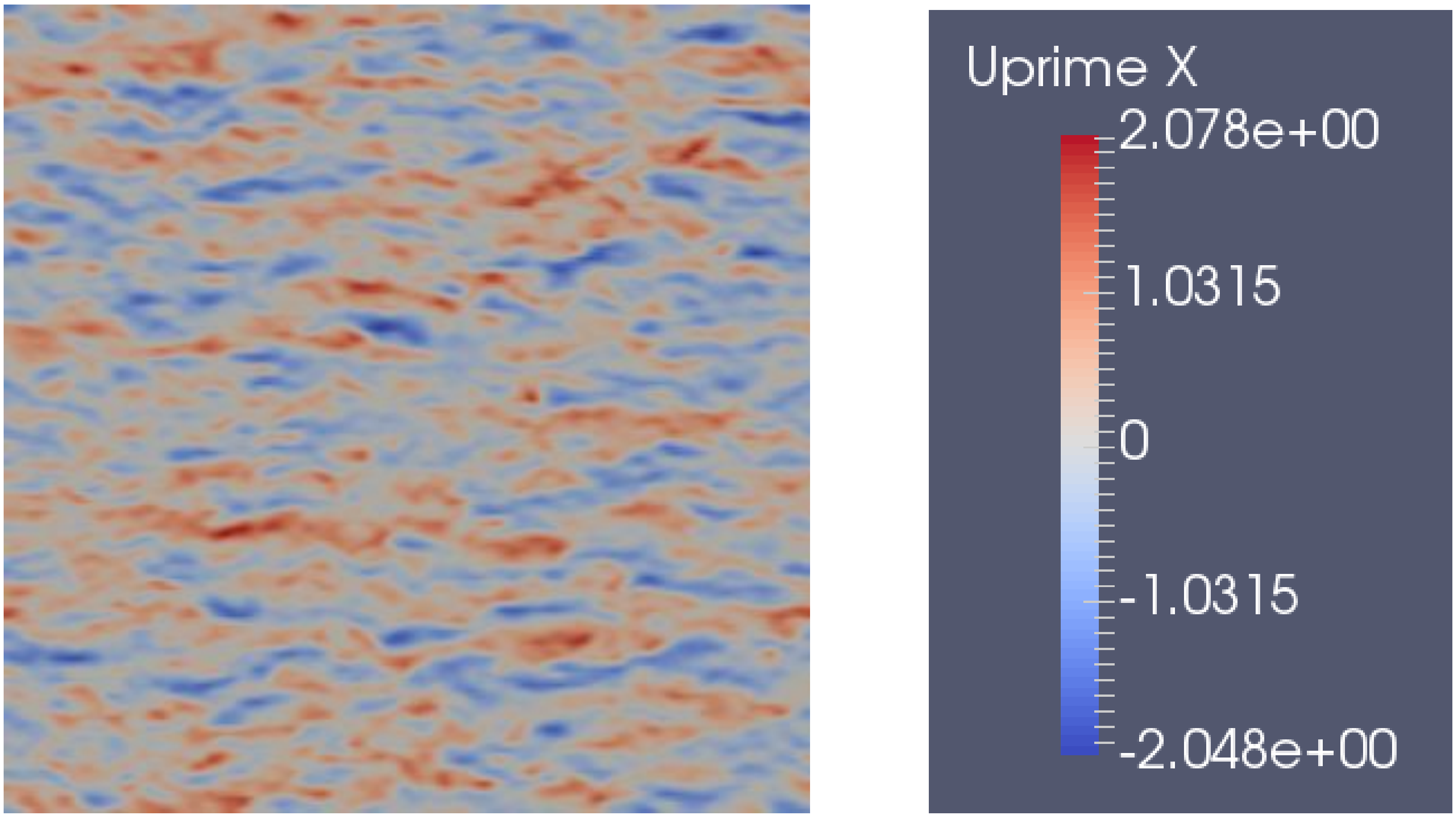


Illustration of temperature for Neutral Stratification ABL test case. t = 20000 s.
Mesh 300x300x102



Streamwise velocity fluctuations at 90 m above surface, mesh 150 x 150 x 51

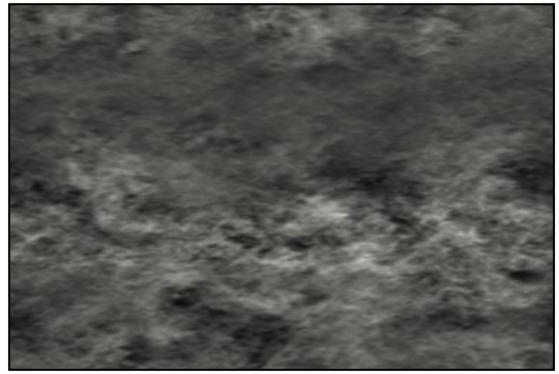


Need to take into account velocity and pressure fluctuations on wind blade

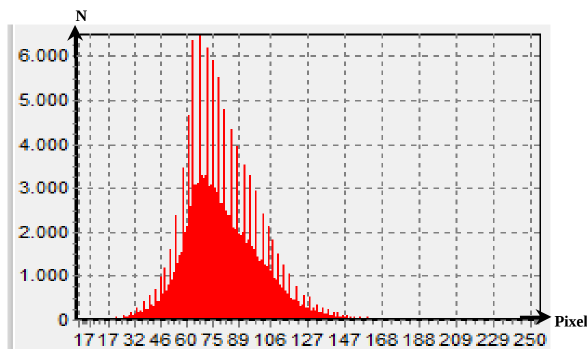
Case of Study: Atmosphere boundary layer

Analysis of the Stably stratified velocity structure Umag_stable_horbw0).

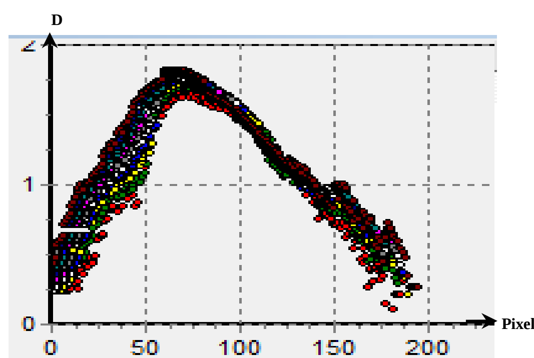
Image of the velocity field



Histogram of the image

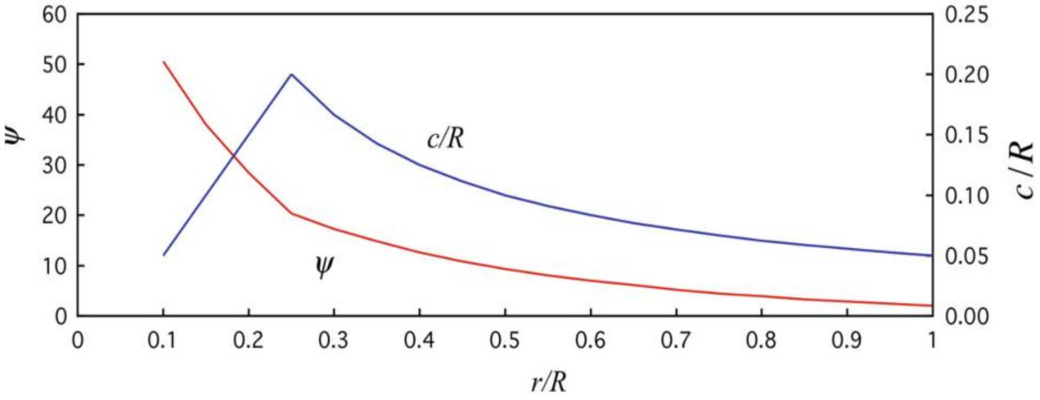


Fractal Dimension

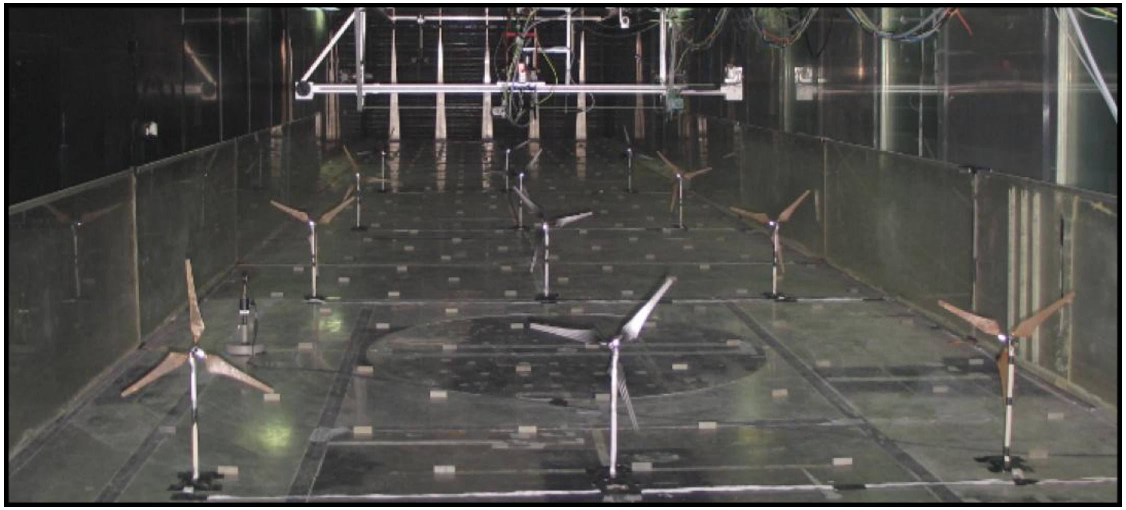


Experiment in EnFlo Laboratory, University of Surrey Guildford (the stable, unstable and reference neutral cases)

Dr=416 mm, Dt=13 mm, Uref=1.5 m/s, TSR=6



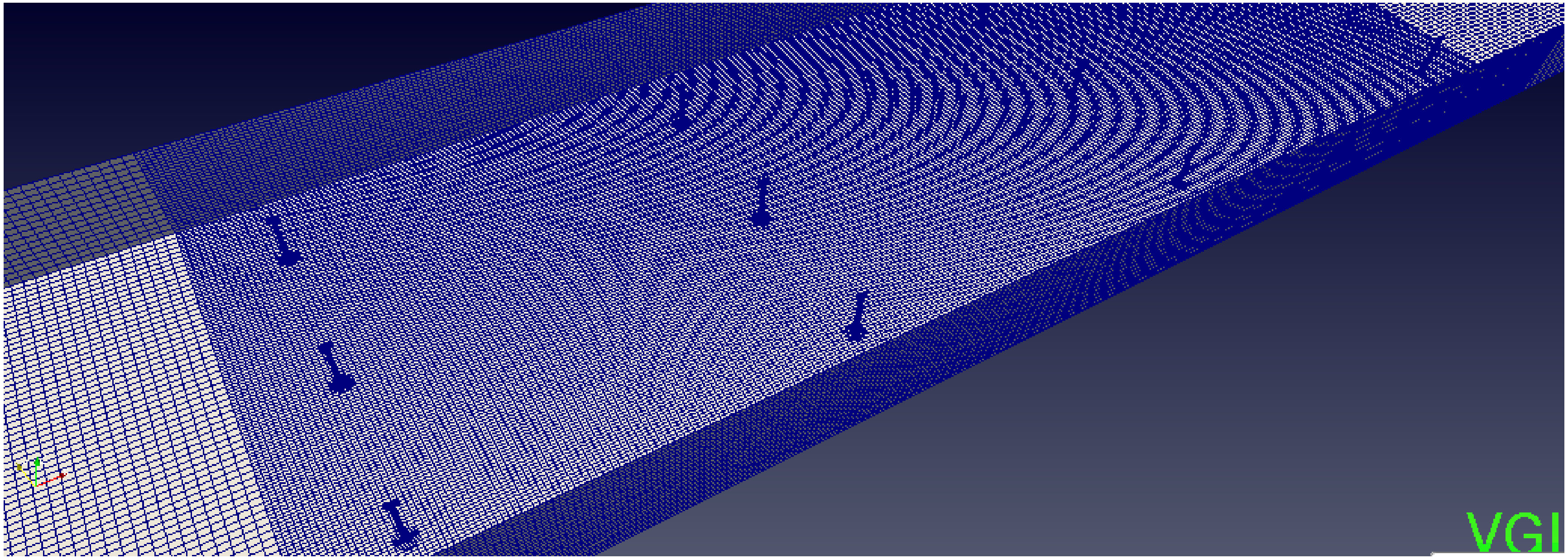
Blade chord and twist

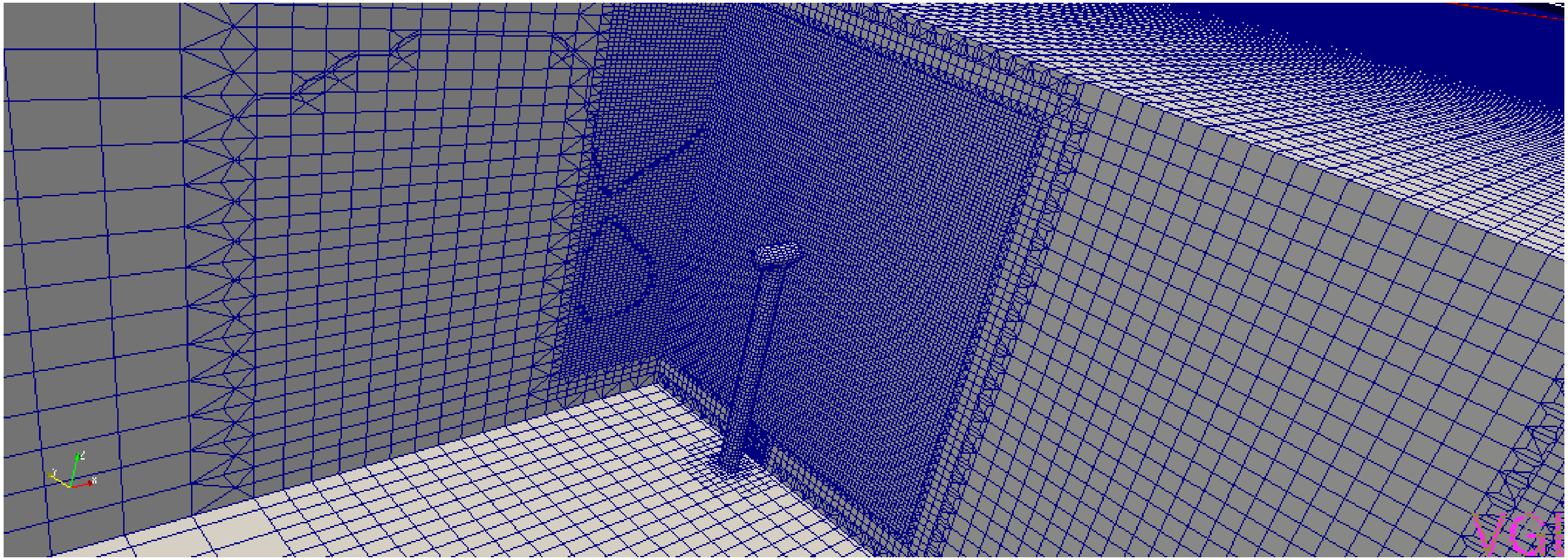


View of wind turbines in the working section, looking upstream. A 3-wide × 4-deep array.

E. Hancock and T. D. Farr 2014 *J. Phys.: Conf. Ser.* **524** 012166.

Numerical Domain and Grid with 4.6, 8 mil. cells





Results for case with Neutral ABL

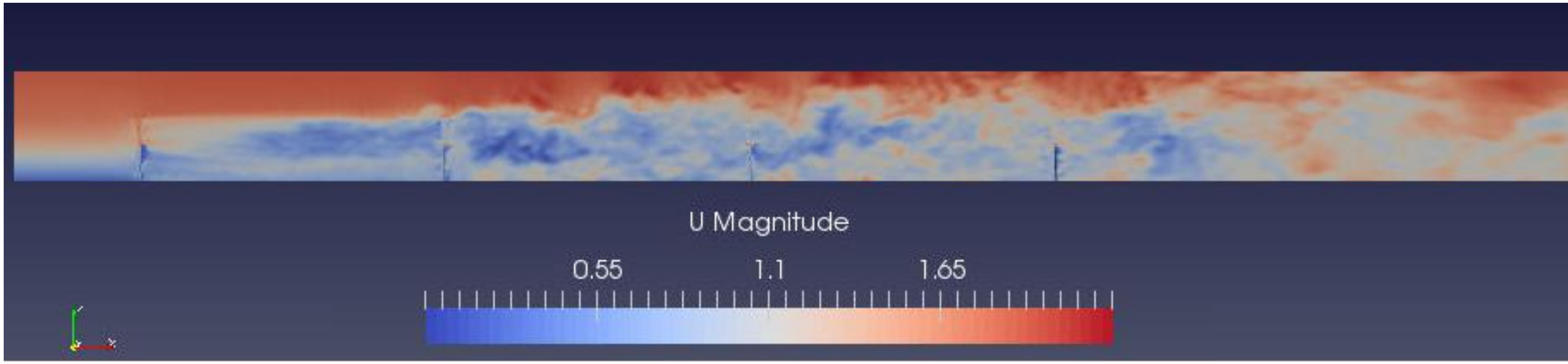


Figure. The velocity field in the longitudinal plane of symmetry of the array for wind turbines (the xOz plane)

I section
I I section
III section
I V section

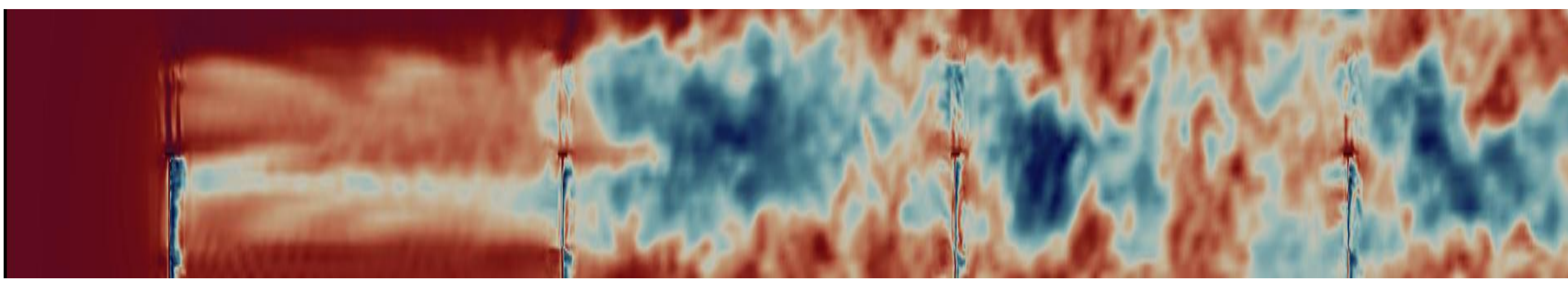


Figure. Propagation of the wind along the sequence of wind turbines

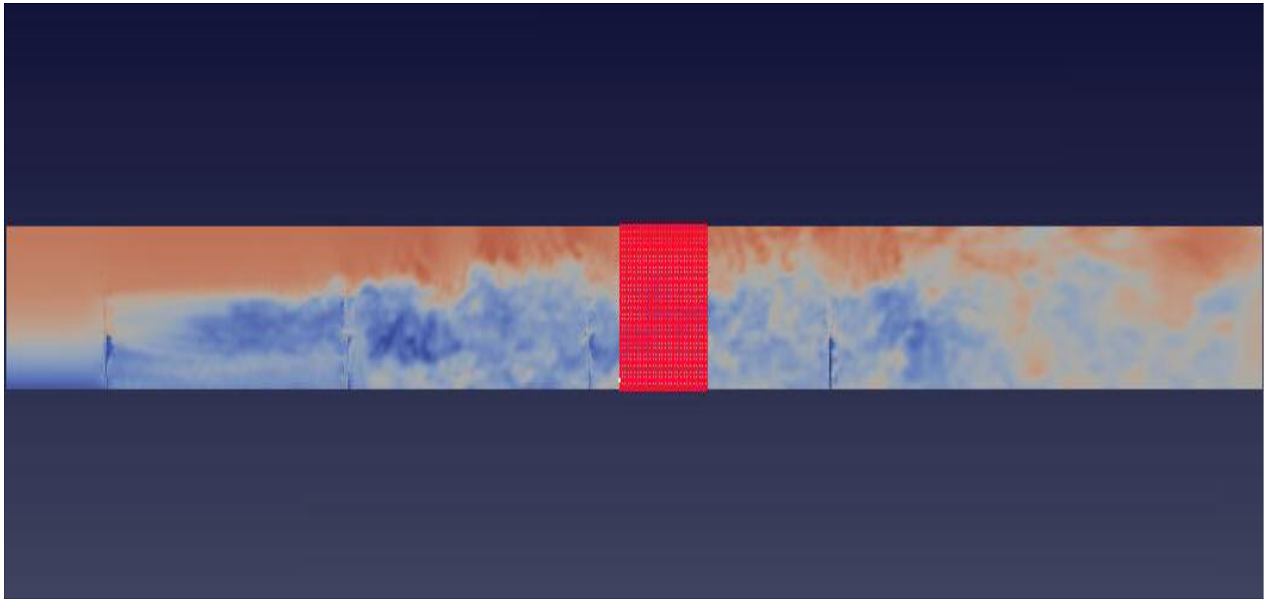
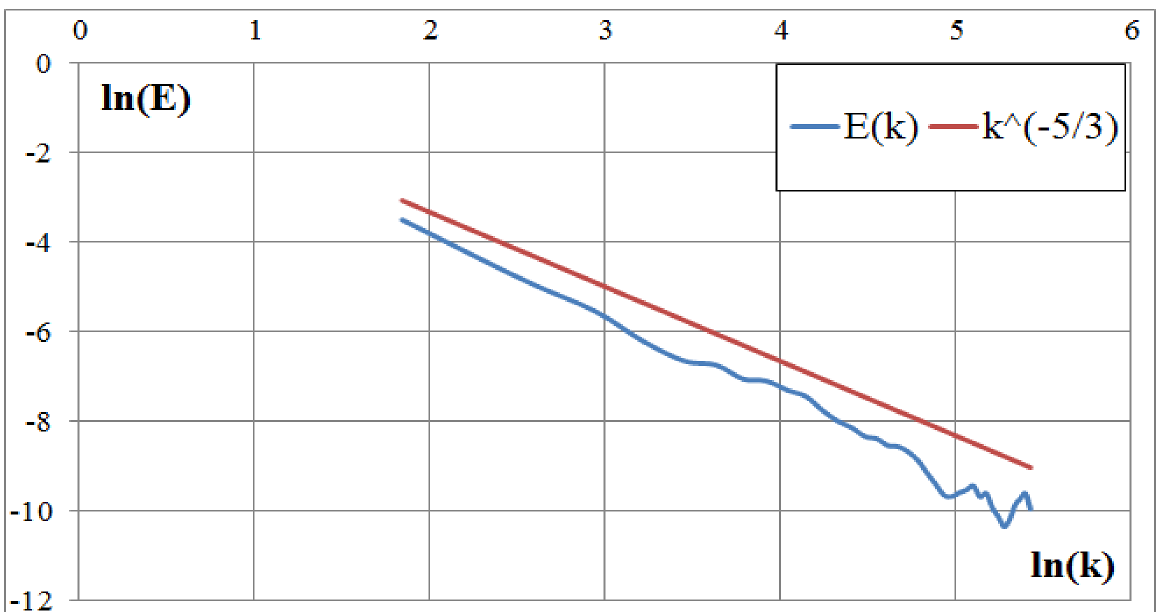
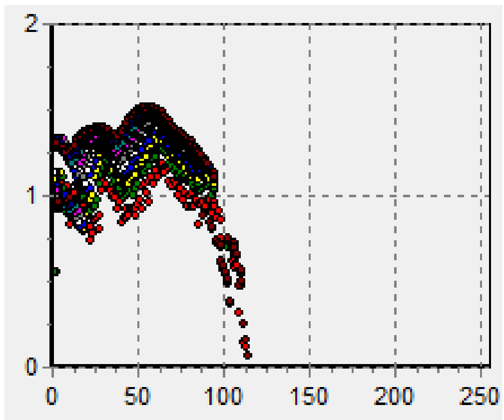


Figure 7: 3D box in numerical domain for calculation E(k)

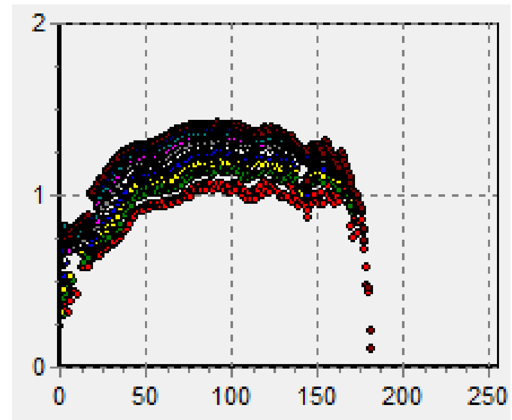


Results of data processing by means of ImaCalc program: the multifractal spectrum *f(α)* for the different sections .

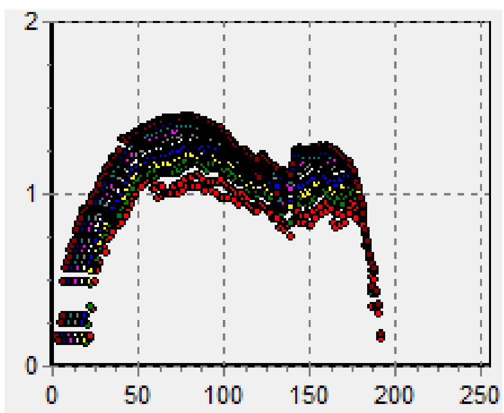
Both the range of velocity values (i) and the shape of the multi fractal spectra change in a non/linear fashion.



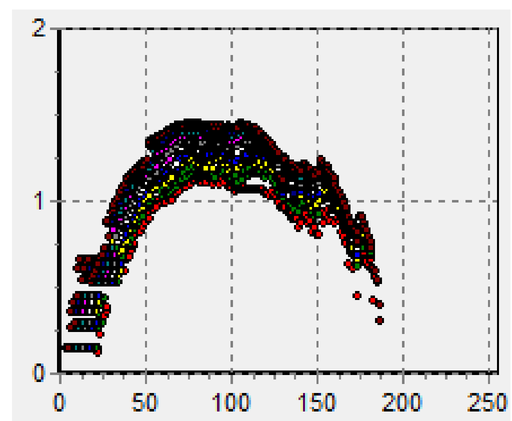
Section 1



Section 2

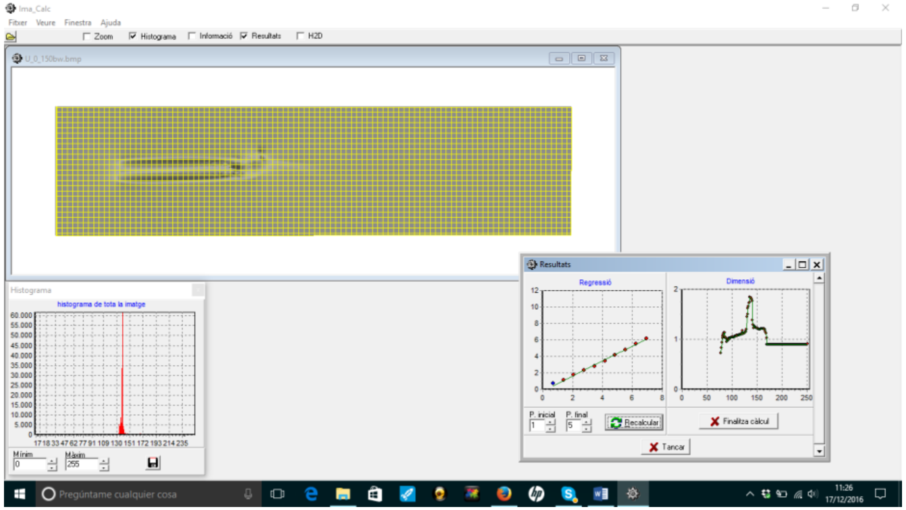


Section 3



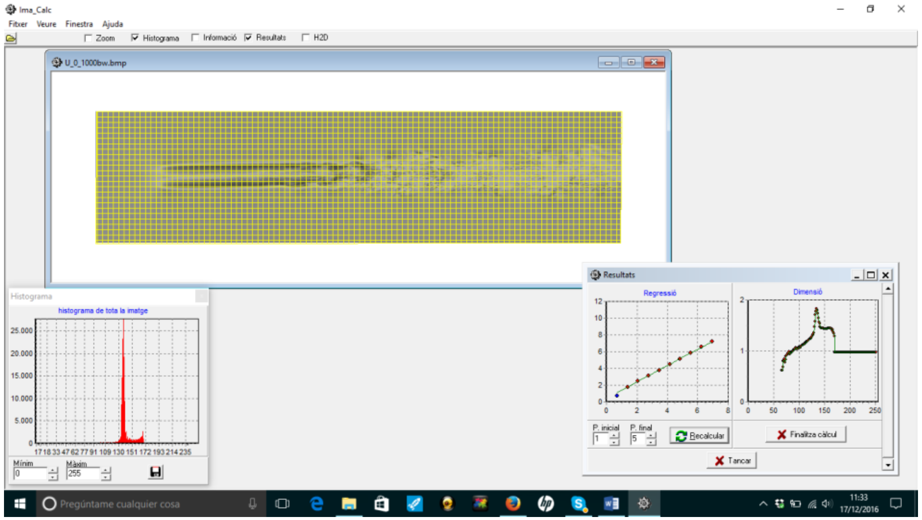
Section 4

MULTIFRACTAL ANALYSIS OF A WAKE FOR A SINGLE WIND TURBINE*



ImaCalc Program

t = 150 seconds



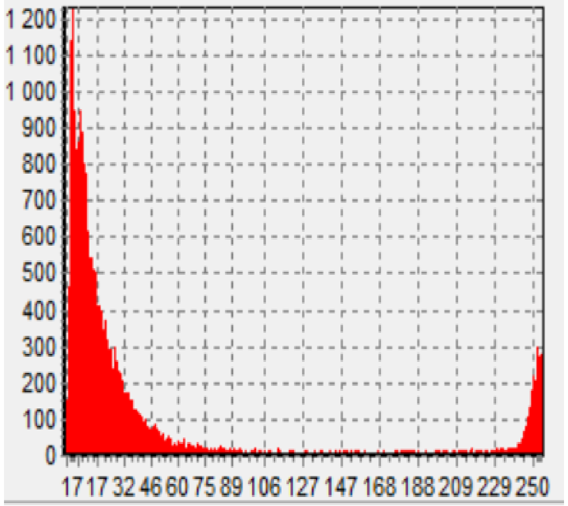
t = 1000 seconds

The intermittency exponent changed for higher values of vorticity at an increasing rate, from 1.1 for low values up to the maximum of 2.1.

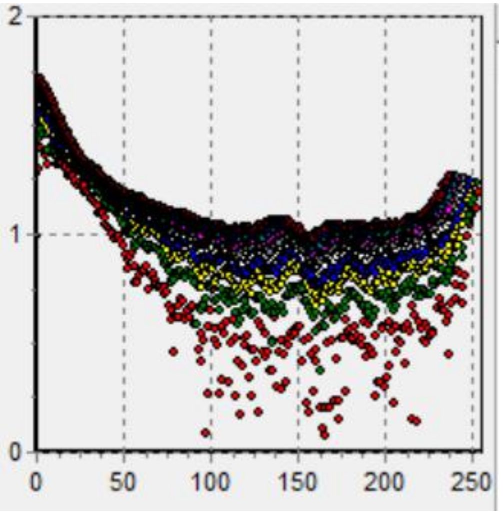
* **S. Strijhak, J.M. Redondo, J. Tellez**. **MULTIFRACTAL ANALYSIS OF A WAKE FOR A SINGLE WIND TURBINE**. DOI: https://doi.org/10.14311/TPFM.2017.035

Case of Study: PLUMES

Analysis of the experimental Plumes
The histogram and multifractal analysis were built using ImaCalc software .



Histogram of Plumes experiments.



Multifractal analysis in ImaCalc software

The aim of the experimental procedure was to generate a turbulent axisymmetric plume, controlling its position and its physical characteristics as buoyancy and momentum fluxes.

A. Matulka, P. López, J. M. Redondo, A. Tarquis. On the entrainment coefficient in a forced plume: quantitative effects of source parameters. Nonlin. Processes Geophys., 21, 269–278, 2014

Some photos
for memory

Barcelona, UPC, April 2016

ISPRAS OPEN CONFERENCE 2017

ISPRAS OPEN CONFERENCE 2017

Carolina, Jorn, Jose Manuel, Matvey

ISPRAS OPEN CONFERENCE 2017: BOAT TRIP

On a tour of the Kremlin

PhD defence of Jackson Tellez-Alvarez in UPC

On EGU conference in Vienna, Austria with Ilias

Jose Manuel liked to cook

Sergei, Jose Manuel, Jackson

Conclusions

- There are a large number of interesting unsolved problems in geophysical flows (plumes, ABL, jets, wakes, rotating turbulence)
- Prof. Jose Manuel Redondo Apraiz left behind a great scientific property (papers, experiments, etc.)
- We propose to hold a scientific seminar in memory of Prof. Jose Manuel Redondo Apraiz. May be in January of 2021