Esercizi vari su equazioni differenziali e integrali

Equazioni differenziali

- $u''' - 3u' + 2u = t$

$$\boxed{C_1 e^t + C_2 t e^t + C_3 e^{-2t} + \frac{t}{2} + \frac{3}{4}}$$

- $u''' - 2u'' + 2u' = t^2$

$$\boxed{C_1 + C_2 e^t \cos t + C_3 e^t \sin t + \frac{t^3}{6} + \frac{t^2}{2} + t}$$

- $u'' + u = e^t \sin(2t)$

$$\boxed{C_1 \cos t + C_2 \sin t - e^t \left[\frac{1}{5}\cos(2t) + \frac{1}{10}\sin(2t)\right]}$$

- $u''' + u' = \cos t$

$$\boxed{C_1 + C_2 \cos t + C_3 \sin t - \frac{t}{2}\cos t}$$

- $u' + 2u = te^t \cos t$

$$\boxed{Ce^{-2t} + \frac{e^t}{50}\left[(5t-3)\sin t + (15t-4)\cos t\right]}$$

- $u'' + (u')^2 - 1 = 0$

$$\boxed{\ln|e^{2t} + C_1| - t + C_2 \quad (u_s = \pm t + \tilde{C}_1)}$$

- $u'' - (1+t)u' - (1-t)u = 0$

$$\boxed{C_1 e^{\frac{t^2}{2}} \operatorname{erf}\left(\frac{t-1}{\sqrt{2}}\right) + C_2 e^{\frac{t^2}{2}}}$$

- $u''' + 3u' = t^2 + 1$

$$\boxed{C_1 + C_2 \cos(\sqrt{3}t) + C_3 \sin(\sqrt{3}t) + \frac{t}{9} + \frac{t^3}{9}}$$

- $u''' - 6u' + 4u = 0$

$$\boxed{C_1 e^{2t} + C_2 e^{(-1+\sqrt{3})t} + C_3 e^{(-1-\sqrt{3})t}}$$

- $u'''' + 8u'' + 16u = 0$

$$\boxed{\begin{aligned} &C_1 \cos(2t) + C_2 \sin(2t) + \\ &\qquad + C_3 t\cos(2t) + C_4 t \sin(2t)\end{aligned}}$$

- $u' - 2tu = t$

$$\boxed{Ce^{t^2}}$$

- $u' - 2tu = t^3$

$$\boxed{Ce^{t^2} - \frac{t^2}{2} - \frac{1}{2}}$$

- $u''' + 9u' = 2t + 1$

$$\boxed{C_1 + C_2 \cos(3t) + C_3 \sin(3t) + \frac{t}{9} + \frac{t^2}{9}}$$

- $u'' - 2u' + u = \cos(4t)$

$$\boxed{C_1 e^t + C_2 t e^t - \frac{15}{289}\cos(4t) - \frac{8}{289}\cos(4t)}$$

- $u''' + 4u'' = 8t + 20$

$$\boxed{C_1 + C_2 t + C_3 e^{-4t} + \frac{9}{4}t^2 + \frac{t^3}{3}}$$

- $u''' - 2u'' - 3u' = t^2 - 3t + 4 + \sin t$

$$\boxed{\begin{aligned} &C_1 + C_2 e^{-t} + C_3 e^{3t} - \frac{t^3}{9} + \frac{13}{18}t^2 - \frac{68}{27}t + \\ &\qquad + \frac{1}{10}\sin t + \frac{1}{5}\cos t\end{aligned}}$$

- $u''' + 5u'' + 4u' = \cos^2\left(\frac{t}{4}\right)$

$$\boxed{\begin{aligned} &C_1 + C_2 e^{-t} + C_3 e^{-4t} + \frac{t}{8} + \\ &\qquad + \frac{12}{65}\sin\left(\frac{t}{2}\right) - \frac{8}{65}\cos\left(\frac{t}{2}\right)\end{aligned}}$$

- $u''' + 4u' = 5\sin(2t)$

$$\boxed{C_1 + C_2 \cos(2t) + C_3 \sin(2t) - \frac{5}{8}t\sin(2t)}$$

- $u' = 5t^3\sqrt{u-u^2}$

  $$\boxed{\frac{1}{2}\sin\left(\frac{5}{4}t^4+C\right)+\frac{1}{2} \quad (u_s=0,\ u_s=1)}$$

- $u''-3u'-10u = 2t^2-t+3+\sin(2t)$

  $$\boxed{C_1e^{-2t}+C_2e^{5t}-\frac{203}{500}+\frac{11}{50}t-\frac{t^2}{5}+\frac{3}{116}\cos(2t)-\frac{7}{116}\sin(2t)}$$

- $2u'''+8u''+8u' = 5-t+\frac{1}{2}\cos^2 t$

  $$\boxed{C_1+C_2e^{-2t}+C_3te^{-2t}-\frac{t^2}{16}+\frac{25}{32}-\frac{1}{128}\cos(2t)}$$

- $u''-4u'+5u = 2t^2+4-\sin^2\left(\frac{t}{2}\right)$

  $$\boxed{C_1e^{2t}\cos t+C_1e^{2t}\sin t+\frac{2}{5}t^2+\frac{16}{25}t+\frac{263}{250}+\frac{1}{16}\cos t-\frac{1}{16}\sin t}$$

- $u' = \frac{2t}{t^2-2t}\tan u$

  $$\boxed{\arcsin\left[C(t-2)^2\right] \quad (u_s=k\pi \text{ con } k\in\mathbb{Z})}$$

- $u'+u\cos t = \sin(2t)$ $\boxed{Ce^{-\sin t}+2(\sin t-1)}$

- $u'+u^2-u=0$ $\boxed{\frac{e^t}{C+e^t} \quad (u_s=0,\ u_s=1)}$

- $u'''-2u''-3u' = \cos(3t)+5+t^2$

  $$\boxed{C_1+C_2e^{-t}+C_3e^{3t}+\frac{1}{90}\cos(3t)-\frac{1}{45}\sin(3t)-\frac{t^3}{9}+\frac{2}{9}t^2-\frac{59}{27}t}$$

- $u'+\frac{2t-1}{t}u = e^{-2t}$ $\boxed{e^{-2t}\left(t\ln|t|+C\right)}$

- $u'''+16u' = \sin(4t)+3t^2+1$

  $$\boxed{C_1+C_2\cos(4t)+C_3\sin(4t)-\frac{1}{32}t\sin(3t)+\frac{t^3}{16}+\frac{5}{128}t}$$

- $u' = (u^2+u)\cos(2t)$

  $$\boxed{\frac{Ce^{\sin t\cos t}}{1-Ce^{\sin t\cos t}} \quad (u_s=0,\ u_s=-1)}$$

Problemi di Cauchy

- $\begin{cases} u''+4u=\sin t \\ u(0)=1 \\ u'(0)=1 \end{cases}$ $\boxed{\cos(2t)+\frac{1}{3}\left[\sin(2t)+\sin t\right]}$

- $\begin{cases} 3u''+5u'-2u=0 \\ u(0)=1 \\ u'(0)=2 \end{cases}$ $\boxed{\frac{12}{7}e^{\frac{t}{3}}-\frac{5}{7}e^{-2t}}$

- $\begin{cases} u''+4u'+13u=0 \\ u(0)=0 \\ u'(0)=1 \end{cases}$ $\boxed{\frac{1}{3}e^{-2t}\sin(3t)}$

- $\begin{cases} u'''-4u'=0 \\ u(0)=0 \\ u'(0)=1 \\ u''(0)=1 \end{cases}$ $\boxed{-\frac{1}{4}+\frac{3}{8}e^{2t}-\frac{1}{8}e^{-2t}}$

- $\begin{cases} u''+u'-6u=t^2+2t \\ u(0)=0 \\ u'(0)=2 \end{cases}$

  $$\boxed{\frac{11}{20}e^{2t}-\frac{58}{135}e^{-3t}-\frac{1}{108}(13+42t+18t^2)}$$

- $\begin{cases} u'=t^2u^2 \\ u(0)=1 \end{cases}$ $\boxed{\frac{3}{3-t^3}}$

- $\begin{cases} u'''+9u'=1+2\cos(3t) \\ u(0)=0 \\ u'(0)=1 \\ u''(0)=0 \end{cases}$

  $$\boxed{\frac{1}{3}\sin(3t)+\frac{t}{9}-\frac{t}{9}\cos(3t)}$$

- $\begin{cases} u'''+2u''-3u'=-3t^2+2t \\ u(0)=0 \\ u'(0)=1 \\ u''(0)=1 \end{cases}$

  $$\boxed{-\frac{1}{27}+\frac{1}{27}e^{-3t}+\frac{t^3}{3}+\frac{t^2}{3}+\frac{10}{9}t}$$

- $\begin{cases} u' = t^{3/2}(u^2+u+1) \\ u(0) = 0 \end{cases}$

  $\boxed{\frac{\sqrt{3}}{2}\tan\left(\frac{\sqrt{3}}{5}x^{\frac{5}{2}}+\frac{\pi}{6}\right)-\frac{1}{2}}$

- $\begin{cases} u' = \dfrac{u}{\sqrt{\dfrac{t^2}{4}+1}} \\ u(0) = 1 \end{cases}$

  $\boxed{\frac{t^2}{2}+1+t\sqrt{\frac{t^2}{4}+1}}$

Integrali

- $\displaystyle\int \frac{e^x+1}{e^x-1}\,\mathrm{d}x$ $\boxed{2\ln\left|\sinh\left(\frac{x}{2}\right)\right|}$

- $\displaystyle\int \frac{x^3}{x^3-x^2-x+1}\,\mathrm{d}x$

  $\boxed{x-\frac{1}{4}\ln|x+1|+\frac{5}{4}\ln|x-1|-\frac{1}{2(x-1)}}$

- $\displaystyle\int \frac{x^3}{x^3-x^2+4x-4}\,\mathrm{d}x$

  $\boxed{x+\frac{1}{5}\left[\ln|x-1|+2\ln|x^2+4|-8\arctan\left(\frac{x}{2}\right)\right]}$

- $\displaystyle\int \frac{x^3}{x^3-x^2+5x-5}\,\mathrm{d}x$

  $\boxed{x+\frac{1}{6}\left[\ln|x-1|+\frac{5}{2}\ln|x^2+5|-5\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)\right]}$

- $\displaystyle\int \frac{\cos^3 x}{\sin x}\,\mathrm{d}x$ $\boxed{\ln|\sin x|+\frac{1}{4}\cos(2x)}$

- $\displaystyle\int x\arctan\left(\frac{x}{2}\right)\mathrm{d}x$ $\boxed{\left(\frac{x^2}{2}+2\right)\arctan\left(\frac{x}{2}\right)-x}$

- $\displaystyle\int x\ln(1+x+x^2)\,\mathrm{d}x$

  $\boxed{\frac{1}{4}\left[(2x^2+1)\ln|1+x+x^2|-2x^2+x-2\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right]}$

- $\displaystyle\int \frac{x^2-x+6}{x^2+2x+2}\,\mathrm{d}x$

  $\boxed{-\frac{3}{2}\ln|x^2+2x+2|+x+7\arctan(x+1)}$

- $\displaystyle\int \frac{1}{x^2}\cos\left(\frac{x+2}{x}\right)\mathrm{d}x$ $\boxed{-\frac{1}{2}\sin\left(1+\frac{2}{x}\right)}$

- $\displaystyle\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x$ $\boxed{\frac{1}{2}\left(\arctan x+\frac{x}{1+x^2}\right)}$

- $\displaystyle\int \frac{x^3}{(1+x^2)^2}\,\mathrm{d}x$ $\boxed{\frac{1}{2}\left[\ln|1+x^2|+\frac{1}{1+x^2}\right]}$

- $\displaystyle\int \frac{e^{3x}}{1+e^{2x}}\,\mathrm{d}x$ $\boxed{e^x-\arctan(e^x)}$

- $\displaystyle\int \frac{x^3+3x^2}{1+x+x^2}\,\mathrm{d}x$

  $\boxed{\frac{x^2}{2}+2x-\frac{3}{2}\ln|1+x+x^2|-\frac{1}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}$

- $\displaystyle\int \sqrt{\frac{2-x}{x+3}}\,\mathrm{d}x$

  $\boxed{5\arcsin\left(\sqrt{\frac{x+3}{5}}\right)+\sqrt{x+3}\sqrt{2-x}}$

- $\displaystyle\int \frac{1}{x^3-4x^2+4x}\,\mathrm{d}x$ $\boxed{\frac{1}{4}\ln\left|\frac{x}{x-2}\right|-\frac{1}{2x-4}}$

- $\displaystyle\int x\sqrt{2x^2-2x+1}\,\mathrm{d}x$

  $\boxed{\frac{\sqrt{2}}{16}\operatorname{arcsinh}(2x-1)+\frac{1}{24}(8x^2-2x+1)\sqrt{2x^2-2x+1}}$

- $\displaystyle\int \frac{\cos x}{1+\cos x}\,\mathrm{d}x$ $\boxed{x-\tan\left(\frac{x}{2}\right)}$

- $\displaystyle\int e^{-3x}\sin(2x)\,\mathrm{d}x$

  $\boxed{-\frac{1}{13}e^{-3x}\left[3\sin(2x)+2\cos(2x)\right]}$

- $\displaystyle\int x^3\sqrt{4+x^2}\,\mathrm{d}x$ $\boxed{(4+x^2)^{\frac{3}{2}}\left(\frac{x^2}{5}-\frac{8}{15}\right)}$

- $$\int \frac{1}{1+x^4}\,dx$$
  $$\frac{\sqrt{2}}{4}\left[\operatorname{arctanh}\left(\frac{\sqrt{2}x}{x^2+1}\right)+ + \arctan\left(\sqrt{2}x+1\right)+\arctan\left(\sqrt{2}x-1\right)\right]$$
- $$\int \frac{1}{x^2}\ln\left(\frac{x-3}{2x}\right)dx \qquad \frac{x-3}{3x}\ln\left(\frac{x-3}{2x}\right)+\frac{1}{x}$$
- $$\int \frac{x^4+x+1}{x^3-2x^2+x}\,dx$$
  $$\frac{x^2}{2}+2x-\frac{3}{x-1}+2\ln|1-x|+\ln|x|$$
- $$\int \frac{x^4}{x^3-1}\,dx$$
  $$\frac{x^2}{2}+\frac{1}{3}\ln|x-1|-\frac{1}{6}\ln\left(x^2+x+1\right)+ + \frac{1}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)$$
- $$\int \frac{x^3}{\sqrt{1+x^2}}\,dx \qquad \frac{1}{3}(x^2-2)\sqrt{1+x^2}$$
- $$\int \frac{x^3}{x^2-7x+12}\,dx$$
  $$7x+\frac{x^2}{2}-27\ln|x-3|+64\ln|x-4|$$
- $$\int \frac{1}{x^2}\ln\left(\frac{x+1}{x-1}\right)dx$$
  $$-\frac{1}{x}\ln\left(\frac{x+1}{x-1}\right)+2\ln|x|-\ln|x^2-1|$$
- $$\int \frac{\cos^3 x}{1+\sin x}\,dx \qquad \sin x+\frac{1}{4}\cos(2x)$$
- $$\int xe^x\cos(2x)\,dx$$
  $$\frac{e^x}{5}\left[\left(\frac{3}{5}+x\right)\cos(2x)+\left(2x-\frac{4}{5}\right)\sin(2x)\right]$$
- $$\int \frac{x^4}{x^2-6x+5}\,dx$$
  $$\frac{x^3}{3}+3x^2+31x+\frac{1}{4}\left(625\ln|5-x|-\ln|1-x|\right)$$
- $$\int e^x\cos(\pi x)\,dx$$
  $$\frac{e^x}{1+\pi^2}\left[\pi\sin(\pi x)+\cos(\pi x)\right]$$
- $$\int \frac{x^8}{x^3-1}\,dx \qquad \frac{x^3}{3}+\frac{x^6}{6}+\frac{1}{3}\ln|x^3-1|$$
- $$\int x^3\cos(2x)\,dx$$
  $$\frac{1}{8}(6x^2-3)\cos(2x)+\frac{1}{4}(2x^3-3x)\sin(2x)$$
- $$\int \frac{e^{-x}}{1+e^x}\,dx \qquad -x-e^{-x}+\ln|1+e^x|$$
- $$\int \frac{x\cos x-\sin x}{x^2}\,dx \qquad \frac{\sin x}{x}$$
- $$\int x^2\ln(2x+3)\,dx$$
  $$-\frac{3}{4}x+\frac{x^2}{4}-\frac{x^3}{9}+\left(\frac{9}{8}+\frac{x^3}{3}\right)\ln(2x+3)$$
- $$\int (1+x^2)^{\frac{5}{2}}\,dx$$
  $$\left(\frac{33}{48}x+\frac{13}{24}x^3+\frac{x^5}{6}\right)\sqrt{1+x^2}+\frac{5}{16}\operatorname{arcsinh}(x)$$
- $$\int \frac{\cos^2 x\sin x}{1+\cos x}\,dx$$
  $$\cos x-\frac{1}{4}\cos(2x)-2\ln\left|\cos\left(\frac{x}{2}\right)\right|$$
- $$\int (x+3)\sqrt{x^2+2x+2}\,dx$$
  $$\frac{1}{3}(x^2+5x+5)\sqrt{x^2+2x+2}+\operatorname{arcsinh}(1+x)$$
- $$\int e^{-\frac{x}{2}}\sin^2 x\,dx$$
  $$\frac{1}{17}e^{-\frac{x}{2}}\left[\cos(2x)-4\sin(2x)-17\right]$$
- $$\int e^{-2x}x^3\,dx \qquad -\frac{1}{8}e^{-2x}(4x^3+6x^2+6x+3)$$
- $$\int x^2\cos^2 x\,dx$$
  $$\frac{x^3}{6}+\frac{x}{4}\cos(2x)+\left(\frac{x^2}{4}-\frac{1}{8}\right)\sin(2x)$$

- $\int x\sqrt{-x^2-4x+3}\,dx$

  $\frac{1}{3}(x^2+x-9)\sqrt{-x^2-4x+3}-7\arcsin\left(\frac{x+2}{\sqrt{7}}\right)$

- $\int \frac{x^4-3x^2-1}{x^3-4x}\,dx$

  $\frac{x^2}{2}+\frac{3}{8}\ln|x^2-4|+\frac{1}{4}\ln|x|$

- $\int \frac{x}{1+\cos x}\,dx$ $\quad x\tan\left(\frac{x}{2}\right)+2\ln\left|\cos\left(\frac{x}{2}\right)\right|$

- $\int \sqrt{\frac{2-3x}{x+5}}\,dx$

  $\frac{17}{\sqrt{3}}\arcsin\sqrt{\frac{3(x+5)}{17}}+\sqrt{x+5}\sqrt{2-3x^2}$

- $\int \frac{1}{4x^3+x^2+x}\,dx$

  $\ln|x|-\frac{1}{2}\ln|4x^2+x+1|-\frac{1}{\sqrt{15}}\arctan\left(\frac{8x+1}{\sqrt{15}}\right)$

- $\int \frac{1}{9x^3-6x^2+x}\,dx$ $\quad \ln\left|\frac{x}{1-3x}\right|+\frac{1}{1-3x}$

- $\int x^3\sqrt{2x^2+3}\,dx$ $\quad \frac{1}{10}(x^2-1)(2x^2+3)^{\frac{3}{2}}$

- $\int \frac{\sin^3 x}{1-\cos x}\,dx$ $\quad -\frac{1}{2}(1+\cos x)^2$

- $\int \arcsin\left(\frac{x-2}{x+2}\right)dx$

  $x\arcsin\left(\frac{x-2}{x+2}\right)-2\sqrt{2x}+4\arctan\left(\sqrt{\frac{x}{2}}\right)$

- $\int \frac{e^{3x}}{1+e^{2x}}\,dx$ $\quad e^x-\arctan(e^x)$

- $\int \frac{1}{\sqrt{e^{-2x}-1}}\,dx$ $\quad -\arctan\left(\sqrt{e^{-2x}-1}\right)$

- $\int \frac{x^2+x+1}{(x-1)^2}\,dx$ $\quad x+\frac{3}{1-x}+3\ln|x-1|$

- $\int \frac{\sin^4 x}{1+\cos x}\,dx$ $\quad \frac{x}{2}-\frac{1}{4}\sin(2x)-\frac{1}{3}\sin^3 x$

- $\int x\sqrt{3-2x+x^2}\,dx$

  $\operatorname{arcsinh}\left(\frac{x-1}{\sqrt{2}}\right)+\frac{1}{6}(3-x+2x^2)\sqrt{3-2x+x^2}$

- $\int \frac{x^2+3x-1}{x(x-1)}\,dx$ $\quad x+\ln|x|+3\ln|x-1|$

- $\int \frac{x^2+2x+3}{\sqrt{1+9x^2}}\,dx$

  $\frac{53}{54}\operatorname{arcsinh}(3x)+\frac{1}{18}(x+4)\sqrt{1+9x^2}$

- $\int xe^{2x}\sin(2x)\,dx$

  $\frac{1}{4}xe^{2x}\left[\sin(2x)-\cos(2x)\right]+\frac{1}{8}e^{2x}\cos(2x)$

Integrali generalizzati

- $\int_{-\infty}^{+\infty}\frac{|x|^\alpha}{\sqrt{1+x^2}}\cos\left(\frac{x}{x+1}\right)dx$

  Converge se $-1<\alpha<0$

- $\int_0^1 \frac{1}{x^\alpha}e^{-\frac{1}{x}}\,dx$ $\quad$ Converge per ogni $\alpha\in\mathbb{R}$

- $\int_0^{+\infty}\frac{x}{\sqrt{x^3+1}}\sin x\,dx$ $\quad$ Converge

- $\int_{-\infty}^{+\infty}\frac{\cos(x^2)}{\sqrt{|x|}}\,dx$ $\quad$ Converge

- $\int_2^{+\infty}\frac{1}{\ln x}\sin\left(\frac{1}{x}\right)dx$ $\quad$ Diverge

- $\int_2^{+\infty}\frac{\sin\left(\frac{1}{x}\right)}{\ln x}\sqrt{\frac{x}{x-2}}\,dx$ $\quad$ Diverge

- $\int_1^{+\infty}\frac{x}{\sqrt{x^3+1}}\sin x\,dx$ $\quad$ Converge

- $\int_0^{+\infty}\frac{\arctan x}{x^2+x}\,dx$ $\quad$ Converge

- $\int_1^{+\infty}\frac{1}{\sqrt{x^3+x^2-2}}\,dx$ $\quad$ Converge

- $\int_{-\infty}^{+\infty}x^4e^{-x^2+10x}\,dx$ $\quad$ Converge

- $\int_0^2\frac{1}{|x^2-1|^\alpha}\,dx$ $\quad$ Converge se $\alpha<1$

- $$\int_0^{+\infty} \frac{\cos\left(\frac{1}{x}\right)}{\sqrt{|x-1|}\sqrt{|x-2|}x^{\frac{1}{3}}}\,\mathrm{d}x$$ Converge
- $$\int_1^{\infty} \frac{1}{\sqrt{x-1}} \sin(x+2)\,\mathrm{d}x$$ Converge
- $$\int_1^{\infty} \frac{1}{\sqrt{x(x-1)}} \sin(\sqrt{x})\,\mathrm{d}x$$ Converge

- Sia $f(t)$ una funzione continua per $t \geq 0$ e tale per cui
$$\lim_{t\to+\infty} f(t) = \lambda \neq 0$$
Determinare l'andamento asintotico di
$$F(x) = \int_0^x f(t)\,\mathrm{d}t$$
per $x \to +\infty$. λx

Altri

- Sia $u \in C^1$ su $x \geq 0$. Se valgono
$$\lim_{x\to+\infty} \left[u'(x) - 2xu(x)\right] = 1$$
$$\lim_{x\to+\infty} u(x) = 0$$
determinare
$$\lim_{x\to+\infty} xu(x)$$
$-\frac{1}{2}$

- Determinare la soluzione particolare dell'equazione
$$u'' + 2tu' + u = 3 + 5t^2$$
$1 + t^2$

- Sia $u(t)$ la soluzione del problema $u' - 2tu = 1$ con dato iniziale $u(0) = 0$. Determinare il comportamento asintotico di $u(t)$ per $t \to +\infty$.
$\frac{\sqrt{\pi}}{2}e^{t^2}$

- Determinare una procedura per rappresentare la soluzione particolare dell'equazione
$$u'' - (2+t)u' + (2t-1)u = f(t)$$

- Determinare una procedura per rappresentare la soluzione particolare dell'equazione
$$u'' - (1+2t)u' + 2(t-1)u = f(t)$$

- Sia $G(x)$ la funzione così definita: $G'(x) = e^{-x^2}$, con $G(+\infty) = 0$. Determinare l'andamento asintotico di $G(x)$ per $x \to \infty$.
$-\frac{e^{-x^2}}{2x}$