Integrali

Classici

$$\int f(x)^\alpha f'(x)\,\mathrm{d}x = \frac{1}{\alpha+1} f(x)^{\alpha+1} + c \qquad (\alpha \neq -1)$$

- $\displaystyle\int (ax+b)^n\,\mathrm{d}x = \frac{1}{a(n+1)}(ax+b)^{n+1} + c \qquad (n \neq -1)$
- $\displaystyle\int x^\alpha\,\mathrm{d}x = \frac{1}{\alpha+1}x^{\alpha+1} + c \qquad (\alpha \neq -1)$
- $\displaystyle\int \mathrm{d}x = x + c$
- $\displaystyle\int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + c$
- $\displaystyle\int \sqrt{x}\,\mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}} + c$
- $\displaystyle\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x} + c$
- $\displaystyle\int |x|\,\mathrm{d}x = \frac{x|x|}{2} + c$

Logaritmici ed esponenziali

$$\int \frac{f'(x)}{f(x)}\,\mathrm{d}x = \ln\left|f(x)\right| + c$$

- $\displaystyle\int \frac{1}{ax+b}\,\mathrm{d}x = \frac{1}{a}\ln|ax+b| + c$
- $\displaystyle\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + c$

$$\int a^{f(x)} f'(x)\,\mathrm{d}x = a^{f(x)}\log_a e + c = \frac{a^{f(x)}}{\ln a} + c$$

- $\displaystyle\int a^x\,\mathrm{d}x = a^x \log_a e + c = \frac{a^x}{\ln a} + c$
- $\displaystyle\int e^x\,\mathrm{d}x = e^x + c$

$$\int \log_a\left(f(x)\right) f'(x)\,\mathrm{d}x = f(x)\log_a\left(\frac{f(x)}{e}\right) + c$$

- $\displaystyle\int \log_a x\,\mathrm{d}x = x\log_a\left(\frac{x}{e}\right) + c$
- $\displaystyle\int \ln\left(f(x)\right) f'(x)\,\mathrm{d}x = f(x)\ln\left(f(x)\right) - f(x) + c$
- $\displaystyle\int \ln x\,\mathrm{d}x = x\ln x - x + c$

Seni, coseni e tangenti

$$\int \sin\left(f(x)\right) f'(x)\,\mathrm{d}x = -\cos\left(f(x)\right) + c$$

- $\displaystyle\int \sin x\,\mathrm{d}x = -\cos x + c$

$$\int \cos\left(f(x)\right) f'(x)\,\mathrm{d}x = \sin\left(f(x)\right) + c$$

- $\displaystyle\int \cos x\,\mathrm{d}x = \sin x + c$

$$\int \tan\left(f(x)\right) f'(x)\,\mathrm{d}x = -\ln\left|\cos\left(f(x)\right)\right| + c$$

- $\displaystyle\int \tan x\,\mathrm{d}x = -\ln|\cos x| + c$

$$I_n = \int \sin^n x\,\mathrm{d}x = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}I_{n-2} + c \qquad (n \neq 0)$$

- $\displaystyle I_2 = \int \sin^2 x\,\mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin(2x) + c$
- $\displaystyle I_3 = \int \sin^3 x\,\mathrm{d}x = -\frac{1}{3}(\sin^2 x + 2)\cos x + c$
- $\displaystyle I_4 = \int \sin^4 x\,\mathrm{d}x = \frac{3}{8}x - \frac{1}{4}\sin^3 x\cos x - \frac{3}{16}\sin(2x) + c$

$$J_n = \int \cos^n x\,\mathrm{d}x = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}J_{n-2} + c \qquad (n \neq 0)$$

- $\displaystyle J_2 = \int \cos^2 x\,\mathrm{d}x = \frac{x}{2} + \frac{1}{4}\sin(2x) + c$
- $\displaystyle J_3 = \int \cos^3 x\,\mathrm{d}x = \frac{1}{3}(\cos^2 x + 2)\sin x + c$
- $\displaystyle J_4 = \int \cos^4 x\,\mathrm{d}x = \frac{3}{8}x + \frac{1}{4}\cos^3 x\sin x + \frac{3}{16}\sin(2x) + c$

$$K_n = \int \tan^n x\,\mathrm{d}x = \frac{\tan^{n-1}x}{n-1} - K_{n-2} + c \qquad (n \neq 1)$$

- $\displaystyle K_2 = \int \tan^2 x\,\mathrm{d}x = \tan x - x + c$
- $\displaystyle K_3 = \int \tan^3 x\,\mathrm{d}x = \frac{1}{2}\tan^2 x + \ln|\cos x| + c$

Reciproci di seni, coseni e tangenti

$$\int \frac{1}{\sin x}\,\mathrm{d}x = \int \csc x\,\mathrm{d}x = \ln\sqrt{\frac{1-\cos x}{1+\cos x}} + c = \ln\left|\frac{\sin x}{\cos x + 1}\right| + c$$
$$= \ln\left|\tan\left(\frac{x}{2}\right)\right| + c$$

$$\int \frac{1}{\cos x}\,\mathrm{d}x = \int \sec x\,\mathrm{d}x = \ln\sqrt{\frac{1+\sin x}{1-\sin x}} + c = \ln\left|\frac{\sin x + 1}{\cos x}\right| + c$$
$$= \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$$

$$\int \frac{1}{\tan x}\,\mathrm{d}x = \int \cot x\,\mathrm{d}x = \ln|\sin x| + c$$

$$I_n = \int \frac{1}{\sin^n x}\,\mathrm{d}x = -\frac{1}{n-1}\left[\frac{\cos x}{\sin^{n-1}x} - (n-2)I_{n-2}\right] + c \qquad (n \neq 1)$$

- $\displaystyle I_2 = \int \frac{1}{\sin^2 x}\,\mathrm{d}x = \int (1 + \cot^2 x)\,\mathrm{d}x = -\cot x + c$

$$J_n = \int \frac{1}{\cos^n x}\,\mathrm{d}x = \frac{1}{n-1}\left[\frac{\sin x}{\cos^{n-1}x} + (n-2)J_{n-2}\right] + c \qquad (n \neq 1)$$

- $\displaystyle J_2 = \int \frac{1}{\cos^2 x}\,\mathrm{d}x = \int (1 + \tan^2 x)\,\mathrm{d}x = \tan x + c$

$$K_n = \int \frac{1}{\tan^n x}\,\mathrm{d}x = -\left[\frac{1}{(n-1)\tan^{n-1}x} + K_{n-2}\right] + c \qquad (n \neq 1)$$

- $\displaystyle K_2 = \int \frac{1}{\tan^2 x}\,\mathrm{d}x = -x - \cot x + c$
- $\displaystyle K_3 = \int \frac{1}{\tan^3 x}\,\mathrm{d}x = -\frac{1}{2}\cot^2 x - \ln|\sin x| + c$

Rapporti di seni e coseni ($n \neq m$)

$$\int \frac{\sin^n x}{\cos^m x}\,\mathrm{d}x = -\frac{1}{n-m}\left[\frac{\sin^{n-1} x}{\cos^{m-1} x} - (n-1)\int \frac{\sin^{n-2} x}{\cos^m x}\,\mathrm{d}x\right] + c$$

- $$\int \frac{\sin x}{\cos^2 x}\,\mathrm{d}x = \frac{1}{\cos x} + c$$

$$\int \frac{\cos^n x}{\sin^m x}\,\mathrm{d}x = \frac{1}{n-m}\left[\frac{\cos^{n-1} x}{\sin^{m-1} x} + (n-1)\int \frac{\cos^{n-2} x}{\sin^m x}\,\mathrm{d}x\right] + c$$

- $$\int \frac{\cos x}{\sin^2 x}\,\mathrm{d}x = -\frac{1}{\sin x} + c$$

Fratte trigonometriche

$$\int \frac{1}{1 \pm \sin x}\,\mathrm{d}x = \tan\left(\frac{x}{2} \mp \frac{\pi}{4}\right) + c_1 = -\frac{2}{\tan\left(\frac{x}{2}\right) \pm 1} + c_2$$

$$\int \frac{1}{1 + \cos x}\,\mathrm{d}x = \tan\left(\frac{x}{2}\right) + c$$

$$\int \frac{1}{1 - \cos x}\,\mathrm{d}x = -\cot\left(\frac{x}{2}\right) + c$$

$$\int \frac{1}{\cos(cx) \pm \sin(cx)}\,\mathrm{d}x = \frac{1}{c\sqrt{2}} \ln\left|\tan\left(\frac{cx}{2} \pm \frac{\pi}{8}\right)\right| + c$$

$$\int \frac{1}{\left[\cos(cx) \pm \sin(cx)\right]^2}\,\mathrm{d}x = \frac{1}{2c} \tan\left(cx \mp \frac{\pi}{4}\right) + c$$

Trigonometriche inverse

$$\int \arcsin x\,\mathrm{d}x = x \arcsin x + \sqrt{1 - x^2} + c$$

$$\int \arccos x\,\mathrm{d}x = x \arccos x - \sqrt{1 - x^2} + c$$

$$\int \arctan x\,\mathrm{d}x = x \arctan x - \frac{1}{2}\ln(1 + x^2) + c$$

$$\int \operatorname{arccot} x\,\mathrm{d}x = x \operatorname{arccot} x + \frac{1}{2}\ln(1 + x^2) + c$$

Funzioni iperboliche e relative inverse

$$\int \sinh x\,\mathrm{d}x = \cosh x + c$$

$$\int \cosh x\,\mathrm{d}x = \sinh x + c$$

$$\int \tanh x\,\mathrm{d}x = \ln(\cosh x) + c$$

$$\int \coth x\,\mathrm{d}x = \ln|\sinh x| + c$$

$$\int \frac{1}{\sinh^2 x}\,\mathrm{d}x = -\int (1 - \coth^2 x)\,\mathrm{d}x = -\coth x + c$$

$$\int \frac{1}{\cosh^2 x}\,\mathrm{d}x = \int (1 - \tanh^2 x)\,\mathrm{d}x = \tanh x + c$$

$$\int \frac{1}{\tanh^2 x}\,\mathrm{d}x = x - \coth x + c$$

$$\int \operatorname{arcsinh} x\,\mathrm{d}x = x \operatorname{arcsinh} x - \sqrt{1 + x^2} + c$$

$$\int \operatorname{arccosh} x\,\mathrm{d}x = x \operatorname{arccosh} x - \sqrt{x^2 - 1} + c$$

$$\int \operatorname{arctanh} x\,\mathrm{d}x = x \operatorname{arctanh} x + \frac{1}{2}\ln(1 - x^2) + c$$

Razionali ed irrazionali ($a > 0$)

$$\int \frac{f'(x)}{a^2 + f(x)^2}\,\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{f(x)}{a}\right) + c_1 = -\frac{1}{a}\operatorname{arccot}\left(\frac{f(x)}{a}\right) + c_2$$

- $$\int \frac{1}{a^2 + x^2}\,\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + c_1 = -\frac{1}{a}\operatorname{arccot}\left(\frac{x}{a}\right) + c_2$$
- $$\int \frac{1}{1 + x^2}\,\mathrm{d}x = \arctan x + c_1 = -\operatorname{arccot} x + c_2$$

$$\int \frac{f'(x)}{a^2 - f(x)^2}\,\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{a + f(x)}{a - f(x)}\right| + c$$

- $$\int \frac{1}{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{a + x}{a - x}\right| + c = \begin{cases} \frac{1}{a}\operatorname{arctanh}\left(\frac{x}{a}\right) + c = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) + c, & \text{se } |x| < a \\ \frac{1}{a}\operatorname{arccoth}\left(\frac{x}{a}\right) + c = \frac{1}{2a}\ln\left(\frac{x+a}{x-a}\right) + c, & \text{se } |x| > a \end{cases}$$
- $$\int \frac{1}{1 - x^2}\,\mathrm{d}x = \frac{1}{2}\ln\left|\frac{1 + x}{1 - x}\right| + c = \begin{cases} \operatorname{arctanh} x + c = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) + c, & \text{se } |x| < 1 \\ \operatorname{arccoth} x + c = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) + c, & \text{se } |x| > 1 \end{cases}$$

$$\int \frac{f'(x)}{\sqrt{a^2 + f(x)^2}}\,\mathrm{d}x = \operatorname{arcsinh}\left(\frac{f(x)}{a}\right) + c$$

- $$\int \frac{1}{\sqrt{a^2 + x^2}}\,\mathrm{d}x = \operatorname{arcsinh}\left(\frac{x}{a}\right) + c_1 = \ln\left(x + \sqrt{a^2 + x^2}\right) + c_2$$
- $$\int \frac{1}{\sqrt{1 + x^2}}\,\mathrm{d}x = \operatorname{arcsinh} x + c = \ln\left(x + \sqrt{1 + x^2}\right) + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - f(x)^2}}\,\mathrm{d}x = \arcsin\left(\frac{f(x)}{a}\right) + c_1 = -\arccos\left(\frac{f(x)}{a}\right) + c_2$$

- $$\int \frac{1}{\sqrt{a^2 - x^2}}\,\mathrm{d}x = \arcsin\left(\frac{x}{a}\right) + c_1 = -\arccos\left(\frac{x}{a}\right) + c_2$$
- $$\int \frac{1}{\sqrt{1 - x^2}}\,\mathrm{d}x = \arcsin x + c_1 = -\arccos x + c_2$$

$$\int \frac{f'(x)}{\sqrt{f(x)^2 - a^2}}\,\mathrm{d}x = \ln\left|f(x) + \sqrt{f(x)^2 - a^2}\right| + c$$

- $$\int \frac{1}{\sqrt{x^2 - a^2}}\,\mathrm{d}x = \ln\left|x + \sqrt{x^2 - a^2}\right| + c = \begin{cases} \operatorname{arccosh}\left(\frac{x}{a}\right) + c_1, & \text{se } x \geq a \\ -\operatorname{arccosh}\left(-\frac{x}{a}\right) + c_1, & \text{se } x \leq -a \end{cases}$$
- $$\int \frac{1}{\sqrt{x^2 - 1}}\,\mathrm{d}x = \ln\left|x + \sqrt{x^2 - 1}\right| + c = \begin{cases} \operatorname{arccosh} x + c_1, & \text{se } x \geq 1 \\ -\operatorname{arccosh}(-x) + c_1, & \text{se } x \leq -1 \end{cases}$$

$$\int f'(x)\sqrt{f(x)^2 + a^2}\,\mathrm{d}x = \frac{a^2}{2}\operatorname{arcsinh}\left(\frac{f(x)}{a}\right) + \frac{f(x)}{2}\sqrt{f(x)^2 + a^2} + c$$

- $$\int \sqrt{x^2 + a^2}\,\mathrm{d}x = \frac{a^2}{2}\operatorname{arcsinh}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{x^2 + a^2} + c_1 = \frac{a^2}{2}\ln\left(x + \sqrt{x^2 + a^2}\right) + \frac{x}{2}\sqrt{x^2 + a^2} + c_2$$

- $\displaystyle\int \sqrt{x^2+1}\,\mathrm{d}x = \frac{1}{2}\left[\operatorname{arcsinh} x + x\sqrt{x^2+1}\right] + c$
$$= \frac{1}{2}\left[\ln\left(x+\sqrt{x^2+1}\right) + x\sqrt{x^2+1}\right] + c$$

$$\int f'(x)\sqrt{f(x)^2-a^2}\,\mathrm{d}x = -\frac{a^2}{2}\ln\left|f(x)+\sqrt{f(x)^2-a^2}\right| + \frac{f(x)}{2}\sqrt{f(x)^2-a^2} + c$$

- $\displaystyle\int \sqrt{x^2-a^2}\,\mathrm{d}x = -\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + \frac{x}{2}\sqrt{x^2-a^2} + c$
- $\displaystyle\int \sqrt{x^2-1}\,\mathrm{d}x = \frac{1}{2}\left[-\ln\left|x+\sqrt{x^2-1}\right| + x\sqrt{x^2-1}\right] + c$

$$\int f'(x)\sqrt{a^2-f(x)^2}\,\mathrm{d}x = \frac{a^2}{2}\arcsin\left(\frac{f(x)}{a}\right) + \frac{f(x)}{2}\sqrt{a^2-f(x)^2} + c$$

- $\displaystyle\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2-x^2} + c$
- $\displaystyle\int \sqrt{1-x^2}\,\mathrm{d}x = \frac{1}{2}\left[\arcsin x + x\sqrt{1-x^2}\right] + c$

$$\int \frac{1}{ax^2+bx+c}\,\mathrm{d}x = \begin{cases} \dfrac{1}{\sqrt{b^2-4ac}}\ln\left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + c, & \text{se } \Delta > 0 \\ -\dfrac{2}{2ax+b} + c, & \text{se } \Delta = 0 \\ \dfrac{2}{\sqrt{4ac-b^2}}\arctan\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right) + c, & \text{se } \Delta < 0 \end{cases}$$

- $\displaystyle\int \frac{1}{(x-a)(x-b)}\,\mathrm{d}x = \frac{1}{a-b}\ln\left|\frac{x-a}{x-b}\right| + c \qquad (a \neq b)$
- $\displaystyle\int \frac{1}{b+ax^2}\,\mathrm{d}x = \frac{1}{\sqrt{ab}}\arctan\left(\sqrt{\frac{a}{b}}x\right) + c$
- $\displaystyle\int \frac{1}{b-ax^2}\,\mathrm{d}x = \frac{1}{2\sqrt{ab}}\ln\left|\frac{\sqrt{ab}+ax}{\sqrt{ab}-ax}\right| + c$

$$\int \frac{mx+n}{ax^2+bx+c}\,\mathrm{d}x = \frac{m}{2a}\ln|ax^2+bx+c| + \left(n-\frac{bm}{2a}\right)\int\frac{1}{ax^2+bx+c}\,\mathrm{d}x$$

$$\int \frac{1}{(ax^2+bx+c)^n}\,\mathrm{d}x = \frac{1}{(n-1)(4ac-b^2)}\left[\frac{2ax+b}{(ax^2+bx+c)^{n-1}} + 2a(2n-3)\int\frac{1}{(ax^2+bx+c)^{n-1}}\,\mathrm{d}x\right]$$

- $\displaystyle\int \frac{1}{(1\pm x^2)^2}\,\mathrm{d}x = \frac{1}{2}\left[\frac{x}{1\pm x^2} + \int\frac{1}{1\pm x^2}\,\mathrm{d}x\right]$

Esponenziali e misti

$$E_n = \int x^n e^{cx}\,\mathrm{d}x = \frac{1}{c}(x^n e^{cx} - nE_{n-1}) + c$$

- $\displaystyle E_1 = \int xe^{cx}\,\mathrm{d}x = e^{cx}\left(\frac{cx-1}{c^2}\right) + c$
- $\displaystyle E_2 = \int x^2e^{cx}\,\mathrm{d}x = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right) + c$
- $\displaystyle E_3 = \int x^3e^{cx}\,\mathrm{d}x = e^{cx}\left(\frac{x^3}{c} - \frac{3x^2}{c^2} + \frac{6x}{c^3} - \frac{6}{c^4}\right) + c$

$$F_n = \int \frac{e^{cx}}{x^n}\,\mathrm{d}x = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + cF_{n-1}\right) + c \qquad (n \neq 1)$$

- $\displaystyle F_2 = \int \frac{e^{cx}}{x^2}\,\mathrm{d}x = -\frac{e^{cx}}{x} + c\,\mathrm{Ei}(cx) + c$

$$\int xe^{cx^2}\,\mathrm{d}x = \frac{1}{2c}e^{cx^2} + c$$

$$\int x^2e^{cx^2}\,\mathrm{d}x = \frac{1}{2c}\left(xe^{cx^2} - \int e^{cx^2}\,\mathrm{d}x\right) + c$$

$$\int e^{cx}\sin^n x\,\mathrm{d}x = \frac{1}{c^2+n^2}\left[e^{cx}\sin^{n-1}x(c\sin x - n\cos x) + n(n-1)\int e^{cx}\sin^{n-2}x\,\mathrm{d}x\right] + c$$

- $\displaystyle\int e^{cx}\sin x\,\mathrm{d}x = \frac{e^{cx}}{c^2+1}(c\sin x - \cos x) + c$

$$\int e^{cx}\cos^n x\,\mathrm{d}x = \frac{1}{c^2+n^2}\left[e^{cx}\cos^{n-1}x(c\cos x + n\sin x) + n(n-1)\int e^{cx}\cos^{n-2}x\,\mathrm{d}x\right] + c$$

- $\displaystyle\int e^{cx}\cos x\,\mathrm{d}x = \frac{e^{cx}}{c^2+1}(c\cos x + \sin x) + c$

$$\int e^{cx}\sin(ax)\,\mathrm{d}x = \frac{e^{cx}}{c^2+a^2}\left[c\sin(ax) - a\cos(ax)\right] + c$$

$$\int e^{cx}\cos(ax)\,\mathrm{d}x = \frac{e^{cx}}{c^2+a^2}\left[c\cos(ax) + a\sin(ax)\right] + c$$

Tutti i limiti precedenti sono generalizzabili se x è sostituita con $f(x)$ e c'è una $f'(x)$ al numeratore. Infatti, se uno applica la sostituzione $f(x) = t$ ($\mathrm{d}t = f(x)\,\mathrm{d}x$), riottiene quanto scritto sopra.

Tips

Razionali fratte:

$$\int \frac{P_{m\geq n}(x)}{P_n(x)}\,\mathrm{d}x = \int Q(x)\,\mathrm{d}x + \int\frac{R(x)}{P_n(x)}\,\mathrm{d}x$$

Metodo delle fratte semplici (o frazioni parziali):

Fattorizzazione	Associamo
$x-a_i$	$\dfrac{A_i}{x-a_i}$
$(x-a_i)^n$	$\dfrac{A_{i,1}}{x-a_i} + \dfrac{A_{i,2}}{(x-a_i)^2} + \ldots + \dfrac{A_{i,n}}{(x-a_i)^n}$
$x^2+a_ix+b_i$ con $\Delta < 0$	$\dfrac{A_ix+B_i}{x^2+a_ix+b_i}$
$(x^2+a_ix+b_i)^n$ con $\Delta < 0$	$\dfrac{A_{i,1}x+B_{i,1}}{x^2+a_ix+b_i} + \ldots + \dfrac{A_{i,n}x+B_{i,n}}{(x^2+a_ix+b_i)^n}$

$$\int\sqrt{\frac{ax+b}{cx+d}}\,\mathrm{d}x \qquad \text{porre: } cx+d = t^2$$

$$\int P_{\mathrm{odd}}(x)\sqrt{P_2(x)}\,\mathrm{d}x \qquad \text{porre: } P_2(x) = t^2$$

Completamento del quadrato ($a > 0$):

$$ax^2+bx+c = \left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 - \frac{b^2}{4a} + c$$

Fratte trigonometriche:
Applicare la sostituzione:

$$t = \tan\left(\frac{x}{2}\right)$$

cosicché:

$$\sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2},\ \tan x = \frac{2t}{1-t^2},\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{1+t^2}$$

Altre sostituzioni:
Quando l'integrando contiene un termine del tipo (i) $\sqrt{a^2-x^2}$, (ii) $\sqrt{a^2+x^2}$ o $1/(a^2+x^2)$, (iii) $\sqrt{x^2-a^2}$, conviene applicare la sostituzione (i) $x = a\sin t$, (ii) $x = a\tan t$, (iii) $x = a/\cos t$, rispettivamente. Prestare attenzione al segno, soprattutto nel caso (iii).

Funzioni iperboliche inverse:

$$\operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad D: x \in \mathbb{R}$$

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2-1}\right) \qquad D: x \in [1, +\infty)$$

$$\operatorname{arctanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad D: x \in (-1, 1)$$

$$\operatorname{arccoth} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) \qquad D: x \in (-\infty, 1) \cup (1, +\infty)$$

Proprietà e formule notevoli

Disuguaglianza triangolare:

$$\int f(x)\,\mathrm{d}x \leq \left|\int f(x)\,\mathrm{d}x\right| \leq \int \left|f(x)\right|\,\mathrm{d}x$$

Teorema fondamentale del calcolo integrale (pt. 1):
Sia $f: [a,b] \to \mathbb{R}$ una funzione limitata ed integrabile. Si definisce funzione integrale di f la funzione F tale che:

$$F(x) = \int_a^x f(x)\,\mathrm{d}x$$

per ogni $x \in [a,b]$. Se F è continua in $[a,b]$ e derivabile nell'intervallo aperto (a,b), allora:

$$F'(x) = f(x)$$

Corollario:

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

Media integrale:

$$m = f(c) = \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x \qquad \text{con } c \in [a,b]$$

Integrazione per parti:
Conviene considerare le funzioni inverse log, arcsin, arctan, ecc. come $f(x)$ ($f(x) = u$), mentre conviene considerare le funzioni dirette exp, sin, tan, ecc. come fattore derivato $g'(x)$ ($g'(x)\,\mathrm{d}x = \mathrm{d}v$). A maggior ragione quando l'altra funzione è un polinomio.

$$\int_a^b f(x)g'(x)\,\mathrm{d}x = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)\,\mathrm{d}x$$

$$\int_a^b u\,\mathrm{d}v = uv\Big|_a^b - \int_a^b v\,\mathrm{d}u$$

Area sottesa ad una curva (con segno):

$$A = \int_a^b f(x)\,\mathrm{d}x$$

- $A_{\text{ellisse}} = 4\frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,\mathrm{d}x = \pi ab$

Volume di un solido data la sezione in funzione di x:

$$V = \int_a^b S(x)^2\,\mathrm{d}x$$

Volume di rotazione attorno all'asse x:

$$V_{\text{rot}} = \pi\int_a^b f(x)^2\,\mathrm{d}x$$

- $V_{\text{ellisse}} = 2\pi\frac{b^2}{a^2}\int_0^a (a^2-x^2)\,\mathrm{d}x = \frac{4}{3}\pi ab^2$

Lunghezza di una curva:

$$L = \pi\int_a^b \sqrt{1 + \left[f'(x)\right]^2}\,\mathrm{d}x$$

Superficie laterale di rotazione attorno all'asse x:

$$S_{\text{lat}} = 2\pi\int_a^b f(x)\sqrt{1+\left[f'(x)\right]^2}\,\mathrm{d}x$$

Area racchiusa da una curva polare:

$$A_{\text{pol}} = \frac{1}{2}\int_a^b r(\vartheta)^2\,\mathrm{d}\vartheta$$

Lunghezza di una curva polare:

$$L_{\text{pol}} = \int_a^b \sqrt{r(\vartheta)^2 + \left[r'(\vartheta)\right]^2}\,\mathrm{d}\vartheta$$

Integrali ellittici

Integrali ellittici completi di 1°, 2° e 3° tipo ($0 \leq k \leq 1$):

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\vartheta}{\sqrt{1-k^2\sin^2\vartheta}} = \int_0^1 \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\vartheta}\,\mathrm{d}\vartheta = \int_0^1 \sqrt{\frac{1-k^2t^2}{1-t^2}}\,\mathrm{d}t$$

$$\Pi(n;k) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\vartheta}{(1-n\sin^2\vartheta)\sqrt{1-k^2\sin^2\vartheta}}$$

Funzioni speciali

Funzione degli errori:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t$$

- $\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x = \sqrt{\pi}$
- $\int_0^{\infty} e^{-x^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2}$
- $\int_{-\infty}^{\infty} e^{-ax^2+bx+c}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2}{4a}+c\right)$
- $\mathcal{N}(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

- $\Pr(\mu-\sigma \leq X \leq \mu+\sigma) = \int_{\mu-\sigma}^{\mu+\sigma} \mathcal{N}(x;\mu,\sigma)\,\mathrm{d}x \approx 68.27\,\%$
- $\Pr(\mu-2\sigma \leq X \leq \mu+2\sigma) = \int_{\mu-2\sigma}^{\mu+2\sigma} \mathcal{N}(x;\mu,\sigma)\,\mathrm{d}x \approx 95.45\,\%$
- $\Pr(\mu-3\sigma \leq X \leq \mu+3\sigma) = \int_{\mu-3\sigma}^{\mu+3\sigma} \mathcal{N}(x;\mu,\sigma)\,\mathrm{d}x \approx 99.73\,\%$

Funzione degli errori complementare:

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}\,\mathrm{d}t$$

Integrali di Fresnel:

$$\mathrm{S}(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) \mathrm{d}t$$

$$\mathrm{C}(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) \mathrm{d}t$$

- $\displaystyle\int_0^\infty \sin\left(x^2\right) \mathrm{d}x = \int_0^\infty \cos\left(x^2\right) \mathrm{d}x = \sqrt{\frac{\pi}{8}}$
- $\displaystyle\int_{-\infty}^\infty \sin\left(x^2\right) \mathrm{d}x = \int_{-\infty}^\infty \cos\left(x^2\right) \mathrm{d}x = \sqrt{\frac{\pi}{2}}$

Seno integrale:

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t} \mathrm{d}t$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t}{t} \mathrm{d}t$$

$$\int_0^\infty \frac{\sin x}{x} \mathrm{d}x = \frac{\pi}{2}$$

Coseno integrale:

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t} \mathrm{d}t$$

$$\mathrm{cin}(x) = \int_0^x \frac{1-\cos t}{t} \mathrm{d}t$$

Esponenziale integrale:

$$\mathrm{Ei}(x) = -\int_{-x}^\infty \frac{e^{-t}}{t} \mathrm{d}t$$

$$\mathrm{Ein}(x) = \int_0^x \frac{\left(1-e^{-t}\right)}{t} \mathrm{d}t$$

Logaritmo integrale:

$$\mathrm{li}(x) = \int_0^x \frac{\mathrm{d}t}{\ln t} \qquad (x \neq 1)$$

$$\mathrm{Li}(x) = \int_2^x \frac{\mathrm{d}t}{\ln t}$$

Altro:

$$\int_0^{\frac{\pi}{2}} \sin\left(\sin x\right) \mathrm{d}t = \int_0^{\frac{\pi}{2}} \sin\left(\cos x\right) \mathrm{d}t = \frac{\pi}{2}\mathbf{H}_0(1)$$

$$\int_0^{\frac{\pi}{2}} \cos\left(\sin x\right) \mathrm{d}t = \int_0^{\frac{\pi}{2}} \cos\left(\cos x\right) \mathrm{d}t = \frac{\pi}{2}J_0(1)$$

$$\int_0^{\frac{\pi}{2}} \cos\left(\tan x\right) \mathrm{d}t = \int_0^{\frac{\pi}{2}} \cos\left(\cot x\right) \mathrm{d}t = \frac{\pi}{2e}$$

$$\int \sin\left(\frac{1}{x}\right) \mathrm{d}x = x\sin\left(\frac{1}{x}\right) - \mathrm{Ci}\left(\frac{1}{x}\right) + c$$

$$\int \cos\left(\frac{1}{x}\right) \mathrm{d}x = x\cos\left(\frac{1}{x}\right) + \mathrm{Si}\left(\frac{1}{x}\right) + c$$

$$\int \frac{\tan x}{x} \mathrm{d}x$$

https://en.wikipedia.org/wiki/Lists_of_integrals#Lists_of_integrals

https://en.wikipedia.org/wiki/Inverse_trigonometric_functions