OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics
J560/04 Paper 4 (Higher Tier)
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

You may use:
- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper

First name	
Last name	
Centre number	Candidate number

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions

1 18 rice cakes weigh a total of 130 g.
There are 329 calories in 100 g of rice cakes.

How many calories are there in one rice cake?

..... calories **[3]**

2 A circular table top has radius 70 cm.

(a) Calculate the area of the table top in cm^2, giving your answer as a multiple of π.

(a) ..... cm^2 **[2]**

(b) The volume of the table top is $17\,150\pi\,cm^3$.

Calculate the thickness of the table top.

(b) ..... cm **[2]**

3 The value of a car £V is given by

$$V = 20\,000 \times 0.9^t$$

where t is the age of the car in complete years.

(a) Write down the value of V when $t = 0$.

(a) £ ..... **[1]**

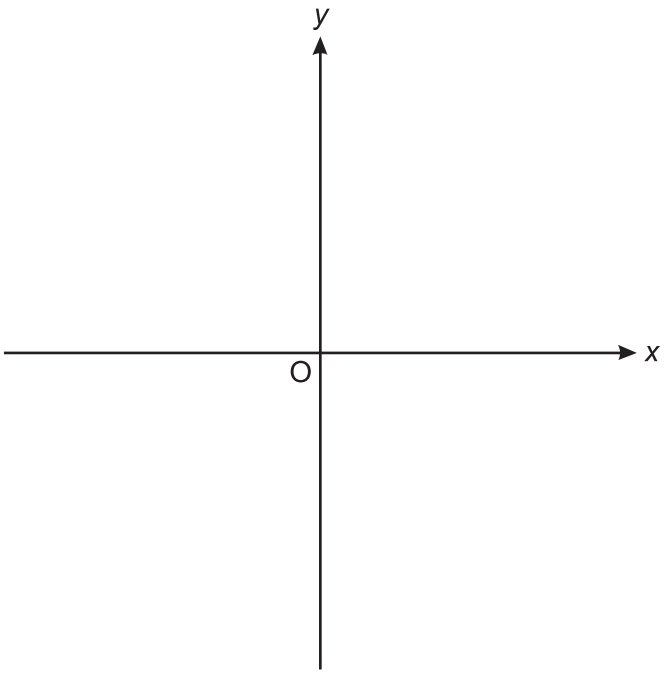
(b) What is the value of V when $t = 3$?

(b) £ ..... **[2]**

(c) After how many complete years will the car's value drop below £10 000?

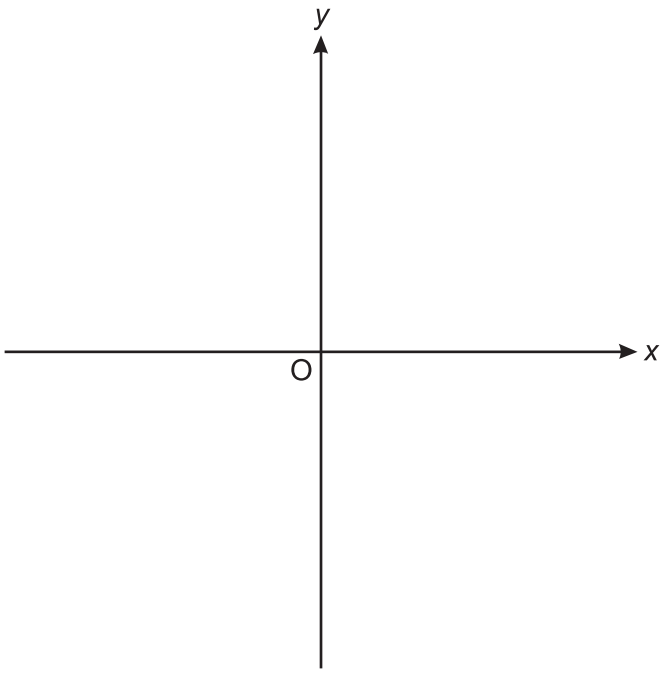
(c) ..... **[2]**

4 **(a)** **(i)** Sketch a graph on the axes below that shows that y is directly proportional to x.



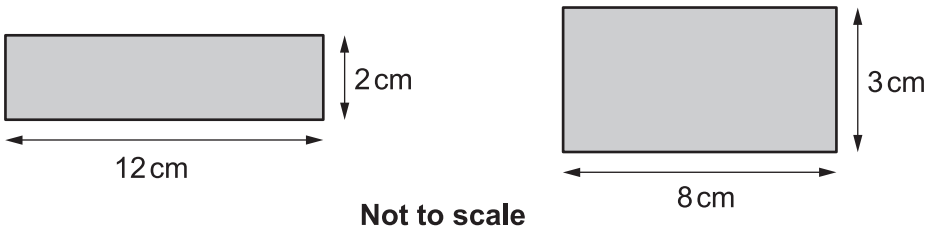
[2]

(ii) Sketch a graph on the axes below that shows $y = x^3$.



[2]

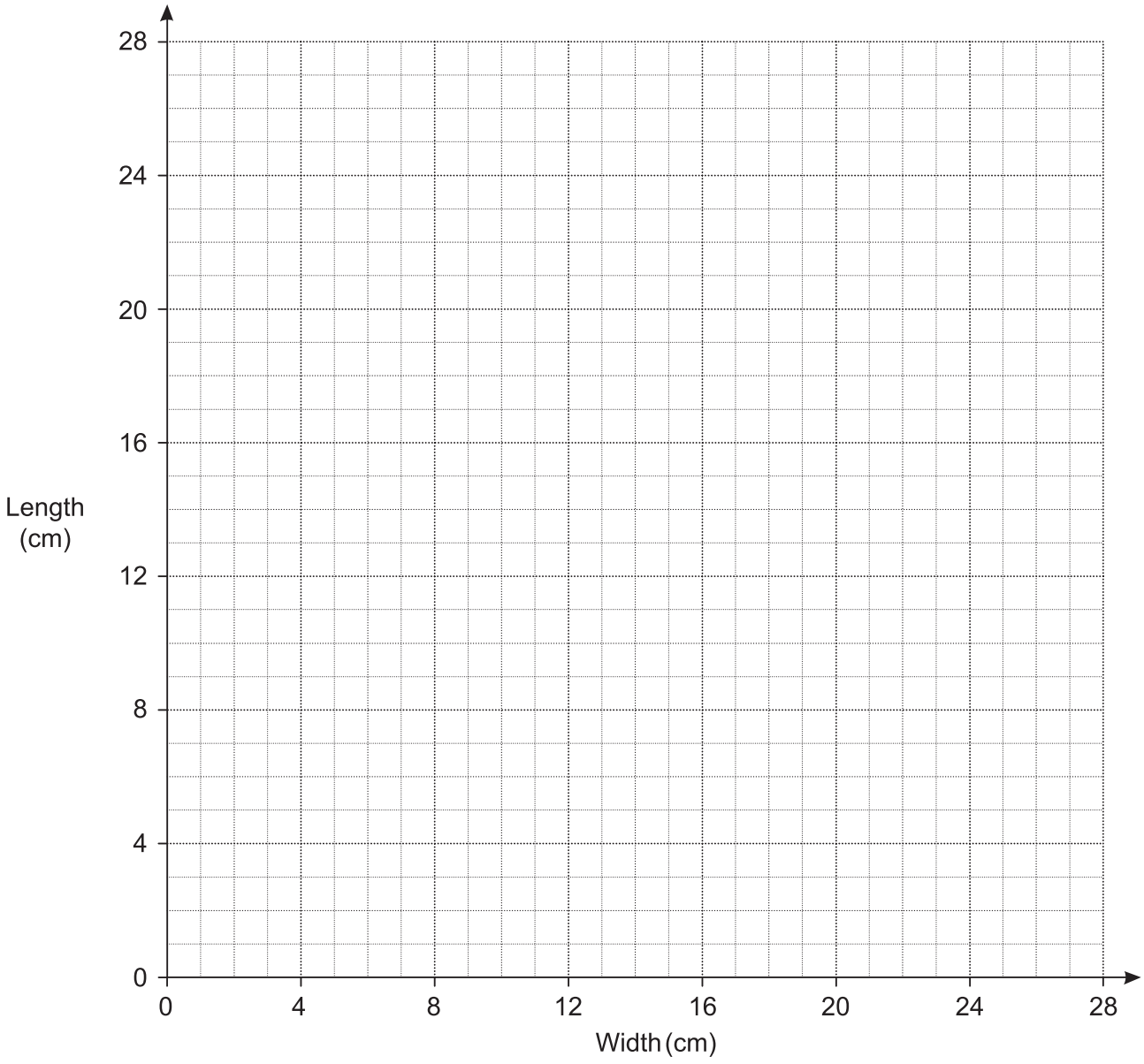
(b) It is possible to draw many rectangles that have area 24 cm^2.
Here are two of them.



Not to scale

(i) Plot the dimensions of these two rectangles on the grid below. **[1]**

(ii) Complete the graph to show the relationship between length and width for rectangles with area 24 cm^2. **[3]**



5 Kieran, Jermaine and Chris play football.

- Kieran has scored 8 more goals than Chris.
- Jermaine has scored 5 more goals than Kieran.
- Altogether they have scored 72 goals.

How many goals did they each score?

Kieran .....

Jermaine .....

Chris .....

[5]

6 Peter makes a large amount of pink paint by mixing red and white paint in the ratio 2 : 3.

Red paint costs £80 per 10 litres.
White paint costs £5 per 10 litres.

Peter sells his pink paint in 10-litre tins for £60 per tin.

Calculate how much **profit** he makes for each tin he sells.

£ ..... **[5]**

7 Dan believes he knows what his brother Ethan is thinking.
He carries out an experiment to test this.

Dan and Ethan sit back-to-back.
Ethan rolls an ordinary fair dice.
Ethan then thinks about the number on the dice while Dan tries to predict this number.

(a) In 300 attempts, how many correct predictions would you expect Dan to make if he was just guessing?

(a) ..... **[2]**

(b) The results of the first 15 attempts are shown in the table.

Ethan's number	2	6	5	3	2	1	5	1	3	4	4	6	1	6	5
Dan's prediction	2	4	3	1	2	6	1	6	4	3	2	6	5	2	3
Matching pair	✓				✓							✓			

Estimate the probability of getting a matching pair using the results of

(i) the first five attempts,

(b)(i) ..... **[1]**

(ii) all 15 attempts.

(ii) ..... **[1]**

(c) Use answers from **(a)** and **(b)** to comment on Dan's belief that he knows what Ethan is thinking.

.....

.....

..... **[2]**

8 **(a)** A function is represented by the following function machine.

(i) A number is input into the machine.
The output is used as a new input.
The second output is 11.

Work out the number that was the **first input.**

(a)(i) ..... **[2]**

(ii) A number is input into the machine.
The output given is the same number.

Work out the number.

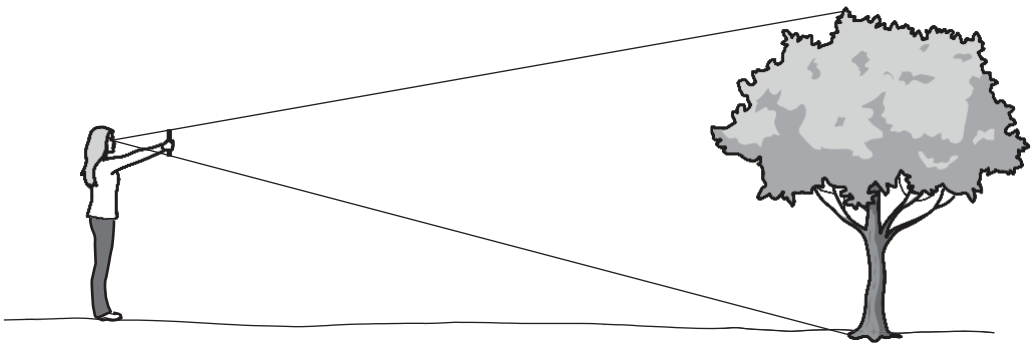
(ii) ..... **[3]**

(b) Another function machine is shown below.

If the Input is 3, the Output is 5.
If the Input is 7, the Output is 25.

Use this information to fill in the two boxes. **[3]**

9 **(a)** Anna estimates the height of a tree.



Anna holds a ruler vertically so the height of the tree is exactly covered by the ruler.
She is 20 metres from the tree.
The ruler is 30 cm long.
The horizontal distance from her eyes to the ruler is 60 cm.

Calculate an estimate of the height of the tree.

(a) ..... m **[3]**

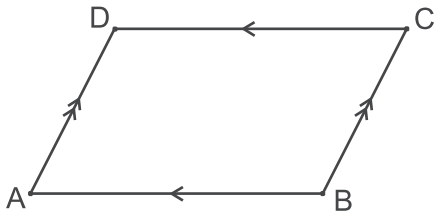
(b) Give two reasons why this method may not be suitable to estimate the height of a very tall building.

1 .....

2 .....

[2]

10 ABCD is a parallelogram.



Prove that triangle ABD is congruent to triangle CDB. **[3]**

11 **(a)** Give **one** reason why 0 is an even number.

.....

..... **[1]**

(b) The lengths of the sides of a **right-angled** triangle are all integers.

Prove that if the lengths of the two shortest sides are even, then the length of the third side must also be even.

.....

.....

.....

.....

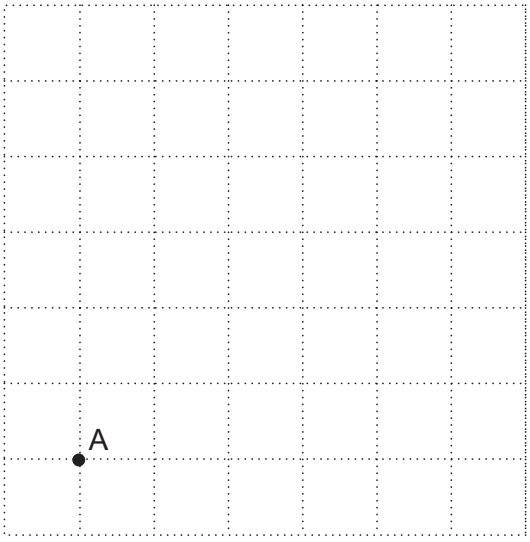
.....

..... **[3]**

12 **(a)** Without using a calculator, show that $\sqrt{20} = 2\sqrt{5}$. **[2]**

(b) The point A is shown on the unit grid below.
The point B is $2\sqrt{5}$ units from A and lies on the intersection of two grid lines.

Mark **one** possible position for B.



[3]

13 The volume of Earth is 1.08×10^{12} km^3.
The volume of Jupiter is 1.43×10^{15} km^3.

How many times larger is the radius of Jupiter than the radius of Earth?
Assume that Jupiter and Earth are both spheres.

[The volume v of a sphere with radius r is $v = \frac{4}{3}\pi r^3$.]

..... **[4]**

14 The table shows the marks gained by 150 students taking an examination.

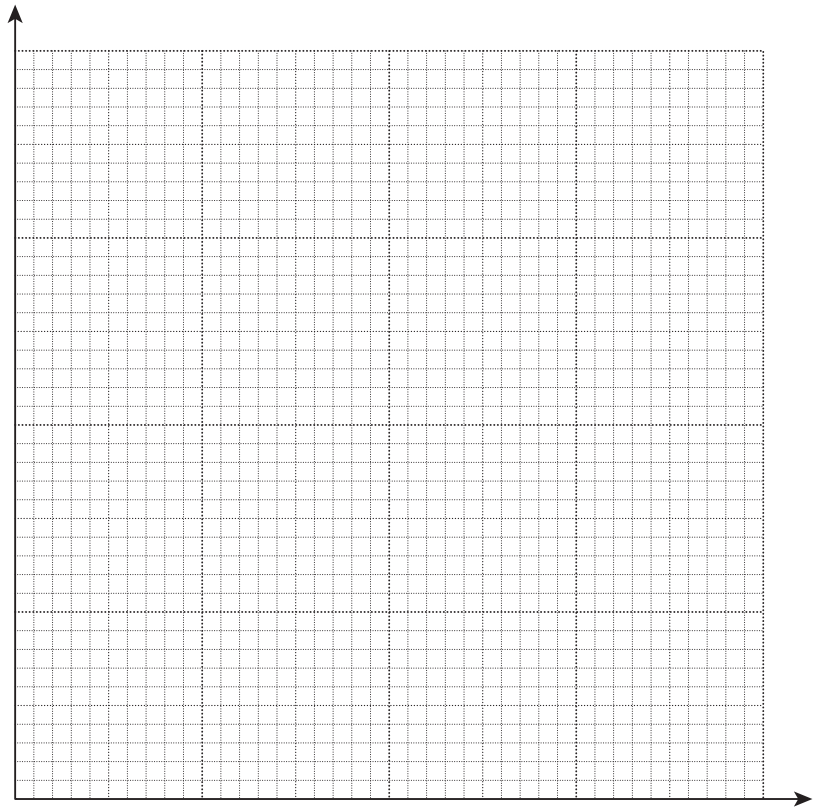
Mark (m)	$0<m\leqslant 10$	$10<m\leqslant 20$	$20<m\leqslant 30$	$30<m\leqslant 40$	$40<m\leqslant 50$	$50<m\leqslant 60$	$60<m\leqslant 70$	$70<m\leqslant 80$
Frequency	9	14	26	27	25	22	17	10

(a) (i) Construct a cumulative frequency table.

Mark (m)	$m \leqslant 10$	$m \leqslant 20$	$m \leqslant 30$	$m \leqslant 40$	$m \leqslant 50$	$m \leqslant 60$	$m \leqslant 70$	$m \leqslant 80$
Cumulative Frequency	9							150

[2]

(ii) Draw the cumulative frequency graph on the grid below.



[4]

(b) Students are to be awarded Gold, Silver, Bronze or Fail.
The students' teacher wishes to award the top 10% of students Gold, the next 60% Silver and the next 20% Bronze.

Use your graph to estimate the lowest mark that Silver will be awarded for.

(b) ..... **[3]**

(c) Explain why the teacher's method will not necessarily award Gold to exactly 10% of the students.

.....

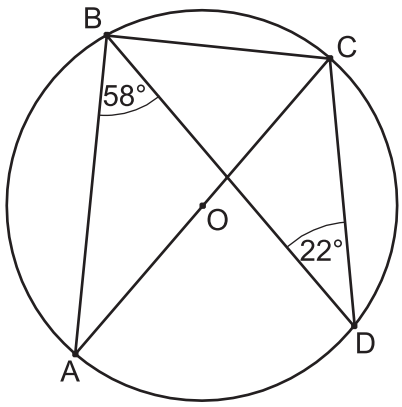
..... **[1]**

15 At a constant temperature, the volume of a gas V is inversely proportional to its pressure p.

By what percentage will the pressure of a gas change if its volume increases by 25%?

..... % **[4]**

16 A, B, C and D are points on the circumference of a circle, centre O.
AC is a diameter of the circle.
Angle ABD = 58°.
Angle CDB = 22°.



Not to scale

Work out the sizes of angle ACD and ACB, giving reasons for your answers.

(a) Angle ACD = ...°

.....

..... **[2]**

(b) Angle ACB = ...°

.....

.....

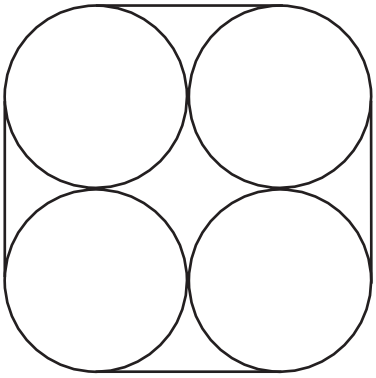
..... **[3]**

17 A restaurant menu has 8 starters, 12 mains and 6 desserts.
A customer can choose from the following meals

- a starter and a main,
- a main and a dessert,
- a starter, a main and a dessert.

Show that there are 744 different ways of choosing a meal at this restaurant. **[3]**

18 Four pencils are held together with a band.
The figure below shows the bottom end of the pencils and the band.



Each of the pencils has diameter 9 mm.

Find the length of the band in this position.

..... mm **[4]**

19 A sequence is defined by the term-to-term rule $u_{n+1} = u_n^2 - 8u_n + 17$.

(a) Given that $u_1 = 4$, find u_2 and u_3.

(a) .... **[2]**

(b) Given instead that $u_1 = 2$, find u_2, u_3 and u_{100}.

(b) .... **[3]**

20 **(a)** Express as a single fraction.

$$\frac{m+1}{n+1} - \frac{m}{n}$$

Simplify your answer.

(a) ..... **[2]**

(b) Using your answer to part **(a)**, prove that if m and n are positive integers and $m < n$, then

$$\frac{m+1}{n+1} - \frac{m}{n} > 0.$$ **[2]**

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Date – Morning/Afternoon

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

SAMPLE MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 100

DRAFT

Subject-Specific Marking Instructions

1. **M** marks are for using a correct method and are not lost for purely numerical errors.
   A marks are for an accurate answer and depend on preceding **M** (method) marks. Therefore **M0 A1** cannot be awarded.
   B marks are independent of **M** (method) marks and are for a correct final answer, a partially correct answer, or a correct intermediate stage.
   SC marks are for special cases that are worthy of some credit.

2. Unless the answer and marks columns of the mark scheme specify **M** and **A** marks etc, or the mark scheme is 'banded', then if the correct answer is clearly given and is not from wrong working **full marks** should be awarded.

   Do not award the marks if the answer was obtained from an incorrect method, ie incorrect working is seen and the correct answer clearly follows from it.

3. Where follow through (**FT**) is indicated in the mark scheme, marks can be awarded where the candidate's work follows correctly from a previous answer whether or not it was correct.

   Figures or expressions that are being followed through are sometimes encompassed by single quotation marks after the word *their* for clarity, e.g. FT 180 × (*their* '37' + 16), or FT 300 – √(*their* '$5^2 + 7^2$'). Answers to part questions which are being followed through are indicated by e.g. FT 3 × *their* (a).

   For questions with FT available you must ensure that you refer back to the relevant previous answer. You may find it easier to mark these questions candidate by candidate rather than question by question.

4. Where dependent (**dep**) marks are indicated in the mark scheme, you must check that the candidate has met all the criteria specified for the mark to be awarded.

5. The following abbreviations are commonly found in GCSE Mathematics mark schemes.

   - **figs 237**, for example, means any answer with only these digits. You should ignore leading or trailing zeros and any decimal point e.g. 237000, 2.37, 2.370, 0.00237 would be acceptable but 23070 or 2374 would not.
   - **isw** means **ignore subsequent working** after correct answer obtained and applies as a default.
   - **nfww** means **not from wrong working**.
   - **oe** means **or equivalent**.
   - **rot** means **rounded or truncated**.
   - **seen** means that you should award the mark if that number/expression is seen anywhere in the answer space, including the answer line, even if it is not in the method leading to the final answer.

- **soi** means **seen or implied**.

6. In questions with no final answer line, make no deductions for wrong work after an acceptable answer (ie **isw**) unless the mark scheme says otherwise, indicated by the instruction 'mark final answer'.

7. In questions with a final answer line following working space:

(i) If the correct answer is seen in the body of working and the answer given on the answer line is a clear transcription error allow full marks unless the mark scheme says 'mark final answer'. Place the annotation ✓ next to the correct answer.

(ii) If the correct answer is seen in the body of working but the answer line is blank, allow full marks. Place the annotation ✓ next to the correct answer.

(iii) If the correct answer is seen in the body of working but a completely different answer is seen on the answer line, then accuracy marks for the answer are lost. Method marks could still be awarded. Use the M0, M1, M2 annotations as appropriate and place the annotation ✗ next to the wrong answer.

8. In questions with a final answer line:

(i) If one answer is provided on the answer line, mark the method that leads to that answer.

(ii) If more than one answer is provided on the answer line and there is a single method provided, award method marks only.

(iii) If more than one answer is provided on the answer line and there is more than one method provided, award zero marks for the question unless the candidate has clearly indicated which method is to be marked.

9. In questions with no final answer line:

(i) If a single response is provided, mark as usual.

(ii) If more than one response is provided, award zero marks for the question unless the candidate has clearly indicated which response is to be marked.

10. When the data of a question is consistently misread in such a way as not to alter the nature or difficulty of the question, please follow the candidate's work and allow follow through for **A** and **B** marks. Deduct 1 mark from any **A** or **B** marks earned and record this by using the MR annotation. **M** marks are not deducted for misreads.

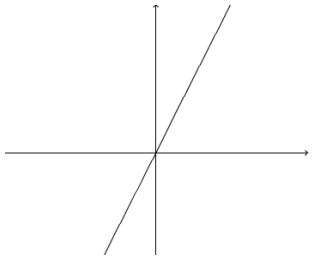
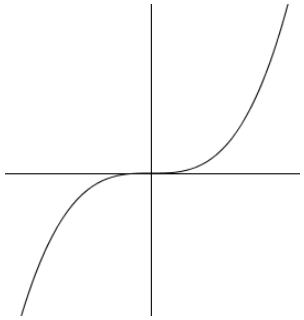
11. Unless the question asks for an answer to a specific degree of accuracy, always mark at the greatest number of significant figures even if this is rounded or truncated on the answer line. For example, an answer in the mark scheme is 15.75, which is seen in the working. The candidate then rounds or truncates this to 15.8, 15 or 16 on the answer line. Allow full marks for the 15.75.

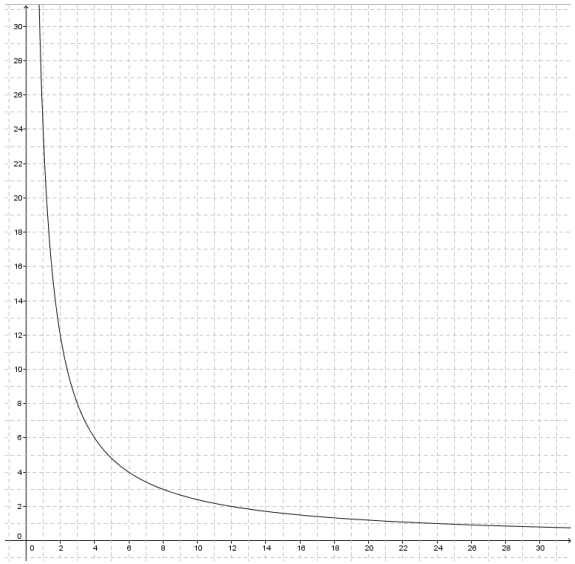
12. Ranges of answers given in the mark scheme are always inclusive.

13. For methods not provided for in the mark scheme give as far as possible equivalent marks for equivalent work. If in doubt, consult your Team Leader.

14. Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned, but if present it must be correct.

Question			Answer	Marks	Part marks and guidance	
1			23.6 – 23.8 Accept 24 provided full method shown	3 1 AO1.3b 2 AO3.1c	**M2** for $\frac{329\times130}{18\times100}$ Or **M1** for any two of $\frac{329}{100}$ or $\frac{130}{100}$ or $\frac{329}{18}$ or 329×130	May be done in stages
2	(a)		4900π	2 1 AO1.2 1 AO1.3a	**M1** for $\pi \times 70^2$ may be implied by 15393.8…	
	(b)		3.5	2 2 AO1.3a	**M1** for $\frac{17150\pi}{\textit{their}\ '4900\pi'}$	**FT** from **(a)**, provided **(a)** is a multiple of π
3	(a)		£20 000	1 1 AO1.3a		
	(b)		£14 580 or £14 600	2 2 AO1.3a	**M1** for $20\,000 \times 0.9^3$	
	(c)		7 years	2 1 AO1.3a 1 AO3.1c	**M1** for 2 trials shown	

Question			Answer	Marks	Part marks and guidance	
4	(a)	(i)	Any straight line through the origin e.g. 	**2** **1 AO1.1** **1 AO2.3b**	**B1** for a straight line	
		(ii)		**2** **1 AO1.1** **1 AO2.3b**	**B1** for a cubic with two turning points	
	(b)	(i)	At least one point plotted correctly	**1** **1 AO2.3b**		

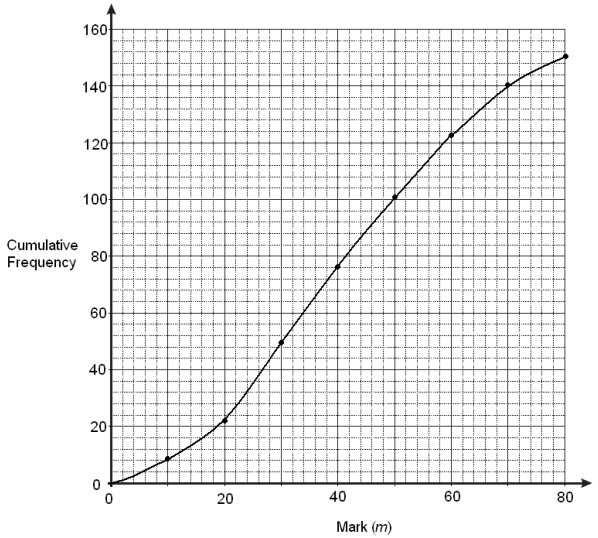
Question			Answer	Marks	Part marks and guidance	
		(ii)		**3** **1 AO2.3b** **1 AO3.1b** **1 AO3.2**	**B2** for at least 5 points correctly plotted OR **B1** for at least 3 points correctly plotted AND **B1** for <u>curve</u> drawn through *their* points	
5			25, 30, 17	**5** **2 AO1.3a** **2 AO3.1d** **1 AO3.3**	**M1** for any two consistent expressions, e.g. $x - 8$, x **M1** for $x - 8 + x + x + 5 = 72$ **oe** **A1** for $x = 25$ **B1** for Kieran 25 or Jermaine 30 or Chris 17	Accept equivalent correct equations

Question			Answer	Marks	Part marks and guidance	
6			£25	**5** **2 AO1.3b** **3 AO3.1d**	**M1** for $10 \times \frac{2}{5}$ = 4 litres red or $10 \times \frac{3}{5}$ = 6 litres white **M1** for red costs £8 per litre or white costs £0.50 per litre **M1** for cost of one 10-litre can is *their* '4' × *their* '8' + *their* '6' × *their* '0.5' **M1** for 60 – *their* '35'	Alternative method: M1 for 2 : 3 = 20 litres red : 30 litres white M1 for 2 × £80 + 3 × £5 = £175 M1 for $\frac{\textit{their}\ '175'}{5}$ = 35 M1 for 60 – *their* '35'
7	(a)		50	**2** **2 AO1.3a**	**B1** for $\frac{1}{6}$	
	(b)	(i)	$\frac{2}{5}$ **oe**	**1** **1 AO2.1b**		
		(ii)	$\frac{1}{5}$ **oe**	**1** **1 AO2.1b**		
	(c)		No evidence that Dan knows what Ethan is thinking as over the 15 trials the relative frequency of $\frac{1}{5}$ is very close to the theoretical probability of $\frac{1}{6}$	**2** **1 AO2.5a** **1 AO3.3**	**M1** for reason not including reference to $\frac{1}{5}$ relative frequency or $\frac{1}{6}$ theoretical probability **FT** *their* **(a)** and **(b)**	
8	(a)	(i)	-1	**2** **1 AO1.3a** **1 AO3.1a**	**M1** for use of – 5 and ÷ 2 **soi** Or **M1** for answer 3	

Question			Answer	Marks	Part marks and guidance	
		(ii)	-5	**3** **1 AO1.3a** **2 AO3.1a**	**M1** for $2x + 5$ **M1** for $x =$ *their* '$2x + 5$' and solve	
	(b)		5, 10	**3** **1 AO1.3a** **2 AO3.1a**	**M1** for $3a + b = 5$ and $7a + b = 25$ **M1** for attempt to solve Or **M1** input increases by 4; output increases by 20 **M1** so one box must have $\times 5$ for the arithmetic sequence	Condone $\frac{x^2+1}{2}$ across the two boxes for 3 marks
9	**(a)**		10 metres	**3** **1 AO1.3a** **2 AO3.1c**	**M1** for correct ratio $\frac{\text{height}}{20} = \frac{30}{60}$ **oe** **M1** rearrange Or **M1** for scale factor 0.5 **M1** for 20×0.5	
	(b)		2 valid reasons, e.g. She would have to be very far from the building. The estimate is likely to be inaccurate due to the scale factors at the distances involved.	**2** **2 AO3.4a**		

Question			Answer	Marks	Part marks and guidance	
10			e.g. BD is common ABD = BDC (alternate angles) AB = CD (parallelogram) So triangles ABD and CBD are congruent by SAS	**3** **1 AO1.1** **2 AO2.4b**	**B2** for two facts with conclusion or **B2** for three facts with conclusion missing or unclear or **B1** for one correct fact	Each fact must be backed up with a reason
11	**(a)**		Any correct reason	**1** **1 AO2.4a**		Exemplar responses: -1 and 1 both odd and either side of 0 Or can be divided by 2 exactly Or numbers that end in 0 are even Or zero remainder when divided by 2 Or next number in pattern of even numbers 8 6 4 2 Or added to an even number it gives even answer and added to odd number gives odd answer
	(b)		e.g. $a^2 + b^2 = c^2$ $a = 2x$ and $b = 2y$ implies $c^2 = 4x^2 + 4y^2$ So c is even	**3** **1 AO2.1a** **1 AO2.4b** **1 AO3.2**	**B1** for use of Pythagoras' theorem **M1** for even × even = even **soi**	

Question			Answer	Marks	Part marks and guidance	
12	(a)		$\sqrt{20} = \sqrt{4 \times 5}$ $= \sqrt{4} \times \sqrt{5}$ $= 2\sqrt{5}$	**2** **2 AO1.3a**	**M1** for $\sqrt{4} \times \sqrt{5}$	
	(b)		Either point which is 4 across and 2 up from A or 2 across and 4 up	**3** **1 AO2.3b** **1 AO3.1a** **1 AO3.2**	**B1** for $a^2 + b^2 = 20$ **B1** for 4 and 16 (or 2 and 4) seen If zero scored **SC1** for correctly marking the position of *their a* and *b*	Condone both correct points marked
13			11 or better	**4** **2 AO1.3b** **1 AO3.1b** **1 AO3.2**	**M1** for $r = \sqrt[3]{\frac{3v}{4\pi}}$ **soi** **A1** for r (Earth) = 6365 km or r (Jupiter) = 69890 km **M1** for $\frac{\textit{their}\,'69890'}{\textit{their}\,'6365'}$	Alternate method: M1 for $\frac{1.43 \times 10^{15}}{1.08 \times 10^{12}}$ A1 for 1324[.074…] M1 for $\sqrt[3]{1324}$

Question			Answer	Marks	Part marks and guidance	
14	(a)	(i)	Table: 9 23 49 76 101 123 140 150	2 2 AO1.3a	**M1** for attempt to accumulate the values	
		(ii)		4 1 AO1.3b 3 AO2.3b	**B1** for labelling axes **B1** for correct curve through points **B1** for at least six points correctly plotted	
	(b)		28 – 32	3 2 AO2.1b 1 AO2.3a	**M1** for 45 or 105 seen **A1** for corresponding answer **FT** *their* graph	
	(c)		The boundaries are set from approximations based on grouped data, not the actual scores obtained by the students	1 1 AO2.5b		
15			20 [decrease](%)	4 1 AO1.1 1 AO1.3b 2 AO3.1d	**M1** for pV = constant **oe** **M1** for $p_{initial}V_{initial} = p_{after}V_{after}$ **oe** **M1** for $1 \times 1 = p_{after} \times 1.25$ **oe**	

Question			Answer	Marks	Part marks and guidance	
16	(a)		58° Subtended on same arc **oe**	**2** **1 AO2.1a** **1 AO2.4b**	**B1** for angle	
	(b)		68° e.g. angle DBC is 32° because the angle in a semicircle is a right angle **oe** so angle ACB is 68° because angles in a triangle sum to 180° **oe**	**3** **2 AO2.1a** **1 AO2.4b**	**B1** for using the angle in a semicircle is a right angle **B1** for using angles in a triangle sum to 180°	
17			Starter and main 8 × 12 Main and dessert 12 × 6 Three courses 8 × 12 × 6 96 + 72 + 576 = 744	**3** **1 AO1.3b** **1 AO2.1a** **1 AO2.2**	**M1** for one correct product **M1** for summing *their* three products	
18			64.3 or $9\pi + 36$ **oe**	**4** **2 AO1.3b** **2 AO3.1d**	**M1** for $\frac{9\pi}{4}$ **soi** **A1** for 9π or 28.2[7…] **M1** for *their* '9π' + 36	
19	(a)		1 **nfww** 10 **nfww**	**2** **1 AO1.2** **1 AO1.3a**	**B1** for each	**FT** *their* 'u_2' for u_3
	(b)		5 **nfww** 2 **nfww** 5 **nfww**	**3** **1 AO1.2** **1 AO1.3a** **1 AO2.1a**	**B1** for each	**FT** *their* 'u_2' for u_3

Question			Answer	Marks	Part marks and guidance	
20	(a)		$\frac{n-m}{n(n+1)}$	**2** **2 AO1.3b**	**M1** for $\frac{n(m+1)-m(n+1)}{n(n+1)}$	
	(b)		$m < n \Rightarrow n - m > 0$ $\Rightarrow \frac{n-m}{n(n+1)} > 0$ $\Rightarrow \frac{m+1}{n+1} - \frac{m}{n} > 0$	**2** **2 AO2.4b**	**M1** for *their* '$\frac{n-m}{n(n+1)}$' > 0	

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3	Total
1	1		2	3
2(a)	2			2
2(b)	2			2
3(a)	1			1
3(b)	2			2
3(c)	1		1	2
4(a)(i)	1	1		2
4(a)(ii)	1	1		2
4(b)(i)		1		1
4(b)(ii)		1	2	3
5	2		3	5
6	2		3	5
7(a)	2			2
7(b)(i)		1		1
7(b)(ii)		1		1
7(c)		1	1	2
8(a)(i)	1		1	2
8(a)(ii)	1		2	3
8(b)	1		2	3
9(a)	1		2	3
9(b)			2	2
10	1	2		3
11(a)		1		1
11(b)		2	1	3
12(a)	2			2
12(b)		1	2	3
13	2		2	4
14(a)(i)	2			2
14(a)(ii)	1	3		4
14(b)		3		3
14(c)		1		1
15	2		2	4
16(a)		2		2
16(b)		3		3
17	1	2		3
18	2		2	4
19(a)	2			2
19(b)	2	1		3
20(a)	2			2
20(b)		2		2
Totals	**40**	**30**	**30**	**100**