Daftar integral dari fungsi eksponensial

Dari Wikipedia bahasa Indonesia, ensiklopedia bebas

Daftar integral (antiderivatif) dari fungsi eksponensial. Untuk daftar lengkap fungsi integral, lihat Tabel integral.

Dalam semua rumus, konstanta *a* diasumsikan bukan nol.

Daftar isi

Integral tak tentu

Integral tak tentu adalah fungsi-fungsi antiderivatif. Sebuah konstanta (yaitu konstanta integrasi) dapat ditambahkan pada sisi kanan dari rumus ini, tetapi tidak dituliskan di sini demi kesederhanaan.

Integral melibatkan hanya fungsi eksponensial

$$\int \mathrm{f}'(x)e^{f(x)}\;\mathrm{d}x = e^{f(x)}$$

$$\int e^{cx}\;\mathrm{d}x = \frac{1}{c}e^{cx}$$

$$\int a^{cx}\;\mathrm{d}x = \frac{1}{c\cdot \ln a}a^{cx} \text{ for } a > 0,\ a \ne 1$$

Integral melibatkan fungsi eksponensial dan pangkat

$$\int xe^{cx}\;\mathrm{d}x = \frac{e^{cx}}{c^2}(cx-1)$$

\int xe^{-cx}\; \mathrm{d}x =x \frac{1}{-c}e^{-cx}

$$\int x^2 e^{cx}\;\mathrm{d}x = e^{cx}\left(\frac{x^2}{c}-\frac{2x}{c^2}+\frac{2}{c^3}\right)$$

$$\int x^n e^{cx}\;\mathrm{d}x = \frac{1}{c}x^n e^{cx} - \frac{n}{c}\int x^{n-1}e^{cx}\mathrm{d}x = \left(\frac{\partial}{\partial c}\right)^n \frac{e^{cx}}{c} = e^{cx}\sum_{i=0}^n (-1)^i \frac{n!}{(n-i)!\,c^{i+1}}\,x^{n-i}$$

$$\int \frac{e^{cx}}{x}\;\mathrm{d}x = \ln|x| + \sum_{n=1}^\infty \frac{(cx)^n}{n\cdot n!}$$

$$\int \frac{e^{cx}}{x^n}\;\mathrm{d}x = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + c\int \frac{e^{cx}}{x^{n-1}}\,\mathrm{d}x\right) \qquad (\text{for } n \neq 1)$$

Integral melibatkan fungsi eksponensial dan trigonometri

$$\int e^{cx}\sin bx\;\mathrm{d}x = \frac{e^{cx}}{c^2+b^2}(c\sin bx - b\cos bx) = \frac{e^{cx}}{\sqrt{c^2+b^2}}\sin(bx-\phi) \qquad \cos(\phi) = \frac{c}{\sqrt{c^2+b^2}}$$

$$\int e^{cx}\cos bx\,\mathrm{d}x = \frac{e^{cx}}{c^2+b^2}(c\cos bx + b\sin bx) = \frac{e^{cx}}{\sqrt{c^2+b^2}}\cos(bx-\phi) \qquad \cos(\phi) = \frac{c}{\sqrt{c^2+b^2}}$$

$$\int e^{cx}\sin^n x\,\mathrm{d}x = \frac{e^{cx}\sin^{n-1}x}{c^2+n^2}(c\sin x - n\cos x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\sin^{n-2}x\,\mathrm{d}x$$

$$\int e^{cx}\cos^n x\,\mathrm{d}x = \frac{e^{cx}\cos^{n-1}x}{c^2+n^2}(c\cos x + n\sin x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\cos^{n-2}x\,\mathrm{d}x$$

Integral melibatkan fungsi kesalahan

$$\int e^{cx}\ln x\,\mathrm{d}x = \frac{1}{c}\left(e^{cx}\ln|x| - \mathrm{Ei}\,(cx)\right)$$

$$\int x e^{cx^2}\,\mathrm{d}x = \frac{1}{2c}\,e^{cx^2}$$

$$\int e^{-cx^2}\,\mathrm{d}x = \sqrt{\frac{\pi}{4c}}\operatorname{erf}(\sqrt{c}x)$$ (erf adalah suatu fungsi error)

$$\int x e^{-cx^2}\,\mathrm{d}x = -\frac{1}{2c}e^{-cx^2}$$

$$\int \frac{e^{-x^2}}{x^2}\,\mathrm{d}x = -\frac{e^{-x^2}}{x} - \sqrt{\pi}\operatorname{erf}(x)$$

$$\int \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\,\mathrm{d}x = \frac{1}{2}\left(\operatorname{erf}\,\frac{x-\mu}{\sigma\sqrt{2}}\right)$$

Integral lain-lain

$$\int e^{x^2}\,\mathrm{d}x = e^{x^2}\left(\sum_{j=0}^{n-1}c_{2j}\,\frac{1}{x^{2j+1}}\right) + (2n-1)c_{2n-2}\int\frac{e^{x^2}}{x^{2n}}\,\mathrm{d}x \quad \text{valid untuk setiap } n>0,$$

di mana $c_{2j} = \frac{1\cdot 3\cdot 5\cdots(2j-1)}{2^{j+1}} = \frac{(2j)\,!}{j!\,2^{2j+1}}$.

(Perhatikan bahwa nilai ekspresi ini *independen* atau tidak tergantung dari nilai n, karena itu tidak muncul dalam integral.)

$$\int \underbrace{x^{x^{\cdot^{\cdot^{x}}}}}_{m}\,dx = \sum_{n=0}^{m}\frac{(-1)^n(n+1)^{n-1}}{n!}\Gamma(n+1,-\ln x) + \sum_{n=m+1}^{\infty}(-1)^n a_{mn}\Gamma(n+1,-\ln x) \qquad (\text{for } x>0)$$

di mana $a_{mn} = \begin{cases} 1 & \text{jika } n=0, \\ \frac{1}{n!} & \text{jika } m=1, \\ \frac{1}{n}\sum_{j=1}^{n} j a_{m,n-j}a_{m-1,j-1} & \text{selainnya} \end{cases}$

dan $\Gamma(x,y)$ adalah fungsi gamma

$$\int \frac{1}{ae^{\lambda x}+b}\,\mathrm{d}x = \frac{x}{b} - \frac{1}{b\lambda}\ln\left(ae^{\lambda x}+b\right)$$ ketika $b\neq 0, \lambda\neq 0,$ dan $ae^{\lambda x}+b>0\,.$

$$\int \frac{e^{2\lambda x}}{ae^{\lambda x}+b}\,\mathrm{d}x = \frac{1}{a^2\lambda}\left[ae^{\lambda x}+b-b\ln\left(ae^{\lambda x}+b\right)\right]$$ ketika $a\neq 0, \lambda\neq 0,$ dan $ae^{\lambda x}+b>0\,.$

Integral tertentu

$$\int_0^1 e^{x\cdot\ln a+(1-x)\cdot\ln b}\,\mathrm{d}x = \int_0^1\left(\frac{a}{b}\right)^x\cdot b\,\mathrm{d}x = \int_0^1 a^x\cdot b^{1-x}\,\mathrm{d}x = \frac{a-b}{\ln a-\ln b}$$ untuk $a>0,\ b>0,\ a\neq b,$

yang merupakan rata-rata logaritme

$$\int_0^\infty e^{ax}\,\mathrm{d}x = \frac{1}{-a} \quad (\operatorname{Re}(a) < 0)$$

$$\int_0^\infty e^{-ax^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad (a > 0)$$ (Integral Gaussian)

$$\int_{-\infty}^\infty e^{-ax^2}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_{-\infty}^\infty e^{-ax^2} e^{-2bx}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}}e^{\frac{b^2}{a}} \quad (a > 0)$$ (lihat Integral suatu fungsi Gaussian)

$$\int_{-\infty}^\infty x e^{-a(x-b)^2}\,\mathrm{d}x = b\sqrt{\frac{\pi}{a}} \quad (\operatorname{Re}(a) > 0)$$

$$\int_{-\infty}^\infty x e^{-ax^2+bx}\,\mathrm{d}x = \frac{\sqrt{\pi}b}{2a^{3/2}}e^{\frac{b^2}{4a}} \quad (\operatorname{Re}(a) > 0)$$

$$\int_{-\infty}^\infty x^2 e^{-ax^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \quad (a > 0)$$

$$\int_{-\infty}^\infty x^2 e^{-ax^2-bx}\,\mathrm{d}x = \frac{\sqrt{\pi}(2a+b^2)}{4a^{5/2}}e^{\frac{b^2}{4a}} \quad (\operatorname{Re}(a) > 0)$$

$$\int_{-\infty}^\infty x^3 e^{-ax^2+bx}\,\mathrm{d}x = \frac{\sqrt{\pi}(6a+b^2)b}{8a^{7/2}}e^{\frac{b^2}{4a}} \quad (\operatorname{Re}(a) > 0)$$

$$\int_0^\infty x^n e^{-ax^2}\,\mathrm{d}x = \begin{cases} \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)/a^{\frac{n+1}{2}} & (n > -1, a > 0) \\ \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}} & (n = 2k, k \text{ integer}, a > 0) \\ \frac{k!}{2a^{k+1}} & (n = 2k+1, k \text{ integer}, a > 0) \end{cases}$$ (!! merupakan faktorial ganda)

$$\int_0^\infty x^n e^{-ax}\,\mathrm{d}x = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}} & (n > -1, a > 0) \\ \frac{n!}{a^{n+1}} & (n = 0, 1, 2, \ldots, a > 0) \end{cases}$$

$$\int_0^1 x^n e^{-ax}\,\mathrm{d}x = \frac{n!}{a^{n+1}}\left[1 - e^{-a}\sum_{i=0}^n \frac{a^i}{i!}\right]$$

$$\int_0^\infty e^{-ax^b}\,dx = \frac{1}{b}\ a^{-\frac{1}{b}}\,\Gamma\left(\frac{1}{b}\right)$$

$$\int_0^\infty x^n e^{-ax^b}\,dx = \frac{1}{b}\ a^{-\frac{n+1}{b}}\,\Gamma\left(\frac{n+1}{b}\right)$$

$$\int_0^\infty e^{-ax}\sin bx\,\mathrm{d}x = \frac{b}{a^2+b^2} \quad (a > 0)$$

$$\int_0^\infty e^{-ax}\cos bx\,\mathrm{d}x = \frac{a}{a^2+b^2} \quad (a > 0)$$

$$\int_0^\infty x e^{-ax}\sin bx\,\mathrm{d}x = \frac{2ab}{(a^2+b^2)^2} \quad (a > 0)$$

$$\int_0^\infty x e^{-ax}\cos bx\,\mathrm{d}x = \frac{a^2-b^2}{(a^2+b^2)^2} \quad (a > 0)$$

$$\int_0^{2\pi} e^{x\cos\theta}\,d\theta = 2\pi I_0(x)$$ (I_0 adalah modifikasi fungsi Bessel dari jenis pertama)

$$\int_0^{2\pi} e^{x\cos\theta + y\sin\theta} d\theta = 2\pi I_0\left(\sqrt{x^2+y^2}\right)$$

Pranala luar

- Wolfram Mathematica Online Integrator (http://www.wolframalpha.com/calculators/integral-calculator/)
- V. H. Moll, The Integrals in Gradshteyn and Ryzhik (http://www.math.tulane.edu/~vhm/Table.html)

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