

Contoh Soal-soal dan Pembahasan Integral

1. $\int (2x^3 + 3x^2 + x + 7)dx = \dots\dots$

Jawab:

pakai rumus : $\int k x^n dx = \frac{k}{n+1} x^{n+1} + c$

$$\begin{aligned}\int (2x^3 + 3x^2 + x + 7)dx &= \frac{2}{4}x^4 + \frac{3}{3}x^3 + \frac{1}{2}x^2 + 7x + c \\ &= \frac{1}{2}x^4 + x^3 + \frac{1}{2}x^2 + 7x + c\end{aligned}$$

2. $\int \sin 3x \sin 2x dx = \dots\dots$

Jawab:

ingat rumus trigonometri : $-2 \sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$

$$\sin \alpha \sin \beta = -\frac{1}{2} (\cos(\alpha + \beta) - \cos(\alpha - \beta))$$

$$= \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\int \sin 3x \sin 2x dx = \int \frac{1}{2} \cos(3x - 2x) dx - \int \frac{1}{2} \cos(3x + 2x) dx$$

$$= \int \frac{1}{2} \cos x dx - \int \frac{1}{2} \cos 5x dx \rightarrow \text{pakai rumus } \int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$$

Sehingga menjadi :

$$= \frac{1}{2} \sin x - \frac{1}{2} \frac{1}{5} \sin 5x + c$$

$$= \frac{1}{2} \sin x - \frac{1}{10} \sin 5x + c$$

$$3. \int x^2 \sqrt{2x^3 + 3} \, dx = \dots\dots$$

Jawab :

cara substitusi:

$$\text{misal: } u = 2x^3 + 3$$

$$\frac{du}{dx} = 6x^2 \rightarrow dx = \frac{du}{6x^2}$$

Sehingga :

$$\begin{aligned} \int x^2 \sqrt{2x^3 + 3} \, dx &= \int x^2 u^{\frac{1}{2}} \frac{du}{6x^2} \\ &= \int \frac{1}{6} u^{\frac{1}{2}} \, du = \frac{1}{6} \frac{1}{1 + \frac{1}{2}} u^{\frac{1}{2} + 1} + c \\ &= \frac{1}{6} \frac{2}{3} u^{\frac{3}{2}} + c = \frac{1}{9} (2x^3 + 3) \sqrt{2x^3 + 3} + c \end{aligned}$$

$$4. \int x^2 \cos x \, dx = \dots\dots$$

Jawab :

$$\text{Pakai rumus integral parsial : } \int u \, dv = uv - \int v \, du$$

$$\text{misal : } u = x^2 \rightarrow du = 2x \, dx$$

$$dv = \cos x \, dx \rightarrow v = \int \cos x \, dx = \sin x$$

Sehingga :

$$\int x^2 \cos x \, dx = x^2 \cdot \sin x - 2 \int x \sin x \, dx$$

$$\int x \sin x \, dx \text{ perlu diparsialkan lagi tersendiri :}$$

$$\text{misal } u = x \rightarrow du = dx$$

$$dv = \sin x \, dx \rightarrow v = \int \sin x \, dx = -\cos x$$

$$\begin{aligned}
 \text{sehingga : } \int x \sin x \, dx &= x \cdot (-\cos x) - \int -\cos x \, dx \\
 &= -x \cos x + \int \cos x \, dx \\
 &= -x \cos x + \sin x + c
 \end{aligned}$$

Maka :

$$\begin{aligned}
 \int x^2 \cos x \, dx &= x^2 \cdot \sin x - 2 \int x \sin x \, dx \\
 &= x^2 \cdot \sin x - 2(-x \cos x + \sin x) + c \\
 &= x^2 \cdot \sin x + 2x \cos x - 2 \sin x + c \\
 &= (x^2 - 2) \cdot \sin x + 2x \cos x + c
 \end{aligned}$$

$$5. \int x \cos(2x^2 + 3) \, dx = \dots$$

jawab:

$$\text{misal : } u = 2x^2 + 3 \rightarrow du = 4x \, dx \rightarrow dx = \frac{du}{4x}$$

sehingga :

$$\begin{aligned}
 \int x \cos(2x^2 + 3) \, dx &= \int x \cos u \frac{du}{4x} \\
 &= \int \frac{1}{4} \cos u \, du \\
 &= \frac{1}{4} \sin u + c \\
 &= \frac{1}{4} \sin(2x^2 + 3) + c
 \end{aligned}$$

$$6. \int_3^4 x (2 + x)^3 \, dx = \dots$$

jawab :

misal : $u = x \rightarrow du = dx$

$$\begin{aligned} dv = (2+x)^3 dx \rightarrow v &= \int (2+x)^3 dx \rightarrow \int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c \\ &= \frac{1}{4} (2+x)^4 \end{aligned}$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned} \int_3^4 x (2+x)^3 dx &= \frac{1}{4} (2+x)^4 \Big|_3^4 - \int_3^4 \frac{1}{4} (2+x)^4 dx \\ &= \frac{1}{4} (2+x)^4 \Big|_3^4 - \frac{1}{4} \frac{1}{5} (2+x)^5 \Big|_3^4 \\ &= \frac{1}{4} (1296 - 625) - \frac{1}{20} (7776 - 3125) \\ &= \frac{671}{4} - \frac{4651}{20} \\ &= \frac{3355 - 4651}{20} = -\frac{1296}{20} = -64 \frac{4}{5} \end{aligned}$$

$$7. \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 x \cos x dx = \dots$$

Jawab:

Cara 1:

$$\text{Pakai rumus : } \int \sin^n(ax+b) \cos(ax+b) dx = \frac{1}{a(n+1)} \sin^{n+1}(ax+b) + c$$

$$\begin{aligned} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 x \cos x dx &= \frac{1}{3} \sin^3 x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= \frac{1}{3} \left(1^3 - \left(\frac{1}{2} \right)^3 \right) = \frac{1}{3} \cdot \frac{7}{8} = \frac{7}{24} \end{aligned}$$

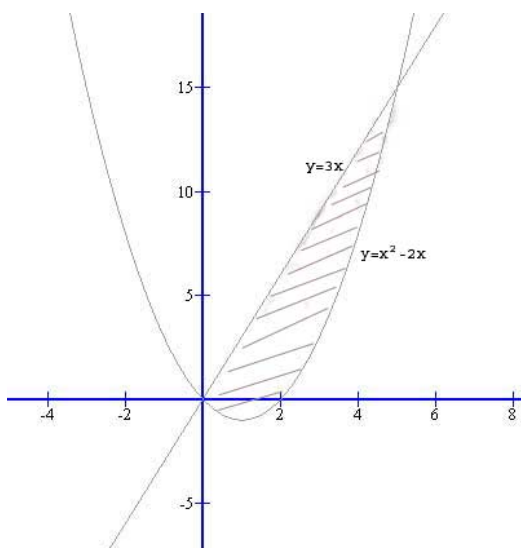
Cara 2:

Cara substitusi :

misal $u = \sin x \rightarrow du = \cos x \, dx$

$$\begin{aligned} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx &= \int u^2 \, du = \frac{1}{3} u^3 \\ &= \frac{1}{3} \sin^3 x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= \frac{1}{3} \left(1^3 - \left(\frac{1}{2} \right)^3 \right) = \frac{1}{3} \cdot \frac{7}{8} = \frac{7}{24} \end{aligned}$$

8. Luas daerah yang diarsir pada gambar adalah :



Jawab :

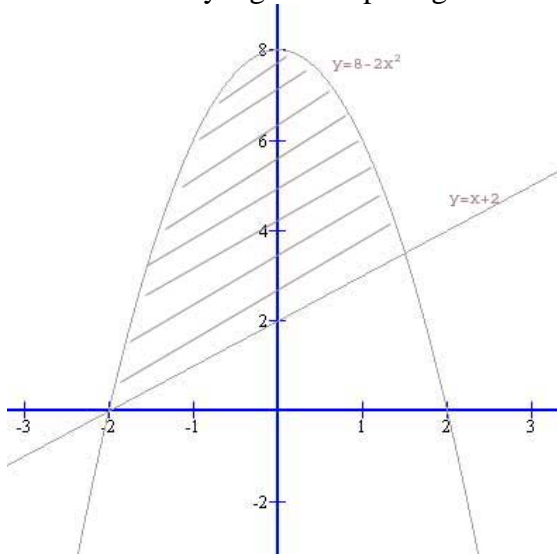
Cari titik potong persamaan $y = 3x$ dan $y = x^2 - 2x$:

$$\begin{aligned} 3x &= x^2 - 2x \\ \Leftrightarrow x^2 - 5x &= 0 \\ \Leftrightarrow x(x - 5) &= 0 \end{aligned}$$

didapat titik potong di $x = 5$ dan $x = 0$

$$\begin{aligned}
 L &= \int_0^5 (3x - (x^2 - 2x)) \, dx \\
 &= \int_0^5 (5x - x^2) \, dx \\
 &= \frac{5}{2}x^2 - \frac{1}{3}x^3 \Big|_0^5 \\
 &= \frac{5}{2}5^2 - \frac{1}{3}5^3 \\
 &= \frac{125}{2} - \frac{125}{3} = \frac{375 - 250}{6} = \frac{125}{6} \\
 &= 20 \frac{5}{6} \text{ satuan luas}
 \end{aligned}$$

9. Luas daerah yang diarsir pada gambar adalah :



Jawab:

cari titik potong kedua persamaan :

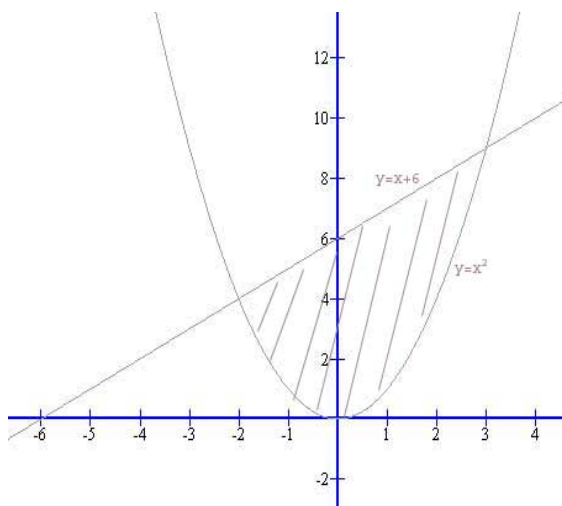
$$\begin{aligned}
 8 - 2x^2 &= x + 2 \\
 \Leftrightarrow 2x^2 + x - 6 &= 0 \\
 \Leftrightarrow (2x - 3)(x + 2) &= 0
 \end{aligned}$$

Didapat titik potong $x = \frac{3}{2}$ dan $x = -2$

$$\begin{aligned}
L &= \int_{-2}^{\frac{3}{2}} ((8-2x^2) - (x+2)) dx \\
&= \int_{-2}^{\frac{3}{2}} (6-2x^2-x) dx \\
&= 6x - \frac{2}{3}x^3 - \frac{1}{2}x^2 \Big|_{-2}^{\frac{3}{2}} \\
&= \left\{ 6 \cdot \frac{3}{2} - \frac{2}{3} \left(\frac{3}{2} \right)^3 - \frac{1}{2} \left(\frac{3}{2} \right)^2 \right\} - \left\{ 6 \cdot -2 - \frac{2}{3}(-2)^3 - \frac{1}{2}(-2)^2 \right\} \\
&= \left\{ 9 - \frac{2}{3} \cdot \frac{27}{8} - \frac{1}{2} \cdot \frac{9}{4} \right\} - \left\{ -12 + \frac{16}{3} - 2 \right\} \\
&= 9 - \frac{54}{24} - \frac{9}{8} + 12 - \frac{16}{3} + 2 \\
&= 23 - \frac{54}{24} - \frac{9}{8} - \frac{16}{3} \\
&= \frac{552-54-27-128}{24} = \frac{343}{24} = 14 \frac{7}{24} \text{ satuan luas}
\end{aligned}$$

10. Volume benda putar yang terjadi jika daerah yang dibatasi kurva $y = x^2$ dan $y = x + 6$. Diputar mengelilingi sumbu x sebesar 360° adalah.....

Jawab:



Titik potong kurva :

$$x^2 = x + 6$$

$$\Leftrightarrow x^2 - x - 6 = 0$$

$$\Leftrightarrow (x-3)(x+2) = 0$$

titik potong di $x = 3$ dan $x = -2$

$$V = \pi \int_{-2}^3 ((x+6)^2 - (x^2)^2) dx$$

$$= \pi \int_{-2}^3 ((x^2 + 12x + 36) - x^4) dx$$

$$= \pi \int_{-2}^3 (-x^4 + x^2 + 12x + 36) dx$$

$$= \pi \left\{ -\frac{1}{5}x^5 + \frac{1}{3}x^3 + 6x^2 + 36x \right\} \Big|_{-2}^3$$

$$= \pi \left\{ \left(-\frac{243}{5} + 9 + 54 + 108 \right) - \left(\frac{32}{5} - \frac{8}{3} + 24 - 72 \right) \right\}$$

$$= \pi \left(-\frac{243}{5} + 171 - \frac{32}{5} + \frac{8}{3} + 48 \right)$$

$$= \pi \left(-\frac{275}{5} + \frac{8}{3} + 219 \right)$$

$$= \pi \left(219 - 55 + \frac{8}{3} \right) = \pi \left(164 + \frac{8}{3} \right)$$

$$= 166 \frac{2}{3} \pi \text{ satuan volume}$$