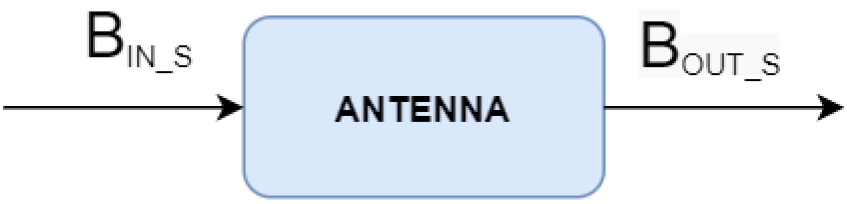
ANALYTICAL STUDY FOR STABILITY

The main goal of this section is to present the analytical study we carried out to find a rough estimate for the upper bound of the user lambda factor.

Given the complexity of the system we thought to analyze a simplified version of it in which the antenna is like a black box. The antenna receives some bytes on one side and sends out some bytes on the other side.



In this diagram we have:

- $\mathbf{B_{IN\_S}}$ = Average number of bytes per second in input
- $\mathbf{B_{OUT\_S}}$ = Average number of bytes per second in output

Let us try and estimate both values starting from B_{IN_S}.

$\mathbf{B_{IN\_S} = \lambda_P * A_{VG\_S\_IN} * N_U}$ where:

- $\mathbf{\lambda_P}$ is the **user lambda**, the factor we are trying to estimate.
- $\mathbf{A_{VG\_S\_IN}}$ is the **average size of packets in input** expressed in Bytes. This value is chosen for each user using a uniform distribution ranging in [1, 75], therefore the average value is 76/2= 38 Byte.
- N_U is the **number of users** in the simulation.

$\mathbf{B_{OUT\_S} = N_{RB\_F} * A_{VG\_S\_RB} * N_{F\_S}}$

- $\mathbf{N_{RB\_F}}$ is the **number of resource block used per frame** sent by the antenna. This value changes from frame to frame depending on the presence, or lack thereof, packets in the queues. Given the fact that we are trying to find the upper bound of user lambda we thought it was fair to assume $\mathbf{N_{RB\_F}}$ = 25 (which is the maximum).
- $\mathbf{A_{VG\_S\_RB}}$ is the **average size of a single resource block**. This value is the hardest to estimate; it varies because of the CQI sent by each user every timeslot, it depends on the user chosen by the antenna, and finally the way the CQI is chosen by each user changes depending on the scenario studied (uniformCQI or binomialCQI). Simplification was necessary, and this is why we went for the more optimistic approach; uniformCQI. In this case, on average $\mathbf{A_{VG\_S\_RB}}$ = $(\sum_{i=1}^{15} CQI[i])/15 = 40{,}6\ B$ (check out the CQI table for more details)

- N_{F_S} is the number of frames sent each second. Given the timeslot T this value is simply equal to 1s/T = 1s / (10^{-3}s) = 1000

Given these estimates it is easy to express the stability condition as:

$$B_{IN\_S} < B_{OUT\_S} \rightarrow$$

$$\lambda_P * A_{VG\_S\_IN} * N_U < \mathrm{N}_{RB\_F} * A_{VG\_S\_RB} * N_{F\_S} \rightarrow$$

$$\lambda_P < \frac{\mathrm{N}_{RB\_F} * A_{VG\_S\_RB} * N_{F\_S}}{A_{VG\_S\_IN} * N_U} \rightarrow$$

$$\lambda_P < \frac{1015000}{38 \cdot N_U}$$

By using this formula, we can calculate the estimated maximum *UserLambda* for each Num_User chosen in the simulations:

#User	25	50	100	150	200
λ_P	1068.421	534.2105	267.1053	178.0702	133.5526
λ_P *0.4	427.3684	213.6842	106.8421	71.22808	53.42104
λ_P *0.33	352.57893	176.2895	88.144749	58.76317	44.07236

As we said previously this is just an estimated maximum value, therefore we expect the system to be stable only at a certain fraction of the values calculated. By studying the experimental results of the configurations chosen we identified the turning point in which the antenna gets saturated:

- For UniformCQI it is around 40% of the estimated values.
- For BinomialCQI it is around 33% of the estimated values.