AILAB
Artificial Intelligence Laboratory

Artificial Intelligence

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Class Details

- Class schedule
  - Wednesday 8:30am–10:30am, room G
  - Thursday 8:30am–10:30am, room G

- Classes are live streamed and recorded
  - The course page on Elly (elly2023.smfi.unipr.it) mentions the live-streaming link, which is the same for all classes
  - The course page on Elly mentions the code to register to the team and access the recordings
  - Only *formally* enrolled students can access the classes and the recordings
  - *It is strictly prohibited to download the recordings*

Communications with the Teacher

- Few simple rules to communicate with the teacher
  - Communications and announcements from the teacher are posted via Elly
  - Meetings with the teacher are requested via e-mail
  - Before requesting a meeting, send an e-mail to the teacher describing the reasons for the meeting request
    - Clearly describe the problems with sufficient details
    - Attach the exercises and/or the programs that originated the meeting request

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Exam Details (I)

- Exam schedule
  - 3 sessions in January–February
  - 3 sessions in June–July
  - 1 session in September

- Exam sessions are composed of
  - A written exam and an oral exam
  - The oral exam follows a successful written exam
  - The oral exam is upon explicit request from the teacher or from the student
  - The oral exam is part of the process, and it is not supposed to only improve the grade of the written exam
  - Written and oral exams must be passed in the same session
  - Written and oral exams cover the whole course program

Exam Details (II)

- Exam enrollment
  - Enrollment is strictly needed to participate to an exam
  - Enrollment is performed using Esse3

unipr.esse3.cineca.it

  - Enrollment is possible only within the period associated with each session
  - Clearing the enrollment for an exam is possible only within the period associated with each session

- Students are not supposed to enroll to all sessions

Didactic Materials

- Classes are the official reference for the course
  - The slides used for the classes will be available via Elly throughout the academic year (after classes)
  - The class recordings will be available via Elly throughout the academic year
- Additional materials will be provided via Elly
  - To expand and complement slides
  - To allow for deeper investigations on selected topics
- Further reading
  - S. Russell, P. Norvig. *Artificial Intelligence: A Modern Approach*, 4th edition, Pearson, 2020

What is Artificial Intelligence?

"In the 1980s, Minsky and Good had shown how neural networks could be generated automatically—self replicated—in accordance with any arbitrary learning program. Artificial brains could be grown by a process strikingly analogous to the development of a human brain."

Arthur C. Clarke, *2001: A Space Odyssey*

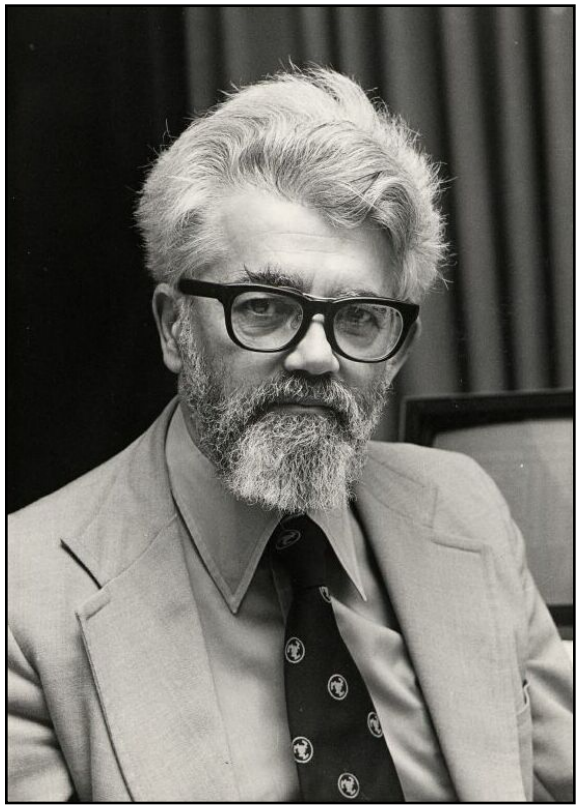
Artificial Intelligence

- An ambitious project that started together with Computer Science
- The goal of the project was to design and develop
  - Machines with intelligent behaviors
  - Machines that could effectively interact with the real world (robots)
- Recently, Artificial Intelligence (AI) often targets
  - Machines that can solve complex problems
  - Machines that exhibit rational behaviors
  - Machines that can dialog with humans in a natural language
  - Machines that can generate creative contents
  - Machines that interact with complex and dynamic worlds (the Internet, the Web, ...)

The Dartmouth Meeting

- Prof. John McCarthy (1927-2011) invents the name *artificial intelligence* in 1955
  - In a letter to propose a meeting at the Dartmouth College to be held in the summer of 1956

- The Dartmouth meeting hosted discussions on the problems that AI have *not yet* solved
  - What is intelligence?
  - What is rationality?
  - Can machine think?
    - If so, how?
    - If not, why not?
  - …



A Characterization of AI

- From a published interview with John McCarthy
  - *Q: What is artificial intelligence?*
  - A: It is the science and engineering of making intelligent machines, especially intelligent computer programs. It is related to the similar task of using computers to understand human intelligence, but AI does not have to confine itself to methods that are biologically observable.
  - *Q: Yes, but what is intelligence?*
  - A: Intelligence is the computational part of the ability to achieve goals in the world. Varying kinds and degrees of intelligence occur in people […]

Other Characterizations

- AI is meant to deal with complex problems that can be (easily) solved by humans and other living beings, but that are not normally described as algorithms
  - E.g., understand natural languages, invent proofs of theorems, play chess

- Several problems that were targeted by AI in the past are no longer considered relevant
  - E.g., compile a Fortran program (~1955), compute the symbolic indefinite integral of a function (~1965), find a known structure in an image (~1970)

What is Easy? What is Hard?

- Several activities that are based on *cognitive capabilities* and that are normally associated with intelligence have already been automatized algorithmically, sot they are no longer relevant for AI

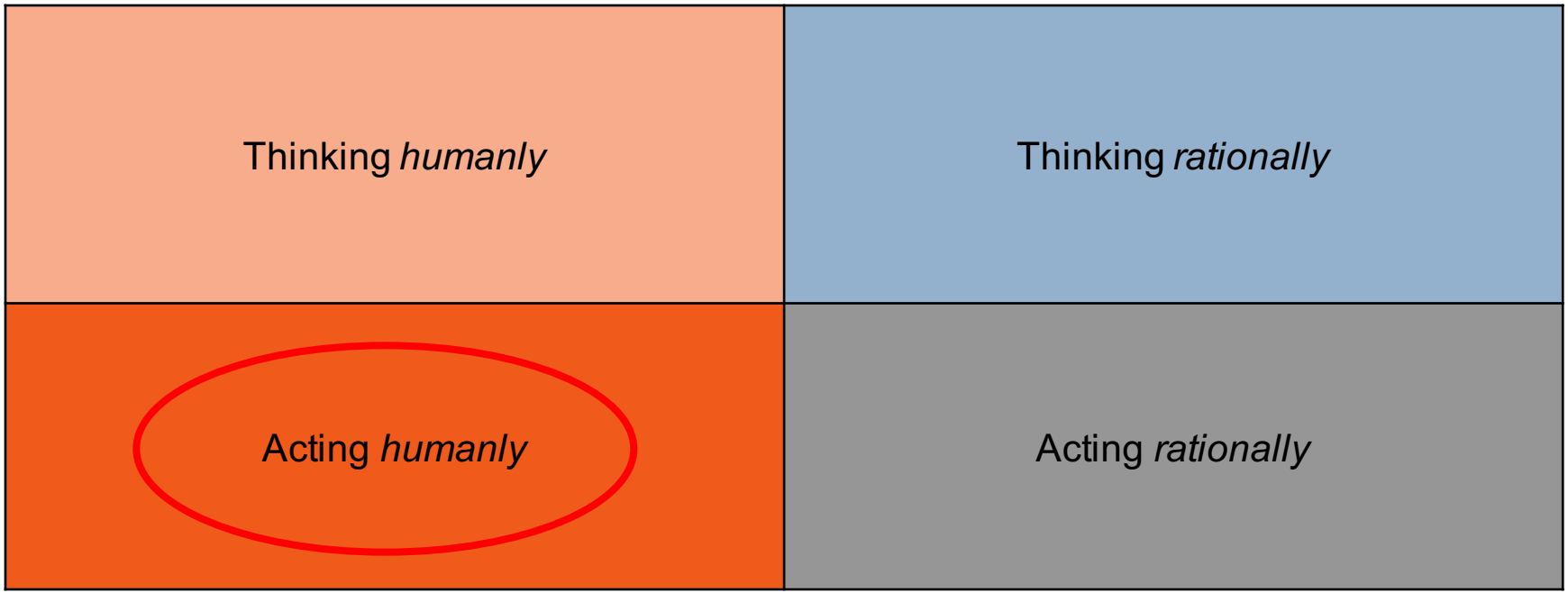
- Several activities that are easily and routinely performed by animals have not yet been automatized algorithmically
  - Walk without bumping into obstacles
  - Fuse information from senses (touch, sight, smell, …)
  - Effectively work together without a centralized control (like bees and ants)

Long Term Goals of AI

- AI can be strong
  - A computer executing an appropriate program can exhibit an intelligence that is indistinguishable from human intelligence
  - The empiricist philosophers Thomas Hobbes stated *"By ratiocination, I mean computation* […] *Ratiocination, therefore, is the same with addition and subtraction."* (Elements of Philosophy, 1656)

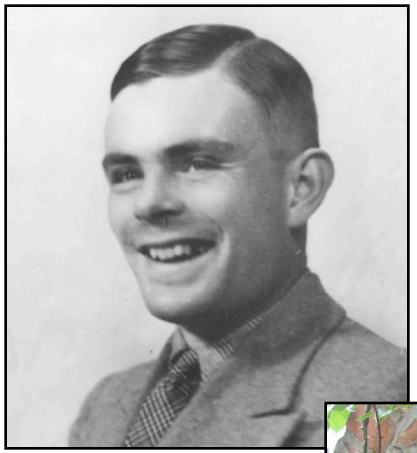
- AI can be weak
  - A computer will never have sufficient resources to exhibit an intelligence that is indistinguishable from human intelligence
  - A computer will never be so complex
    - Computers will simulate selected processes of the human mind

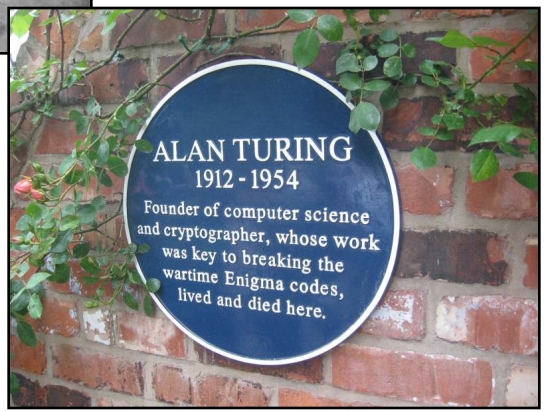
Four Viewpoints



Turing Test (I)

- What are the essential characteristics of intelligent behavior?
- Dr. Alan Turing (1912-1954) proposed an empirical test in 1950
- The Turing test is still relevant to understand the goals of AI





Turing Test (II)

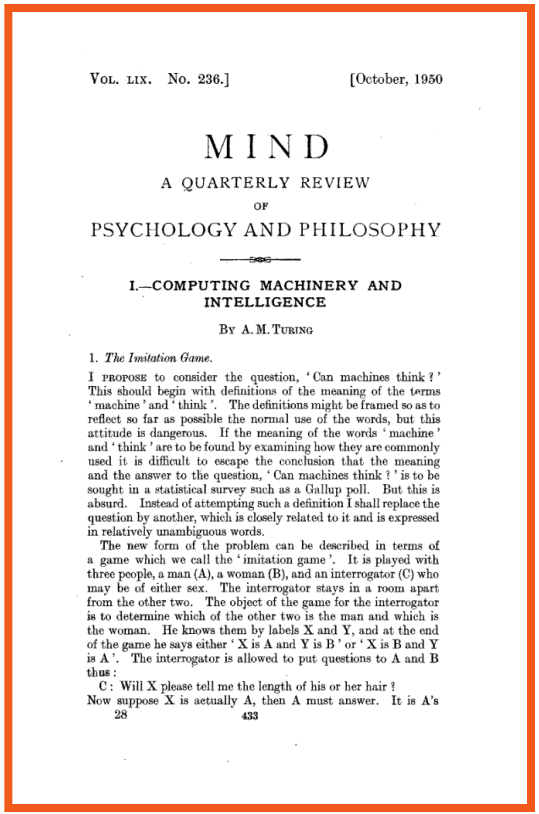
- The article Computing Machinery and Intelligence (Mind, 1950) starts with a section entitled

  The Imitation Game

- The section starts as follows

  I propose to consider the question, "Can machines think?"

- The article describes a test to assess if a machine can be considered intelligent
  - It provides an empirical definition of intelligence



VOL. LIX. No. 236.] [October, 1950

MIND

A QUARTERLY REVIEW

OF

PSYCHOLOGY AND PHILOSOPHY

I.—COMPUTING MACHINERY AND INTELLIGENCE

BY A. M. TURING

1. *The Imitation Game.*

I PROPOSE to consider the question, 'Can machines think?' This should begin with definitions of the meaning of the terms 'machine' and 'think'. The definitions might be framed so as to reflect so far as possible the normal use of the words, but this attitude is dangerous. If the meaning of the words 'machine' and 'think' are to be found by examining how they are commonly used it is difficult to escape the conclusion that the meaning and the answer to the question, 'Can machines think?' is to be sought in a statistical survey such as a Gallup poll. But this is absurd. Instead of attempting such a definition I shall replace the question by another, which is closely related to it and is expressed in relatively unambiguous words.

The new form of the problem can be described in terms of a game which we call the 'imitation game'. It is played with three people, a man (A), a woman (B), and an interrogator (C) who may be of either sex. The interrogator stays in a room apart from the other two. The object of the game for the interrogator is to determine which of the other two is the man and which is the woman. He knows them by labels X and Y, and at the end of the game he says either 'X is A and Y is B' or 'X is B and Y is A'. The interrogator is allowed to put questions to A and B thus:

C: Will X please tell me the length of his or her hair?

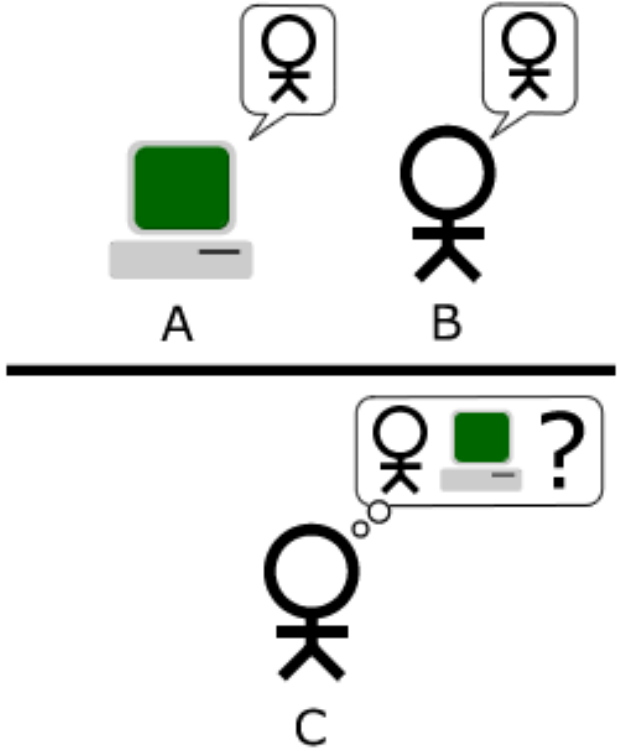
Now suppose X is actually A, then A must answer. It is A's

28 433

Turing Test (III)

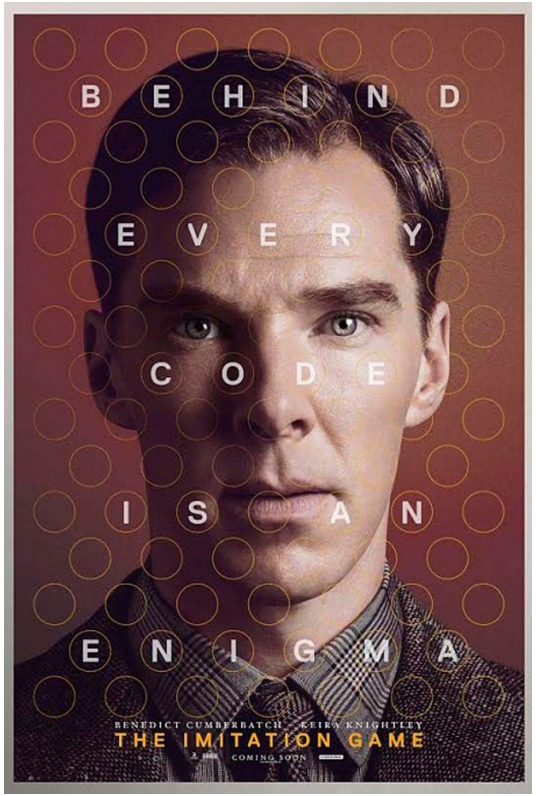
- *A* is a machine designed and developed to play chess
  - E.g., IBM *Deep Blue*
- *B* and *C* are two chess players (not newbies, not champions)
- *C* is not in the same room where *A* and *B* are
- *C* does not known if the current opponent is either *A* (the machine) or *B* (the human)

- *A* can be considered *as intelligent as B* if *C* cannot decide which is the current opponent



Acting Humanly

- The Turing test provides an empirical definition for the term *intelligent behavior*
  - In the original test, Turing did not talk about chess players, which inherently require rationality
- The Turing test advocates an anthropocentric approach
  - The goal is to act as a person
  - Intelligent machines play *imitation games*

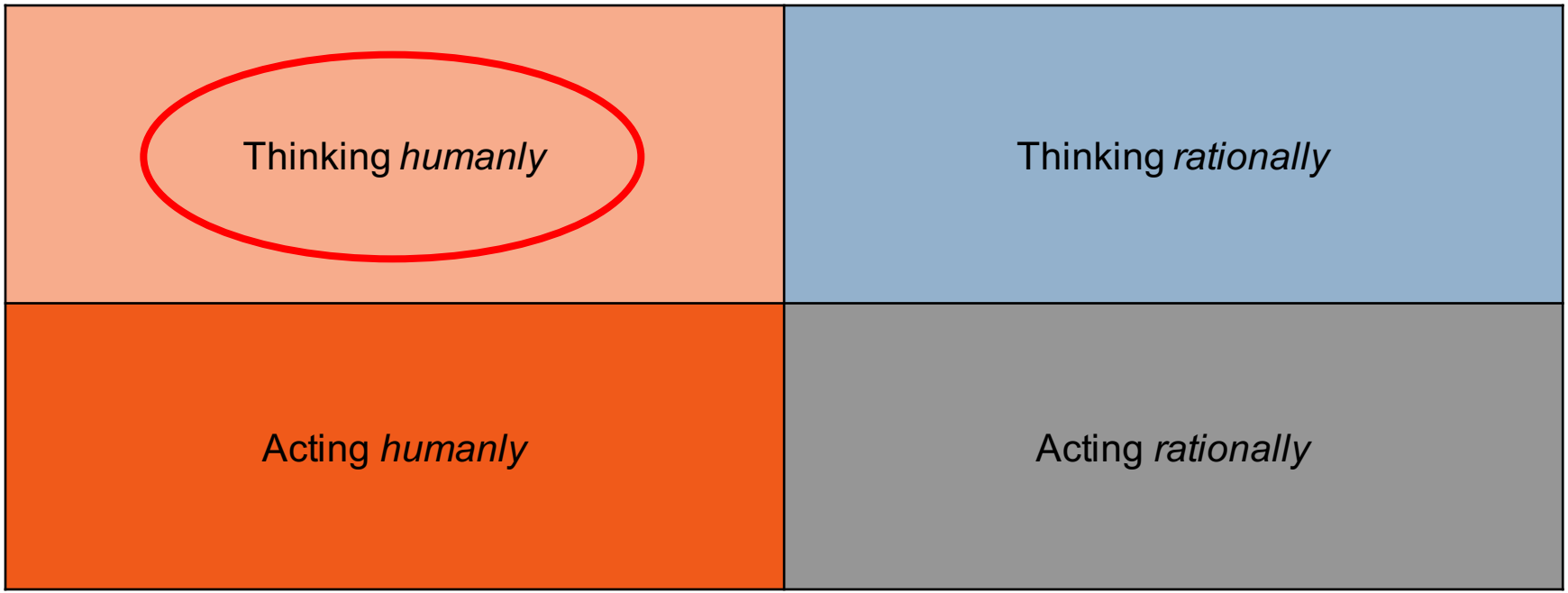


Eliza (J. Weizenbaum, 1966)

- The Turing test has been approached several times by several computer programs
- The computer program Eliza behaves as a Rogerian psychotherapist
- Eliza simply
  - Identifies relevant parts of input sentences
  - Rephrases input sentences to continue the conversation

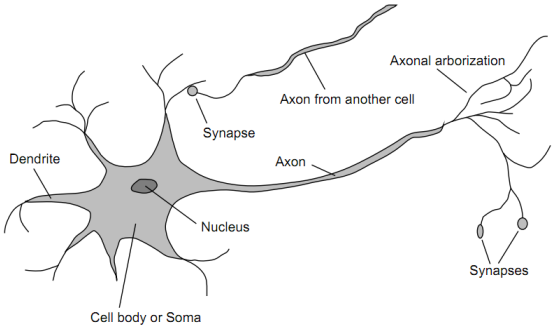
```
> Hello, I am Eliza.
* Hi
> How are you today...What would you like to discuss?
* I have problems at school
> Can you elaborate on that?
* I don't pass exams
> Do you wish to be able to pass exams?
* Yes, sure
> I see.
* Math exams are too difficult for me
> Come, come, elucidate your thoughts.
* I don't see a meaning in math
> Do you wish to be able to see a meaning in math?
```

Four Viewpoints

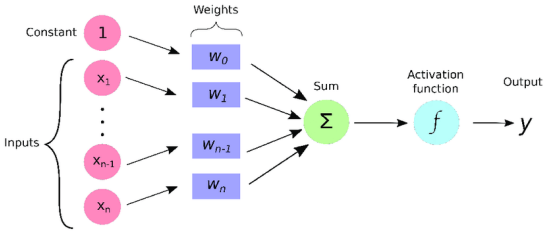


Thinking Humanly

- A possible approach to imitate human behavior is based on the possibility to simulate the organ that originates thoughts
  - The brain is the organ where thoughts originate

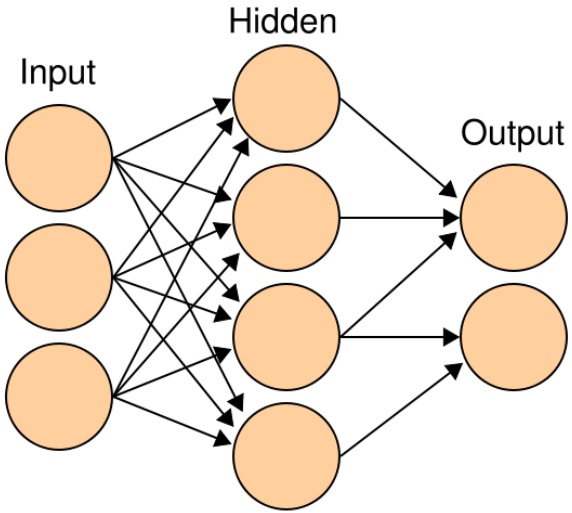


- Using this approach, the goal is to build *electronic brains* to simulate human brains
  - The simulation is at the cell level
  - The simulation includes simulated neurons, axons, soma, …



Neural Networks (I)

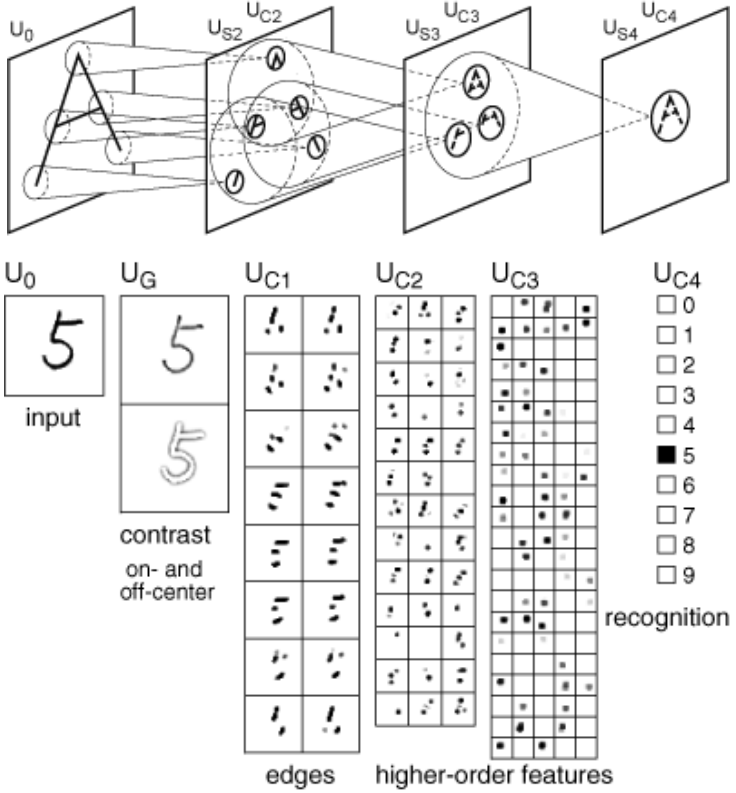
- Each *unit* simulates a neuron
- Units are connected in a *network*
  - That receives *percepts* from *sensors*
  - That provides *stimuli* to *actuators*
- The network is trained to *learn* the desired behavior
  - Normally, it is too complex to be explicitly programmed
  - Useful neural networks have billions of neurons



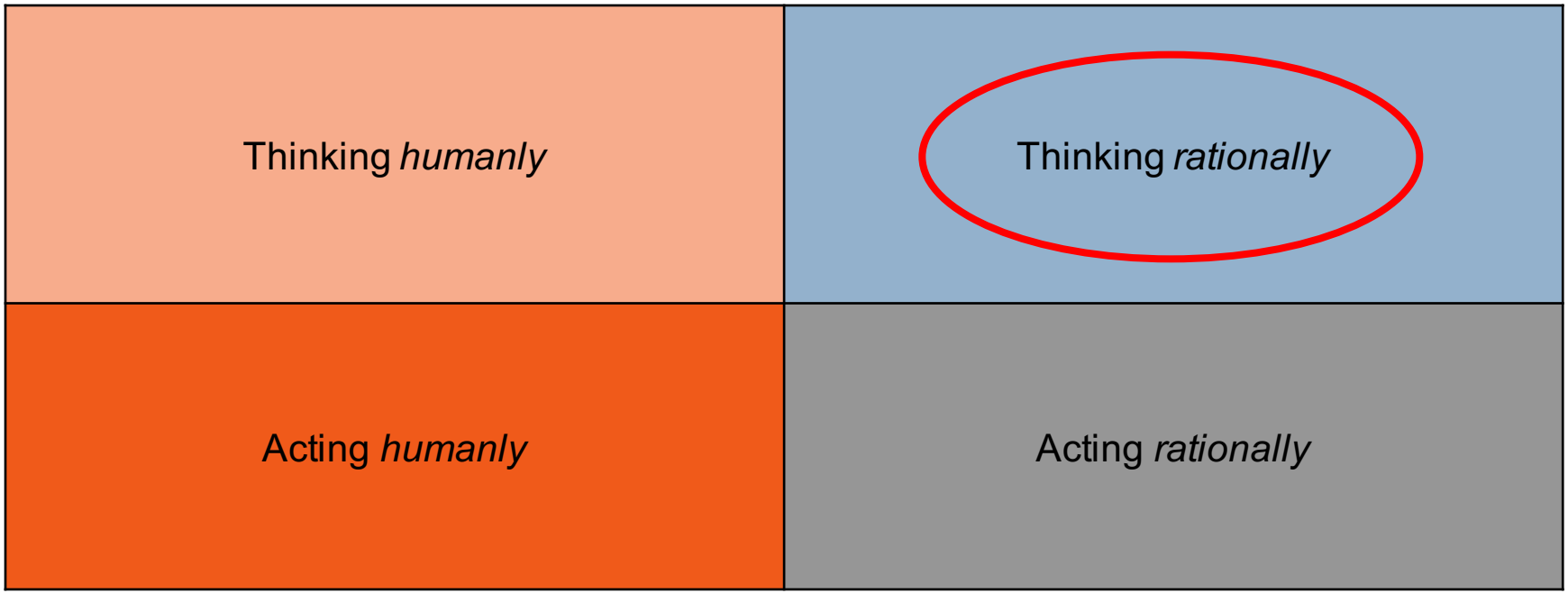
Neural Networks (II)

- *Neocognitron* (K. Fukushima, 1980)
  - Neural network trained to recognize handwritten characters

- It is an example of a
  - *Convolutional Neural Network* (*CNN*) because each neuron receives input from a local subset of the precedent neurons
  - *Deep network* (i.e., a network with several layers) that leverages deep learning



Four Viewpoints



Thinking Rationally

- Human beings are not always rational
  - They are heavily influenced by habits, hopes, unreachable goals, unsolvable problems, …
- Rationality is described by logic languages or equivalent formalisms
  - Assume that $A \rightarrow B$ is considered true
  - B must be considered true as soon as A is known to be true
- Logic formalisms are used for deductions

Expert Systems

- A computer program written in terms of a *knowledge base*, which is a (possibly dynamic) set of *facts* and *inference rules*
  - Facts:

$$mother\_of(ann,\ bob),\ sister\_of(claire,\ ann)$$

  - Inference rules:

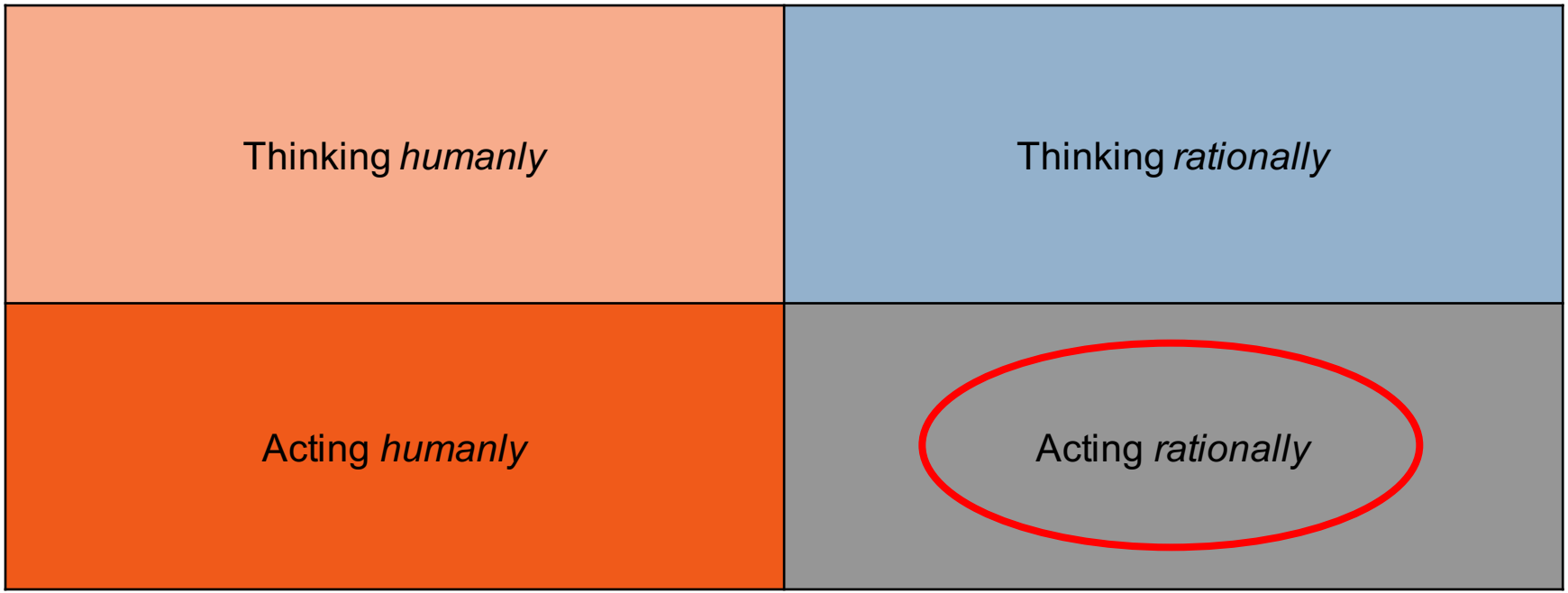
$$mother\_of(X,\ Y) \wedge sister\_of(Z,\ X) \rightarrow aunt\_of(Z,\ Y)$$

- The knowledge base of an expert system can be used to *infer* new facts from known facts

MYCIN (E. Shortliffe, 1970s)

- MYCIN is an expert system for medical diagnosis that
  - Contains more than 600 inference rules
  - Can ask yes/no questions
  - Provides a list of diagnosis and prescriptions
- An extensive experimental campaign measured that MYCIN was correct in 69% of the cases
  - It was correct more often than the doctors that provided the inference rules

Four Viewpoints



Acting Rationally

- The machines that act rationally within an *environment* are known as intelligent agents
  - An agent wants the *world* (agent and environment) in certain states, and it acts *autonomously* to bring the world there
  - An agent is characterized by its behavior and the mechanisms used to obtain its behavior are irrelevant
    - Agents are often based on logic languages
    - Agents often use deduction to decide what to do (i.e., to decide which action to perform on the environment)
    - Agents are not requested to imitate human behaviors
    - Agents are normally requested to be rational

SHRDLU (T. Winograd, 1971)

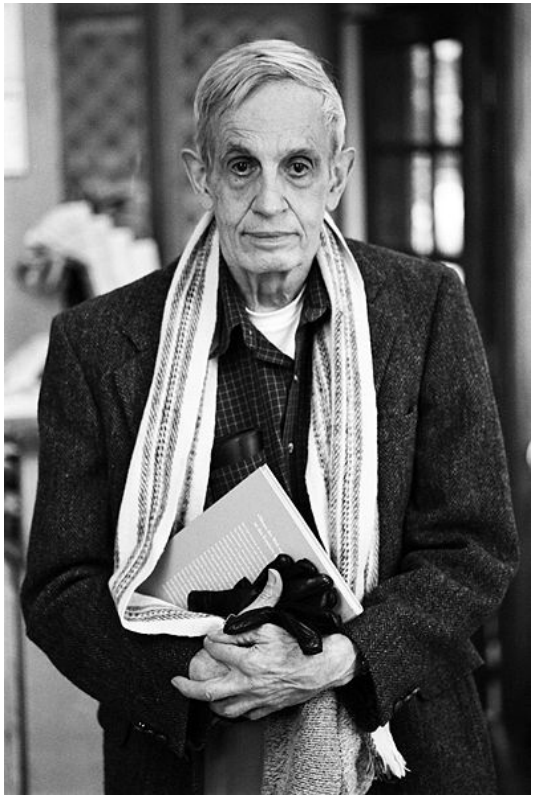
- The computer program SHRDLU controls an emulated robot arm to interact in the blocks world
- The user can interact with SHRDLU using a natural language (English)
  - SHRDLU associates a meaning with user sentences
  - SHRDLU can ask questions to better understand the user
- SHRDLU has been considered for several years the prototypical AI program

```
* Pick up a big red block.
> OK.
* Find a block which is taller than
the one you are holding and put it
into the box.
> BY "IT", I ASSUME YOU MEAN THE
BLOCK WHICH IS TALLER THAN THE ONE I
AM HOLDING.
> OK.
* What does the box contain?
> THE BLUE PYRAMID AND THE BLUE
BLOCK.
* What is the pyramid supported by?
> THE BOX.
* How many blocks aren't in the box?
> FOUR OF THEM.
* Is at least one of them narrower
than the one which I told you to
pick up?
> YES, THE RED CUBE.
```

Game Theory

- Game theory is used to design and develop intelligent agents, especially when the world is a multi-agent system
  - It was popularized by the works of John Nash (1928-2015)
  - It is studied and used, differently, by AI and Economics

- Game theory is used to reason on rational behaviors by describing rational behaviors in terms of the earned utility



Game Theory and AI

- AI often restricts its scope to games with the following characteristics
  - The game includes only two players that play alternate turns
  - The game has simple and formalized rules
  - The game is played in a completely accessible environment, and therefore, players have *perfect knowledge* of the state of the game
  - The game (often) have strict time constraints

- For example
  - Checkers, chess, … are often considered in AI
  - Poker, bridge, … are not often considered in AI

Real Agents (I)

- *Chinook* won the *Man-Machine Checkers Championship*
- It is still possible to play against Chinook

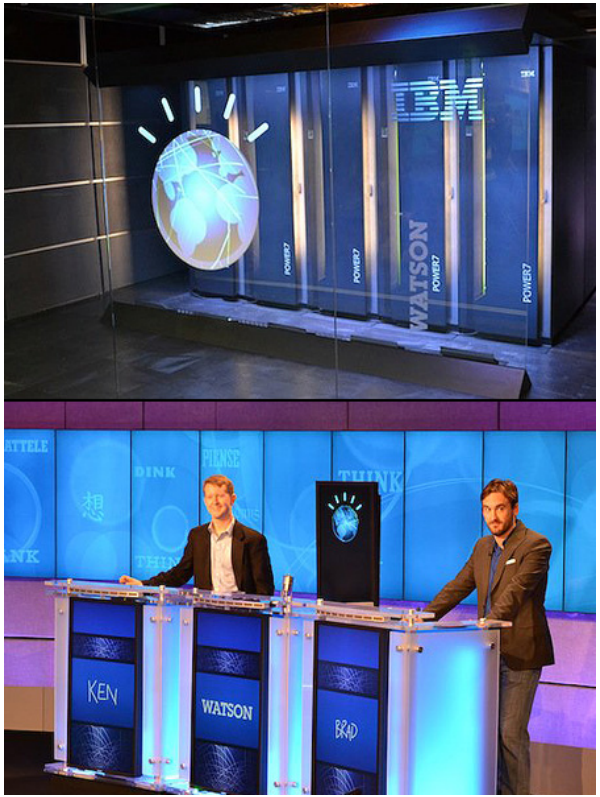
  www.cs.ualberta.ca/~chinook
- IBM *Deep Blue* won against Garry Kasparov in 1996 and received worldwide attention also from general media

Real Agents (II)

- Go was considered unfeasible for computers
  - Chess: *branching factor* > 40, more than 50 turns per game
  - Go: branching factor > 250, more than 350 turns per game
- *AlphaGo* is an AI player for Go
  - In October 2015, AlphaGo won 5-0 against the European (human) champion
- AlphaGo uses neural networks to support a game-theoretic algorithm to decide next move

Real Agents (III)

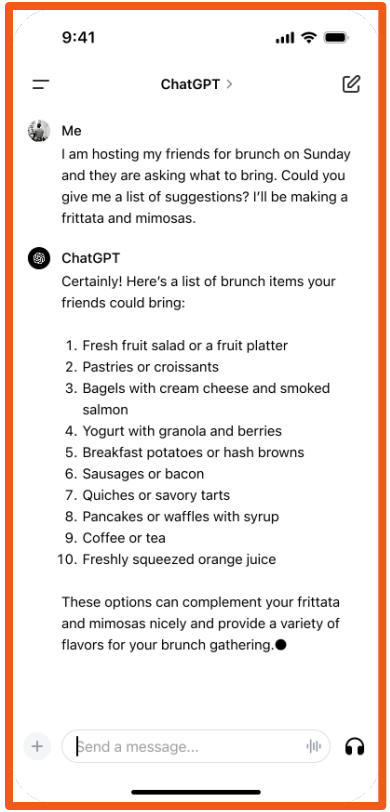
- *Watson* is a computer program that can answer questions expressed in English
  - It stores ~$200 \cdot 10^6$ documents including a dump of Wikipedia
- In 2011, Watson participated to three special episodes of *Jeopardy!* winning against several opponents
  - Watson was not connected to the Internet during the show recordings



Real Agents (IV)

- *GPTs* (*Generative Pre-trained Transformers*) are a family of related *LLMs (Large Language Models)*
- LLMs are deep neural networks trained to complete texts
  - They are trained on a huge amount of texts produced by humans (typically, more than 1T words of documents)
  - They are fine tuned using reinforcement learning feedback from humans
- LLMs are examples of generative AI agents

- Several alternatives to GPTs exist (e.g., LLAMA, Gemini, …)



Constraint Satisfaction Problems

"Were you to ask me which programming paradigm is likely to gain most in commercial significance over the next 5 years I'd have to pick Constraint Logic Programming (CLP)…"

BYTE, 1992

CSPs (I)

- Constraint Satisfaction Problems (CSPs) form a class of problems that is inherently related to the class of *search problems*
  - Search problems over trees
  - Search problems over graphs

- The class of CSPs can be understood as a specific class of search problems that is
  - *Sufficiently general* to include several interesting problems
  - *Sufficiently specific* to enable efficient solving algorithms and general-purpose *heuristics*

CSPs (II)

- Essentially, a *CSP* is a triple
  - A finite and nonempty set symbols called *variables*

$$V = \{ X_1 , X_2, \ldots, X_n \}$$

  - A finite and nonempty set of domains, one for each variable

$$D = \{ \mathrm{dom}(X_1), \mathrm{dom}(X_2), \ldots, \mathrm{dom}(X_n) \}$$

  - A finite set of *constraints* over variables such that a constraint C is

$$C \subseteq \mathrm{dom}(X_1) \times \mathrm{dom}(X_2) \times \ldots \times \mathrm{dom}(X_n)$$

CSPs (III)

- Normally, constraints are called
  - *Unary*, when they involve just one variable
  - *Binary*, when they involve two variables
  - *Global*, otherwise

- A *solution* to a CSP is a mapping that associates each variable *X* to a value in *dom(X)* such that all constraints are jointly *satisfied*
  - A CSP can have zero, one, or several solutions
  - Often, suitable restrictions of the domains of variables intended to include at least one solution are sufficient (*satisfiability testing*)

CSPs (IV)

- For example, consider the following CSP
  $V = \{ X_1, X_2 \}$
  $\text{dom}(X_1) = [1..100], \text{dom}(X_2) = \text{dom}(X_1)$
  $X_1 < 5, X_1 + X_2 = 10$

- The first constraint ensures $\text{dom}(X_1) = \{ 1, 2, 3, 4 \}$
  But, for the second constratint $X_2 = 10 - X_1$
  And therefore, $\text{dom}(X_2) = \{ 6, 7, 8, 9 \}$

- The CSP has 4x4=16 possible solutions
  $(X_1=1,X_2=6)$, $(X_1=1,X_2=7)$, $(X_1=1,X_2=8)$, $(X_1=1,X_2=9)$
  $(X_1=2,X_2=6)$, $(X_1=2,X_2=7)$, $(X_1=2,X_2=8)$, $(X_1=2,X_2=9)$
  …

Solutions and Assignments

- An *assignment* is a mapping that associates some variables of the CSP with values taken from respective domains

- An assignment is
  - *Partial*, when it does not include all variables
  - *Inconsistent*, when at least one constraint is not satisfied

- A solution is a *complete* and *consistent* assignment
  - Therefore, solutions can be found by extending assignments in search for complete and consistent assignments

Types of CSPs

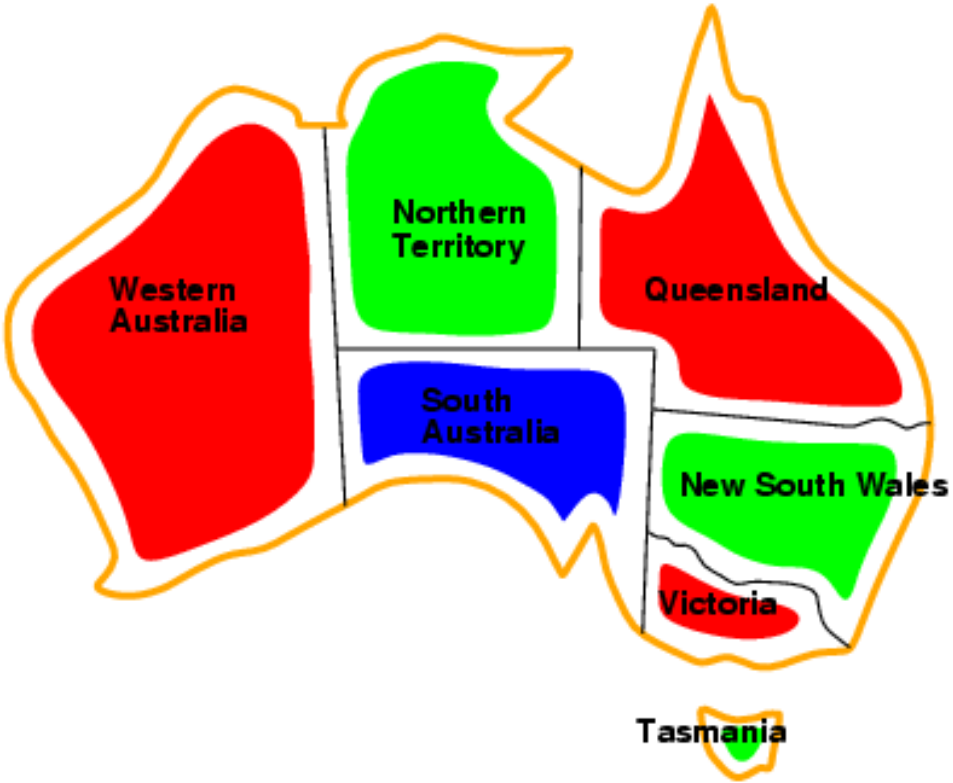
- CSPs with *discrete* domains
  - *Finite* domains with cardinality less that or equal to *d*
    - *n* variables, $O(d^n)$ complete assignments
  - *Countably infinite* domains
    - Natural numbers, integers, strings, …
- CSPs with *continuous* (*uncountably infinite*) domains
  - Real numbers, complex numbers, …
  - Constraints are normally expressed in terms of equations, inequalities, …

Map Coloring (I)

- Variables
  - WA, NT, Q, NSW, V, SA, T
- One domain
  - D = { R, G, B }
- Constraints
  - Neighboring zones must have different colors
  - WA ≠ NT, WA ≠ SA,
    NT ≠ SA, NT ≠ Q,
    SA ≠ Q, …

Map Coloring (II)

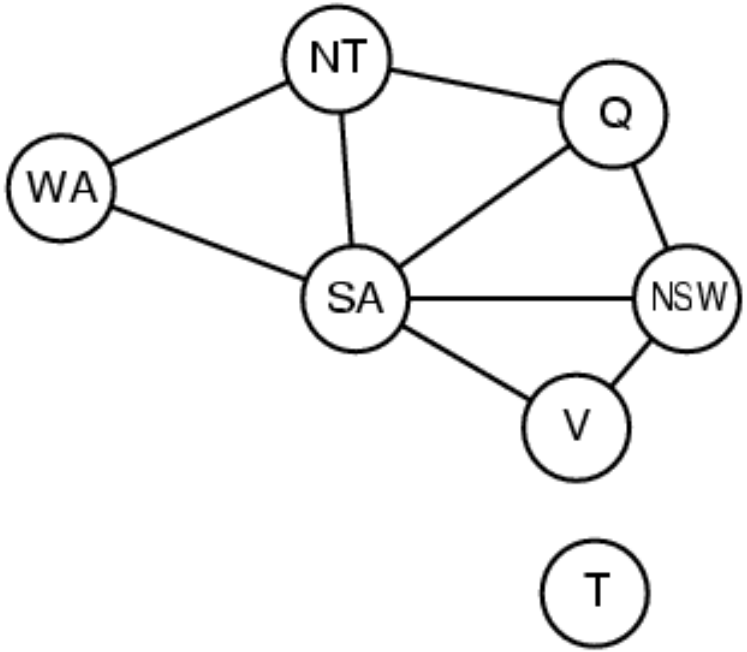
- Variables
  - WA, NT, Q, NSW, V, SA, T
- One domain
  - D = { R, G, B }
- Constraints
  - Neighboring zones must have different colors
  - WA ≠ NT, WA ≠ SA,
    NT ≠ SA, NT ≠ Q,
    SA ≠ Q, …



Constraint Graph (I)

- A CSP is
  - *Unary,* if it has only unary constraints
  - *Binary,* if it has unary and binary constraints

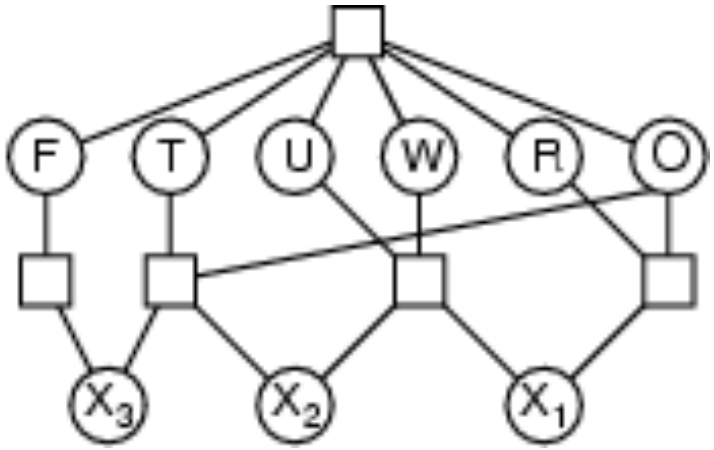
- Given a binary CSP, it is always possible to define a corresponding constraint graph
  - It is an undirected graph
  - The nodes are the variables
  - The arcs are the constraints



Constraint Graph (II)

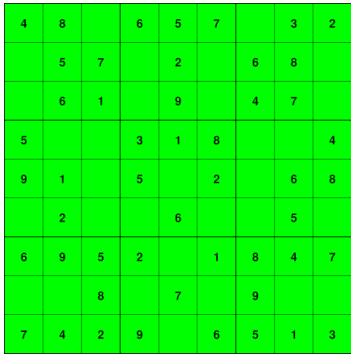
- Variables
  - $T\ W\ O\ F\ U\ R$
  - $X_1\ X_2\ X_3$
- One domain
  - D = [0..9]
- Constraints
  - $O + O = R + 10\ X_1$
  - $X_1 + W + W = U + 10\ X_2$
  - $X_2 + T + T = O + 10\ X_3$
  - $X_3 = F,\ T \neq 0,\ F \neq 0$
  - *alldifferent*(F,T,U,W,R,O)

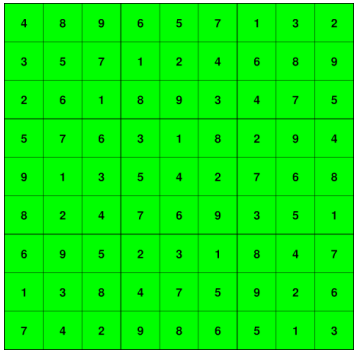
```
  T W O
+ T W O
-------
F O U R
```



A Real-World Case: Sudoku Puzzles

- Variables
  - The 81 cells in the grid
- One domain
  - D = [1..9]
- Constraints
  - Values in each row do not repeat
  - Values in each column do not repeat
  - Values in each 3x3 block do not repeat
  - Some values are fixed





Real-World CSPs

- Assignment problems
  - For example, who teaches what class?
- Timetabling problems
  - For example, which class is offered when and where?
- Hardware configuration
- Transportation scheduling
- Factory scheduling

- Note that many real-world problems involve real-valued variables and are associated with optimization criteria

Generate & Test (I)

- A CSP can be solved searching for at least one complete and consistent assignment

- *Generate & Test (G&T)* algorithm
  - All partial assignments are generated until all complete assignments are found
  - Each complete assignment is tested for consistency

- Normally, the G&T algorithm is the least efficient algorithm because it tests for consistency very late (only for complete assignments)

Generate & Test (II)

```
void solveGT(int index) {
    if(index == n) {
        if(areConstraintNotViolated())
            solutionFound();
    } else {
        if(variable[index] != FREE)
            solveGT(index + 1);
        else {
            for(int i = 0; i < d; i++) {
                variable[index] = dom[i];

                solveGT(index + 1);
            }

            variable[index] = FREE;
        }
    }
}
```

Standard Backtracking (I)

- *Standard BackTracking (SBT)* algorithm
  - All partial assignments are possibly (not necessarily) generated
  - Partial assignments are tested for consistency
    - Inconsistencies can be tested even if only some variables are assigned
  - A solution is found as soon as a complete assignment is found
    - The test for consistency is coupled with the assignment of free variables

Standard Backtracking (II)

```
void solveSBT(int index) {
    if(index == n)
        solutionFound();
    else {
        if(variable[index] != FREE)
            solveSBT(index + 1);
        else {
            for(int i = 0; i < d; i++) {
                variable[index] = dom[i];

                if(areConstraintNotViolated())
                    solveSBT(index + 1);
            }

            variable[index] = FREE;
        }
    }
}
```

Standard Backtracking (III)

- If the CSP has *n* variables, the solutions are at depth *n* in the *search tree*
  - A *depth-first search (children nodes are explored before the parent node)* can be used
  - Depth-first search uses linear memory usage
  - Depth-first search terminates if the tree is finite

- Each node of the search tree corresponds to the assignment of one variable
  - *Branching factor b=d*, where *d* is the cardinality of domains
  - Therefore, the tree has d^n leafs

Standard Backtracking (IV)

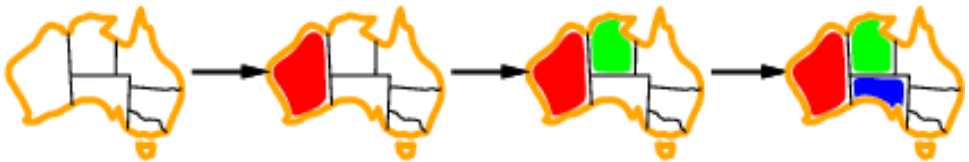
```
function BACKTRACKING-SEARCH( csp) returns a solution, or failure
    return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING( assignment,csp) returns a solution, or
failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE( Variables[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment according to Constraints[csp] then
            add { var = value } to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failue then return result
            remove { var = value } from assignment
    return failure
```

Search Heuristics

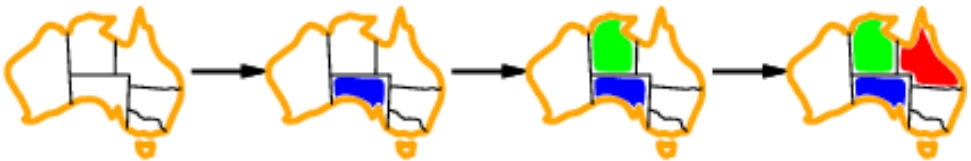
- The effectiveness of SBT can be improved using general-purpose search heuristics to
  - Choose which variable to use in next assignment
    - Function `SELECT-UNASSIGNED-VARIABLE`
  - Choose which value to use in next assignment
    - Function `ORDER-DOMAIN-VALUES`
  - Identify inconsistencies as soon as possible
    - The sooner inconsistencies are identified, the better it is
  - Choose which variable to set free in case of backtracking
    - These heuristics are not treated in this course, and only *chronological backtracking* is studied

Which Variable to Assign Next

- *Choose the variable with less values in the domain (min-domain variable heuristic)*

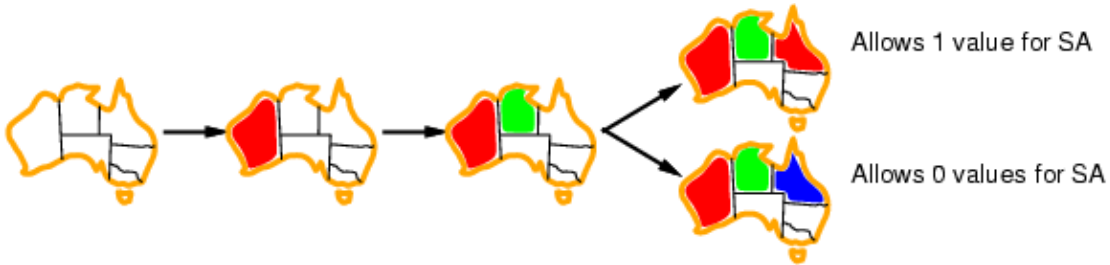


- *Choose the variable involved in more constraints (max-degree variable)*



Which Value to Assign Next

- Normally, min-domain variables are considered and one of the max-degree variables among them is finally chosen
- *Once a variable is chosen for the next assignment, the value that violates the fewest constraints is taken (min-conflict value)*



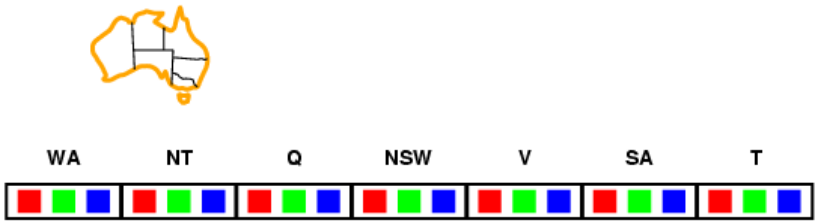
Constraint Propagation

- Given a CSP, the domains of variables can be reduced by reasoning on constraints
  - The process of reasoning on constraints to reduce (*filter*) the domains of variables is called *constraint propagation*

- Constraints can be propagated
  - At the beginning of the search to reduce the search space and better apply heuristics
  - After the assignment of a value to a variable to reduce the search space and better apply heuristics
  - Actually, the assignment of a value to a variable is nothing but the addition of a new constraints that can be propagated

Forward Checking (I)

- The assignment of a value to a variable *X* can be used to remove inconsistent values from the domains of all variables that share constraints with *X*



- Backtracking occurs when the domain of a variable reduces to the empty set because at least a constraint is violated

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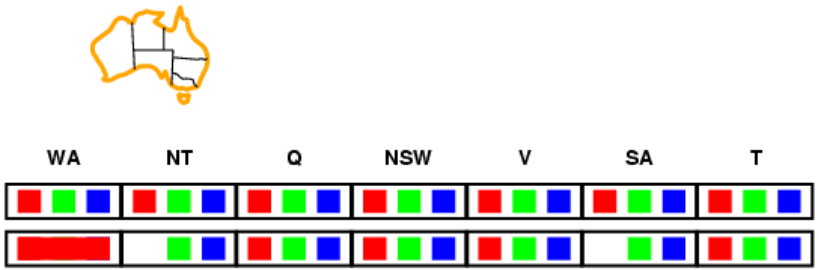

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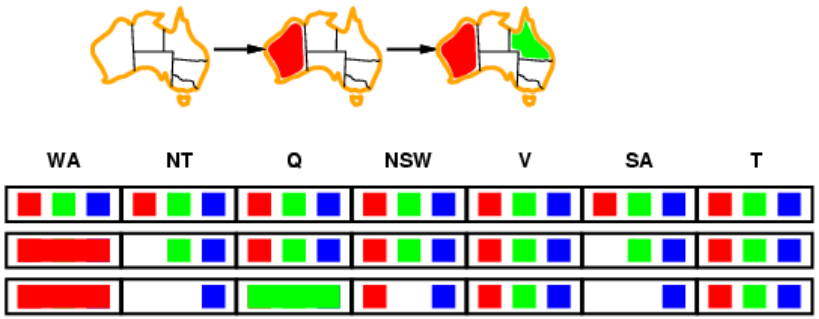

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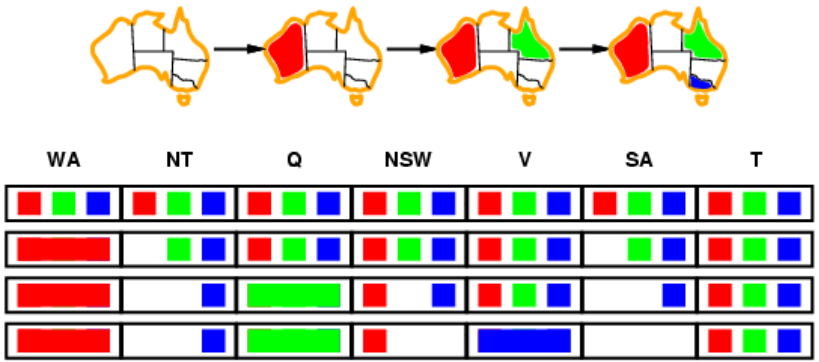
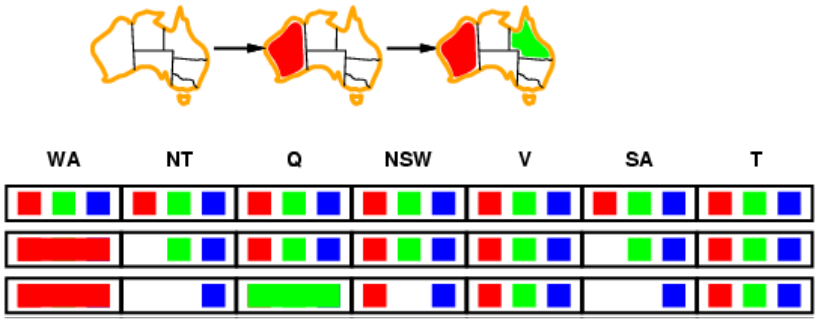


- Backtracking occurs when the domain of a variable reduces to the empty set because at least a constraint is violated

Forward Checking (V)

- Forward checking is a simple constraint propagation algorithm that is not able to capture all inconsistencies
  - For example, NT and SA cannot be both colored blue



Arc Consistency (I)

- Forward checking propagates constraints only from the last assigned variable *X* to the free variables that share constraints with *X*
  - It does not further propagate the reduction of the domains to other variables
- Note that the reduction of the domain of a variable can be also considered as an added constraint
  - For example, if *v* is removed from the domain of *X*, then the added constraint is $X \neq v$
  - The *arc consistency algorithm* can be used to further propagate constraints

Arc Consistency (II)

- Note that
  - Given a generic CSP, an equivalent binary CSP can be found
  - Given a binary CSP, unary constraints can be enforced and removed
  - Therefore, a generic CSP can be always associated with a constraint graph that has no self-loops
- A binary CSP is called *arc consistent* if the domains of variables ensure that all arcs in the constraint graph are consistent
  - The directed arc $X \rightarrow Y$ is consistent if and only if for all values $x \in \text{dom}(X)$ there exists a value $y \in \text{dom}(Y)$ such that the constraint is satisfied
  - The undirected arc $X \text{—} Y$ is consistent if and only if $X \rightarrow Y$ and $Y \rightarrow X$ are both consistent

Arc Consistency (III)

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

  while *queue* is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if RM-INCONSISTENT-VALUES(X_i, X_j) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add (X_k, X_i) to *queue*

---

function RM-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff remove a value
  removed $\leftarrow$ *false*
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy constraint(X_i, X_j)
      then delete x from DOMAIN[X_i]; *removed* $\leftarrow$ *true*
  return *removed*

Arc Consistency (IV)

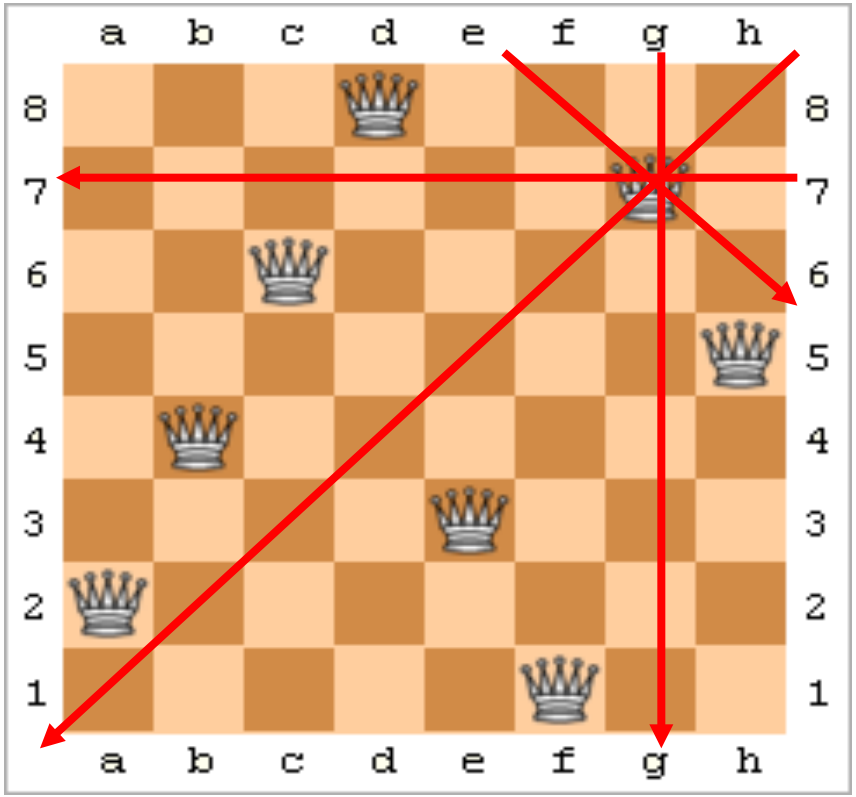
- The *MAC* (*Maintaining Arc Consistency*) algorithm is a variant of the SBT algorithm that enforces arc consistency before every assignment
  - It can use one of the several arc consistency algorithms available in the literature
  - It can benefit from appropriate search heuristics
- However, note that arc consistency does not capture all inconsistencies
  - It considers only binary constraints (in turns)
  - It can be generalized to *path consistency* to better capture inconsistencies

Parametric CSPs

- Parametric CSPs can be used to study the performance of solving algorithms
  - The number of variables of the CSP should be allowed to increase
  - The number of values in the domains of variables should be allowed to increase
  - The size of the CSP (number of variables and number of values in domains) should be controlled by a single parameter

- Parametric CSPs allows studying the asymptotic performance of studied algorithms both in terms of time and memory

n-Queens (I)



- A problem popularized by
  - *Carl Friedrich Gauss* (1777-1855) for *n*=8
  - *Edsger Dijkstra* (1930-2002), which used it to describe the SBT algorithm
- Given an *n*x*n* chessboard, the problem is to position *n* queens such that they do not lay
  - In the same row
  - In the same column
  - In the same diagonal

n-Queens (II)

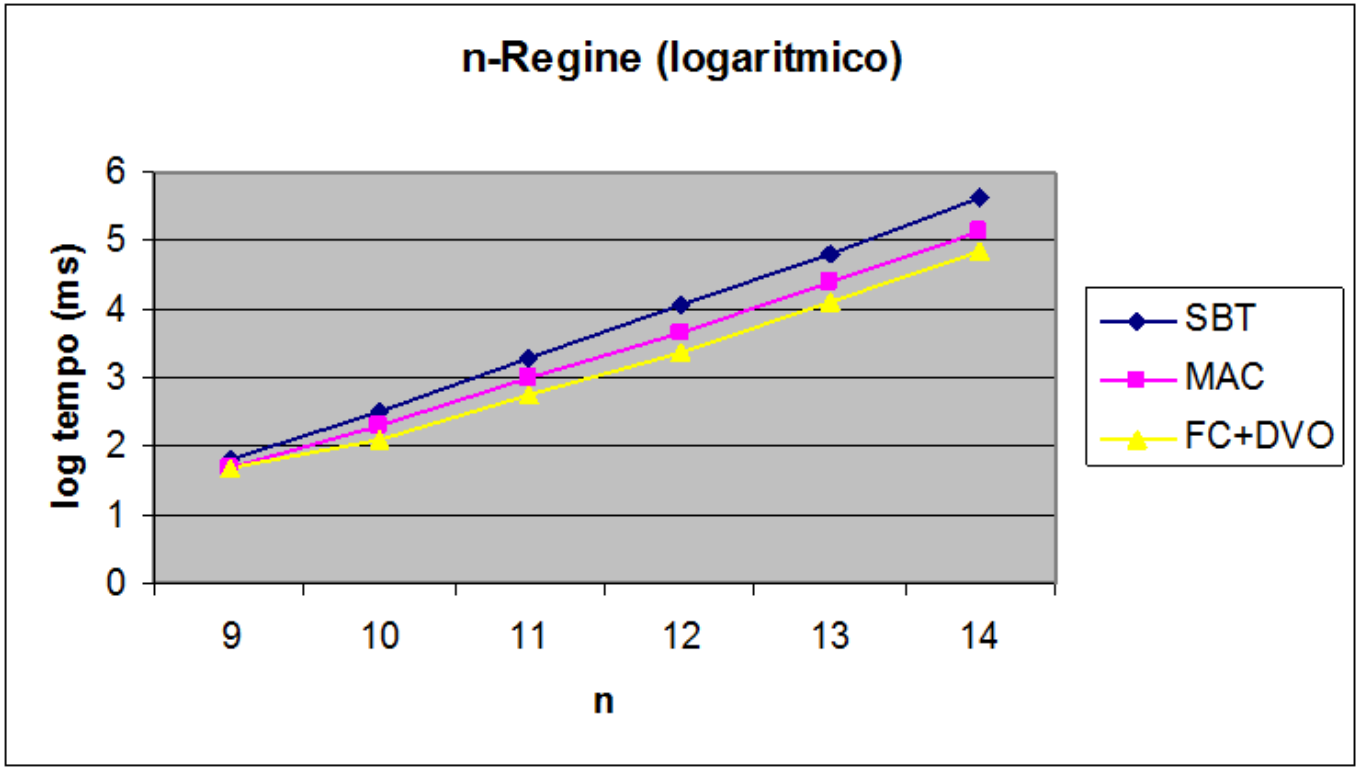
n	# solutions
4	2
5	10
6	4
7	40
8	92
9	352
10	724
11	2,680
12	14,200
13	73,712
14	365,596
15	2,279,184

- The problem is not easy
  - The number of valid arrangements of the queens increases rapidly with *n*
  - But, for example, even if for *n*=8 there are 16,777,216 possible arrangements, only 92 of them are solutions
- Note that the *n*-queens problem is classic, but it is no longer considered representative of complex real-world problems

n-Queens (III)

n	G&T (ms)	SBT (ms)	MAC (ms)	FC+DVO (ms)
4	0	0	0	0
5	0	0	0	16
6	32	0	0	31
7	625	0	0	32
8	15.188	0	15	31
9	321.828	63	47	47
10	> 1 ora	328	204	125
11		1.953	984	547
12		12.016	4.391	2.328
13		62.172	24.219	12.547
14		427.906	143.203	72.047

n-Queens (IV)


n-Regine (logaritmico)
log tempo (ms)
0
1
2
3
4
5
6
9
10
11
12
13
14
n
SBT
MAC
FC+DVO

MiniZinc (I)

- MiniZinc is a language for the description of CSPs
  - It is independent of the adopted solving algorithm
  - It supports CSPs over the integers, the reals, the Booleans, and even over sets of them
- MiniZinc is a language of a family of languages that include
  - *Zinc*, which is a higher level language that can be effectively translated to MiniZinc
  - *FlatZinc*, which is minimalistic and used as a lower level alternative to MiniZinc
    - Note that MiniZinc is normally translated to FlatZinc

MiniZinc (II)

- The parts of a simple MiniZinc program are
  - Parameters of the CSP (can be overridden before execution)
    - `int: nc = 3;`
  - Variables of the CSP and their domains
    - `var 1..nc: wa;`
  - Constraints of the CSP
    - `constraint wa != nt;`
  - Type of problem (and optimization criteria, if needed)
    - `solve satisfy;`
    - `solve maximize 400*b + 450*c;`
  - Output procedure
    - `output ["wa=", show(wa)];`

Map Coloring in MiniZinc

```
% Map coloring
int: nc = 3;
var 1..nc: wa;
var 1..nc: nt;
…
constraint wa != nt;
…
solve satisfy;
output ["wa=", show(wa), …];
```

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