ESERCIZI ESAME

giovedì 26 ottobre 2023 14:13

$$\boxed{R(\underbrace{(a,b)}_{x},\underbrace{(c,d)}_{y}) \iff ad=cb} \qquad \boxed{\frac{a}{b}=\frac{c}{d}}$$

RIFLESSIVA: $R(x,x)$

$$R((a,b),(a,b)) \iff ab=ab \quad \checkmark \qquad \frac{a}{b}=\frac{a}{b} \quad \checkmark$$

SIMMETRICA

$$\underbrace{R(x,y)}\dashrightarrow \;\Rightarrow R(y,x)$$

$$R((a,b),(c,d)) \iff ad=cb \iff \qquad \frac{a}{b}=\frac{c}{d} \iff \frac{c}{d}=\frac{a}{b}$$

$$cb=ad \iff R((c,d),(a,b)) \iff R(y,x) \quad \checkmark$$

TRANSITIVA $\quad R(x,y)\wedge R(y,z) \Rightarrow R(x,z)$

$x=(a,b) \qquad z=(e,f)$

$y=(c,d)$

$$R((a,b),(c,d))\wedge R((c,d),(e,f)) \iff \underbrace{ad=cb \wedge cf=ed}_{\star}$$

$$\overset{?}{\Rightarrow} \quad \underbrace{af=eb}_{TH} \iff R((a,b),(e,f)) \iff R(x,z)$$

$$\star \begin{cases} ad=cb \\ cf=ed \end{cases} \qquad \begin{cases} a\cancel{d}=\dfrac{e\cancel{d}}{f}\cdot b \\ c=\dfrac{ed}{f} \end{cases} \Rightarrow af=eb \quad \checkmark$$

$$\frac{a}{b}=\frac{c}{d} \wedge \frac{c}{d}=\frac{e}{f} \Rightarrow \frac{a}{b}=\frac{e}{f} \quad \checkmark$$

ESERCIZIO 2.

$$D = \{2, 4, 6, 10, 12, 20, 30, 60\}$$

$$R(x,y) \iff x \mid y \iff \exists m \in \mathbb{Z} : y = mx$$

RIFLESSIVA: $R(x,x) \iff x \mid x \iff \exists m \in \mathbb{Z} : x = \overset{m=1}{m}x$ ✓

TRANSITIVA: $R(x,y) \wedge R(y,z) \Rightarrow R(x,z)$

$$x \mid y \wedge y \mid z \iff \exists m,q \in \mathbb{Z} : y = mx \wedge z = qy$$

$$\begin{cases} y = mx \\ z = q \cdot y = q \cdot (mx) = \underbrace{(q \cdot m)}_{\in \mathbb{Z}} \cdot x \end{cases} \qquad (q \cdot m) = j$$

$$\overset{\exists j \in \mathbb{Z}:}{\Rightarrow} z = jx \iff x \mid z$$

$$\iff R(x,z) \ ✓$$

ANTISIMMETRICA: $R(x,y) \wedge R(y,x) \Rightarrow x = y$

$$x \mid y \wedge y \mid x \iff \exists m,q \in \mathbb{Z} : y = mx \wedge x = qy$$

$$\begin{cases} y = mx \\ x = qy = q(mx) = (q \cdot m) \cdot x \end{cases}$$

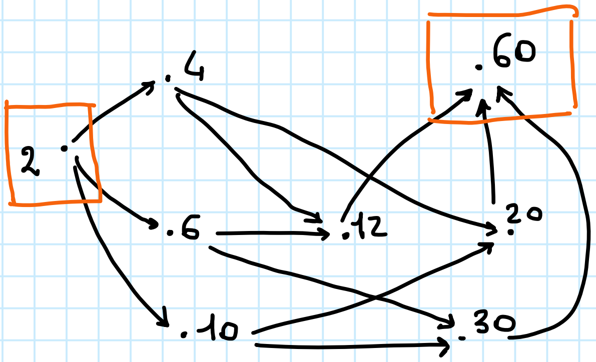
$$x = \underbrace{(q \cdot m)}_{} x$$

$$q \cdot m = 1$$

$q = m = 1 \rightarrow x = y$ ✓

$q = m = -1 \rightarrow x = -y$ non possibile $x, y \in D$

$$D = \{2, 4, 6, 10, 12, 20, 30, 60\}$$



R è un ordine PARZIALE

$$4 \nmid 6 \wedge 6 \nmid 4$$

Ha minimali e massimali ?

$$\downarrow \qquad\qquad \downarrow$$

$$2 \qquad\qquad 60$$

ESERCIZIO 3.

$$P(n): \boxed{\sum_{i=0}^{n} \left(\frac{1}{2}\right)^i = \frac{2^{n+1}-1}{2^n}}$$

$$\left[\sum_{i=0}^{n} q^i = \frac{1-q^{n+1}}{1-q} \qquad q = \frac{1}{2} \right.$$

Dim:

$\longrightarrow$ CASO BASE : $n = 0$

$$\sum_{i=0}^{0} \left(\frac{1}{2}\right)^i = \left(\frac{1}{2}\right)^0 = 1 \overset{\checkmark}{=} \frac{2^{0+1}-1}{2^0} = \frac{2^{n+1}-1}{2^n}$$

$\longrightarrow$ PASSO INDUTTIVO:

$$P(n) \Longrightarrow P(n+1)$$

$$\underbrace{\sum_{i=0}^{n} \left(\frac{1}{2}\right)^i = \frac{2^{n+1}-1}{2^n}}_{HP} \boxed{\Longrightarrow} \sum_{i=0}^{n+1} \left(\frac{1}{2}\right)^i \overset{?}{=} \underbrace{\frac{2^{n+2}-1}{2^{n+1}}}_{*}$$

$$\sum_{i=0}^{n+1} \left(\frac{1}{2}\right)^i = \sum_{i=0}^{n} \left(\frac{1}{2}\right)^i + \underbrace{\left(\frac{1}{2}\right)^{n+1}}_{i=n+1} \overset{HP}{=} \frac{2^{n+1}-1}{2^n} + \frac{1}{2^{n+1}} =$$

$$= \frac{2(2^{n+1}-1)+1}{2^{n+1}} = \frac{2^{n+2}-2+1}{2^{n+1}} = \left.\frac{2^{n+2}-1}{2^{n+1}}\right] * \quad \square$$

ESERCIZIO 1.

$$R(x,y) \iff \exists k \in \mathbb{Z} : x - y = k$$

$$\iff x - y \in \mathbb{Z}$$

RIFLESSIVA: $R(x,x) \iff \exists k \in \mathbb{Z} . \ x - x = k$

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sì perché $x - x = 0$ e $0 \in \mathbb{Z}$.

SIMMETRICA: $R(x,y) \Rightarrow R(y,x)$

$$\exists k \in \mathbb{Z} : x - y = k \Rightarrow y - x = \underbrace{-k}_{\in \mathbb{Z}} \Rightarrow R(y,x)$$

TRANSITIVA. $R(x,y) \wedge R(y,z) \Rightarrow R(x,z)$

$$\exists k, j \in \mathbb{Z} : x - y = k \ \wedge\ y - z = j \iff \begin{cases} x - y = k \Rightarrow x = k + j + z \\ y - z = j \Rightarrow y = j + z \end{cases}$$

$$x - z = \underbrace{k + j}_{\in \mathbb{Z}} \iff \exists q \in \mathbb{Z} : x - z = q \iff R(x,z) \ \checkmark$$

ESERCIZIO 2.

$$D_{16} = \{1, 2, 4, 8, 16\}$$

R è d'ordine   HW: dim RIFL., ANTI-SIMM e TRANSITIVA

$$\boxed{1} \to 2 \to 4 \to 8 \to \boxed{16}$$

minimale   massimale

R è di ordine TOTALE

ESERCIZIO 3.

$$\sum_{i=1}^{n} (2i + 3) = ?$$

$$\boxed{\sum_{i=1}^{n} (2i+3)} = \sum_{i=1}^{n} 2i + \sum_{i=1}^{n} 3 = 2 \cdot \sum_{i=1}^{n} i + 3 \sum_{i=1}^{n} 1 =$$

$$= \cancel{2} \cdot \frac{n(n+1)}{\cancel{2}} + 3 \cdot n = \boxed{n(n+4)} \quad P(n)$$

Dim.

$\hookrightarrow$ CASO BASE: $n = 1$

$$\sum_{i=1}^{1}(2i+3) = 2\cdot 1+3 = 5 \overset{\checkmark}{=} 1(1+4) = n(n+4)$$

↪ PASSO INDUTTIVO:

$$P(n) \implies P(n+1)$$

$$\underbrace{\sum_{i=1}^{n}(2i+3) = n(n+4)}_{HP} \boxed{\implies} \underbrace{\sum_{i=1}^{n+1}(2i+3) \overset{?}{=} (n+1)(n+5)}_{\star}$$

$$\sum_{i=1}^{n+1}(2i+3) = \sum_{i=1}^{n}(2i+3) + \underbrace{2(n+1)+3}_{i=n+1} \overset{HP}{=} n(n+4)+2(n+1)+3$$

$$= n^2 + \underbrace{4n+2n}_{6n} + \underbrace{2+3}_{5} = n^2 + \underbrace{5n+n}_{6n} + 5 = \underbrace{(n+1)(n+5)}_{\star} \quad \square$$

TOTALE: 27 PUNTI

16/27 sufficienza

Prova I + Prova II } MEDIA ⟶ VOTO "finale" ≥ 18

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