CORREZIONE PROVA PARZIALE

martedì 7 novembre 2023 14:59

I TURNO

1) R relazione su $\mathbb{N}$

$$\forall x, y \in \mathbb{N} : R(x,y) \iff x \bmod 5 = y \bmod 5$$

R è di equivalenza?

$27 \bmod 5 = 2$

- RIFLESSIVA

$\boxed{\forall x \in \mathbb{N}} : R(x,x)$?

$R(x,x) \iff x \bmod 5 = x \bmod 5$ ✓

- SIMMETRICA

$\boxed{\forall x, y \in \mathbb{N}} : R(x,y) \Rightarrow R(y,x)$?

$R(x,y) \iff x \bmod 5 = y \bmod 5 \overset{\Rightarrow}{\iff} y \bmod 5 = x \bmod 5$
$\iff R(y,x)$

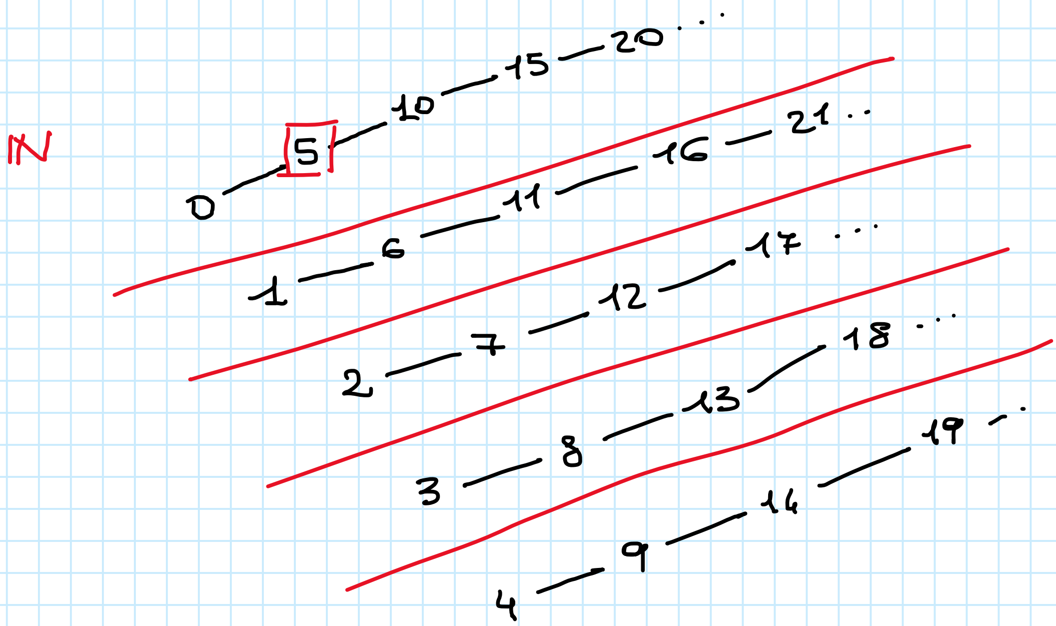
- TRANSITIVA

$\boxed{\forall x, y, z \in \mathbb{N}} : (R(x,y) \wedge R(y,z)) \Rightarrow R(x,z)$?

$R(x,y) \wedge R(y,z) \iff (x \bmod 5 = y \bmod 5) \wedge$
$(y \bmod 5 = z \bmod 5)$

$\Rightarrow x \bmod 5 = z \bmod 5 \iff R(x,z)$

Sì R è di equivalenza.
Classi di equivalenza? 5



2) $D_{63} = \{1, 3, 7, 9, 21, 63\}$

$\forall x, y \in D_{63} : R(x,y) \iff x \mid y$

$\exists m \in \mathbb{Z} \quad y = mx$

R è d'ordine?

- RIFLESSIVA

$\forall x \in D_{63} : R(x,x)$?

$R(x,x) \iff x \mid x \iff \exists m \in \mathbb{Z} : x = mx$?

$m = 1 \qquad x = x \quad \checkmark$

- ANTI-SIMMETRICA

$\forall x, y \in D_{63} : (R(x,y) \wedge R(y,x)) \Rightarrow x = y$?

$R(x,y) \wedge R(y,x) \iff x \mid y \wedge y \mid x \iff$

$(\exists m \in \mathbb{Z} : y = mx) \wedge (\exists q \in \mathbb{Z} : x = qy) \iff$

$\begin{cases} y = mx \\ x = qy \end{cases} \iff \Big\{ \; y = m \cdot (qy) \iff y = (m \cdot q) y$

$\Rightarrow \quad m \cdot q = 1 \qquad m, q \in \mathbb{Z} \qquad m = \frac{1}{q}$

$\Rightarrow \quad m \cdot q = 1 \qquad m, q \in \mathbb{Z} \qquad m = \frac{1}{q}$

$m = q = 1$ $\qquad$ $m = q = -1$

NON accettabile

$$\Rightarrow \begin{cases} y = 1 \cdot x \\ x = 1 \cdot y \end{cases} \Rightarrow \quad x = y$$

· TRANSITIVA

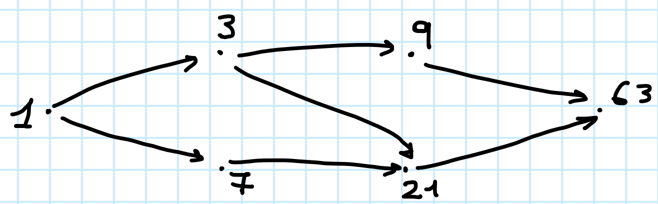
$$\forall x, y, z \in D_{63} : (R(x,y) \wedge R(y,z)) \Rightarrow R(x,z) \ ?$$

$$R(x,y) \wedge R(y,z) \Leftrightarrow x | y \wedge y | z \Leftrightarrow$$

$$(\exists m \in \mathbb{Z} : y = m x) \wedge (\exists q \in \mathbb{Z} \cdot z = q \cdot y) \Leftrightarrow$$

$$\begin{cases} y = m x \\ z = q y \end{cases} \quad \begin{cases} \\ z = q(m x) \end{cases} \Rightarrow z = \underbrace{(q \cdot m)}_{\in \mathbb{Z}} x \Leftrightarrow R(x,z)$$

R è d'ordine.



R è parziale. CONTROESEMPIO $3 \nmid 7 \wedge 7 \nmid 3$

$\Rightarrow$ non è TOTALE

$$(\exists x, y \in D_{63} : x \nmid y \wedge y \nmid x)$$

R ha elementi MINIMALI? sì: 1 (è anche minimo)

R ha elementi massimali? sì: 63 (è anche massimo)

3) $\sum_{i=1}^{m}(6i^2+2i) = ?$

$$\boxed{\sum_{i=1}^{m}(6i^2+2i)} = 6\cdot\sum_{i=1}^{m} i^2 + 2\sum_{i=1}^{m} i =$$

$$= \cancel{6}\cdot\frac{m(m+1)(2m+1)}{\cancel{6}} + \cancel{2}\cdot\frac{m(m+1)}{\cancel{2}} =$$

$$= m(m+1)(2m+2) = \boxed{2m(m+1)^2}$$

Dim:

CASO BASE: $m=1$

$$\sum_{i=1}^{1}(6i^2+2i) = 6\cdot 1^2+2\,1 = 8 = 2\cdot 1(1+1)^2 = 2m(m+1)^2 \quad \checkmark$$

PASSO INDUTTIVO:

$$\boxed{P(m)}^{HP} \Longrightarrow \boxed{P(m+1)}\ ?$$

$$\sum_{i=1}^{m+1}(6i^2+2i) = \sum_{i=1}^{m}(6i^2+2i) + 6(m+1)^2 + 2(m+1) =$$

$$\overset{HP}{=} 2m(m+1)^2 + 6(m+1)^2 + 2(m+1) =$$

$$= 2(m+1)[m(m+1)+3(m+1)+1] =$$

$$= 2(m+1)[m^2+\underbrace{m+3m}_{4m}+\underbrace{3+1}_{4}] = 2(m+1)(m+2)^2$$

$$\downarrow$$

$$2(m+1)(m+2)^2$$

II TURNO

1) $\forall x,y \in \mathbb{N} : R(x,y) \iff \exists k \in \mathbb{Z} : x - y = 5k$

$$\begin{cases} x = 5 \cdot q + r \\ y = 5 \cdot q' + r' \end{cases}$$

$$x - y = 5(\underbrace{q - q'}_{\in \mathbb{Z}})$$

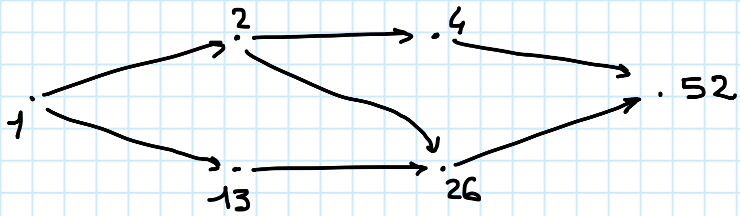
$R(x,y) \iff r = r'$

$\Updownarrow$

$x \bmod 5 = y \bmod 5$

2) $D_{52} = \{1, 2, 4, 13, 26, 52\}$

$R(x,y) \iff x | y$



R è parziale. Minimale: 1

Massimale: 52

3) $\sum_{i=1}^{n} (3^i + i) = ?$

$$\sum_{i=1}^{n} 3^i = -3^0 + \sum_{i=0}^{n} 3^i = \frac{1 - 3^{n+1}}{-2} - 3^0 = \frac{3^{n+1} - 1}{2} - 1$$

$$\|$$

$$3^0 + \underbrace{(3^1 + 3^2 + \dots + 3^n)}$$

$$\|$$

$$3^0 + \sum_{i=1}^{n} 3^i$$

$$\boxed{\sum_{i=1}^{n}(3^i+i)} = \sum_{i=1}^{n} 3^i + \sum_{i=1}^{n} i = \frac{3^{n+1}-1}{2} - 1 + \frac{n(n+1)}{2}$$

$$= \frac{3^{n+1}-1-2+n(n+1)}{2} = \boxed{\frac{3^{n+1}-3+n(n+1)}{2}}$$

Dim.:

CASO BASE : $n=1$

$$\sum_{i=1}^{1}(3^i+i) = 3^1+1 = 4 \longleftarrow 4 = \frac{3^2-3+2}{2}$$

PASSO INDUTTIVO:

$$P(n) \Longrightarrow P(n+1)$$

$$\boxed{\sum_{i=1}^{n+1}(3^i+i)} = \sum_{i=1}^{n}(3^i+i) + 3^{n+1} + (n+1) =$$

$$\underset{HP}{=} \frac{3^{n+1}-3+n(n+1)}{2} + 3^{n+1} + (n+1) =$$

$$= \frac{3^{n+1}-3+n(n+1)+2\cdot 3^{n+1}+2\cdot(n+1)}{2} =$$

$$= \frac{3^{n+1}\overbrace{(1+2)}^{3} - 3 + (n+1)(n+2)}{2} =$$

$$= \boxed{\frac{3^{n+2}-3+(n+1)(n+2)}{2}}$$