

Rank of matrix

if simple elements haru cha vani find garnai lai turn matrix to echelon form ani tespachi aaucha

if number haru complex cha calculate garnai time lagcha vani

use calc to find determinant of 3

if $d_3 = 0$, d_3 not equal = vaye rank is 3

find d_2 , if $d_2 = 0$ find every d_2 inside matrix, rank 2

Topics for Linear Algebra

1. Transformation
2. Rank, Spaces and Span
3. Orthogonality & Projections
4. SVD and Eigen Decomposition
5. Least Squares

Projection of vector b onto vector a :

$$\text{proj}_a b = \frac{a \cdot b}{a \cdot a} a$$

- This formula gives you the vector projection of b onto a .
- The result is a vector in the direction of a .

Suppose we have:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

We want to solve:

$$Ax = b$$

But this system is overdetermined (3 equations, 2 unknowns).

So we solve:

$$A^T A x = A^T b$$

Which gives us the least squares solution.

$$A^T A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

Step 2c: Solve the normal equation:

$$\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$y = 1.5x - 1.33$$

Find the Eigen vector of the matrix $A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$.
Corresponding to the eigen value $\lambda = 3$.

Add row

$$\begin{bmatrix} 3-4+4 \\ 1-2+4 \\ 1-1+3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

jaile trace sangai milcha vanne hunna kaile kai add row

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Which of the following is an eigenvalue of A , and a corresponding eigenvector?

- A. Eigenvalue = 5, Eigenvector = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- B. Eigenvalue = 6, Eigenvector = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- C. Eigenvalue = 2, Eigenvector = $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- D. Eigenvalue = 7, Eigenvector = $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Eigen values

The eigen values of Matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

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sum of eigen value = sum of trace (diagonal)

- (a) 6, 0
- ✓ (b) 6, 1
- (c) 3, 2
- (d) 1, 2

Given $[A]_{3 \times 3} \rightarrow$ Eigen value?

$$\lambda^3 - (\text{Trace of } A)\lambda^2 + (\text{Sum of principle diagonal minors})\lambda - |A| = 0$$

Ex:- $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$$\lambda^3 - (18)\lambda^2 + (5 + 20 + 20)\lambda - 0 = 0$$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\lambda = 0, 3, 15$$