## sng-team-assignment

## April 3, 2024

```
[]: #Our team decided to use as a bonus other variables, for example we took (asu
      →target) Real Personal Income (and as xvars) IP: Financial Products and
      →Nonindustrial Supplies, IP: Consumer Goods
[]: # Installing packages
     import pandas as pd
     from numpy.linalg import solve
     import numpy as np
[]: # Setting directory to the csv file
     from google.colab import drive
     drive.mount('/content/drive')
     SNG_Team = '/content/drive/My Drive/comput/current.csv'
     # Loading the dataframe
     df = pd.read_csv(SNG_Team)
     df cleaned = df.drop(index=0)
     df_cleaned.reset_index(drop=True, inplace=True)
     df_cleaned['sasdate'] = pd.to_datetime(df_cleaned['sasdate'], format='\m/\%d/\%Y')
     df cleaned
```

Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force\_remount=True).

```
[]:
                          RPI W875RX1 DPCERA3M086SBEA
                                                           CMRMTSPLx
           sasdate
        1959-01-01
                     2583.560
                                2426.0
                                                15.188 2.766768e+05
    1
        1959-02-01
                     2593.596
                                2434.8
                                                15.346 2.787140e+05
    2
        1959-03-01
                     2610.396
                                2452.7
                                                15.491 2.777753e+05
    3
        1959-04-01 2627.446
                                2470.0
                                                15.435 2.833627e+05
        1959-05-01
                     2642.720
                                2486.4
                                                15.622 2.853072e+05
    776 2023-09-01 19111.748 15741.9
                                               116.594 1.507530e+06
    777 2023-10-01 19145.402 15784.6
                                               116.663 1.505477e+06
    778 2023-11-01 19213.108
                              15859.9
                                               117.127 1.514733e+06
    779 2023-12-01 19251.946 15899.0
                                                117.773 1.530296e+06
    780 2024-01-01 19377.558 15948.8
                                                117.639
                                                                 NaN
```

```
INDPRO
                                IPFPNSS
                                           IPFINAL
                                                      IPCONGD
          RETAILx
0
      18235.77392
                     21.9665
                                           22.2688
                                                      31.7011
                                23.3891
1
      18369.56308
                     22.3966
                                23.7048
                                           22.4617
                                                      31.9337
2
      18523.05762
                     22.7193
                                23.8483
                                           22.5719
                                                      31.9337
3
                                                      32.4374
      18534.46600
                     23.2032
                                24.1927
                                           22.9026
4
                     23.5528
                                24.3936
                                           23.1231
                                                      32.5925
      18679.66354
776
    705304.00000
                    103.2096
                               101.0935
                                          101.3665
                                                     102.1034
777
     703528.00000
                    102.3722
                               100.5292
                                          100.5527
                                                     101.1664
778
     703336.00000
                    102.6710
                               100.9362
                                          101.2159
                                                     101.8557
779
     706180.00000
                    102.6715
                               100.8332
                                          101.2843
                                                     101.9884
780
     700291.00000
                    102.5739
                               100.9984
                                          101.7258
                                                     102.6235
     DNDGRG3M086SBEA
                       DSERRG3M086SBEA
                                          CES0600000008
                                                          CES2000000008
0
               18.294
                                                    2.13
                                                                    2.45
                                 10.152
1
               18.302
                                 10.167
                                                    2.14
                                                                    2.46
2
                                                    2.15
               18.289
                                 10.185
                                                                    2.45
3
               18.300
                                 10.221
                                                    2.16
                                                                    2.47
4
               18.280
                                 10.238
                                                    2.17
                                                                    2.48
. .
                                                                   34.55
776
              120.395
                                123.976
                                                   29.90
                                                   29.97
                                                                   34.67
777
              120.040
                                124.228
778
                                                   30.26
                                                                   34.96
              119.325
                                124.551
779
              119.193
                                124.917
                                                   30.45
                                                                   35.01
780
                                                   30.56
                                                                   35.21
              118.745
                                125.662
                                DTCOLNVHFNM
     CES3000000008
                     UMCSENTx
                                               DTCTHFNM
                                                             INVEST
                                                                      VIXCLSx
0
               2.04
                                                            84.2043
                           NaN
                                    6476.00
                                               12298.00
                                                                          NaN
1
               2.05
                           NaN
                                    6476.00
                                               12298.00
                                                            83.5280
                                                                          NaN
2
               2.07
                                    6508.00
                                               12349.00
                                                            81.6405
                                                                          NaN
                           {\tt NaN}
3
               2.08
                           NaN
                                    6620.00
                                               12484.00
                                                            81.8099
                                                                          NaN
4
               2.08
                          95.3
                                    6753.00
                                               12646.00
                                                            80.7315
                                                                          NaN
                •••
776
              26.62
                          67.9
                                  508808.61
                                              913938.95
                                                          5074.6108
                                                                      15.0424
                                                                      19.0462
777
              26.65
                                  513229.64
                          63.8
                                              918210.64
                                                          5015.5456
778
              26.89
                          61.3
                                  517434.30
                                              922552.40
                                                          4999.7208
                                                                      13.8563
779
                                                          5077.4222
              27.14
                          69.7
                                  522366.13
                                              928336.14
                                                                      12.6960
780
                                         NaN
                                                     {\tt NaN}
                                                          5105.3504
             27.22
                           NaN
                                                                      13.3453
```

[781 rows x 128 columns]

<google.colab.\_quickchart\_helpers.SectionTitle at 0x785a9b6ecb20>

from matplotlib import pyplot as plt
\_df\_32['RPI'].plot(kind='hist', bins=20, title='RPI')
plt.gca().spines[['top', 'right',]].set\_visible(False)

from matplotlib import pyplot as plt

```
_df_33['W875RX1'].plot(kind='hist', bins=20, title='W875RX1')
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_34['DPCERA3M086SBEA'].plot(kind='hist', bins=20, title='DPCERA3M086SBEA')
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_35['CMRMTSPLx'].plot(kind='hist', bins=20, title='CMRMTSPLx')
plt.gca().spines[['top', 'right',]].set_visible(False)
<google.colab._quickchart_helpers.SectionTitle at 0x785a8dd41c30>
from matplotlib import pyplot as plt
_df_36.plot(kind='scatter', x='RPI', y='W875RX1', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_37.plot(kind='scatter', x='W875RX1', y='DPCERA3M086SBEA', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_38.plot(kind='scatter', x='DPCERA3M086SBEA', y='CMRMTSPLx', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_39.plot(kind='scatter', x='CMRMTSPLx', y='RETAILx', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
<google.colab._quickchart_helpers.SectionTitle at 0x785a9b5b3e50>
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['RPI']
 plt.plot(xs, ys, label=series_name, color=palette[series_index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_40.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
= plt.ylabel('RPI')
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['W875RX1']
```

```
plt.plot(xs, ys, label=series_name, color=palette[series_index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df sorted = df 41.sort values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('W875RX1')
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['DPCERA3M086SBEA']
 plt.plot(xs, ys, label=series name, color=palette[series index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_42.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('DPCERA3M086SBEA')
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['CMRMTSPLx']
 plt.plot(xs, ys, label=series name, color=palette[series index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_43.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('CMRMTSPLx')
<google.colab._quickchart_helpers.SectionTitle at 0x785a9b5b0fa0>
from matplotlib import pyplot as plt
_df_44['RPI'].plot(kind='line', figsize=(8, 4), title='RPI')
plt.gca().spines[['top', 'right']].set_visible(False)
from matplotlib import pyplot as plt
_df_45['W875RX1'].plot(kind='line', figsize=(8, 4), title='W875RX1')
```

```
plt.gca().spines[['top', 'right']].set_visible(False)
    from matplotlib import pyplot as plt
    _df_46['DPCERA3M086SBEA'].plot(kind='line', figsize=(8, 4),_
     →title='DPCERA3M086SBEA')
    plt.gca().spines[['top', 'right']].set_visible(False)
    from matplotlib import pyplot as plt
    _df_47['CMRMTSPLx'].plot(kind='line', figsize=(8, 4), title='CMRMTSPLx')
    plt.gca().spines[['top', 'right']].set_visible(False)
[]: # Extract transformation codes
     transformation_codes = df.iloc[0, 1:].to_frame().reset_index()
     transformation_codes.columns = ['Series', 'Transformation_Code']
[]: import numpy as np
     # Function to apply transformations based on the transformation code
     def apply_transformation(series, code):
         if code == 1:
             # No transformation
             return series
         elif code == 2:
             # First difference
             return series.diff()
         elif code == 3:
             # Second difference
             return series.diff().diff()
         elif code == 4:
             # Log
             return np.log(series)
         elif code == 5:
             # First difference of log
             return np.log(series).diff()
         elif code == 6:
             # Second difference of log
             return np.log(series).diff().diff()
         elif code == 7:
             # Delta (x t/x \{t-1\} - 1)
             return series.pct_change()
         else:
             raise ValueError("Invalid transformation code")
     # Applying the transformations to each column in df_cleaned based on
      ⇔transformation_codes
     for series_name, code in transformation_codes.values:
         df_cleaned[series_name] = apply_transformation(df_cleaned[series_name].
      ⇔astype(float), float(code))
```

```
df_cleaned = df_cleaned[2:]
    df_cleaned.reset_index(drop=True, inplace=True)
    df cleaned.head()
[]:
         sasdate
                       RPI
                             W875RX1 DPCERA3M086SBEA CMRMTSPLx
                                                                   RETAILx \
    0 1959-03-01 0.006457
                            0.007325
                                             0.009404 -0.003374 0.008321
    1 1959-04-01 0.006510
                            0.007029
                                            -0.003622
                                                        0.019915 0.000616
    2 1959-05-01 0.005796
                                             0.012043
                                                                  0.007803
                            0.006618
                                                        0.006839
    3 1959-06-01 0.003068
                            0.003012
                                             0.003642 -0.000097
                                                                  0.009064
    4 1959-07-01 -0.000580 -0.000762
                                            -0.003386
                                                        0.012155 -0.000330
          INDPRO
                  IPFPNSS
                            IPFINAL
                                      IPCONGD ...
                                                  DNDGRG3M086SBEA \
    0 0.014306 0.006035 0.004894 0.000000
                                                        -0.001148
    1 0.021075 0.014338 0.014545 0.015650
                                                         0.001312
    2 0.014955 0.008270 0.009582 0.004770 ...
                                                        -0.001695
    3 0.001141 0.007034 0.007128 -0.004767
                                                         0.003334
    4 -0.024240
                 0.001168 0.008249
                                     0.013054 ...
                                                        -0.001204
       DSERRG3M086SBEA CES0600000008
                                       CES2000000008 CES3000000008
                                                                     UMCSENTx
    0
              0.000292
                            -0.000022
                                           -0.008147
                                                                          NaN
                                                           0.004819
    1
              0.001760
                            -0.000022
                                            0.012203
                                                          -0.004890
                                                                          NaN
    2
                                                                          NaN
             -0.001867
                            -0.000021
                                           -0.004090
                                                          -0.004819
    3
              0.001946
                            -0.004619
                                            0.003992
                                                                          NaN
                                                           0.004796
             -0.000013
                             0.000000
                                           -0.004040
                                                          -0.004796
                                                                          NaN
       DTCOLNVHFNM DTCTHFNM
                                       VIXCLSx
                                INVEST
    0
          0.004929 0.004138 -0.014792
                                            NaN
    1
          0.012134 0.006734 0.024929
                                            NaN
    2
          0.002828 0.002020 -0.015342
                                            NaN
    3
          0.009726 0.009007 -0.012252
                                            NaN
         -0.004631 -0.001000 0.029341
                                            NaN
    [5 rows x 128 columns]
    <google.colab._quickchart_helpers.SectionTitle at 0x785a8ddd8d30>
    from matplotlib import pyplot as plt
    _df_16['RPI'].plot(kind='hist', bins=20, title='RPI')
    plt.gca().spines[['top', 'right',]].set_visible(False)
    from matplotlib import pyplot as plt
    _df_17['W875RX1'].plot(kind='hist', bins=20, title='W875RX1')
    plt.gca().spines[['top', 'right',]].set_visible(False)
    from matplotlib import pyplot as plt
    _df_18['DPCERA3M086SBEA'].plot(kind='hist', bins=20, title='DPCERA3M086SBEA')
    plt.gca().spines[['top', 'right',]].set_visible(False)
```

```
from matplotlib import pyplot as plt
_df_19['CMRMTSPLx'].plot(kind='hist', bins=20, title='CMRMTSPLx')
plt.gca().spines[['top', 'right',]].set_visible(False)
<google.colab._quickchart_helpers.SectionTitle at 0x785a9e0c6b30>
from matplotlib import pyplot as plt
df 20.plot(kind='scatter', x='RPI', y='W875RX1', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_21.plot(kind='scatter', x='W875RX1', y='DPCERA3M086SBEA', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_22.plot(kind='scatter', x='DPCERA3M086SBEA', y='CMRMTSPLx', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
from matplotlib import pyplot as plt
_df_23.plot(kind='scatter', x='CMRMTSPLx', y='RETAILx', s=32, alpha=.8)
plt.gca().spines[['top', 'right',]].set_visible(False)
<google.colab. quickchart helpers.SectionTitle at 0x785a8de093f0>
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 vs = series['RPI']
 plt.plot(xs, ys, label=series name, color=palette[series index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_24.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('RPI')
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['W875RX1']
 plt.plot(xs, ys, label=series name, color=palette[series index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_25.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
```

```
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('W875RX1')
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['DPCERA3M086SBEA']
 plt.plot(xs, ys, label=series name, color=palette[series index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_26.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('DPCERA3M086SBEA')
from matplotlib import pyplot as plt
import seaborn as sns
def _plot_series(series, series_name, series_index=0):
 palette = list(sns.palettes.mpl_palette('Dark2'))
 xs = series['sasdate']
 ys = series['CMRMTSPLx']
 plt.plot(xs, ys, label=series name, color=palette[series index % len(palette)])
fig, ax = plt.subplots(figsize=(10, 5.2), layout='constrained')
df_sorted = _df_27.sort_values('sasdate', ascending=True)
_plot_series(df_sorted, '')
sns.despine(fig=fig, ax=ax)
plt.xlabel('sasdate')
_ = plt.ylabel('CMRMTSPLx')
<google.colab._quickchart_helpers.SectionTitle at 0x785a8de09390>
from matplotlib import pyplot as plt
_df_28['RPI'].plot(kind='line', figsize=(8, 4), title='RPI')
plt.gca().spines[['top', 'right']].set_visible(False)
from matplotlib import pyplot as plt
_df_29['W875RX1'].plot(kind='line', figsize=(8, 4), title='W875RX1')
plt.gca().spines[['top', 'right']].set_visible(False)
from matplotlib import pyplot as plt
_df_30['DPCERA3M086SBEA'].plot(kind='line', figsize=(8, 4),_
 →title='DPCERA3M086SBEA')
plt.gca().spines[['top', 'right']].set_visible(False)
```

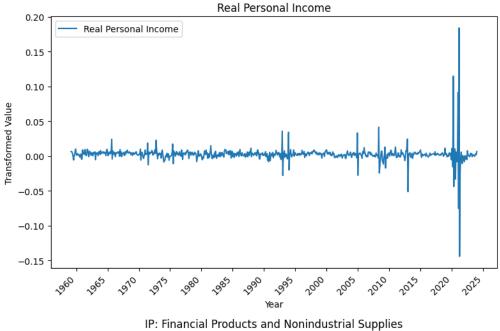
```
_df_31['CMRMTSPLx'].plot(kind='line', figsize=(8, 4), title='CMRMTSPLx')
    plt.gca().spines[['top', 'right']].set_visible(False)
[]: import matplotlib.pyplot as plt
     import matplotlib.dates as mdates
     series_to_plot = ['RPI', 'IPFPNSS', 'IPCONGD']
     series_names = ['Real Personal Income',
                     'IP: Financial Products and Nonindustrial Supplies',
                     'IP: Consumer Goods'l
     # Create a figure and a grid of subplots
     fig, axs = plt.subplots(len(series_to_plot), 1, figsize=(8, 15))
     # Iterate over the selected series and plot each one
     for ax, series_name, plot_title in zip(axs, series_to_plot, series_names):
         if series_name in df_cleaned.columns:
             dates = pd.to_datetime(df_cleaned['sasdate'], format='%m/%d/%Y')
             ax.plot(dates, df_cleaned[series_name], label=plot_title)
             ax.xaxis.set_major_locator(mdates.YearLocator(base=5))
             ax.xaxis.set_major_formatter(mdates.DateFormatter('%Y'))
            ax.set_title(plot_title)
            ax.set_xlabel('Year')
            ax.set_ylabel('Transformed Value')
            ax.legend(loc='upper left')
            plt.setp(ax.xaxis.get_majorticklabels(), rotation=45, ha='right')
         else:
```

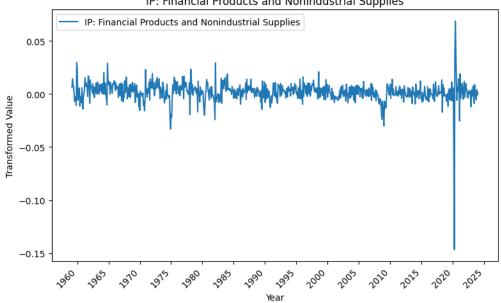
ax.set\_visible(False) # Hide plots for which the data is not available

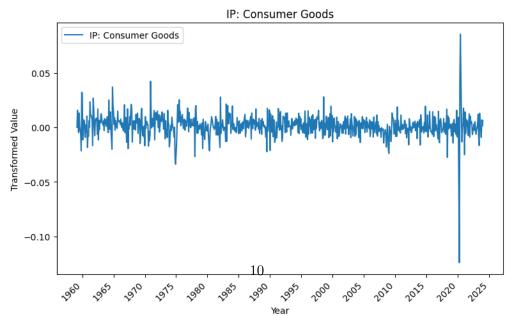
from matplotlib import pyplot as plt

plt.tight\_layout()

plt.show()







```
[]: Yraw = df_cleaned['RPI']
     Xraw = df_cleaned[['IPFPNSS', 'IPCONGD']]
     num_lags = 4 ## this is p
     num_leads = 1 ## this is h
     X = pd.DataFrame()
     ## Add the lagged values of Y
     col = 'RPI'
     for lag in range(0,num lags+1):
             # Shift each column in the DataFrame and name it with a lag suffix
             X[f'{col}_lag{lag}'] = Yraw.shift(lag)
     for col in Xraw.columns:
         for lag in range(0,num_lags+1):
             # Shift each column in the DataFrame and name it with a lag suffix
             X[f'{col}_lag{lag}'] = Xraw[col].shift(lag)
     ## Add a column on ones (for the intercept)
     X.insert(0, 'Ones', np.ones(len(X)))
     ## X is now a DataFrame
    X.head()
[]:
        Ones RPI_lag0
                        RPI_lag1 RPI_lag2 RPI_lag3 RPI_lag4
                                                                 IPFPNSS_lag0
         1.0 0.006457
                             NaN
                                                  NaN
                                                            NaN
                                                                     0.006035
                                       NaN
     1
         1.0 0.006510 0.006457
                                                  NaN
                                                            NaN
                                       NaN
                                                                     0.014338
         1.0 0.005796 0.006510 0.006457
                                                  NaN
                                                            NaN
                                                                     0.008270
     3
         1.0 0.003068 0.005796 0.006510
                                            0.006457
                                                            NaN
                                                                     0.007034
         1.0 -0.000580 0.003068 0.005796
                                            0.006510 0.006457
                                                                     0.001168
                     IPFPNSS lag2 IPFPNSS lag3
                                                                 IPCONGD lag0
        IPFPNSS lag1
                                                  IPFPNSS lag4
     0
                 NaN
                               NaN
                                             NaN
                                                            NaN
                                                                     0.00000
     1
            0.006035
                               NaN
                                             NaN
                                                            NaN
                                                                     0.015650
     2
            0.014338
                          0.006035
                                              NaN
                                                            NaN
                                                                     0.004770
     3
            0.008270
                          0.014338
                                        0.006035
                                                            NaN
                                                                    -0.004767
     4
            0.007034
                          0.008270
                                        0.014338
                                                       0.006035
                                                                     0.013054
        IPCONGD_lag1
                      IPCONGD_lag2
                                    IPCONGD_lag3
                                                   IPCONGD_lag4
     0
                 NaN
                               NaN
                                             NaN
                                                            NaN
     1
            0.000000
                               NaN
                                             NaN
                                                            NaN
     2
            0.015650
                           0.00000
                                             NaN
                                                            NaN
     3
            0.004770
                           0.01565
                                          0.00000
                                                            NaN
           -0.004767
                           0.00477
                                          0.01565
                                                            0.0
```

```
[]: y = Yraw.shift(-num_leads)
[]: 0
            0.006510
           0.005796
     1
     2
           0.003068
     3
          -0.000580
          -0.005653
    774
           0.001759
    775
           0.003530
    776
           0.002019
    777
           0.006503
    778
                 NaN
    Name: RPI, Length: 779, dtype: float64
[]: ## Save last row of X (converted to numpy)
     X T = X.iloc[-1:].values
     ## Subset getting only rows of X and y from p+1 to h-1
     ## and convert to numpy array
     y = y.iloc[num_lags:-num_leads].values
     X = X.iloc[num lags:-num leads].values
     X_T
[]: array([[ 1.00000000e+00, 6.50344580e-03, 2.01939211e-03,
              3.53017207e-03, 1.75935785e-03, -3.00450660e-04,
              1.63700864e-03, -1.02096759e-03, 4.04040151e-03,
            -5.59759862e-03, -1.84411458e-03, 6.20787002e-03,
              1.30197557e-03, 6.79041985e-03, -9.21933939e-03,
            -1.94613030e-03]])
[]: from numpy.linalg import solve
     # Solving for the OLS estimator beta: (X'X)^{-1} X'Y
     beta_ols = solve(X.T @ X, X.T @ y)
     ## Produce the One step ahead forecast
     ## % change month-to-month INDPRO
     forecast = X T@beta ols*100
     forecast
[]: array([0.15461092])
[]: def calculate_forecast(df_cleaned,
                            p = 4,
                            H = [1,4,8],
```

```
end_date = '12/1/2021',
                   target = 'RPI',
                   xvars = ['IPFPNSS', 'IPCONGD']):
## Subset df_cleaned to use only data up to end_date
rt_df = df_cleaned[df_cleaned['sasdate'] <= pd.Timestamp(end_date)]
## Get the actual values of the target at different steps ahead
Y actual = []
for h in H:
    os = pd.Timestamp(end_date) + pd.DateOffset(months=h)
    Y_actual.append(df_cleaned[df_cleaned['sasdate'] == os][target]*100)
    ## Now Y contains the true values at T+H (multiplying * 100)
Yraw = rt_df[target]
Xraw = rt_df[xvars]
X = pd.DataFrame()
## Add the lagged values of Y
for lag in range(0,p):
    # Shift each column in the DataFrame and name it with a lag suffix
    X[f'{target}_lag{lag}'] = Yraw.shift(lag)
for col in Xraw.columns:
    for lag in range(0,p):
        X[f'{col}_lag{lag}'] = Xraw[col].shift(lag)
## Add a column on ones (for the intercept)
X.insert(0, 'Ones', np.ones(len(X)))
## Save the last row of X (converted to a `numpy` array)
X_T = X.iloc[-1:].values
## While the X will be the same, Y needs to be leaded differently
Yhat = []
for h in H:
    y_h = Yraw.shift(-h)
    ## Subset getting only rows of X and y from p+1 to h-1
    y = y_h.iloc[p:-h].values
    X_{-} = X.iloc[p:-h].values
    # Solving for the OLS estimator beta: (X'X)^{-1} X'Y
    beta_ols = solve(X_.T @ X_, X_.T @ y)
    ## Produce the One step ahead forecast
    ## % change month-to-month RPI
    Yhat.append(X_T@beta_ols*100)
## Now calculate the forecasting error and return
return np.array(Y_actual) - np.array(Yhat)
```

```
[]: t0 = pd.Timestamp('12/1/2021')
e = []
T = []
for j in range(0, 10):
    t0 = t0 + pd.DateOffset(months=1)
    print(f'Using data up to {t0}')
    ehat = calculate_forecast(df_cleaned, p = 4, H = [1,4,8], end_date = t0)
    e.append(ehat.flatten())
    T.append(t0)

## Create a pandas DataFrame from the list
edf = pd.DataFrame(e)
## Calculate the RMSFE, that is, the square root of the MSFE
np.sqrt(edf.apply(np.square).mean())
```

```
Using data up to 2022-01-01 00:00:00
Using data up to 2022-02-01 00:00:00
Using data up to 2022-03-01 00:00:00
Using data up to 2022-04-01 00:00:00
Using data up to 2022-05-01 00:00:00
Using data up to 2022-06-01 00:00:00
Using data up to 2022-07-01 00:00:00
Using data up to 2022-08-01 00:00:00
Using data up to 2022-09-01 00:00:00
Using data up to 2022-10-01 00:00:00
```

[]: 0 0.708419 1 0.439215 2 0.426919 dtype: float64