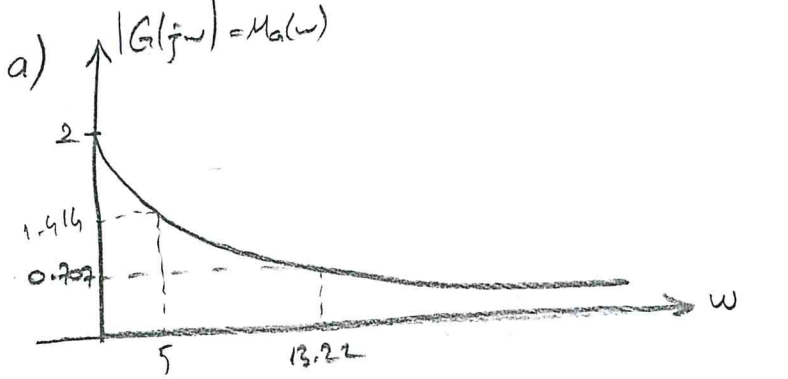
KONTROL TEORİSİ

1) i) $G(s) = \frac{10}{s+5}$

$M_G(w) = G(s)\Big|_{s=jw} = \left|\frac{10}{jw+5}\right| = \frac{10}{\sqrt{5^2+w^2}} = M_G(w)$

$\angle G(jw) = 0 - \arctan\frac{w}{5} = -\arctan\frac{w}{5}$

ii) a)



$w = 0 \Rightarrow \frac{10}{\sqrt{5^2+0^2}} = \frac{10}{5} = 2$

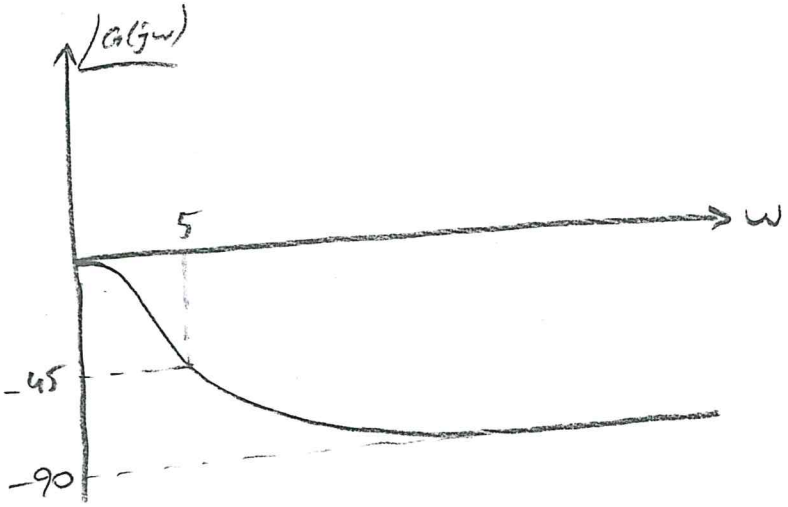
$\frac{10}{\sqrt{5^2+w^2}} = \frac{1}{\sqrt{2}}$

$10\cdot\sqrt{2} = \sqrt{5^2+w^2}$

$200 = 25 + w^2$

$175 = w^2$

$w = 13.22$ rad/sn.



$\angle G(jw) = -\arctan\frac{w}{5}$

$-\arctan\frac{w}{5} = 45$

$\arctan\frac{w}{5} = -45$

$\frac{\tan 45}{1} = \frac{w}{5}$

$\boxed{w = 5}$

$-\arctan\frac{w}{5} = 90°$

$\arctan\frac{w}{5} = -90°$

$\tan(-90°) = \frac{w}{5}$

$\boxed{w = \infty}$

iii) b) i) $r(t) = 10\cos(t+60) \rightarrow \boxed{w = 1}$

$10\cos(t+60) \rightarrow \boxed{G(s)} \rightarrow 10 \cdot |G(s)| \cdot \cos(t+60+\angle G(s))$

$w=1$;

$10 \cdot \frac{10}{\sqrt{w^2+5^2}} \cdot \cos\left(t+60-\arctan\frac{w}{5}\right) \Rightarrow w=1 \Rightarrow$

$10 \cdot \frac{10}{\sqrt{1^2+25}} \cos\left(t+60-\underbrace{\arctan\frac{1}{5}}_{11.3}\right) \Rightarrow \frac{100}{\sqrt{26}} \cos(t+60-11.3^\circ)$

$\Rightarrow \underbrace{3.84\sqrt{26}\cos(t+48.9^\circ)}_{19.58}$

ii) $w = 10$

$10 \cdot \frac{10}{\sqrt{w^2+5^2}} \cdot \cos\left(10t+60-\arctan\frac{w}{5}\right) \Rightarrow$

$10 \cdot \frac{10}{\sqrt{125}} \cdot \cos\left(10t+60-\underbrace{\arctan\frac{10}{5}}_{63.43}\right) \Rightarrow \underbrace{4\sqrt{5}}_{8.94} \cos(10t-3.43^\circ)$

iii) $w = 20$;

$10 \cdot \frac{10}{\sqrt{20^2+5^2}} \cos\left(20t+60-\underbrace{\arctan\frac{20}{5}}_{75.96}\right) \Rightarrow 4.85 \cos(20t-15.96^\circ)$

iv) $w = 50$;

$10 \cdot \frac{10}{\sqrt{50^2+5^2}} \cdot \cos\left(50t+60-\underbrace{\arctan\frac{50}{5}}_{84.28}\right) \Rightarrow 1.99 \cos(50t-24.28^\circ)$

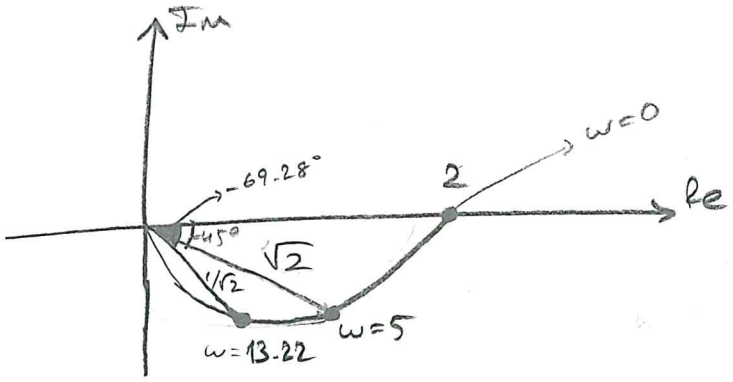
v) $w = 100$;

$10 \cdot \frac{10}{\sqrt{100^2+5^2}} \cos\left(100t+60-\underbrace{\arctan\frac{100}{5}}_{87.13}\right) \Rightarrow 0.99 \cos(100t-27.13^\circ)$

vi) $w = 200$

$10 \cdot \frac{10}{\sqrt{200^2+5^2}} \cos\left(200t+60-\underbrace{\arctan\frac{200}{5}}_{88.57}\right) \Rightarrow 0.05 \cos(200t-28.57^\circ)$

iii) $G(j\omega) = \dfrac{10}{\sqrt{\omega^2+5^2}}$ $\quad \omega=0 \Rightarrow \dfrac{10}{5} = 2$



$-45° = -\arctan\left(\dfrac{\omega}{5}\right) \Rightarrow \underline{\omega = 5}$

$\dfrac{10}{\sqrt{5^2+5^2}} = \dfrac{10}{5\sqrt{2}} = \sqrt{2}$

$\dfrac{1}{\sqrt{2}} = \dfrac{10}{\sqrt{\omega^2+5^2}}$

$10.\sqrt{2} = \sqrt{\omega^2+5^2}$

$200 = \omega^2 + 25$

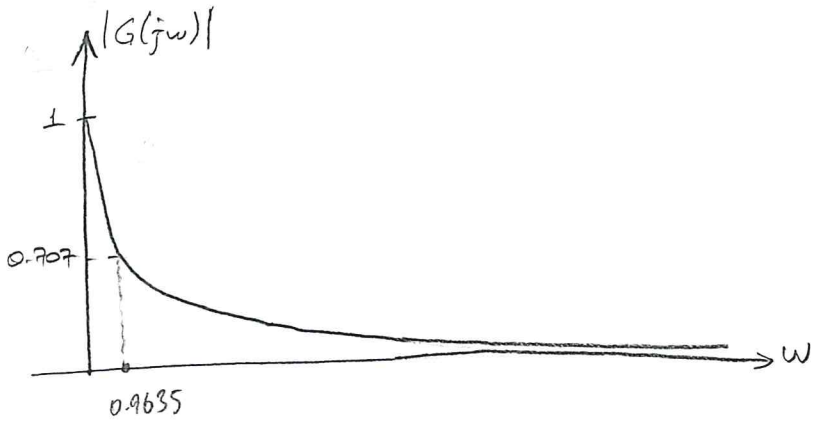
$\underline{\omega = 13.22} \quad \Rightarrow -\arctan\left(\dfrac{13.22}{5}\right)$

$\Rightarrow -69.28°$

iv)

2) i) $G(s) = \frac{5}{s^2+6s+5} \Rightarrow G(j\omega) = \frac{5}{(j\omega)^2+6(j\omega)+5} = \frac{5}{-\omega^2+6j\omega+5} = \frac{5}{(-\omega^2+5)+6j\omega}$

$|G(j\omega)| = \frac{|5|}{\sqrt{(5-\omega^2)^2+(6\omega)^2}} = \frac{5}{\sqrt{\omega^4+26\omega^2+25}}$



$\frac{5}{\sqrt{\omega^4+26\omega^2+25}} = \frac{1}{\sqrt{2}}$

$(5\sqrt{2})^2 = \left(\sqrt{\omega^4+26\omega^2+25}\right)^2$

$50 = \omega^4+26\omega^2+25$

$0 = \omega^4+26\omega^2+25$, $x=\omega^2 \Rightarrow$

$x^2+26x-25=0 \Rightarrow$

$x_1 = 0.9284 = \omega_1^2 \Rightarrow \omega_1 = 0.9635$

$x_2 = -26.9284 = \omega_2^2$ (impossible)

$\omega = \sqrt{5} \rightarrow -90°$

$\angle G(j\omega) = 0 - \arctan\left(\frac{6\omega}{5-\omega^2}\right)$

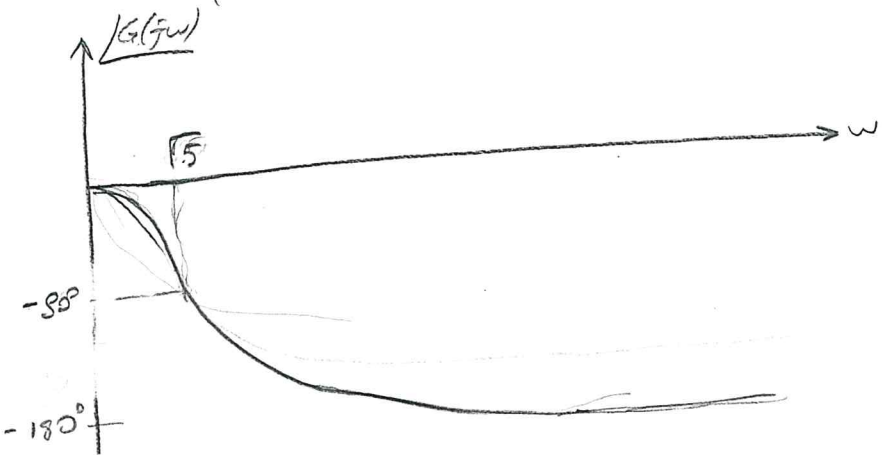
$\angle G(j\omega) = -\arctan\left(\frac{6\omega}{5-\omega^2}\right) \Rightarrow -\arctan\left(\frac{6\omega}{5-\omega^2}\right) = 45° \Rightarrow \arctan\left(\frac{6\omega}{5-\omega^2}\right) = -45 \Rightarrow$

$\frac{6\omega}{5-\omega^2} = \tan(-45) \Rightarrow \frac{6\omega}{5-\omega^2} = -1 \Rightarrow 6\omega = \omega^2-5 \Rightarrow \omega^2-6\omega-5=0$

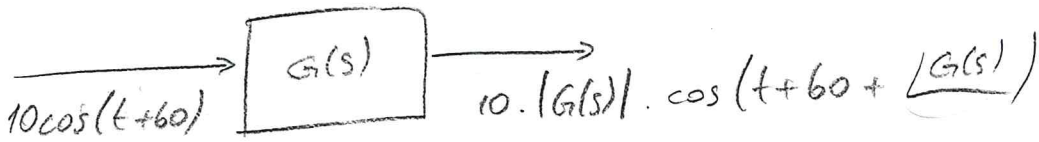
$\omega_1 = 6.74$

$\omega_2 = -0.74$ (impossible)

$\Rightarrow -\arctan\left(\frac{6\omega}{5-\omega^2}\right) = 90° \Rightarrow \frac{6\omega}{5-\omega^2} = \tan(-90) \Rightarrow \boxed{\omega = \infty}$



ii) b) i) $r(t) = 10\cos(t+60) \rightarrow \boxed{\omega = 1}$



$\omega = 1$;

$$10 \cdot \frac{5}{\sqrt{\omega^4+26\omega^2+25}} \cdot \cos\left(t+60-\arctan\frac{6\omega}{5-\omega^2}\right) \Rightarrow \underbrace{10\cdot\frac{5}{\sqrt{52}}}_{72.11} \cdot \cos\left(t+60-\underbrace{\arctan\frac{6}{4}}_{56.30^\circ}\right)$$

$$\Rightarrow 72.11\cos(t+3.69^\circ)$$

ii) $\omega = 10$;

$$\underbrace{10\cdot\frac{5}{\sqrt{10^4+26\cdot10^2+25}}}_{0.44}\cos\left(10t+60-\underbrace{\arctan\left(\frac{60}{-95}\right)}_{-32.27^\circ=147.73^\circ}\right) \Rightarrow 0.44\cos(10t-87.73^\circ)$$

iii) $\omega = 20$;

$$\underbrace{10\cdot\frac{5}{\sqrt{20^4+26\cdot20^2+25}}}_{0.12}\cos\left(20t+60-\underbrace{\arctan\left(\frac{120}{-395}\right)}_{-16.89^\circ=163.11^\circ}\right) \Rightarrow 0.12\cos(10t-103.11^\circ)$$

iv) $\omega = 50$;

$$\underbrace{10\cdot\frac{5}{\sqrt{50^4+26\cdot50^2+25}}}_{0.019}\cos\left(50t+60-\underbrace{\arctan\left(\frac{300}{-2495}\right)}_{-6.85^\circ=173.15^\circ}\right) \Rightarrow 0.019\cos(50t-113.15^\circ)$$

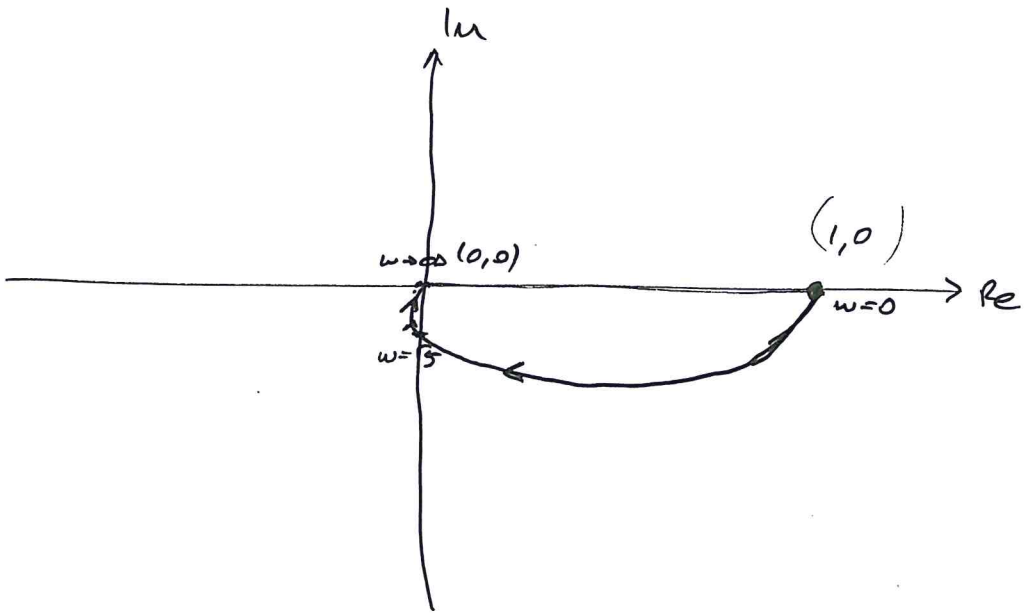
v) $\omega = 100$;

$$\underbrace{10\cdot\frac{5}{\sqrt{100^4+26\cdot100^2+25}}}_{4.99\times10^{-3}}\cos\left(100t+60-\underbrace{\arctan\left(\frac{600}{-9995}\right)}_{-3.43^\circ=176.57^\circ}\right) \Rightarrow 4.99\times10^{-3}\cdot\cos(100t-116.57^\circ)$$

vi) $\omega = 200$;

$$\underbrace{10\cdot\frac{5}{\sqrt{200^4+26\cdot200^2+25}}}_{\cdots}\cos\left(200t+60-\underbrace{\arctan\left(\frac{1200}{-39995}\right)}_{-1.71^\circ=178.29^\circ}\right) \Rightarrow 1.24\times10^{-3}\cos(200t-118.29^\circ)$$

iii) $G(j\omega) = \dfrac{5}{\sqrt{\omega^4 + 26\omega^2 + 25}}$



iv)

iii) $G(j\omega) = \dfrac{5}{\sqrt{\omega^4 - 6\omega^2 + 25}}$ $\Rightarrow, \omega = 0 \Rightarrow G(j\omega) = 1$

$\angle G(j\omega) = -\arctan\left(\dfrac{2\omega}{5-\omega^2}\right)$ $\quad -\arctan\left(\dfrac{2\omega}{5-\omega^2}\right) = 45°$

i) $\dfrac{2\omega}{5-\omega^2} = \tan(-60) = \dfrac{-\sin 60}{\cos 60} = \dfrac{-0.86}{0.5}$ $= -1.72$

$\dfrac{2\omega}{5-\omega^2} = -1.72$

$\Rightarrow 1.72\omega^2 - 2\omega - 8.6 = 0$.

$\omega_1 = 8.5551$

~~$\omega_2 = -5.1151$~~ (impossible)

① $\dfrac{2\omega}{5-\omega^2} = \tan(-45) = \dfrac{\sin(-45)}{\cos(-45)} = \dfrac{-\sin 45}{\cos 45} = \dfrac{-0.707}{0.707} = -1$

$\dfrac{2\omega}{5-\omega^2} = -1 \Rightarrow \omega^2 - 2\omega - 5 = 0$

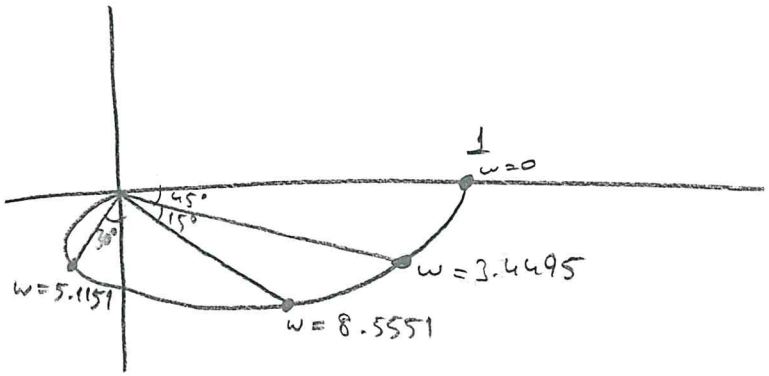
$\omega_1 = 3.4495$

~~$\omega_2 = -1.4495$~~ (impossible)

③ $\dfrac{2\omega}{5-\omega^2} = \dfrac{-\sin 120}{\cos 120} = \dfrac{-0.86}{-0.5} = 1.72 \Rightarrow -1.72\omega^2 - 2\omega + 8.6 = 0$

$\omega_1 = 5.1151$

~~$\omega_2 = -8.5551$~~ (impossible)



ii) b) i) $r(t) = 10\cos(t+60) \rightarrow \omega = 1$

$\omega = 1$;

$10 \cdot \dfrac{5}{\sqrt{\omega^4 - 6\omega^2 + 25}} \cdot \cos\left(t + 60 - \arctan\left(\dfrac{2\omega}{5-\omega^2}\right)\right) \Rightarrow 11.18\cos(t + 26.30)$

11.18 ; 33.69

ii) $\omega = 10$; $0.5150\cos(10t + 48.11°)$

iii) $\omega = 20$; $0.1259\cos(20t + 54.21°)$

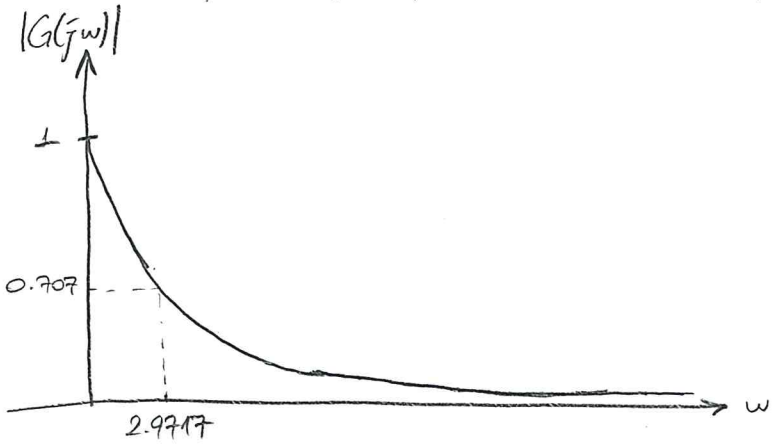
iv) $\omega = 50$; $0.020\cos(50t + 57.70)$

v) $\omega = 100$; $5.001 \times 10^{-3}\cos(100t + 58.85°)$

vi) $\omega = 200$; $1.25 \times 10^{-3}\cos(200t + 59.42°)$

3) i) $G(s) = \frac{5}{s^2+2s+5} \Rightarrow G(j\omega) = \frac{5}{(j\omega)^2+2(j\omega)+5} = \frac{5}{-\omega^2+2j\omega+5} = \frac{5}{(5-\omega^2)+2j\omega}$

$$M_G(\omega) = |G(j\omega)| = \frac{|5|}{\sqrt{(5-\omega^2)^2+(2\omega)^2}} = \frac{5}{\sqrt{\omega^4-6\omega^2+25}}$$



$$\frac{5}{\sqrt{\omega^4-6\omega^2+25}} = \frac{1}{\sqrt{2}}$$

$$(5.\sqrt{2})^2 = \left(\sqrt{\omega^4-6\omega^2+25}\right)^2$$

$$50 = \omega^4-6\omega^2+25$$

$$0 = \omega^4-6\omega^2-25, \quad x=\omega^2 \Rightarrow$$

$$x^2-6x-25 = 0 \Rightarrow x_1 = 8.8310$$

~~$x_2 = -2.8310$~~ (impossible).

$$x_1 = \omega^2 \Rightarrow \omega = 2.9717$$

$$\angle G(j\omega) = 0 - \arctan\left(\frac{2\omega}{5-\omega^2}\right)$$

$$\angle G(j\omega) = -\arctan\left(\frac{2\omega}{5-\omega^2}\right)$$

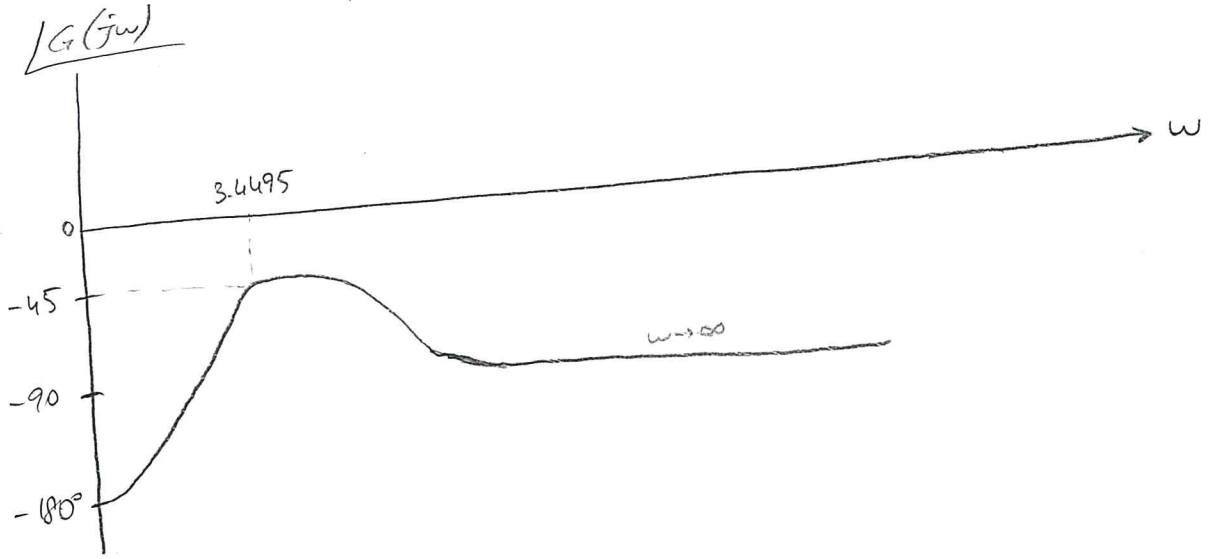
$$-\arctan\left(\frac{2\omega}{5-\omega^2}\right) = 45^\circ \Rightarrow \arctan\left(\frac{2\omega}{5-\omega^2}\right) = -45^\circ \Rightarrow \frac{2\omega}{5-\omega^2} = \tan(-45^\circ)$$

$$\Rightarrow \frac{2\omega}{5-\omega^2} = -1 \Rightarrow 2\omega = \omega^2-5 \Rightarrow \omega^2-2\omega-5 = 0 \; ; \; \omega_1 = 3.4495$$

~~$\omega_2 = -1.4495$~~ (impossible)

$$\Rightarrow -\arctan\left(\frac{2\omega}{5-\omega^2}\right) = 90^\circ \Rightarrow \omega = \infty$$

$$\Rightarrow -\arctan\left(\frac{2\omega}{5-\omega^2}\right) = 180^\circ \Rightarrow \omega = 0$$



4) $G(s) = \dfrac{52}{(s^2+4s+13)(s+4)} \Rightarrow G(j\omega) = \dfrac{\angle}{-j\omega^3-8\omega^2+29j\omega+52}$

3) iv)