Decimal digit	(BCD) 8421	Excess-3	84-2-1	2421
0	0000	0011	0000	0000
1	0001	0100	0111	0001
2	0010	0101	0110	0010
3	0011	0110	0101	0011
4	0100	0111	0100	0100
5	0101	1000	1011	1011
6	0110	1001	1010	1100
7	0111	1010	1001	1101
8	1000	1011	1000	1110
9	1001	1100	1111	1111

Three bit Gray code

Decimal	Cyclic
0	0000
1	0001
2	0011
3	0010
4	0110
5	0100
6	1100
7	1110
8	1010
9	1000

Four-bit Gray code

Gray code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

Decimal	Binary	Gray
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

except for the most significant bit position, all columns are "reflected" about the midpoint; in the most significant bit position, the top half is all 0's and the bottom half all 1's.

A decimal number can be converted to Gray code by first converting it to binary. The binary number is converted to the Gray code by performing a modulo-2 sum of each digit
(starting with the least significant digit) with its adjacent digit. For example, if the binary
representation of a decimal number is

Code conversion:
if the binary representation of a decimal number is $b_3 \quad b_2 \quad b_1 \quad b_0$

$$G_3 = b_3$$
$$G_2 = b_3 \oplus b_2$$
$$G_1 = b_2 \oplus b_1$$
$$G_0 = b_1 \oplus b_0$$

$\oplus$ indicates exclusive-OR operation

$$0 \oplus 0 = 0$$
$$0 \oplus 1 = 1$$
$$1 \oplus 0 = 1$$
$$1 \oplus 1 = 0$$

As an example let us convert **decimal 14 to Gray code**.

Decimal	Binary			
	b_3	b_2	b_1	b_0
14	1	1	1	0

G_3	G_2	G_1	G_0
1	0	0	1

$$G_3 = b_3 = 1$$
$$G_2 = b_3 \oplus b_2 = 1 \oplus 1 = 0$$
$$G_1 = b_2 \oplus b_1 = 1 \oplus 1 = 0$$
$$G_0 = b_1 \oplus b_0 = 1 \oplus 0 = 1$$

The conversion of a Gray code word to its decimal equivalent is done by following this sequence in reverse. In other words, the Gray code word is converted to binary and

G_3	G_2	G_1	G_0
1	1	1	0

$$b_3 = G_3 = 1$$

$$G_2 = b_3 \oplus b_2$$

$$\therefore b_2 = G_2 \oplus b_3 \quad (\text{since } G_2 \oplus b_3 = b_3 \oplus b_2 \oplus b_3)$$

$$= 1 \oplus 1 \quad (\text{since } G_2 = 1 \text{ and } b_3 = 1)$$

$$= 0$$

$$G_1 = b_2 \oplus b_1$$

$$\therefore b_1 = G_1 \oplus b_2$$

$$= 1 \oplus 0$$

$$= 1$$

$$G_0 = b_1 \oplus b_0$$

$$\therefore b_0 = 1$$

Thus the binary equivalent of the Gray code word 1110 is **1011**, which is equal to Decimal 11.

Single-Variable Theorems

$$0 \cdot 0 = 0$$

$$1 + 1 = 1$$

$$1 \cdot 1 = 1$$

$$0 + 0 = 0$$

$$0 \cdot 1 = 1 \cdot 0 = 0$$

$$1 + 0 = 0 + 1 = 1$$

If $x = 0$, then $\overline{x} = 1$

If $x = 1$, then $\overline{x} = 0$

$$x \cdot 0 = 0$$

$$x + 1 = 1$$

$$x \cdot 1 = x$$

$$x + 0 = x$$

$$x \cdot x = x$$

$$x + x = x$$

$$x \cdot \overline{x} = 0$$

$$x + \overline{x} = 1$$

$$\overline{\overline{x}} = x$$

Commutative

$$x \cdot y = y \cdot x$$

$$x + y = y + x$$

Associative

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$x + (y + z) = (x + y) + z$$

Distributive

$$x \cdot (y + z) = x \cdot y + x \cdot z$$

$$x + y \cdot z = (x + y) \cdot (x + z)$$

Absorption

$$x + x \cdot y = x$$

TABLE 2.2 BOOLEAN ALGEBRA POSTULATES AND THEOREMS

Expression	Dual
$P2(a) : a + 0 = a$	$P2(b) : a \cdot 1 = a$
$P3(a) : a + b = b + a$	$P3(b) : ab = ba$
$P4(a) : a + (b + c) = (a + b) + c$	$P4(b) : a(bc) = (ab)c$
$P5(a) : a + bc = (a + b)(a + c)$	$P5(b) : a(b + c) = ab + ac$
$P6(a) : a + \bar{a} = 1$	$P6(b) : a \cdot \bar{a} = 0$
$T1(a) : a + a = a$	$T1(b) : a \cdot a = a$
$T2(a) : a + 1 = 1$	$T2(b) : a \cdot 0 = 0$
$T3 : \quad \bar{\bar{a}} = a$	
$T4(a) : a + ab = a$	$T4(b) : a(a + b) = a$
$T5(a) : a + \bar{a}b = a + b$	$T5(b) : a(\bar{a} + b) = ab$
$T6(a) : ab + a\bar{b} = a$	$T6(b) : (a + b)(a + \bar{b}) = a$
$T7(a) : ab + a\bar{b}c = ab + ac$	$T7(b) : (a + b)(a + \bar{b} + c) = (a + b)(a + c)$
$T8(a) : \overline{a + b} = \bar{a}\bar{b}$	$T8(b) : \overline{ab} = \bar{a} + \bar{b}$
$T9(a) : ab + \bar{a}c + bc = ab + \bar{a}c$	$T9(b) : (a + b)(\bar{a} + c)(b + c) = (a + b)(\bar{a} + c)$
$T10(a) : f(x_1, x_2, \ldots, x_n) = x_1 f(1, x_2, \ldots, x_n) + \bar{x}_1 f(0, x_2, \ldots, x_n)$	
$T10(b) : f(x_1, x_2, \ldots, x_n) = [x_1 + f(0, x_2, \ldots, x_n)][\bar{x}_1 + f(1, x_2, \ldots, x_n)]$	

$$x \cdot (x + y) = x$$

$$x \cdot y + x \cdot \overline{y} = x$$

$$(x + y) \cdot (x + \overline{y}) = x$$

Combining

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\overline{x + y} = \overline{x} \cdot \overline{y}$$

DeMorgan's theorem

$$x + \overline{x} \cdot y = x + y$$

$$x \cdot (\overline{x} + y) = x \cdot y$$

$$x \cdot y + y \cdot z + \overline{x} \cdot z = x \cdot y + \overline{x} \cdot z$$

$$(x + y) \cdot (y + z) \cdot (\overline{x} + z) = (x + y) \cdot (\overline{x} + z)$$

Consensus