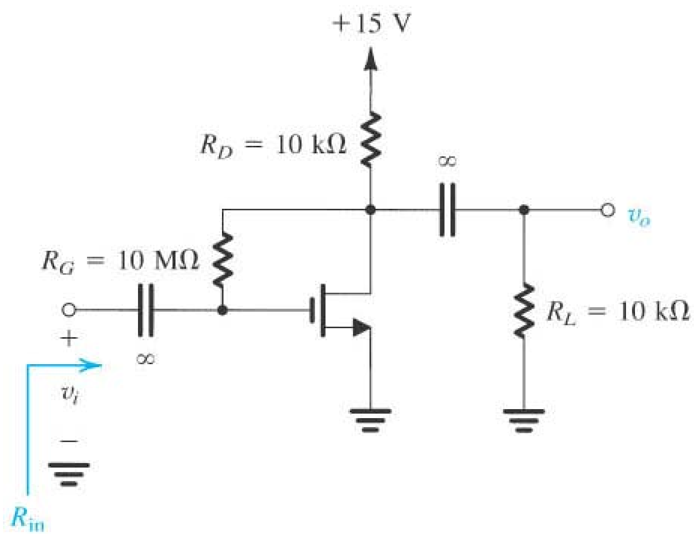
Lecture 29: MOSFET Small-Signal Amplifier Examples.

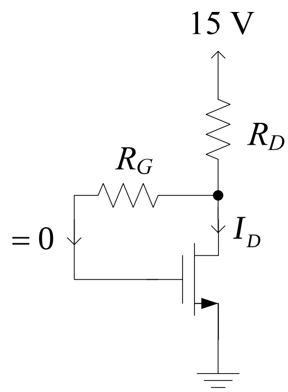
We will illustrate the analysis of small-signal MOSFET amplifiers through two examples in this lecture.

Example N29.1 (text example 4.10). Determine A_v (neglecting the effects of R_G), R_{in}, and R_{out} for the circuit below given that $V_t = 1.5$ V, $k_n' W/L = 0.25$ mA/V^2, and $V_A = 50$ V.



(Fig. 4.38a)

The first step is to determine the DC operating point. The DC equivalent circuit is:



Since $V_{GD} = 0 < V_t$ the MOSFET is operating in the saturation mode if $I_D \neq 0$. Assuming operation in the saturation mode the DC drain current from (4.22) is

$$I_D = \frac{1}{2} k_n{}' \frac{W}{L} \left(V_{GS} - V_t\right)^2 \left(1 + \lambda V_{DS}\right) \tag{4.22}$$

Notice in the circuit that $V_{GS} = V_{DS}$, so we will eventually create a triatic equation in V_{GS} for I_D.

However, the last factor in (4.22) will be quite small for large V_A (small λ). So, for simplicity we will neglect r_o giving

$$I_D \approx \frac{1}{2} k_n{}' \frac{W}{L} \left(V_{GS} - V_t\right)^2 \tag{4.20}$$

For this DC circuit

$$I_D = \frac{1}{2} \cdot 0.25 \times 10^{-3} \left(V_{GS} - 1.5\right)^2 = 1.25 \times 10^{-4} \left(V_{GS} - 1.5\right)^2$$

Notice in the circuit that $V_{GS} = V_{DS}$ so that this last equation becomes

$$I_D = 0.125 \left(V_{DS} - 1.5\right)^2 \text{ mA} \qquad (4.73),(1)$$

Also, by KVL

$$V_{DS} = 15 - R_D I_D = 15 - 10,000 I_D \qquad (4.74),(2)$$

Substituting (2) into (1)

$$I_D = 1.25 \times 10^{-4} \left(15 - 10,000 I_D - 1.5\right)^2$$

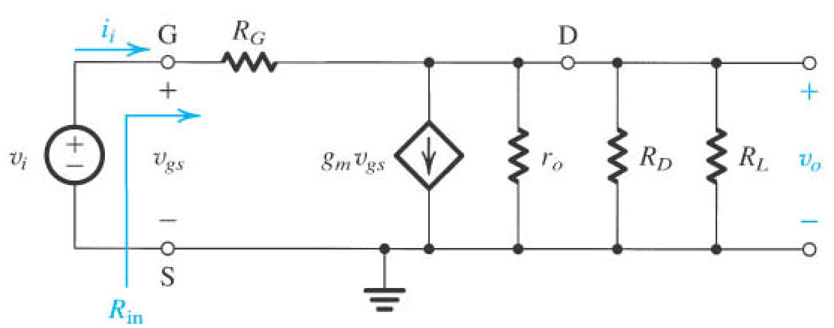
Solving this equation gives

$$I_D = 1.06 \text{ mA} \Rightarrow V_{DS} = 4.4 \text{ V} (= V_{GS})$$

or
$$I_D = 1.72 \text{ mA} \Rightarrow V_{DS} = -2.2 \text{ V} (= V_{GS})$$

This latter result is not consistent with the assumption of operation in the saturation mode since $V_{GS} < V_t = 1.5$ V. So the proper solution for I_D is the first ($I_D = 1.06$ mA).

Next, we construct the small-signal equivalent circuit. We'll use the π small-signal model of the MOSFET with r_o included:



(Fig. 4.38b)

✓ $g_m = k_n' \dfrac{W}{L}\left(V_{GS} - V_t\right) = 0.25 \times 10^{-3} \left(4.4 - 1.5\right) = 0.725 \text{ mS}$

✓ $r_o = \dfrac{V_A}{I_D} = \dfrac{50}{1.06 \text{ mA}} = 47.2 \text{ k}\Omega$

Recall from the previous lecture that the proper I_D in the r_o calculation is that with λ = 0, which is what we ended up calculating earlier.

To compute the small-signal voltage gain, we start at the output (assuming R_G is extremely large $R_G \gg r_o \parallel R_D \parallel R_L$)

$$v_o \approx -g_m v_{gs}\left(r_o \parallel R_D \parallel R_L\right)$$

At the input notice that $v_{gs} = v_i$. Therefore

$$A_v = \frac{v_o}{v_i} \approx -g_m\left(r_o \parallel R_D \parallel R_L\right) = -g_m(4{,}521) = -3.28 \text{ V/V}$$

Notice that the assumption $R_G \gg r_o \parallel R_D \parallel R_L$ is met and hugely exceeded since 10 MΩ >> 4,521 Ω.

For the input resistance R_{in} calculation, we cannot set $v_{gs} = 0$ – and subsequently open circuit the dependent current source – since this would artificially force $R_{\text{in}} = 0$. Rather, we need to determine i_i as a function of v_i and use this in the definition:

$$R_{\text{in}} \equiv \frac{v_i}{i_i}$$

The dependent current source will remain in these calculations.

Proceeding, at the input of the small-signal equivalent circuit shown above

$$i_i = \frac{v_i - v_o}{R_G} = \frac{v_i}{R_G}\left(1 - \frac{v_o}{v_i}\right) = \frac{v_i}{R_G}\left(1 - A_v\right)$$

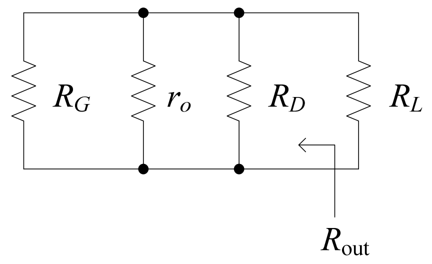
Therefore,

$$i_i = \frac{v_i}{R_G}(1 + 3.28)$$

Consequently, using this expression we find that

$$R_{\text{in}} = \frac{v_i}{i_i} = \frac{R_G}{4.28} = 2.34 \text{ M}\Omega$$

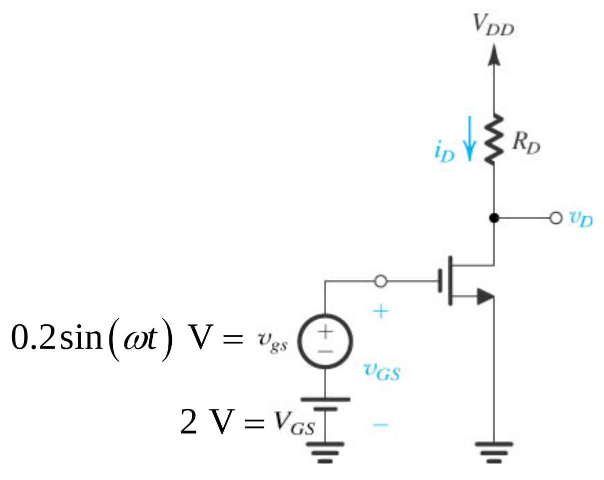
Lastly, to determine the output resistance, we can set $v_{gs} = 0$ in the small-signal equivalent circuit above, which will open circuit the dependent current source leading to the equivalent circuit:



from which we see that

$$R_{\text{out}} = R_G \parallel r_o \parallel R_D = 8.24 \text{ k}\Omega$$

Example N29.2 (text exercise 4.23). Determine the following quantities for the conceptual MOSFET small-signal amplifier of Fig. 4.34 given that $V_{DD} = 5$ V, $R_D = 10$ kΩ, and $V_{GS} = 2$ V.



(Fig. 4.34)

The MOSFET characteristics are $V_t = 1$ V, $k_n' = 20$ μA/V^2, $W/L = 20$, and $\lambda = 0$.

(a) Determine I_D and V_D. We see from the circuit that $V_{GS} > V_t$. Therefore, the MOSFET is operating in the saturation or triode mode. We'll assume saturation. In that case

$$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} \cdot 20 \times 10^{-6} \cdot 20 (2-1)^2 = 0.2 \text{ mA}$$

and
$$V_D = V_{DD} - I_D R_D = 3 \text{ V}$$

Let's check if the MOSFET is operating in the saturation mode:

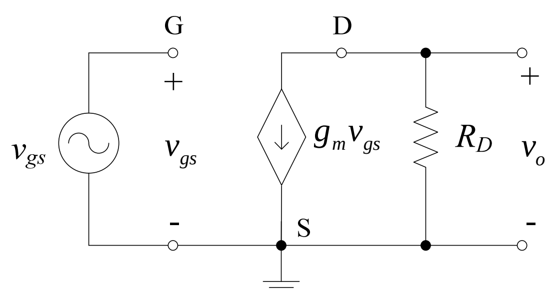
$$V_{GD} = 2 - 3 = -1 < V_t$$

Therefore, the MOSFET is indeed saturated, as assumed.

(b) Determine g_m. Using (4.61)

$$g_m = k_n' \frac{W}{L} (V_{GS} - V_t) = 20 \times 10^{-6} \cdot 20 \cdot (2-1) = 4.0 \text{ mS}$$

(c) Determine the voltage gain A_v. We begin by first constructing the small-signal equivalent circuit



Directly from this circuit,

$$v_o = -g_m v_{gs} R_D$$

so $A_v = \dfrac{v_o}{v_{gs}} = -g_m R_D = -0.4\times10^{-3}\cdot 10\times10^{3} = -4 \text{ V/V}$

(d) If $v_{gs} = 0.2\sin(\omega t)$ V, find v_d and the max/min v_D.

$$A_v \equiv \frac{v_o}{v_{gs}} \Rightarrow v_d = A_v v_{gs} = -4\cdot 0.2\sin(\omega t)$$

Therefore, $v_d = -0.8\sin(\omega t)$ V

Hence, $v_D\big|_{\max} = V_D + V_d = 3 + 0.8 = 3.8 \text{ V}$

while $v_D\big|_{\min} = V_D - V_d = 3 - 0.8 = 2.2 \text{ V}$

(e) Determine the second harmonic distortion. From (4.57) or (6) in the previous lecture notes, the drain current is given as

$$i_D = I_D + k_n'\frac{W}{L}(V_{GS}-V_t)v_{gs} + \frac{1}{2}k_n'\frac{W}{L}v_{gs}^2$$

or $$i_D = I_D + 20\times10^{-6}\cdot 20(2-1)v_{gs} + \frac{1}{2}\cdot 20\times10^{-6}\cdot 20 v_{gs}^2$$

$$= I_D + 0.4\times10^{-3}v_{gs} + 0.2\times10^{-3}v_{gs}^2$$

Substituting $v_{gs} = 0.2\sin(\omega t)$ into this equation gives

$$i_D = I_D + 80\times10^{-6}\sin(\omega t) + 8\times10^{-6}\sin^2(\omega t)$$

Using the trigonometry identity

$$\sin^2(\omega t) = 1/2 - 1/2\cos(2\omega t)$$

this last expression becomes

$$i_D = 200 + 80\sin(\omega t) + 4 - 4\cos(2\omega t) \ \mu\text{A}$$

or $$i_D = 204 + 80\sin(\omega t) - 4\cos(2\omega t) \ \mu\text{A}$$

The first term in i_D is I_D, the DC current. We see that there is a slight shift upward in value by 4 μA.

The third term in i_D is the second harmonic term because it varies with time at twice the frequency of the input signal. The second harmonic distortion is

$$\frac{4}{80}\times 100\% = 5\ \%$$