and the motor torque provides the torque to drive the belts plus the disturbance or undesired load torque, so that

$$T_m = T + T_d.$$

The torque T drives the shaft to the pulley, so that

$$T = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + r(T_1 - T_2).$$

Therefore,

$$\frac{dx_3}{dt} = \frac{d^2\theta}{dt^2}.$$

Hence,

$$\frac{dx_3}{dt} = \frac{(T_m - T_d)}{J} - \frac{b}{J}x_3 - \frac{2kr}{J}x_1,$$

where

$$T_m = \frac{K_m}{R}v_2, \quad \text{and} \quad v_2 = -k_1k_2\frac{dy}{dt} = -k_1k_2x_2.$$

Thus, we obtain

$$\frac{dx_3}{dt} = \frac{-K_mk_1k_2}{JR}x_2 - \frac{b}{J}x_3 - \frac{2kr}{J}x_1 - \frac{T_d}{J}. \tag{3.119}$$

Equations (3.117)–(3.119) are the three first-order differential equations required to describe this system. The matrix differential equation is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -1 & r \\ \frac{2k}{m} & 0 & 0 \\ \frac{-2kr}{J} & \frac{-K_mk_1k_2}{JR} & \frac{-b}{J} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{-1}{J} \end{bmatrix} T_d. \tag{3.120}$$

The signal-flow graph and block diagram models representing the matrix differential equation are shown in Figure 3.26, where we include the identification of the node for the torque disturbance torque T_d.

We can use the flow graph to determine the transfer function $X_1(s)/T_d(s)$. The goal is to reduce the effect of the disturbance T_d, and the transfer function will show us how to accomplish this goal. Using Mason's signal-flow gain formula, we obtain

$$\frac{X_1(s)}{T_d(s)} = \frac{-\frac{r}{J}s^{-2}}{1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2},$$

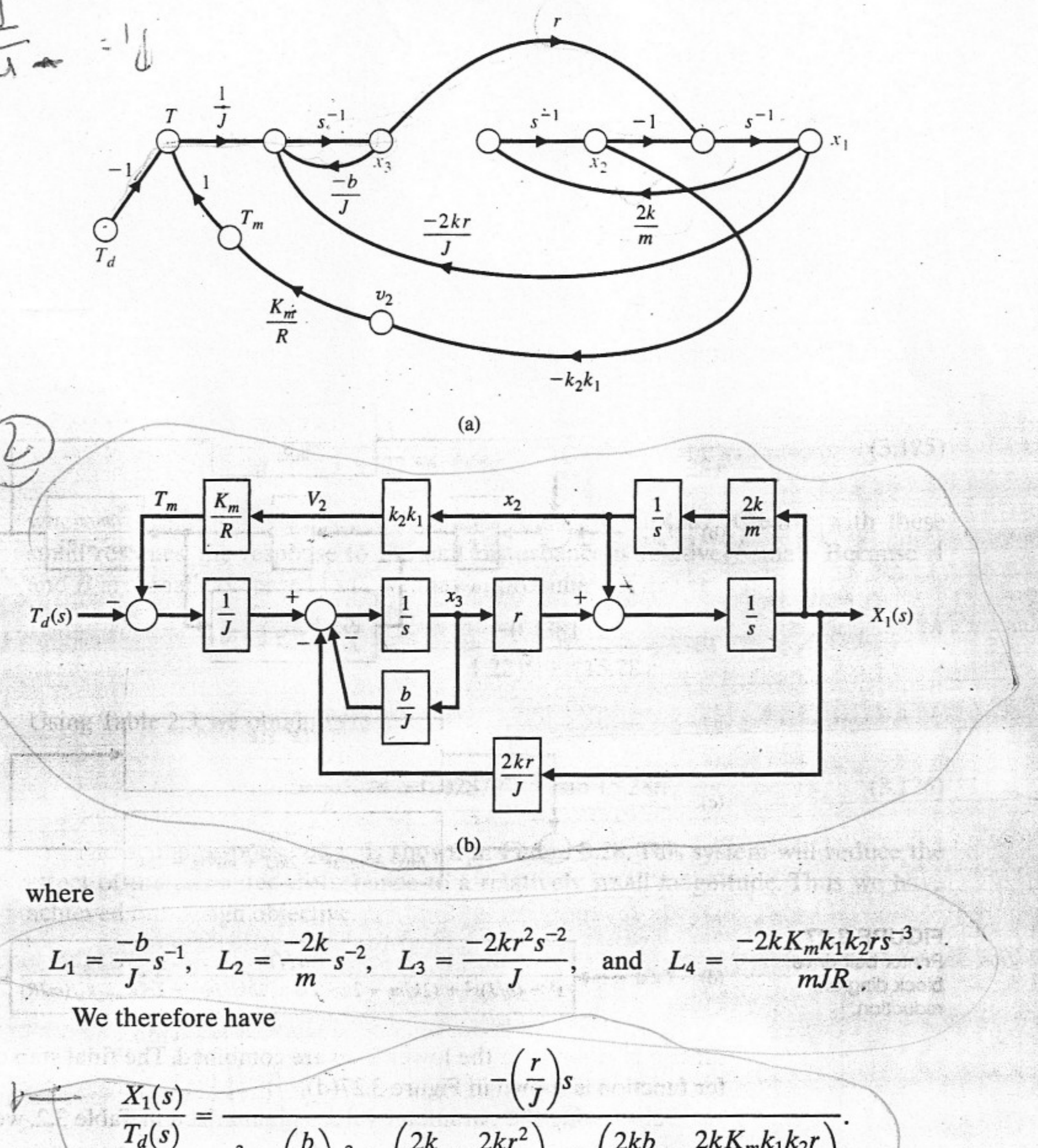

FIGURE 3.26 Printer belt drive. (a) Signal-flow graph. (b) Block diagram model.

where

$$L_1 = \frac{-b}{J}s^{-1},\quad L_2 = \frac{-2k}{m}s^{-2},\quad L_3 = \frac{-2kr^2s^{-2}}{J},\quad \text{and}\quad L_4 = \frac{-2kK_mk_1k_2rs^{-3}}{mJR}.$$

We therefore have

$$\frac{X_1(s)}{T_d(s)} = \frac{-\left(\frac{r}{J}\right)s}{s^3 + \left(\frac{b}{J}\right)s^2 + \left(\frac{2k}{m} + \frac{2kr^2}{J}\right)s + \left(\frac{2kb}{Jm} + \frac{2kK_mk_1k_2r}{JmR}\right)}.$$

We can also determine the closed-loop transfer function using block diagram reduction methods, as illustrated in Figure 3.27. Remember, there is no unique path to follow in reducing the block diagram; however, there is only one correct solution in the end. The original block diagram is shown in Figure 3.26(b). The result of the first step is shown in 3.27(a), where the upper feedback loop has been reduced to a single transfer function. The second step illustrated in Figure 3.27(b) then reduces the two lower feedback loops to a single transfer function. In the third step shown in Figure 3.27(c), the lower feedback loop is closed and then the remaining transfer

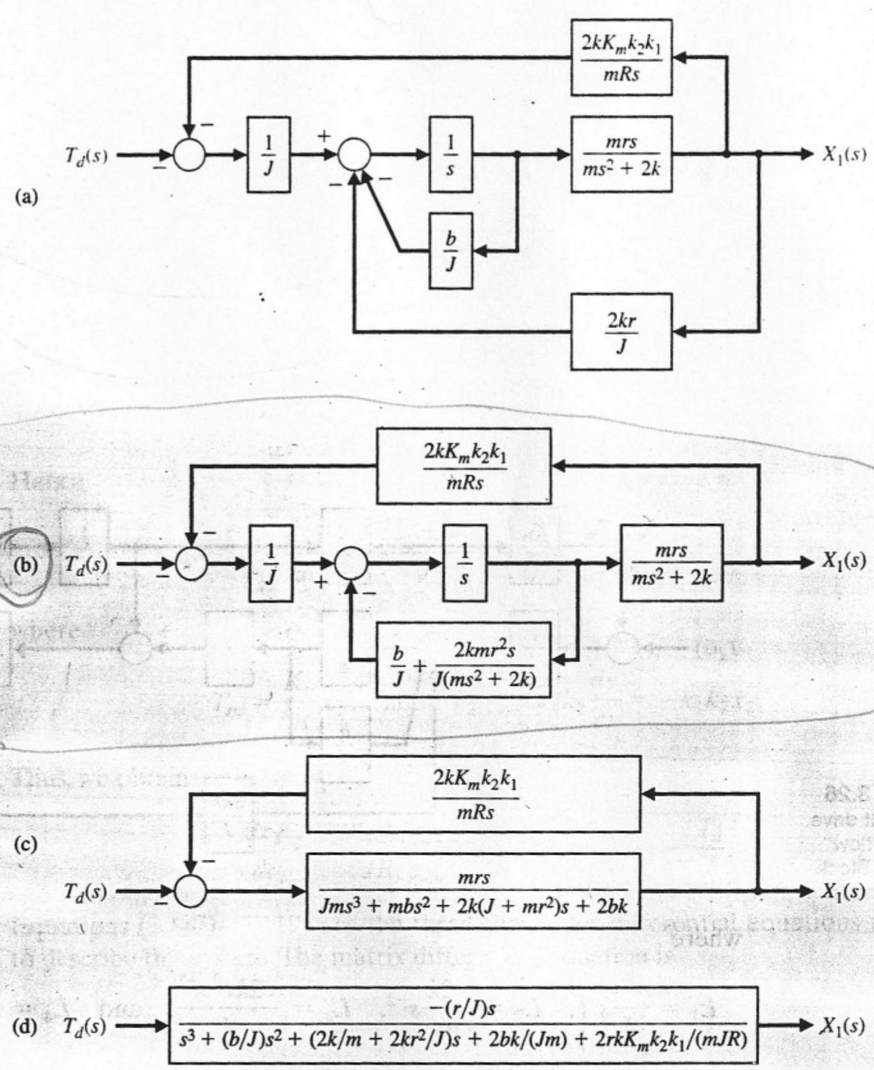

FIGURE 3.27 Printer belt drive block diagram reduction.

functions in series in the lower loop are combined. The final step closed-loop transfer function is shown in Figure 3.27(d).

Substituting the parameter values summarized in Table 3.2, we obtain

$$\frac{X_1(s)}{T_d(s)} = \frac{-15s}{s^3 + 25s^2 + 14.5ks + 1000k(0.25 + 0.15k_2)}. \tag{3.121}$$

We wish to select the spring constant k and the gain k_2 so that the state variable x_1 will quickly decline to a low value when a disturbance occurs. For test purposes, consider a step disturbance $T_d(s) = a/s$. Recalling that $x_1 = r\theta - y$, we thus seek a small magnitude for x_1 so that y is nearly equal to the desired $r\theta$. If we have a perfectly stiff belt with $k \to \infty$, then $y = r\theta$ exactly. With a step disturbance, $T_d(s) = a/s$, we have

$$X_1(s) = \frac{-15a}{s^3 + 25s^2 + 14.5ks + 1000k(0.25 + 0.15k_2)}. \tag{3.122}$$