EEM 323

ELECTROMAGNETIC WAVE THEORY II

PLANE WAVES IN;

LOW-LOSS DIELECTRICS

GOOD CONDUCTORS

2013 – 2014 FALL SEMESTER

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Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

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veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

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PLANE WAVES IN LOSSY MEDIA

Good dielectrics = low loss media
(very small σ)

Good conductors = high loss media
(very large σ)

What we will see in the lecture is;

For both good dielectrics and good conductors, Maxwell's equations can be approximated for obtaining simpler solutions.

For media; neither good conductor nor good dielectric, Maxwell's equations have no simplifications (no approximations).

General Case:
(Neither good conductor nor good dielectric)

$$\bar{J} = \sigma \bar{E}$$

$$\nabla \times \bar{H} = \bar{J} + j\omega\epsilon \bar{E} = (\sigma + j\omega\epsilon)\bar{E}$$

$$= j\omega\underbrace{\left(\epsilon + \frac{\sigma}{j\omega}\right)}_{\epsilon_c}\bar{E}$$

$$\epsilon_c = \epsilon + \frac{\sigma}{j\omega} = \epsilon' - j\epsilon''$$

$$\tan\delta_c = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon}$$

Plane Waves

$$\nabla^2 \bar{E} + k^2 \bar{E} = 0 \qquad k = \omega\sqrt{\mu\epsilon} = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

↑ speed of light in that medium

TEM waves

$$\bar{E}(\bar{R}) = \bar{E}_0 \, e^{-j\bar{k}\cdot\bar{R}} = \bar{E}_0 \cdot e^{-jk\hat{a}_n\cdot\bar{R}} \quad (V/m)$$

$$\bar{H}(\bar{R}) = -\frac{1}{j\omega\mu} \nabla \times \bar{E}(\bar{R})$$

$$\bar{H}(\bar{R}) = \frac{1}{\eta}\hat{a}_n \times \bar{E}(\bar{R}) \qquad \eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\nabla^2 \bar{E} + k_c^2 \bar{E} = 0$$

k_c for conducting media.

$$\gamma = jk_c = j\omega\sqrt{\mu\epsilon_c}$$

$$= \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j\omega\epsilon}\right)^{1/2}$$

lossless media $\sigma = 0$, $\Rightarrow \alpha = 0 \Rightarrow \gamma = j\beta$

$$\beta = k = \omega\sqrt{\mu\epsilon}$$

$$\Rightarrow \nabla^2 \bar{E} + \underset{-\gamma^2}{k^2} \bar{E} = 0$$

$$\bar{E} = \bar{E}_0 \, e^{-\alpha z} \cdot e^{-j\beta z}$$ $+z$ propagation

β; phase constant (rad/m)

α; attenuation constant (Np/m)

$1\ Np \Rightarrow 1m$ kayıp $e^{-1} = 0{,}368$

Good dielectrics = low loss media (very small σ)

$$\gamma = \alpha + j\beta$$

$$\boxed{\alpha \cong \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}}$$

$$\boxed{\beta \cong \omega\sqrt{\mu\epsilon}\left[1 + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]}$$

$$\eta_c \cong \sqrt{\frac{\mu}{\epsilon}}\left(1 + \frac{\sigma}{j\omega\epsilon}\right)^{-1/2}$$

$$\boxed{\eta_c \cong \sqrt{\frac{\mu}{\epsilon}}\left(1 + j\frac{\sigma}{2\omega\epsilon}\right) \quad (\Omega)}$$

Good conductors = high loss media (very large σ)

$$\gamma = \alpha + j\beta$$

$$\boxed{\alpha = \beta = \sqrt{\pi f \mu \sigma}}$$

$$\boxed{\eta_c \cong \frac{\alpha}{\sigma} + j\frac{\alpha}{\sigma} \quad (\Omega)}$$

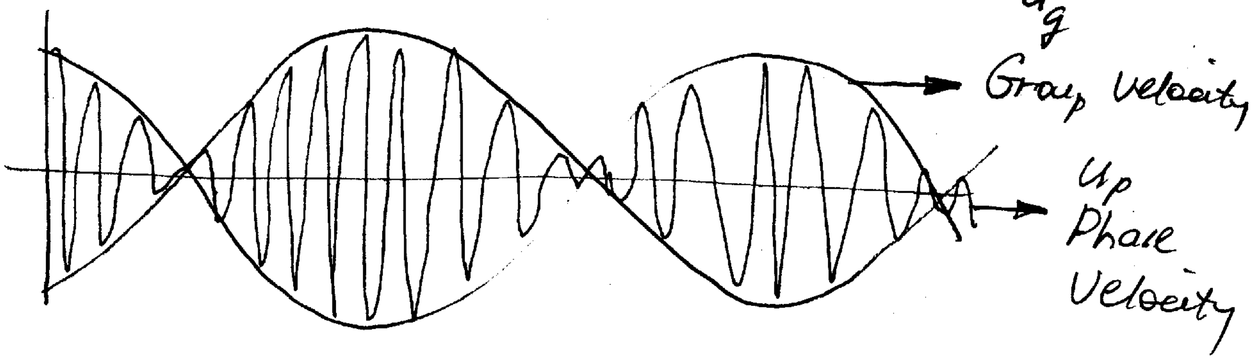
$$\boxed{\lambda = \frac{2\pi}{\beta} = 2\sqrt{\frac{\pi}{f\mu\sigma}} \quad (m)}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad (m)$$ Skin Depth

$$\boxed{\delta = \frac{1}{\beta} = \frac{\lambda}{2\pi} \quad (m)}$$

Group Velocity:

is the velocity of the wave-packet's envelope.



EX two travelling waves ; equal amplitude
slightly different frequencies
$\omega_0+\Delta\omega,\ \omega_0-\Delta\omega$,

$$E(z,t)= E_0 \cdot \cos\left[(\omega_0+\Delta\omega)t - (\beta_0+\Delta\beta)z\right] + E_0 \cos\left[(\omega_0-\Delta\omega)t - (\beta_0-\Delta\beta)z\right]$$

$$\boxed{E(z,t) = 2E_0 \cdot \cos(t.\Delta\omega - z.\Delta\beta) \cdot \cos(\omega_0 t-\beta_0 z)}$$

envelope function. $\Delta\omega \ll \omega_0$ oscillating wave.

Phase Velocity: is the velocity of the phase of the waveform, defined in a specific direction.
For an electric propagating along the z axis, witha radial frequency ω_o, its waveform can be defined as

$$E(z;t) = E_o\cos(\omega_o t - \beta_o z)$$

set $\omega_0 t - \beta_0 z = \text{constant}$

$$\omega_0\, dt - \beta_0\, dz = 0$$

$$\Rightarrow \frac{dz}{dt} = \frac{\omega_0}{\beta_0}$$

$$\boxed{u_p = \frac{dz}{dt} = \frac{\omega_0}{\beta_0}}$$

Let us now calculate the **group velocity** using the envelope function of the waveform.

Set $t.\Delta\omega - z.\Delta\beta = \text{constant}$

$$u_g = \frac{dz}{dt} = \frac{\Delta\omega}{\Delta\beta} = \frac{1}{\Delta\beta/\Delta\omega}$$

$$\boxed{u_g = \frac{1}{d\beta/d\omega}}$$

$\frac{du_p}{d\omega} = 0 \Rightarrow u_g = u_p$ NO DISPERSION

$\frac{du_p}{d\omega} < 0 \Rightarrow u_g < u_p$ NORMAL DISPERSION

$\frac{du_p}{d\omega} > 0 \Rightarrow u_g > u_p$ ANOMALOUS DISPERSION

Electromagnetic Power and the Poynting Vector

$\bar{P} = \bar{E} \times \bar{H}$ (W/m^2) Noktasal

$\oint \bar{E} \times \bar{H} \cdot d\bar{s}$ (W) Toplam Güç ~~Kaybı~~ Kazancı

$\bar{P}_{av} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$ (W/m^2)

ortalama

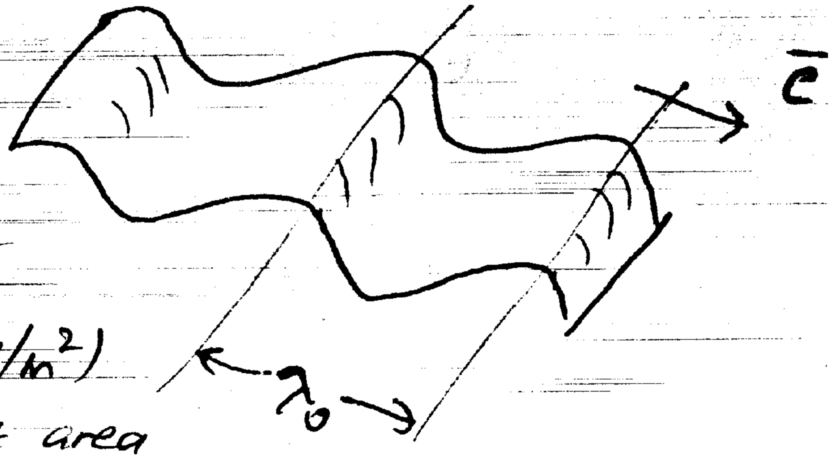
Wavenumber and wave propagation vector:

$|\bar{k}_0| = k_0 = \frac{2\pi}{\lambda_0}$

For Transverse Electromagnetic waves:

TEM wave

$\bar{E} \times \bar{H} \perp \bar{k}, \hat{a}_n$



Poynting Vector

$\bar{E} \times \bar{H} = \bar{\mathcal{P}}$ (W/m^2)

power flow / unit area

$$\bar{\mathcal{P}}_{av} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$$

time average rate of energy flow per unit area.

$$\bar{\mathcal{P}}_{av,n} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\} \cdot \hat{a}_n$$

" in the direction $\hat{a}_n$

Poynting Theorem

$$-\oint \bar{\mathcal{P}} \cdot d\bar{S} = \frac{\partial}{\partial t} \int_V (W_e + W_m)\, dv + \int_V P_\sigma\, dv$$

$W_e = \frac{1}{2} \epsilon E^2$ electric energy density

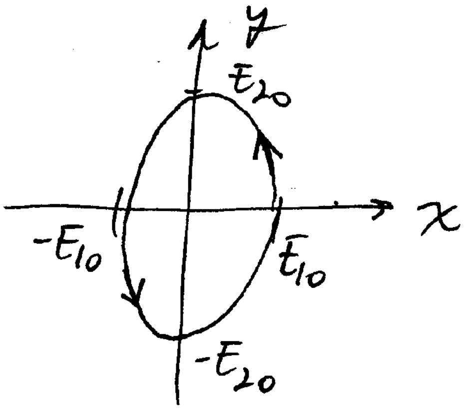
$W_m = \frac{1}{2} \mu H^2$ magnetic energy density

$P_\sigma = \sigma E^2 = J^2/\sigma$: ohmic power density

EXAMPLE (Polarization of EM waves):

$$\bar{E}(z) = \hat{a}_x E_{10} e^{-j\beta z} - \hat{a}_y j E_{20} . e^{-j\beta z}$$

$$\mathcal{E}(z,t)\Big|_{z=0} = \hat{a}_x E_{10} \cos\omega t + \hat{a}_y E_{20} \sin\omega t$$



counter-clockwise

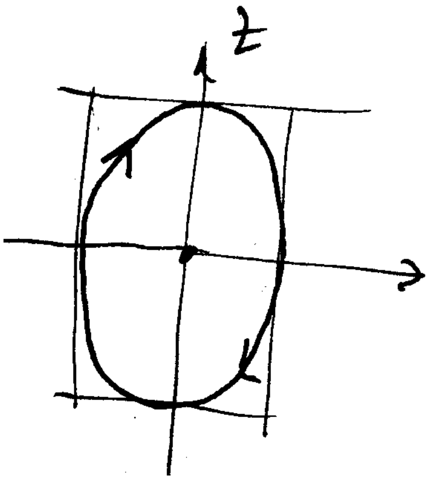
elliptically if $E_{10} \neq E_{20}$

circular if $E_{10} = E_{20}$

EXAMPLE (Polarization of EM waves):

$$\bar{E}(x) = \hat{a}_y E_{10} e^{j\beta x} + \hat{a}_z j . E_{20} e^{j\beta x}$$

$$\mathcal{E}(x,t)\Big|_{x=0} = \hat{a}_y E_{10} \cos\omega t - \hat{a}_z E_{20} . \sin\omega t$$



clockwise

elliptical if $E_{10} \neq E_{20}$

circular if $E_{10} = E_{20}$

Ex Linearly polarized uniform plane wave (+z propagation) in sea water.

$$\bar{E}\Big|_{z=0} = \hat{a}_x \; 100 . \cos(10^7 \pi t) \quad V/m$$

$\epsilon_r = 80$, $\mu_r = 1$ $\sigma = 4$ (S/m) (sea water)

(a) attenuation constant (zayıf. katsayısı)
phase constant, intrinsic impedance, phase velocity
wavelength, skin depth.
(faz katsayısı) (sızma derinliği)

(b) Distance at which $|\bar{E}|$ is 1% of $|\bar{E}|\Big|_{z=0}$.

(c) write $\mathcal{E}(z,t)$ and $\mathcal{H}(z,t)$ at $z = 0.8$ (m).

$$\bar{E}\Big|_{z=0} = \hat{a}_x \; 100 . \cos(\underbrace{10^7 \pi}_{} . t)$$

$$\omega = 10^7 . \pi , \; f = \frac{10^7 . \pi}{2\pi} = 5 . 10^6 \; Hz = 5 \; MHz.$$

$$\epsilon_c = \epsilon - \frac{j\sigma}{\omega} = \epsilon \left(1 + \frac{\sigma}{j\omega\epsilon} \right)$$

$$\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{\omega \epsilon_0 \epsilon_r} = \frac{4}{10^7 . \pi \left(\frac{1}{36\pi} . 10^{-9} \right) 80} = \boxed{180 \gg 1}$$

Good Conductor

(a) Attenuation Constant

$$\alpha = \sqrt{\pi f \mu \sigma} = \sqrt{5\pi . 10^6 . (4\pi . 10^{-7})\, 4} = 8.89 \ (Np/_m)$$

Phase Constant

$$\beta = \sqrt{\pi f \mu \sigma} = \alpha = 8.89 \ (rad/_m)$$

Intrinsic Impedance

$$\eta_c = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\sqrt{\frac{\pi (5 . 10^6)(4\pi . 10^{-7})}{4}}$$

$$\eta_c = \pi . e^{j\pi/4} \ (\Omega)$$

Phase velocity

$$v_p = \frac{\omega}{\beta} = \frac{10^7 . \pi}{8.89} = 3.53 . 10^6 \ (m/s)$$

Wavelength

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{8.89} = 0.707 \ (m.)$$

Skin Depth

$$\delta = \frac{1}{\alpha} = \frac{1}{8.89} = 0.112 \ (m.).$$

(b) Amplitude decreases to 1%. of 1. at $z=0$

$$e^{-\alpha . z_1} = 0.01 \qquad z_1 = \frac{1}{\alpha} \ln 100 = \frac{4.605}{8.89}$$

$$e^{+\alpha . z_1} = 100 \qquad \boxed{z_1 = 0.518 \text{ m.}}$$

(c) Phasor Notation $\quad \bar{E}(z) = \hat{a}_x \, 100 . \, e^{-\alpha z} . \, e^{-j\beta z}$

$$\bar{\mathcal{E}}(z,t) = Re\{ \bar{E}(z) \, e^{j\omega t} \}$$

$$= Re\{ \hat{a}_x \, 100 . \, e^{-\alpha z} . \, e^{-j\beta z} . \, e^{j\omega t} \}$$

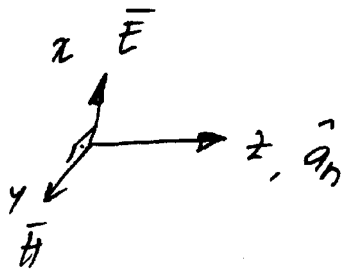
$$= \hat{a}_x \, 100 . \, e^{-\alpha z} . \cos(\omega t - \beta z)$$

At $z = 0.8$ m.

$$\bar{E}(0.8, t) = \hat{a}_x . \, 100 . \, e^{-0.8\alpha} . \cos(10^7 \pi t - 0.8\beta)$$

$$\boxed{\bar{E}(0.8,t) = \hat{a}_x \; 0.082 . \cos(10^7 . \pi t - 7.11) \quad V/m}$$

$\mathcal{H} = ?$ $\quad \bar{H} = ?$



$\bar{H} \perp \bar{E}$ ve $\bar{H} \perp \hat{a}_n$

$\Rightarrow \bar{H} = \hat{a}_y H_y$.

NOTE: $\mathcal{H}_y(z,t) \neq \mathcal{E}_x(z,t) / \eta_c$ complex

$$H_y(z) = \frac{E_x(z)}{\eta_c} \qquad \mathcal{H}_y(z,t) = Re\left\{ \frac{E_x(z)}{\eta_c} e^{j\omega t} \right\}$$

$f = 5$ MHz. in sea water.

$z=0$ --- 100
$z=0.8$ -- 0.082
$z=0.518$ 1.

Loss is even higher at higher frequencies.

Phasors

$$H_y(z=0.8) = \frac{100 \cdot e^{-0.8\alpha} \, e^{-j0.8\beta}}{\pi \cdot e^{j\pi/4}} = \frac{0.082 \cdot e^{-j7.11}}{\pi \cdot e^{j\pi/4}}$$

$$H_y(z=0.8) = 0.026 \cdot e^{-j1.61}$$

$$\bar{H}(z=0.8) = \hat{a}_y \; 0.026 \cdot e^{-j1.61}$$

$$\boxed{\bar{\mathcal{H}}(z=0.8,t) = \hat{a}_y \; 0.026 \cdot \cos(10^7 \cdot \pi \cdot t - 1.61) \quad (A/m)}$$

$$\bar{P} = \bar{E} \times \bar{H} \quad (W/m^2)$$

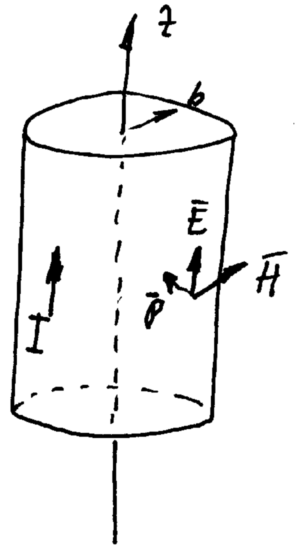
Phasor

Poynting Vector

Power Flow / unit area

$$\boxed{-\oint_S \bar{P} \cdot d\bar{s} = \frac{\partial}{\partial t} \int_V (\underbrace{w_e}_{\frac{1}{2}\epsilon E^2} + \underbrace{w_m}_{\frac{1}{2}\mu H^2}) \, dv + \int_V \underbrace{p_\sigma}_{\sigma E^2 = J^2/\sigma} \, dv}$$

Poynting's Theorem



EX Verify Poynting's Theorem.

Cylinder of radius b,
conductivity σ
total uniform current I

DC situation

current is uniform $\Rightarrow \bar{J} = \hat{a}_z \dfrac{I}{\pi b^2}$

$$\Rightarrow \bar{E} = \frac{\bar{J}}{\sigma} = \hat{a}_z \frac{I}{\sigma \pi b^2}$$

On the surface of the wire

$$\bar{H} = \hat{a}_\phi \frac{I}{2\pi b}$$

Poynting vector on the surface

$$\bar{P} = \bar{E} \times \bar{H} = (\hat{a}_z \times \hat{a}_\phi) \frac{I^2}{2\sigma \pi^2 b^3} = -\hat{a}_r \frac{I^2}{2\sigma \pi^2 b^3}$$

$$-\oint_S \bar{P} \cdot \bar{ds} = -\oint_S \bar{P} \cdot \hat{a}_r \, ds = \frac{I^2}{2\sigma \pi^2 b^3} \cdot 2\pi b \cdot \ell$$

cylinder length of ℓ.

$$\boxed{-\oint_S \bar{P} \cdot \bar{ds} = I^2 \left(\frac{\ell}{\sigma \pi b^2}\right) = I^2 R}$$

$R = \ell / \sigma S$.

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Instantaneous Power (anlık ~~güç~~
Average Power Density ve ~~ortalama~~ güç yoğunlukları)
ortalama

$$\bar{E}(z) = \hat{a}_x\, E_x(z) = \hat{a}_x E_0\, e^{-(\alpha+j\beta)z}$$

$$\Rightarrow \bar{\mathcal{E}}(z,t) = Re\{\bar{E}(z)\, e^{j\omega t}\} = \hat{a}_x\, E_0\, e^{-\alpha z}\, Re\{e^{j(\omega t-\beta z)}\}$$

$$\boxed{\bar{\mathcal{E}}(z,t) = \hat{a}_x\, E_0\, e^{-\alpha z} \cdot \cos(\omega t-\beta z)}$$

Uniform Plane Wave
$+z$ propagation $(\hat{a}_n = \hat{a}_z)$

$$\Rightarrow \bar{H}(z) = \hat{a}_y\, H_y(z) = \hat{a}_y \frac{E_0}{|\eta|}\, e^{-\alpha z} \cdot e^{-j(\beta z+\theta_\eta)}$$

$$\eta = |\eta| \cdot e^{j\theta_\eta}$$

$$\Rightarrow \bar{\mathcal{H}}(z,t) = Re\{\bar{H}(z)\, e^{j\omega t}\}$$

$$\boxed{\bar{\mathcal{H}}(z,t) = \hat{a}_y \frac{E_0}{|\eta|}\, e^{-\alpha z} \cdot \cos(\omega t-\beta z-\theta_\eta)}$$

$$\boxed{\bar{\mathcal{P}}(z,t) = \bar{\mathcal{E}}(z,t) \times \bar{\mathcal{H}}(z,t)}$$

Anlık Güç Yoğunluğu
Instantaneous Power Density

$$\bar{\mathcal{P}}(z,t) = \hat{a}_z \frac{E_0^2}{2|\eta|} \cdot e^{-2\alpha z} \left[\cos\theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)\right]$$

NOTE: $\mathrm{Re}\{\bar{E}(z)e^{j\omega t}\} \times \mathrm{Re}\{\bar{H}(z)e^{j\omega t}\} \neq \mathrm{Re}\{\bar{E}(z)\times\bar{H}(z)\cdot e^{j\omega t}\}$

Time-average Poynting Vector $\bar{\mathcal{P}}_{av}(z)$

$$\boxed{\boxed{\bar{\mathcal{P}}_{av}(z) = \frac{1}{T}\int_0^T \bar{\mathcal{P}}(z,t)\,dt} = \hat{a}_z \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cdot \cos\theta_\eta \quad (W/m^2)}$$

$$T = \frac{2\pi}{\omega}$$

Similarly

$$\boxed{\bar{\mathcal{P}}_{av}(z) = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\}} \quad (W/m^2)$$

Power and Poynting Vector

$\bar{\mathcal{P}} = \bar{E}\times\bar{H} \quad (W/m^2)$ Power flow/unit area

$$-\oint_S \bar{\mathcal{P}}\cdot d\bar{s} = \frac{\partial}{\partial t}\int_v (w_e + w_m)\,dv + \int_v p_\sigma\,dv$$

$$w_e = \frac{1}{2}\epsilon E^2$$

$$w_m = \frac{1}{2}\mu H^2$$

$$p_\sigma = \sigma E^2 = J^2/\sigma$$

average power density

$$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\} \quad (W/m^2)$$