EEM 323

ELECTROMAGNETIC WAVE THEORY II

PLANE ELECTROMAGNETIC WAVE PROPAGATION – 2

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Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

TIME-HARMONIC FIELDS

EXAMPLE

Given the electric field is x polarized and propagating in the +z axis. Solve vector wave equation and comment on your results.

$$\nabla^2 \bar{E} + k^2 \bar{E} = 0$$

k: wavenumber

$$k = w\sqrt{\mu\epsilon} = w/v_p \quad (rad/m)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) E_x = 0$$

scalar

Scalar wave equation (simple media)

E is constant along x axis

E varies only along the z axis (propagation)

$$\frac{\partial E_x}{\partial x} = 0 \qquad \frac{\partial E_x}{\partial y} = 0$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0}$$

Then the solution is.

$$E_x(z) = E_x^+(z) + E_x^-(z)$$

$$= E_o^+ e^{-jkz} + E_o^- e^{+jkz}$$

arbitrary constants

$$\bar{E}(z) = \underbrace{\hat{a}_x E_x^+(z)}_{+z \text{ travelling wave}} + \underbrace{\hat{a}_x E_x^-(z)}_{-z \text{ travelling wave}}$$

Let us analyze the first term

$$\bar{E}(z) = \hat{a}_x E_x^+(z) = \hat{a}_x E_o^+ e^{jkz}$$

$$\bar{E}(z,t) = \hat{a}_x E_x^+(z,t) = \hat{a}_x \, Re\left[E_x^+(z) e^{jwt}\right]$$

$$= \hat{a}_x \, Re\left[E_o^+ e^{j(wt-kz)}\right]$$

$$= \hat{a}_x E_o^+ \cos(wt-kz)$$

<u>Homework</u> Plot $E(z,t)$ and $H(z,t)$ as a fcn of z for $t=0, \frac{\pi}{2w}, \frac{\pi}{w}$

You will realize that EM wave is traveling in $+z$ direction.

Now, calculate its <u>phase velocity</u>:

$$\text{Phase} = wt - kz$$

$$\text{Phase velocity} = u_p = ?$$

For a reference point <u>traveling with</u> the wave

$$\text{Phase} = \text{constant} = wt - kz$$

$$\frac{d}{dt}(\text{Phase}) = \underbrace{\frac{d}{dt}(wt - kz)}_{w - k\underbrace{\frac{dz}{dt}}_{u_p}} = 0$$

$$u_p = \frac{dz}{dt} = \frac{w}{k} = \frac{1}{\sqrt{\mu\epsilon}} \quad \text{Phase velocity}$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c \quad \text{speed of light in vacuum.}$$

Note that,

$$u_p = \underset{\text{frequency}}{f}\ \underset{\text{wavelength}}{\lambda} \quad \leftarrow \text{Phase velocity}$$

$$\frac{w}{k} = \underset{w/2\pi}{f}\ \lambda \quad ,$$

$$\boxed{k = \frac{2\pi}{\lambda}} \quad \text{wave number.}$$

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HOMEWORK Recalculate all values for $-z$ traveling wave.

EXAMPLE Now, calculate $\overline{H}$

$$\nabla \times \overline{E} = -jw\mu\, \overline{H}$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & \partial/\partial z \\ E_x^+(z) & 0 & 0 \end{vmatrix} = -jw\mu\left(\hat{a}_x H_x^+ + \hat{a}_y H_y^+ + \hat{a}_z H_z^+\right)$$

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$$\nabla \times \bar{E} = \hat{a}_x (0) - \hat{a}_y \left(-\frac{\partial E_x^+(z)}{\partial z}\right) + \hat{a}_z (0)$$

$$H_x^+ = 0$$

$$H_y^+ = \frac{1}{-j\omega\mu} \frac{\partial E_x^+(z)}{\partial z}$$

$$H_z^+ = 0$$

$$\frac{\partial E_x^+(z)}{\partial z} = \frac{\partial}{\partial z}\left(E_0^+ e^{-jkz}\right) = -jk \underbrace{E_0^+ e^{-jkz}}_{E_x^+(z)}$$

$$\bar{H} = \hat{a}_y H_y^+(z) = \hat{a}_y \underbrace{\frac{k}{\omega\mu}}_{1/\eta} E_x^+(z)$$

$$\boxed{\bar{H} = \hat{a}_y \frac{1}{\eta} E_x^+(z)}$$

$$\eta = \frac{\omega\mu}{k} = \frac{\cancel{\omega}\mu}{\cancel{\omega}\sqrt{\mu\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$$ Intrinsic Impedance

$$\boxed{\bar{H}(z,t) = \hat{a}_y \frac{E_0^+}{\eta} \cos(\omega t - kz)}$$

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EXAMPLE

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Given uniform plane wave

$\bar{E} = \hat{a}_x E_x$ propagating in a lossless simple medium in $+z$-direction.

$(\epsilon_r = 4,\ \mu_r = 1,\ \sigma = 0)$

Frequency = 100 MHz

E_x is sinusoidal having a maximum value of 10^{-4} (V/m) at $t = 0$ and $z = 1/8$ (m).

a) Calculate $\bar{E}(z,t)$

b) $\bar{H}(z,t)$

c) Determine the locations where E_x is a positive maximum when $t = 10^{-8}$ (s).

$$k = \omega\sqrt{\mu\epsilon} = \omega \underbrace{\sqrt{\mu_0\epsilon_0}}_{1/c} \underbrace{\sqrt{\mu_r\epsilon_r}}_{4}$$

$$k = \frac{2\pi \cdot 10^8}{3 \cdot 10^8}\sqrt{4} = \frac{4\pi}{3} \quad (\text{rad/m})$$

a) Select $\cos(\omega t)$ as the reference, 95

$$\bar{E}(z,t) = \hat{a}_x E_x = \hat{a}_x 10^{-4} \cos(2\pi 10^8 t - kz + \psi)$$

unknown phase constant

Given $E(z,t)\Big|_{\substack{t=0 \\ z=1/8}} = +10^{-4}$

$\cos(\omega t + \psi)$ is maximum

$$\Rightarrow \underset{\substack{\uparrow \\ 0}}{\omega t} - k\underset{\substack{\uparrow \\ 1/8}}{z} + \psi = 0$$

$$\Rightarrow \psi = kz = \left(\frac{4\pi}{3}\right)\left(\frac{1}{8}\right) = \frac{\pi}{6} \ (\text{rad})$$

$$\bar{E}(z,t) = \hat{a}_x 10^{-4} \cos\left(2\pi 10^8 t - \frac{4\pi}{3} z + \frac{\pi}{6}\right)$$

(V/m)

b) $\bar{H} = \hat{a}_y H_y = \hat{a}_y \dfrac{E_x}{\eta}$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{\frac{\mu_r}{\epsilon_r}} = \underbrace{\sqrt{\frac{\mu_0}{\epsilon_0}}}_{\eta_0} \frac{1}{\sqrt{\epsilon_r}} = \frac{\overset{120\pi}{\eta_0}}{\underbrace{\sqrt{\epsilon_r}}_{2}}$$

$\eta = 60\pi$

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$$\bar{H}(z,t) = \hat{a}_y \frac{10^{-4}}{60\pi} \cos\left[2\pi 10^8 t - \frac{4\pi}{3}\left(z - \frac{1}{8}\right)\right] \ (A/m)$$

c) Given $t = 10^{-8}$ (s) Cosine fcn = 1

Then its phase should equal to $\pm 2n\pi$

$$2\pi\, 10^8 (10^{-8}) - \frac{4\pi}{3}\left(z_m - \frac{1}{8}\right) = \pm 2n\pi$$

$$z_m = \frac{13}{8} \pm \frac{3}{2} n \ \ (m) \qquad n = 0, 1, 2, \quad z_m > 0$$

$$\boxed{\lambda = \frac{2\pi}{k} = \frac{2\pi}{(4\pi/3)} = \frac{3}{2} \ (m)}$$

$$\boxed{z_m = \frac{13}{8} \pm n\lambda \ \ (m)}$$

Next:

Doppler effect

Polarization of EM waves