EEM 323

ELECTROMAGNETIC WAVE THEORY II

FARADAY INDUCTION

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Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

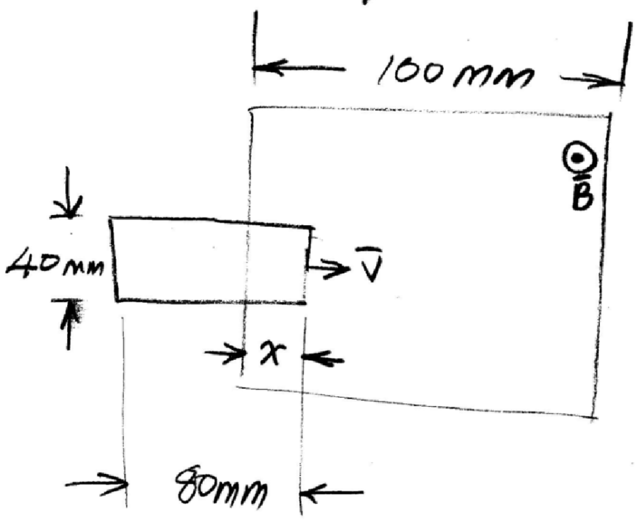
[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

FLUX CUTTING EMF

68A

FARADAY INDUCTION

EXAMPLE (Flux cutting Emf)



Given $\bar{B} = 250$ mT. inside large coil.

$V = 125$ mm/sec (smaller coil)

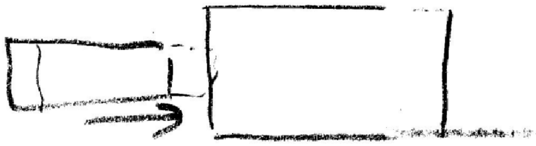
Find Emf, plot Emf vs. distance x in $-20 \le x \le 120$ mm.

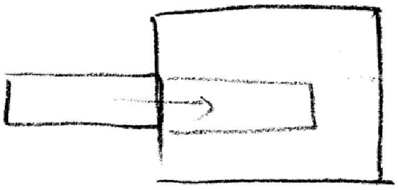
$$\text{Emf} = \mathcal{V} = -\frac{d\Phi}{dt} = \oint \bar{E} \cdot d\bar{l} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$$

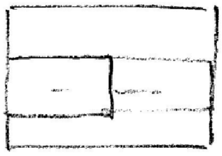
Analyze what happens when x is increased from $x = -20$ to $x = 120$?

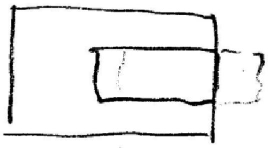
How many regions do you think we would observe?

68B

A  $-20 \le x \le 0$ mm.

B  $0 \le x \le 80$ mm.

C  $80 \le x \le 100$ mm

D  $100 \le x \le 120$

A: $\Phi = 0$ $\quad$ $EMF = 0$ $\quad$ $125 \cdot 10^{-3}$

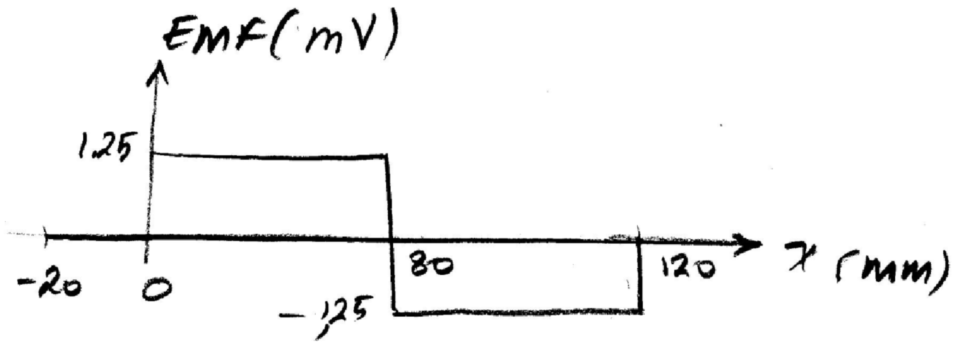
B: $EMF = -\dfrac{d\Phi}{dt} = B\,h\,\dfrac{dx}{dt} = 250 \cdot 10^{-3}\; 40 \cdot 10^{-3}$

$= 12500\, 10^{-3}\, (V)$

$= 1.25\, (mV)$

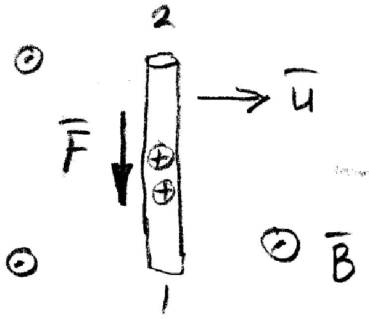
C: $\Phi = \text{constant}$ $\quad$ $EMF = \dfrac{d\Phi}{dt} = 0$

D: $EMF = -1.25\, (mV)$



MOTIONAL EMF

69B



$\bar{F}_c$: Coulombian force of attraction due to the velocity of charges in static magnetic field.

$$\bar{F}_c = q(\bar{u} \times \bar{B})$$

Then $(\bar{F} = q\bar{E})$

$$\bar{E} = \bar{u} \times \bar{B}$$

We now can find Emf due to f_c.

$$\boxed{Emf_c = V_c = \int_c \bar{E} \cdot d\bar{l} = \int_1^2 (\bar{u} \times \bar{B}) \cdot d\bar{l}}$$

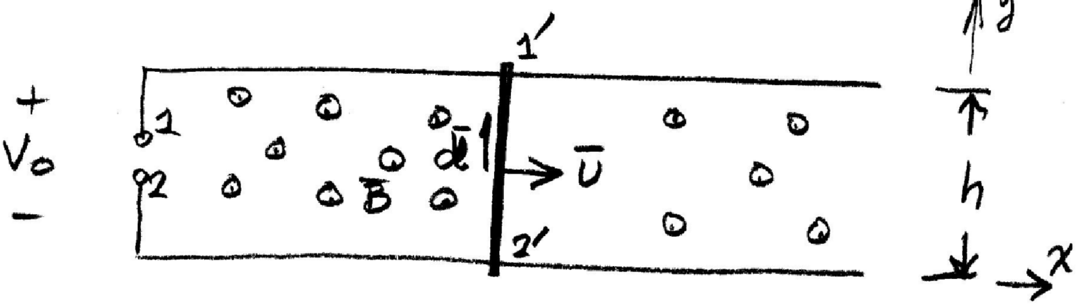
For closed circuits

$$\boxed{V_c = \oint (\bar{u} \times \bar{B}) \cdot d\bar{l} \quad (V)}$$

EXAMPLE Moving conductor 69C

$$\text{Emf}_m = v = \oint_c (\bar{u} \times \bar{B}) \cdot d\bar{l} \quad (\text{Volts})$$

assume $\bar{B}$ is constant



Given $\bar{B} = \hat{a}_z B_o$

$\bar{u}$: velocity of the bar

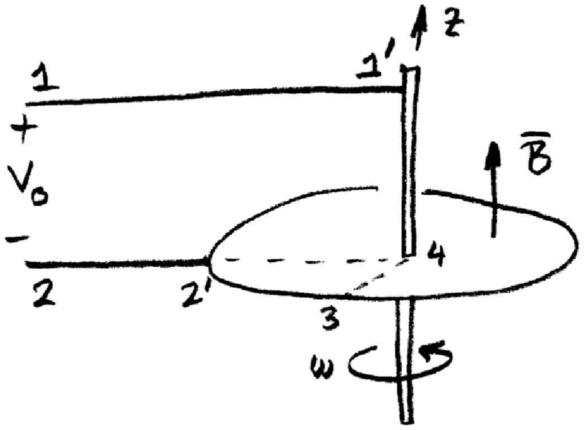
$V_o = ?$, P_e (electric power) ?

$$V_o = \oint_c (\bar{u} \times \bar{B}) \cdot d\bar{l} = \int_{2'}^{1'} \underbrace{(\hat{a}_x \bar{u} \times \hat{a}_z B_o)}_{-uB_o \hat{a}_y} \cdot \hat{a}_y \, dl$$

$$= -uB_o h$$

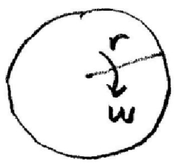
$$P_e = I^2 R = \left(\frac{uB_o h}{R}\right)^2 R = \underbrace{\frac{(uB_o h)^2}{R}}_{V_o^2/R} \quad (W)$$

EXAMPLE Faraday's Disc Generator



$$\bar{B} = \hat{a}_z B_o \qquad V_o = ?$$

$$V_o = \oint (\bar{u} \times \bar{B}) \cdot \bar{dl}$$



$$= \int_3^4 \Big(\big[\underbrace{\hat{a}_\phi (r w)}_{u} \big] \times \hat{a}_z B_o \Big) \cdot \hat{a}_r \, dr$$

In 1 secs. f revolutions per sec

Distance: $2\pi r$

Then $2\pi r . f = r(2\pi f) = r w$

$$V_o = w B_o \int_b^0 r \, dr$$

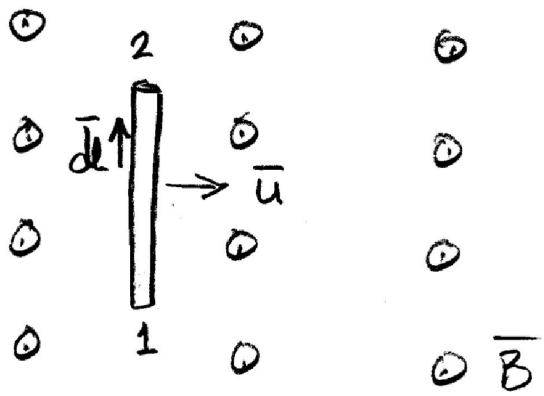
$$= -\frac{w B_o b^2}{2} \quad \text{(Volts)}.$$

TOTAL ELECTROMOTIVE FORCE (EMF)

EMF = Flux derivative EMF $\rightarrow -\dfrac{d\Phi}{dt}$

+ Motional EMF $\rightarrow$? (Assume Φ is constant)

MOTIONAL ELECTROMOTIVE FORCE (EMF)



$\bar{B}$: magnetic flux density = constant!

$\bar{u}$: velocity of the conducting wire

Note: conductor carries both (+) and (−) charges!

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EMF = Electromotive force is due to;

1) Time varying $B \Rightarrow$ $\text{EMF} = -\dfrac{d\Phi}{dt}$

2) Coulomb force of attraction due to charges with non zero velocity

$$\Rightarrow \text{EMF} = \oint (\bar{u} \times \bar{B}) \cdot d\bar{l}$$

Question

What if we observe both cases at the same time?

$\Rightarrow$ Add them together. (Let us rename them)

$$\text{EMF}_{total} = V_{tot} = \begin{array}{l}\text{Induced EMF} \\ + \text{Motional EMF}\end{array}$$

$$V_{total} = \underbrace{V_{induced}}_{-\int_s \frac{\partial B}{\partial t} \cdot d\bar{s}} + \underbrace{V_{motional}}_{\oint_c (\bar{u} \times \bar{B}) \cdot d\bar{l}}$$

$$\boxed{V_{tot} = V_{ind} + V_{mot}} \quad (V)$$

71 B

ⓐ Calculate $Emf_{induced}$ when the loop is at rest when its rotated α with the z-axis.

ⓑ Calculate Emf motional when the loop is rotating w about the x-axis.

ⓒ Calculate total Emf when loop is rotating and B is time varying.

Given loop is h by W, and

$$\bar{B} = \hat{a}_y \, B_0 \sin(wt).$$

$$\Phi = B_0 \, h \, W \sin(wt) \cdot \cos\alpha$$

$$V_{ind} = Emf_{ind} = \frac{-d\Phi}{dt} = -B_0 \, \underbrace{hW}_{S} \, w \cos(wt) \cos\alpha$$

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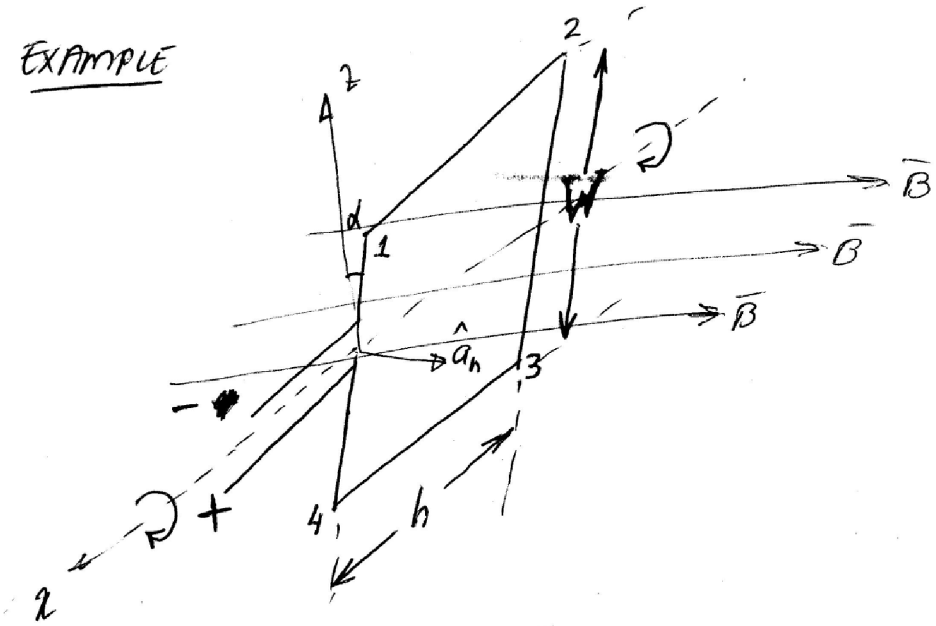
EXAMPLE



(h) by (W) rectangular conducting loop

$\bar{B} = \hat{a}_y \, B_0 . \sin wt$

71 B

ⓐ Calculate $Emf_{induced}$ when the loop is at rest when its rotated α with the z-axis.

ⓑ Calculate Emf motional when the loop is rotating w about the x-axis.

ⓒ Calculate total Emf when loop is rotating and B is time varying.

Given loop is h by W, and

$\bar{B} = \hat{a}_y \, B_0 \sin(wt)$.

(a) $\Phi = \int \bar{B} \cdot d\bar{s} = \int \left(\hat{a}_y B_0 \sin wt \cdot \hat{a}_n \, ds \right)$

$$\hat{a}_y \cdot \hat{a}_n = \cos\alpha$$

$$\Phi = B_0 \, h \, W \, \sin(wt) \cdot \cos\alpha$$

$$V_{ind} = EMF_{ind} = \frac{-d\Phi}{dt} = -B_0 \, \overset{S}{hW} \, w \cos(wt) \cos\alpha$$

(b)

$$V_{mot} = \oint (\bar{v} \times \bar{B}) \cdot d\bar{l}$$

$$\int_2^1 \left[\left(\hat{a}_n \frac{W}{2} w \right) \times \left(\hat{a}_y B_0 \sin wt \right) \right] \cdot \left(\hat{a}_x \, dx \right)$$

$$+ \int_3^4 \left[\left(-\hat{a}_n \frac{W}{2} w \right) \times \left(\hat{a}_y B_0 \sin wt \right) \right] \cdot \left(\hat{a}_x \, dx \right)$$

$$= 2 \left(\frac{W}{2} w B_0 \cdot \sin wt \cdot \sin\alpha \right) h$$

Sides (2-3) and (4-1) do not contribute!..

$$\boxed{V_{mot} = B_0 w (hW) \cdot \sin wt \cdot \sin\alpha}$$

(c) TOTAL EMF

72B

$$Emf_{total} = Emf_{ind} + Emf_{mot}$$

$$= B_0 (hW)\, w \sin(wt)(\sin wt \sin\alpha - \cos wt \cos\alpha)$$

What happens if $wt = \alpha$?

$$Emf_{tot} = -B_0 (hW) w \left[\underbrace{\cos^2(wt) - \sin^2(wt)}_{\cos(2wt)} \right]$$

$$= -B_0 (hW)\, w \cos(2wt)$$

Alternative Solution for (c)

Do not assume B constant and motionless,

Do not separate the problem into two!

Assume B is time-varying, flux is time varying and there is motion!

$$\Phi(t) = \bar{B}(t) \cdot [\hat{a}_n(t)\, \overset{hW}{S}]$$

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$$\bar{B}(t) = \hat{a}_y B_o \sin wt$$

$$\hat{a}_y \cdot \hat{a}_n = \cos \alpha \quad , \quad \alpha = wt$$

$$\Phi(t) = B_o S \sin wt \cos wt$$

$$= \frac{1}{2} B_o S \sin(2wt)$$

$$Emf_{tot} = -\frac{d\Phi(t)}{dt} = -\frac{d}{dt}\left(\frac{1}{2} B_o S \sin 2wt\right)$$

$$= -B_o S w . \cos(2wt)$$