EEM 323

ELECTROMAGNETIC WAVE THEORY II

TIME VARYING FIELDS

2013 – 2014 FALL SEMESTER

Prof. S. Gökhun Tanyer

DEPARTMENT OF ELECTRICAL–ELECTRONICS ENGINEERING

FACULTY OF ENGINEERING, BASKENT UNIVERSITY

Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Static $\bar{E}$	Static $\bar{H}$
$\nabla \times \bar{E} = 0$	$\nabla \cdot \bar{B} = 0$
$\nabla \cdot \bar{D} = \rho$	$\nabla \times \bar{H} = \bar{J}$
$\bar{D} = \epsilon \bar{E}$ (linear media)	$\bar{B} = \mu \bar{H}$

Static CASE
Uncoupled Equations

ρ constant $\longrightarrow$ $\bar{E}$ $\longleftarrow$ unrelated

$\bar{J}$ constant $\longrightarrow$ $\bar{H}$ $\longleftarrow$ independent

ρ : time varying $\longrightarrow$ $\bar{E}$ is time varying

$\bar{J}$: time varying $\longrightarrow$ $\bar{H}$ is time varying

$$\bar{E} \rightleftarrows \bar{H}$$

E and H are governed by coupled equations.

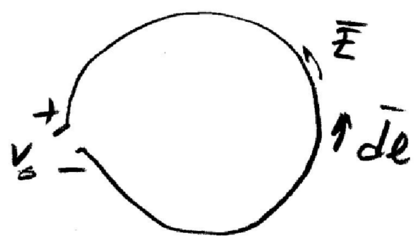
EMF - Electromotive Force (V) AND FARADAY'S LAW OF INDUCTION

Static E and H

$$\boxed{\oint \bar{E}\cdot d\bar{l} = 0}$$

E is conservative.

$\Rightarrow V_0 = 0$



Faraday's Induction Law

$$\boxed{\nabla \times \bar{E} = -\frac{\partial}{\partial t}\bar{B}} \quad \text{---}(1)$$

$\nabla \times \bar{E} \neq 0 \Rightarrow \bar{E} \neq \nabla V$

Why? $\Rightarrow \nabla \times \nabla V = 0$!

FARADAY LAW OF EM INDUCTION

Integrate Eqn. (1)

$$\int_S \nabla \times \bar{E}\cdot d\bar{s} = \boxed{\oint \bar{E}\cdot d\bar{l} = \underbrace{-\int_S \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}}_{-N\frac{d\Phi}{dt}}}$$

Flux: $\Phi = \int \bar{B}\cdot d\bar{s} \Rightarrow -N\dfrac{d\Phi}{dt}$

6TH

EXAMPLE

Stationary Circuit – Time-varying Magnetic Field

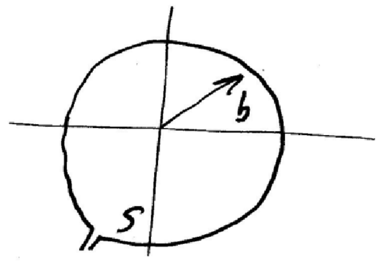
$$V = \oint_c \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_s \bar{B} \cdot d\bar{s}$$

EMF

$$\boxed{EMF, V = -\frac{d\Phi}{dt}}$$

* Induction Motor
* Current measurement loop – clamper

OR loop of N turns, xy plane,



$$\bar{B} = \hat{a}_z B_0 \cdot \cos\left(\frac{\pi}{2b} r\right) \sin(\omega t)$$

EMF=?

$$\Phi = \int_s \bar{B} \cdot d\bar{s} = \int_0^b \left(\hat{a}_z B_0 \cdot \cos\left(\frac{\pi}{2b} r\right) \cdot \sin \omega t\right) \cdot \left(\hat{a}_z 2\pi r \, dr\right)$$

$$= \frac{8b^2}{\pi}\left(\frac{\pi}{2} - 1\right) B_0 \cdot \sin(\omega t)$$

N turns ...

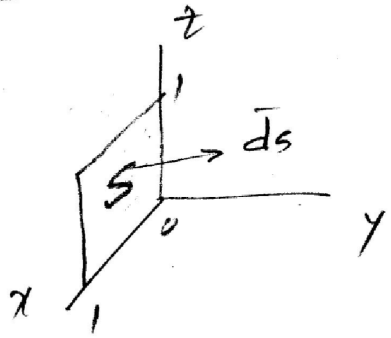
$$V = -N \frac{d\Phi}{dt}$$

$$\boxed{EMF, V = -\frac{8N}{\pi} b^2 \left(\frac{\pi}{2} - 1\right) B_0 \cdot \omega \cdot \cos(\omega t) \quad (V)}$$

EXAMPLE

loop of N turns on the x-z plane

$$\bar{B} = \hat{a}_z B_0 \cos\left(\frac{\pi}{2b} r\right) \sin(\omega t)$$



$$\bar{ds} = \hat{a}_y \, dx \, dz$$

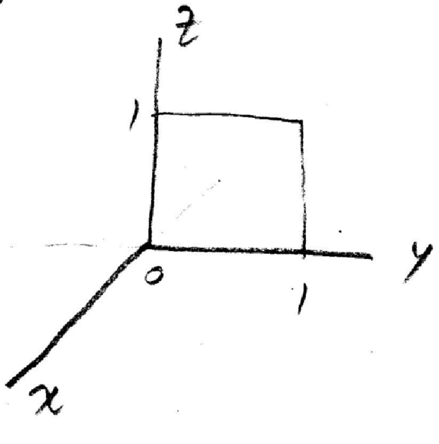
$$\bar{B} \cdot \bar{ds} = 0$$

$$\Phi = \int_S \bar{B} \cdot \bar{ds} = 0 \quad (V)$$

No induction

EXAMPLE

N turns, S on the y-z plane



$$\bar{B} = \hat{a}_x B_0 \sin(\omega t) + \hat{a}_y B_1 \sin(\omega t) + \hat{a}_z B_2 \sin(\omega t)$$

$$\bar{ds} = \hat{a}_x \, dy \, dz$$

67C

$$\bar{B}\cdot\bar{ds} = \bar{B}\cdot\hat{e}_x\, dy\, dz$$

$$= B_0 \sin(wt)\, dy\, dz$$

$$\Phi = \int_s \bar{B}\cdot\bar{ds}$$

$$= \int_0^1\int_0^1 B_0 \sin(wt)\, dy\, dz$$

$$= B_0 \sin(wt)$$

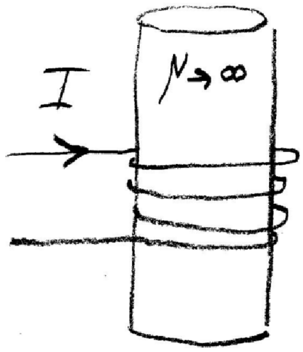
$$\text{EMF} = V = -N \frac{d\Phi}{dt} = -wNB_0 \cos(wt)\ (V)$$

67D

Exercise 6.1 Omitted lecture notes

$$\int \bar{B} \cdot \bar{ds} = \Phi \quad \text{flux (magnetic)}$$

For linear media: $\bar{B} = \mu \bar{H}$



$$\int \mu \bar{H} \cdot \bar{ds} = \Phi$$

$$\int \bar{H} \cdot \bar{ds} = \underbrace{\frac{1}{\mu}}_{R} \Phi$$

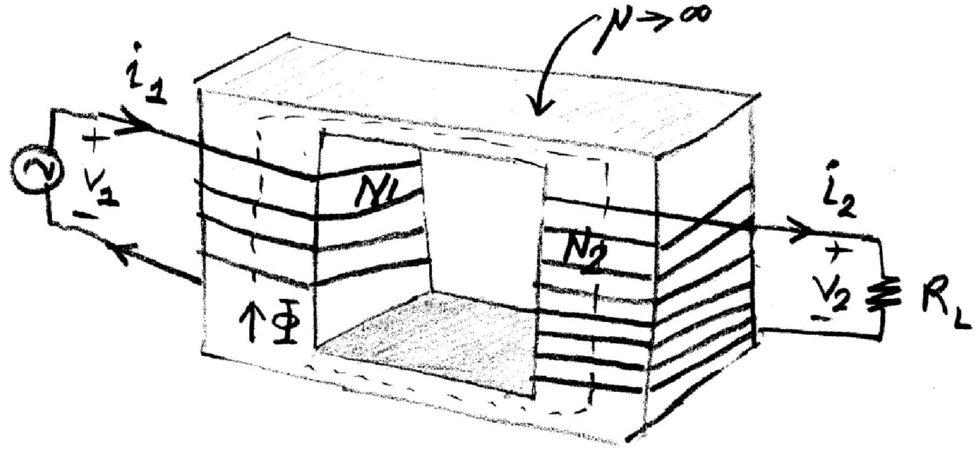
$$\int_S \bar{H} \cdot \bar{ds} = \oint_C \bar{H} \cdot \bar{dl} = R \Phi$$

Stokes

If $\mu \to \infty$, $R \to 0 \Rightarrow R\Phi \cong 0$

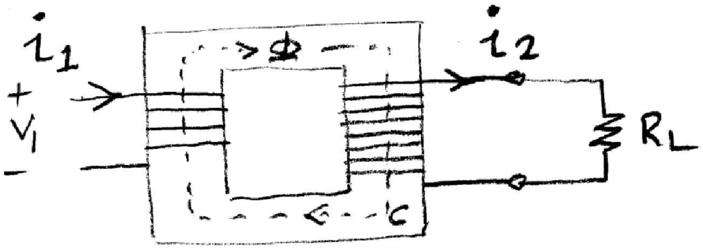
$$\oint \bar{H} \cdot \bar{dl} = 0$$ when coil is wound around a very high μ material

EXAMPLE: Transformers



We now have two coils wound on a transformer.

$$\mu \to \infty \Rightarrow \oint \bar{H} \cdot \bar{dl} = I_{enc} = 0$$

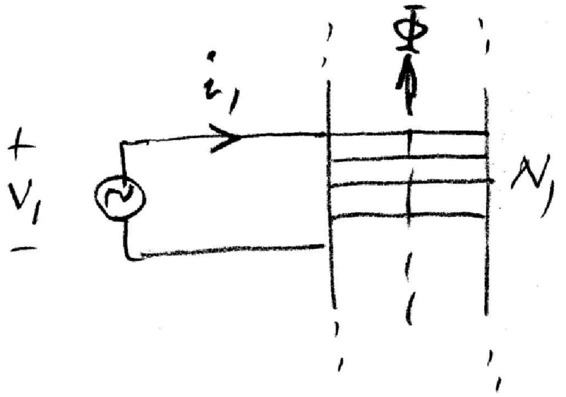


$$\oint_C \bar{H} \cdot \bar{dl} = I_{enc} = N_1 i_1 - N_2 i_2 = \cancel{\mathcal{R}}^{\,0} \Phi = 0$$

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}}$$

67F

What about the ratio between the voltages?



Recall $EMF = \mathcal{V} = -\dfrac{d\Phi}{dt}$ (V)

We now have

$$\boxed{V_1 = N_1 \frac{d\Phi_1}{dt} \quad (V) \quad (\text{right hand rule})}$$

Similarly

$$\boxed{V_2 = N_2 \frac{d\Phi_2}{dt} \quad (V)}$$

$\Phi_1 = \Phi_2$ then,

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

67 G

EXAMPLE

Input impedance of a transformer

Calculate $(R_1)_{effective} = \dfrac{v_1}{i_1}$

Recall

$$v_1 = \frac{N_1}{N_2} v_2$$

$$i_1 = \frac{N_2}{N_1} i_2$$

Then

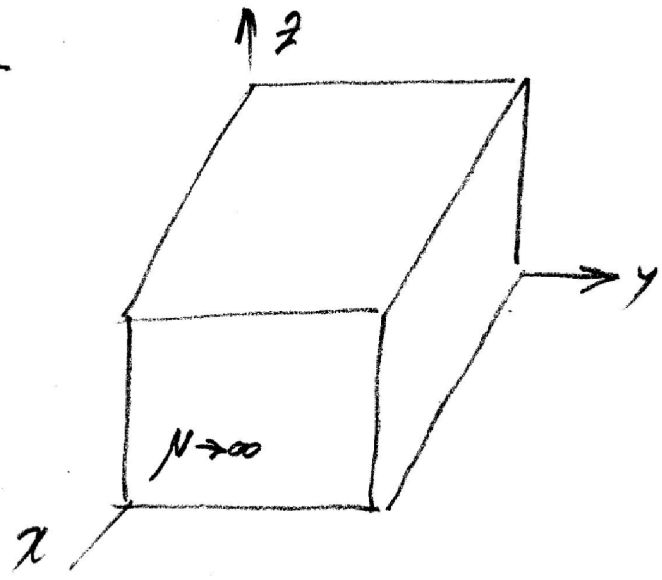
$$(R_1)_{eff} = \frac{(N_1/N_2)\, v_2}{(N_2/N_1)\, i_2} = \left(\frac{N_1}{N_2}\right)^2 \underbrace{\frac{v_2}{i_2}}_{R_L}$$

In general

$$\boxed{(Z_1)_{eff} = \left(\frac{N_1}{N_2}\right)^2 Z_L}$$

EXAMPLE

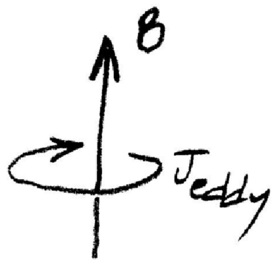
67 H



Given: $\bar{B} = \hat{a}_x B_0$

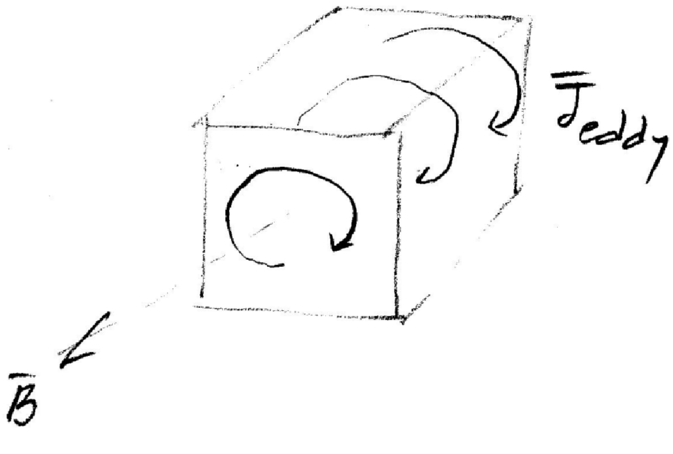
Eddy currents occur when $\sigma \neq 0$.

Also, given



opposite of Right Hand Rule.

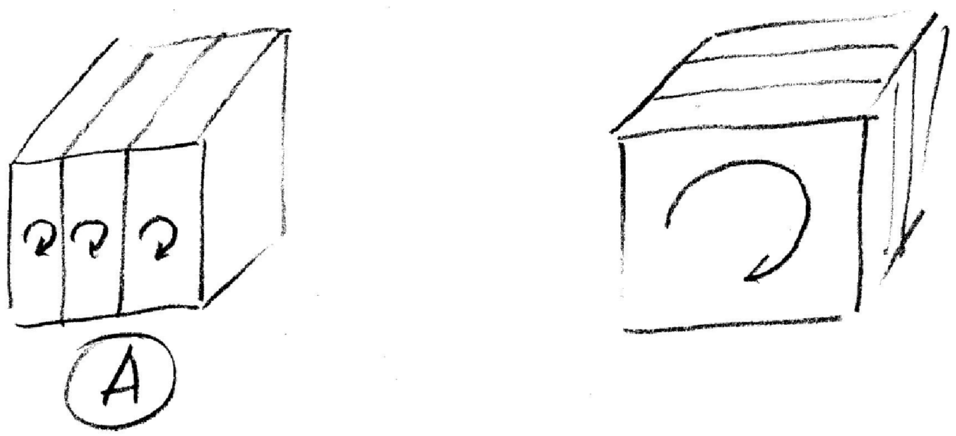
Draw J_{eddy} for the problem given figure.



67 I

EXAMPLE Lamination of the magnetic core.

How should you slice the core with high μ and low σ to reduce the EDDY CURRENTS which cause OHMIC POWER LOSS and heat loss.



$\Rightarrow$ Total eddy-current power loss decreases as the number of laminations increases.