INDEX

Absolute summability, 51, 108
  for suddenly-applied exponential, 51–52
Accumulator, 19, 31, 34
  difference equation representation of, 34–35
  as time-invariant system, 20
Adaptive signal processing, 4
Additivity property, 18
Aliasing, 144–45
  prefiltering to avoid, 185–87
  in reconstruction of undersampled sinusoidal signal, 148–49
  sampling/reconstruction of sinusoidal signal, 147
Aliasing distortion, 144–45
All-pass decomposition, 280–82
All-pass systems, 274–79
  defined, 274
  first-and second-order, 275
  uses of, 278
Alternation theorem, 489–91
Amplitude spectrum, 49
Analog channel vocoder, 5
Analog signals, 8
  digital processing of, 185–201
    analog-to-digital (A/D) conversion, 187–93
    analysis of quantization errors, 193–97
    prefiltering to avoid aliasing, 185–87
Analog-to-digital (A/D) conversion, 187–93
  oversampled:
    with direct quantization, 201–5
    with noise shaping, 206–10
  sample-and-hold system, 188–90
Analytic signal, 776
Antialiasing filter, 186
Aperiodic discrete-time sinusoids, 15
Approximation:
  Butterworth, 458–60
  Chebyshev, 460–63
  elliptic, 463–65
  equiripple, 492–93
Attenuation, and group delay, effects of, 343–45
Autocorrelation, 65–66
Autocorrelation invariance, 526
Autocorrelation sequence:
  of $h[n]$, 67–68
  spectrum analysis of random signals using estimates of, 743–48
Autocovariance sequence, 65–66
Autoregressive (AR) linear random process, 771
Autoregressive moving-average (ARMA) linear random process, 771
Auxiliary conditions, and linearity/time invariance/causality of a system, 39
Averaging of periodograms, 737–38
  computation using the DFT, 739

Backward difference, 21, 32, 33–34
Bandlimited differentiator, ideal, 159
Bandlimited signal, reconstruction of, from its samples, 150–53
Bandpass filters, FIR equiripple approximation of, 507–10
Bandpass sampling, 799–801
Bandpass signals, representation of, 796–99
Bandstop filters, frequency response of, 44
Bartlett windows, 468–71, 744, 750–54
Basic sequences/sequence operations, 11–16
  combining, 13–16
  exponential sequences, 13–14
  unit sample sequence, 11
  unit step sequence, 11–13
Bilateral z-transform, 95
Bilinear transformation, 450–54, 517
  Butterworth approximation, 458–60
  of a Butterworth filter, 454–57
  Chebyshev approximation, 460–63
  design examples, 454–65
  elliptic approximation, 463–65
  and warping, 453–54
Binary number system, 371–74
Bit-reversed order, 642–43
Blackman window, 468fn, 469
Block convolution, 585–87
  overlap-add method of, 723
  using time-dependent Fourier transform, 722–23
Block floating point, 668
Block diagram representation of linear constant-coefficient difference equations, 341–48
Block processing, 693
Bounded-input bounded-output (BIBO), 21
Butterflies, 676
Butterworth approximation, 458–60
Butterworth filter:
  bilinear transformation of, 454–57
  impulse invariance with, 446–49

Canonic form implementation, 346, 347
Cascade connection, 29
Cascade form structures:
  coefficient quantization effects, 379–82, 386–90
  FIR systems, 390, 412
  IIR systems, 356–59, 379–82, 403–10
Cauchy integral theorem, 776
Cauchy principal values, 781–82
Cauchy-Riemann conditions, 776
Causal all-pass systems, 274–78

Causal generalized linear-phase systems, 297–311
type I FIR linear-phase systems, 298
type II FIR linear-phase systems, 298
type III FIR linear-phase systems, 298
type IV FIR linear-phase systems, 298–99
Causality, 21
and auxiliary conditions, 39
and region of convergence (ROC), 110–11
Causal sequences, 31, 129, 777–82
defined, 778
exponential sequence, 779–80
finite-length sequence, 779
sufficiency of Fourier transform for, 777–82
exponential sequence, 779–80
finite-length sequence, 779
Characteristic of floating point number, 412
Charge-coupled devices (CCDs), 2
Charge transport devices (CTDs), 2
Chebyshev approximation, 460–63
Chirp transform algorithm (CTA), 656–61
parameters, 661
Circular convolution:
with delayed impulse sequence, 572–73
and discrete Fourier transform, 571–75
N-point circular convolution, 571
of two rectangular pulses, 573–75
Circular shift of a sequence, discrete Fourier transform, 564–67
Coefficient quantization, 377–39
of an optimum FIR filter, 386–90
in FIR systems, 384–86
in IIR systems, 377–82
maintaining linear phase, 390–91
poles of quantized second-order sections, 382–84
Commutative property of convolution, 29
Compensation for zero-order hold, 506–7
Complex-conjugate pole pair, direct form implementation of, 382–84
Complex cepstrum, 138, 788–89
Complex exponential sequences, 14–15, 40
Complex sequences:
defined, 789
Hilbert transform relations for, 789–801
Compressor, 20–21, 167–68
Computationally realizable systems, 242
Conjugate-antisymmetric sequence, 55–56
Conjugate-symmetric sequence, 55–56
Conjugation property, z-transform, 123
Continuous phase, 343
Continuous-to-discrete-time (C/D) conversion, 141–49
Continuous-time complex exponentials and sinusoids, compared to discrete-time complex exponentials and sinusoids, 14
Continuous-time IIR filter design, 442–43
Continuous-time processing, of discrete-time signals, 163–67
Continuous-time signals, 8
aliasing, 144–45
in reconstruction of undersampled sinusoidal signal, 148–49
sampling/reconstruction of sinusoidal signal, 147
bandlimited signal reconstruction, from its samples, 150–53
continuous-time processing of discrete-time signals, 163–67
moving-average system with noninteger delay, 165–67
noninteger delay, 164–65
digital processing of analog signals, 185–201
analog-to-digital (A/D) conversion, 187–93
analysis of quantization errors, 193–97
digital-to-analog (D/A) conversion, 197–201
prefiltering to avoid aliasing, 185–87
discrete-time processing of, 153–63
changing sampling rate using, 167–78
impulse invariance, 160–63
linear time-invariant discrete-time systems, 154–60
frequency-domain representation of, 142–49
multirate signal processing, 179–84
interchange of filtering and downsampling/upsampling, 179–80
polyphase decompositions, 180–82
polyphase implementation of decimation filters, 182–83
polyphase implementation of interpolation filters, 183–84
Nyquist frequency/Nyquist rate, 146–47
periodic sampling, 140–42
sampling of, 140–239
sampling rate change by noninteger factor, 176–78
sampling rate increase by integer factor, 172–76
sampling rate reduction by integer factor, 167–72
unit impulse function, 142–43
Convergence:
Fourier transform, 50–53
z-transform, 96, 105–11
Convolution property, z-transform, 124–26
Convolution of sequences, 22–28 124–25
block, 585–88, 721–22
circular, 571–75
linear, 576–88
periodic, 548–50
Convolution sum:
analytic evaluation of, 26–28
computation of, 25–26

defined, 23
Convolution-sum expression, 23–25
Convolution theorem, 60–61
Fourier transform, 60–61
z-transform, 124–25
CORDIC rotator algorithm, 677, 684
Correlation, computing using DFT, 746–48
Coupled form, for second-order system, 384
Coupled form oscillator, 425
Critically sampled filter bank, 770
Cross-correlation, 69–70

DB (decibels), 254
DCT-1, 590–93
relationship between DFT and, 593–94
See also Discrete cosine transform (DCT)
DCT-2, 590–93
energy compaction property of, 595–98
relationship between DFT and, 594–95
See also Discrete cosine transform (DCT)
Dead bands, 416
Decimation, 172
Decimation filters, polyphase implementation of, 182–83
Decimation-in-frequency algorithms, 646–52
alternate forms, 650–52
in-place computation, 650
Decimation-in-time algorithms, 635–46
alternate forms, 643–46
in-place computations, 640–43
Decimator, 172
Decomposition:
all-pass, 280–82
of a linear-phase system, 308–11
Delay distortion, 242–43
Deterministic autocorrelation sequence, 67–68
DFS, *See* Discrete Fourier series (DFS)
DFT, *See* Discrete Fourier transform (DFT)
Difference equations, recursive computation of, 37–38
Differentiation in frequency, 60
Differentiation property, z-transform, 122–23
Differentiators, 516
Kaiser window design of, 482–85
Digital filters, 439
Digital processing of analog signals, 185–201
analog-to-digital (A/D) conversion, 187–93
analysis of quantization errors, 193–97
digital-to-analog (D/A) conversion, 197–201
prefiltering to avoid aliasing, 185–87
Digital signal processing, 9
early use of digital computers in, 5–6
Digital signals, 8
Digital-to-analog (D/A) conversion, 185, 197–201
oversampling and noise shaping in, 210–13
Dirac delta function, 142–43
Direct form I implementation, of an LTI system, 346–48
Direct form II implementation, of an LTI system, 347–48
Discrete cosine transform (DCT), 589–99
applications of, 598–99
DCT-1, 590–93
relationship between DFT and, 593–94
DCT-2, 590–93
energy compaction property of, 595–98
relationship between DFT and, 594–95
definitions of, 589–90
multiplication of, 599
Discrete Fourier series (DFS):
duality in, 544–45
of a periodic impulse train, 544
of a periodic regular pulse train, 545–46
properties of, 546–51
duality property, 547
linearity property, 546
periodic convolution, 548–51
shift of a sequence, 546–47
symmetry properties, 547
representation of periodic sequences, 543–46
Discrete Fourier transform (DFT), 541–692
autocorrelation sequence, spectrum analysis of random signals using estimates of, 743–48
chirp transform algorithm (CTA), 656–61
parameters, 661
computation of, 629–52
decimation-in-frequency algorithms, 646–52
decimation-in-time algorithms, 635–46
Goertzel algorithm, 633–35
correlation, computing using DFT, 746–48
defined, 541
DFT analysis of sinusoidal signals, 697–714
oversampling/linear interpolation for frequency estimation, 713–14
spectral sampling, 703–14
with 32-point Kaiser window and zero-padding, 711–12
using a Kaiser window, 708–10
windowing, 698–703
discrete cosine transform (DCT), 589–99
applications of, 598–99
DCT-1 and DCT-2, 590–93
definitions of, 589–90
energy compaction property of DCT-2, 595–98
multiplication of, 599
relationship between DFT and DCT-1, 593–94
relationship between DFT and DCT-2, 594–95
discrete Fourier series (DFS), 543–46
duality in, 544–45
of a periodic impulse training, 544
of a periodic regular pulse train, 545–46
properties of, 546–51
fast Fourier transform (FFT) algorithms, 629–92
algorithms for more general values of N, 655
coefficients, 654–55
indexing, 652–54

finite register length, effects of, 661–68
Fourier analysis of signals using, 693–97
relationship between DFT values, 697
as Fourier representation of finite-duration sequences, 559–64
implementation of, using convolution, 655–61
implementing linear time-invariant systems using, 582–88
linear convolution, 576–88
circular convolution as linear convolution with aliasing, 577–82
of two finite-length sequences, 277
periodic sequences, representation of, 542–46
of periodic signals, 551–55
periodogram, 730–42
averaging of, 737–39
defined, 732
explicit computation of, 732
modified, 732
periodogram analysis, 731, 739–42
power spectrum, 731–32, 734
properties of, 733–37
power spectrum estimation:
computing using DFT, 746–48
example based on estimation of autocorrelation sequence, 748–54
properties of, 564–76
circular convolution, 571–75
circular shift of a sequence, 564–67
duality property, 567–68
linearity property, 564
symmetry properties, 568–70
sampling the Fourier transform, 555–59
time-dependent Fourier analysis of nonstationary signals, 723–30
radar signals, 728–30
speech signals, 720–28
time-dependent Fourier transform, 714–22
block convolution using, 722–23
defined, 714
effect of the window in, 717–18
of a linear chirp signal, 715–17
sampling in time/frequency, 718–22
Winograd Fourier transform algorithm (WFTA), 655–56
Discrete Hilbert transforms, 775–810
analytic signal, 776, 789
bandpass sampling, 799–801
bandpass signals, representation of, 796–99
Cauchy principal values, 781–82
causal sequences:
defined, 778
exponential sequence, 779–80
finite-length sequence, 779
sufficiency of the Fourier transform for, 777–82
Hilbert transformers, 791
design of, 792–95
Kaiser window design of, 793–95
Hilbert transform relations, 775–801
for complex sequences, 789–801
magnitude, relationships between phase and, 788–89
Poisson's formulas, 776
Discrete-time differentiators, 482–85
Discrete-time filter, determining specifications for, 440–42
Discrete-time Fourier transform (DTFT), 48fn
Discrete-time impulse, 11
Discrete-time random signals, 65–70
Discrete-time signal processing, 2
continuous signals, 2
future of, 7
history of, 6–7
Discrete-time signals, 8–93
basic sequences/sequence operations, 11–16
combining basic sequences, 13–16
complex exponential sequence, 14
exponential sequences, 13–14
sinusoidal sequences, 13
unit sample sequence, 11
unit step sequence, 11–13
defined, 8
discrete-time systems, 16–22
graphic depiction of, 10
sampling frequency, 10
sampling period, 10
as sequences of numbers, 9–16
signal-processing systems, classification of, 8–9
Discrete-time sinusoids, periodic/aperiodic, 15
Discrete-time specifications, 443
Discrete-time systems, 16–22
causality, 21
coefficient quantization, 377–39
of an optimum FIR filter, 386–90
in FIR systems, 384–86
in IIR systems, 377–82
maintaining linear phase, 390–91
poles of quantized second-order sections, 382–84
defined mathematically, 16–17
discrete-time random signals, 65–70
autocorrelation/autocovariance sequence, 65–66
deterministic autocorrelation sequence, 67–68
power density spectrum, 68
random process, 65
white noise, 69
finite-precision numerical effects, 370–77
numerical representations, 371–74
quantization in implementing systems, 374–77
FIR systems, basic network structures for, 366–70
cascade form, 367–68
direct form, 367
linear-phase FIR system structures, 368–70

fixed-point realizations of IIR digital filters, zero-input limit cycles in, 413–18
Fourier transform theorems, 58–65
convolution theorem, 60–61
differentiation in frequency, 60
frequency shifting, 59
linearity of Fourier transform, 59
modulation or windowing theorem, 61–62
Parseval's theorem, 60
time reversal, 60
time shifting, 59
frequency-domain representation of, 40–48
eigenfunctions for linear time-invariant systems, 40–45, 61
frequency response of the ideal delay system, 41
frequency response of the moving-average system, 44–45
ideal frequency-selective filters, 43–44
sinusoidal response of linear time-invariant systems, 42–43
suddenly-applied complex exponential inputs, 46–48
ideal delay system, 17
IIR systems, basic structures for, 354–63
cascade form, 356–59
direct forms, 354–56
parallel form, 359–61
instability, 21–22
testing for, 22
limit cycles:
avoiding, 417–18
due to overflow, 416–17
due to round-off, 414–16
due to truncation, 414–16
in first-order system, 414–16
linear constant-coefficient difference equations, 34–40
block diagram representation of, 341–48
difference equation representation of the accumulator, 34–35
difference equation representation of the moving-average system, 35–36
recursive computation of difference equations, 37–38
signal flow graph representation of, 348–53
linear systems, 18
accumulator system, 19
nonlinear system, 19
linear time-invariant systems, 22–28, 154–60
convolution sum, 23–28
eigenfunctions for, 40–45
properties of, 28–34
memoryless systems, 18
moving average, 17–18
and processing of continuous-time signals, 153
representation of sequences by Fourier transforms, 48–54
absolute summability for suddenly-applied exponential, 51–52
Fourier transform of complex exponential sequences, 54
Fourier transform of a constant, 53
inverse Fourier transform, 48
square-summability for the ideal lowpass filter, 52–53
round-off, limit cycles due to, 414–16
round-off noise, 391–413
analysis of cascade IIR structure, 403–10
analysis of direct-form FIR systems, 410–12
analysis of direct-form IIR structures, 391–99
first-order system, 396–97
floating-point realizations of discrete-time systems, 412–13
interaction between scaling and, 402–3
scaling in fixed-point implementations of IIR systems, 399–403
second-order system, 397
sinusoidal response of, 42–43
stability, 21–22
testing for, 22
structures for, 340–438
symmetric properties of Fourier transform, 55–58
conjugate-antisymmetric sequence, 55–56
conjugate-symmetric sequence, 55–56
even function, 55
even sequence, 55
illustration of, 57–58
odd function, 55
odd sequence, 55
time-invariant systems, 20–21
accumulator as, 20
compressor system, 20–21
transposed forms, 363–66
for a basic second-order section, 364–65
for first-order system with no zeros, 363–64
truncation, limit cycles due to, 414–16
Discrete-to-continuous-time (D/C) conversion, 152–53
Distortion compensation, 282–83
Doppler frequency, 729–30
Downsampling, 168–72
interchange of filtering and, 179–80
Duality property:
discrete Fourier series (DFS), 544–45, 547
discrete Fourier transform (DFT), 567–68

Eigenfunctions, 40
Eigenvalues, 40–41
See also Frequency response
Elliptic approximation, 463–65, 828–29
Elliptic continuous-time filters, 463–65, 828–29
Energy density spectrum, 60
Equiripple approximations, 492–93
Even function, 55
Even sequence, 55
Expander, 172
Exponential multiplication, 121–22
Exponential sequences, 13–14, 779–80
left-sided, 99–100
right-sided, 98
sum of, 100–101
Extraripple case, 493

Fast Fourier transform (FFT)
algorithms, 2–3, 6, 576, 629–92
algorithms for more general values of N, 655
coefficients, 654–55
decimation-in-frequency algorithms, 646–52
decimation-in-time FFT algorithms, 635–46
indexing, 652–54
Feedback, in IIR systems, 361–63
FFT, *See* Fast Fourier transforms
Filter design, 439–540
bilinear transformation, 450–65
discrete-time filter, determining specifications for, 440–42
discrete-time IIR filter design, from a continuous-time filters, 442–65
FIR discrete-time filter design, 510–11
FIR equiripple approximation examples, 503–10
bandpass filter, 507–10
compensation for zero-order hold, 506–7
lowpass filter, 503–6
IIR discrete-time filter design, 510–11
impulse invariance, 443–49
impulse invariance with a Butterworth filter, 446–49
optimum approximations of FIR filters, 486–502
alternation theorem, 489–91
characteristics of optimum FIR filters, 501–2
equiripple approximations, 492
extraripple case, 493
optimal type II lowpass filters, 497–98
optimal type I lowpass filters, 491–97
Parks-McClellan algorithm, 498–501, 510
techniques, 439–540
windowing, 465–78
Bartlett window, 468–71
Blackman window, 468fn, 469
Hamming window, 468–71
Hanning window, 468–71
incorporation of generalized linear phase, 469–73
Kaiser window method, 474–78
linear-phase lowpass filter example, 472–73
properties of commonly used windows, 468–69
rectangular window, 468
Filtering, interchange of down-sampling/upsampling and, 179–80
Filters:
antialiasing, 186
continuous-time IIR, 442–43
finite impulse response (FIR), 442
frequency-selective, 439–540
generalized linear-phase, 316
ideal frequency-selective, 43–44, 241–42
linear-phase, 316
optimal type II lowpass, 497–98
optimal type I lowpass, 491–97
sharp-cutoff antialiasing filters, 186–87
transversal, 367
Finite-duration sequences, 105, 571
Fourier representation of, 559–64
See also Discrete Fourier transform (DFT)
Finite impulse response (FIR) filters, 442
Finite-length sequences, 103–4, 117, 779
periodic sequence, 783–87
and power expansion, 117
sufficiency theorems for, 782–87
Finite-precision numerical effects, 370–77
numerical representations, 371–74
quantization in implementing systems, 374–77
Finite register length, effects of, 661–68
FIR equiripple approximation examples, 503–10
bandpass filter, 507–10
compensation for zero-order hold, 506–7
lowpass filter, 503–6
FIR filters:
optimum approximations of, 486–502
alternation theorem, 489–91
characteristics of optimum FIR filters, 501–2
equiripple approximations, 492
extraripple case, 493
optimal type II lowpass filters, 497–98
optimal type I lowpass filters, 491–97
First backward difference, 88, 524
First-order all-pass systems, 275
FIR linear-phase systems:
examples of, 300–305
locations of zeros for, 306–8
relation of minimum-phase systems to, 308–11
type III linear-phase system, 302
type II linear-phase system, 302
type I linear-phase system, 300–302
type IV linear-phase system, 302–5
FIR systems:
basic network structures for, 366–70
cascade form, 367–68
direct form, 367
linear-phase FIR system structures, 368–70
coefficient quantization in, 384–86
compensation of, 283–87
Fixed-point realizations of IIR digital filters, zero-input limit cycles in, 413–18
Floating-point realizations of discrete-time systems, 412–13
Flow graph, determination of system function from, 352–53
Flow graph reversal/transposition, 363
Forward difference, 21, 31, 33
Fourier transform, 63
of complex exponential sequences, 54
of a constant, 53
linearity of, 59
magnitude of, 49
of a periodic impulse train, 552
of periodic signals, 551–55

relationship between Fourier series coefficients and, 554–55
phase of, 49
sampling, 555–59
Frequency:
differentiation in, 60
Doppler, 729–30
Nyquist, 146–47
sampling, 10, 140–41
Frequency-domain representation:
of discrete-time signals/systems, 40–48
of sampling, 142–49
Frequency response, 40–41
determining impulse response from, 64
of the ideal delay system, 41
of the moving-average system, 44–45
Frequency-response compensation, 282–87
Frequency response for rational system functions, 253–70
multiple poles or zeros, 265–70
second-order FIR system, 268
second-order IIR system, 265–67
third-order IIR system, 268–70
single zero or pole, 258–265
Frequency-selective filters, 439–540
See also Filter design
Frequency shifting, 59
Fricative sounds, 724

Gain in *dB*, 254
Gain of the system, 241
Generalized linear phase, 295–97
linear systems with, 291–311
Generalized linear-phase filters, 316
Gibbs phenomenon, 468
Goertzel algorithm, 629, 633–35
Group delay:
and attenuation, effects of, 243–45
defined, 243

Hamming window, 468–71
Hanning window, 468–71
High-pass filters, 44
Kaiser window method design of, 479–82
Hilbert transformers, 791
design of, 792–95
Kaiser window design of, 793–95
See also Discrete Hilbert transforms
Hilbert transform relations, 775–76
for complex sequences, 789–801
See also Discrete Hilbert transforms
Homogeneous difference equation, 36
Homogeneous solution, 36

Ideal 90-degree phase shifter, 790–91
Ideal continuous-time bandlimited differentiator, discrete-time implementation of, 158–59
Ideal continuous-time lowpass filtering, using discrete-time lowpass filter, 155–57
Ideal continuous-to-discrete (C/D) converter, 141, 154, 185
Ideal delay system, 17, 31
frequency response of, 41
Ideal discrete-to-continuous (D/C) converter, 152–153, 185
Ideal frequency-selective filters, 43–44, 241–42
Ideal highpass filters:
frequency response of, 44, 242
impulse response of, 242
Ideal lowpass filters, 43
frequency response of, 43, 241
impulse response of, 242
with linear phase, 242–43
Ideal reconstruction filter, 150–51
Ideal signal reconstruction system, 150–51
IIR systems:
basic structures for, 354–63
cascade form, 356–59
direct forms, 354–56
parallel form, 359–61
coefficient quantization in, 377–82
feedback in, 361–63
Image processing, application of multidimensional digital processing to, 4
Impulse, 11
Impulse invariance, 160–63, 517
applied to continuous-time systems with rational system functions, 162–63
with a Butterworth filter, 446–49
discrete-time lowpass filter obtained by, 162
filter design by, 443–49
Impulse response:
determining from frequency response, 64
for a difference equation, determining, 64–65
Impulse responses, 31–32
Impulse train modulation, 142
Infinite-duration impulse response (IIR) systems, 32
Infinite impulse response (IIR) filters, 442
Infinite sum, expressing closed form, 97–98
Initial-rest conditions, 38
Initial-value theorem, 126
Innermost nonzero pole, 105
Inspection method for inverse z-transform, 111–12
Instability, 21–22
testing for, 22
Interpolation, 173
frequency-domain illustration of, 174
linear, 175–76
by filtering, 175
impulse response for, 175
Interpolator, 173
Inverse by partial fractions, 115–16
Inverse DFT algorithm, 676
Inverse Fourier transform, 48, 63
Inverse of non-rational z-transform, 122–23
Inverse systems:
defined, 33–34
system functions for, 248–50
Inverse transform, by a power series expansion, 117
Inverse z-transform, 111–18
inspection method, 111–12
inverse by partial fractions, 115–16

partial fraction expansion, 112–16
second-order z-transform, 113–14

Kaiser window, 474–82
DFT analysis of sinusoidal signals using, 708–10
Kaiser window design, of Hilbert transformers, 793–95
Kaiser window method, 474–78
examples of FIR filter design by, 478–85
discrete-time differentiators, 482–85
highpass filter, 479–82
and lowpass filter design, 476–78
relationship to other windows, 478

Leakage, 701
Left-sided exponential sequence, 99–100
Left-sided sequence, 105
power series expansion for, 118
L'Hopital's rule, 151
Limit cycles:
avoiding, 417–18
due to overflow, 416–17
due to round-off, 414–16
due to truncation, 414–16
in first-order system, 414–16
Linear chirp signal, time-dependent Fourier transform of, 715–17
Linear constant-coefficient difference equations, 34–40
block diagram representation of, 341–48
impulse response for rational system functions, 250–53
first-order IIR system, 251–52
simple FIR system, 252–53
inverse systems, 248–50
for first-order system, 249–50
with zero in the ROC, 250
region of convergence (ROC), determining, 247–48
second-order system, 246–47
signal flow graph representation of, 348–53
stability and causality, 247–48
system functions for systems characterized by, 245–53
Linear convolution, 576–88
using discrete Fourier transform (DFT), 576–88
Linearity:
and auxiliary conditions, 39
of Fourier transform, 59
Linearity property:
discrete Fourier series (DFS), 546
discrete Fourier transform (DFT), 564
z-transform, 119
Linear noise models, 391–412
cascade form IIR systems, 403–410
direct form FIR systems, 410–12
direct form IIR systems, 391–403
parallel form IIR sytmes, 410
Linear phase, 291–311
causal generalized linear-phase systems, 297–311
type I FIR linear-phase systems, 298
type II FIR linear-phase systems, 298
type III FIR linear-phase systems, 298
type IV FIR linear-phase systems, 298–99
decomposition of a linear-phase system, 308–11
FIR linear-phase systems:
examples of, 300–305
locations of zeros for, 306–8
relation of minimum-phase systems to, 308–11
type III linear-phase system, 302
type II linear-phase system, 302
type I linear-phase system, 300–302
type IV linear-phase system, 302–5
generalized linear phase, 295–97
ideal lowpass filter with, 293–95
systems with, 292–95
Linear-phase filters, 316
Linear-phase FIR system structures, 368–70
Linear predictive coding (LPC), 4
Linear quantizers, 191
Linear system, 78–79
Linear systems, 18
accumulator system, 19
nonlinear system, 19
Linear time-invariant discrete-time systems, 22–28, 154–60
all-pass systems, 274–79
causal all-pass systems, 274–78
defined, 274
first- and second-order all-pass systems, 275
uses of, 278
convolution sum, 23–28
analytic evaluation of, 26–28
computation of, 25–26
defined, 23
direct form II implementation of, 347–48
direct form I implementation of, 346–47
eigenfunctions for, 40–45
frequency response of, 241–45
ideal frequency-selective filter, 241–42
phase distortion and delay, 242–45
frequency response for rational system functions, 253–70
multiple poles and zeros, 265–70
single zero or pole, 258–65
generalized linear phase, linear systems with, 291–311
ideal continuous-time bandlimited differentiator, discrete-time implementation of, 158–59
ideal continuous-time lowpass filtering, using discrete-time lowpass filter, 155–57
minimum-phase systems, 280–91
frequency-response compensation, 282–87
minimum-phase and all-pass decomposition, 280–82
properties of, 287–91
properties of, 28–34

cascade connection, 29
constraints of linearity/time invariance, 30–31
finite-duration impulse response (FIR) systems, 32–33
impulse responses, 31–32
infinite-duration impulse response (IIR) systems, 32
parallel connection, 30
relationship between magnitude and phase, 270–74
transform analysis of, 240–339
Log magnitude, 254
Long division, power series expansion by, 118

Magnitude:
Fourier transform, 49
relationships between phase and, 788–89
Magnitude distortions, 241
Magnitude response, 241
Magnitude spectrum, 49
Magnitude-squared function, 254, 258, 457
Mantissa, 412
Matlab, 3
Maximum entropy methods (MEM spectral analysis), 4
Maximum-phase sequence, 335
Memoryless systems, 18
MEM spectral analysis, 4
Microelectronics, 6–7
Microprocessors:
and discrete-time signals, 7
rapid evolution of, 2
Minimax criteriod, 489
Minimum energy-delay property, minimum-phase systems, 288–91
Minimum group-delay property, minimum-phase systems, 288
Minimum phase-lag property, minimum-phase systems, 287–88
Minimum-phase systems, 250, 280–91
frequency-response compensation, 282–87
minimum-phase and all-pass decomposition, 280–82
properties of, 287–91
minimum energy-delay property, 288–91
minimum group-delay property, 288
minimum phase-lag property, 287–88
relation of FIR linear-phase systems to, 308–11
Modified periodogram, 732
Modulation theorem, 61–62
Moving average, 17–18, 31
Moving-average (MA) linear random process, 771
Moving-average system:
difference equation representation of, 35–36
with noninteger delay, 165–67
Multidimensional signal processing, 3–4
Multiplication by an exponential sequence, 121–22
Multirate signal processing, 179–84
interchange of filtering and downsampling/upsampling, 179–80
polyphase decompositions, 180–82
polyphase implementation of decimation filters, 182–83
polyphase implementation of interpolation filters, 183–84
Multistage noise shaping (MASH), 210

Narrowband time-dependent Fourier analysis, 725
Nonanticipative system, 21
Noncausal window, 716
Noncomputable network, 363
Noninteger delay, 164–65
moving-average system with, 165–67
Nonlinear system, 19
Non-rational z-transform, inverse of, 122–23
Nonstationary signals:
time-dependent Fourier analysis of, 723–30
radar signals, 728–30
speech signals, 720–28
N-point circular convolution, 571
Nyquist frequency, 146–47
Nyquist rate, 146–47, 153
Nyquist sampling theorem, 146

Odd function, 55
Odd sequence, 55
Offset binary coding scheme, 191
One's complement, 371
One-sided z-transform, 95
Optimal type II lowpass filters, 497–98
Optimal type I lowpass filters, 491–97
Optimum approximations of FIR filters, 486–502
alternation theorem, 489–91 and polynomials, 290–91
characteristics of optimum FIR filters, 501–2
equiripple approximations, 492
extraripple case, 493
optimal type II lowpass filters, 497–98
optimal type I lowpass filters, 491–97
Parks-McClellan algorithm, 498–501
Outermost finite pole, 105
Overflow oscillations, in second-order system, 416–17
Overlap-add method, 586–87, 680
Overlap-save method, 587, 680
Oversampled A/D conversion:
with direct quantization, 201–5
with noise shaping, 206–10
Oversampled D/A conversion, and noise shaping, 210–13

Parallel connection, 30
Parallel form structures:
coefficient quantization effects, 382
IIR systems, 359–61
Roundoff noise effects, 410
Parks-McClellan algorithm, 487, 498–501, 506, 510
Parseval's relation:
discrete Fourier transform, 621
Fourier transform, 60
Parseval's theorem, 60, 400
Partial fractions, inverse by, 115–16
Periodic conjugate-antisymmetric components, 570
Periodic conjugate-symmetric components, 570

Periodic convolution:
defined, 548
discrete Fourier series (DFS), 548–51
Periodic discrete-time sinusoids, 15
Periodic impulse train, 143
Periodic sampling, 140–42
Periodic sequences, representation of, 542–46
Periodic signals, Fourier transform of, 551–55
Periodogram, 730–42
averaging of, 737–39
defined, 732
explicit computation of, 732
modified, 732
periodogram analysis, 731, 739–42
example of, 739–42
power spectrum, 731–32, 734
properties of, 733–37
Periodogram analysis, 731, 739–42
Phase:
Fourier transform, 49
relationships between magnitude and, 788–89
Phase distortion and delay, 242–45
Phase response, 241
Phase shift, 241
Phase spectrum, 49
Plosive sounds, 724
Poisson's formulas, 776
Poles of X(z), 298
Polyphase decompositions, 180–82
Polyphase implementation:
of decimation filters, 182–83
of interpolation filters, 183–84
Power density spectrum, 68
of quantization noise, 207–9
Power series expansion, 116–18
by long division, 118
finite-length sequence, 117
inverse transform by, 117
for a left-sided sequence, 118
Power spectrum, 731–32, 734
estimation:
computing using DFT, 746–48
example based on estimation of autocorrelation sequence, 748–54
Prefiltering, to avoid aliasing, 185–87
Product of two sequences, 11

Quantization errors, 372
analysis of, 193–97
for sinusoidal signal, 194–95
Quantization levels, 191–94
coding of, 191
Quantization noise:
example of, 195
power density spectrum of, 207–9

Radar signals:
clutter, 729
Doppler frequency, 729–30
time-dependent Fourier analysis of, 728–30
Random process, 65
Random signals, 65–70, 811–23
spectrum analysis of, 730–54
Rational system functions:
frequency response for, 250–70
multiple poles and zeros, 265–70
single zero or pole, 258–65
impulse response for, 250–53
first-order IIR system, 251–52
simple FIR system, 252–53
Reconstruction of sinusoidal signal, 147
undersampled, aliasing in, 148–49
Rectangular window, 471
Recursive computation of difference equations, 37–38
Recursive representation, 35
Region of convergence (ROC), 96, 105–11
and causality, 110–11
properties of, for z-transform, 105–11
and stability, 110–11
Region of convergence (ROC), 96–97
Right-sided exponential sequence, 98
Right-sided sequence, 105, 108
Rotations, 684
Round-off, limit cycles due to, 414–16

Sample-and-hold circuits, 188–89
Sampled-data Delta-Sigma modulator, 206
Sampling:
bandpass, 799–801
downsampling, 168–72, 179–80
Fourier transform, 555–59
frequency-domain representation of, 142–49
periodic, 140–42
spectral, 703–14
in time/frequency, 718–22
upsampling, 172–76, 179–80
Sampling the Fourier transform, 555–59
Sampling frequency, 10, 140–41
Sampling period, 10, 140–42
Sampling rate change by noninteger factor, 176–78
Sampling rate compressor, 167–68
Sampling rate expander, 172
Sampling rate increase by integer factor, 172–76
Sampling rate reduction by integer factor, 167–72
Second-order all-pass systems, 275
Second-order z-transform, 113–14
Sequence operations, 11–16
Sequences:
autocorrelation, of h[n], 67–68
autocovariance, 65–66
causal, 31, 129, 777–82
circular shift of, 564–67
complex, 789–801
complex exponential, 14
conjugate-antisymmetric, 55–56
conjugate-symmetric, 55–56
delayed impulse sequence, 572–73
deterministic autocorrelation, 67–68
even, 55
exponential, 13–14, 779–80
finite-duration, 105, 571
finite-length, 103–4, 779
left-sided, 99–100, 105, 118
maximum-phase, 335
odd, 55
periodic, 542–46
representation of, by Fourier transforms, 48–54
right-sided, 98, 105, 108
shifted exponential, 120–21

sinusoidal, 13
two-sided, 108–10
unit sample, 11
unit step, 11–13
Sharp-cutoff antialiasing filters, 186–87
Shifted exponential sequence, 120–21
Shift of sequences, 59
circular, 564–67
periodic, 546–47
Shifting:
frequency, 59
time, 59, 120–21
Shift of a sequence, discrete Fourier series (DFS), 546–47
Short-time Fourier transform, *See* Time-dependent Fourier transform
Time-dependent Fourier transform, 714–22
Signal:
defined, 8
independent variable in mathematical representation of, 8
Signal expression processing, 3
Signal interpretation, 2–3
Signal modeling, 4
Signal processing, defined, 1–2
Signal processing microprocessors, processing capability of, 1
Signal-processing systems, classification of, 8–9
Sign bit, 371
Sign and magnitude, 371
Sink nodes, 349
Sinusoidal response, of linear time-invariant systems, 42–43
Sinusoidal sequences, 13
Sinusoidal signals, DFT analysis of, 697–714
oversampling/linear interpolation for frequency estimation, 713–14
spectral sampling, 703–14
with 32-point Kaiser window and zero-padding, 711–12
using a Kaiser window, 708–10
windowing, 698–703
Source nodes, 349
Spectral analysis, 4, 6
Spectral sampling:
effect of, 703–14
illustration of, 703–6
with frequencies matching DFT frequencies, 706–8
Spectrum:
amplitude, 49
energy density, 60
magnitude, 49
phase, 49
power, 731–32, 734
power density, 68, 207–9
Speech signals:
time-dependent Fourier analysis of, 720–28
narrowband spectrogram, 728
wideband spectrogram, 725–28
Split-radix FFT (SRFFT), 683
Square-summability, for the ideal lowpass filter, 52–53
SRFFT, 683
Stability, 21–22, 30–31, 247–48
and region of convergence, 110–11
testing for, 22
Stationary, use of term, 66
Stationary random signals, Fourier analysis of, 730–42
Steady-state response, 46
Stochastic signals, *See* Random signals
Structures for discrete-time systems, 340–418
block diagram representation of linear constant-coefficient difference equations, 341–48
cascade form, *See* Cascade form structures
direct form, *See* Direct form structures
finite precision numerical effects, *See* Quantization
FIR systems, 366–70
IIR systems, 354–63
parallel form, *See* Parallel form structures
signal flow graph representation of linear constant-coefficient difference equations, 348–53
transposed forms, 363–66
Suddenly-applied complex exponential inputs, 46–48
Suddenly-applied exponential, absolute summability for, 51–52
Sufficiency theorems for finite-length sequences, 782–88
Summability:
absolute, 51, 108
for suddenly-applied exponential, 51–52
square-, 52–53
Superposition, principal of, 22–24
Surface acoustic wave (SAW), 2
Switched-capacitor technologies, 2
Symmetry properties:
discrete Fourier series (DFS), 547
discrete Fourier transform, 568–70
discrete-time Fourier transform (DTFT), 55–58
Synthesis formula, 48
System function, 241
Systems, 374–77

Tapped delay line structure, 367
Telecommunications, and discrete-time signal processing, 7
Time-dependent Fourier analysis of nonstationary signals, 723–30
radar signals, 728–30
speech signals, 720–28
Time-dependent Fourier transform, 693, 714–22
block convolution using, 722–23
defined, 714
effect of the window in, 717–18
of a linear chirp signal, 715–17
sampling in time/frequency, 718–22
Time invariance, and auxiliary conditions, 39
Time-invariant systems, 20–21, 77
accumulator as, 20
compressor system, 20–21
Time-reversal property, z-transform, 123–24
Time reversal theorem, 60
Time-shifting property:
discrete Fourier series, 546

discrete Fourier transform, 564–67
Fourier transform, 59
z-transform, 120–21
shifted exponential sequence, 120–21
Time-shifting theorem, 59
Transient response, 46–47
Transposed forms, 363–66
for a basic second-order section, 364–65
for first-order system with no zeros, 363–64
Transversal filter structure, 367
Trapezoidal approximation, 525
Truncation, limit cycles due to, 414–16 "Twicing," 528
Two-dimensional signal processing techniques, 3–4
Two's complement, 371
Two-sided exponential sequence, 102
Two-sided sequences, 108–10
Type I FIR linear-phase systems, 298
Type II FIR linear-phase systems, 298
Type III FIR linear-phase systems, 298
Type IV FIR linear-phase systems, 298–99

Undersampled sinusoidal signal, reconstruction of, 148–49
Uniform convergence, of z-transform, 97
Unilateral z-transform, 95
Unit circle, 95–96
Unit impulse function, 142–43
sifting property of, 143
Unit sample sequence, 11
Unit step sequence, 11–13
Unweighted approximation error, 503
Upsampling, 172–76
interchange of filtering and, 179–80

Video coding, 3
Vocal tract, 724
Vocoder simulations, 5
Voiced sounds, 724
Von Hann, Julius, 468fn

Warping, and bilinear transformation, 453–54
"Whitening" procedure, 333
White noise, 69
Wideband spectrogram, 725
Windowing, 465–78, 693
Bartlett window, 468–71
Blackman window, 468fn, 469
effect of, 698–703
on Fourier analysis of sinusoidal signals, 698–701
Hamming window, 468–71
Hanning window, 468–71
incorporation of generalized linear phase, 469–73
Kaiser window, 474–78
linear-phase lowpass filter example, 472–73
rectangular window, 468
Windowing theorem, 61–62
Winograd Fourier transform algorithm (WFTA), 655–56

Zero-input limit cycles, defined, 376, 413
Zero-order hold, compensation for, 506–7
Zero padding, 583
Zeros of $X(z)$, 98
z-plane, 96
z-transform, 94–139
common pairs, 104
defined, 95
evaluating a convolution using, 125
finite-duration sequence, 105
finite-length sequence, 103–4
infinite sum, expressing closed form, 97–98
inverse z-transform, 111–18
inspection method, 111–12
partial fraction expansion, 112–16
left-sided sequence, 99–100, 105
power series expansion, 116–18
by long division, 118
finite-length sequence, 117
inverse transform by, 117
for a left-sided sequence, 118
region of convergence properties for, 105–11
region of convergence (ROC), 96–97
right-sided sequence, 98, 105
sum of two exponential sequences, 100–101
transform properties, 119–26
conjugation property, 123
convolution property, 124–26
differentiation property, 122–23
exponential multiplication, 121–22
initial-value theorem, 126
inverse of non-rational z-transform, 122–23
linearity property, 119
time-reversal property, 123–24
time-shifting property, 120–21
two-sided sequence, 102, 105
uniform convergence of, 97
unit circle, 95–96
z-transform operator Z, 95
z-transform pairs, 104