EE141 Digital Signal Processing (Fall 2003)

Solution Manual

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Text: A.V. Oppenheim, R.W. Schafer, and J. R. Buck, *Discrete-Time Signal Processing*, 2nd edition, Prentice-Hall, 1999.

HW#1: P2.1 (a), (c), (e), (g); P2.4; P2.24

P2.1

(a) T(x[n]) = g[n]x[n];

- Stable if g[n] is bounded;
- Causal – output is not decided by future input;
- Linear – T(ax[n] + by[n]) = ag[n]x[n] + bg[n]y[n] = aT(x[n]) + bT(y[n]);
- Time variant – T(x[n-m]) = g[n]x[n-m];
- Memoryless – output only depends on x[n] with same n;

(c) $T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$;

- Stable;
- Causal only if $n_0 = 0$, else non-causal;
- Linear – T(ax[n] + by[n]) = aT(x[n]) + bT(y[n]);
- Time Invariant – $T(x[n-m]) = \sum_{k=n-m-n_0}^{n-m+n_0} x[k] = \sum_{k'=n-n_0}^{n+n_0} x[k'-m]$;
- Memoryless only if $n_0 = 0$;

(e) $T(x[n]) = e^{x[n]}$;

- Stable;
- Causal;
- Nonlinear – T(ax[n] + by[n]) $\neq$ aT(x[n]) + bT(y[n]);
- Time Invariant – $T(x[n-m]) = e^{x[n-m]}$;
- Memoryless – output only depends on x[n] with same n;

(g) T(x[n]) = x[-n];

- Stable;
- Non-Causal – output depends on future input;
- Linear – T(ax[n] + by[n]) = aT(x[n]) + bT(y[n]);
- Time Variant;
- Not Memoryless;

P2.4

As $y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n-1]$, the Fourier transform is

$$Y(e^{jw})-\frac{3}{4}Y(e^{jw})e^{-jw}+\frac{1}{8}Y(e^{jw})e^{-2jw}=2X(e^{jw})e^{-jw}$$

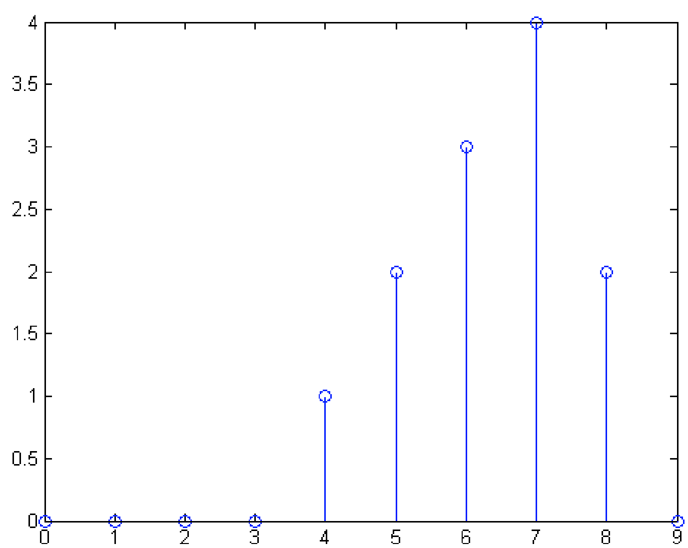
When $x[n]=\delta[n]\Rightarrow X(e^{jw})=1$, thus

$$Y(e^{jw})=\frac{2e^{-jw}}{1-(3/4)e^{-jw}+(1/8)e^{-2jw}}=8\left(\frac{1}{1-(1/2)e^{-jw}}-\frac{1}{1-(1/4)e^{-jw}}\right)$$

$$y[n]=8\left(\left(\frac{1}{2}\right)^n-\left(\frac{1}{4}\right)^n\right)u[n]$$

P2.24

As h[n] = [1 1 1 1 –2 –2] for n from 0 to 5 and x[n] = u[n-4], the system response is:
y[n] = x[n] * h[n] = [0 0 0 0 1 2 3 4 2 0 …..]; The sketch is shown as follows:



HW#2: P2.5; P2.18; P2.29 (a), (c), (e)

P2.5

(a) The roots for polynomial $1-5z^{-1}+6z^{-2}=0$ are 2 and 3, so the homogeneous response for the system is:

$$y[n]=A_1 2^n+A_2 3^n$$

(b) As $y[n]-5y[n-1]+6y[n-2]=2x[n-1]$ and $x[n]=\delta[n]$, the impulse response of the system is:

$$H(e^{jw})=\frac{2e^{-jw}}{1-5e^{-jw}+6e^{-2jw}}=2\left(\frac{1}{1-3e^{-jw}}-\frac{1}{1-2e^{-jw}}\right)$$

$$\Rightarrow h[n]=2(3^n-2^n)u[n]$$

(c) As $y[n]-5y[n-1]+6y[n-2]=2x[n-1]$ and $x[n]=u[n]$, the step response of the system is:

$$Y(z)=H(z)X(z)=\frac{2z^{-1}}{\left(1-5z^{-1}+6z^{-2}\right)\left(1-z^{-1}\right)}=\frac{1}{1-z^{-1}}-\frac{4}{1-2z^{-1}}+\frac{3}{1-3z^{-1}},|z|>3$$

$$\Rightarrow y[n]=(3^{n+1}-2^{n+2}+1)u[n]$$

<u>P2.18</u>

(a) $h[n]=(1/2)^n u[n]$
Causal, the output of the system does not depend on future input;

(b) $h[n]=(1/2)^n u[n-1]$
Causal, the output of the system does not depend on future input;

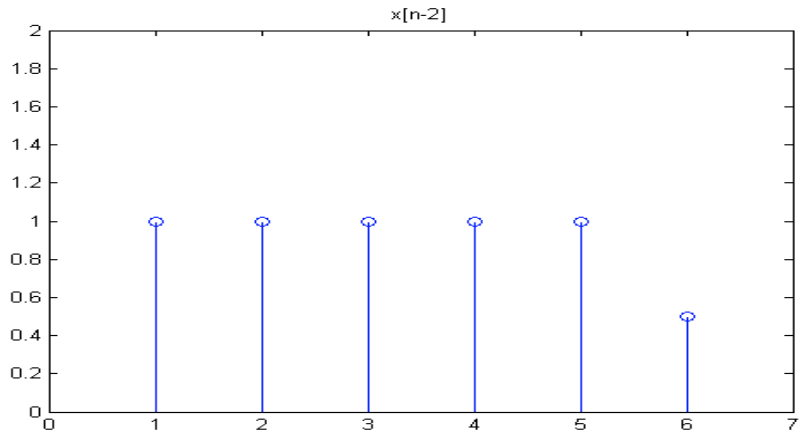
(c) $h[n]=(1/2)^{|n|}$
Non-Causal, the output of the system depends on future input;

(d) $h[n]=u[n+2]-u[n-2]$
Non-Causal, the output of the system depends on future input;

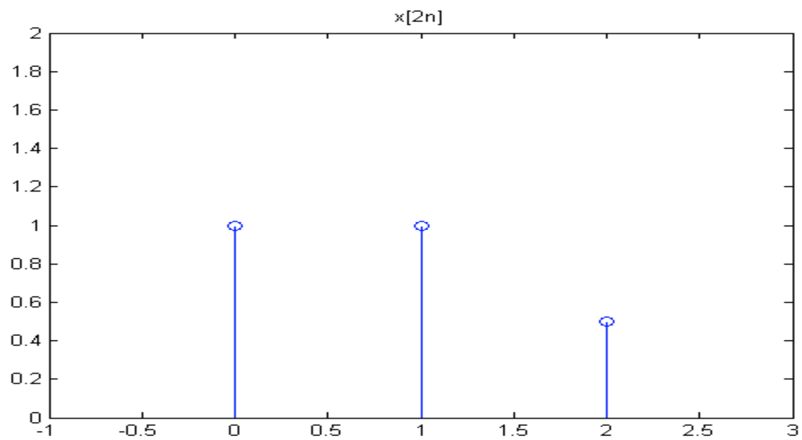
(e) $h[n]=(1/3)^n u[n]+3^n u[-n-1]$
Non-Causal, the output of the system depends on future input;

<u>P2.29</u>

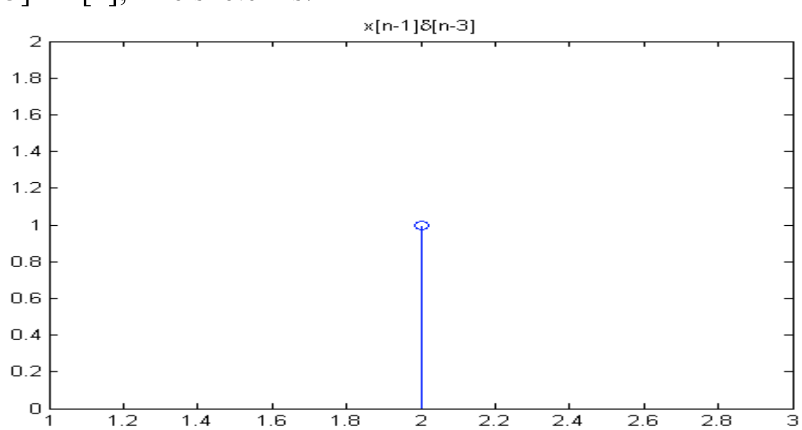
As x[n] = [1 1 1 1 1 1/2] for n from –1 to 4,
(a) x[n-2] = [1 1 1 1 1 1/2] for n from 1 to 6; The sketch is:



(c) x[2n] = [1 1 1/2] for n from 0 to 2; The sketch is:



(e) x[n-1]δ[n-3] = x[2]; The sketch is:



<u>HW#3: P2.40, P2.41, P3.27 (a), (c)</u>

<u>P2.40</u>

$x[n] = \cos(\pi n)u[n] = (-1)^n u[n]$

$h[n] = (\frac{j}{2})^n u[n]$

$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} (\frac{j}{2})^k u[k](-1)^{n-k} u[n-k] = (-1)^n \sum_{k=0}^{n} (-\frac{j}{2})^k$

$= (-1)^n \frac{1-(-j/2)^{n+1}}{1-(-j/2)}$

Since $\lim_{n\to\infty} \frac{1-(-j/2)^{n+1}}{1-(-j/2)} = \frac{1}{1+j/2}$

The steady state response to the excitation $x[n] = (-1)^n u[n]$ is

$(-1)^n \frac{1}{1+j/2} = \frac{\cos(\pi n)}{1+j/2}$

P2.41

Given a periodic impulse train $x[n] = \sum_{k=-\infty}^{\infty} \delta[n+kN]$, we can write its Fourier transform as

$$X(e^{j\omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k / N) \qquad (1)$$

(Refer to *Signal and Systems*, 2nd edition by A.V. Oppenheim and A.S. Willsky, Page 371 for its proof)

In problem, 2.41,*N*=16, so its Fourier transform is

$$X(e^{j\omega}) = \frac{2\pi}{16} \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k / 16) \qquad (2)$$

Let $Y(e^{j\omega})$ denotes the output of the system, then

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \qquad (3)$$

If $|\omega| < 3\pi/8$, $H(e^{j\omega}) = e^{-j\omega 3}$

$$= e^{-j\omega 3} \frac{2\pi}{16} \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k / 16) = \frac{2\pi}{16} [\delta(\omega) + e^{j3\pi/8}\delta(\omega + \pi/8) + e^{-j3\pi/8}\delta(\omega - \pi/8)] \quad (4)$$

If $|\omega| \geq 3\pi/8$, $H(e^{j\omega}) = 0$, thus $Y(e^{j\omega}) = 0$, (5)

So $Y(e^{j\omega}) = \frac{2\pi}{16}[\delta(\omega) + e^{j3\pi/8}\delta(\omega + \pi/8) + e^{-j3\pi/8}\delta(\omega - \pi/8)]$ (6)

Take the inverse Fourier transform, we can get

$$y[n] = \frac{1}{16}(1 + e^{j3\pi/8}e^{-n\pi/8} + e^{-j3\pi/8}e^{n\pi/8}) = \frac{1}{16}(1 + e^{-(n-3)\pi/8} + e^{(n-3)\pi/8})$$
$$= \frac{1}{16}(1 + 2\cos\frac{\pi}{8}(n-3)) = \frac{1}{16} + \frac{1}{8}\cos(\frac{\pi}{8}(n-3)) \qquad (7)$$

Note: Take a look at (3), $H(e^{j\omega})$ is band limited, $X(e^{j\omega})$ is infinite pulse train. If we multiply them together, we can only consider those pulses falling into the band $(-3\pi/8, 3\pi/8)$. The rest pulses are cancelled due to the multiplication with 0. There are three pulses falling into the band $\frac{2\pi}{16}\delta(\omega)$, $\frac{2\pi}{16}\delta(\omega+\pi/8)$, $\frac{2\pi}{16}\delta(\omega-\pi/8)$, so we get (6)

P3.27

(a)

$$X(z)=\frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})}=\frac{A}{(1+\frac{1}{2}z^{-1})^2}+\frac{B}{1+\frac{1}{2}z^{-1}}+\frac{C}{1-2z^{-1}}+\frac{D}{1-3z^{-1}}$$

X (z)'s poles are z=-1/2, 2, 3, if it is stable, the ROC is $|z|\in(1/2,2)$

$$A=X(z)(1+\frac{1}{2}z^{-1})^2\Big|_{z=-1/2}=\frac{1}{(1-2z^{-1})(1-3z^{-1})}\Big|_{z=-1/2}=\frac{1}{35}$$

$$C=X(z)(1-2z^{-1})\Big|_{z=2}=\frac{1}{(1+\frac{1}{2}z^{-1})^2(1-3z^{-1})}\Big|_{z=2}=-\frac{1568}{1225}$$

$$D=X(z)(1-3z^{-1})\Big|_{z=3}=\frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})}\Big|_{z=3}=\frac{2700}{1225}$$

Also, Let $z^{-1}=0$ at both sides,

$$X(z)\Big|_{z^{-1}=0}=1=A+B+C+D$$

Thus, $B=1-A-C-D=\frac{58}{1225}$

$$X(z)=\frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})}=\frac{1/35}{(1+\frac{1}{2}z^{-1})^2}+\frac{58/1225}{1+\frac{1}{2}z^{-1}}-\frac{1568/1225}{1-2z^{-1}}+\frac{2700/1225}{1-3z^{-1}}$$

Since the ROC is $|z|\in(1/2,2)$,

$$x[n]=\frac{1}{35}(n+1)(-\frac{1}{2})^n u[n+1]+\frac{58}{1225}(-\frac{1}{2})^n u[n]+\frac{1568}{1225}2^n u[-n-1]-\frac{2700}{1225}3^n u[-n-1]$$

Note: To get the inverse Z-Transform of second-order term or multiple order term, we can use the differentiation property $nx[n]\leftrightarrow -z\frac{dX(z)}{dz}$ (Refer to page122 of textbook for its proof)

E.g. right side sequence $x[n]=a^n u[n]\leftrightarrow\frac{1}{1-az^{-1}}$ (ROC: $|z|>|a|$)

$$na^n[n]\leftrightarrow -z[{d(\frac{1}{1-az^{-1}})}\Big/{dz}]=\frac{az^{-1}}{(1-az^{-1})^2}\text{, So } na^{n-1}[n]\leftrightarrow\frac{z^{-1}}{(1-az^{-1})^2}$$

(c)

$$x[n]\leftrightarrow X(z)=\frac{z^3-2z}{z-2}=z^2+2z+\frac{2}{1-2z^{-1}}$$

X(z) has its only pole at z=2. If x[n] is a left-sided sequence, the ROC is $|z|<2$

$$x[n]=\delta[n+2]+2\delta[n+1]-2(2)^n u[-n-1]$$

or $x[n]=\delta[n+2]+2\delta[n+1]-2^{n+1}u[-n-1]$

P3.1

(g)

$(\frac{1}{2})^n(u[n]-u[n-10]=\sum_{n=0}^{9}(\frac{1}{2})^n\delta[n]$ is a finite length sequence, so its ROC is $|z|\neq 0$. The solution in the textbook is right.

HW#4: P3.6 (d), (e); P3.20; P4.1; P4.3

P3.6

(d) $X(z)=\dfrac{1-(1/2)z^{-1}}{1-(1/4)z^{-2}}$ $\quad\quad |z|>1/2$

- Partial Fraction Expansion:

$$X(z)=\frac{1-(1/2)z^{-1}}{1-(1/4)z^{-2}}=\frac{1-(1/2)z^{-1}}{(1-(1/2)z^{-1})(1+(1/2)z^{-1})}=\frac{1}{1+(1/2)z^{-1}}$$

$$\Rightarrow x[n]=\left(-\frac{1}{2}\right)^n u[n]$$

- Power Series Expansion:

$$X(z)=\frac{1-(1/2)z^{-1}}{1-(1/4)z^{-2}}=1-(1/2)z^{-1}+(1/4)z^{-2}-(1/8)z^{-3}+(1/16)z^{-4}+\cdots$$

$$\Rightarrow x[n]=\left(-\frac{1}{2}\right)^n u[n]$$

- Fourier Transform exists as the ROC including unit circle.

(e) $X(z)=\dfrac{1-az^{-1}}{z^{-1}-a}$ $\quad\quad |z|>|1/a|$

- Partial Fraction Expansion:

$$X(z)=\frac{1-az^{-1}}{z^{-1}-a}=\frac{1}{z^{-1}-a}-\frac{az^{-1}}{z^{-1}-a}=\frac{1}{z^{-1}-a}-a-\frac{a^2}{z^{-1}-a}=\frac{-1/a}{1-(1/a)z^{-1}}-a+\frac{a}{1-(1/a)z^{-1}}$$

$$\Rightarrow x[n]=-\left(\frac{1}{a}\right)^{n+1}u[n]-a\delta[n]+\left(\frac{1}{a}\right)^{n-1}u[n]=-\left(\frac{1}{a}\right)^{n+1}u[n]+\left(\frac{1}{a}\right)^{n-1}u[n-1]$$

- Power Series Expansion:

$$X(z)=\frac{1-az^{-1}}{z^{-1}-a}=\frac{1}{z^{-1}-a}-\frac{az^{-1}}{z^{-1}-a}$$

$$\frac{1}{z^{-1}-a}=\left(-\frac{1}{a}\right)\left(1+\frac{1}{a}z^{-1}+\frac{1}{a^2}z^{-2}+\frac{1}{a^3}z^{-3}+\cdots\right)\Rightarrow x1[n]=\left(-\frac{1}{a}\right)\left(\frac{1}{a}\right)^{n}u[n]=-\left(\frac{1}{a}\right)^{n+1}u[n]$$

$$-\frac{az^{-1}}{z^{-1}-a}=z^{-1}+\frac{1}{a}z^{-2}+\frac{1}{a^2}z^{-3}+\frac{1}{a^3}z^{-4}+\cdots\Rightarrow x2[n]=\left(\frac{1}{a}\right)^{n-1}u[n-1]$$

$$\Rightarrow x[n]=x1[n]+x2[n]=-\left(\frac{1}{a}\right)^{n+1}u[n]+\left(\frac{1}{a}\right)^{n-1}u[n-1]$$

- Fourier Transform exists when the ROC including unit circle, which means $|a|<1$.

P3.20

(a) As the ROC of *X*(z) is |z| > 3/4, and the ROC of *Y*(z) is |z| > 2/3, the ROC of *H*(z) should be |z| > 2/3 ;

(b) As the ROC of *X*(z) is |z| < 1/3, and the ROC of *Y*(z) is 1/6 < |z| < 1/3, the ROC of *H*(z) should be |z| > 1/6 ;

P4.1

As $x_c(t)=\sin[2\pi(100t)]$ and T = 1/400 sec,

$$x[n]=x_c(nT)=\sin[2\pi(100nT)]=\sin\left(\frac{n\pi}{2}\right)$$

P4.3

As $x_c(t)=\cos[4000\pi t]$ and $x[n]=\cos\left[\frac{n\pi}{3}\right]$,

(a) Let $x[n]=x_c(nT)\Rightarrow T=\frac{1}{12{,}000}$

(b) T is not unique, for example, $T=\frac{5}{12{,}000}$

HW#5: P4.5; P4.7; P5.2; P5.3

P4.5

(a) From Nyquist Sampling theorem, to avoid aliasing in the C/D converter, the sampling frequency $\Omega_s=\frac{1}{T_s}\geq 2\Omega_m=2*5000Hz=10^4\,Hz$, so $T_s\leq 10^{-4}\,s$

(b) $\Omega_{cutoff} = \frac{f_{cutoff}}{2\pi}\Omega_s = \frac{\pi/8}{2\pi}10^4 = 625Hz$

(c) $\Omega_{cutoff} = \frac{f_{cutoff}}{2\pi}\Omega_s = \frac{\pi/8}{2\pi}2\times 10^4 = 1250Hz = 1.25kHz$

Note: The relation between digital frequency f and analog frequency Ω is $\frac{\Omega}{\Omega_s} = \frac{f}{2\pi}$, where Ω_s is the sampling frequency, f is in radians.

P4.7

(a)

$x_c(t) = s_c(t) + \alpha s_c(t-\tau_d)$

$X_c(j\Omega) = S_c(j\Omega)(1+\alpha e^{-j\Omega\tau_d})$

Consider sampling, $x[n] = x_c(nT)$, in frequency domain (refer to Eq4.19 in textbook, P147),

$$X\left(e^{j\Omega T}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$

$$X\left(e^{j\omega}\right) = X\left(e^{j\Omega T}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right) = \frac{1}{T}(1+\alpha e^{-j\Omega\tau_d})\sum_{k=-\infty}^{\infty} S_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$

$$= \frac{1}{T}(1+\alpha e^{-j\omega\tau_d/T})\sum_{k=-\infty}^{\infty} S_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

(b)

$H(e^{j\omega}) = 1 + \alpha e^{-j\omega\tau_d/T}$

(c)

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}(1+\alpha e^{-j\omega\tau_d/T})e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jn\omega}d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi}\alpha e^{-j\omega(n-\tau_d/T)}d\omega$$

$$= \frac{\sin(n\pi)}{n\pi} + \alpha\frac{\sin[(n-\tau_d/T)\pi]}{(n-\tau_d/T)\pi}$$

if $\tau_d = T$, $h[n] = \frac{\sin(n\pi)}{n\pi} + \alpha\frac{\sin[(n-1)\pi]}{(n-1)\pi} = \delta[n] + \alpha\delta[n-1]$

if $\tau_d = T/2$, $h[n] = \frac{\sin(n\pi)}{n\pi} + \alpha\frac{\sin[(n-1/2)\pi]}{(n-1)\pi} = \delta[n] + \alpha\frac{\sin[(n-1/2)\pi]}{(n-1)\pi}$

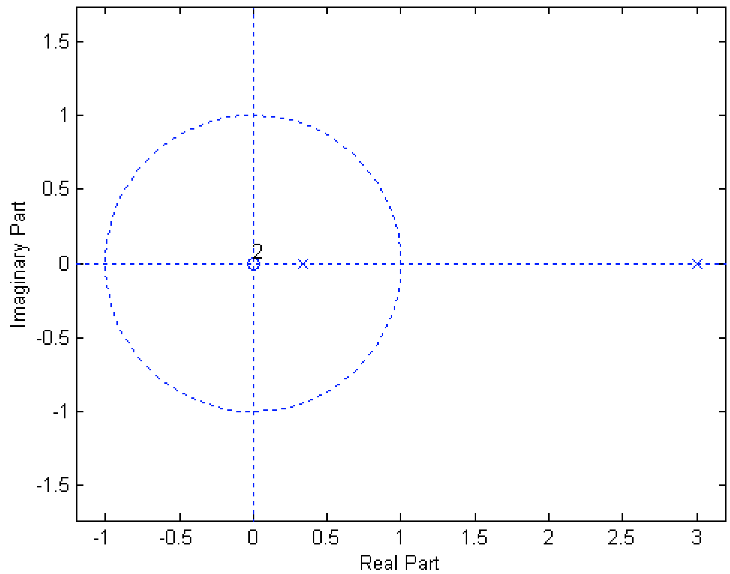
P5.2

$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n]$

$z^{-1}Y(z) - \frac{10}{3}Y(z) + zY(z) = X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{(1-(1/3)z^{-1})(1-3z^{-1})} = \frac{-3/8}{1-(1/3)z^{-1}} + \frac{3/8}{1-3z^{-1}}$$

(a) H (z) has two zeroes: 0, ∞; two poles: 1/3, 3



(b) The system is stable, so the ROC includes the unit circle. The ROC is 1/3<|z|<3

$$h[n] = -\frac{3}{8}(\frac{1}{3})^n u[n] - \frac{3}{8} 3^n u[-n-1]$$

P5.3

$$y[n-1] + \frac{1}{3} y[n-2] = x[n]$$

$$z^{-1}Y(z) + \frac{1}{3} z^{-2} Y(z)) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} + (1/3)z^{-2}} = \frac{1}{z^{-1}(1+(1/3)z^{-1})} = \frac{z}{1+(1/3)z^{-1}}$$

The poles are: $1/3$, there are two ROC, $0 <|z|< 1/3$, $|z|> 1/3$

(1) $0 <|z|< 1/3$: $h[n] = -(-\frac{1}{3})^{n+1} u[-n-2]$, choose (d)

(2) $|z|> 1/3$: $h[n] = (-\frac{1}{3})^{n+1} u[n+1]$, choose (a)

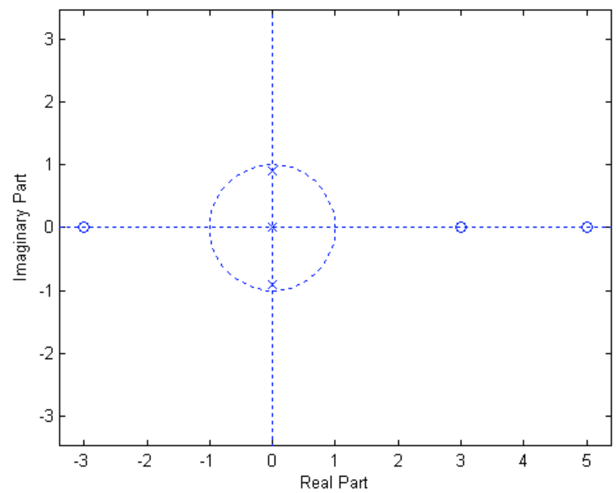
HW#6: P5.10; P5.12; P5.15

P5.10

As one of the zeros of H(z) is at $z = \infty$, the corresponding pole of Hi(z) will be also at infinity. The existence of a pole at $z = \infty$ implies that the system is not causal.

P5.12

(a)



As the poles of H(z) are 0.9, -0.9, ROC includes the unit circle, the system is stable.

(b)

$$H(z) = \frac{(1+0.2z^{-1})(1+3z^{-1})(1-3z^{-1})}{(1+j0.9z^{-1})(1-j0.9z^{-1})}$$

$$= \underbrace{\frac{-9(1+0.2z^{-1})(1+(1/3)z^{-1})(1-(1/3)z^{-1})}{(1+j0.9z^{-1})(1-j0.9z^{-1})}}_{H_1(z)} \underbrace{\frac{(z^{-1}+1/3)(z^{-1}-1/3)}{(1+(1/3)z^{-1})(1-(1/3)z^{-1})}}_{H_{ap}(z)}$$

$$= H_1(z)H_{ap}(z)$$

P5.15

Generalized Linear Phase – GLP;

Linear Phase – LP;

(a) As $h[n] = 2\delta[n] + \delta[n-1] + 2\delta[n-2] \Rightarrow H(jw) = (1+4\cos w)e^{-jw}$, $A(jw) = 1+4\cos w$ and $\alpha = 1, \beta = 0$, it is a GLP, but not LP as $A(jw)$ is not always nonnegative for all w;

(b) It is not a GLP or LP as it is not a symmetric filter;

(c) As $h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2] \Rightarrow H(jw) = (3+2\cos w)e^{-jw}$, $A(jw) = 3+2\cos w$ and $\alpha = 1, \beta = 0$, it is a GLP and also LP as $A(jw)$ is always nonnegative for all w;

(d) As $h[n] = \delta[n] + \delta[n-1] \Rightarrow H(jw) = 2\cos(w/2)e^{-j(w/2)}$,

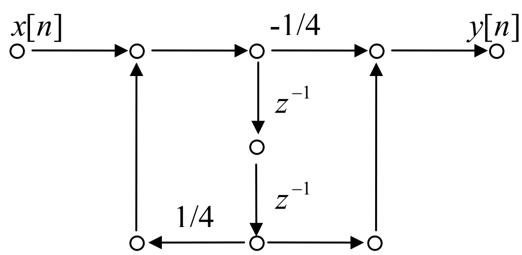
$A(jw)=2\cos(w/2)$ and $\alpha=1/2, \beta=0$, it is a GLP, but not LP as $A(jw)$ is not always nonnegative for all w;

(e) As $h[n]=\delta[n]-\delta[n-2] \Rightarrow H(jw)=2\sin we^{-j(w-\pi/2)}$,
$A(jw)=2\sin w$ and $\alpha=1, \beta=\pi/2$, it is a GLP, but not LP as $A(jw)$ is not always nonnegative for all w;

<u>HW#7: P6.7; P6.8; P6.11; P6.25; P7.15</u>
<u>P6.7</u>
The difference equation is: $y[n]-\frac{1}{4}y[n-2]=x[n-2]-\frac{1}{4}x[n]$

Z-Transform: $Y(z)-\frac{1}{4}Y(z)z^{-2}=X(z)z^{-2}-\frac{1}{4}X(z)$

Transfer function: $H(z)=\frac{Y(z)}{X(z)}=\frac{-\frac{1}{4}+z^{-2}}{1-\frac{1}{4}z^{-2}}$

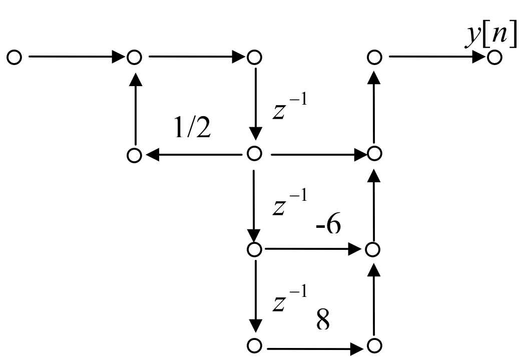


<u>P6.8</u>
$y[n]-2y[n-2]=3x[n-1]+x[n-2]$

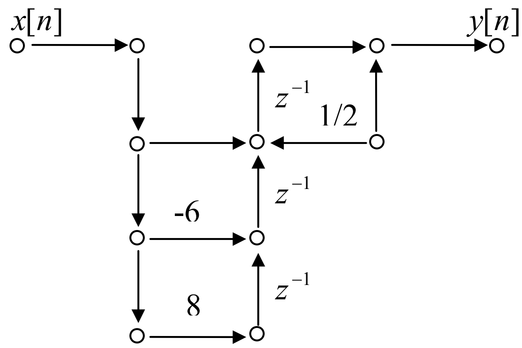
<u>P6.11</u>

$$H(z)=\frac{z^{-1}(1-2z^{-1})(1-4z^{-1})}{1-\frac{1}{2}z^{-1}}=\frac{z^{-1}-6z^{-2}+8z^{-3}}{1-\frac{1}{2}z^{-1}}$$

(a)



(b)



P6.25

(a) $H(z)=[H_1(z)+H_2(z)]H_3(z)=[\dfrac{1-\frac{1}{2}z^{-1}}{1-\frac{3}{8}z^{-1}+\frac{7}{8}z^{-2}}+(1+2z^{-1}+z^{-2})](\dfrac{1}{1-z^{-1}})$

The Z-Transfer function:

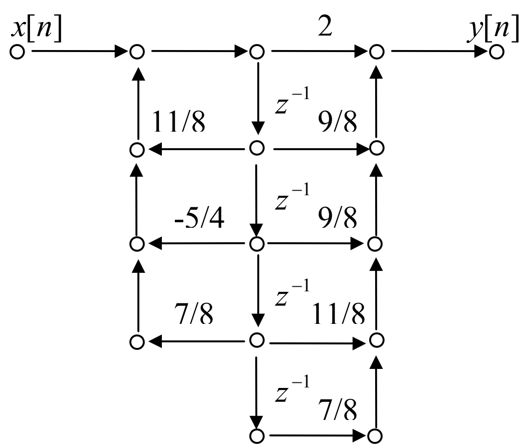
$$H(z)=\frac{2+\frac{9}{8}z^{-1}+\frac{9}{8}z^{-2}+\frac{11}{8}z^{-3}+\frac{7}{8}z^{-4}}{1-\frac{11}{8}z^{-1}+\frac{5}{4}z^{-2}-\frac{7}{8}z^{-3}}$$

(b)

The difference equation:

$$y[n]-\frac{11}{8}y[n-1]+\frac{5}{4}y[n-2]-\frac{7}{8}y[n-3]=2x[n]+\frac{9}{8}x[n-1]+\frac{9}{8}x[n-2]+\frac{11}{8}x[n-3]+\frac{7}{8}x[n-4]$$

(c)



P7.15

Specifications:

(a) Pass band ripple: $\delta_p=0.05$, $A_p=20\log\delta_p=-26.02dB$

Stop band ripple: $\delta_s=0.1$, $A_s=20\log\delta_s=-20dB$

Pass band edge: $\omega_p=0.25\pi$

Stop band edge: $\omega_s=0.35\pi$

Cutoff: $\omega_c = 0.3\pi$

The peak approximate error $20\log_{10}\delta < -26.02dB$

Among the windows in Table7.1 (Page 471), Hanning, Hamming, Blackman can be used

(b)

Hanning: $0.1\pi = \frac{8\pi}{M}$, $M = 80$, $L = M + 1 = 81$

Hamming: $0.1\pi = \frac{8\pi}{M}$, $M = 80$, $L = M + 1 = 81$

Blackman: $0.1\pi = \frac{12\pi}{M}$, $M = 120$, $L = M + 1 = 121$

Note that the estimation is not accurate. We can use MATLAB to find the minimum of filter order to meet the requirements.