EEM 323

ELECTROMAGNETIC WAVE THEORY II

NORMAL INCIDENCE

AT PLANAR BOUNDARY

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Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

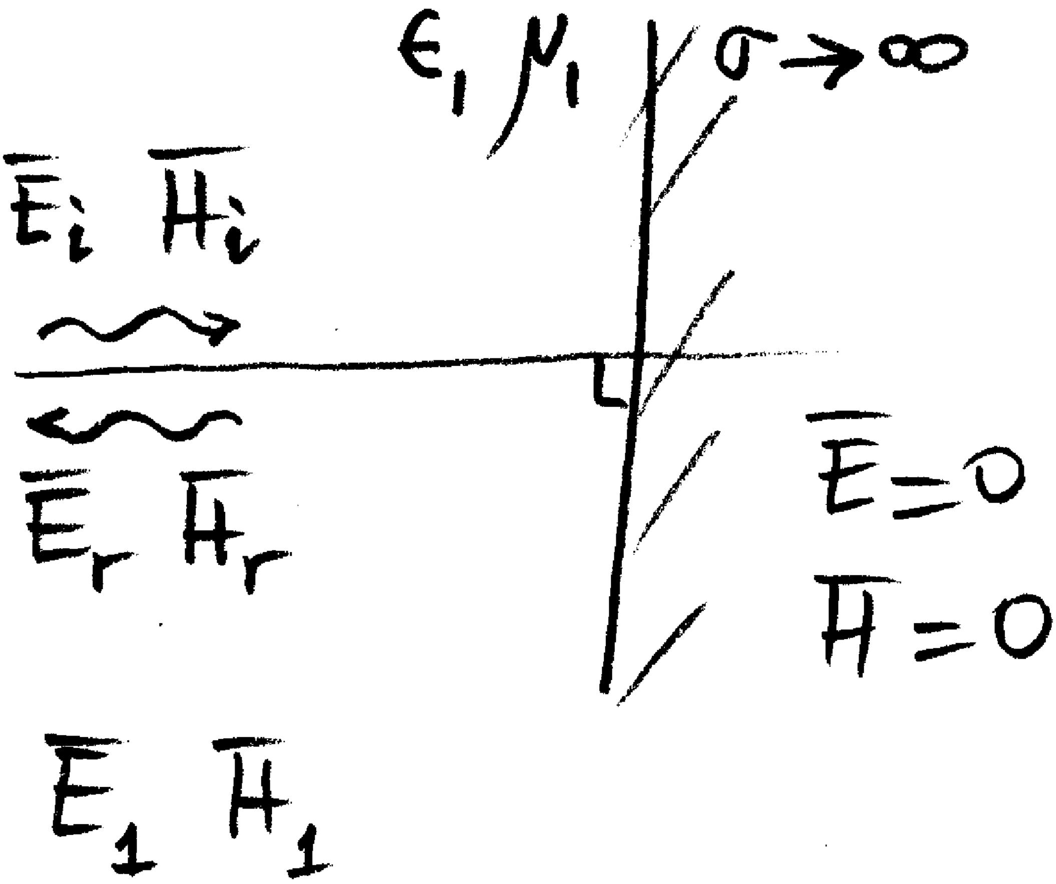
[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

NORMAL INCIDENCE AT PLANAR BOUNDARY OF

Dielectric – Perfect conductor boundary:

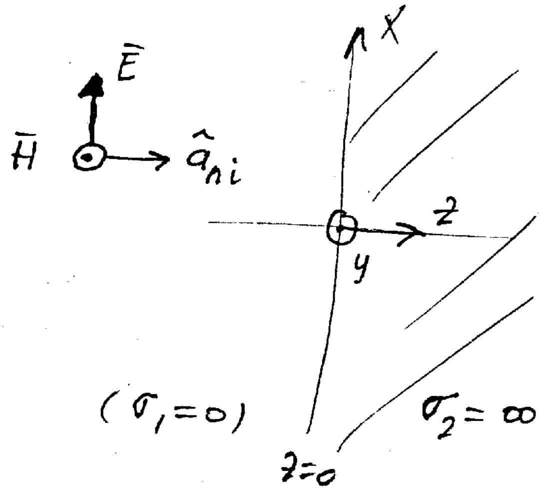


Problem:
Given the incident E field, calculate all the other fields

• NORMAL INCIDENCE AT A COND. BOUNDARY

$$E_i(z) = \hat{a}_x E_{io} e^{-j\beta_1 z}$$

$$H_i(z) = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}$$



reflected waves?

$$E_r(z) = \hat{a}_x E_{ro} e^{j\beta_1 z}$$

$$E_1(z) = E_i(z) + E_r(z) = \hat{a}_x \left(E_{io} e^{-j\beta_1 z} + E_{ro} e^{j\beta_1 z} \right)$$

Cond. Surface : Boundary condition

$$E_1(z=0) = \hat{a}_x (E_{io} + E_{ro}) = E_2(0) = 0$$

$$\Rightarrow \boxed{E_{ro} = -E_{io}}$$

HW(BONUS)

plot $|E|$ vs. z in time. (multiplot)

$$E_r(z) = -\hat{a}_x E_{io} e^{+j\beta_1 z}$$

$$E_1(z) = \hat{a}_x E_{io} \underbrace{\left(e^{-j\beta_1 z} - e^{j\beta_1 z} \right)}_{-2j \sin\beta_1 z}$$

TOTAL FIELD :

$$\boxed{E_1(z) = -\hat{a}_x \, j 2E_{io} \sin\beta_1 z}$$

Standing Wave

EX y-polarized uniform plane wave $(\bar{E}_i\ \bar{H}_i)$

frequency 100 MHz, in air, normal incidence to $x=0$.

$x=0$; perfectly conducting plane boundary.

$|\bar{E}_i| = 6\ (mV/m)$

(a) Phasor and instantaneous expressions $\bar{E}_i\ \bar{H}_i$

(b) $\bar{E}_r\ \bar{H}_r$ reflected wave.

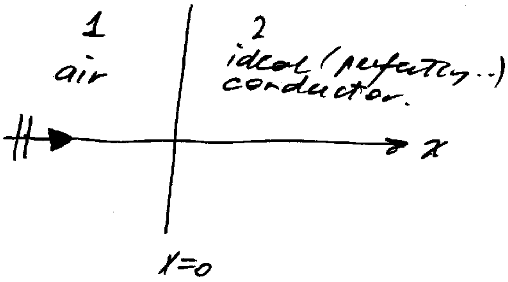
(c) $\bar{E}_1\ \bar{H}_1$ total wave

(d) determine the location nearest to the conducting plane where $\bar{E}_1 = 0$.

GKhangur

$$\omega = 2\pi f = 2\pi \cdot 10^8\ (rad/s.)$$

$$\beta_1 = k_0 = \frac{\omega}{c} = \frac{2\pi \cdot 10^8}{3 \cdot 10^8} = \frac{2\pi}{3}\ (rad/s)$$



$$\eta_1 = \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ (\Omega)$$

(a)
$$\bar{E}_i(x) = \hat{a}_y\ 6 \cdot 10^{-3} \cdot e^{-j\frac{2\pi}{3}x}\ (V/m)$$

$$\bar{H}_i(x) = \frac{1}{\eta_1}\ \hat{a}_x \times \bar{E}_i(x) = \hat{a}_z\ \frac{10^{-4}}{2\pi} \cdot e^{-j\frac{2\pi x}{3}}\ (A/m)$$

Phasor Form

Instantaneous Expressions (Anlık Değerleri)

$$\bar{\mathcal{E}}_i(x,t) = Re\{\bar{E}_i(x)\,e^{j\omega t}\}$$

$$= \hat{a}_y\, 6.10^{-3} . \cos\left(2\pi.10^8 . t - \frac{2\pi}{3}x\right) \quad (V/m)$$

$$\bar{\mathcal{H}}_i(x,t) = \hat{a}_z\, \frac{10^{-4}}{2\pi} . \cos\left(2\pi.10^8 . t - \frac{2\pi}{3}x\right) \quad (A/m)$$

(b) $\bar{E}_r(x) = -\hat{a}_y\, 6.10^{-3} . e^{j\frac{2\pi}{3}x} \quad (V/m)$

$$\bar{H}_r(x) = \frac{1}{\eta_1}(-\hat{a}_x) \times \bar{E}_r(x) = \hat{a}_z\, \frac{10^{-4}}{2\pi}\, e^{j\frac{2\pi}{3}x} \quad (A/m)$$

Instantaneous Expressions

S. Gökhun Tanyer

$$\bar{\mathcal{E}}_r(x,t) = Re\{\bar{E}_r(x)\,e^{j\omega t}\} = -\hat{a}_y\, 6.10^{-3} . \cos\left(2\pi.10^8 . t + \frac{2\pi}{3}x\right)$$

$$\bar{\mathcal{H}}_r(x,t) = Re\{\bar{H}_r(x)\,e^{j\omega t}\} = \hat{a}_z\, \frac{10^{-4}}{2\pi} \cos\left(2\pi.10^8\, t + \frac{2\pi}{3}x\right)$$

(c) Total Wave (Standing Wave)

$$\bar{E}_1(x) = \bar{E}_i(x) + \bar{E}_r(x) = +\hat{a}_y\, 6.10^{-3}\left(e^{-j\frac{2\pi}{3}x} - e^{j\frac{2\pi}{3}x}\right)$$

$$\bar{E}_1(x) = -\hat{a}_y\, j12 . 10^{-3} . \sin\left(\frac{2\pi}{3}x\right) \quad (V/m)$$

$$\bar{H}_1(x) = \bar{H}_i(x) + \bar{H}_r(x) = \hat{a}_z\, \frac{10^{-4}}{\pi} \cos\left(\frac{2\pi}{3}x\right) \quad (A/m)$$

Instantaneous Expressions

$$\bar{\mathcal{E}}_1(x,t) = \mathrm{Re}\{\bar{E}_1(x)\, e^{j\omega t}\} = \hat{a}_y\ 12 \cdot 10^{-3} \sin\left(\frac{2\pi}{3}x\right) \cdot \sin(2\pi \cdot 10^8 t)$$

$$\bar{\mathcal{H}}_1(x,t) = \mathrm{Re}\{\bar{H}_1(x)\, e^{j\omega t}\} = \hat{a}_z\ \frac{10^{-4}}{\pi} \cos\left(\frac{2\pi}{3}x\right) \cdot \cos(2\pi \cdot 10^8 t)$$

(d) Standing Wave - Nulls

$$x_1 = -\frac{\lambda_1}{2} = -\frac{\pi}{\beta_1} = -\frac{3}{2} \ (m)$$

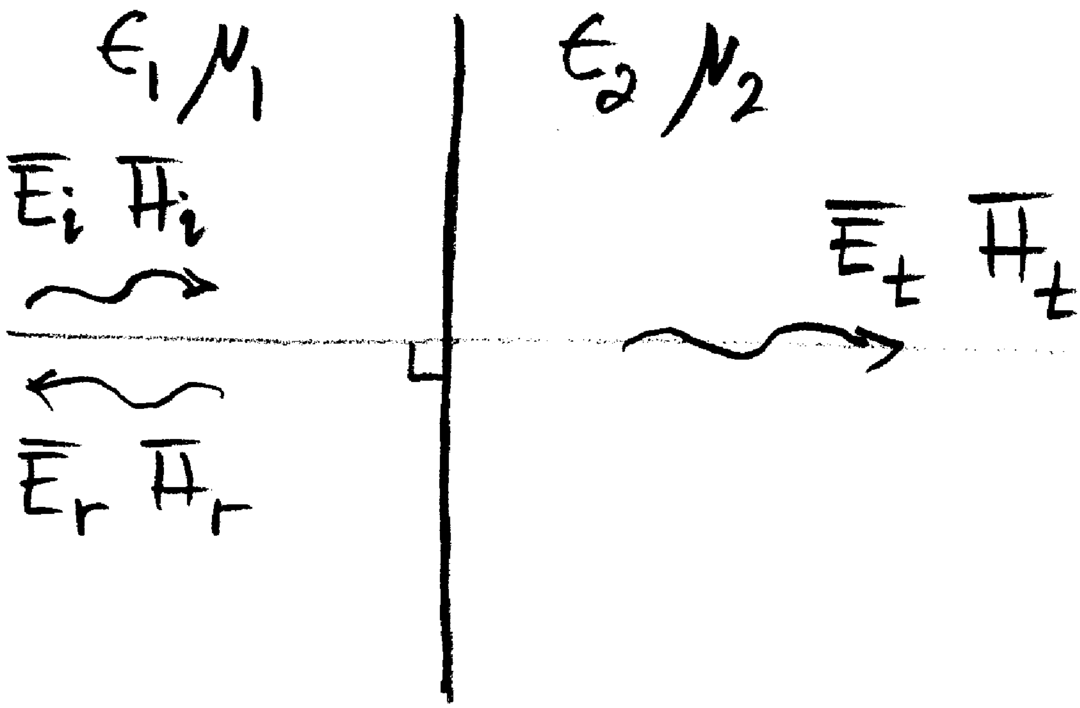
ÖDEV: Second, Third -- null $x_1\ x_2\ x_3\ \cdots = ?$

ÖDEV:

Durağan dalga çözümünü Manyetik alan için hesaplayınız.

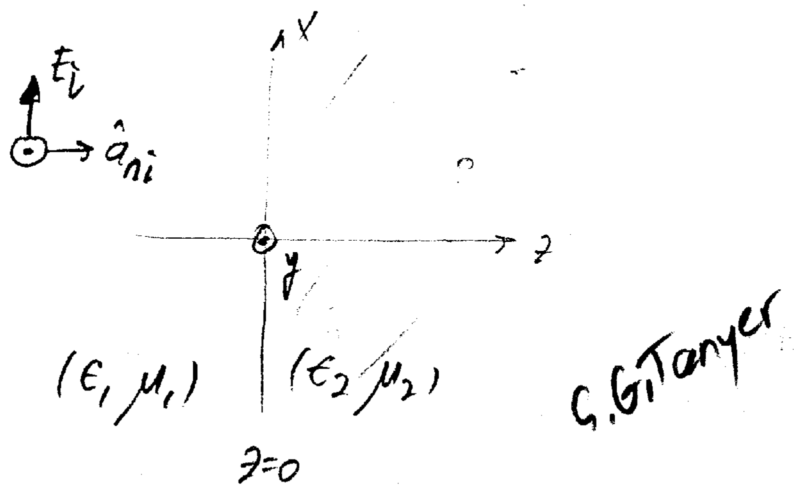
NORMAL INCIDENCE AT PLANAR BOUNDARY OF

Dielectric – Conductor boundary:



Problem:

Given the incident E field, calculate all the other fields



$$\bar{E}_i(z) = \hat{a}_x\, E_{i0} \cdot e^{-j\beta_1 z}$$ Incident Wave

$$\bar{H}_i(z) = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

$$\bar{E}_r(z) = \hat{a}_x\, E_{r0}\, e^{j\beta_1 z}$$ Reflected Wave

$$\bar{H}_r(z) = (-\hat{a}_z) \times \frac{1}{\eta_1} \bar{E}_r(z) = -\hat{a}_y \frac{E_{r0}}{\eta_1} e^{j\beta_1 z}$$

$$E_t(z) = \hat{a}_x \cdot E_{t0} \cdot e^{-j\beta_2 z}$$ Transmitted Wave

$$H_t(z) = \hat{a}_z \times \frac{1}{\eta_2} \bar{E}_t(z) = \hat{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z}$$

QUESTION:

- Given only the incident Electric field, what are the fields in both regions?
- How many equations do we need?

Boundary Conditions at $z=0$ (valid for tangential components only)

$$E_i(0) + E_r(0) = E_t(0) \Rightarrow \boxed{E_{io} + E_{ro} = E_{to}}$$

$$H_i(0) + H_r(0) = H_t(0) \Rightarrow \boxed{\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}}$$

$$\boxed{E_{ro} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{io}}$$

$$\boxed{E_{to} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{io}}$$

Reflection Coefficient

$$\boxed{\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

$$\boxed{\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

$$\boxed{1 + \Gamma = \tau}$$

If medium 2 is perfect conductor,

$\Rightarrow \Gamma = -1 \quad , \quad \tau = 0$

$\Rightarrow E_{ro} = -E_{io} \quad , \quad E_{to} = 0$

$\Rightarrow$ Total reflection and Standing wave

If medium 2 <u>is not</u> a perfect conductor,

$\Rightarrow$ Partial reflection, partial transmission

<u>Note:</u> Given $\bar{E}_i$ $\Rightarrow$ Find $\bar{E}_r$ $\bar{E}_t$

$\Rightarrow$ find $\bar{H}_i$ $\bar{H}_r$ $\bar{H}_t$

$\Rightarrow$ find $\bar{E}_1$ $\bar{H}_1$

$\bar{E}_2$ $\bar{H}_2$

$$\bar{H}_i(z) = \frac{(\hat{a}_z \times \bar{E}_{io})}{\eta_1} e^{-j\beta_1 z}$$

$$\bar{H}_r(z) = \frac{(-\hat{a}_z \times \bar{E}_{io})}{\eta_1} (\Gamma) e^{j\beta_1 z}$$

$$\bar{H}_t(z) = \frac{(\hat{a}_z \times E_{io})}{\eta_2} (\tau) e^{-j\beta_2 z}$$

REVIEW:

Assume $|\Gamma| \neq 0$, $\neq 1$ yani and $0 < |\Gamma| < 1$

$|\tau| \neq 0$, $\neq 1$ yani and $0 < |\tau| < 1$

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$$\bar{E}_1(z) = \bar{E}_i(z) + \bar{E}_r(z) = \hat{a}_x E_{i0}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)$$

$$= \hat{a}_x E_{i0}\left[(1+\Gamma)e^{-j\beta_1 z} + \Gamma\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\right]$$

$$= \hat{a}_x E_{i0}\left[(1+\Gamma)e^{-j\beta_1 z} + \Gamma(j2\sin\beta_1 z)\right]$$

$$\boxed{\bar{E}_1(z) = \hat{a}_x E_{i0}\left[\tau \cdot e^{-j\beta_1 z} + \Gamma(j2\sin\beta_1 z)\right]}$$

Standing Wave Ratio

$$\boxed{S = \frac{|E|_{max}}{|E|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

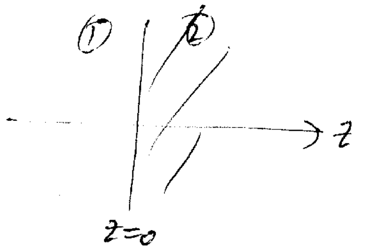
$$\boxed{|\Gamma| = \frac{S-1}{S+1}}$$

Ex η_1 real, η_2 real $\Rightarrow$ Γ and τ real.

(dissipationless media).

Γ is positive or negative

$\boxed{\Gamma > 0 \quad (\eta_2 > \eta_1)}$ Ç.G.T.



$$\max\{|\bar{E}_1(z)|\} = E_{i0}(1+\Gamma)$$

QUESTION: If the reflection coefficient is positive and +1, what would be the standing wave function?

maximum occurs when $2\beta_1 z_{max} = -2n\pi \quad (n=0,1,2..)$

$$z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2}$$

minimum occurs when $|\bar{E}_1(z)| = E_{i0}(1-\Gamma)$

$$2\beta_1 \cdot z_{min} = -(2n+1)\pi$$

$$z_{min} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4}$$

QUESTION: If the reflection coefficient is negative and -1, what would be the standing wave function?

$\boxed{\Gamma < 0 \quad (\eta_2 < \eta_1).}$

$$\max\{|\bar{E}_1(z)|\} = E_{i0}(1-\Gamma)$$

$$\min\{|\bar{E}_1(z)|\} = E_{i0}(1+\Gamma)$$

locations of min and max are interchanged.

Ex uniform plane wave, lossless media

η_1, η_2 intrinsic impedances, plane boundary

$\Rightarrow$ Obtain time-average power densities in ①, ②.

$$\bar{P}_{av} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$$

$$\bar{E}_1(z) = \hat{a}_x E_{io} e^{-j\beta_1 z}(1+\Gamma e^{j2\beta_1 z})$$

$$\bar{H}_1(z) = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}(1-\Gamma e^{j2\beta_1 z})$$

$$\bar{P}_{av,1} = \hat{a}_z \frac{E_{io}^2}{2\eta_1} Re\{(1-\Gamma e^{j2\beta_1 z})(1-\Gamma e^{-j2\beta_1 z})\}$$

$$= \hat{a}_z \frac{E_{io}^2}{2\eta_1} Re\{(1-\Gamma^2) + \Gamma(e^{j2\beta_1 z} - e^{-j2\beta_1 z})\}$$

$$= \hat{a}_z \frac{E_{io}^2}{2\eta_1} Re\{(1-\Gamma^2) + j\, 2\Gamma \cdot \sin 2\beta_1 z\}$$

SGT.

$$\boxed{\bar{P}_{av,1} = \hat{a}_z \frac{E_{io}^2}{2\eta_1}(1-\Gamma^2)}$$

Γ } real since ① ② lossless.

Similarly

ÖDEV

$$\boxed{\bar{P}_{av,2} = \hat{a}_z \frac{E_{io}^2}{2\eta_2} \tau^2}$$

We should have

$$\bar{P}_{av,1} = \bar{P}_{av,2} \Rightarrow \boxed{1-\Gamma^2 = \frac{\eta_1}{\eta_2}\tau^2}$$

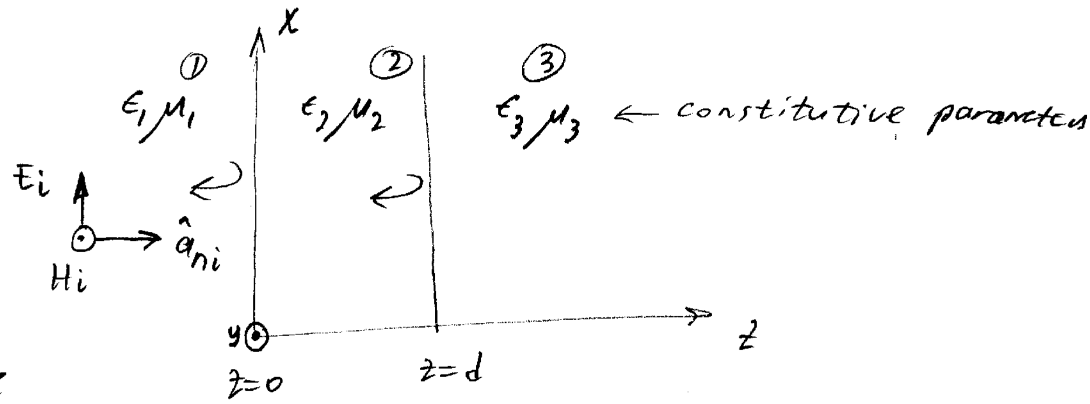
ÖDEV Show that it's true.

QUESTION:

NORMAL INCIDENCE AT PLANAR BOUNDARY OF MULTIPLE DIELECTRIC INTERFACES

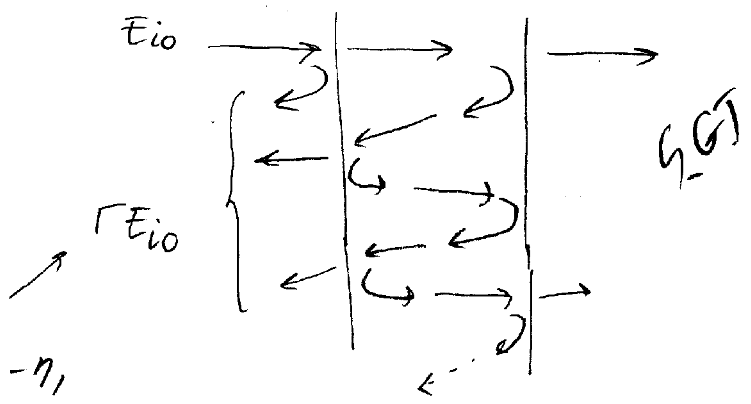
- How would you approach the problem?
- How many unknowns are there and how many do you need?
- How many equations are there?
- If there are enough equations, how would you solve the problem?

NORMAL INCIDENCE AT MULTIPLE DIELECTRIC INTERFACES



normal uniform plane wave

$$\left.\begin{aligned}\bar{E}_i &= \hat{a}_x \cdot E_{io}\, e^{-j\beta_1 z} \\ \bar{E}_r &= \hat{a}_x\, E_{ro} \cdot e^{j\beta_1 z}\end{aligned}\right\} \quad \bar{E}_1 = \hat{a}_x \left(E_{io}\, e^{-j\beta_1 z} + E_{ro} \cdot e^{j\beta_1 z} \right)$$



NOTE: $\Gamma \neq \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

QUESTION: How many methods/approaches can you think of to solve this problem?

1. Calculate all recursive reflections and transmissions, one by one.
Que: How many steps do you think you need to solve?
Que: If it is infinite, can you define a method for convergence? Can you define the constraints for the right time to stop?

2. Write down the total E and H fields and apply the boundary conditions, and see what you can do..

Let us skip the first method (you think of it later).

Now let us solve the fields in all three media.

Let us number the fields according to the media number; 1, 2 and 3.

For media (1):

$$\bar{E}_1 = \hat{a}_x \left(E_{i0}\, e^{-j\beta_1 z} + E_{r0} \cdot e^{j\beta_1 z} \right)$$

$$\bar{H}_1 = \hat{a}_y \, \frac{1}{\eta_1} \left(E_{i0}\, e^{-j\beta_1 z} - E_{r0} \cdot e^{j\beta_1 z} \right)$$

For media (2):

Medium 2: Forward and Backward Waves

$$\bar{E}_2 = \hat{a}_x \left(E_2^+ e^{-j\beta_2 z} + E_2^- \cdot e^{j\beta_2 z} \right)$$

$$\bar{H}_2 = \hat{a}_y \frac{1}{\eta_2} \left(E_2^+ e^{-j\beta_2 z} - E_2^- \cdot e^{j\beta_2 z} \right)$$

For media (3):

Medium 3: Only a forward wave in (+z) direction

$$\bar{E}_3 = \hat{a}_x E_3^+ e^{-j\beta_3 z}$$

$$\bar{H}_3 = \hat{a}_y \frac{E_3^+}{\eta_3} \cdot e^{-j\beta_3 z}$$

Question:
Define the known and the unknown variables (fields).

Unknowns E_{ro}, E_2^+, E_2^-, E_3^+

Boundary Conditions

@ $z=0$ $\quad E_1(0) = E_2(0)$

$\quad H_1(0) = H_2(0)$

@ $z=d$ $\quad E_2(d) = E_3(d)$

$\quad H_2(d) = H_3(d)$