EEM 323

ELECTROMAGNETIC WAVE THEORY II

MAXWELL'S EQUATIONS

Homogeneous vector wave equation

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Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

DISPLACEMENT CURRENT

$$\boxed{\text{Given} \quad \nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t} \quad (A/m^3)} \quad *$$

Remember $\nabla\times\bar{H} = \bar{J}$

$$\nabla\cdot\nabla\times\bar{H} = \nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\boxed{\rho_v = \nabla\cdot\bar{D}}$$

$$= -\frac{\partial}{\partial t}\,\nabla\cdot\bar{D}$$

$$= -\nabla\cdot\frac{\partial \bar{D}}{\partial t} \quad (A/m^3)$$

$$\nabla\cdot\nabla\times\bar{H} = 0 = \nabla\cdot\bar{J} + \frac{\partial}{\partial t}\nabla\cdot\bar{D}$$

Physical current AND/OR charges moving in time create nonzero magnetic curl, and that is:

$$\boxed{\nabla\times\bar{H} = \underbrace{\bar{J}}_{\bar{J}_c} + \underbrace{\frac{\partial \bar{D}}{\partial t}}_{\bar{J}_D} \quad (A/m^2)}$$

$\bar{J}_D$ ← displacement current (current on C)

$\bar{J}_c$ ← conduction current (current on R).

Recalling Faraday's Law of Induction:

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

We now have

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$$

Let us include the following for completeness

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot \bar{B} = 0$$

Review of the wave generating steps,

$$\begin{tikzcd}
\frac{\partial \rho}{\partial t} \arrow[r] & \bar{J}_D \arrow[r] & \bar{D} \arrow[ddrr] & & \\
& & & & \\
\bar{J}_c \arrow[r] & \bar{H} \arrow[r] & \bar{B} \arrow[r] & \bar{E} \arrow[r] & \bar{D} \arrow[lll, bend left]
\end{tikzcd}$$

infinite loop in the time varying case !.

These four equations are known as,
MAXWELL'S EQUATIONS.

James Clerk Maxwell (1831 - 1879)

MAXWELL'S EQUATIONS (Point form)

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot \bar{B} = 0$$

Now, let us integrate all four equations (1 thru 4) one by one using any selection of surface S and volume V.

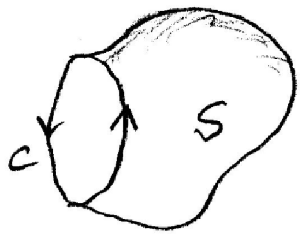
Maxwell's first equation:

$$\int_S (\nabla \times \bar{E}) \cdot \bar{ds} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

$$\Big\downarrow \text{Stoke's}$$

$$\oint_c \bar{E} \cdot \bar{dl} = -\frac{d}{dt} \underbrace{\int_S \bar{B} \cdot \bar{ds}}_{\Phi \text{ mag. flux}} = -\frac{d\Phi}{dt}$$

$$\oint_c \bar{E} \cdot \bar{dl} = -\frac{d}{dt} \int_S \bar{B} \cdot \bar{ds}$$



Maxwell's second equation

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$$

$$\int_S (\nabla \times \bar{H}) \cdot \bar{ds} = \underbrace{\int_S \bar{J}_c \cdot \bar{ds}}_{I_c} + \int_S \frac{\partial \bar{D}}{\partial t} \cdot \bar{ds}$$

$$\Big\downarrow \text{Stoke's}$$

$$\oint_c \bar{H} \cdot \bar{dl} = I_c + \frac{d}{dt} \int_S \bar{D} \cdot \bar{ds}$$

Maxwell's third equation:

$$\nabla \cdot \bar{D} = \rho_v$$

$$\int_v \nabla \cdot \bar{D} \, dv = \underbrace{\int_v \rho_v \, dv}_{Q}$$

div

$$\oint_s \bar{D} \cdot \bar{ds} = Q$$

Maxwell's last (fourth) equation:

$$\nabla \cdot \bar{B} = 0$$

$$\int_v \nabla \cdot \bar{B} \, dv = 0$$

$$\oint_s \bar{B} \cdot \bar{ds} = 0$$

MAXWELL'S EQUATIONS (Integral form)

Faraday's law of induction:

$$\oint_C \bar{E} \cdot d\bar{l} = -\frac{d}{dt}\int_S \bar{B} \cdot d\bar{s}$$

Ampere's circuital law:

$$\oint_C \bar{H} \cdot d\bar{l} = I_c + \frac{d}{dt}\int_S \bar{D} \cdot d\bar{s}$$

Gauss law:

$$\oint_S \bar{D} \cdot d\bar{s} = Q$$

Observation:
There is no isolated magnetic charge in the universe.

$$\oint_S \bar{B} \cdot d\bar{s} = 0$$

(*) D. K. Cheng, Field and Wave Electromagnetics.

Electromagnetic Boundary Conditions:

Boundary conditions for the static case is still valid.

(*) D. K. Cheng, Field and Wave Electromagnetics.

If both media are lossless and there are no sources on the interface

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If medium 1 is dielectric and medium 2 a perfect conductor

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MAXWELL'S EQUATIONS IN

Source–free region:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \bullet \mathbf{D} = 0$$

$$\nabla \bullet \mathbf{B} = 0$$

Linear, Isotropic region:

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Maxwell's equations in; Linear, Isotropic and source-free region:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

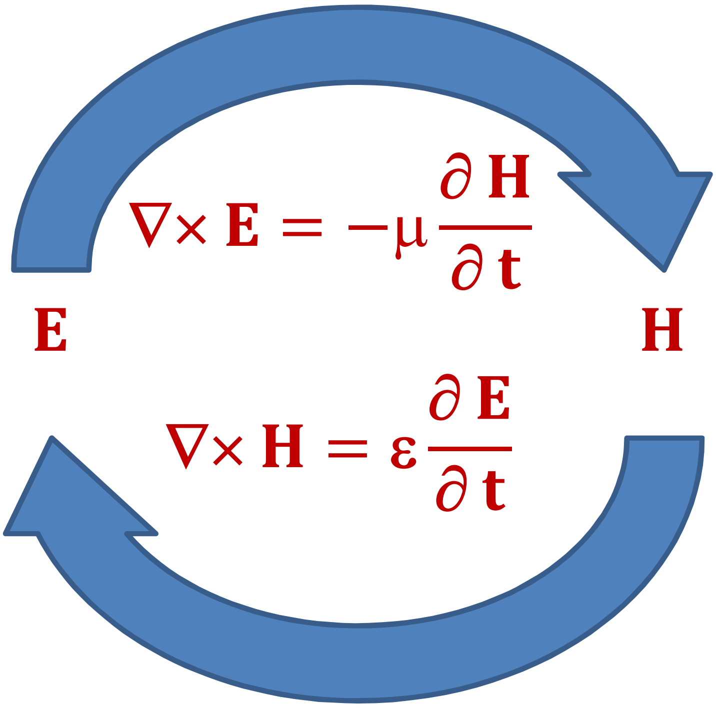
$$\nabla \bullet \mathbf{E} = 0$$

$$\nabla \bullet \mathbf{H} = 0$$

Homogeneous Vector Wave Equation

Interdependence of E and H: chain reaction of field generation can be shown as follows.

If E varies in time, it generates a time-varying H



Time-varying H generates time-varying E,

and vice versa ...

Recall

$$\nabla\times\bar{E} = -\mu\frac{\partial\bar{H}}{\partial t}$$

$$\nabla\times\bar{H} = \epsilon\frac{\partial\bar{E}}{\partial t}$$

and take the curl of the first equation

$$\nabla\times(\nabla\times\bar{E}) = \nabla\times\left(-\mu\frac{\partial\bar{H}}{\partial t}\right)$$

$$= -\mu\frac{\partial}{\partial t}(\nabla\times\bar{H})$$

$$= -\mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}$$

Let us now examine the left hand side of the equation

$$\nabla\times\nabla\times\bar{E} = \nabla(\underbrace{\nabla\cdot\bar{E}}_{0}) - \nabla^2\bar{E} = -\nabla^2\bar{E}$$

And now write both together

Homogeneous Vector Wave Equation for E

$$\boxed{\nabla^2\bar{E} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} = 0}$$

Note that the speed of light in medium is u,

$$\mu\varepsilon = 1/u^2$$

$$u = 1/\sqrt{\mu\varepsilon}$$

HW: Show that

$$\nabla^2\bar{H} - \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2} = 0$$