EEM 323

ELECTROMAGNETIC WAVE THEORY II

OBLIQUE INCIDENCE

AT PLANAR BOUNDARY

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Önemli not: Ders notlarındaki şekillerin hazırlanmasında internet ortamından faydalanılmıştır. Özellikle belirtilmeyen tüm şekil, tablo, eşitlik ve denklemler vb. "D. K, Fundamentals of Engineering Electromagnetics, Addison-Wesley Inc." ile "D. K, Field and Wave Electromagnetics, Mc-Graw Hill Inc." kitabından taranarak elde edilmiştir. Alıntıların kaynağına kolay ulaşılabilmesi maksadıyla numarası ve altyazıları da gösterilmektedir.

DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

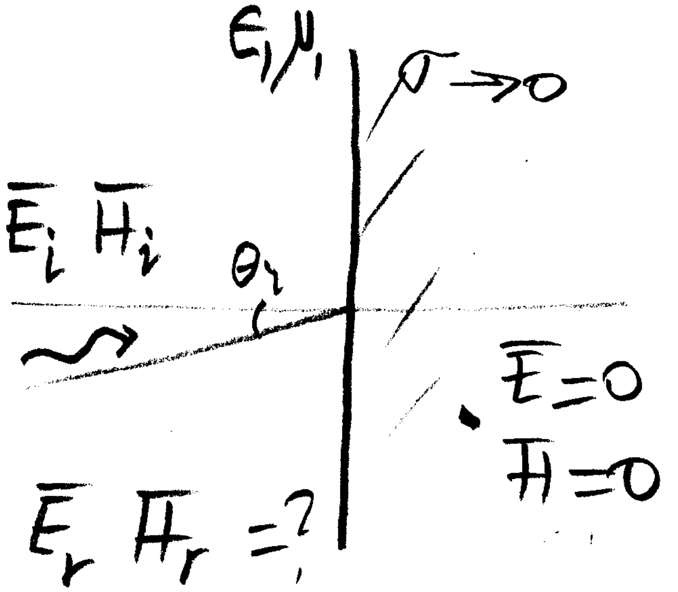
[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

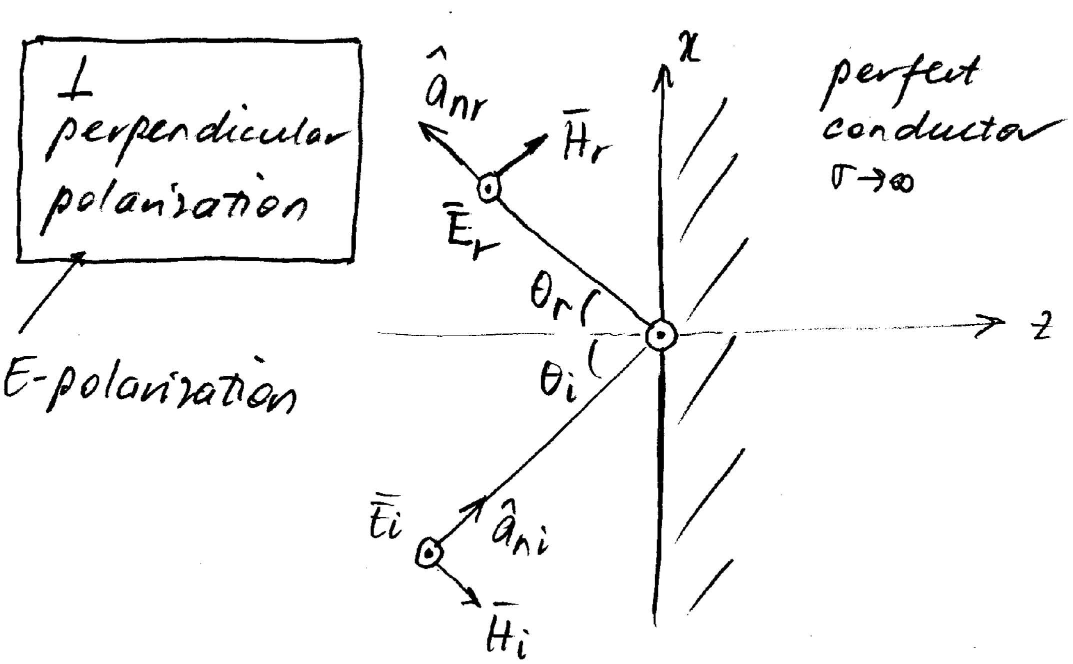
[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

OBLIQUE INCIDENCE AT PLANAR BOUNDARY OF

Dielectric – Conductor:

Perpendicular polarization:
E field is perpendicular to the plane of the plane of incident / reflected rays.





Incident $\hat{a}_{ni} = \hat{a}_x \sin\theta_i + \hat{a}_z \cos\theta_i$

$$\bar{E}_i(x,z) = \hat{a}_y E_{io} \cdot e^{-j\beta_1 (x \cdot \sin\theta_i + z \cdot \cos\theta_i)}$$

$$\bar{H}_i(x,z) = \frac{1}{\eta_1}\left[\hat{a}_{ni} \times \bar{E}_i(x,z)\right]$$

$$= \frac{E_{io}}{\eta_1}\left(-\hat{a}_x \cos\theta_i + \hat{a}_z \sin\theta_i\right) \cdot e^{-j\beta_1 (x \sin\theta_i + z \cos\theta_i)}$$

Reflected $\hat{a}_{nr} = \hat{a}_x \sin\theta_r - \hat{a}_z \cos\theta_r$

$$\bar{E}_r(x,z) = \hat{a}_y E_{ro} \cdot e^{-j\beta_1 (x \cdot \sin\theta_r - z \cdot \cos\theta_r)}$$

$$\bar{H}_r(x,z) = \frac{1}{\eta_1}\left[\hat{a}_{nr} \times E_r(x,z)\right]$$

$$= \frac{E_{ro}}{\eta_1}\left(+\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right) \cdot e^{-j\beta_1 (x \sin\theta_r - z \cos\theta_r)}$$

QUESTION:

- Given only the incident Electric field, what are the fields in both regions?
- How many equations do we need?

Let us use those boundary conditions.

BOUNDARY CONDITIONS

B.C. #1

$$\bar{E}_1(x,0) = E_i(x,0) + E_r(x,0)$$

$$\bar{E}_1(x,0) = \hat{a}_y \left(E_{io}\, e^{-j\beta_1 x \sin\theta_i} + E_{ro}\, e^{-j\beta_1 x \sin\theta_r} \right) = 0$$

to satisfy B.C#1 $\forall x$

$$\boxed{E_{io} = -E_{ro}}$$

$$e^{-j\beta_1 x \sin\theta_i} = e^{-j\beta_1 x \sin\theta_r} \Rightarrow \sin\theta_i = \sin\theta_r$$

Snell's law of Reflection $\boxed{\theta_i = \theta_r}$

$$\Rightarrow \bar{E}_r(x,z) = -\hat{a}_y E_{io}\, e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)}$$

$$\Rightarrow \bar{H}_r(x,z) = \frac{1}{\eta_1}\left(\hat{a}_{nr} \times \bar{E}_r(x,z)\right)$$

$$= \frac{E_{io}}{\eta_1}\left(-\hat{a}_x \cos\theta_i - \hat{a}_z \sin\theta_i\right) e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)}$$

$$\Rightarrow \bar{E}_1(x,z) = -\hat{a}_y\, j2E_{io} \sin(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}$$

$$\Rightarrow H_1(x,z) = -2\frac{E_{io}}{\eta_1}\Big[\hat{a}_x \cos\theta_i \cos(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i} + \hat{a}_z\, j\sin\theta_i \sin(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}\Big]$$

HOMEWORK:

Analyze your solution to understand that;

- There is a standing wave pattern in the direction normal to the boundary,
- The standing wave pattern is due to E_{1y} and H_{1x},
- Their corresping patterns are in $\sin(\beta_{1z} z)$ and $\cos(\beta_{1z} z)$ where $\beta_{1z} = \beta\cos(\theta_i)$
- Average power propagates along the z – axis.

HOMEWORK:

Calculate the distance of the nulls from the z = 0 plane.

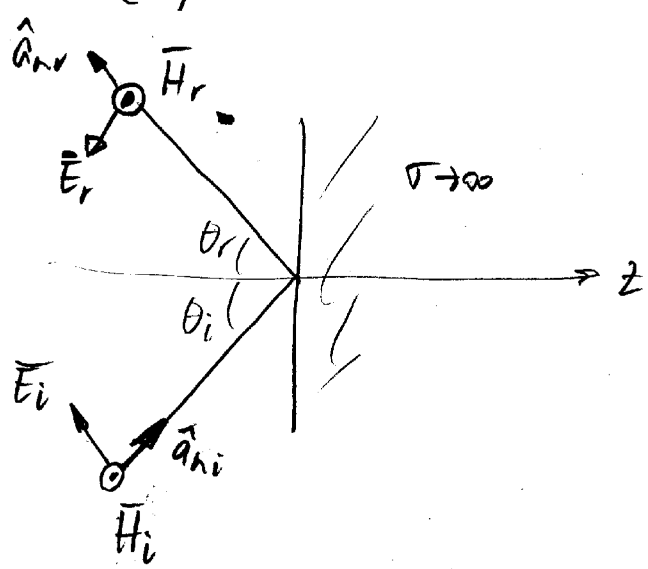
$\bar{E}_1 = 0 \quad \forall x$ when $\sin(\beta_1 z \cos\theta_i) = 0$

$$\Rightarrow \beta_1 \cdot z \cdot \cos\theta_i = \frac{2\pi}{\lambda_1} \cdot z \cdot \cos\theta_i = -m\pi \qquad m = 1, 2, 3$$

$$\Rightarrow \boxed{z = -\frac{m\lambda_1}{2 \cdot \cos\theta_i} \qquad m = 1, 2, 3, \ldots}$$

HOMEWORK:

4 ÖDEV: Solve for the case: "//" Parallel Polarization

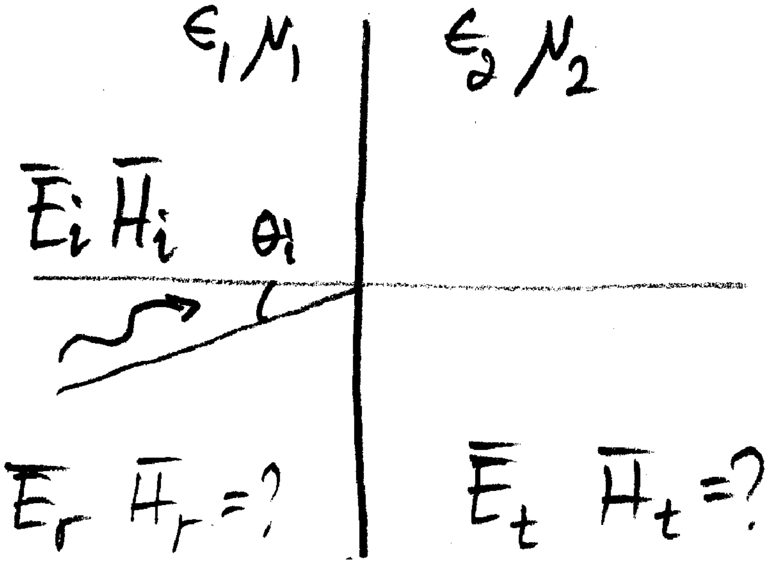


OBLIQUE INCIDENCE AT PLANAR BOUNDARY OF

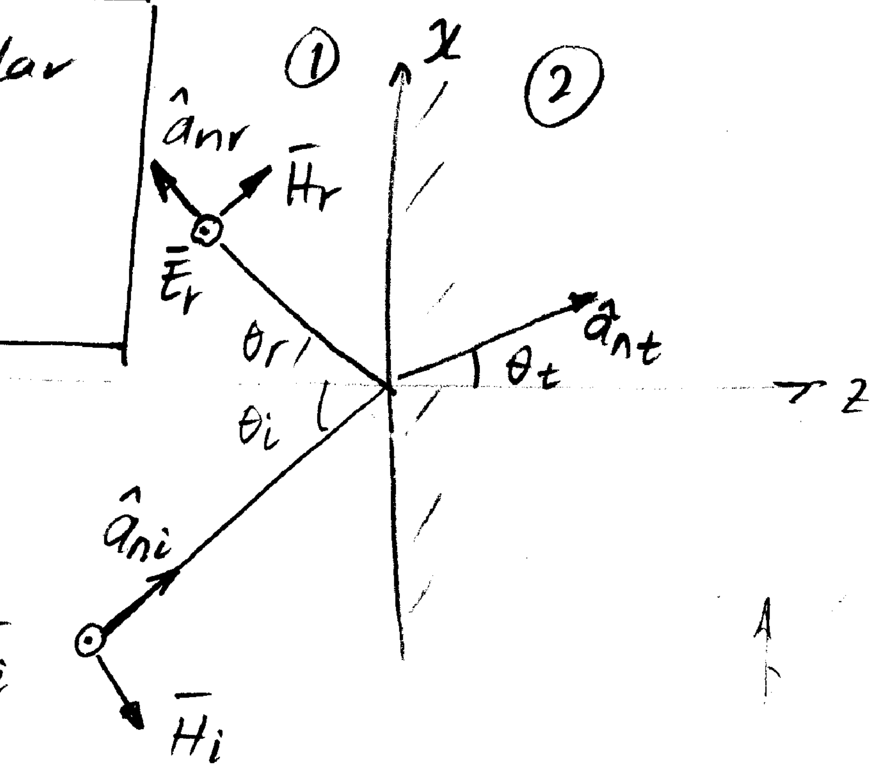
Dielectric – Dielectric:

Paralel polarization:

E field is parallel to the plane of the plane of incident / reflected rays.



⊥ perpendicular polarization (dik)



Recall the general form of plane wave:

$$e^{-j\bar{k}\cdot\bar{R}}$$

$$= e^{-jk\hat{a}_n\cdot\bar{R}}$$

$\bar{R} = (x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)$

$\hat{a}_n = (\hat{a}_x \sin\theta_i + \hat{a}_z \cos\theta_i)$

Direction vector:

NOTE: $\hat{a}_{ni} = \hat{a}_x \sin\theta_i + \hat{a}_z \cos\theta_i$

inc $\bar{E}_i(x,z) = \hat{a}_y E_{i0} \cdot e^{-j\beta_1(x\cdot\sin\theta_i + z\cdot\cos\theta_i)}$

Given the incident electric field, we can calculate the magnetic field.

$$\bar{H}_i(x,z) = \frac{E_{i0}}{\eta_1}(-\hat{a}_x \cos\theta_i + \hat{a}_z \cdot \sin\theta_i)\cdot e^{-j\beta_1(x\sin\theta_i + z\cdot\cos\theta_i)}$$

NOTE: lossless case $\beta_1 = k_1$

Now let us write the form of the reflected fields.

$$\hat{a}_{nr} = \hat{a}_x \sin\theta_r - \hat{a}_z \cos\theta_r$$

$$\bar{E}_r(x,z) = \hat{a}_y E_{ro} \cdot e^{-j\beta_1(x \cdot \sin\theta_r - z\cos\theta_r)}$$

$$\bar{H}_r(x,z) = \frac{E_{ro}}{\eta_1}\left(\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r\right) \cdot e^{-j\beta_1(x\sin\theta_r - z \cdot \cos\theta_r)}$$

Similarly, the transmitted fields can be written as

trans. $\hat{a}_{nt} = \hat{a}_x \cdot \sin\theta_t + \hat{a}_z \cos\theta_t$

$$\bar{E}_t(x,z) = \hat{a}_y E_{to} \cdot e^{-j\beta_2(x \cdot \sin\theta_t + z \cdot \cos\theta_t)}$$

$$\bar{H}_t(x,z) = \frac{E_{to}}{\eta_2}\left(-\hat{a}_x \cos\theta_t + \hat{a}_z \sin\theta_t\right) \cdot e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

Question:

How many unknowns do we have?

How many equations do we need?

So, what are those boundary conditions?

Unknowns $E_{ro}, E_{to}, \theta_r, \theta_t$.

BOUNDARY CONDITIONS.

B.C. #1:

$$E_{iy}(x,0) + E_{ry}(x,0) = E_{ty}(x,0)$$

$$\Rightarrow \boxed{E_{io} \cdot e^{-j\beta_1 x \cdot \sin\theta_i} + E_{ro} \cdot e^{-j\beta_1 \cdot x\sin\theta_r} = E_{to} \cdot e^{-j\beta_2 \cdot x\sin\theta_t}}$$

B.C. #2 :

$$H_{ix}(x,0) + H_{rx}(x,0) = H_{tx}(x,0)$$

$$\boxed{\frac{1}{\eta_1}\left(-E_{i0}\cdot\cos\theta_i\, e^{-j\beta_1 x\sin\theta_i} + E_{r0}\cdot\cos\theta_r\cdot e^{-j\beta_1 x\sin\theta_r}\right) = -\frac{E_{t0}}{\eta_2}\cos\theta_t\cdot e^{-j\beta_2 x\sin\theta_t}}$$

We should be careful to the third boundary equation. Note that boundary conditions (1) and (2) should be satisfied for all values of x on the boundary.

BC. #3 :

BC. #1 & #2 should be satisfied for all X. then,

$$e^{-j\beta_1 x\sin\theta_i} = e^{-j\beta_1 x\cdot\sin\theta_r} = e^{-j\beta_2 x\sin\theta_t}$$

$$\Rightarrow \beta_1 x\sin\theta_i = \beta_1 x\cdot\sin\theta_r = \beta_2 \cdot x\sin\theta_t$$

$$\Rightarrow \boxed{\theta_i = \theta_r}$$ Snell's law of Reflection (Yansıma)

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{\eta_1}{\eta_2}}$$ Snell's law of Refraction (Kırılım)

$$\Rightarrow \quad E_{io} + E_{ro} = E_{to}$$

and

$$\frac{1}{\eta_1}(E_{io}+E_{ro})\cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t$$

Finally we obtain the reflection and transmission coefficients for the oblique incidence for the perpendicular polarization case.

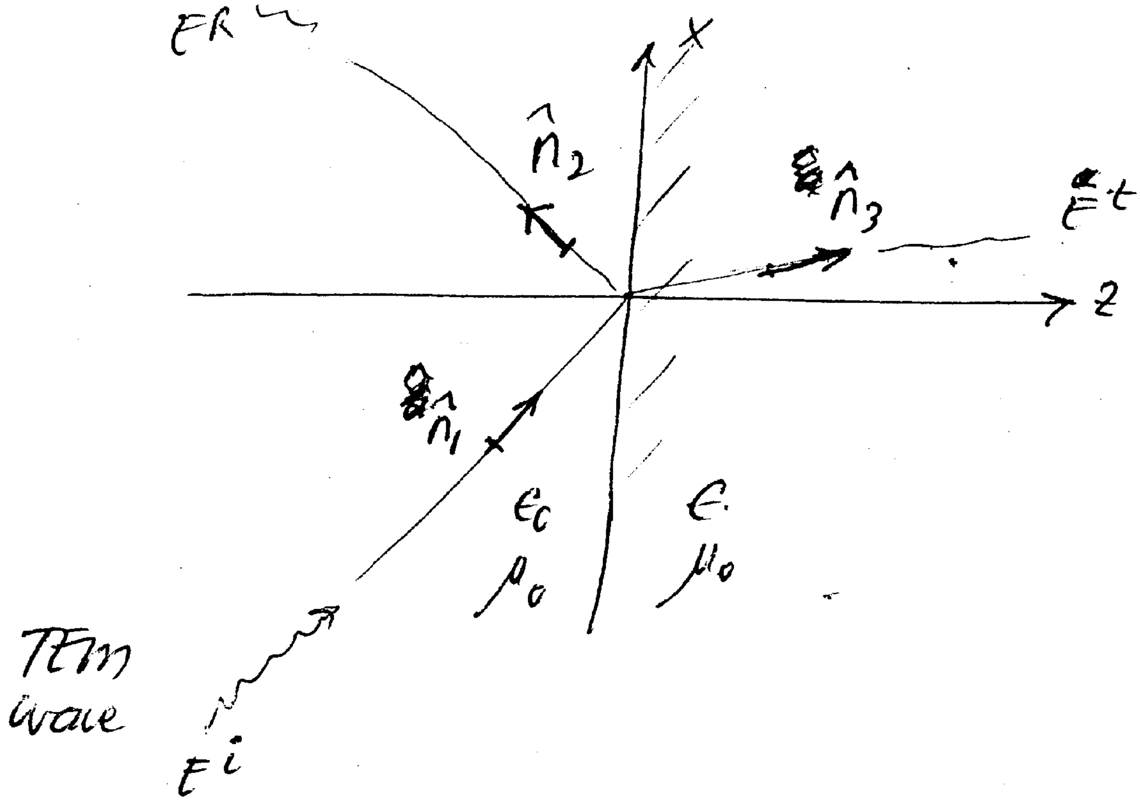
$$\boxed{\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$$\boxed{\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

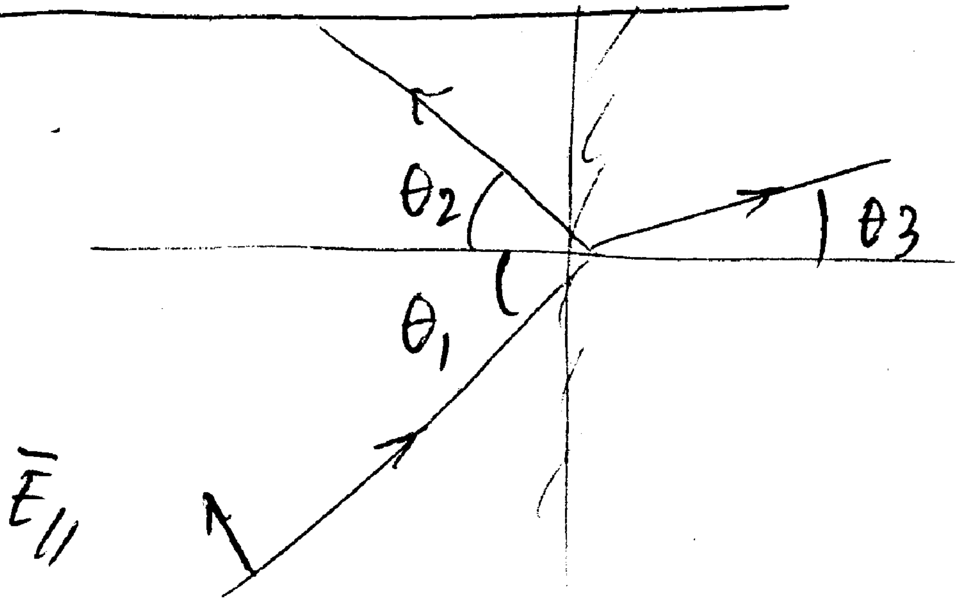
$$\boxed{1 + \Gamma_\perp = \tau_\perp}$$

ÖDEV

Reflection from a dielectric interface



Parallel Polarization



In general,

$$\bar{E}_i = \bar{E}_1 e^{-jk_0 \hat{n}_1 \cdot \bar{r}} \qquad \bar{H}_i = Y_0 \hat{n}_1 \times \bar{E}_i$$

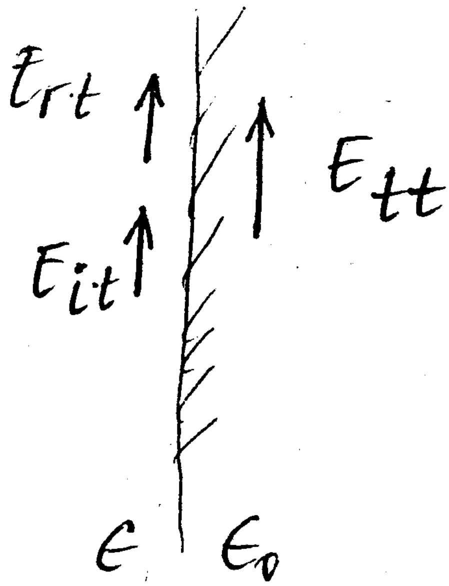
$$\bar{E}_r = \bar{E}_2 e^{-jk_0 \hat{n}_2 \cdot \bar{r}} \qquad \bar{H}_r = Y_0 \hat{n}_2 \times \bar{E}_r$$

$$\bar{E}_t = \bar{E}_3 e^{-jk_0 \hat{n}_3 \cdot \bar{r}} \qquad \bar{H}_t = Y \hat{n}_3 \times \bar{E}_t$$

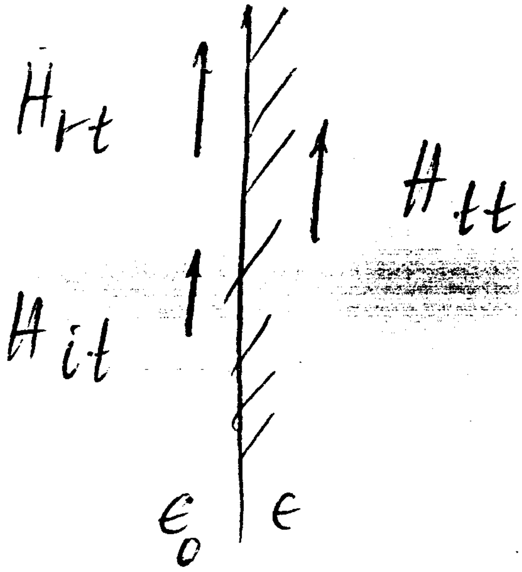
$E_2 \quad E_3 \quad n_2 \quad n_3$ unknowns

Boundary Conditions

B.C.(1) Continuity of tangent E:



B.C. (2) Continuity of tangent H



B.C. (3) Continuity of of all fields at the z = 0 plane

$$k_o n_{1x} = k_o n_{2x} = k_o n_{3x} \qquad \text{(B.C: 3.1)}$$

and

$$k_o n_{1y} = k_o n_{2y} = k_o n_{3y} = 0 \qquad \text{(B.C: 3.2)}$$

where

$$n = k/k_o.$$

Using B.C. (3):

$$\sin(\theta_1) = \sin(\theta_2)$$

Then

$$\theta_1 = \theta_2$$

REFLECTION LAW

$$\sin(\theta_1) = n\sin(\theta_3)$$

SNELL'S LAW
OF REFRACTION

B.C. (1) and (2): yield

$E_{1x} = -E_1 \cos(\theta_1)$	$E_{1z} = -E_1 \sin(\theta_1)$
$E_{2x} = E_2 \cos(\theta_2)$	$E_{2z} = E_2 \sin(\theta_2)$
$E_{3x} = E_3 \cos(\theta_3)$	$E_{3z} = -E_3 \sin(\theta_3)$

B.C. (1):

$$E_{1x} + E_{2x} = E_{3x}$$

$$E_1 cos(\theta_1) + E_2 cos(\theta_2) = E_3 cos(\theta_3)$$

B.C. (2) (H has only y component):

$$(1/n_o)E_1 - E_2 = (1/n)\, E_3$$

Bu kısımdan sonra ders notları ilave edilecektir.