EEM 323

ELECTROMAGNETIC WAVE THEORY II

TIME HARMONIC FIELDS

Vector wave equation

Scalar wave equation

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DERS KİTABI

[1] David Keun Cheng, *Fundamentals of Engineering Electromagnetics*, Addison-Wesley Publishing, Inc., 1993.
veya David Keun Cheng, Çeviri: Adnan Köksal, Birsen Saka, *Mühendislik Elektromanyetiğinin Temelleri – Fundamentals of Engineering Electromagnetics*, Palme Yayınları.

KAYNAK / YARDIMCI KİTAPLAR:

[2] David Keun Cheng, *Field and Wave Electromagnetics*, Addison-Wesley Publishing, Inc. *veya* David Keun Cheng, Çeviri: Mithat İdemen, *Elektromanyetik Alan Teorisinin Temelleri – Field and Wave Electromagnetics*, Literatür Yayıncılık.

[3] Stanley V. Marshall, Richard E. DuBroff, Gabriel G. Skitek, *Electromagnetic Concepts and Applications*, Dördüncü Basım, Prentice Hall International, Inc., 1996.

[4] Joseph A. Edminister, Elektromanyetik, 2. Baskıdan çeviri, Çevirenler: M. Timur Aydemir, E. Afacan, K. C. Nakipoğlu, Schaum's Outlines, McGraw Hill Inc., Nobel Yayın Dağıtım, Ankara, 2000.

TIME-HARMONIC FIELDS

Now let us assume that the field has only a single frequency component (other frequency terms can be analyzed similarly).

Then, we can use **Phasor Representation** for the fields,

The fields are now called **the time-harmonic fields** of frequency f (in Hertz), and ω (in radians).

$$\mathbf{E}(x,y,z;t) = Re\{\mathbf{E}(x,y,z)e^{j\omega t}\}$$

$$\mathbf{H}(x,y,z;t) = Re\{\mathbf{H}(x,y,z)e^{j\omega t}\}$$

and similarly for all the other vector fields are shown in **vector phasors**.

Phasors:

- Are not functions of f

  (they are valid only for a single value of f)
- Instantaneous time functions cannot contain complex numbers

  (for example for the electric field, we should observe values in terms of Volt/meters, a real value !)
- Any electromagnetic expression containing j must necessarily be a relation of phasors.

Now let us place the phasor representation for the fields into Maxwell's equations

A reminder

$$\mathbf{E}(x, y, z; t) = Re\{\mathbf{E}(x, y, z)e^{j\omega t}\}$$

$$\mathbf{H}(x, y, z; t) = Re\{\mathbf{H}(x, y, z)e^{j\omega t}\}$$

The time derivatives are simply a multiplicative factor of $j\omega$

$$\nabla\times \mathbf{E} = -\mathrm{j}\omega\ \mathbf{B}$$

$$\nabla\times \mathbf{H} = \mathrm{j}\omega\ \mathbf{D}$$

For a linear and isotropic media

$$\mathbf{B} = \mu\ \mathbf{H}$$

$$\mathbf{D} = \varepsilon\ \mathbf{E}$$

Maxwell's equations becomes

Linear, Isotropic, Time-harmonic:

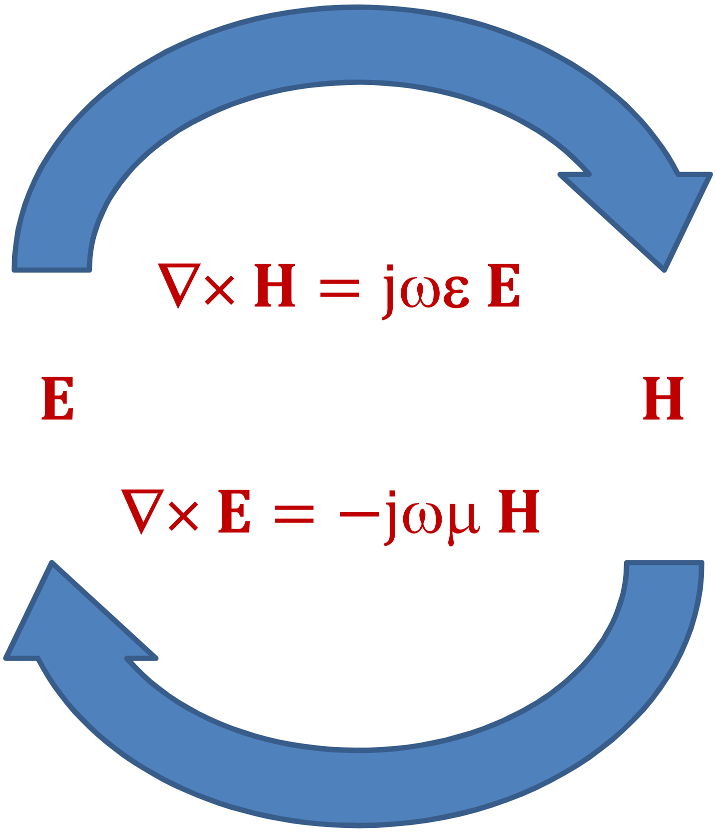
$$\nabla\times \mathbf{E} = -\mathrm{j}\omega\mu\ \mathbf{H}$$

$$\nabla\times \mathbf{H} = \mathrm{j}\omega\varepsilon\ \mathbf{E}$$

Interdependence of $\mathbf{E}$ and $\mathbf{H}$:

Chain reaction of field generation:

If $\mathbf{E}$ varies in time, it generates a time-varying $\mathbf{H}$



$$\nabla\times \mathbf{H} = j\omega\varepsilon\, \mathbf{E}$$

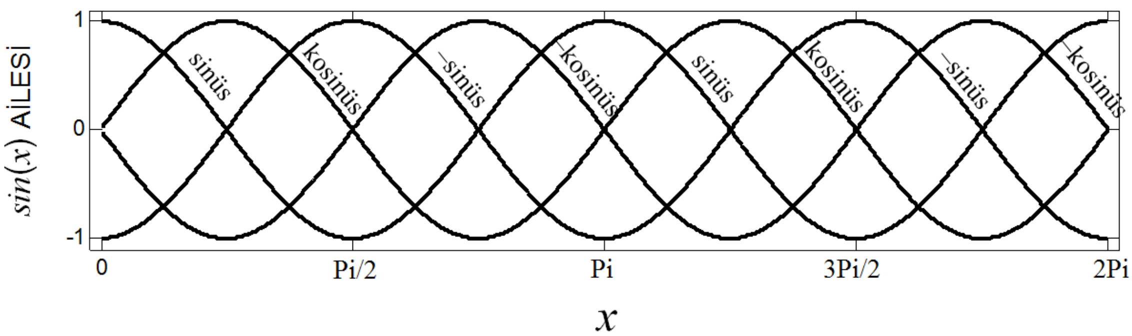
$$\nabla\times \mathbf{E} = -j\omega\mu\, \mathbf{H}$$

Time-varying $\mathbf{H}$ generates time-varying $\mathbf{E}$,

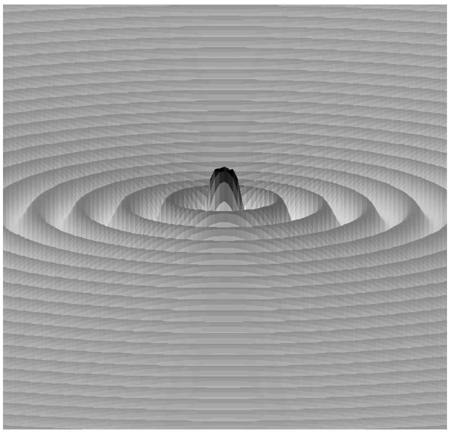
and so on ...

Since this time variation is only sinusoidal,

this **chain reaction** can last forever if there is no energy loss !..



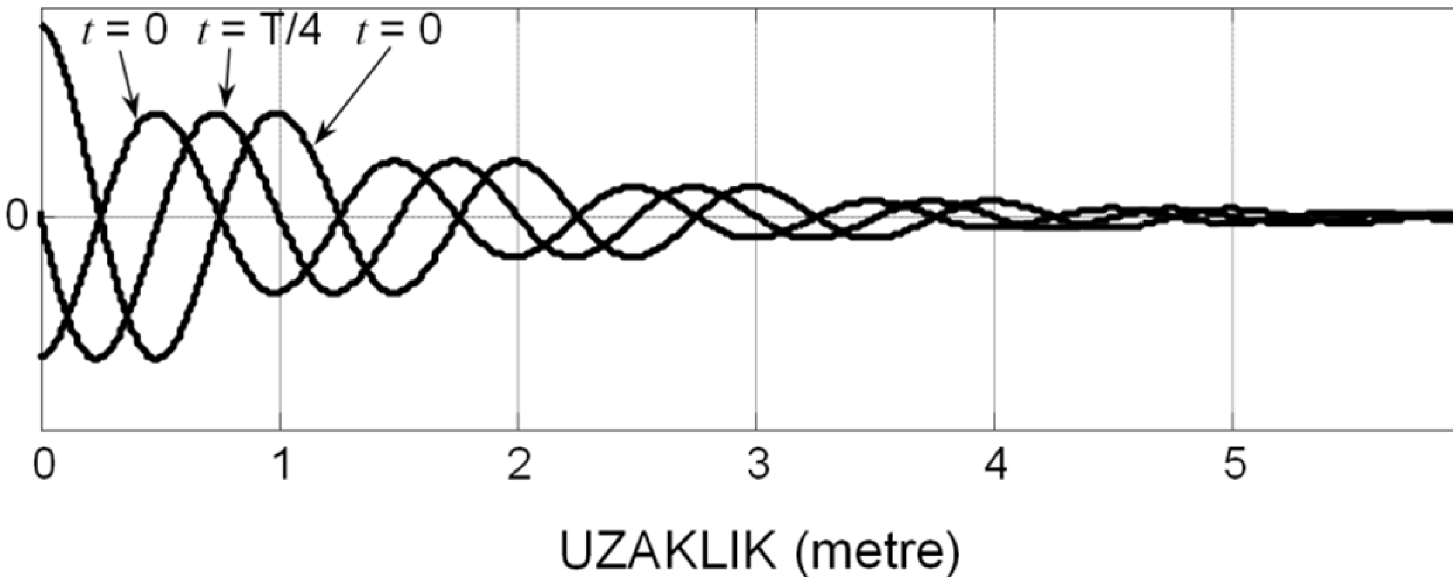
This is why it is called the **Electro – Magnetic Wave**.



[REF: S. G. Tanyer, Müziğin Doğası – Matematiğin Sesi]

Lossless assumption:

In general, EM wave could generate currents in the propagating material which causes power dissipation. EM wave could spread while propagating. Both will result in attenuation of the wave's amplitude.



[REF: S. G. Tanyer, Müziğin Doğası – Matematiğin Sesi]

If the material is assumed to be **lossless**, then power dissipation is assumed to be negligible, and zero.

EM wave equation in

Linear, Isotropic, Time-harmonic and Lossless media:

$$\nabla\times \mathbf{E} = -j\omega\mu\, \mathbf{H} \quad \rightarrow \quad \mathbf{H} = \frac{j}{\omega\mu}\nabla\times \mathbf{E}$$

$$\nabla\times \mathbf{H} = j\omega\varepsilon\, \mathbf{E} \quad \rightarrow \quad \nabla\times\left\{\frac{j}{\omega\mu}\nabla\times \mathbf{E}\right\} = j\omega\varepsilon\, \mathbf{E}$$

Then, we have (note that ω and μ are constants)

$$\nabla\times\left\{\frac{j}{\omega\mu}\nabla\times \mathbf{E}\right\} = j\omega\varepsilon\, \mathbf{E} \quad \rightarrow \quad \nabla\times\{\nabla\times \mathbf{E}\} = \omega^2\mu\varepsilon\, \mathbf{E}$$

Using

$$\nabla\times\{\nabla\times \mathbf{E}\} = \nabla\times\nabla\times \mathbf{E} = \nabla(\nabla\bullet \mathbf{E}) - \nabla^2\mathbf{E} = -\nabla^2\mathbf{E}$$

$$\nabla^2\mathbf{E} = -\,\omega^2\mu\varepsilon\, \mathbf{E} \quad \rightarrow \quad \nabla^2\mathbf{E} + \omega^2\mu\varepsilon\, \mathbf{E} = 0$$

Helmholtz wave equation in

Lossless (nonconducting), Simple (linear and isotropic) media:

$$\nabla^2\mathbf{E} + k^2\, \mathbf{E} = 0$$

where

$$k = \omega\sqrt{\mu\varepsilon} = 2\pi/\lambda \qquad \text{(rad/m)}$$

$n = k/k_0$ is the index of refraction

Note that

$u_{p} = 1/\sqrt{\mu\varepsilon}$ is the speed of light in the media, and

$1/\sqrt{\mu_0\varepsilon_0} = c = 300{,}000$ (km/sec) is the speed in free space (vacuum)

HOMEWORK:

1. Derive Helmholtz wave equation in linear, isotropic and source-free region for the magnetic field intensity $\boldsymbol{H}$.
2. Now, for the wave equation (both for $\boldsymbol{E}$ and $\boldsymbol{H}$),

   Let $\lambda \to 0$

   Analyze this limiting case for Maxwell's equations

   Recalculate Helmholtz wave equation

   Comment on your results.

EXAMPLE (6-9) 83

Nonconducting dielectric medium $\epsilon = 9\epsilon_0$ and μ_0

Given, $\bar{E}(z,t) = \hat{a}_y \, 5 \cos(10^9 t - \beta z)$ (V/m)

Find the magnetic field intensity $\bar{H}$ and β.

$$\bar{E}(z) = \hat{a}_y \, 5 \, e^{-j\beta z}$$

$$\bar{H}(z) = -\frac{1}{j\omega\mu_0} \nabla \times \bar{E}$$

$$= -\frac{1}{j\omega\mu_0} \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 5e^{-j\beta z} & 0 \end{vmatrix}$$

$$= -\frac{1}{j\omega\mu_0} \left(-\hat{a}_x \frac{\partial}{\partial z} E_y \right)$$

$$= -\frac{1}{j\omega\mu_0} \left(\hat{a}_x \, j\beta \, 5 \, e^{-j\beta z} \right)$$

$$\bar{H}(z) = -\hat{a}_x \frac{\beta}{\omega\mu_0} 5 \, e^{-j\beta z}$$

Now calculate β, 84

Recall $\sigma = 0$, $\bar{J} = 0$, thus

$$\bar{E}(z) = \frac{1}{j\omega\epsilon} \nabla \times \bar{H} = \frac{1}{j\omega\epsilon}\left(\hat{a}_y \frac{\partial}{\partial z} H_x\right)$$

$$= \hat{a}_y \frac{\beta^2}{\omega^2 \mu_0 \epsilon} 5 e^{-j\beta z}$$

$\bar{E}(z)$ was given to equal $\hat{a}_x 5 e^{-j\beta z}$, then

$$\frac{\beta^2}{\omega^2 \mu_0 \epsilon} = 1 \Rightarrow \beta = \omega\sqrt{\mu_0 \underset{\uparrow \; 9\epsilon_0}{\epsilon}}$$

$$\beta = 3\omega \underbrace{\sqrt{\mu_0 \epsilon_0}}_{1/c} = 3\omega/c$$

$$\beta = \frac{3 \times 10^9}{3 \times 10^8} = 10 \;(\text{rad/m})$$

$$\lambda = 2\pi/\beta = \pi/5 \;(\text{meters})$$

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$$\bar{H}(z) = -\hat{a}_x \frac{5(10)}{10^9 (4\pi 10^{-7})} e^{-j10z}$$

$$= -\hat{a}_x \; 0.0398 \; e^{-j10z}$$

$$\bar{H}(z,t) = -\hat{a}_x \; 0{,}0398 \cos(10^9 t - 10z) \quad (A/m)$$

REVIEW

Maxwell's equations in source-free nonconducting (lossless) media (Review) 86

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\nabla \cdot \bar{E} = 0$$

$$\nabla \cdot \bar{H} = 0$$

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$$\nabla^2 \bar{E} - \underbrace{\mu\epsilon}_{1/u_p^2} \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

$$\left.\begin{aligned} \nabla^2 \bar{E} - \frac{1}{u_p^2} \frac{\partial^2 \bar{E}}{\partial t^2} &= 0 \\ \nabla^2 \bar{H} - \frac{1}{u_p^2} \frac{\partial^2 \bar{H}}{\partial t^2} &= 0 \end{aligned}\right\}$$

Homogeneous vector wave eqns for E and H

HELMHOLTZ'S EQUATIONS (SCALAR WAVE EQUATIONS)

Maxwell's equations in source-free nonconducting (lossless) media for time-harmonic fields (Homogeneous Helmholtz's equations for phasors E_s and H_s):

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$$\nabla^2 \bar{E} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

switch to phasor notation

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\mu\epsilon = 1/u^2$$

$$w^2 \mu\epsilon = k^2$$

k: wavenumber

$$k = 2\pi/\lambda$$

$$\left.\begin{aligned} \nabla^2 E_s + k^2 E_s &= 0 \\ \nabla^2 H_s + k^2 H_s &= 0 \end{aligned}\right\} \text{Scalar wave equation}$$

E_s and H_s are phasors.

Scalar wave equations represent propagating waves!

A FLASH FORWARD TO THE LECTURE ON PLANE WAVE PROPATION

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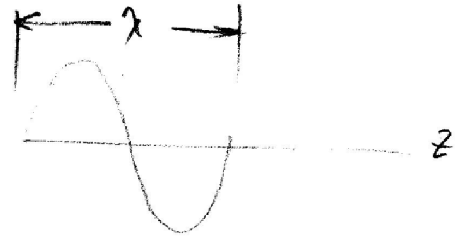

$$\cos(\underbrace{wt - kz}_{\text{constant}})$$

$$wt - kz = \text{constant}$$

$$\frac{d}{dt}(wt - kz) = 0$$

$$w - k \underbrace{\frac{dz}{dt}}_{u} = 0$$

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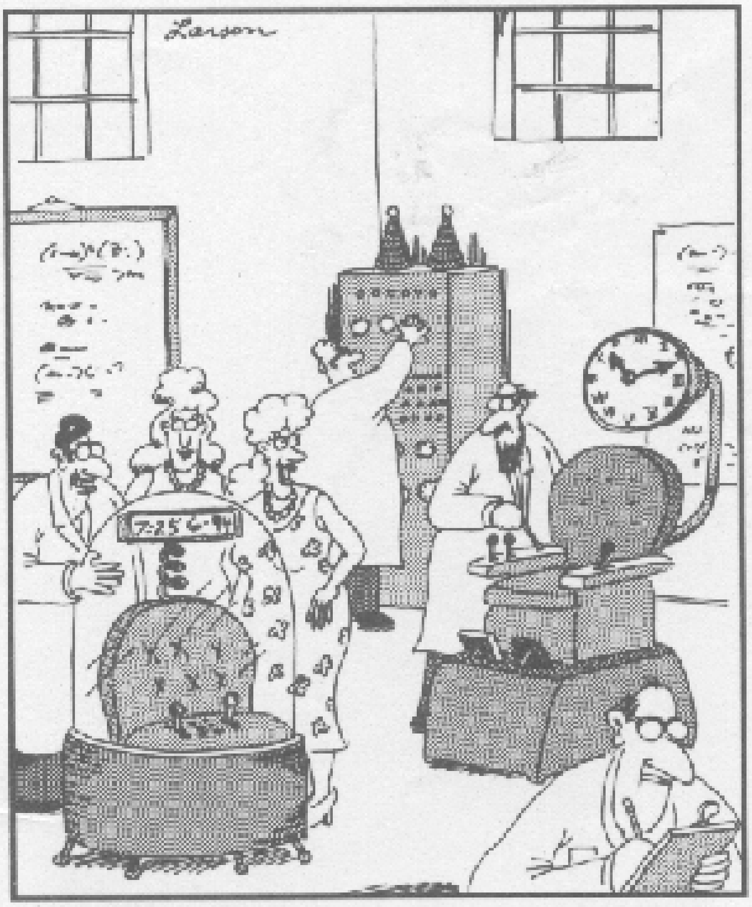
$$u = w/k$$

$$u = \lambda . f$$

$$\lambda f = 2\pi f / k$$

$$k = \lambda / 2\pi$$



"Oh, Professor DeWitt! Have you seen Professor Weinberg's time machine? ... It's digital!"

The Far Side

JANUARY

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Monday