Appendix A

A Brief Introduction to Magnetics

A.1 Where Magnetic Fields Come From

Magnetic fields result from the motion of electrical charges. The two most common effects that generate magnetic fields are electron spin and moving charges. Some atoms, such as iron, nickel, and cobalt, have imbalances in the total spin of the electrons in their electron shells, and this effect is ultimately responsible for these materials' "magnetic" properties. Moving charges, such as those that form an electrical current, also develop a magnetic field.

The relationships of electric current and magnetic field in empty space under steady-state conditions can be described by:

$$\oint_S \vec{B} \bullet ds = 0 \qquad \textbf{(Equation A.1)}$$

$$\oint_C \vec{B} \bullet dl = \mu_0 I \qquad \textbf{(Equation A.2)}$$

where $\vec{B}$ is magnetic field, I is electrical current density, and μ_0 is the permeability of empty space. Note that $\vec{B}$ is a vector quantity, meaning that it has three separate parts, independently representing field in the *x*, *y*, and *z* directions. Vector notation, although somewhat terse and confusing, is frequently used in electromagnetics because it is vastly less confusing, and more useful, than many alternative representations.

Translated into English, equation A-1 states that the total flux coming through any closed surface, such as a sphere, must equal zero. This means that magnetic poles must come in pairs; for every north pole there must be an south pole of equal magnitude. Another implication of this is that magnetic flux lines always form loops. Equation A-2 states that if you integrate magnetic field along a closed loop (integrating only the field components that are tangent to the loop), the integral will be proportional to the electrical current enclosed by the loop. While the ideas expressed by these equations are simple, using them to predict magnetic fields based on the magnitude and path of an electrical current can be extremely complicated.

Fortunately, it is possible to derive equations that take geometry and current as inputs, and yield magnetic field. One such equation, called the Biot-Savart law, is given by:

$$B = \frac{\mu_0 I}{4\pi}\oint_C \frac{dl \times \vec{R}}{|\vec{R}|^3}$$ **(Equation A.3)**

The vector $\vec{R}$ defines the distance and direction from the point on the current path one is integrating to the point in space where one wants to know the magnetic field. This geometric relationship is shown in Figure A-1. The three components of the $\vec{R}$ vector are the differences in the x, y, and z coordinates between the two points. The absolute value of $\vec{R}$ is simply the linear distance between the two points. Note that the multiplication is between two vector quantities, and is called the cross-product.

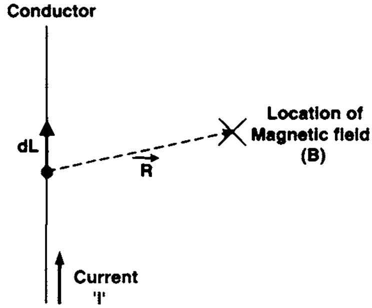

Figure A-1: Geometric interpretation of $\vec{R}$ vector in Biot-Savart integral.

If one has enough skill in evaluating integrals, the Biot-Savart law provides a handy means of predicting magnetic fields from electrical currents. Two of the simpler and more useful results that can be derived from this law are:

The magnetic field around an infinitely long, straight conductor, carrying a current I at a radius of r is given by:

$$B = \frac{\mu_0 I}{2\pi r}$$ **(Equation A.4)**

The magnetic field at the center of a closed circular loop of wire of radius r carrying a current of I is given by:

$$B = \frac{\mu_0 I}{r} \qquad \textbf{(Equation A.5)}$$

Illustrations of these two cases are given in Figure A-2.

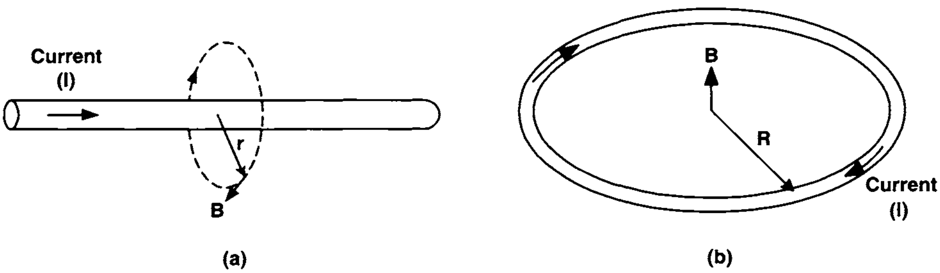

Figure A-2: Magnetic field around wire (a), and inside loop (b).

The above two cases are significant not because you will encounter them in pure form, but because they are useful as quick-and-dirty approximations for many more complex situations. Because the math can get very complicated very fast for problems with even relatively simple geometries, exact analytic solutions for problems in magnetics are both rare and difficult to come by.

A.2 Magnetic Materials

Because all materials contained charged particles, everything exhibits some magnetic properties of one kind or another. For the purposes of this discussion, we will only consider ferromagnetic materials. This group of materials includes iron, nickel, and cobalt, as well as many of their alloys.

Ferromagnetic materials generally are not homogenous down to the atomic level, but consist of clusters of atoms called domains. These domains behave like tiny permanent magnets, each domain having its own north and south pole. In an macroscopically sized, "unmagnetized" piece of a ferromagnetic material, the orientations of all of these domains is random (Figure A-3a), so their individual magnetic fields cancel each other out, and you don't see a net magnetic field. In the case of a permanent magnet, these domains are mostly lined up in the same direction (Figure A-3b), so that the domains' fields reinforce each other.

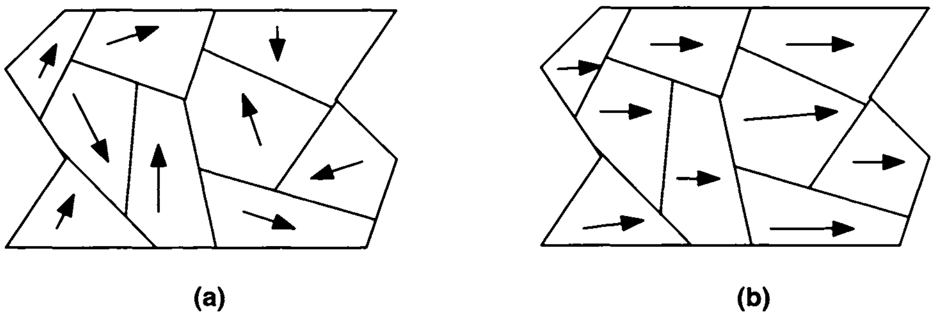

Figure A-3: Magnetic domains in unmagnetized (a) and magnetized (b) materials.

In a given piece of material, the "easiest" (lowest energy) state for the material to be in is with the domains aligned randomly. To get them to align from a random orientation requires that one apply an external magnetic field. One way of doing this is to wind a coil of wire around the material and apply electrical current to the coil. Because the exact properties of each domain vary somewhat, they don't all flip into alignment at once. As you increase the current in the coil, more and more domains will line up. As more domains line up, you get a stronger magnetic field in the material. At some point, all of the domains are lined up, and applying more current beyond this point doesn't produce a stronger field. At this point the material is said to be saturated. The relationship between magnetizing current and the resulting magnetic field is shown in Figure A-4.

Figure A-4. Relationship between a magnetizing current and resulting field.

Because the amount of magnetizing force you get out of a coil is dependent not only on the current, but also on the number of turns and the geometry, magnetizing force is usually described by a field quantity of its own called magnetic field strength, denoted by the symbol H. H is measured in oersteds. In empty space a magnetizing field of one oersted will result in a magnetic field of one gauss.

In a ferromagnetic material, however, a magnetizing field of one oersted can result in a magnetic field of several thousand gauss. This is because the domains are providing the field, and just have to be coaxed into moving into the desired orientations. This is why you get a much stronger electromagnet by wrapping a coil around a steel nail than you do by just using the coil. The ratio between the change in B versus the change in H ($\delta B/\delta H$) is called relative permeability. Materials such as cold-rolled steels can have relative permeability ranging from 100–10,000. Specialty magnetic alloys such as permalloy or mu-metal can have permeabilities ranging up to 100,000.

Once the domains in a ferromagnetic material have been oriented, in many cases they will want to stay that way when the magnetizing field is removed. To get them to go back to a net unoriented, demagnetized state may require one to reverse the magnetizing field a small amount. If one alternately sweeps a magnetizing field (H) positive and negative, and plots the resulting flux (B) versus magnetizing field (H), one gets a hysteresis diagram like the one shown in Figure A-5.

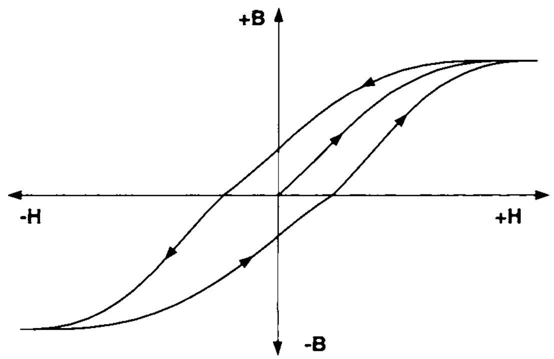

Figure A-5: B-H curve showing magnetic hysteresis.

Some materials, such as soft steels and some ferrites, take very little coaxing to demagnetize. Others, such as samarium-cobalt compounds, take enormous amounts of reverse-magnetizing force to drive them back to a demagnetized state. Materials that strongly resist demagnetization by a reversed magnetizing field are said to have high coercivity. High-coercivity materials are used to make permanent magnets and magnetic recording media. Low-coercivity materials such as steel are used in applications where it is necessary to be able to easily control or vary the amount of flux, such as electromagnets and electrical transformers.

Because one of the more important characteristics of a permanent magnet is how resistant it is to demagnetization, the materials used are often described by only the second quadrant of the hysteresis plot, where magnetizing force (H) is negative and magnetic flux density (B) is positive. This results in the B-H curve commonly used to describe magnet materials, an example of which is shown in Figure A-6.

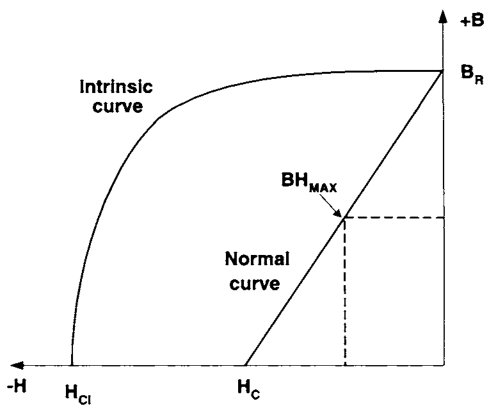

Figure A-6: Second quadrant B-H curves for permanent magnet material.

One may notice that there are two curves in Figure A-6. These are the intrinisic and the normal curves. The relationship between the two curves is given by:

$$B_{normal} = B_{intrinsic} + H \qquad \textbf{(Equation A-6)}$$

The normal curve represents the actual levels of B that appear in the material when subjected to an external magnetizing field of H. The intrinsic curve represents the flux contributed by the magnetic material. There are four points on these curves that are considered important for both characterization of magnetic materials and for magnetic design. These are:

B_r – Remanent induction, which is the flux density (B) present in a closed ring of this material in a saturated state. Measured in gauss.

H_{ci} – Intrinsic coercive force. Used as a measure both of how resistant a material is to demagnetization, and also how much magnetizing force (H) is required to magnetize it to saturation. Measured in oersteds.

H_c – Coercive force. The amount of reverse magnetizing force (H) required to drive the flux density in a closed ring of the saturated material to zero. Also measured in oersteds.

BH_{max} – Maximum energy product. This is the maximum product of B and H along the normal curve. This represents the amount of mechanical work that can be stored as potential energy in the magnet's field. This characterization parameter is therefore very important to people who design electromechanical devices such as motors. The higher a material's BH_{max}, the less of it you need to build a motor of a given power capacity. Measured in mega-gauss-oersteds (MGOe).

If you were to take a gaussmeter and hold it up to the face of a fully "charged" magnet with a B_r specified at 10,000 gauss, you will see considerably less than 10,000 gauss. This is because the B_r figure only appears as actual field when the magnet is in a closed magnetic circuit. In the case of a bar magnet, the empty space the flux must travel through to go from one pole to another effectively "opens" the magnetic circuit. In terms of the B-H curve for that material, this addition of a gap and the resulting reduction in flux (B) corresponds to riding down and left along the normal curve. The point at which the magnetic system rests on the normal curve is called the "operating point." Because the effect of adding a gap to a magnetic system is similar to applying a demagnetizing force (H), the resulting reduction in B is referred to as self-demagnetization. The degree to which a particular magnet experiences this phenomenon is dependent both on the properties of the material of which the magnet is made and also on the magnet's geometry. For a given material, the degree to which self-demagnetization effects occur is inversely dependent on the magnet's length-to-width ratio (when magnetized along the length). When made of comparable materials, a short, thick magnet will tend to provide lower flux densities (B) measured at the magnet face than a long skinny one will.

Calculating the flux density produced at a given point in space by a magnet of a particular geometry and material can be a nontrivial task, especially for complex geometries. For simple geometries it is possible to obtain closed-form equations that provide estimates for the cases of the flux density (B) along the axis of both a cylindrical magnet (Equation A-7) and a rectangular magnet (Equation A-8), based on the magnet's physical dimensions and Br [Dext98a]. Refer to Figure A-7 for an illustration of the dimensions for these cases.

Cylindrical Magnet:

$$B = \frac{B_r}{2}\left[\frac{d+l}{\sqrt{(d+l)^2 + r^2}} - \frac{d}{\sqrt{d^2 + r^2}}\right]$$ **(Equation A-7)**

Rectangular Magnet:

$$B = \frac{B_r}{\pi}\left[\tan^{-1}\left(\frac{d+l}{tw}\sqrt{t^2 + w^2 + (d+l)^2}\right) - \tan^{-1}\left(\frac{d}{tw}\sqrt{t^2 + w^2 + d^2}\right)\right]$$

(Equation A-8)

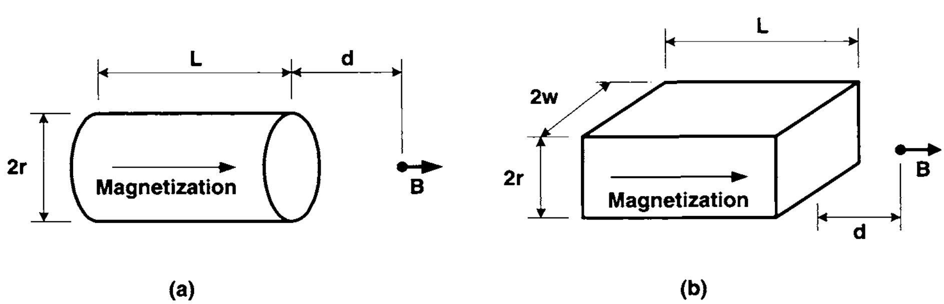

Figure A-7: Cylindrical magnet for Equation A-7 and rectangular magnet for Equation A-8.

A.3 Some Permanent Magnet Materials

Several families of materials are commonly used to make permanent magnets. Table A-1 lists the magnetic properties of a few example materials from the more common families.

Table A-1: Common magnet materials and key properties [Dext98b].

Material	BH_{max} (MGOe)	B_r (G)	H_c (Oe)	H_{ci} (Oe)
Ceramic8 (ferrite)	3.5	3850	2950	3050
Alnico5 (cast)	5.5	12800	640	640
Alnico8 (cast)	5.3	8200	1650	1860
SmCo16	16	8300	7500	18000
SmCo28	28	10900	6500	7000
NdFeB31	31	11200	11000	25000
NdFeB44	44	13500	11000	12000

Beyond magnetic properties, magnetic materials also have other characteristics that suit them for particular applications. The following are a few other properties to be considered when deciding to use a particular material.

- Temperature stability
- Service temperature range
- Corrosion resistance
- Hardness and brittleness