ROYAL SCHOOL OF ARTILLERY

BASIC SCIENCE & TECHNOLOGY SECTION
GUNNERY STAFF/CAREER COURSES

THE DOPPLER PRINCIPLE

INTRODUCTION

1. The **Doppler Effect** is a familiar, though perhaps not always recognised, feature of modern life. It is most commonly illustrated by the whistle of a passing train, which rises in pitch as it approaches and drops in pitch as it recedes. A similar effect is experienced with the sound of engines when standing at the side of a road carrying fast moving traffic. A good example is the motor racing circuit.

2. The phenomenon was first discovered by an Austrian physicist named **Christian Johan Doppler**. He realised that the change in pitch is due simply to the fact that the number of sound waves striking the eardrum per second changes because of the source's motion.

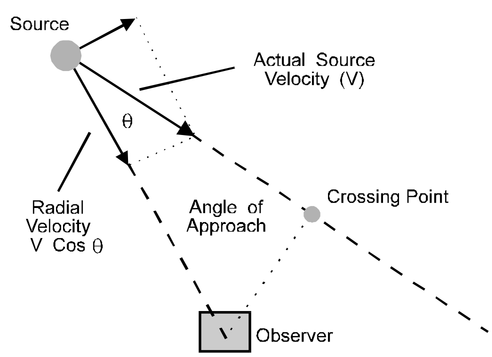


Fig.1. Source Actual Velocity and Radial Velocity

3. As Doppler suggested, the Doppler effect applies to light, or any wave energy, as well as to sound. When light from a moving source reaches the eye, there is a shift in frequency - that is, colour - when the source is moving fast enough. For instance, if the source is travelling towards us, more light waves are crowded into each second and the light perceived shifts towards the high frequency violet end of the visible spectrum. On the other hand, if the source is moving away, fewer waves arrive per second and the light shifts towards the low frequency end of the spectrum.

4. Astronomers have been studying the spectra of the stars for a long time, and they are well acquainted with the normal picture. They have been able to calculate the velocity of stars moving towards or away from us (i.e., radial velocity) by measuring the displacement of the usual spectral lines towards the violet end or the red end of the spectrum.

5. This term radial velocity is an important one. It is obvious that the source of wave energy, whether it be sound or electromagnetic, may not be moving directly towards or away from the observer, as shown at Fig.1. The path may be such that the source is passing at some minimal distance, usually known as the crossing point.

6. This means that the velocity of the source object along its path, usually called the **actual velocity**, is not the velocity towards, or away from, the observer. Velocity along the line between observer and source is always along a radial line emerging from the observer and is thus referred to as the **radial velocity**. Simple vector resolution techniques allow the radial velocity to be calculated from the actual velocity at any observed point. In the case of Fig.1, if the **actual velocity** (V) of the source is **500 m/sec** and the **angle of approach** (θ) is **60°**, then the **radial velocity** component is:-

$$\begin{aligned} \mathbf{Vradial} &= \mathbf{Vactual \times Cos\ 60°} \\ &= \mathbf{250\ m/sec.} \end{aligned}$$

7. The energy sources referred to so far have all been independent with a known standard against which to measure their frequency shift. However, where an object does not emit a known usable signal, a signal must be provided. This is normally achieved at radar frequencies by illuminating the target with a radar beam source and observing the Doppler shift on the echo signal. However, the question now arises as to how much Doppler shift can be expected from a given source, and the answer to that question is the next to be addressed.

CALCULATION OF DOPPLER SHIFT

8. In dealing with the actual frequency shift that occurs on a given source of wave energy, it is necessary to consider four different cases. They are:-

   a. Observer moving towards the source (approaching).
   b. Observer moving away from the source (receding).
   c. Source moving towards the observer (approaching).
   d. Source moving away from the observer (receding).

   Source waves of velocity v=2,000 m/sec will be observed together with two wavelengths of λ=0.5m and λ=0.25m. From the wave relationship f =v/λ these two wavelengths produce frequencies of 4kHz and 8 kHz. The source or observer velocity will be 10 m/sec as the case may be.

Receiver moving towards source

9. Fig.2 is a representation of the situation where a receiver is moving towards the wave source. One of the waves in question is 0.5m in length (λ) and at a frequency (f) of 4kHz has a period, or duration, of 1/4000th of a second (0.25ms). The leading edge of this single wave travelling left to right at 2,000m/s is designated 'A' and the trailing edge 'B'. The diagram shows the situation where the leading edge is just arriving at the receiver, which is moving from right to left at 10m/s.

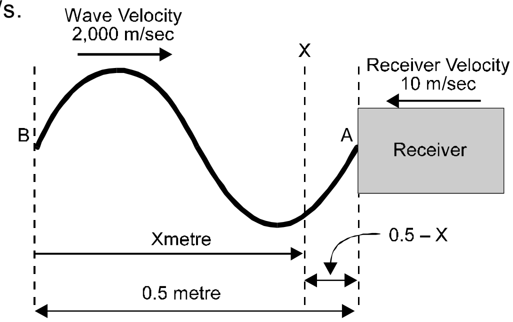


Fig.2. Receiver approaching source

10. Obviously, since the trailing edge of the wave, 'B', is still moving left to right, and the receiver is still moving right to left, point 'B' on the wave will reach the receiver at some point 'X' between their start positions, some time later. The question is how much later?

11. The answer to this can be deduced from fairly simple algebra. Point 'B' moves right towards 'X' at 2,000m/s and, since distance=speed×time, the distance X=2,000×t, where 't' is the time taken to reach 'X', whatever that is. At the same time, the receiver moves right to left at 10m/s and will cover the distance to 'X', (0.5–X), in the same 't' seconds. Thus 0.5–X=10×t, and, since both situations are clearly true of the same event, making 'X' the subject of each equation:-

	$X = 2000t$
and	$0.5 - X = 10t$
giving	$X = 0.5 - 10t$
Thus since	$X = 2000t$ and $X = 0.5 - 10t$
then	$2000t = 0.5 - 10t$
	$2010t = 0.5 = 1/2$
and	$t = \frac{1}{2 \times 2010} = \frac{1}{4020}$ .sec

12. It is clear that the single wave arrives at the receiver over a period of t=1/4020 seconds, and the distance over which point 'B' moves in order for the whole wave to be received is shorter than if the receiver had stood still. The wave has effective length of 1000/2010 metres, which can be calculated from the same equations above by transposing to make 't' the subject in each case instead of 'X'.

13. From the fundamental wave relationship f=1/T, where 'T' is the time period for one cycle, the frequency at which the wave is received may be calculated as f = 1/(1/4020), which is 4020Hz. Thus the transmitted frequency of 4000Hz has become a received frequency of 4020Hz and a Doppler Shift of 20Hz has occurred. This is normally specified in the form:-

$$f_R = f_{TX} + f_D$$

Where f_R is the frequency received, f_{TX} is the frequency transmitted and f_D is the Doppler shift component.

14. If the receiver had been moving at a different velocity, say 20m/s, then, because the wave velocity is the same, the equations would become:-

	$X = 2000t$	(Wave velocity=2,000m/s)
and	$0.5 - X = 20t$	(Receiver velocity=20m/s)
giving	$X = 0.5 - 20t$	
thus since	$X = 2000t$ and $X = 0.5 - 20t$	
then	$2000t = 0.5 - 20t$	
	$2020t = 0.5 = 1/2$	
and	$t = \frac{1}{2 \times 2020} = \frac{1}{4040}$ .sec	

Giving a received frequency of 4040Hz and a Doppler shift component of 40Hz, which is doubled. Thus the Doppler shift is directly proportional to the receiver velocity.

15. Moving the receiver at 10m/s again but this time altering the wavelength of propagation to λ=0.25m, the transmitted frequency is then 8,000Hz for a wave velocity of 2,000m/s. Now the equations would be:-

	$X = 2000t$	(Wave velocity=2,000m/s)
and	$0.25 - X = 10t$	(Receiver velocity=10m/s)
giving	$X = 0.25 - 10t$	(Wavelength=0.25m)
thus since	$X = 2000t$ and $X = 0.25 - 10t$	
then	$2000t = 0.25 - 10t$	
	$2010t = 0.25 = 1/4$	
and	$t = \frac{1}{4 \times 2010} = \frac{1}{8040}$. sec	

Giving a received frequency of 8040Hz and a Doppler shift component of 40Hz, which is doubled for that at f_{TX}=4kHz of para 13. Thus the Doppler shift is directly proportional to both the transmitted frequency and the receiver velocity.

16. On the other hand, keeping the receiver moving at 10m/s but now altering the velocity of propagation to 4,000m/s, the transmitted frequency is then 8,000Hz for a wavelength of 0.5m. Once again the equations would take the form:-

	$X = 4000t$	(Wave velocity=4,000m/s)
and	$0.5 - X = 10t$	(Receiver velocity=10m/s)
giving	$X = 0.5 - 10t$	
thus since	$X = 4000t$ and $X = 0.5 - 10t$	
then	$4000t = 0.5 - 10t$	
	$4010t = 0.5 = 1/2$	
and	$t = \frac{1}{2 \times 4010} = \frac{1}{8020}$. sec	

17. This gives a received frequency of f_R = 8020Hz, which has the components f_{TX} = 8,000Hz and f_D = 20 Hz and the Doppler shift at f_{TX}=8kHz is halved against the shift produced at the lower wave velocity of para 15. From the discussion thus far, it can be deduced that the Doppler shift is directly proportional to receiver velocity and transmitter frequency but inversely proportional to wave velocity. The changes are normally expressed in the form:-

$$f_R = f_{TX} + f_D$$

where $f_D = f_{TX} \times \frac{Vr}{Vw}$ (Receiver velocity) (Wave velocity)

18. Thus, looking at the received frequency and Doppler shift produced in the cases discussed so far:-

a. For the receiver moving at **10m/s** with a wave propagating at **2,000m/s** and frequency of **4kHz** the Doppler shift component becomes:-

$$f_R = f_{TX} + f_D$$

where $f_{TX} = 4{,}000$ Hz

and $f_D = f_{TX} \times \dfrac{Vr}{Vw}$

$= 4{,}000 \times \dfrac{10}{2{,}000}$

$= 20$ Hz

Thus $f_R = 4{,}000 + 20$

$= 4020$ Hz

b. For the receiver moving at **20m/s** with a wave propagating at **2,000m/s** and frequency of **4kHz** the Doppler shift component alone becomes:-

$$f_D = f_{TX} \times \frac{Vr}{Vw} = 4{,}000 \times \frac{20}{2{,}000} = 40 \text{ Hz}$$

c. For the receiver moving at **10m/s** with a wave propagating at **2,000m/s** and frequency of **8kHz** (wavelength = 0.25m) the Doppler shift component becomes:-

$$f_D = f_{TX} \times \frac{Vr}{Vw} = 8{,}000 \times \frac{10}{2{,}000} = 40 \text{ Hz}$$

d. For the receiver moving at **10m/s** with a wave propagating at **4,000m/s** and frequency of **8kHz** the Doppler shift component is:-

$$f_D = f_{TX} \times \frac{Vr}{Vw} = 8{,}000 \times \frac{10}{4{,}000} = 20 \text{ Hz}$$

19. When waves that are members of the EM spectrum become involved, the velocity at which they travel is the velocity of light, c = approximately 300×10^6 m/s. So taking an example that might occur in Doppler radar systems, where f_{TX} is 3GHz and Vr is 250m/s:-

$$f_R = f_{TX} + f_D$$

where $f_{TX} = 3$ GHz

and $f_D = f_{TX} \times \dfrac{Vr}{Vw}$

$= 3 \times 10^9 \times \dfrac{250}{300 \times 10^6}$

$= 2{,}500$ Hz

Thus $f_R = 3$ GHz $+ 2{,}500$ Hz

$= 3{,}000{,}002{,}500$ Hz

As can be seen from this last example, Doppler shift increases with frequency transmitted and receiver velocity. It is also evident that the higher the wave velocity the lower is the Doppler shift. Fortunately, EM waves travel at a limited maximum constant velocity.

Receiver moving away from source

20. Fig.3 shows the situation where a receiver is moving away from the wave source. One of the waves in question is 0.5m (500mm) in length and at a frequency of 4kHz has a period, or duration, of 1/4000th of a second (0.25ms). The leading edge of this single wave travelling left to right at 2,000m/s is designated 'A' and the trailing edge 'B'. The diagram shows the situation where the leading edge is just arriving at the receiver, which is moving from left to right at 10m/s.

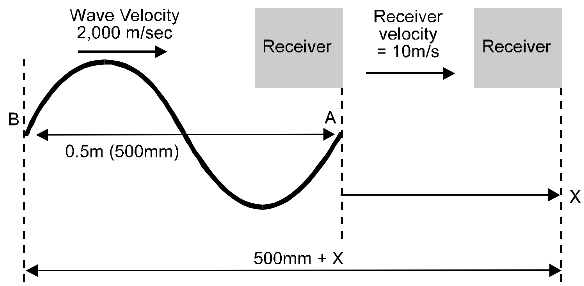


Fig.3. Receiver moving away from source

21. Obviously, since the trailing edge of the wave, 'B', is still moving left to right, and the receiver is still moving left to right, point 'B' on the wave will reach the receiver at some point 'X' after their start positions, some time later. The question again is how much later?

22. The answer to this can again be deduced from fairly simple algebra. Point 'B' moves right towards 'X' at 2,000m/s and, since distance=speed×time, the distance 0.5+X=2,000×t, where 't' is the time taken to reach 'X', whatever that is. At the same time, the receiver moves left to right at 10m/s and will cover the distance to 'X' in the same 't' seconds. Thus X=10×t, and, since both situations are clearly true of the same event, making 'X' the subject of each equation:-

$X = 10t$

and $0.5 + X = 2{,}000t$

giving $X = 2{,}000t - 0.5$

Thus since $X = 2000t - 0.5$ and $X = 10t$

then $2000t - 0.5 = 10t$

$1990t = 0.5 = 1/2$

and $t = \dfrac{1}{2 \times 1990} = \dfrac{1}{3980}$ sec

23. It is clear that the single wave arrives at the receiver over a period of t=1/3980 seconds, and the distance over which point 'B' moves in order for the whole wave to be received is longer than if the receiver had stood still. The wave has an effective length of 0.5 + 5/1990 metres, which can be calculated from the same equations above by transposing to make 't' the subject in each case instead of 'X'.

24. From the fundamental wave relationship f=1/T, where 'T' is the time period for one cycle, the frequency at which the wave is received may be calculated as f = 1/(1/3980), which is 3980Hz. Thus the transmitted frequency of 4000Hz has become a received frequency of 3980Hz and a Doppler Shift has occurred of –20Hz. The amount of shift is identical to that for receiver towards source and is calculated in the same way using:-

$f_R = f_{TX} - f_D$ (Note the negative sign)

where $f_D = f_{TX} \times \dfrac{Vr}{Vw}$ (Receiver velocity) / (Wave velocity)

Source moving towards receiver

25. Fig.4 is a representation of the situation where the source is moving towards a receiver. One of the waves in question is 0.5m (500mm) in length and at a frequency of 4kHz has a period, or duration, of 1/4000th of a second (0.25ms). The leading edge of this single wave travelling left to right at 2,000m/s is designated 'A' and the trailing edge 'B'. The diagram shows the situation over the period during which one cycle is produced.

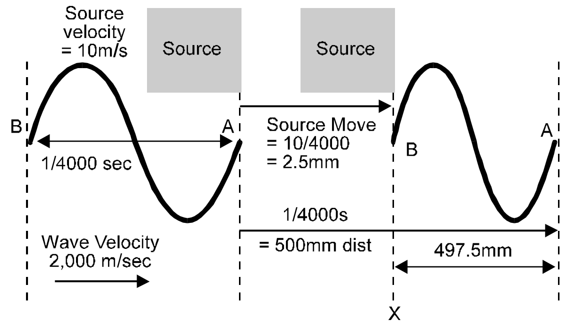


Fig.4. Source moving towards receiver

26. As the source is forming point 'A' on the wave, which moves away at 2,000m/s, it is also moving in the same direction. Over the period during which one cycle is produced (1/4000s=0.25ms), the source moves to 'X' a distance of X=speed×time =10×1/4000 = 2.5mm, where point 'B' occurs. This means that the wave is now compressed in space to a length of 500mm–2.5mm = 497.5mm.

27. The waves relationship, velocity = frequency × wavelength ($V = f \times \lambda$) indicates that the frequency received is now:-

$$f = V/\lambda = 2{,}000/497.5\text{mm} = 4{,}020.10 \text{ Hz}$$

but $f_R = f_{TX} + f_D$

Where f_R is the frequency received, f_{TX} is the frequency transmitted and f_D is the Doppler shift component. So in this case f_{TX} = 4,000 Hz and f_D = 20.10 Hz. There is an obvious difference between this and the case of the receiver moving towards the source of para 13. When compared to that the doppler shift is of the order:-

$$= \frac{20.10 \times 100\%}{20} = 100.5\%$$

Which gives an error of about 0.5% if it is assumed that the calculation for doppler shift is carried out in the same way as for the receiver in motion with respect to the source, discussed in paras 9–24, and given by the function:-

$$f_D = f_{TX} \times \frac{Vt}{Vw} \quad \begin{matrix}\text{(Transmitter velocity)}\\ \text{(Wave velocity)}\end{matrix}$$

28. It would seem, providing the error introduced for the case of the source moving towards the receiver is acceptable, that where a source is approaching a receiver, or the receiver is approaching the source, the same calculation can be used to extract the Doppler shift. On the other hand, if the error is not acceptable, its effect at radar wavelengths should be assessed. This will be discussed later.

Source moving away from receiver

29. Fig.4 is a representation of the situation where the source is moving away from a receiver. One of the waves in question is 0.5m (500mm) in length and at a frequency of 4kHz has a period, or duration, of 1/4000th of a second (0.25ms). The leading edge of this single wave travelling left to right at 2,000m/s is designated 'A' and the trailing edge 'B'. The diagram shows the situation over the period during which one cycle is produced.

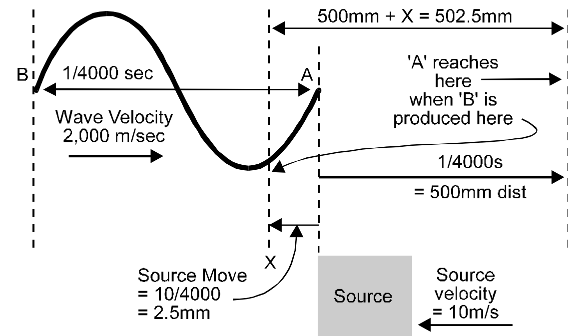


Fig.5. Source moving away from receiver

30. As the source is forming point 'A' on the wave, which moves away left to right at 2,000m/s, it is also moving right to left at 10m/s. Over the period during which one cycle is occurs (1/4000s=0.25ms), the source moves to 'X' a distance of X=speed×time =10×1/4000 = 2.5mm, where point 'B' is produced. This means that the wave is now stretched in space to a length of 500mm+2.5mm = 502.5mm.

31. The waves relationship, velocity = frequency × wavelength ($V = f \times \lambda$) indicates that the frequency received is now:-

$$f = V/\lambda = 2{,}000/502.5\text{mm} = 3{,}980.10 \text{ Hz} \quad \text{(To two decimal places)}$$

but $f_R = f_{TX} + f_D$

Where f_R is the frequency received, f_{TX} is the frequency transmitted and f_D is the Doppler shift component. So in this case f_{TX} = 4,000 Hz and f_D = –19.90 Hz. Once again when compared to the case of para 13, the Doppler shift is of the order:-

$$= \frac{19.90 \times 100\%}{20} = 99.5\%$$

Which again gives an error of about 0.5% if it is assumed that the calculation for doppler shift is carried out in the same way as for the receiver in motion with respect to the source, discussed in paras 9–24, and given by the function:-

$$f_D = f_{TX} \times \frac{Vt}{Vw} \quad \begin{matrix}\text{(Transmitter velocity)}\\ \text{(Wave velocity)}\end{matrix}$$

Errors caused by source movement

32. The last two situations considered, where the source is moving with respect to the observer, have thrown up errors using the Doppler shift formula above. Whilst the errors are relatively small in the circumstances discussed, it is now necessary to examine the errors and look to see if they will

cause problems within radar system designs.

33. The source is deemed to be moving at 100m/s towards the observer and if the wave velocity is $c = 300\times10^6$m/s (velocity of light), the effect is illustrated at Fig.6. One cycle at 2GHz gives a wavelength λ=c/f of = 150mm. A time interval of 1/2GHz = 0.5ns per cycle means that the source moves just 0.5ns × 100 = 50nm in the same time. As a result, the compression of the wave reduces its length to 150mm–50nm = 149.99995mm. Using the formula for frequency f=c/λ =300×10^6/149.99995 giving f_R = 2.000000667GHz. Subtracting the 2GHz transmitted gives 666.66Hz which is the

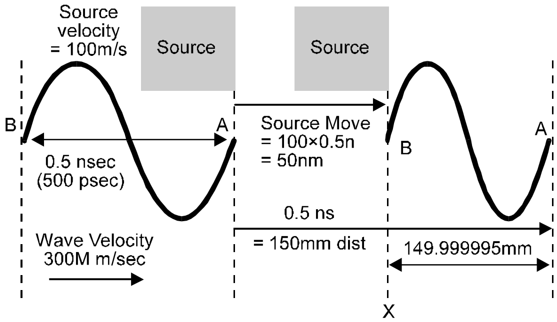


Fig.6. Doppler shift at velocity of light

actual Doppler shift.

34. By using the Doppler shift formula, the frequency change becomes:-

$$f_D = f_{TX} \times \frac{Vt}{Vw} \quad \text{(Transmitter velocity)} \quad \text{(Wave velocity)}$$

$$= 2GHz \times \frac{100}{300\times10^6}$$

$$= 666.6666667 \text{ Hz}$$

When the two figures are compared, the ratio of the calculated Doppler shift to the actual one is of the order:-

$$= \frac{666.6666667 \text{ Hz} \times 100}{666.66 \text{ Hz}}$$

$$=100.001\%$$

Which is an error of about 0.001% and a marked improvement over the figure at the wave velocity of 2,000 m/s. It is also true that the higher the velocity the lower is the error. All current radar systems are able to accommodate this sort of error with little or no problem in velocity accuracy.

Note. The calculations here are based upon the use of a standard Casio type calculator which has 10 digit display. This means that the accuracy of the figure for Doppler shift produced by using the new wavelength dimension (666.66Hz) is a little lower than it could be. When the same calculation is carried out using a machine with 15 digit accuracy, the error falls even further to 0.0000333%.

35. In conclusion, all four cases looked at show results that are consistent with the formulae derived in paragraphs 17 and 24. Thus:-

$f_R = f_{TX} + f_D$ (Approacher)

$f_R = f_{TX} - f_D$ (Receder)

and generally:-

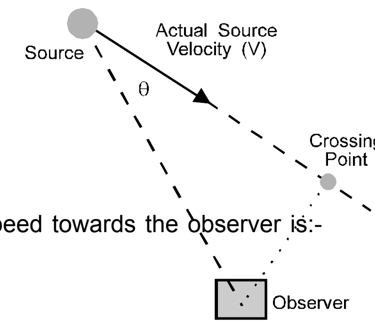
$f_R = f_{TX} \pm f_D$ (Either case)

where $f_D = f_{TX} \times \frac{Vr}{Vw}$ (Receiver velocity) (Wave velocity)

SELF TEST QUESTIONS

Read each question carefully and then select the one answer that you believe to be fully correct

1. The source shown in the following diagram has an angle of approach θ = 28°. Actual source velocity is 450 m/sec. The component of speed towards the observer is:-



a. 397 m/sec
b. 211 m/sec
c. 239 m/sec
d. 250 m/sec

2. In considering the Doppler Shift that occurs on EM wave source transmissions. It is true to say that the shift:-

a. only occurs at the violet end of the spectrum
b. increases with decreasing wavelength
c. is not affected by the angle of approach
d. is not affected by wave velocity

3. A speeding boat travelling at 30m/s produces a 3kHz surface wave travelling at a velocity of 1500m/s that passes a float which bobs up and down as a result. The number of times per second the float will oscillate up and down is:-

a. 60
b. 3,060
c. 1530
d. 30

4. A radar receiver is moved at 18 kilometres per hour directly towards a transmitter that is operating at 12GHz. The doppler shift that will be present on the received signal is:-

a. 20 Hz
b. 40 Hz
c. 100 Hz
d. 200 kHz

5. A receiver is moved directly away from its transmitter along a radial path at a velocity of 150 m/s which produces a doppler shift of 4 kHz. The frequency of transmission is:-

a. 4 GHz
b. 12 GHz
c. 8 GHz
d. 3 GHz

6. A source of EM waves emits 10GHz when moving at a velocity of 500m/s across the front of a receiver. The angle of approach is 46°. This means that the received frequency is:-

a. 11,578 Hz
b. 1,578 Hz
c. 33.333 kHz
d. 11,578 kHz