ROYAL SCHOOL OF ARTILLERY

BASIC SCIENCE & TECHNOLOGY SECTION

Pulsed Doppler Radar

INTRODUCTION

Surveillance radars, used for ground-based air defence (GBAD), look especially for radar echoes from aircraft and helicopters. However, there are many other objects in the surveillance area that reflect radar signals and it is necessary to distinguish between these 'clutter' signals and the 'target' signals. Doppler radars seek to exploit the Doppler shift in the radar echo as a means of recognising echoes that are returned from an 'interesting' target. Furthermore, the amount of Doppler shift can be measured and used to determine the radial velocity of the target.

The Doppler shift is covered in another BST Handout. Any approaching target will give a radar return with a slightly higher frequency than that which was transmitted whereas any receding target will give a return at a slightly lower frequency. Ground clutter will be at the same frequency as the transmitted frequency, as will the return from any other stationary object. Thus, any echo with a Doppler shift must have come from a moving target and that target can be identified and tracked.

Pulsed radar is used because it can also determine the target's range (150 m for each µs of delay between transmitted pulse and its echo).

Pulsed Doppler radar operates to combine both functions so that both range and velocity of a target can be obtained. This handout explains how it is accomplished - it is not as straightforward as it seems!

HOW MUCH DOPPLER SHIFT?

Under practical conditions, the amount of Doppler shift is a small amount – it might be a few kHz – when compared with the radar frequency in use, which might be a few GHz. This means that the Doppler shift will be of order 0.000 01%, or one part in one million. (This is because the velocity of an aeroplane might be 300 ms^{-1} which is one-millionth of the speed of radar waves, 300 million ms^{-1}.) The two formulae that can be used to calculate the Doppler shift in a radar echo are:

$$f_d = \frac{2\,v_r}{\lambda_t} \quad \text{or} \quad \frac{2\,v_r\,f_t}{c}$$

Where $\mathbf{f_d}$ is the Doppler shift, $\mathbf{v_r}$ is the radial velocity of the target, **c** is the velocity of light, $\mathbf{f_t}$ is the frequency that the radar transmits and $\boldsymbol{\lambda_t}$ is its wavelength. V_r is V × Cos θ, where θ is the angle between the track of the aeroplane and the line of sight from the radar.

Example: The Doppler shift for a radar operating at a frequency of 3 GHz that has a target with a radial velocity of 50 ms^{-1} will be:

$$f_d = \frac{2\,v_r}{\lambda_t} = \frac{2 \times 50}{0.1} = 1 \text{ kHz}$$

Note: λ_t = 10 cm or 0.1 m. Remember the rule of thumb that GHz × cm = 30: this is a quick way of converting between frequency and wavelength.

DETERMINING THE DOPPLER SHIFT

Pulsed radars typically have a duty cycle of a few percent - in other words they might spend 98% of the time listening for an echo and only 2% of the time actually transmitting. The Doppler shift will, therefore, only exist for the duration of the radar's pulse - in the echo from the target. This means that we do not have continuous access to the Doppler shift, it is only available for measurement for the duration of the echo. This duration is equal to the pulse length that the radar transmitted.

From the above calculation, you will see that a typical Doppler shift is 1 kHz. This means that one cycle of the Doppler frequency lasts for 1 ms or 1 000 µs (Period = 1 / f). The radar emits energy in pulses which might last for a few micro-seconds (many radars have a pulse duration, or pulse width, of a few micro-seconds).

During one pulse from the radar, say 5 µs, the Doppler shift is available for measurement - in other words, the radar receives a 5 µs fragment of a sine wave whose cycle lasts for 1 000 µs. This is about 1/200th of the sine wave and it is not enough to determine its frequency with any degree of accuracy. To appreciate this problem, draw one cycle of a sine wave and then select one small part (say, 1/20th of it) - see how difficult it would be to reconstruct the whole sine wave from such a small fragment. Figure One illustrates this - the fragment is about 1/20th of the wave (a fragment that was 1/200th of the wave would not even be visible on this scale).

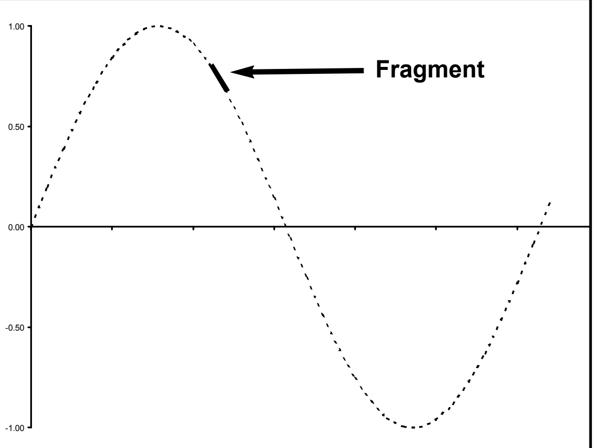


Figure 1: Reconstructing a Sinusoid from a Fragment Representing about 1/20th of a Cycle

This leads us to the conclusion that it is practically impossible to measure the Doppler shift using a single pulse from a radar - because the pulse duration (e.g. 5 µs) is too short for us to observe any changes over the duration of one cycle of the Doppler shift (e.g. 1000 µs).

CW DOPPLER RADAR

A CW radar does not use pulses and so this type of radar does not suffer from the above problem - provided that the dwell-time is sufficient to span one cycle of the Doppler frequency (e.g. a few milli-seconds). This is not normally a problem with CW radar. However, the CW radar gives no indication of range - which is a requirement of a surveillance radar.

The CW radars that you might encounter - MVMD (Muzzle Velocity Measuring Device) and RGU (Radar Gathering Unit) do not measure range at all - but they are not surveillance radars!

PULSED DOPPLER RADAR

A single pulse cannot be used to determine the Doppler shift because the time interval is too short. We can make the time interval longer by using a group of pulses to determine the Doppler shift.

A typical pulse repetition frequency (prf) might be 10 kHz. This means that the radar emits ten thousand pulses each second - the pulse interval is 100 µs (T=1/PRF). If we were to use a group of ten pulses then the time for which we observe the Doppler shift will be 10 × 100 µs or 1 000 µs. This time equals the period of our typical Doppler frequency - so we now have sufficient data to measure it. This is illustrated in Figure Two.

The grey line, in Figure Two, represents what the Doppler shift would look like if it were present as a continuous wave. The radar emits pulses and the height of a series of pulses traces out the Doppler frequency, as indicated. For other Doppler frequencies (i.e. targets with a different radial velocity) and different PRFs, the picture is similar. The heights of a series of pulses trace out the Doppler waveform.

PROCESSING THE RADAR ECHO

The Doppler shift that we are trying to measure represents only about 0.000 01% of the transmitted frequency. Such a small change is virtually impossible to measure directly and the only effective way to measure it is to compare the frequency of the radar echo with the frequency that was originally transmitted. This requires that the oscillator that was used to produce the transmitted pulse must be available during the listening period so that it can be used in this comparison.

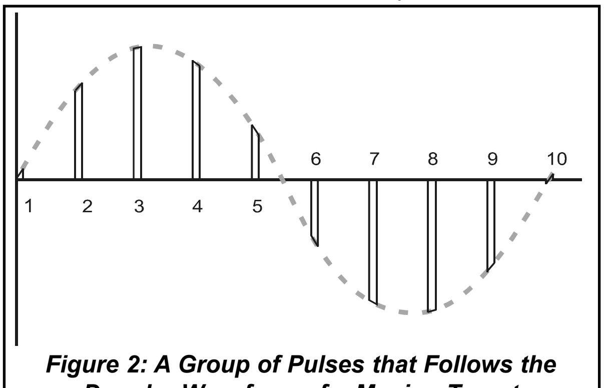


Figure 2: A Group of Pulses that Follows the Doppler Waveform of a Moving Target

The coherent oscillator (COHO) is used as the source of the transmitted pulse. This oscillator runs continuously and produces a precisely controlled sinusoidal waveform whose frequency and phase remain as steady as possible.

COHERENT DETECTION

A ‘detector’ is the part of a radio or radar receiver that extracts information from the received signal. In a car radio, the output signal from the detector becomes, when amplified and fed into the loudspeakers, the music that you hear. In a radar receiver, the output signal from the detector contains information about the target.

When processing a group of pulses, in order to determine the Doppler shift, a special detector, called a ‘phase-sensitive detector’ or ‘phase detector’, is required. This has two signals as inputs: the coherent oscillator and the (processed) echo from the target. The detector multiplies the signals together and passes the output through a low-pass filter to give its output.

Although this process might seem complicated, it is necessary to extract the Doppler shift from the group of pulses and to produce the type of output that is illustrated in Figure Two. If a non-coherent detector were used then all the pulses of Figure Two would be the same height, regardless of any Doppler shift. The target would still be identified but moving targets would appear to be identical to stationary targets. This would not be much use in a surveillance radar as low-flying and stealth targets would be very difficult to separate from clutter (i.e. reflections from the ground, buildings and rain).

Effect of Phase: when two sinusoidal waves are multiplied together then the result is affected by their phase. This is illustrated in Figures Three, Four and Five where a wave of amplitude 3 units has been multiplied by a wave of amplitude 2 units. The waves have the same frequency but different phases. The results are as follows:

- If they are in phase then the result of multiplication is always positive. This is because for one half-cycle both signals are positive and for the next half-cycle then both signals are negative. There will either be two positives or two negatives to be multiplied together. See Figure Three for the waveforms.
- If they are in anti-phase then the result of multiplication is always negative. This is because whenever one is positive then the other (being in anti-phase) will be negative. There will always be a positive value to be multiplied by a negative value. See Figure Four for the waveforms.
- If they have a phase difference of 90° (quadrature phase, or one-quarter of a cycle) then the result is alternately positive and negative - averaging to zero. See Figure Five for the waveforms.
- In general, the resultant output depends on the product of their rms voltages multiplied by the Cosine of the phase difference.

EFFECT OF TARGET MOTION

In the diagrams of Figures Three, Four and Five, one waveform represents the coherent oscillator whilst the other represents the echo from the target (the echo has been amplified, of course). Figure Three, represents the situation where the echo has returned in-phase with the coherent oscillator. This means that the radar wave must have travelled a distance equal to an exact whole number of wavelengths in its journey to the target and back. This represents a pulse like pulse number three in Figure Two.

Why will pulse number four have a different height from pulse number three? The following paragraphs explain why pulse four is smaller than pulse three:

- The target is moving towards the radar so the next pulse that the radar emits will have a shorter distance to travel.
- In our 'typical' radar, the PRF is 10 kHz, so the next pulse will be only 100 µs later.
- Our target is moving towards the radar at 50 ms^{-1} so it will move a distance = speed × time = 50 × 100 µs = 5 mm closer towards the radar between one pulse and the next.
- The distance that the radar signal travels changes by 10 mm or 1 cm, because it is a two-way journey.
- The wavelength in use is 10 cm (3 GHz, see page One) so the change in distance represents 1/10 of a wavelength.
- One-tenth of a wavelength reduction in path length will cause a phase change of 36°
- The output of a phase detector depends on the Cosine of the phase angle: Cos 36° is 0.81 so the height of pulse four should be about 81% of the height of pulse three.
- If a non-coherent detector were used then, as explained above, both pulses would be of the same height.

The argument, above, does not seem to require any Doppler shift. However, the phase of the echo has been advanced by the reduction in path length - this means that the echo has a higher frequency because it gains phase compared to the original wave. Any wave whose phase advances compared to another wave must have a higher frequency than that wave. This is just Doppler shift viewed from another angle.

Stationary Target: this would have no Doppler shift and, therefore, all pulses would have the same height. Whatever phase was returned by the first pulse would be maintained for subsequent pulses.

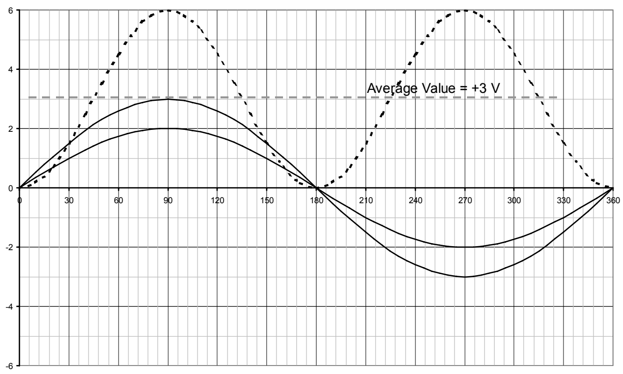


Figure 3: Coherent Detection: Phase Angle Zero

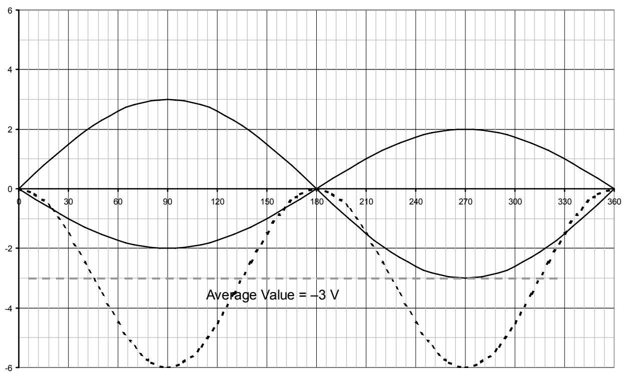


Figure 4: Coherent Detection: Phase Angle 180°

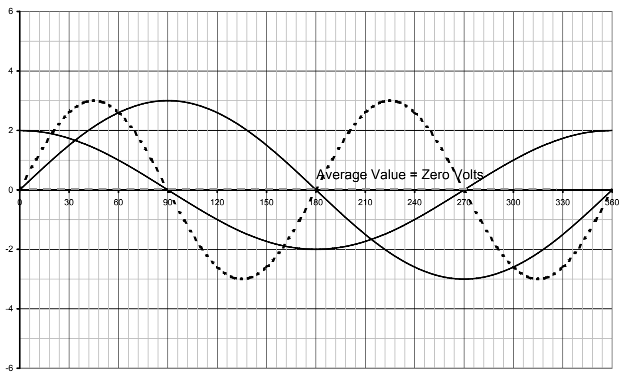


Figure 5: Coherent Detection: Phase Angle 90°

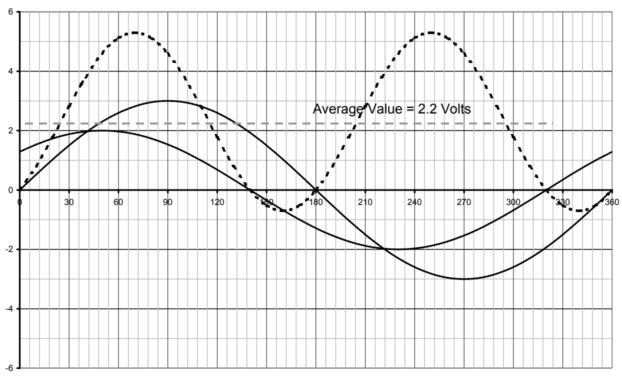


Figure 6: Coherent Detection: Phase Angle 36°

BLOCK DIAGRAM OF A COHERENT RADAR

Figure Seven shows the main parts of the radio-frequency section of a pulsed Doppler radar (similar in form to that used in the surveillance radar of Rapier). The function of each block is as follows:

- **STALO**: the **ST**able **L**ocal **O**scillator, this produces a continuous sine-wave whose frequency must remain constant for the duration of the group of pulses that are used to determine the Doppler shift (this is called the Coherent Processing Interval - CPI). In this example, its frequency is 2.97 GHz. For frequency agility, the STALO can be switched to a different frequency for the next group of pulses.
- **COHO**: the **COH**erent **O**scillator, this produces another continuous sine wave whose phase and frequency must remain steady throughout the CPI. In this example, its frequency is 30 MHz (or 0.03 GHz).
- **TX Mixer**: a mixer may add or subtract frequencies - this one adds them. The result, 2.97 GHz + 0.03 GHz is the transmitter's output frequency of 3 GHz. Since mixers operate at low powers, this must be amplified before transmission.
- **Travelling Wave Tube, TWT**: this is a powerful amplifier of micro-waves and it takes the small output from the TX Mixer and increases its power to many kilo-Watts. (A Klystron may also be used but TWT devices have greater frequency agility and stability.)
- **Pulse Modulator**: the COHO and STALO run continuously - but this is a pulse radar. The pulse modulator turns on the TWT when a pulse is to be transmitted and then turns it off again until the next pulse. In our 'typical' radar, it would be on for 5 µs every 100 µs.
- **Duplexer or T/R Switch**: this routes the signal from TXT to antenna when transmitting and from antenna to receiver during receive. It also prevents any of the transmitter's signal from leaking into the receiver to avoid damaging the sensitive circuits there.

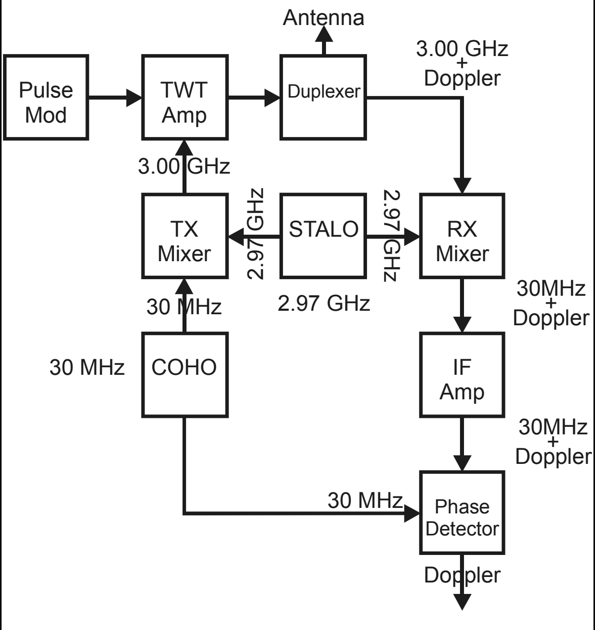
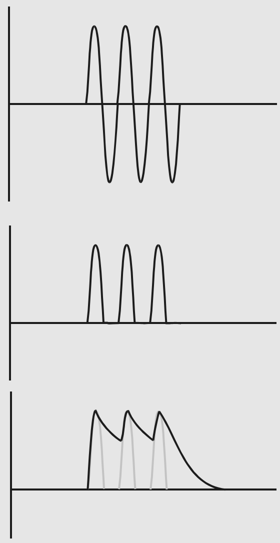


Figure 7: Block Diagram of Part of a Pulsed Doppler Radar System

ENVELOPE DETECTOR (NON-COHERENT)

This is a very simple type of radio wave detector that is still used in many systems. It uses a diode to rectify the signal and, hence, convert its amplitude to a dc voltage. The top diagram shows an idealised radar echo - usually there will be hundreds of cycles in the echo, but just three are shown here, for clarity.



When this wave is fed through an envelope detector then the the diode removes one half of the sine wave, as shown in the middle diagram. Phase is not used - the amplitude of the wave determines the size of the output.

When the wave is fed through a low-pass filter (often just a capacitor and a resistor) then the output resembles the bottom diagram. It might look a bit uneven but remember that in a real system there would normally be hundreds of cycles in the pulse so the ripples would actually be much less.

The resulting signal would be used, for example, to produce a bright spot on a radar display

- **RX Mixer**: when the echo returns, at 3 GHz plus any Doppler, the signals are mixed with the STALO. This mixer subtracts frequencies and the signal becomes 30 MHz plus any Doppler. Note that we used the same STALO signal twice - this ensures that we get back to 30 MHz without loss of information.
- **IF Amplifier**: the signal is very small after mixing and needs considerable amplification before the phase detector. The IF Amp increases the power of the signal and, by means of filters, removes any signals that are not around 30 MHz - these might have come from other sources, such as interference.
- **Phase Detector**: This multiplies the echo signal, 30 MHz + Doppler by the COHO signal, 30 MHz. The result is a single output for each radar echo. When a number of pulses are used (Coherent Processing Interval) then the tops of these pulses trace out the Doppler waveform - as show in Figure Two.

What Next?: the Doppler signal is now ready for processing by the computer to determine its frequency and, hence, the radial velocity of the target. This will usually involve converting the signal from analogue to digital and then some considerable computation of the data. See later for more details of this process.

REDUCING CLUTTER

We have seen that, when a coherent system is used than a series of pulses from a moving target will trace-out the Doppler waveform. A series of pulses from a stationary target will all have the same height because the Doppler frequency is zero. The diagram of Figure Eight shows the sort of signals that might be obtained from a moving target in front of a large hill. Since the hill is much bigger than the target then it reflects much more radar signal and gives very big pulses into the receiver. The small target gives small pulses. In practice, the energy from the target might be many thousand times smaller than that from the clutter.

The key to the elimination of clutter is to note that the height of its pulses remains constant - because it is not moving. The height of the target's echoes, which have been coherently detected, follow the Doppler waveform and are changing on every pulse.

If we delay the signals by exactly one pulse interval (100 μs in our example radar) and then subtract the delayed signals from the 'live' signals then the clutter signals will cancel because they are the same every time. The signals from moving targets will, unfortunately, be reduced but not by very much. Overall, there is a significant advantage in doing this.

Figure Nine shows a circuit called a 'delay-line canceller'. This simple circuit can be used to reduce the signal from clutter by a very large amount whilst not having very much effect on echoes from moving targets. In modern radars, the delay line is implemented using a computer to shuffle data through a sequence of memory locations as this enables a variable delay to be used. The implementation using computer RAM is necessary when different PRFs are used, because the delay must be equal to 1/PRF and it is much easier to change the computer program to give a different delay than it would be to change to a different delay line.

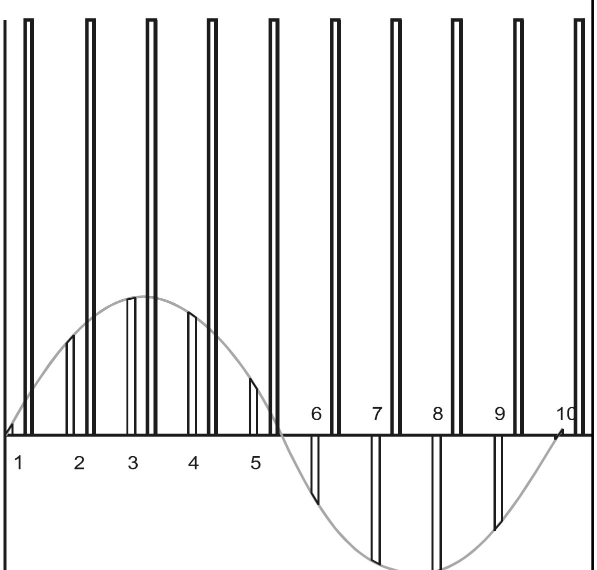


Figure 8: A Group of Pulses from a Moving Target with Echoes from (Stationary) clutter

LIMITATION OF DOPPLER MEASUREMENT

Once the Doppler waveform has been obtained at the end of the coherent processing interval, it must be analysed to determine its frequency content. Targets that have moving parts - propellors, helicopter and turbine blades - and targets that are vibrating - especially helicopter bodies, return multiple Doppler frequencies as their various parts are moving at different speeds. Two adjacent targets, within the radar beam simultaneously, might also return different Doppler frequencies. The analysis must be able to take account of these factors.

Unfortunately, the more detailed the analysis that is required then the longer the time that the radar must dwell on the target. This is because the time that is taken to collect sufficient information (coherent processing interval) must be increased if more detailed information is required.

In our example radar, with 3 GHz carrier frequency, 10 kHz PRF and 5 μs pulses, a target of radial velocity 50 ms^{-1} gave a Doppler shift of 1 kHz (see Page One). A target with a radial velocity of 45 ms^{-1} would give a Doppler shift of 900 Hz. To distinguish between 1 000 Hz and 900 Hz - a difference (called Δf) of 100 Hz - requires that we dwell on the target for 1/100th second. This would require a coherent pulse interval containing 100 pulses - and a very large amount of computer power to analyse the results. The rotation rate of the antenna would have to be reduced to give that much time on the target and that would significantly reduce the update rate for the system.

Formulae: the observation time, t_0, that is required to distinguish between two Doppler frequencies that differ by Δf_d is:

$$t_0 = \frac{1}{\Delta f_d}$$

The observation time, t_0, that is required to distinguish between two radial velocities that differ by Δv_r is:

$$t_0 = \frac{\lambda}{2\,\Delta v_r}$$

The observation time, t_0, is the coherent processing interval that the radar must use. Its duration is given by:

$$t_0 = n\,T$$

Where 'T' is the pulse interval or 1/PRF.

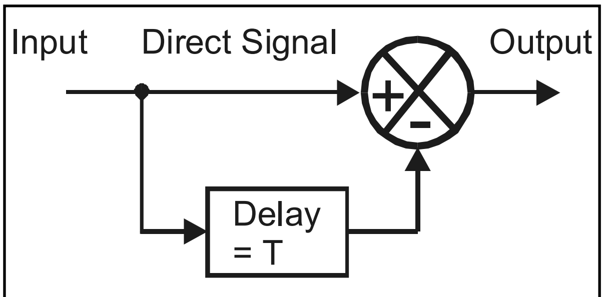


Figure 9: Delay-Line Canceller

Example:

A radar operating at 6 GHz, with a PRF of 5 kHz is required to classify targets according to their radial velocities. The radial velocity must be measured to an accuracy of 20 ms^{-1}. How long a period of time must the radar dwell on this target to achieve this and how many pulses must be used in the coherent processing interval?

(a) Δv_r is 20 ms^{-1}. Using the formula from above, and converting 6 GHz into a wavelength of 5 cm, the observation time required is:

$$t_0 = \frac{\lambda}{2\,\Delta v_r}$$

$$= \frac{0.05 \text{ metres}}{2 \times 20 \text{ ms}^{-1}}$$

$$= 1.25 \text{ ms}$$

(b) A PRF of 5 kHz has pulses spaced 1/5000th second apart, or 0.2 ms. The number of pulses, n, required in the CPI is, therefore given by the equation above:

t_0 = n T which transposes for ‘n’
n = 1.25 ÷ 0.2
= 6.25
= 7 since it must be a whole number of pulses.

BLIND PHASE

You have seen in Figure Two how the coherently-detected radar echoes trace out the Doppler waveform during the coherent processing interval. Under some circumstances, it is possible that the pulses might coincide with the zero points of the Doppler waveform. This would arise when the echo signals always arrived with a phase angle of either 90° or 270° - either of these phases produces a zero output from a phase detector (See Figure Five.)

For this to occur, the range to the target must be an odd multiple of λ/8 (so that the radar waves travel an odd multiple of λ/4 - to give the 90° or 270° phase) and the radial velocity of the target must produce a Doppler shift that is half the PRF, equal to the PRF, 1.5 times the PRF, etc. When this occurs then the output from the phase detector is zero for every pulse.

A target that fits the above category is invisible to the radar. As a target manoeuvres, it could fade in and out of view to the radar if, by chance, it moves as described in the previous paragraph. This is illustrated in Figure Eleven (b) where the pulses arrive at the radar very close to the zero point of the Doppler Waveform.

For our example radar, with a wavelength of 10 cm and a PRF of 10 kHz then blind phase may occur if the target has a radial velocity of 250 ms^{-1} – quite likely for an aeroplane.

Blind phase may also occur when the target moves at multiples of that speed: 500 ms^{-1}, 750 ms^{-1}, 1 000 ms^{-1}, etc.

The solution to blind phase is to use two parallel channels at the phase-detector stage radar receiver. One channel passes through as already described but the second one receives the signal from the COHO via a circuit that changes its phase by 90°. The outputs from the two channels can be squared, added and square-

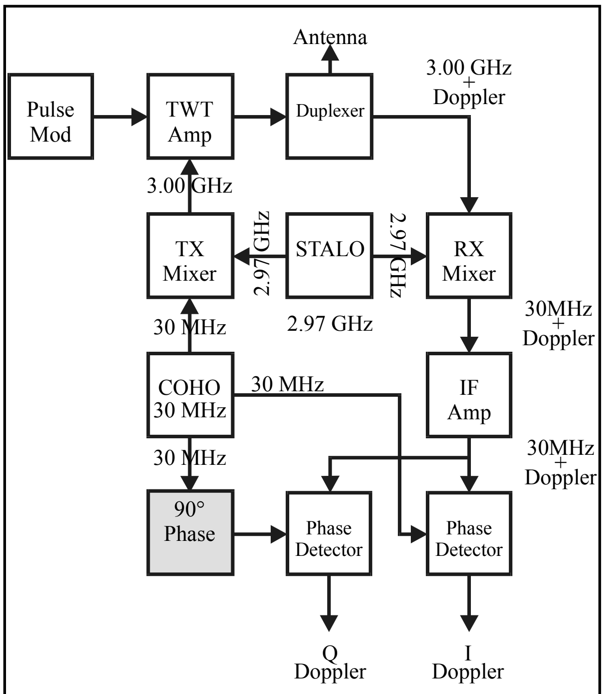


Figure 10: Block Diagram of Part of a Pulsed Doppler Radar System with In-Phase & Quadrature Channels

rooted to give the overall output (Pythagoras - right-angled triangle). One channel is called the In-Phase, or 'I', channel, whilst the second channel, one-quarter of a cycle out of phase, is called the Quadrature, or 'Q', channel.

Using 'I' and 'Q' channels, at 90° to each other ensures that should one channel encounter a blind phase then the other channel will be able to detect the echo. In fact, if one channel has a blind phase of 90° (like Figure Five) then the other must have a phase of either Zero° (like Figure Three) or 180° (like Figure Four). Either way, the other channel is unlikely to miss the target.

The formula for calculating the speeds, $V_{bp,}$ at which blind phase might occur is:

$$V_{bp} = \frac{n \times \lambda \times PRF}{4} \qquad \text{'n' is a whole number}$$

Note that blind phase requires that the range has a specific value, an odd-number of $\lambda/8$ metres, in addition to the radial velocity requirement.

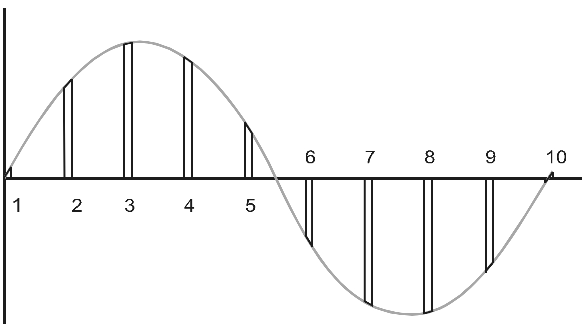


Figure 11 (a): 'Normal' Doppler Waveform for Radial Velocity of 50 ms^{-1}. F_d = 1 kHz, PRF = 10 kHz

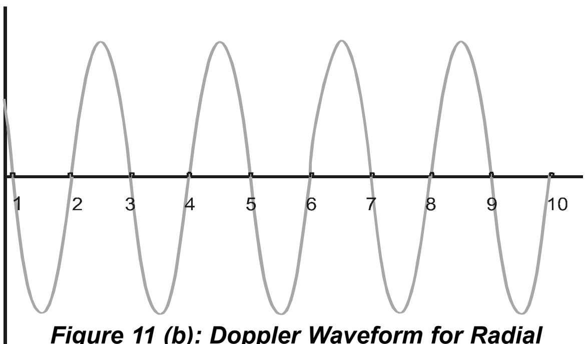


Figure 11 (b): Doppler Waveform for Radial Velocity of 250 ms^{-1}. F_d = 5 kHz, PRF = 10 kHz at Blind Phase

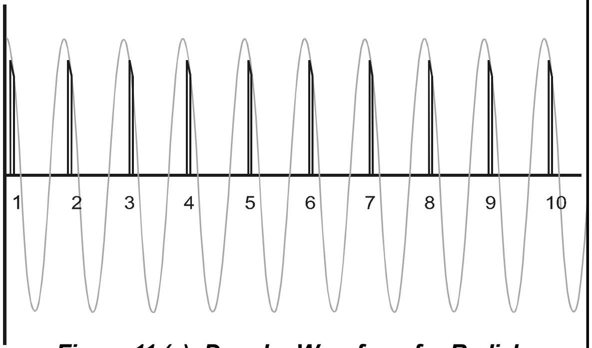


Figure 11 (c): Doppler Waveform for Radial Velocity of 500 ms^{-1}. F_d = 10 kHz, PRF = 10 kHz at Blind Speed

BLIND SPEEDS

In Figure Two, the tops of the processed radar echoes trace out the Doppler Waveform. The radar echoes occur at the PRF of the radar whilst the Doppler waveform is produced by the motion of the target. The two are independent and can, therefore, be of any size, shape and position. The previous section described 'Blind Phase' where the conditions are such that the echoes back from the target arrive during the time when the Doppler waveform crosses zero. This makes them invisible to the radar. A blind speed occurs when the Doppler frequency is an exact multiple of the PRF.

At a blind speed, the radar echoes arrive at exactly the same part of the Doppler waveform every time. Each phase-detected pulse is the same height as the next - it looks like clutter and is rejected.

Figure Eleven (a) indicates the 'ideal' Doppler processing where the group of pulses traces out the Doppler waveform. Here, the PRF was 10 kHz and the Doppler shift was 1 kHz – the pulses easily trace out the Doppler waveform.

However, if the target were approaching the same radar at 500 ms^{-1} then the Doppler shift would be ten times greater, as shown in Figure Eleven (c). At this speed, the Doppler shift is 10 kHz and it equals the PRF. You can see from the figure that the group of echoes no longer traces out the Doppler waveform. In fact, all the pulses are the same height and would be rejected as stationary clutter. Unlike blind phase, where the range had to be an odd number of $\lambda/8$, this can occur at any range.

The radar is blind to a target that has a radial velocity of 500 ms^{-1} – when the Doppler shift $f_d = 2\ v_r/\lambda$ is equal to the PRF. The same effect will occur when the radial velocity is any whole-number multiple of this velocity. This gives us a series of blind speeds of 500 ms^{-1}, 1 000 ms^{-1}, 1 500 ms^{-1}, etc.

Each blind speed is double a speed at which blind phase can occur. The formula for blind speed, v_{bs}, is:

$$V_{bs} = \frac{n \times \lambda \times PRF}{2} \qquad \text{'n' is a whole number}$$

Remember that these are radial velocities. Radial velocity is equal to: target velocity × Cos θ - where 'θ' is the angle between the track of the target and the line if sight from the radar). This means that the target's actual speed need not be equal to a blind speed - so a manoeuvring target, where the angle θ is varying rapidly, could fail to be seen by the radar each time its radial velocity happened to match a blind speed.

Remember: the first blind speed is when the Doppler shift is equal to the PRF of the radar. Other blind speeds occur at whole-number multiples of the first blind speed. As before, the actual speed of the target does not have to equal a blind speed - blind speeds occur when the radial velocity (V × Cos θ) equals a blind speed. A manœuvring target could pass through a blind speed as it changes course.

DOPPLER AMBIGUITIES

The Doppler shift causes a series of radar pulses to trace out the Doppler waveform - when they are coherently detected. This gives a series of 'samples' of the Doppler waveform that can be used to re-construct the waveform. The process by which this is done is called a 'Fast Fourier Transform'. This is a complex series of calculations, performed by a computer and the result is a spectrum of the signal - a list of the frequencies that it contains. Those frequencies are the Doppler shift(s) in the echo.

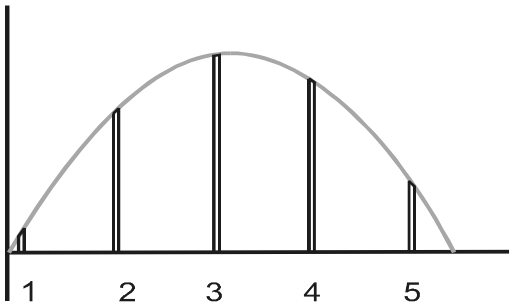


Figure 12 (a): Lowest frequency Doppler Waveform for Radial Velocity of 50 ms^{-1}. F_d = 1 kHz, PRF = 10 kHz

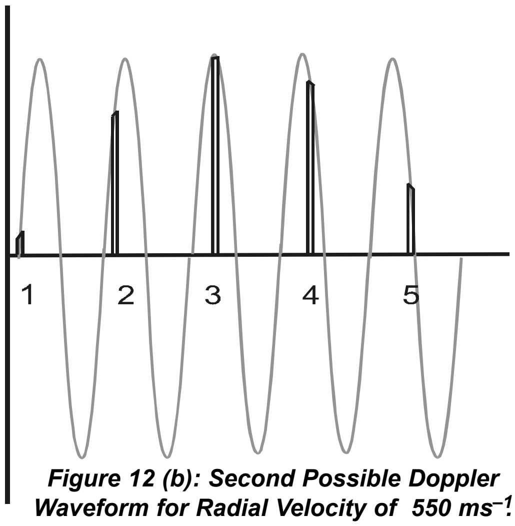


Figure 12 (b): Second Possible Doppler Waveform for Radial Velocity of 550 ms^{-1}. F_d = 11 kHz, PRF = 10 kHz

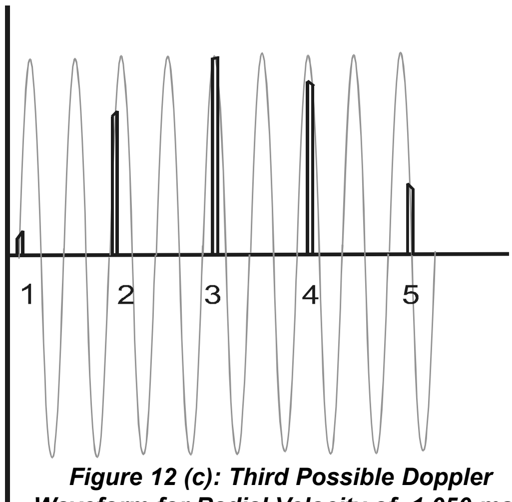


Figure 12 (c): Third Possible Doppler Waveform for Radial Velocity of 1 050 ms^{-1}. F_d = 21 kHz, PRF = 10 kHz

However, for any given set of points at a constant frequency (like our radar pulses), there is an infinite number of sine waves that can fit them. This is illustrated in Figure Twelve, where just three are shown, as follows:

- Figure Twelve (a) shows the 'obvious' way to fit a sine wave through the tops of the radar echoes. This results in a Doppler frequency of 1 kHz and a radial velocity of 50 ms^{-1}.
- Figure Twelve (b) shows a Doppler frequency of 11 kHz (1 kHz plus the PRF – 10 kHz – of the radar). This sine wave fits the required pattern of echoes just as well as the 1 kHz did. This frequency corresponds to a radial velocity of 550 ms^{-1}, which is bigger than the previous velocity by the value of the first blind speed.
- Figure Twelve (c) shows a Doppler frequency of 21 kHz (1 kHz plus twice the PRF). Again, this frequency fits the pattern and corresponds to a radial velocity of 1 050 ms^{-1}. This is equal to the second blind speed plus the 'apparent' speed.

There is an infinite number of possible radial velocities that will fit any chosen pattern of echoes. Therefore, it is true to say that for this radar, an indicated radial velocity of 50 ms^{-1} could equally well be 550 ms^{-1}, 1 050 ms^{-1}, etc. There is no way of telling from the waveforms that we have used. All Doppler measurements such as this must produce a series of possible radial velocities and the true radial velocity could equally well be any of them.

In the case of this radar being used to measure the velocity of a car, or other ground-based vehicle, then we could use common sense to declare the radial velocity to be 50 ms^{-1} (because cars do not travel at 550 ms^{-1} or Mach 1.6) and reject the other alternatives. In the case of a fighter aircraft this assumption cannot be made.

The formulae for the possible radial velocities of a target, taking into account this Doppler Ambiguity is as follows:

$$V_r = \text{Apparent } V_r + n \times V_{bs}$$

$$V_r = \text{Apparent } V_r + \frac{n \times \lambda \times PRF}{2}$$

Blind Speed: this is a useful parameter to know because its value affects many features of the performance of the radar:

- The first blind speed occurs when the Doppler shift equals the PRF. Doppler radars usually have very high PRFs to make the blind speed as high as possible.
- A target whose radial velocity equals a whole-number multiples of the blind speed appears to have a radial velocity of zero and are cannot easily be distinguished from clutter.

- The blind phase occurs at velocities that are odd-numbered multiples of half of the blind speed. E.g. 0.5, 1.5, 2.5, 3.5, etc times the blind speed.
- All indicated radial velocities could be bigger than the the indicated figure by any whole-number multiple of the blind speed.

The consequences of this, for a radar with a first blind speed of, say, 150 ms^{-1} would be:

- Targets whose radial velocity was 75 ms^{-1}, 225 ms^{-1}, 275 ms^{-1}, etc., would sometimes produce echoes with blind phase and this would cause them to be indetectable by the radar for some of the time.
- Targets whose radial velocity was 150 ms^{-1}, 300 ms^{-1}, 450 ms^{-1}, etc., would always produce echoes at a blind speed and this would cause them to be indetectable by the radar for all of the time.
- Any actual radial velocity could be bigger than the indicated figure by 150 ms^{-1}, 300 ms^{-1}, 450 ms^{-1}, etc., because of the Doppler ambiguity.

[Note that CW radars, such as MVMD and police speed traps, do not have blind speeds and ambiguities like those described here for pulsed Doppler radars. The blind speed is an inevitable consequence of trying to reconstitute a Doppler shift from a series of samples that were obtained with each radar pulse. CW radars do not use pulses so they do not take samples of the Doppler waveform. However, the limitations describes above for the observation time do apply to CW radars.]

Example:

A radar using a frequency of 5 GHz and a PRF of 2 kHz detects a target with an apparent radial velocity of 25 ms^{-1}. What are the possible radial velocities of this target?

(1) The first blind speed is $\lambda \times PRF/2 = 60\ ms^{-1}$.

(2) The true radial velocity of the target could be the indicated value plus any whole-number multiple of 60 ms^{-1}. Possible values: 25 ms^{-1}, 85 ms^{-1}, 145 ms^{-1}, 205 ms^{-1}, etc. This measurement cannot tell which is correct.

RESOLVING DOPPLER AMBIGUITIES

The practical solution to determining the target's radial velocity is to use more than one PRF and to change the PRF in the time that the radar scans the target. This will give two sets of possible radial velocities and the value that appears in both series must be the true answer.

For example, the calculation above showed that when a radar with a frequency of 5 GHz and a PRF of 2 kHz scans a target then it determines the radial velocity to be one of the series: 25, 85, 145, 205 ms^{-1}, etc. Suppose that radar now switches to a PRF of 2.2 kHz – and the radar indicates the target's radial velocity to be 13 ms^{-1}. The blind speed for this PRF is 72 ms^{-1}, so the possible values are now: 13, 85, 157, 229 ms^{-1}, etc. Clearly, the true radial velocity must be 85 ms^{-1} because it appears in both lists of possible answers.

In practice, there might be many more changes of PRF to ensure that the ambiguity is resolved. Changing PRFs also resolves range ambiguities (see another BST handout for details) and is used to increase immunity to enemy counter measures

RADIAL VELOCITY RANGES

When the Doppler waveform is analysed to produce the Doppler frequencies that it contains (FFT) then the result appears in one of a series of velocity bands rather than an actual velocity. The number of bands is equal to the number of pulses used in the measurement. The width of each band is calculated by dividing the first blind speed by the number of pulses in the group.

In our example radar, which uses groups of ten pulses, there will be ten velocity bands (called velocity bins) and each target will be allocated to one of them. The first blind speed was 500 ms^{-1}, so each band will span 50 ms^{-1}. In other words, targets will be classified on a ten-point scale from Zero to 500 ms^{-1}.

This is a consequence of the influence of t_o, the observation time on the velocity resolution, Δv_r. The first blind speed is when the Doppler shift equals the PRF which is the same as when the periodic time of the Doppler shift equals the pulse interval of the radar. If we use a group of 'n' pulses then we are using an observation time of 'n' pulse intervals – so the smallest Doppler shift that we can identify is 'n' times less than the blind speed.

DOPPLER PROCESSING

Figure B1 shows a schematic diagram of the sort of processing that takes place in a digital, pulsed Doppler radar receiver. The signal from the phase detector enters at the top, right-hand corner of the diagram. This signal will resemble that shown in Figure B2, which is identical to Figure Two - that we have already considered.

The PRF of our ‘typical’ radar is 10 kHz - this means that it emits one pulse every 100 µs. The maximum unambiguous range of this radar is 15 km (150 metres for each micro-second of pulse interval). The output from the phase detector is connected to a ‘Sample and Hold’ circuit that is controlled by a 100 kHz signal (Point ‘C’ on the diagram). This will operate once every ten micro-seconds, or ten times during the pulse interval. Each time it operates, the sample is passed to the Analogue to Digital Converter. This acts as the range gate controller for this radar. The first sample & hold will operate between 10 µs and 20 µs after the transmission of the pulse - so it will detect echoes from targets between 1.5 km and 3 km from the radar.

The A to D converter is also controlled by the 100 kHz signal, at Point ‘B’. It converts the phase-detected signal to digital and, because it runs at 100 kHz (one conversion every ten micro-second)s, it will produce ten samples of the received radar signal each time a pulse is transmitted. The digital output is passed to the end of a row of computer memory (RAM). This is configured to shift the data one place to the left when triggered by a signal at Point ‘A’. It is called a ‘Shift Register’ because the data can be shifted left or right.

There should be ten memory cells (range gates) across each row, in the centre of the diagram. To keep the diagram uncluttered, some cells have not been drawn. There will also be many more such memory cells in which data from other groups of pulses is stored.

The data at the right-hand end of the shift register arrive at the rate of one every ten micro-seconds, from the A to D converter. At Point ‘A’, the 100 kHz clock signal causes the data to shift left by one step each time. After 100 µs, there will be 10 numbers in the shift register, each corresponding to the echo that was received during the time periods 10 - 20 µs, 20 - 30 µs, 30 - 40 µs, etc., after the pulse was transmitted. These correspond to the radar echoes that were received from ranges of 1500 - 3000 m, 3000 - 4500 m, 4500 - 6000 m etc., up the the maximum range of 15 km. (There is no data from ranges zero to 1500 m because the pulse length of 10 µs means that the minimum range of this radar is 1500 m. No echoes can be received from less than 1500 m because the transmitter is still sending out the radar pulse.)

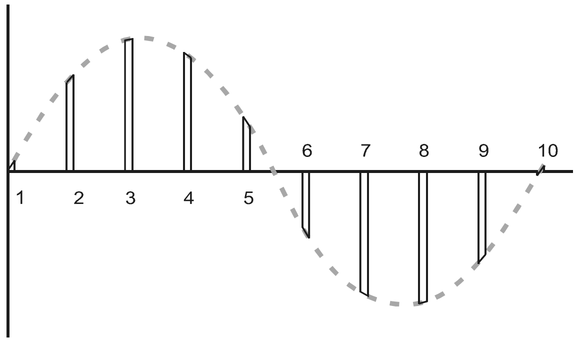


Figure B2: A Group of Pulses that Follows the Doppler Waveform of a Moving Target

These memory locations, where the echoes are stored for each time interval are called range bins - they are computer memory where the data from each different range is dumped for analysis.

After 100 µs has elapsed, the 10 kHz signal operates and the data in the shift register are moved down into the top row of the main store. At the same time, the data already in the store move down by one row and the bottom row of data is discarded.

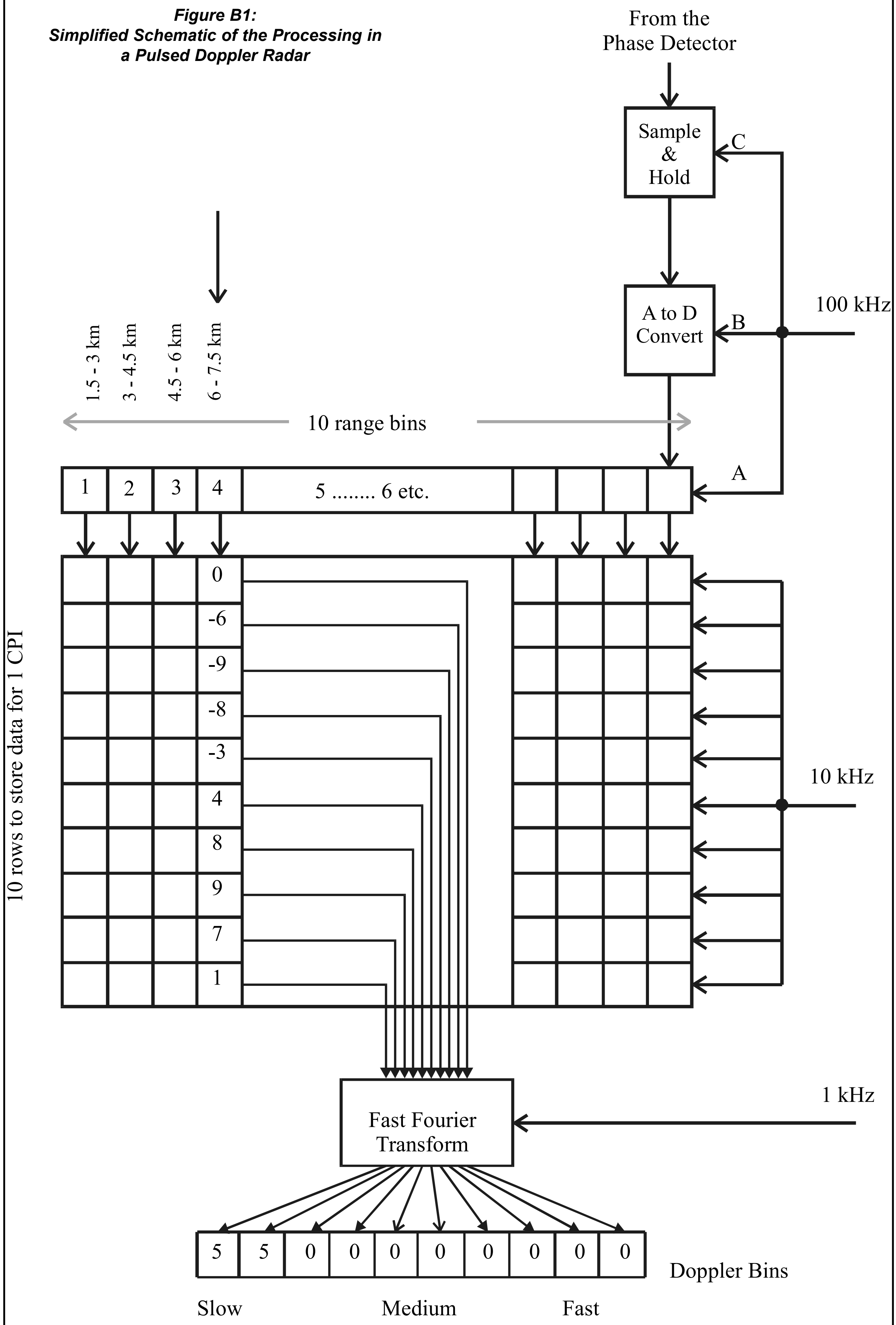
All the above has happened in 100 µs - before the next radar pulse is transmitted. After ten pulses have been transmitted, the ten rows of memory store will be loaded with data and the 1 kHz signal actuates the Fast Fourier Transform computation. There are ten of these to be calculated, one for each range bin.

So far, we have collected data from one azimuth and ten pulses on that azimuth. Already we have collected 100 items of data from that azimuth. Whilst the analysis proceeds, the radar continues to transmit, perhaps using a different PRF, and the radar antenna continues to turn. Our example radar emits ten thousand pulses per second and each pulse generates one hundred items of data. That might seem a lot of data to a human but it is only a few Mega-Bytes and not a great deal to a modern computer.

Processing the Doppler: If our target were at a range of about 7 km then its echo would arrive back about 47 µs after the radar pulse were transmitted and it would end up in Range Bin 4. After ten pulses, the column of numbers in Range Bin 4 would be the echoes from our target. These numbers have been included in the diagram. If you compare these numbers with the heights of the echoes shown in Figure B1 then you should be able to see that they are the same. The number at the bottom of the column corresponds with pulse number one in Figure B2 and the pulse at the top of the column, the last pulse of the set, corresponds with pulse number ten.

These numbers are fed into the Fast Fourier Transform circuit which, because it has ten inputs (samples of the Doppler waveform) is able to analyse them into ten velocity ranges (bins). In this radar, the first blind speed is 500 ms^{-1} so each velocity bin spans 50 ms^{-1}. Our target, with a radial velocity of 50 ms^{-1}, falls partly in the 1 - 50 and partly in the 50 - 100 bins and this is indicated by the number ‘5’ in each bin. After each group of radar pulses, this analysis has to be carried out for each of the ten columns of data. This occurs in our example radar every ten pulses, at the rate of one kilo-Hertz.

Figure B1:
Simplified Schematic of the Processing in a Pulsed Doppler Radar



SUMMARY OF TERMS AND FORMULAE

Doppler Shift

The Doppler Shift, f_d, for a radial velocity v_r, is given by:

$$\mathbf{f_d = \frac{2\,v_r}{\lambda} \quad or \quad \frac{2\,v_r\,f_t}{c}}$$

Where λ is the wavelength and f_t the frequency of operation of the radar.

Doppler Resolution

The ability to distinguish between targets that have similar radial velocities is called Doppler Resolution. The smallest change in the Doppler frequency that the radar can detect is determined by the observation time, t_o.
The Doppler Resolution, Δf_d, is given by:

$$\Delta f = \frac{1}{t_0}$$

The velocity resolution, Δv_r - the smallest change in radial velocity that can be determined, corresponds to the Doppler resolution:

$$\Delta v_r = \frac{\lambda}{2\,t_0}$$

This is also the same as dividing the first blind speed (V_B) by the number of radar pulses (n) that are used for each Doppler measurement.

$$\Delta v_r = \frac{V_B}{n}$$

The observation time, t_o, is the time taken by the radar to collect the Doppler information. In a pulse radar, with pulse interval T (T = 1/PRF), the observation time is:

$$t_0 = n\,T$$

Where 'n' is the number of pulses in the coherent processing interval.

Blind Speeds

A target with a Doppler shift that is equal to any whole-number multiple of the PRF cannot be seen by the radar. These are called blind speeds and are given by the following formula:

$$V_{bs} = \frac{n \times \lambda \times PRF}{2} \qquad \text{'n' is a whole number}$$

Any radial velocity measured by the radar could be the measured speed plus any blind speed.

Blind Phase

A target travelling with a radial velocity that equals any odd multiple of half of the blind speed can produce echoes at a blind phase - these echoes arrive at the zero points of the Doppler waveform. The speed for a blind phase is:

$$V_{bp} = \frac{n \times \lambda \times PRF}{4} \qquad \text{'n' is an odd number}$$

SELF-TEST QUESTIONS

1. A radar uses a frequency of 10 GHz to monitor the radial velocity of a target. When the target moves towards the radar at 125 ms^{-1} then the Doppler shift on the radar echo will be approximately:

a. 8.3 kHz
b. 4.2 kHz
c. 1.25 kHz
d. 10 kHz

2. The time duration of a group of identical radar pulses, used to determine Doppler shift, is called a:

a. second trace echo
b. coherent processing interval
c. pulse duration
d. pulse interval

3. When echoes from a moving target are detected using a non-coherent detector then the echoes:

a. trace out the Doppler waveform.
b. indicate the blind phase
c. reveal moving targets
d. have no useful Doppler information

4. When echoes are coherently detected then it is necessary to give the detector a sample of the:

a. amplitude of the radar carrier wave.
b. output from the travelling wave tube
c. coherent oscillator
d. stable local oscillator.

5. When two signals in anti-phase are fed into a phase detector then the output will be:

a. positive all the time
b. negative all the time
c. alternating between positive and negative
d. always zero.

6. A delay-line canceller is used to reduce the effects of:

a. noise
b. Doppler shift
c. blind speeds
d. stationary clutter

6. A radar needs to distinguish between targets whose Doppler shifts differ by 500 Hz. This radar would have to collect data from the target for a minimum period of:

a. 500 ms
b. 500 μs
c. 2 ms
d. 2 μs

7. A radar with a PRF of 8 kHz uses 8 pulses in a coherent processing interval (CPI). The duration of its CPI will be:

a. 64 μs
b. 1 μs
c. 8 ms
d. 1 ms

8. A pulsed Doppler radar with a PRF of 4 kHz might encounter blind phase when the Doppler shift of the target is:

a. 4 kHz
b. 1 kHz
c. 6 kHz
d. 8 kHz

9. To counter the problems caused by blind phase, a pulsed Doppler radar would use:

a. coherent detection
b. fast Fourier transforms
c. a delay-line canceller
d. in-phase and quadrature processing channels.

10. A pulsed Doppler radar operates at 6 GHz with a PRF of 5 kHz. Its first blind speed will be:

a. 30 ms^{-1}.
b. 100 ms^{-1}.
c. 125 ms^{-1}.
d. 62.5 ms^{-1}.

Answers

1. $fd = 2v/\lambda = 2 \times 125 \div 0.03 = 8.333$ kHz (a)
2. Coherent Processing Interval (b)
3. Must use a coherent detector for Doppler (d)
4. Need the coherent oscillator to detect (c)
5. They are multiplied - always one + & one - (b)
6. Delay line canceller reduces static clutter (d)
7. $t_o = 1/\Delta f = 1/500$ 2 ms (c)
8. Blind phase at odd-numbers of half BS (c)
9. I & Q channels avoid blind phase (d)
10. BS when f_d = PRF. 5 kHz = $2v/\lambda$ = 125 ms^{-1} (c)

SELF-TEST QUESTIONS

11. A pulsed Doppler radar must use groups of pulses in a CPI in order to:

   a. produce accurate range measurements.
   b. eliminate blind speeds.
   c. observe the target over a longer period of time.
   d. eliminate blind phase.

12. A radar with a first blind speed of 500 ms^{-1} detects a target with an apparent radial velocity of 100 ms^{-1}. The radial velocity of the target might also be:

   a. 200 ms^{-1}.
   b. 300 ms^{-1}.
   c. 400 ms^{-1}.
   d. 600 ms^{-1}.

13. When the radial velocity of a target is equal to a blind speed then the radar echoes from the target emerge from the phase detector:

   a. at the zero crossing point of the Doppler waveform.
   b. with a constant height, like clutter.
   c. at the peak of the Doppler waveform.
   d. with a varying height, following the Doppler waveform.

14. The first blind speed of a pulsed Doppler radar occurs when the radial velocity of the target causes:

   a. a Doppler shift equal to the pulse duration.
   b. echoes to return in anti-phase to the coherent oscillator.
   c. a Doppler shift equal to the PRF.
   d. a Doppler shift equal to half the PRF.

15. A pulsed Doppler radar with a first blind speed of 200 ms^{-1} and a CPI spanning twenty pulses can measure radial velocities to a maximum accuracy of:

   a. 10 ms^{-1}.
   b. 20 ms^{-1}.
   c. 40 ms^{-1}.
   d. 80 ms^{-1}.

16. A pulsed Doppler radar operates at two, different PRFs to give two, different blind speeds. A target has an indicated radial velocity of 100 ms^{-1} when the blind speed is 300 ms^{-1} and 40 ms^{-1} when the blind speed is 360 ms^{-1}. The true radial velocity of the target is:

   a. 140 ms^{-1}.
   b. 60 ms^{-1}.
   c. 400 ms^{-1}.
   d. 460 ms^{-1}.

17. A pulsed Doppler radar would switch between several different PRFs in order to:

   a. eliminate ambiguities in radial velocity.
   b. make more accurate measurements of velocity.
   c. reduce the observation time, t_o.
   d. determine the blind speed.

18. A pulsed Doppler radar operates at 7.5 GHz with PRFs of 10 kHz and 12 kHz. Its first blind speeds for each PRF are:

   a. 400 and 480 ms^{-1}.
   b. 200 and 240 ms^{-1}.
   c. 100 and 120 ms^{-1}.
   d. 75 and 90 ms^{-1}.

19. The Coherent Oscillator (COHO) must:

   a. change frequency every pulse.
   b. oscillate at the transmitted frequency.
   c. maintain a precise frequency.
   d. oscillate at the PRF

Answers

11. Longer time = more accurate Doppler (c)
12. Add 1 × Blind Speed. 100 + 500 (d)
13. Constant height echoes at blind speed (b)
14. Doppler shift = PRF at 1st blind speed (c)
15. Divide 1st blind speed by 'n', 'n'=20 (a)
16. Series (1) is 100,400,700. Series (2) is 40,400, 760. Common value is 400. (c)
17. Switch PRF to resolve ambiguity (see Q16) (a)
18. Blind speed = nλPRF/2 (b)
19. Coherent Osc. keeps constant freq. (c)

Teaching Objectives		Comments
J.03.01 Calculate Doppler shifts and radial velocities		
J.03.01.01	Use either $F_d = 2\ v_r\ f/c$ or $F_d = 2\ v_r/\lambda$ to calculate the Doppler Shift	v_r is the radial velocity.
J.03.01.02	Use three-dimensional geometry to calculate the radial velocity of a target.	
J.03.02 Describe the link between observation time and Doppler resolution		
J.03.02.01	Describe Doppler resolution, Δv, as the smallest difference in radial velocities that can be recognised.	E.g. distinguishing a radial velocity of 200 ms^{-1} from 220 ms^{-1} needs resolution of 20 ms^{-1}.
J.03.02.02	Convert a Doppler resolution in radial velocity into a resolution in Doppler shift, Δf.	Using $\Delta f = 2\ \Delta v/\lambda$
J.03.02.03	State that the time for which the target must be observed to achieve Δv is $1/\Delta f$	
J.03.02.04	State that Doppler measurements require many pulses on the target to achieve the required observation time.	
J.03.03 Describe the spectrum of a train of identical pulses.		
J.03.03.01	Describe the terms: pulse duration, pulse interval and pulse repetition frequency.	τ, T and prf, respectively.
J.03.03.02	Sketch the spectrum for 'n' pulses.	Spectrum line width = $1/(n\tau)$, effective bandwidth = $1/2\tau$, line spacing = 1/T
J.03.03.03	State that the width of a spectral line is $1/(n\tau)$.	Leads to Coherent Processing Intervals.
J.03.03.04	State that the effective bandwidth of the pulse train is $1/2\tau$.	Based on half-power.
J.03.03.05	State that the line spacing is 1/T	
J.03.03.06	Sketch the spectrum for a radar carrier modulated by a pulse train.	As above, but with two sidebands.
J.03.04 Describe the effects on non-coherent detection on a Doppler-shifted echo		
J.03.04.01	Describe non-coherent detection as a means of extracting the magnitude of the echo – regardless of its phase.	E.g. simple diode, capacitor & resistor envelope detector.
J.03.04.02	State that the Doppler shift makes no difference to the output of the non-coherent detector.	The observation time is too short.
J.03.04.03	Recognise the waveforms in the various stages of a non-coherent detector.	

Teaching Objectives		Comments
J.03.05 Describe the process of coherent detection of a Doppler-shifted echo		
J.03.05.01	State that the Doppler shift is extracted by comparing the echo with the original signal over a period of time.	
J.03.05.02	Recognise the block diagram of a coherent system.	
J.03.05.03	State that a coherent detector multiplies the original carrier by the echo.	Demo using Excel. Output depends on the Cosine of the phase difference.
J.03.05.04	Describe the changes in the phase relationship between transmitted and received waves for a target at different ranges.	
J.03.05.05	Describe the changes in phase relationship between transmitted and received waves for a moving target.	
J.03.06 Describe the process of clutter rejection		
J.03.06.01	Describe clutter as echoes from any unwanted targets.	Include: terrain, buildings, trees and precipitation.
J.03.06.02	State that the Doppler shift from stationary clutter is practically zero and that the coherently-detected returns will vary little from pulse to pulse.	
J.03.06.03	Describe the operation of a delay-line canceller as a clutter-rejection filter.	
J.03.07 Describe the problems of blind phase, blind speed and Doppler ambiguities		
J.03.07.01	Identify blind phase.	Path length is an odd-multiple of $\lambda/4$ and f_d is odd-multiple of ½ prf.
J.03.07.02	Describe the use of I & Q channels to eliminate blind phase.	
J.03.07.03	Identify blind speeds.	Any path length and f_d = integer multiple of prf.
J.03.07.04	Describe the ambiguous nature of pulsed Doppler measurements.	Speed = indicated speed + integer multiple of blind speed.
J.03.07.05	Describe how speed ambiguities and blind speeds are resolved by scheduled changes in prf.	
J.03.08 Recognise the components of a block diagram showing Doppler processing		
J.03.08.01	Identify each element of the block diagram of pulsed Doppler processing schematic diagram.	
J.03.08.02	Describe the function of each block	