DWR The Radar Range Equation



ROYAL SCHOOL OF ARTILLERY

BASIC SCIENCE & TECHNOLOGY SECTION



The Radar Range Equation

INTRODUCTION

The primary job of an air defence radar is to detect and extract information from the echoes that it receives from airborne targets. In order to achieve this, the radar carries the following tasks:

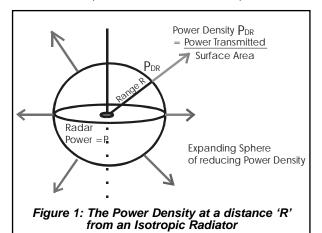
- the radar generates a microwave signal (e.g. 3 kW at a frequency of 3 GHz or a wavelength of 10 cm).
- the signal is transmitted via an antenna.
- the radar determines how long it takes for the signal to travel out to the target and for the reflection to travel back (each µs delay corresponds to 150 m of range to the target.)
- the same antenna collects the echo and presents it to the receiver for detection (the echo might be only 100 nW).
- the receiver detects and analyses the echo.

The Radar Range Equation is a mathematical analysis of what happens to the strength of the radar signal during the above processes. This analysis is important because its results reveal much about the factors that affect the performance of a radar system. The equation usually appears in a form that can be used to predict the maximum range at which the radar would be able to detect a particular target.

This handout analyses each of the five processes, above, to determine their effect on the overall performance of the radar.

TRANSMITTER POWER

The radar transmitter usually contains a powerful amplifier device such as a Klystron (FSB2) or Travelling-Wave Tube (FSC). These devices can produce radar signals of very high powers (e.g. many kW). However, for a portable or semi-portable radar then it might not be practical to use enormous power as this causes other problems such as the requirement for



cooling and electrical power. It is generally not necessary to operate a medium-range, radar system at more than a few kilo-Watts of peak power. (Remember that the average power is: [peak-power] × [duty-cycle] so a radar with a peak power of 3 kW and a duty cycle of 10% would have an average power of only 300 W.)

ANTENNA

The task of the transmitting antenna is to transfer the outgoing signal from the waveguide or co-axial cable inside the transmitter to the 'free space' outside. The antenna also focuses the outgoing signal into a beam and this has the effect of concentrating the signal in a particular direction. The focusing effect of the antenna increases the power of the signal in the direction of the beam and also decreases its power in all other directions. The effectiveness of the antenna is measured using a factor called its 'gain' and this is explained below:

Isotropic Antenna: the simplest antenna is one that does not form any sort of beam - instead, it radiates power equally in all directions. This is called an 'isotropic antenna'. When we measure the gain of an antenna then we are comparing its performance against that of an isotropic antenna, as follows:

- The power that is transmitted from an isotropic source emerges from a point and spreads out (at the speed of light) evenly through the surrounding space, as if on the surface of an expanding balloon. This is illustrated in Figure One.
- The surface area of the expanding sphere is uniformly illuminated by the isotropic transmission. The number of Watts reaching each square metre of this expanding sphere is called the Power Density. it is measured in Wm⁻².
- As the signal travels further away from the antenna then the surface area of the sphere gets bigger; the power is spread out over a larger area and the Power Density decreases.

The power density at any range will be the power transmitted from the centre of the sphere, P_t , divided by the area over which it is spread. The surface area of a sphere is given by $4\pi R^2$, where R is the range of interest. Thus the power density at range R is:-

$$P_{DR} = \frac{P_t}{4\pi R^2} \quad W \, m^{-2}$$

This is illustrated in Figure One. It shows that the power density reduces as you get further away from the

antenna because the power that was transmitted is spreading more and more thinly as the range increases.

Example: An isotropic antenna radiates 3 kW. The power density at a range of 1 000 m would be:

$$P_{DR} = \frac{P_t}{4\pi R^2} W m^{-2}$$

$$= \frac{3000}{4 \times \pi \times 1000^2} W m^{-2}$$

$$P_{DR} = 239 \,\mu\text{W} m^{-2}$$

Thus, one kilo-metre away from an isotropic radiator that radiates 3 kW, the power density is 239 μW per square metre.

EFFECTS OF TRANSMITTER AERIAL GAIN

The concept of an isotropic radiator is largely theoretical and real aerial systems have some sort of directional properties. The gain (G) of any transmitter aerial system is defined as:

G = Power Density produced by antenna
Power Density of isotropic source

This represents the factor of increase in the power delivered to a distant point when the antenna is used, compared to isotropic radiation.

In radar systems the transmitted power is deliberately formed into a narrow beam to concentrate it in a direction of interest. This means that there is an increase in the power density along the line in which it is pointing. This is illustrated in Figure Two.

For example, an antenna system with a gain of 100 would transmit 100 times the power of an isotropic source into the centre of its beam. Note that no more power is made because much less power goes in the other directions.

When the gain of the antenna is taken into account then power density formula for $P_{\rm DT}$, at range, R, becomes;

 P_{DT} = Power of isotropic radiator x Aerial gain

$$P_{DT} = P_{DR} \times G$$

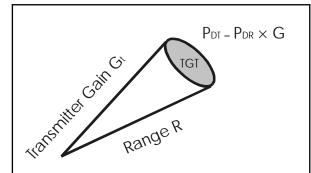


Figure 2: Power Density at the Target, depends on the Beam-Forming Properties of the Antenna

$$P_{DT} = \frac{P_t \times G}{4\pi R^2}$$

Example: An antenna with a gain of 100 radiates 3 kW. The power density at a range of 1 000 m would be:

$$P_{DT} = \frac{P_{t} \times G}{4\pi R^{2}} \quad W \, m^{-2}$$

$$= \frac{3000 \times 100}{4 \times \pi \times 1000^{2}} \quad W \, m^{-2}$$

$$P_{DT} = 23.9 \quad mW \, m^{-2}$$

Thus, one kilo-metre away from this antenna, the power density is 23.9 mW per square metre, one-hundred times more than the power that was produced by an isotropic radiator with the same power input.

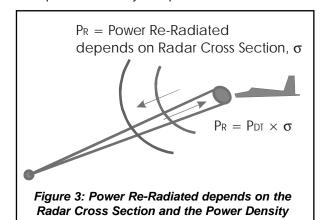
Inverse-Square Law: note that range is squared in the formula for power density. This means that if the range is doubled then the power density reduces by a factor four (two-squared). If the above calculations were repeated for a range of 10 km then the figures obtained would be one-hundred times smaller.

EFFECTS OF TARGET CHARACTERISTICS

The next factor to be considered in the design of a radar system is the target characteristics. Some targets are large with plenty of reflecting surface whereas others are small with little reflecting surface. As far as the radar is concerned there is only one point of interest and that is the amount of power that is reflected.

When the target passes through the beam of radar power then some of the energy will be intercepted by the target and re-radiated (this could be described as a reflection). The target can now be considered to be a secondary radiator of power. The power that the target re-radiates has all come from the radar and the amount that is re-radiated depends on the power density in the vicinity of the target and the dimensions of that part of the target that participates in the re-radiation.

The ability of a target to reflect radar power is not necessarily related to its actual size. For example, a small piece of metal (a conductor) will probably reflect much more radar power than a large piece of plastic (an insulator). Radar absorbent material (RAM) is used to coat parts of stealthly aeroplanes in order to reduce



radar reflections. Also, the shape of the material has an effect - corners make particularly good reflectors, for example.

Radar Cross Section: Radar Cross Section, σ , is a fictional area of a target intercepting a quantity of power which, when scattered equally in all directions produces an echo at the radar equal to that from the target. It has units in m^2 and it does not bear a simple relationship to its actual cross sectional area. It is also know as "equivalent echoing area" (EEA).

The *power density* multiplied by the *radar cross section* gives the number of Watts that is reflected or retransmitted by the target. This power forms the return signal from the target (the radar echo) - but it still has to travel back to the radar to be detected.

A real target is liable to provide different returns depending upon the angle from which it is viewed, ie, its aspect. As the reflected signal varies in magnitude the target is said to 'glint'. This effect can produce problems for equipments involved in tracking and is discussed in another handout entitled "Tracking Systems". For the discussion here glint is ignored.

If the target σ , is multiplied by the power density arriving at the target, P_{DT}, the power re-radiated by the target is found in Watts. ie,

$$(Wm^{-2}) \times m^2 = W$$

This is illustrated in Figure Three. Hence, the power reradiated by the target is:

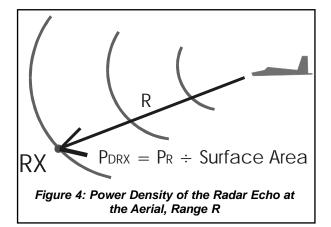
$$P_{R} = P_{DT} \times \sigma$$

$$P_{R} = P \times G \times \sigma W$$

$$4\pi R^{2}$$

Example: An antenna produces a power density of 23.9 mW m $^{-2}$ in the vicinity of a target of EEA 15 m 2 . The amount of power that would be re-radiated by the target would be:

$$P_R$$
 = Power Density × EEA
= 23.9 mW m⁻² × 15
= 359 mW.



POWER DENSITY AT THE RECEIVING AERIAL

When the reflected target signal reaches the radar receiver it will have a certain power density, depending upon the range from which it has travelled. The reflected power has emerged from the target on its own ever expanding bubble. There is no reason to suppose that this power will, magically, know that it is supposed to head towards the radar receiver - so we must treat the target as another isotropic radiator.

In view of the fact that EM waves travel at the velocity of light and that target speeds are substantially lower than that, then the target is considered to have remained stationary during the process of reflecting the radar signal. This means that the return distance is the same as the target range that was used for the outgoing signal. In this case the power density at the radar aerial, as illustrated at Figure Four, becomes:-

$$P_{DRX} = Power returned by Target \div 4\pi R^{2}$$

$$P_{DRX} = P_{R} \div 4\pi R^{2}$$

$$P_{DRX} = \frac{P \times G \times \sigma}{4\pi R^{2}} \div 4\pi R^{2}$$

$$P_{DRX} = \frac{P \times G \times \sigma}{4\pi R_{2} \times 4\pi R^{2}} \qquad (Wm^{-2})$$

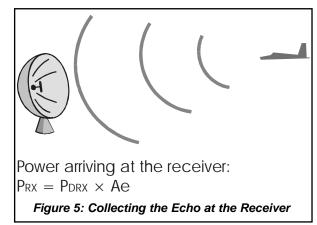
Example: A target re-radiates 359 mW when illuminated by a radar from a range of 1 km. The power density that this produces in the vicinity of the radar is:

$$P_{DRX}$$
 = Power returned by Target $\div 4\pi R^2$
 P_{DRX} = 359 mW $\div 4\pi 1000^2$
 P_{DRX} = 28.6 nWm⁻².

Thus, each square metre in the vicinity of the radar receives a power of 28.6 nW. This small amount of power has to be collected by the receiving antenna and sent to the receiver.

RECEIVER AERIAL PERFORMANCE

When the target echo arrives back at the receiver aerial, the most important thing to do is to 'catch' as much of that signal as is possible. In order to do that then the catching area must be as large as possible.



This implies that the biggest practicable aerial reflector should be used. This is shown in Figure Five.

In most pulsed radars, the receiving and transmitting aerials are one and the same but in CW radar they are usually separate. For the purposes of this illustration, pulse operation will be assumed and the same antenna is used for both transmitting and receiving.

Effective Aperture: The effective aperture (A_e) , is the area from which the receiving antenna collects power. The physical aperture of an antenna can be found by measuring its frontal area (e.g. a rectangular antenna that measures 1.2 m by 0.5 m would have a physical aperture of 0.6 m²). However, most antennae systems are only about 75% efficient so the effective aperture would be 75% of 0.6, or 0.45 m². [The figure of 75% is typical - but some antenna might be better. The value is usually found by testing the antenna, not from theoretical calculations.]

Power Collected: the power collected by the receiving antenna can be found by multiplying the power density at the aerial by the effective aperture of the antenna. This is the signal power that the antenna collects. and delivers to the receiver. Thus, aerial signal power at the receiver feeder (Watts) = power density of the returning signal (Wm⁻²) × effective aperture of the reflector (m²), giving:-

Receiver Signal Power:

$$P_{RX} = P_{DRX} \times A_{e}$$

Example: The power density in the vicinity of the radar is 28.6 nWm⁻² and the effective aperture of the antenna is 0.45 m². The power collected by the antenna will be:

$$P_{RX} = P_{DRX} \times A_{e}$$

= 28.6 nWm⁻² × 0.45 m²

This 15.1 nW is the power of the radar echo in the example illustrated. It is not a particularly large amount of power but well within the capability of a radar receiver to detect when the amount of noise is not too high.

The formula that describes the journey of the radar signal since it left the transmitter is now:

TABLE 1: RADAR PARAMETERS

R	= Range	(1 km)
P_{t}	= Peak Power of Radar	(3 kW)
G	= Gain of Antenna	(100)
P_{DR}	= Power Density at 1 km	(23.9 mWm ⁻²)
σ	= Radar Cross Section of Target	(15 m ²)
P_{r}	= Power re-radiated from the Tgt	(359 mW)
PDRX	= Power Density in echo	(28.6 nWm ⁻²)
Ae	= Effective Aerial Aperture	(0.45 m ²)
P_{RX}	= Power arriving at the receiver	(15.1 nW)

$$P_{RX} = \frac{P_{t} \times G \times \sigma \times A_{e}}{4\pi R^{2} \times 4\pi R^{2}}$$

and this represents the 15.1 nW that returns to the receiver from our example target. The figures that we used are listed in Table One.

GAIN AND APERTURE

There is a link between the gain of an antenna and its effective aperture. When an antenna has a large aperture then it also has a large gain. The link can also be written as an equation:

$$G = \frac{4\pi A_e}{\lambda^2}$$

Taking the previous equation, we can substitute for 'G' to get:

$$P_{RX} = P_t \times 4\pi A_e \times \sigma \times A_e$$

$$4\pi R^2 \times 4\pi R^2 \times \lambda^2$$

The ' 4π ' on the top can be cancelled with a ' 4π ' on the bottom, the two ' A_e ' factors combine to make A_e^2 , the two ' R^2 ' factors combine to make R^4 and the end result is:

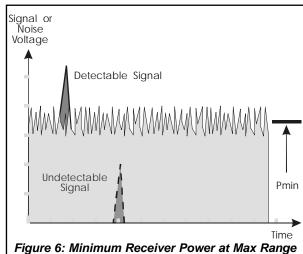
$$P_{RX} = \frac{P_{t} \times \sigma \times A_{e}^{2}}{4\pi \times R^{4} \times \lambda^{2}}$$

This equation predicts that the power received from a radar target depends on the inverse fourth power of the range. Two implications of this are below:

- If a target moves away from, say, 3 km to 6 km then the power of its radar echo reduces by a factor 2⁴ or sixteen times.
- To double the range of a radar system then you would need to increase the transmitter power by a factor sixteen.

MINIMUM RECEIVER POWER

To be detected by the radar receiver then the signal must be distinguishable from the noise. All electronic devices produce noise - most comes from the random



motion of electrons caused by thermal energy. A very weak signal can be obscured by noise and pass unnoticed. For the radar to detect a signal then it must be able to recognise it amongst the noise. This is illustrated, in simple form, in Figure Six.

The smallest signal that the radar receiver can detect is called the minimum receiver power and is given the symbol P_{min} . The significance of this figure is that it can be used to predict the maximum range (R_{max}) at which the radar can detect a target. This occurs when, for a target at the maximum range, the power in the echo just equals the minimum receiver power. In equation form, this is when when:

$$P_{min} = \frac{P_t \times (Ae)^2 \times \sigma}{4\pi \times \lambda^2 \times R_{max}^4}$$

MAXIMUM RANGE

The last equation can be transposed to make the subject radar range, R, in terms of the other factors, as follows:

$$R_{\text{max}}^{4} = \frac{P_{\text{t}} \times (Ae)^{2} \times \sigma}{4\pi \times \lambda^{2} \times P_{\text{min}}}$$

By taking the fourth-root of both sides then the final version of the radar range equation is obtained:

$$R_{\text{max}} = \sqrt[4]{\frac{P_{\text{t}} \times (A_{\text{e}})^2 \times \sigma}{4\pi \times P_{\text{min}} \times \lambda^2}}$$

The maximum range is largely determined by the power of the received signal compared to the power of the noise present in the receiver. A major feature of any receiver is its ability to extract the signal from the noise. This is illustrated in Figure Six where one signal (on the right) is so much smaller than the noise that the signal cannot be detected.

LIMITATIONS OF THE RADAR EQUATION

In deriving the radar equation it should be noted that no allowance has been made for siting or atmospheric conditions (e.g. heavy rain) and it has been assumed that aerial noise is negligible. Also, radars using split beam techniques or conical scan have a loss in gain due to those factors.

FACTORS THAT DETERMINE MAXIMUM RANGE

The Maximum Range at which a radar system will be able to detect a specified target will depend on the following factors:

- the maximum power of the radar transmitter P_t.
- the aperture (and gain) of the antenna.
- · effect of target reflection characterstics.
- · Wavelength of radar wave used.
- Minimum Power that the receiver can distinguish from the noise.

NUMERICAL EXAMPLES

We shall calculate the maximum range for two different radar to see how the values affect the result

Example One (Long Range):

This radar utilises:

- High Peak Power, P_{pk} = 25 kW
- Large Effective Aerial Aperture, Ae = 150 m²
- Short Wavelength, $\lambda = 0.1 \text{ m}$
- A large TGT, $\sigma = 40 \text{ m}^2$
- A low P_{min} for the receiver, $P_{min} = 0.4 \mu W$

Substituting these values into the radar equation gives the equation:

$$R_{\text{max}} = \sqrt[4]{\frac{25kW \times (150)^2 \times 40}{4\pi \times 0.4 \mu W \times 0.1^2}}$$

using your calculator to compute the answer, press the following calculator keys:

4 Shift x^y ((25 Shift 6 × 150 x^2 × 40) ÷ (4 × Shift Exp × 0.4 Shift 4 × 0.1 x^2)) =

to get the answer 25 866 m (Round to 25.9 km)

Example Two (Short Range)

This radar utilises:

- Lower Peak Power, P_{pk} = 10 kW
- Smaller Effective Aerial Aperture, Ae = 20 m²
- Longer Wavelength, $\lambda = 0.3 \text{ m}$
- A smaller TGT, σ = 10 m²
- A higher P_{min} for the receiver, $P_{min} = 4 \mu W$

Substituting these values into the radar equation gives the equation:

$$R_{\text{max}} = \sqrt[4]{\frac{10kW \times (20)^2 \times 10}{4\pi \times 4\mu W \times 0.3^2}}$$

using your calculator to compute the answer, press the following calculator keys:

4 Shift x^y ((10 Shift 6 × 20 x^2 × 10) ÷ (4 × Shift Exp × 4 Shift 4 × 0.3 x^2)) =

to get the answer 1 724 m (Round to 1.7 km)

FORMULA SUMMARY

The power density at range R from an isotropic radiator is:-

$$P_{DR} = \frac{P_t}{4\pi R^2}$$
 W m⁻²

The gain (G) of any transmitter aerial system is defined as:

When the gain of the antenna is taken into account then power density formula for $P_{\rm DT}$, at range, R, becomes;

$$P_{DT} = \frac{P_t \times G}{4\pi R^2}$$
 (W m⁻²)

The power re-radiated by the target is:

$$P_{R} = P \times G \times \sigma$$

$$4\pi R^{2}$$
 (W)

The power density at the radar's receiving aerial is:

$$P_{DRX} = \frac{P \times G \times \sigma}{4\pi R_2 \times 4\pi R^2}$$
 (Wm⁻²)

Signal power collected by the receiver antenna

$$P_{RX} = \frac{P_{t} \times G \times \sigma \times A_{e}}{4\pi R^{2} \times 4\pi R^{2}}$$
 (W)

Antenna gain:

G =
$$\frac{4\pi A_e}{\lambda^2}$$
 (no units, it's a ratio)

Power collected by the antenna:

$$P_{RX} = \frac{P_t \times \sigma \times A_e^2}{4\pi \times R^4 \times \lambda^2}$$

The complete Radar Range Equation:

$$R_{\text{max}} = \sqrt[4]{\frac{P_{\text{t}} \times (A_{\text{e}})^2 \times \sigma}{4\pi \times P_{\text{min}} \times \lambda^2}}$$

SYMBOL SUMMARY

The symbols used to represent the various quantities will be different in different publications. Therefore, it is important that you remember not the symbol (e.g. σ) but what it represents (e.g. Radar Cross-Section).

The symbols used in this handout are as follows:

Symbol	Meaning	Unit
R	Range	m
P_t	Power transmitted	W
P_{DR}	Iso Power Density at range 'R'	Wm ⁻²
G	Gain of antenna	(no unit)
P_{DT}	Power Density at target	Wm ⁻²
σ	Radar cross-section	m^2
P_r	Power re-radiated by target	W
P _{DRX}	Power density at receiver	Wm ⁻²
A _e	Effective aperture of antenna	m^2
P _{RX}	Power collected by antenna	W
P _{min}	Receiver sensitivity	W
λ	Wavelength	m

SELF TEST QUESTIONS

- 1. The power emitted by an isotropic radiator is considered to:
 - a. be highly directional.
 - b. cover the surface of an expanding sphere centred on the source.
 - c. form a shape similar to a tracking beam.
 - d. move away from the source at the speed of sound.
- 2. A 10 kW transmitter operates through an isotropic radiator in space. The power density that would be measured 5 km from the source is:
 - a. 31.83 nWm⁻²
 - b. 159.15 Wm⁻²
 - c. 31.83 µWm⁻²
 - d. $100 \mu Wm^{-2}$
- 3. An isotropic radiator delivers a power density of 200 μ Wm⁻² at a range of 10 km. The power transmitted is approximately:
 - a. 250 kW
 - b. 25 W
 - c. 25 kW
 - d. 80 kW
- 4. A certain radar aerial delivers a power density of $150\,\mu\text{Wm}^{-2}$ to a target which when illuminated by an isotropic radiator at the same range would be covered by a power density of 0.1 μWm^{-2} . The gain of this radar aerial is:
 - a. 1500
 - b. 150
 - c. 15
 - d. 1.5
- 5. A 15 kW radar transmitter delivers 12 mWm⁻² to a target located at a range of 15 km. The gain of this aerial is:
 - a. 2.262
 - b. 2.262
 - c. 151
 - d. 15.1
- 6. A 40 kW radar transmitter uses an aerial with a gain rating of 725. The power density it would deliver to a target located 5 km away is:
 - a. 92.3 μWm⁻²
 - b. 92.3 mWm⁻²
 - c. 175.6 μWm⁻²
 - d. 461.5 μW/m2

- 7. The figure used for the Radar Cross Section of a target:
 - a. is equal to the cross-sectional area of the target
 - b. depends upon the transmitter power of the radar.
 - c. determines how much power will be reflected.
 - d. depends on the gain of the receiving antenna.
- 8. The radar equation encompasses the term P_{\min} . This term is meant to convey the minimum:-
 - a. signal level required to illuminate a target at any range
 - b. input signal level from which the receiver is able to produce a usable output
 - c. transmitter power required to illuminate a target
 - d. power transmitted by the radar
- 9. A 200 kW, search radar using wavelength 0.03 m is required to detect targets with a Radar Cross Section of 1 m² out to maximum range. The effective aperture of the antenna is 214 m² and the minimum signal level the receiver can process is 1 μ W. The maximum range at which the radar can operate is:
 - a. 900 km
 - b. 60 km
 - c. 30 km
 - d. 15 km
- 10. A 50 kW, search radar using wavelength 0.1 m is required to detect targets with a Radar Cross Section of $20m^2$ out to maximum range. The effective aperture of the antenna is 75 m² and the minimum signal level the receiver can process is 0.5 μ W. The maximum range at which the radar can operate is:
 - a. 17.3 km
 - b. 8.4 km
 - c. 5.9 km
 - d. 3.1 km

Answers

1 b 2 c 3 a 4 a 5 a 6 b 7 c 8 b 9 c 10 a

	Teaching Objectives	Comments		
H.06.01 Describe the factors that determine the Power Density near the Target.				
H.06.01.01	Describe the properties of an isotropic radiator.	Expanding bubble, equal Power Density in all directions		
H.06.01.02	Calculate the power density at range R for an isotropic radiator.	Dependence on R is inverse Square Law		
H.06.01.03	Describe the concept of antenna gain.			
H.06.01.04	Calculate the power density at range R for an antenna of gain 'G'.	Dependence on R is inverse Square Law		
	H.06.02 Describe the factors that determine the Po	ower reflected by the Target.		
H.06.02.01	Explain the concept of Radar Cross Section of a target	Area from which power is intercepted, m ² .		
H.06.02.02	Describe the effect of the shape and material of a target on its RCS.	Conductors reflect, corners reflect. Insulators do not reflect well. Radar absorbent material.		
H.06.02.03	Calculate the power re-radiated from the target.	Power reflected = Power Density \times RCS		
H.06.03 Describe the factors that determine the Power collected by the Receiving Antenna.				
H.06.03.01	Describe the re-radiated power as originating from an isotropic source.			
H.06.03.02	Explain how the power density of the echo varies with Range.	A second inverse-square.		
H.06.03.03	Explain the concept of Effective Aerial Aperture.	Collecting area of receiving antenna multiplied by efficiency (typical 75%)		
H.06.03.04	Calculate the power collected by an antenna.			
	H.06.04 Relate Antenna Gain to its Effective A	perture and Wavelength.		
H.06.04.01	State the relationship between Ae , Gain and Wavelength			
H.06.04.02	Substitute for 'G' in the power equation.			
	H.06.05 Relate Power Received to Rec	eiver Sensitivity.		
H.06.05.01	State that the main limitation is the minimum power that the receiver can identify a target above the noise	Receiver sensitivity.		
H.06.05.02	State that when the received power equals the receiver's sensitivity then the target is at maximum range.			
	H.06.06 Use the Radar Range l	Equation		
H.06.06.01	Recognise the radar range equation and describe the significance of each of the quantities that it contains.			
H.06.06.02	Calculate the maximum range for a radar given the relevant parameters.			