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T4 ________________	**E**	F4 ________________

2016
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Summary Sheet

Water Supply Capacity and Intervention Plan Modeling

Summary

With the constant consuming of water, water scarcity problem has attracted the attention of the whole socity. In this paper, we present models for measuring and forecasting water supply capacity, and design an intervention plan for regions suffering from water shortage.

For Task 1, we use the amount of water resource to forecast the water supply, and the water consumption to forecast the demand of water. Thereby, We formulate the ratio of water resource to water comsumption as the standard to measure water supply capacity.

For Task 2, we pick North China, which is a typical region that shorts for water, and analyzed its water scarcity situation from several aspects, such as the total amount of water resource, population, water utilization and water quality.

For Task 3, we estimate the variation of the factors, then substitute them into the model of water supply capacity to forecast the water situation in next 15 years. We collect relevant datas from past, such as living water consumption and industry water consumption, then build $GM(1,1)$ model and obtain the approximate value of water resources by Grey theory. We find that water scarity will become even worse in North China.

For Task 4, we use China's provinces as vertices to build complete bipartite graph, and dispatch water resources of the nation by minimal-cost maximal flow and Kruskal algorithm. We transport water from rich regions to poor regions, which eased the water scarcity in North China. We also calculate the water resource distribution after dispatching, and analyze its effect to the whole nation.

For Task 5, after applying the intervenstion plan of water diversion, we find that the water resource available per capita of extermely water scarcity regions has significantly increased, while the same factor in regions with abundant water resources slightly decreased respectively. That is, it balanced the uneven spatial distribution of water resources.

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1 Introduction

1.1 Background

With the constant consuming of water resource, men began to think about a serious question: Will we meet the depletion of water supplies on this planet? Currently, numerous people from distinct regions all over the world are suffering from water shortage. To ease the situation, and help people from drylands access to clean water, we need to ponder over this problem from two aspects: Firstly, what caused the shortage of water resource? Secondly, which method could be applied to ease the situation?

The possible causes of water shortage are hard to count, but they could be classified as physical shortage and economical shortage. The methods must take those causes into consideration, and forecast about the water resource situation of the region in the future.

1.2 Our Work

1. Construct a model to measure the water supply capacity of a certain region. We must consider the dynamic nature of the factors that affect both supply and demand, and make reasonable representations for comparsion.
2. Analyze the degree of water shortage of a region from the view of water shortage and water utilzation. Then forecast what the water situation will be in next 15 years using the previous model.
3. Design an intervention plan that carefully considered all the factors of water scarcity. Use the intervention and the model to forecast water availability in the future.

2 Water Scarcity in North China

As we can see in Table 1, China's renewable water resources per capita is in severe shortage comparing to world average level. In UN water scarcity map[1], almost the entire North China region (the region at the north of Kunlun Mountains - Qinling Mountains - Huaihe River Line) is in badly water shortage.

[1]http://www.unep.org/dewa/vitalwater/jpg/0222-waterstress-overuse-EN.jpg

	Total renewable water resources per capita (m^3/inhab/year)
China	1993.00
World	19174.99

Table 1: Comparison of Renewable Water Resources between China and World

2.1 Physical Causes

The poor water resource in North China could not meet the demand of the people.

- **The total amount of water resource is small.**
  The total amount of water resource in North China occupies 19% in China.[2] The water resource of each river basin in China is shown in Table 2. Less rainfall and uneven spatial and temporal distribution. The air in north is dry, so water evaporates fast; the terrain in north is mainly rocky mountains and deserts, which leads to low water storage capacity and sever soil erosion, make water scarcity even worse.

- **The population is large.**
  The poplation of North China occupies 46% of the national population. The water resources per capita in China is less that 2200 cubic meter, while the number in north is only 990 cubic meter, even less than one eighth of world average. Regional statistic of water resources and social economy in China is shown in Figure 1.[3] The extremely poor water resource cannot match the demand of water from large population and industry.

River Basin	Surface Water Resource ($10^9\,m^3$)	Groundwater Resource ($10^9\,m^3$)	Total Amount of Water Resource ($10^9\,m^3$)	National Percentage %	Regional Percentage %	Water Production Modulus ($10^5\,m^3/km^2$)
National	27115	8288	28124	-	-	29.46
Song Liao River	1653	625	1928	6.9	36	15.56
Hai River	288	265	421	1.5	7.9	13.24
Huai River	741	393	961	3.4	17.9	28.95
Yellow River	661	406	744	2.6	13.9	9.3
Inner-land Rivers	1164	862	1304	4.6	24.3	3.86

Table 2: Water Resource of River Basins

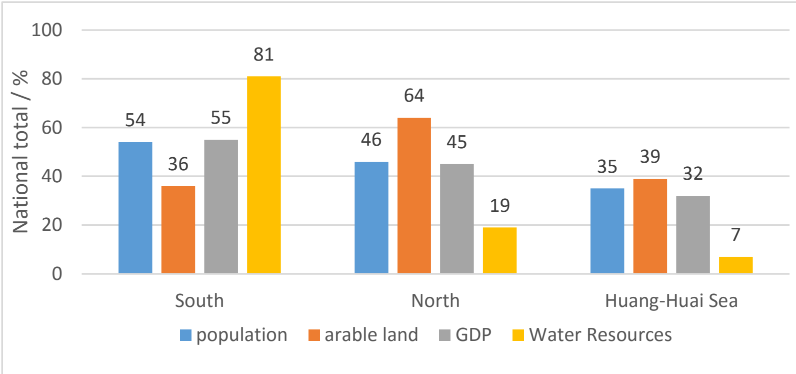


Figure 1: Regional Statistic of Water Resources and Social Economy in China

2.2 Economical Causes

Besides the insufficient water resource, some other reasons also leads to the disablity in taking full advantage of the water resource.

- **Water-utilization-caused water shortage.**
  The North China is the grain production base of China, cultivated area occupies 64% of the nation. But the economy and technology of North China is relatively backward, and the irrigation facilities are not in good condition. Most areas use flood irrigation with soil ditch, but up to 60% of the water lost due to seepage. In North China, the effective utilization coefficient of gravity irrigation water is only about 0.4, and the effective utilization coefficient of pumped irrigation water is only about 0.65.[1]

- **Water-quality-caused water shortage.**
  The pollution of water is serious, which results in unavailable clean water resource. Recently, northern rivers ranked top five in China's seven major polluted water systems.[5] As the water flows of northern rivers are small, the self-cleaning ability of the rivers is poor. Once the rivers are polluted, it's hard to recover and always leads to serious consequences.

3 The Model of Water Supply Capacity

Water supply capacity C could be measued by the ratio of water supply S to water demand D. As we use the amount of water resource to forecast the water supply, and water consumption to forecast the demand of water, thereby, We formulate the ratio of water resource R to water comsumption U as the standard to measure water supply capacity.

$$C = S/D = R/U$$

3.1 Water Supply

Water supply is measured by the amount of water resources. We could calculate water resources by the following formula:

$$R = RS + RG - RO$$

In which RS represents for surface water resources, dynamic water resources in rivers, lakes and glaciers that can be updated annually; RG represents for groundwater resources, water resources in saturated underground aquifers; and RO represents for the overlap between surface water and groundwater.

3.2 Water Demand

Water demand is measured by the amount of water consumption. We could calculate ware consumption by the following formula:

$$U = UL + UI + UA + UE$$

In which UL represents for living water consumption, includes urban and rural water consumption; UI represents for industry water consumption, excluding recycle used water in factories; UA represents for agriculture water consumption, mainly irrigation water consumptions; and UE represents for environment water consumption, particularly refers to the manually added water into rivers and lakes.

4 The Model of Water Situation Forecast

4.1 Analysis

This problem requires analysis of the water situation of North China in next 15 years. We have already got the model to measure the water supply capacity of a region, so we could forecast the water situation by predicting the factors in this model. We use a $GM(1,1)$-based Grey theory model to forecast the water situation of North China.

4.2 Model Construction

Grey theory could estimate and forecast the development pattern of system behavior based on existing information by using GM model. In fact, it regarded "random procedure" as "grey procedure", "random variables" as "grey variables", and mainly use the $GM(1,1)$ model in grey system theory to process.

We collected various datas, such as the total amount of water resource, water consumption of living, industry, agriculture and environment, which are all factors that affects water supply capacity. Then we use Grey theory to estimate the tendency of the datas in next 15 years. The caculation process of the forecast shown below pick living water consumption as example.

4.2.1 Data Verify

Firstly, we must verify whether the data we use satisfies the requirements to construct the model. Reference data is listed in Table 3.

Assume that reference data set is $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(10)) = (246, 250.6, \ldots, 259.4)$.

(1) Calculate the stepwise ratio of the number sequence

$$\lambda(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)} \, (k = 2, 3, \ldots, 10)$$

$$\lambda = (\lambda(2), \lambda(3), \lambda(10)) = (0.9816, 0.9905, \ldots, 0.9788)$$

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Amount of Water	246	250.6	253	261.5	270.1	274.5	288.3	250.2	253.9	259.4

Table 3: Living Water Consumption of Past 10 Years

(2) Grade ratio judgement

While all the $\lambda(k) \in [0.9521, 1.1523]$ lies in the containable cover $\Theta = (e^{-\frac{2}{n+1}}, e^{\frac{2}{n+2}}), n = 10$, we could use $x^{(0)}$ as well defined $GM(1,1)$ model.

4.2.2 $GM(1,1)$ Modeling

Accumulate $x(0)$, we get number sequence $x^{(1)} = (x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(10))$, while:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)\,(k = 1, 2, \dots, 10)$$

We get the result: $x^{(1)} = (246, 496.6, \dots, 2607.5)$

Define the Grey derivative as

$$dx^{(1)}(k) = x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1)$$

Let $z^{(1)}$ be the close series of mean $x^{(1)}$, that is

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)\,(k = 2, 3, \dots, 10)$$

Define $GM(1,1)$ grey differential equations medel as

$$dx^{(1)}(k) + az^{(1)}(k) = b$$

, that is

$$x^{(0)}(k) + az^{(1)}(k) = b$$

When $k = 2, 3, \dots, 10$,

$$\begin{cases} x^{(0)}(z) + az^{(1)}(z) = b, \\ x^{(0)}(3) + az^{(1)}(3) = b, \\ \dots \\ x^{(0)}(10) + az^{(1)}(10) = b. \end{cases}$$

Assume

$$\mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(10) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} -2^{(1)}(2), 1 \\ -2^{(1)}(3), 1 \\ \dots \\ -2^{(1)}(n), 1 \end{bmatrix}$$

$$u = (a, b)^T$$

, then $GM(1,1)$ model could be represented as matrix equation $Y = Bu$.

Using least squares method we could get

$$\widehat{u} = (\widehat{a}, \widehat{b})^T = (B^T B)^{-1} B^T Y = [-0.0021, 259.4399]$$

According to differential equation

$$\frac{dx^{(1)}}{dt} - 0.0021 x^{(1)} = 259.4399$$

We solved that

$$x^{(1)}(t+1) = (x^{(0)}(1) - \frac{b}{a})e^{-at} + \frac{b}{a} = 125437 * e^{(0.0021t)} - 125191$$

According to

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1)\,(k = 2, 3, \dots, 25)$$

, take $t = 10, 11, \dots, 24$, we could get the forcast amount of water supply in next 15 years:

$$(x^{(1)}(11), x^{(1)}(12), x^{(1)}(13), \dots, x^{(1)}(25)) = (265.118, 265.668, \dots, 272.923)$$

Specific results is shown in Table 4.

Year	2015	2016	2017	2018	2019	2020	2021	2022
Prediction	265.118	265.668	266.219	266.772	267.325	267.88	268.435	268.992
Year	2023	2024	2025	2026	2027	2028	2029	
Prediction	269.55	270.109	270.67	271.231	271.794	272.358	272.923	

Table 4: Forecast of living water consumption in 15 years

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Original Value	246	250.6	253	261.5	270.1	274.5	288.3	250.2	253.9	259.4
Model Value	246	260.2	260.8	261.3	261.8	262.4	262.9	263.5	264.0	264.6
Residual	0	9.62	7.76	-0.20	-8.26	-12.12	-25.37	13.28	10.12	5.17
Relative Error	0.000	0.038	0.031	-0.001	-0.031	-.0.44	-0.088	0.053	0.040	0.020

Table 5: $GM(1,1)$ Model Validating

4.3 Model Validating

When $t = 0, 1, \ldots, 9$, we could obtain the living water consumption from 2005 to 2014 estimated by Grey theory. Compare them to the original value, we could calculate the errors of the model.

Error test indicators are shown in Table 5.

From Table 5, we found that the error of $GM(1,1)$ is small enough to ignore.

4.4 Forecast of Other Factors

By the same Grey theory method, we could get the forecast of industry water consumption (Table 6), agriculture water consumption (Table 7), environment water consumption (Table 8) and total amount of water resource (Table 9).

Year	2015	2016	2017	2018	2019	2020	2021	2022
Prediction	337.835	337.04	336.246	335.455	334.666	333.878	333.092	332.308
Year	2023	2024	2025	2026	2027	2028	2029	
Prediction	331.526	330.746	329.968	329.191	328.417	327.644	326.873	

Table 6: Forecast of industry water consumption in 15 years

Year	2015	2016	2017	2018	2019	2020	2021	2022
Prediction	2195.65	2229.5	2263.87	2298.77	2334.21	2370.2	2406.74	2443.84
Year	2023	2024	2025	2026	2027	2028	2029	
Prediction	2481.52	2519.77	2558.62	2598.07	2638.12	2678.79	2720.09	

Table 7: Forecast of agriculture water consumption in 15 years

Year	2015	2016	2017	2018	2019	2020	2021	2022
Prediction	78.1804	81.1765	84.2874	87.5175	90.8714	94.3539	97.9698	101.724
Year	2023	2024	2025	2026	2027	2028	2029	
Prediction	105.623	109.67	113.873	118.237	122.768	127.473	132.358	

Table 8: Forecast of environment water consumption in 15 years

Due to the influence of climate, the precipitation of past 10 years shows a raising trend, which raised the total amount of water resources as well. With the rapid economic development, people's living standard has improved a lot, which leads to the increase of water consumption; the development of industry also requires more water resources.

4.5 Regional Water Situation Forecast

We substitute forecast datas into the model of water supply capacity, that is the ratio of water resource to total water consumption, then we get the forecast of the water supply capacity of North China in next 15 years (Table 10).

From Table 10 we could see that, the water supply capacity of North China in next 15 years is constantly decreasing, which means that the demand of water is harder to satisfy. If no proper interventions are applied to ease the shortage of

Year	2015	2016	2017	2018	2019	2020	2021	2022
Prediction	5096.02	5149.88	5204.31	5259.31	5314.9	5371.07	5427.84	5485.21
Year	2023	2024	2025	2026	2027	2028	2029	
Prediction	5543.18	5601.77	5660.98	5720.81	5781.27	5842.38	5904.12	

Table 9: Forecast of the total amount of water resources

Year	2015	2016	2017	2018	2019	2020	2021	2022
Prediction	1.7714	1.7677	1.7638	1.7598	1.7558	1.7516	1.7474	1.7431
Year	2023	2024	2025	2026	2027	2028	2029	
Prediction	1.7386	1.7341	1.7295	1.7248	1.7201	1.7152	1.7102	

Table 10: Forecast of water supply capacity

water and increase water supply capacity, people would suffer from more and more serious water scarcity in the future.

5 The Model of Intervention Plan

5.1 Assumption

- Assume that the cost of construct pipelines is only linear correlated to distance.
- Assume that water demand per capita is constant value.

5.2 Model Construction

The main stream water diversion methods are inner-basin water transfer and inter-basin water transfer. Considering that south basin usually owns rich water resources, and north basin generally faces water shortage, we plan to design a strategy that could optimally dispatch water resources of the south to the north. Using water consumption per capita as standard, we could guarantee that the amount of water of north and south tends to be stable.

We noticed that the population of Inner Mongolia is relatively small, so a special case comes as a consequence: the water resource per capita of Inner Mongolia is adequate, though the total water resource is poor. On the other hand, due to the large population base, North China suffers from serious lack of water. Thus we choose and calculate the water supply per capita.

We construct a **complete bipartite graph** from all the points of the regions with rich water resource to all the points of the regions with poor water resource, set the capacity of the edges to infinity, and set the cost as the distance between the corresponding two regions. Then we introduce source S and sink T. Add edges from S to the regions with rich water resource, set the capacity of the edges as the amount of water available, and set the cost of the edge to 0. Add edges from the regions with poor water resource to T, set the capacity of the edges as the amount of water lacked (absolute value), and set the cost of the edge to 0. Run **minimum-cost maximum flow** on this graph, then we could obtain the minimum distance of water diversion when the water supply reaches a balanced situation.

Then we integrate the data and sort edges by the ratio of the amount of water transported to its distance. We must delete some edges with low ratio, as they costs too much to transport water. Aftering deleting those expensive edges, we will find the edges remaining formed several connected component.

As water circulates in each connected component, but not between connect components, we deal with each connected component separately. Firstly, we link all the points in a connected component to each other to construct a complete graph. Then we run Kruskal algorithm on this graph to calculate the minimum spanning tree, and use the edges of the minimum spanning tree as water supply pipelines. So that we could ensure that the total length of pipelines in one connect component is minimum.

5.3 Model Solution

Firstly, we collected the water resource situation of China's provinces, shown in Table 11.[4]

Then we run minimum-cost maximum flow on the graph we built using the method that we introduced before. We get a route map of water transportation, shown in Table 12.

After sorting all the edges, we display the route map with different range of ratio of the amount of water transported to edge's distance, shown in Figure 2. we assume that it's proper the delete all the edges with ratio less than 0.02, as the cost to deliver unit water is unaffordable.

After execute Kruskal algorithm on each connected component, we get a set of minimal spanning trees, that is the water diversion plan shown in Figure 3.

5.4 Model Evaluation

The dispatch of water resource aims at transfer water from regions with abundant water resource to regions with poor water resource. It could ease the situation of water shortage in North China.

5.4.1 Strengths

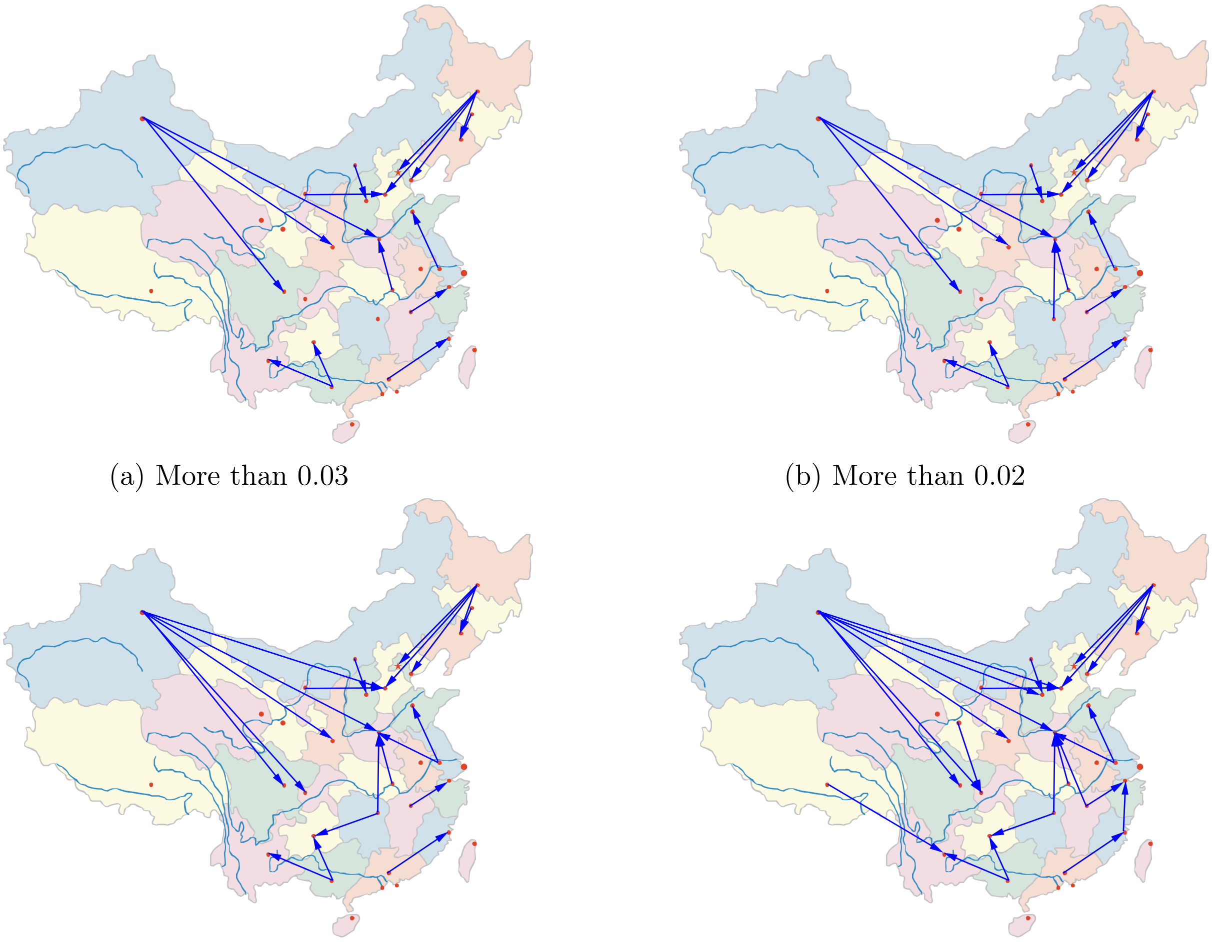
- It eases the contradiction between water supply and demand, and assists people in North China to obtain clean water resources. Further more, it ensures the industries retain necessary water resource supply, and promotes the economic development of North China.
- Timely water supply could decrease the over exploitation of groundwater in arid areas, and alleviate the ground subsidence problem. It takes a giant step forward to prevent the seawater intrusion in coastal areas, which leads to terrible deterioration of groundwater quality. It contributes to the protection of the availablity of clean water resource.

Province	No.	Total Water Supply ($10^9\,m^3$)	Water Consumption Per Capita (m^3/person)	Population (10^4)	Water Resource Available ($10^9\,m^3$)
Beijing	1	37.5	175.7	2152	37.50
Tianjin	2	24.1	161.2	1517	24.10
Hebei	3	192.8	262.0	7384	192.80
Shanxi	4	71.4	196.1	3648	71.40
Neimenggu	5	182.0	727.6	2505	182.00
Liaoning	6	141.8	322.9	4391	141.80
Jilin	7	133.0	483.3	2752	133.00
Heilongjiang	8	364.1	949.7	3833	364.10
Shanghai	9	105.9	437.6	2426	105.90
Jiangsu	10	591.3	743.8	7960	591.30
Zhejiang	11	192.9	350.2	5508	192.90
Anhui	12	272.1	449.3	6083	272.10
Fujian	13	205.6	542.5	3806	205.60
Jiangxi	14	259.3	572.2	4542	259.30
Shandong	15	214.5	219.8	9789	214.50
Henan	16	209.3	222.1	9436	209.30
Hubei	17	288.3	496.5	5816	288.30
Hunan	18	332.4	495.1	6737	332.40
Guangdong	19	442.5	414.2	10724	442.50
Guangxi	20	307.6	649.4	4754	307.60
Hainan	21	45.0	500.7	903	45.00
Chongqing	22	80.5	270.0	2991	80.50
Sichuan	23	236.9	291.6	8140	236.90
Guizhou	24	95.3	271.9	3508	95.30
Yunnan	25	149.4	317.9	4714	149.40
Xizang	26	30.5	967.3	318	30.50
Shaanxi	27	89.8	238.3	3775	89.80
Gansu	28	120.6	466.2	2591	120.60
Qinghai	29	26.3	453.8	583	26.30
Ningxia	30	70.3	1068.6	662	70.30
Xinjiang	31	581.8	2550.7	2298	581.80
Average	-	196.6	524.8	4395.03	196.61
Sum	-	6094.8	16268.2	136246	6094.80

Table 11: Water Resource Situation of Provinces

Water Outgoing City No.	Water Incoming City No.	Amount of Water ($10^9\, m^3$)	Distance (km)	Ratio of Water to Distance
10	15	223	663	0.33635
5	4	69	527	0.13093
14	11	51	562	0.09075
8	6	45	546	0.08242
20	24	49	623	0.07865
20	25	45	879	0.05119
31	16	150	3027	0.04955
17	16	28	569	0.04921
8	1	58	1230	0.04715
30	3	40	948	0.04219
31	23	111	2914	0.03809
13	19	33	901	0.03663
8	2	43	1207	0.03563
31	27	79	2541	0.03109
7	6	9	294	0.03061
8	3	46	1510	0.03046
18	16	19	853	0.02227
31	3	51	3026	0.01685
31	22	49	3202	0.01530
10	16	10	666	0.01502
18	24	12	1105	0.01086
21	23	16	1798	0.00890
31	4	22	2825	0.00779
26	25	16	2246	0.00712
10	9	2	295	0.00678
21	19	4	601	0.00666
14	16	5	961	0.00520
28	22	4	1275	0.00314
13	11	2	688	0.00291

Table 12: Route Map of Water Transportation



(a) More than 0.03 (b) More than 0.02

(c) More than 0.01 (d) All edges

Figure 2: Edges selection by different ratio

- It could balance the water resource distribution of the nation, and solve the uneven spatial distribution problem in China.

5.4.2 Weaknesses

- The dispatch of water resource only moves water resource physically, but cannot increase the total amount of water resource, so it's not a fundamental solution of the water shortage problem.
- With the rapid growth of population, economic development, and urbanization process, water demand will constantly increase with a high growth rate.
- The task of water pollution prevention and treatment is not successful in China. The situation even turns to be worse. !cite
- China have been suffered from serious water shortage for a long time. A representative instance is that China's water resource per capita only reached one quarter of world level. The dispatch of water resource could satisfy the urgent

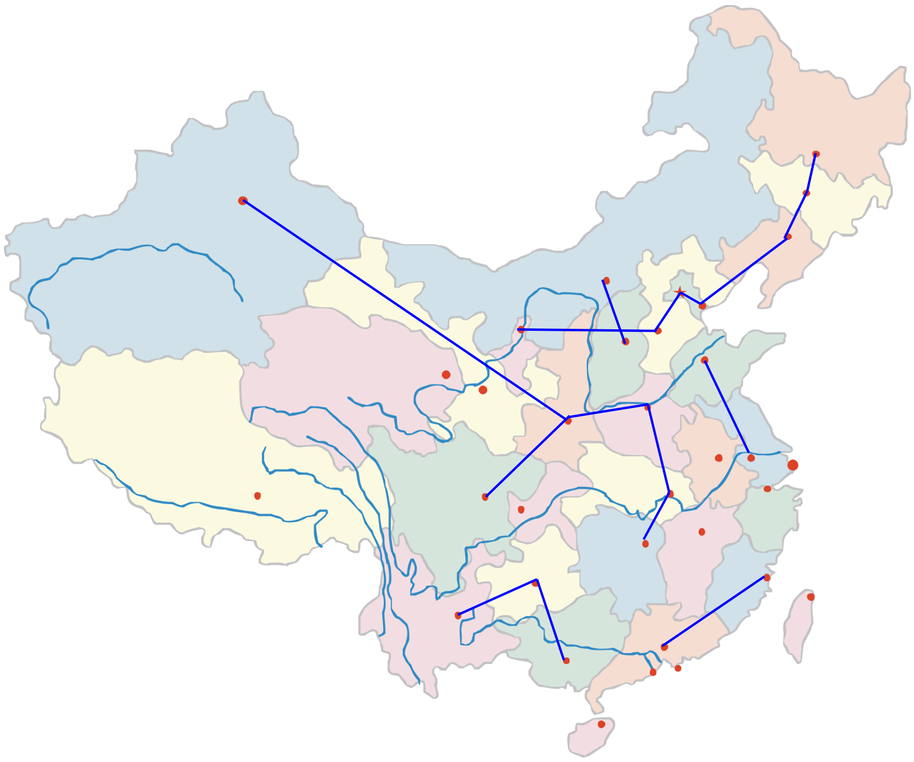


Figure 3: Water Diversion Plan

needs of water resource in North China, although it is only a temporate but not permanente solution. Considering that the water resource in South China is limited, and the amount of water demand is huge due to large population and densely covered industry, it is not realistic to transport water from south to north in a long term.

5.5 Water Availability Forecast

In accordance with internationally recognized standards, mild water shortage means water resources per capita lower than $3000m^3$; moderate water shortage means water resources per capita lower than $2000m^3$; severe water shortage means water resources per capita lower than $1000m^3$; extremely water shortage means water resources per capita lower than $500m^3$. There are 16 provinces in severe water shortage, and 6 provinces in extermely water shortage in China at present. !cite

After applying the intervention plan of water diversion, we obtain the new water consumption per capita, as shown in Table 13.

From Table 13 we could draw the conclusion: As national water resource is in an insufficient state, most regions still suffers from severe water scarcity. But meanwhile, the water resource consumption per capita of extermely water shortage areas

Province	Beijing	Tianjin	Hebei	Shanxi	Neimenggu	Liaoning	Jilin
Before	175.7	161.2	262	196.1	727.6	322.9	483.3
After	443.8	442.3	377.6	384.9	451.1	445.9	450.6
Province	Heilongjiang	Shanghai	Jiangsu	Zhejiang	Anhui	Fujian	Jiangxi
Before	949.7	437.6	743.8	350.2	449.3	542.5	572.2
After	569	436.5	450.1	442.8	447.3	453.5	458.6
Province	Shandong	Henan	Hubei	Hunan	Guangdong	Guangxi	Hainan
Before	219.8	222.1	496.5	495.1	414.2	649.4	500.7
After	457.1	430.6	447.6	465.2	443.4	449.3	498.3
Province	Chongqing	Sichuan	Guizhou	Yunnan	Xizang	Shannxi	Gansu
Before	270	291.6	271.9	317.9	967.3	238.3	466.2
After	269.1	427.4	411.3	412.4	959.1	447.2	465.5
Province	Qinghai	Ningxia	Xinjiang				
Before	453.8	1068.6	2550.7				
After	451.1	457.7	1052.2				

Table 13: Water Consumption Per Capita

has significantly increased. On the other hand, the water resource consumption per capita of areas with abundant water resources has slightly decreased respectively. That is, our intervention plan of water diversion balanced the uneven spatial distribution of water resources.

References

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Appendices

Appendix A Source Code of Grey Theory

```
clc
x0 = [246 250.6 253 261.5 270.1 274.5 288.3 250.2 253.9 259.4];
n = length(x0);
lamda = x0(1 : n − 1)./x0(2 : n); % calculate level ratio
range = minmax('lamda'); % calculate the range of level ratio
x1 = cumsum(x0); % accumulate
B = [−0.5 ∗ (x1(1 : n − 1) + x1(2 : n)), ones(n − 1, 1)];
Y = x0(2 : n);
u = B \ Y;
x = dsolve('Dx + a ∗ x = b','x(0) = x0'); % solve differential equations
x = subs(x, {'a', 'b', 'x0'}, {u(1), u(2), x1(1)});
vpa(x, 6);
forecast = subs(x,'t',[0:n+14]);
a = [0, forecast(1 : n + 14)];
b = forecast − a;
res = vpa('b', 6);
```

Appendix B Source Code of Minimum-cost Maximum Flow

```
#include <cmath>
#include <cstdio>
#include <iostream>
#include <cstring>
#include <algorithm>
#include <queue>
using namespace std;
const int MAXN = 2200;
const int MAXM = 400010;
const int INF = 0x7fffffff;
const int SPEND = 50000;
char name[35][50];
struct Edge {
    int to, next, cap, flow, cost;
} edge[MAXM];
int head[MAXN], tol;
int pre[MAXN], dis[MAXN];
bool vis[MAXN];
int N;
void init(int n) {
    N = n;
    tol = 0;
    memset(head, −1, sizeof(head));
}
void addedge(int u, int v, int cap, int cost) {
```

```
    edge[tol].to = v;
    edge[tol].cap = cap;
    edge[tol].cost = cost;
    edge[tol].flow = 0;
    edge[tol].next = head[u];
    head[u] = tol++;
    edge[tol].to = u;
    edge[tol].cap = 0;
    edge[tol].cost = -cost;
    edge[tol].flow = 0;
    edge[tol].next = head[v];
    head[v] = tol++;
}
bool spfa(int s, int t) {
    queue<int> q;
    for (int i = 0; i < N; i++) {
        dis[i] = INF;
        vis[i] = false;
        pre[i] = -1;
    }
    dis[s] = 0;
    vis[s] = true;
    q.push(s);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        vis[u] = false;
        for (int i = head[u]; i != -1; i = edge[i].next) {
            int v = edge[i].to;
            if (edge[i].cap > edge[i].flow && dis[v] > dis[u] + edge[i].cost) {
                dis[v] = dis[u] + edge[i].cost;
                pre[v] = i;
                if (!vis[v]) {
                    vis[v] = true;
                    q.push(v);
                }
            }
        }
    }
    if (pre[t] == -1)
        return false;
    else
        return true;
}
// return max flow, min cost stores in &cost
int MinCostMaxFlow(int s, int t, int &cost) {
    int flow = 0;
    cost = 0;
    while (spfa(s, t)) {
        int Min = INF;
        for (int i = pre[t]; i != -1; i = pre[edge[i ^ 1].to]) {
            if (Min > edge[i].cap - edge[i].flow)
                Min = edge[i].cap - edge[i].flow;
        }
        for (int i = pre[t]; i != -1; i = pre[edge[i ^ 1].to]) {
```

```
            edge[i].flow += Min;
            edge[i ^ 1].flow -= Min;
            cost += edge[i].cost * Min;
            cout << edge[i].to << " ";
        }
        flow += Min;
        cout << s << " " << Min << endl;
    }
    return flow;
}
double cst1[35];
int b[35], cst2[35][35], l1[35], l2[35];
int main() {
    freopen("t.in", "r", stdin);
    freopen("t.out", "w", stdout);
    int n, n1, n2, k, ans1, ans2;
    n1 = n2 = 0;
    n = 31;
    init(33);
    for (int i = 1; i <= 31; i++) {
        scanf("%s%d", name[i], &k);
    }
    name[0][0] = 's';
    name[32][0] = 'e';
    for (int i = 1; i <= 31; i++) {
        scanf("%d%lf", &k, &cst1[i]);
        if (cst1[i] > 0)
            l1[n1++] = i;
        else
            l2[n2++] = i;
    }
    for (int i = 1; i <= 31; i++) scanf("%d", &b[i]);
    for (int i = 1; i <= 31; i++)
        for (int j = 1; j <= 31; j++) scanf("%d", &cst2[b[i]][b[j]]);
    for (int i = 0; i < n1; i++) addedge(0, l1[i], cst1[l1[i]], 0);
    for (int i = 0; i < n2; i++) addedge(l2[i], 32, -1 * cst1[l2[i]], 0);
    for (int i = 0; i < n1; i++)
        for (int j = 0; j < n2; j++) {
            addedge(l1[i], l2[j], min(cst1[l1[i]], -1 * cst1[l2[j]]),
                    (int)cst2[l1[i]][l2[j]]);
        }
    ans1 = MinCostMaxFlow(0, 32, ans2);
    cout << ans1 << " " << ans2 << endl;
    return 0;
}
```

Appendix C Source Code of Kruskal Algorithm

```
#include <cstdio>
#include <cstring>
#include <iostream>
#include <algorithm>
```

```
#include <cmath>
using namespace std;
const int mx = 50;
// u[i], v[i], w[i] means two end points and weight respectively
// n is the number of vertices, m is the number of edges
int n, m;
int u[mx], v[mx], w[mx], p[mx], r[mx];
int cmp(const int i, const int j) {  // indirect sort func
    return w[i] < w[j];
}
int find(int x) {  // union-find
    return p[x] == x ? p[x] : find(p[x]);
}
int kruskal() {
    int ans = 0;
    for (int i = 0; i < n; i++) p[i] = i;  // init union-find
    for (int i = 0; i < m; i++) r[i] = i;  // init edge no.
    sort(r, r + m, cmp);
    for (int i = 0; i < m; i++) {
        int e = r[i];
        int x = find(u[e]);
        int y = find(v[e]);
        if (x != y) {
            ans += w[e];
            p[x] = y;
            cout << u[e] << " " << v[e] << " " << w[e] << endl;
        }
    }
    return ans;
}
int main() {
    freopen("t3.out", "r", stdin);
    freopen("t4.out", "w", stdout);
    int k, pos = 0;
    n = 6;
    m = n * (n - 1);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) {
            scanf("%d", &k);
            if (i == j) continue;
            u[pos] = i;
            v[pos] = j;
            w[pos++] = k;
        }
    cout << n << " " << m << endl;
    kruskal();
    return 0;
}
```