

Hyperbolic Heterogeneous Graph Transformer

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Abstract—In heterogeneous graphs, we can observe complex structures such as tree-like or hierarchical structures. Recently, the hyperbolic space has been widely adopted in many studies to effectively learn these complex structures. Although these methods have demonstrated the advantages of the hyperbolic space in learning heterogeneous graphs, most existing methods still have several challenges. They rely heavily on tangent-space operations, which often lead to mapping distortions during frequent transitions. Moreover, their message-passing architectures mainly focus on local neighborhood information, making it difficult to capture global hierarchical structures and long-range dependencies between different types of nodes. To address these limitations, we propose Hyperbolic Heterogeneous Graph Transformer (HypHGT), which effectively and efficiently learns heterogeneous graph representations entirely within the hyperbolic space. Unlike previous message-passing based hyperbolic heterogeneous GNNs, HypHGT naturally captures both local and global dependencies through transformer-based architecture. Furthermore, the proposed relation-specific hyperbolic attention mechanism in HypHGT, which operates with linear time complexity, enables efficient computation while preserving the heterogeneous information across different relation types. This design allows HypHGT to effectively capture the complex structural properties and semantic information inherent in heterogeneous graphs. We conduct comprehensive experiments to evaluate the effectiveness and efficiency of HypHGT, and the results demonstrate that it consistently outperforms state-of-the-art methods in node classification task, with significantly reduced training time and memory usage.

Index Terms—Heterogeneous Graph Representation Learning, Hyperbolic Graph Embedding, Graph Transformer.

I. INTRODUCTION

HETEROGENEOUS GRAPHS, which consist of multiple types of nodes and links, effectively represent various real-world scenarios such as academic networks [1]–[3], social networks [4]–[7], and molecular structures [8]–[10]. In heterogeneous graphs, we can observe various complex structures such as tree-like or hierarchical structures. However, representing such complex structures in the Euclidean space is challenging due to its limited ability to capture the hierarchical and power-law characteristics inherent in heterogeneous graphs. Therefore, it is necessary to use an embedding space that can naturally capture such structural patterns beyond the limitations of the Euclidean space [11]–[13].

The hyperbolic space, which has a constant negative curvature, expands exponentially from a north pole. This property enables it to represent hierarchical and complex structures more effectively than Euclidean space [14]–[18]. Due

to this characteristic of the hyperbolic space, many recent studies [14], [19]–[24] have used the hyperbolic space as an embedding space to effectively capture such complex structures. SHAN [22] proposed hyperbolic heterogeneous Graph Neural Network (GNN) that captures complex structures within sampled simplicial complexes from heterogeneous graphs and uses graph attention mechanisms in the hyperbolic space to learn multi-order relations. HHGAT [23] proposed hyperbolic heterogeneous graph attention networks to learn semantic information and complex structures by leveraging metapaths, which are defined as the ordered sequence of node and link types that represent semantic structures within heterogeneous graphs. MSGAT [24] further extended HHGAT by using multiple hyperbolic spaces to effectively learn diverse power-law structures corresponding to different metapaths.

Although existing models have achieved remarkable performance in heterogeneous graph representation learning within the hyperbolic space, they still suffer from several challenges:

- **Challenge 1 (Over-frequent mapping).** Previous hyperbolic heterogeneous GNNs heavily rely on the tangent space to perform hyperbolic graph convolution operations, which often leads to mapping distortions during exponential and logarithmic mappings between the hyperbolic and tangent spaces.
- **Challenge 2 (Prior-knowledge requirement).** They rely on predefined structures sampled from heterogeneous graphs, such as metapath instances, which carefully defined based on domain-specific knowledge for each dataset, thereby restricting generalization and adapt to unseen relational patterns.
- **Challenge 3 (High computational cost).** Due to the use of complex hyperbolic operations and structure-based sampling processes, these models often require substantial computational costs in terms of both time and memory during training.
- **Challenge 4 (Absence of global information).** Since these models [22]–[24] only aggregate information from local neighbors, they struggle to capture the global hierarchical structures inherent in heterogeneous graphs.

Figure 1 illustrates the main challenges in previous hyperbolic heterogeneous GNNs and the corresponding approaches proposed in this work. Challenges 1 and 3 arise from frequent tangent–hyperbolic mappings and the high computational costs of hyperbolic graph convolution, Challenge 2 stems from dependence on predefined metapaths, and Challenge 4 re-

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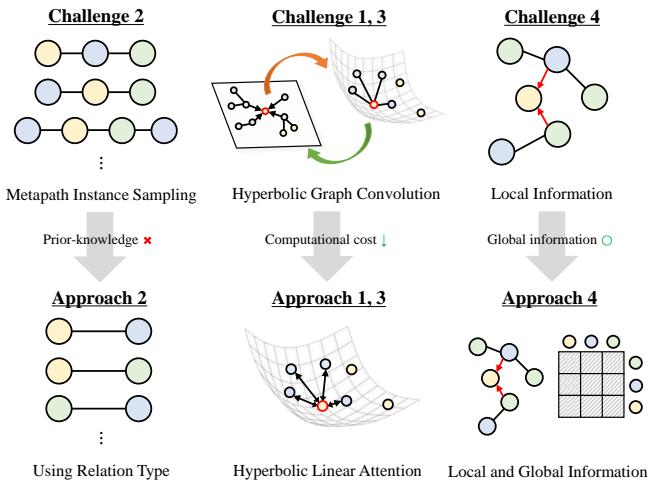


Fig. 1: Challenges (top) and corresponding approaches (bottom) in previous hyperbolic heterogeneous GNNs addressed by the proposed HypHGT.

flects the inability to capture global information due to local neighbor information aggregation. HypHGT addresses these challenges by performing efficient operations entirely within the hyperbolic space through linear self-attention (Approach 1 and 3), replacing predefined metapaths with relation-type modeling (Approach 2), and capturing both local and global dependencies via transformer-based architecture (Approach 4). The details of these approaches are described as follows.

Approach 1. HypHGT learns heterogeneous graph representations entirely within the hyperbolic space, avoiding unnecessary tangent-hyperbolic mappings except for the initialization of node features and the final projection required for downstream tasks. By performing all operations directly in the hyperbolic space, HypHGT effectively reduces mapping distortions during the exponential and logarithmic mappings between the hyperbolic and tangent spaces.

Approach 2. We propose a relation-aware hyperbolic attention mechanism that learn distinct relation-specific structures within relation-specific hyperbolic spaces. This enables HypHGT to effectively capture semantic heterogeneity across different link types and to learn diverse complex structures without relying on predefined structures such as metapath instances.

Approach 3. To address the high computational cost caused by softmax-based self-attention mechanisms in transformer architectures, we use a linear attention mechanism defined directly in the hyperbolic space. It can reduce both computational time and memory consumption.

Approach 4. HypHGT integrates hyperbolic transformer layers that learn global information with heterogeneous GNN layers that aggregate local neighbor information. This architecture enables HypHGT to learn comprehensive representations that capture both local relationships and global hierarchical structures inherent in heterogeneous graphs.

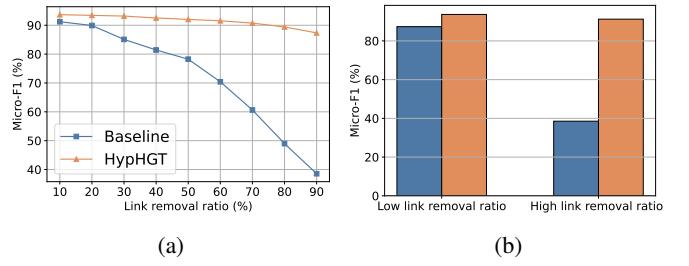


Fig. 2: Comparison between a message-passing-based hyperbolic heterogeneous GNN and HypHGT under increasing graph sparsity. (a) Node classification performance on the ACM dataset under increasing link removal ratios. (b) Node classification performance comparison under high- and low-sparsity levels.

Our study is motivated by the empirical observation that message-passing-based hyperbolic heterogeneous GNNs degrade substantially on sparse heterogeneous graphs, due to limited local neighborhood information.

Figure 2 illustrates a comparison between message-passing-based hyperbolic heterogeneous GNN with HypHGT under increasing graph sparsity. Although baseline model uses the hyperbolic space, their reliance on local neighborhood aggregation limits their ability to effectively capture global hierarchical structures. As graph sparsity increases, the neighborhood information becomes limited, leading to rapid performance degradation. In contrast, HypHGT leverages a transformer-based architecture to directly capture long-range dependencies, enabling more robust learning global hierarchical structures even when local connectivity is severely reduced.

In summary, the transformer-based architecture allows HypHGT to overcome the limitations of message-passing-based hyperbolic heterogeneous GNNs by capturing long-range dependencies and modeling global hierarchical structures beyond local neighborhoods. Moreover, due to the exponential growth of the hyperbolic space with a negative curvature, nodes at different hierarchical levels can be embedded with geometrically consistent separations, enabling the transformer-based architecture to better capture and preserve their hierarchical structural properties. Furthermore, since HypHGT uses a linear self-attention mechanism, it achieves efficient heterogeneous graph learning while preserving the expressive power required to represent complex structures.

The main contributions of our work can be summarized as follows:

- We propose a novel model for learning heterogeneous graph representations, HypHGT. To the best of our knowledge, HypHGT is the first transformer-based embedding model for heterogeneous graphs in the hyperbolic space.
- We conduct comprehensive experiments to evaluate the effectiveness and efficiency of HypHGT. The experimental results demonstrate that HypHGT outperforms state-of-the-art methods in various downstream tasks with

heterogeneous graphs.

- We analyze the impact of hyperbolic spaces on the HypHGT performance, and observe that the learnable curvatures in relation-specific hyperbolic spaces are adaptively optimized according to the structural distributions of relation types.

The remainder of this paper is organized as follows. In Section II, we provide an overview of relevant works. In Section III, we introduce the preliminaries and theoretical background. In Section IV, we present the proposed HypHGT in detail. In Section V, we provide experimental results and in-depth analyses. Finally, in Section VI, we conclude the paper with a summary of key findings. Table I summarizes notations used throughout the paper.

II. RELATED WORK

HypHGT is related to three broader areas of research, namely Euclidean heterogeneous GNNs, hyperbolic heterogeneous GNNs, and hyperbolic transformer.

A. Euclidean Heterogeneous GNNs

GNNs [25]–[28] have demonstrated effectiveness for graph representation learning, and recent research has focused on extending them to heterogeneous graphs to learn heterogeneous information within such graphs. Most recent studies leverage metapaths to capture semantic information inherent in heterogeneous graphs.

HAN [29] introduced a hierarchical graph attention network that aggregates information at two different levels: node-level and semantic-level attention. At the node level, HAN aggregates information from metapath-based neighbors, while at the semantic level, it aggregates the semantic information learned from different metapaths. However, during node-level attention, the intermediate nodes within metapath instances are not considered. To address this limitation, MAGNN [30] extended HAN by considering intermediate nodes into the learning process, thereby enhancing the ability to capture richer semantic information within metapaths. Nevertheless, both HAN and MAGNN require predefined metapaths to learn semantic information inherent in heterogeneous graphs, and defining appropriate metapaths without sufficient domain knowledge remains a challenging task across different datasets.

To address this challenge, recent studies proposed automatic metapath selection. GTN [31] introduced graph transformer layers that learn a soft selection of link types and composite relations to automatically generate informative metapaths. GraphMSE [32] further proposed an automatic metapath selection framework based on semantic feature space alignment.

In contrast, several heterogeneous GNNs do not rely on predefined metapath construction. Instead, they employ link-type dependent graph convolution operations to implicitly capture semantic information within heterogeneous graphs. HGT [33] introduced node- and link-type dependent attention mechanisms to model diverse relation types in heterogeneous graphs, while Simple-HGN [34] extended graph attention

networks by incorporating link-type information into the calculation of attention scores.

Although aforementioned Euclidean heterogeneous GNNs have achieved remarkable progress in heterogeneous graph representation learning, they still struggle to effectively capture the hierarchical structures commonly observed in such graphs.

B. Hyperbolic Heterogeneous GNNs

Since Euclidean space inherently has limitations in effectively representing the complex structures within heterogeneous graphs, recent studies have proposed hyperbolic heterogeneous GNNs that use the hyperbolic space as an embedding space. SHAN [22] proposed a simplicial hyperbolic attention network to capture complex structural patterns within sampled simplicial complexes, which represent multi-order relations inherent in heterogeneous graphs. HHGAT [23] used the hyperbolic space to capture power-law structures based on metapaths, enabling effective learning of semantic information within heterogeneous graphs. Although HHGAT demonstrated remarkable performance in heterogeneous graph representation learning, relying solely on a single hyperbolic space presents limitations when learning the diverse power-law structures within heterogeneous graphs. To overcome this limitation, MSGAT [24] extended HHGAT by using multiple hyperbolic spaces to effectively capture diverse power-law structures corresponding to different metapaths.

Previous hyperbolic heterogeneous GNNs have demonstrated the effectiveness of the hyperbolic space in learning complex structures within heterogeneous graphs. However, they suffer from mapping distortions caused by frequent transitions between the hyperbolic and tangent spaces. Their complex hyperbolic operations lead to increased computational costs, such as computatinal time and memory consumption.

C. Hyperbolic Transformer

Transformers [35]–[37] have recently been extended to the hyperbolic space, to enhance their ability to capture complex structures that are difficult to capture in Euclidean space. The hyperbolic space, with its constant negative curvature and exponential expansion can provide a natural representation for complex structures. Due to these properties, various hyperbolic Transformer models [38]–[40] have been proposed. Hyperbolic Transformer [41] proposed a hyperbolic attention network that uses hyperbolic distance to compute attention scores and performs hyperbolic aggregation through the Einstein midpoint to obtain representations in the hyperbolic space. Similarly, HYBONET [42] computes attention scores based on the Lorentzian distance and performs hyperbolic aggregation using the Lorentz centroid. Since the self-attention mechanism in these models has quadratic time complexity, their scalability is limited. To address this limitation, Hypformer [43] proposed a hyperbolic transformer architecture with linear-time complexity to ensure scalability. Moreover, all computations and representation learning in Hypformer are performed entirely within the fully hyperbolic space, reducing distortions caused

TABLE I: Notations.

Notation	Explanation
\mathcal{G}	A Heterogeneous graph
ϵ	A relation type ($\forall \epsilon \in T_{\mathcal{E}}$)
$T_{\mathcal{E}}$	The set of relation types
$\mathbb{L}^{n,c}$	n -dimensional Lorentz model with curvature c ($c < 0$)
\mathbb{R}^n	n -dimensional Euclidean space
$\mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c}$	Tangent space at point \mathbf{x}
$\exp_{\mathbf{x}}^c(\cdot)$	Exponential map at point \mathbf{x} , $\exp_{\mathbf{x}}^c : \mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c} \rightarrow \mathbb{L}^{n,c}$
$\log_{\mathbf{x}}^c(\cdot)$	Logarithmic map at point \mathbf{x} , $\log_{\mathbf{x}}^c : \mathbb{L}^{n,c} \rightarrow \mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c}$
HT(\cdot)	Hyperbolic transformation
HR(\cdot)	Hyperbolic readjustment and refinement
$\phi(\cdot)$	Kernel-function for linear-attention

by frequent mappings between the hyperbolic and tangent spaces.

In this work, we extend the fully hyperbolic Transformer architecture with linear-time complexity to heterogeneous graphs, enabling effective and efficient heterogeneous graph representation learning entirely within the hyperbolic space.

III. PRELIMINARIES

In this section, we introduce key concepts related to heterogeneous graphs and the Lorentz model adopted in this work.

A. Heterogeneous Graph

Definition 1 (Heterogeneous graph). A heterogeneous graph [29] is defined as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, f_v(\cdot), f_e(\cdot))$. \mathcal{V} is a set of nodes, \mathcal{E} is a set of links, $f_v(\cdot) : \mathcal{V} \rightarrow T_{\mathcal{V}}$ is a node type mapping function, and $f_e(\cdot) : \mathcal{E} \rightarrow T_{\mathcal{E}}$ is a link type mapping function, where $T_{\mathcal{V}}$ and $T_{\mathcal{E}}$ are sets of node types and link types, respectively, with $|T_{\mathcal{V}}| + |T_{\mathcal{E}}| > 2$.

B. Lorentz model

In hyperbolic geometry, the Lorentz model [42], [43] is widely adopted for its numerical stability and computational efficiency in performing geometric operations. The mathematical formulations underlying the Lorentz model are briefly described in Definitions 2-6.

Definition 2 (Lorentz Model). An n -dimensional Lorentz model with negative curvature c ($c < 0$) is defined by Riemannian manifold $\mathbb{L}^{n,c} = (\mathcal{L}^n, \mathbf{g}^c)$. \mathcal{L}^n is the upper sheet of hyperboloid (hyper-surface) $\mathbf{g}^c = \text{diag}(1/c, 1, \dots, 1)$ is the Riemannian metric tensor. Each point in $\mathbb{L}^{n,c}$ can be represented as $\mathbf{x} = \begin{bmatrix} x_t \\ \mathbf{x}_s \end{bmatrix}$, where $\mathbf{x} \in \mathbb{R}^{n+1}$, $x_t \in \mathbb{R}$, and $\mathbf{x}_s \in \mathbb{R}^n$. The set of points $\mathbb{L}^{n,c}$ is defined as follow:

$$\mathbb{L}^{n,c} := \{\mathbf{x} \in \mathbb{R}^{n+1} \mid \langle \mathbf{x}, \mathbf{x} \rangle_{\mathcal{L}} = 1/c, x_t > 0\}. \quad (1)$$

Here, $\langle \mathbf{x}, \mathbf{y} \rangle_{\mathcal{L}} = -x_t y_t + \mathbf{x}_s^T \mathbf{y}_s = \mathbf{x}^T \mathbf{g}^c \mathbf{y}$ is the Lorentzian inner product. The Lorentz model is an upper hyper-surface in an $(n+1)$ dimensional Minkowski space with origin point $(\sqrt{-1/c}, 0, \dots, 0)$. Special relativity provides a physical interpretation of the Lorentz model by associating

the last n components \mathbf{x}_s with spatial dimensions and the 0-th component x_t with the time dimension.

Definition 3 (Tangent space). Given $\mathbf{x} \in \mathbb{L}^{n,c}$, the orthogonal space of \mathbf{x} with respect to the Lorentzian inner product is the tangent space $\mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c}$ at point \mathbf{x} .

$$\mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c} = \{\mathbf{y} \in \mathbb{R}^{n+1} \mid \langle \mathbf{y}, \mathbf{x} \rangle_{\mathcal{L}} = 0\}. \quad (2)$$

Definition 4 (Exponential and logarithmic maps). The exponential map $\exp_{\mathbf{x}}^c(\cdot) : \mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c} \rightarrow \mathbb{L}^{n,c}$ is a function that maps any tangent vector from the tangent space at point \mathbf{x} onto the hyperbolic manifold. The exponential map can be formulated as follow:

$$\exp_{\mathbf{x}}^c(\mathbf{y}) = \cosh\left(\sqrt{|c|}\|\mathbf{y}\|_{\mathcal{L}}\right)\mathbf{x} + \frac{\sinh\left(\sqrt{|c|}\|\mathbf{y}\|_{\mathcal{L}}\right)}{\sqrt{|c|}\|\mathbf{y}\|_{\mathcal{L}}}\mathbf{y}. \quad (3)$$

The logarithmic map $\log_{\mathbf{x}}^c(\cdot) : \mathbb{L}^{n,c} \rightarrow \mathcal{T}_{\mathbf{x}}\mathbb{L}^{n,c}$ plays an opposite role, and it can be formulated as follow:

$$\log_{\mathbf{x}}^c(\mathbf{z}) = \frac{\cosh^{-1}(c\langle \mathbf{x}, \mathbf{z} \rangle_{\mathcal{L}})}{\sinh(\cosh^{-1}(c\langle \mathbf{x}, \mathbf{z} \rangle_{\mathcal{L}}))}(\mathbf{z} - c\langle \mathbf{x}, \mathbf{z} \rangle_{\mathcal{L}}\mathbf{x}). \quad (4)$$

Hypformer [43] introduced Hyperbolic Transformation with Curvatures (HT) and Hyperbolic Readjustment and Refinement with Curvatures (HR). HT enables the curvature to be learnable while preserving relative ordering and remaining disentangled from the normalization term, whereas HR allows the incorporation of several fundamental operations beyond linear transformation, such as dropout, activation functions, and layer normalization.

Definition 5 (Hyperbolic transformation). Given a point $\mathbf{x} \in \mathbb{L}^{d,c_1}$ (implies $\mathbf{x} \in \mathbb{R}^{d+1}$), the hyperbolic transformation is defined as following equations:

$$\text{HT}(\mathbf{x}; f_t, W, c_1, c_2) := (\alpha, \beta)^T, \quad (5)$$

$$\alpha = \sqrt{\frac{c_1}{c_2}\|f_t(\mathbf{x}; W)\|_2^2 - \frac{1}{c_2}}, \quad (\text{time dimension})$$

$$\beta = \sqrt{\frac{c_1}{c_2}}f_t(\mathbf{x}; W), \quad (\text{spatial dimension})$$

where $f_t(\mathbf{x}; W) = W^T \mathbf{x} + b$ denotes the linear transformation with transformation matrix $W \in \mathbb{R}^{(d+1) \times d'}$ and bias b . Note that c_1 and c_2 denote the curvatures before and after the transformation.

Definition 6 (Hyperbolic Readjustment and Refinement). Given a point $\mathbf{x} \in \mathbb{L}^{d,c_1}$, the hyperbolic readjustment and refinement are defined as following equations:

$$\text{HR}(\mathbf{x}; f_s, W, c_1, c_2) := (\alpha, \beta)^T, \quad (6)$$

$$\alpha = \sqrt{\frac{c_1}{c_2}\|f_s(\mathbf{x}_{[1:]})\|_2^2 - \frac{1}{c_2}}, \quad (\text{time dimension})$$

$$\beta = \sqrt{\frac{c_1}{c_2}}f_s(\mathbf{x}_{[1:]}), \quad (\text{spatial dimension})$$

where $f_s(\cdot)$ denotes a function applied to the spatial dimensions, such as dropout, activation function, and LayerNorm.

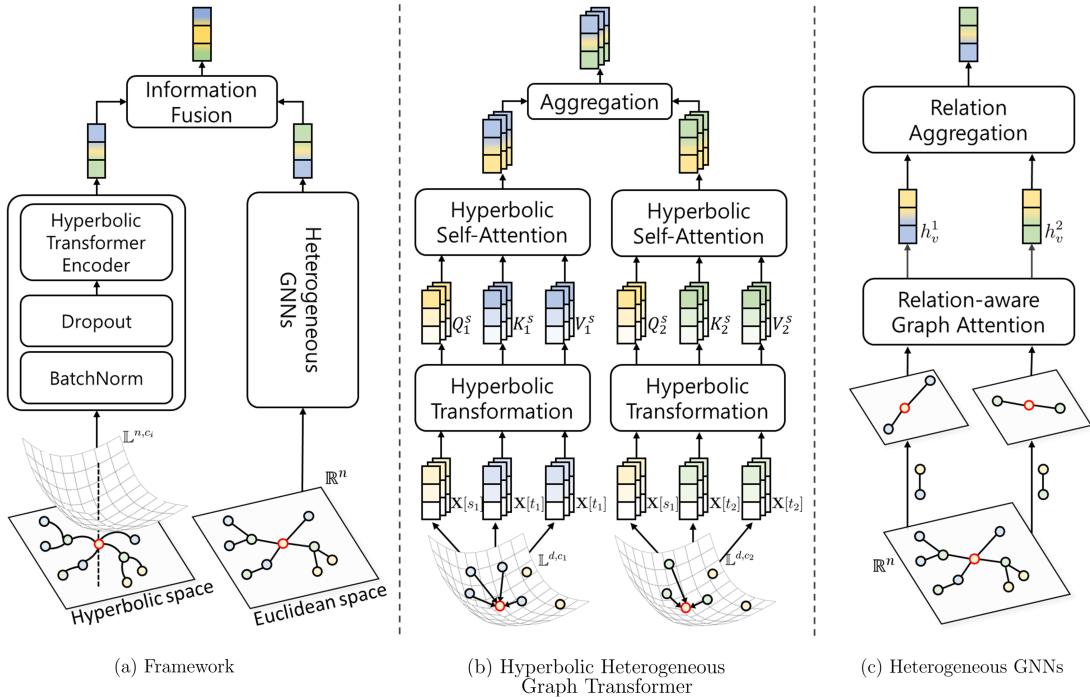


Fig. 3: (a) The overall framework of the proposed HypHGT, which consists of two main components: (b) Hyperbolic Heterogeneous Graph Transformer and (c) Heterogeneous GNNs.

IV. METHODOLOGY

In this section, we provide a detailed introduction to the proposed HypHGT.

A. Overview

Figure 3 illustrates the overall process of HypHGT. It consists of four main steps as follow:

- 1) First, a relation-aware hyperbolic heterogeneous attention mechanism is proposed to effectively capture the hierarchical structures corresponding to each relation type. Note that, our proposed attention mechanism has linear time complexity.
 - 2) Second, the representations learned for each relation type through the relation-aware hyperbolic heterogeneous attention mechanism are aggregated to obtain embeddings that captures global structural information.
 - 3) Third, the heterogeneous GNN layers are used to learn local neighbor information according to each relation type. The relation-specific neighbor representations are aggregated to obtain embeddings that represent the local neighbor information.
 - 4) Fourth, the final node embeddings are obtained by combining the local neighbor information with the global structural information through the information fusion module.

B. Hyperbolic Heterogeneous Graph Transformer

- 1) *Exponential map to an input Lorentz model*: The first step is to map Euclidean node features $x \in \mathbb{R}^n$ to an input Lorentz model \mathbb{L}^{n,c_i} via Exponential map $\exp^{c_i}(x)$:

$\mathcal{T}_o \mathbb{L}^{n,c_i} \rightarrow \mathbb{L}^{n,c_i}$, where n is a dimension of node features and c_i is a curvature of \mathbb{L}^{n,c_i} . To adopt exponential map, we assume x in the tangent space $\mathcal{T}_o \mathbb{L}^{n,c_i}$ at point $o := 0$. We denote the projection of x of into \mathbb{L}^{n,c_i} as $\mathbf{x} \in \mathbb{L}^{n,c_i}$.

2) *Relation-aware hyperbolic attention mechanism*: For given $\mathbf{X} \in \mathbb{L}^{n,c_i}$ which denotes a set of node features \mathbf{x} , we perform hyperbolic attention mechanisms between source and target nodes, where each attention mechanism is conducted within the Lorentz model corresponding to its relation type between them. Specifically, each attention mechanism is distinguished by the source node, target node, and relation type, enabling relation-specific representation learning in the hyperbolic space. We denote specific relation between a source node and a target node as $(s_{\tau(s)}, r_\epsilon, t_{\tau(t)})$, where $s_{\tau(s)}$ and $t_{\tau(t)}$ denote source and target nodes with node type $\tau(s)$ and $\tau(t)$, respectively, and r_ϵ indicates relation between them with relation type ϵ .

Given a specific relation $(s_{\tau(s)}, r_e, t_{\tau(t)})$, and \mathbf{X} , we first perform dropout and BatchNorm operations via HR function as follow equations:

$$\mathbf{X} = \text{HR}(\mathbf{X}, f_{BatchNorm}^\epsilon), \quad (7)$$

$$\mathbf{X} = \text{HR}(\mathbf{X}, f_{\text{dropout}}^\epsilon), \quad (8)$$

where $f_{BatchNorm}^\epsilon$ and $f_{dropout}^\epsilon$ denote relation-specific batch normalization and dropout function, respectively. Then, with transformation matrices W_ϵ^Q , W_ϵ^K , and $W_\epsilon^V \in \mathbb{R}^{(n+1) \times d}$, we first transform it to Q_ϵ , K_ϵ , and V_ϵ . Here, d denotes the dimension of latent representations. This process can be

formulated as below equations:

$$Q_\epsilon = \text{HT}(\mathbf{X}[s_{\tau(t)}]; f_t, W_\epsilon^Q, c_i, c_\epsilon), \quad (9)$$

$$K_\epsilon = \text{HT}(\mathbf{X}[t_{\tau(t)}]; f_t, W_\epsilon^K, c_i, c_\epsilon), \quad (10)$$

$$V_\epsilon = \text{HT}(\mathbf{X}[t_{\tau(t)}]; f_t, W_\epsilon^V, c_i, c_\epsilon), \quad (11)$$

where Q_ϵ , K_ϵ , and $V_\epsilon \in \mathbb{L}^{d,c_\epsilon}$. Here, c_ϵ is the learnable curvature of relation-specific hyperbolic space $\mathbb{L}^{d,c_\epsilon}$.

During the computation of attention output and weighted-sum aggregation, the time dimension in the Lorentz model is not included in the process. Therefore, only the spatial dimensions are used for these operations. After slicing the values along the spatial dimension, the ReLU-based kernel function is applied to ensure non-negativity and enable the decomposition of the softmax function into a linear form. This function transforms features and allows the attention mechanism to be computed in linear time, while maintaining the kernel function properties of the original softmax-based attention mechanism. This process can be formulated as follows:

$$\phi(\mathbf{X}) = \frac{\text{ReLU}(\mathbf{X}_{[1:]}) + \alpha}{\|\beta\|}, \quad (12)$$

$$Q_\epsilon^s, K_\epsilon^s, V_\epsilon^s = \phi(Q_\epsilon), \phi(K_\epsilon), \phi(V_\epsilon), \quad (13)$$

where $\phi(\cdot)$ is a kernel function, α is a small positive constant that is added to prevent numerical instability when the feature values or their norms approach zero, and β is a learnable normalization parameter used to stabilize feature scaling after the transformation. This design prevents the kernel values from exploding or vanishing during the attention mechanism, especially in the hyperbolic space, where feature magnitudes are highly sensitive to curvature.

After feature transformation with kernel function, the relation-specific attention output is computed as follows:

$$H_\epsilon^s = \frac{Q_\epsilon^s (K_\epsilon^{sT} V_\epsilon^s)}{Q_\epsilon^s (K_\epsilon^{sT} \mathbf{1})}, \quad (14)$$

where $\mathbf{1}$ is an all-ones vector and $H_\epsilon^s \in \mathbb{L}^{d,c_\epsilon}$ is the relation-specific attention output for relation type ϵ .

Given the attention output H_ϵ^s , the separated time dimension is concatenated back with H_ϵ^s (i.e., the spatial dimension) to reconstruct the full Lorentz representation, as follows:

$$H_\epsilon^t = \sqrt{\|H_\epsilon^s\|_2^2 - 1/c_\epsilon}, \quad (15)$$

$$H_\epsilon = \text{concat}(H_\epsilon^t, H_\epsilon^s), \quad (16)$$

where H_ϵ^t represents the time dimension component reconstructed to satisfy the Lorentz manifold constraint. By concatenating the reconstructed time-dimension component with the spatial-dimension component (i.e., H_ϵ^s), we obtain the Lorentz representations H_ϵ , ensuring that the relation-specific embeddings lie on the relation-specific hyperbolic space $\mathbb{L}^{d,c_\epsilon}$.

3) *Relation aggregation*: After obtaining the relation-specific representations $H_\epsilon \in \mathbb{L}^{d,c_\epsilon}$ from each relation-specific hyperbolic space, we first transform them into a unified output hyperbolic space to ensure geometric consistency across different relations types. This transform is performed through

HT function defined for each relation type as follow:

$$H'_\epsilon = \text{HT}(H_\epsilon; f_t, W_o, c_\epsilon, c_o) \quad (\forall \epsilon \in T_E), \quad (17)$$

where $H'_\epsilon \in \mathbb{L}^{d,c_o}$ are transformed relation-specific representations, $W_o \in \mathbb{R}^{(d+1) \times d}$ is the transformation matrix, and c_o is the curvature of the output Lorentz model \mathbb{L}^{d,c_o} . This transformation aligns all relation-specific representations within a unified Lorentz model, making them compatible with aggregation.

After the transformation, the representations from all relation types are aggregated through a mean operation to obtain the unified representation as follow:

$$H_T = \frac{1}{|T_E|} \sum_{\epsilon \in T_E} \log_o^{c_o}(H'_\epsilon), \quad (18)$$

where $H_T \in \mathbb{R}^{|\mathcal{V}| \times d}$ denotes output representations from hyperbolic transformer, T_E represents a set of relation types, and $\log_o^{c_o}(\cdot) : \mathbb{L}^{d,c_o} \rightarrow \mathcal{T}_o \mathbb{L}^{d,c_o}$ indicates logarithmic map, which projects H'_ϵ from \mathbb{L}^{d,c_o} onto its tangent space $\mathcal{T}_o \mathbb{L}^{d,c_o}$ at the point $\mathbf{o} := \mathbf{0}$.

4) *Multi-head attention mechanism*: We introduce multi-head attention in the hyperbolic space to enhance representations from hyperbolic heterogeneous graph transformer. Specifically, we divide attention mechanisms into K independent attention mechanisms, conduct them in parallel, and then concatenate the representations from each attention mechanisms to obtain final representation $H_T \in \mathbb{R}^{|\mathcal{V}| \times d}$. Here, \mathcal{V} denotes a set of nodes. This process can be formulated as:

$$H_T = \parallel_{k=1}^K \left(\frac{1}{|T_E|} \sum_{\epsilon \in T_E} \log_o^{c_o}(H'^k_\epsilon) \right). \quad (19)$$

C. Heterogeneous GNNs

1) *Heterogeneous graph attention mechanism*: While hyperbolic heterogeneous graph transformer effectively captures global structural information across different relation types, it is also essential to aggregate local neighbor information to enhance the semantic completeness of node representations. In heterogeneous graphs, local neighbors often contain rich type-dependent information that inherent relational semantics, which may not be fully addressed in hyperbolic heterogeneous graph transformer.

To address this, we use heterogeneous GNNs that perform localized message-passing among different types of neighbors. For a given embedding target node v and its neighbors $\mathcal{N}_\epsilon(v)$ connected through the relation type ϵ , the attention score $\alpha_{v,u}^\epsilon$ between v and the node $u \in \mathcal{N}_\epsilon(v)$ is computed as:

$$\alpha_{v,u}^\epsilon = \frac{\exp(\mathbf{a}_\epsilon^\top \text{LeakyReLU}(W_\epsilon \cdot [h_v || h_u]))}{\sum_{k \in \mathcal{N}(v)} \exp(\mathbf{a}_\epsilon^\top \text{LeakyReLU}(W_\epsilon \cdot [h_v || h_k]))} \quad (20)$$

where $h_v, h_u \in \mathbb{R}^n$ is denotes features of node v and u , respectively, $W_\epsilon \in \mathbb{R}^{d \times 2n}$ denotes relation-specific transformation matrix and \mathbf{a}_ϵ is a learnable attention vector used to compute weights for relation type ϵ .

Algorithm 1 Hyperbolic heterogeneous graph transformer

Require: The set of node features X , The set of relation types $T_{\mathcal{E}}$, The number of attention heads K .

- 1: **(1) Exponential map to input Lorentz model**
- 2: $\mathbf{X} \leftarrow \exp_o^{c_i}(X)$;
- 3: **for** $k = 1, \dots, K$ **do**
- 4: **(2) Relation-aware hyperbolic attention mechanism**
- 5: **for** $\epsilon \in T_{\mathcal{E}}$ **do**
- 6: $\mathbf{X} \leftarrow \text{HR}(\mathbf{X}, f_{BatchNorm}^{\epsilon})$;
- 7: $\mathbf{X} \leftarrow \text{HR}(\mathbf{X}, f_{dropout}^{\epsilon})$;
- 8: $Q_{\epsilon} \leftarrow \text{HT}(\mathbf{X}[s_{\tau(t)}]; f_t, W_{\epsilon}^Q, c_i, c_{\epsilon})$;
- 9: $K_{\epsilon} \leftarrow \text{HT}(\mathbf{X}[t_{\tau(t)}]; f_t, W_{\epsilon}^K, c_i, c_{\epsilon})$;
- 10: $V_{\epsilon} \leftarrow \text{HT}(\mathbf{X}[t_{\tau(t)}]; f_t, W_{\epsilon}^V, c_i, c_{\epsilon})$;
- 11: $Q_{\epsilon}^s, K_{\epsilon}^s, V_{\epsilon}^s \leftarrow \phi(Q_{\epsilon}), \phi(K_{\epsilon}), \phi(V_{\epsilon})$;
- 12: $H_{\epsilon}^s \leftarrow \frac{Q_{\epsilon}^s (K_{\epsilon}^{sT} V_{\epsilon}^s)}{Q_{\epsilon}^s (K_{\epsilon}^{sT} \mathbf{1})}$;
- 13: $H_{\epsilon}^t \leftarrow \sqrt{\|H_{\epsilon}^s\|_2^2 - 1/c_{\epsilon}}$;
- 14: $H_{\epsilon} \leftarrow \text{concat}(H_{\epsilon}^t, H_{\epsilon}^s)$;
- 15: $H_{\epsilon}^k \leftarrow \text{HT}(H_{\epsilon}; f_t, W_o, c_{\epsilon}, c_o)$;
- 16: **end for**
- 17: **end for**
- 18: **(3) Relation-aggregation and Multi-head attention**
- 19: $H_{\mathcal{T}} \leftarrow \|\sum_{k=1}^K \frac{1}{|T_{\mathcal{E}}|} \sum_{\epsilon \in T_{\mathcal{E}}} \log_o^{c_o}(H_{\epsilon}^k)\right)$;
- 20: **return** $H_{\mathcal{T}}$;

After obtaining attention scores, the relation-specific embedding of the target node v for relation type ϵ is obtained by aggregating the neighbor features weighted by these attention scores:

$$h_v^{\epsilon} = \sigma \left(\sum_{u \in \mathcal{N}(v)} \alpha_{v,u}^{\epsilon} W_{\epsilon} h_u \right), \quad (21)$$

where σ denotes a activation function and $h_v^{\epsilon} \in \mathbb{R}^d$ denotes the relation-specific embedding.

2) *Relation aggregation:* Finally, to obtain a unified node representation that aggregates information from all relation types, the relation-specific embeddings are aggregated through a mean operation:

$$h'_v = \frac{1}{|T_{\mathcal{E}}|} \sum_{\epsilon \in T_{\mathcal{E}}} h_v^{\epsilon}, \quad (22)$$

where $h'_v \in \mathbb{R}^d$ denotes embeddings of node v obtained from Heterogeneous GNNs. This aggregation enables HypHGT to learn type-dependent neighbor information while preserving the heterogeneity of relations.

Furthermore, heterogeneous GNNs performs a multi-head attention mechanism, which can be formulated as follows.

$$h'_v = \|\sum_{k=1}^K \left(\frac{1}{|T_{\mathcal{E}}|} \sum_{\epsilon \in T_{\mathcal{E}}} h_v^{\epsilon,k} \right)\right). \quad (23)$$

Algorithm 2 Heterogeneous GNNs and Information Fusion

Require: Embedding target node v , The set of neighbor nodes of v $\mathcal{N}(v)$, The set of relation types $T_{\mathcal{E}}$, The number of attention heads K , lambda λ .

- 1: **(1) Heterogeneous graph attention**
- 2: **for** $\epsilon \in T_{\mathcal{E}}$ **do**
- 3: **for** $k = 1, \dots, K$ **do**
- 4: **for** $u \in \mathcal{N}(v)$ **do**
- 5: $a_{v,u}^{\epsilon} \leftarrow \mathbf{a}_{\epsilon}^{\top} \text{LeakyReLU}(W_{\epsilon} \cdot [h_v \| h_u])$;
- 6: $\alpha_{v,u}^{\epsilon} \leftarrow \text{softmax}(\exp(a_{v,u}^{\epsilon}))$
- 7: **end for**
- 8: $h_v^{\epsilon} \leftarrow \sigma \left(\sum_{u \in \mathcal{N}(v)} \alpha_{v,u}^{\epsilon} W_{\epsilon} h_u \right)$
- 9: **end for**
- 10: **(2) Relation aggregation**
- 11: $h_v'^k \leftarrow \frac{1}{|T_{\mathcal{E}}|} \sum_{\epsilon \in T_{\mathcal{E}}} h_v^{\epsilon}$
- 12: **end for**
- 13: $h'_v \leftarrow \|\sum_{k=1}^K h_v'^k\|$
- 14: **(3) Information fusion**
- 15: $z_v \leftarrow \lambda \cdot (H_{\mathcal{T}}(v)) + (1 - \lambda) \cdot h'_v$
- 16: **return** z_v ;

D. Information Fusion

To obtain final node representations, we integrate the global structural information learned from the hyperbolic heterogeneous graph transformer and the local neighbor information learned from the heterogeneous GNNs. This information fusion allows HypHGT to capture both global hierarchical semantics and local relationships within heterogeneous graphs. For given a embedding target node v , the corresponding final node embedding z_v is defined through this information fusion as follows:

$$z_v = \lambda \cdot (H_{\mathcal{T}}(v)) + (1 - \lambda) \cdot h'_v, \quad (24)$$

where, $H_{\mathcal{T}}(v) \in H_{\mathcal{T}}$ and h'_v represent the embeddings of node v which obtained from hyperbolic heterogeneous graph transformer and heterogeneous GNNs, respectively, and $\lambda \in [0, 1]$ is a hyperparameter that adjust magnitude of global and local information. In this paper, we set λ to 0.5.

E. Model Training

We use the following linear transformation with non-linear activation function $f(\cdot)$ to map node embeddings $z_v \in \mathbb{R}^d$ into a vector space with the desired output dimension, conducting node classification task:

$$f(z_v) = \sigma(W_o \cdot z_v), \quad (25)$$

where $W_o \in \mathbb{R}^{d_o \times d}$ denotes transformation matrix, d_o denotes the dimension of output vector, and σ is the non-linear activation function.

For node classification, we trained HypHGT by minimizing cross-entropy loss \mathcal{L} expressed as

$$\mathcal{L} = - \sum_{v \in V_t} \sum_{c=1}^C y_v[c] \cdot \log(f(z_v)[c]), \quad (26)$$

where v_t is the target node set extracted from the labeled node set, C is the number of classes, y_v is the one-hot encoded label vector for node v , and $f(z_v)$ is a vector predicting the label probabilities of node v .

F. Time Complexity

HypHGT consists of two main components for heterogeneous graph representation learning: hyperbolic heterogeneous graph transformer and heterogeneous GNNs. As shown in Equation 14, the hyperbolic heterogeneous graph transformer performs an inner product between K_ϵ^{sT} and V_ϵ^s within the Lorentz model corresponding to each relation type ϵ . This process has a time complexity of $O(|T_\epsilon| \cdot d^2 \cdot |\mathcal{T}|)$, where T_ϵ and \mathcal{T} denotes a set of relation types and target nodes, respectively. Subsequently, an inner product is performed between Q_ϵ^s and the output of $K_\epsilon^{sT} V_\epsilon^s$, which has a time complexity of $O(|T_\epsilon| \cdot d^2 \cdot (|\mathcal{S} + \mathcal{T}|))$, where \mathcal{S} denotes set of source nodes. Therefore, the total time complexity of hyperbolic heterogeneous graph transformer is $O(|\mathcal{S} + \mathcal{T}|) \approx O(N)$. Meanwhile, the time complexity of heterogeneous GNNs is $O(|T_\epsilon| \cdot (N+E))$. Consequently, our proposed model HypHGT has a linear-time complexity overall, which enables efficient and scalable heterogeneous graph representation learning even on large-scale graphs.

V. EXPERIMENTAL EVALUATIONS

In this section, we analyze the efficiency of our proposed model HypHGT, through with four real-world heterogeneous graph datasets and synthetic heterogeneous graphs. We compare HypHGT with several state-of-the-art heterogeneous graph embedding models. Our experiments are conducted to answer the following research questions (RQs).

- **RQ1** : Does HypHGT outperform the state-of-the-art methods in node classification task?
- **RQ2** : How does each main component of HypHGT contribute to the learning of heterogeneous graph representations?
- **RQ3** : How do multiple hyperbolic spaces in HypHGT affect heterogeneous graph representation learning?
- **RQ4** : Is HypHGT more efficient than existing heterogeneous graph embedding models?

A. Datasets

To evaluate the performance of HypHGT on node classification task, we use three-real world heterogeneous graph datasets. Table II shows the statistics of these datasets. Additionally, detailed description of the datasets are provided as below:

- **IMDB¹** is an online database pertaining to movies and television programs. It comprises three types of nodes {Movie (M), Director (D), Actor (A)} and two types of links {MD, MA}. The movie type nodes are labeled into three classes based on the movie's genre {Action, Drama,

TABLE II: Statistics of real-world datasets

Dataset	# Nodes	# Links	# Classes	# Features
IMDB	Movie (M) : 4,661	M-D : 4,661	3	1,256
	Director (D) : 2,270	M-A : 13,983		
	Actor (A) : 5,841			
DBLP	Author (A) : 4,057	A-P : 19,645	4	334
	Paper (P) : 14,328	P-C : 14,328		
	Conference (C) : 20			
ACM	Paper (P) : 3,020	P-A : 9,936	3	1,902
	Author (A) : 5,912	P-S : 3,025		
	Subject (S) : 57			

Comedy}. The features of nodes are represented as bag-of-words of keywords.

- **DBLP²** is a citation network that comprises three types of nodes {Author (A), Paper (P), Conference (C)} and two types of links {AP, PC}. The author type nodes are labeled into four classes based on the author's research area {Database, Data Mining, Machine Learning, Information Retrieval}. The features of nodes are represented as bag-of-words of keywords.
- **ACM³** is a citation network that comprises three types of nodes {Paper (P), Author (A), Subject (S)} and two types of links {PA, PS}. The paper type nodes are labeled into three classes based on the paper's subject area {Database, Wireless Communication, Data Mining}. The features of nodes are represented as bag-of-words of plots.

B. Competitors

We compare HypHGT with several state-of-the-art GNNs categorized four groups:

- i) **Euclidean homogeneous models**: GCN, and GAT.
- ii) **Hyperbolic homogeneous models**: HGCN, and Hypformer.
- iii) **Euclidean heterogeneous models**: HAN, MAGNN, GTN, HGT, and Simple-HGN.
- iv) **Hyperbolic heterogeneous models**: SHAN, HHGAT, and MSGAT.

For homogeneous models, features are processed to be homogeneous for pair comparison with heterogeneous models. Details of the competitors are provided as follows:

- **GCN** [44] performs graph convolution operations in the Fourier domain for homogeneous graphs.
- **GAT** [45] introduces graph attention mechanisms into the graph convolution operation for homogeneous graphs.
- **HGCN** [20] proposes graph neural networks that uses the hyperbolic space as an embedding space to effectively learn complex structures.
- **Hypformer** [43] proposes a hyperbolic graph transformer architecture in which operations are performed directly within the hyperbolic space, reducing mapping distortions and achieving efficient representation learning with linear-time complexity.

²<https://dblp.uni-trier.de/>

³<https://dl.acm.org/>

¹<https://www.imdb.com/>

TABLE III: Experimental results (%) for the node classification task. The best and second-best performers are bolded and underlined, respectively.

Dataset	Metric	i		ii		iii					iv			
		GCN	GAT	HGCN	Hypformer	HAN	MAGNN	GTN	HGT	Simple-HGN	SHAN	HHGAT	MSGAT	HypHGT
IMDB	Macro-F1	54.38	57.06	57.98	65.12	59.26	61.13	62.73	62.87	67.02	66.75	66.38	<u>68.91</u>	70.47
	Micro-F1	54.89	57.53	58.68	68.15	60.10	61.89	64.26	63.29	69.31	69.99	70.28	<u>70.45</u>	72.56
DBLP	Macro-F1	89.53	91.29	92.70	94.18	92.81	93.62	93.83	93.96	94.11	94.46	93.76	94.51	95.68
	Micro-F1	90.06	92.15	93.39	94.36	93.43	94.28	94.18	94.02	94.73	94.98	94.56	<u>95.28</u>	96.04
ACM	Macro-F1	88.76	88.95	90.26	93.81	91.26	91.29	92.54	91.79	93.40	93.71	93.62	<u>93.84</u>	94.24
	Micro-F1	88.53	89.06	90.71	93.90	92.47	92.70	92.56	92.07	93.13	<u>94.32</u>	93.14	93.95	94.35

- **HAN** [29] proposes a graph attention network for heterogeneous graphs, which incorporates both node-level and semantic-level attention mechanisms.
- **MAGNN** [30] introduces intra-metapath and inter-metapath aggregation to incorporate intermediate semantic nodes and multiple metapaths, respectively.
- **GTN** [31] transforms heterogeneous graphs into multiple metapath graphs through Graph Transformer layers, facilitating more effective node representations.
- **HGT** [33] proposes a heterogeneous subgraph sampling algorithm to model Web-scale graph data and utilize node and link-type dependent parameters to capture heterogeneous relations.
- **Simple-HGN** [34] extends graph attention mechanism by including link type information into attention score calculation, enabling Simple-HGN to handle multiple type of links in heterogeneous graphs.
- **SHAN** [22] proposes hyperbolic heterogeneous graph attention networks to learn multi-order relations from simplicial complexes sampled from heterogeneous graphs.
- **HHGAT** [23] introduces hyperbolic heterogeneous graph attention networks with a single hyperbolic space to learn complex structures within heterogeneous graphs.
- **MSGAT** [24] proposes multi hyperbolic space-based heterogeneous graph attention networks to learn various complex structures based on metapaths.

C. Implementation Details

For the baselines including HypHGT, we randomly initialize model parameters and use the adamW [46] optimizer with a learning rate of 0.0001 and weight decay of 0.0005. We set the dropout rate to 0.5, and the dimensional of final node embedding to 64. For multi-head attention baselines, the number of attention heads for each model was set to 8. For HypHGT, the number of heads in the hyperbolic transformer was set to 2, while the number of heads in the heterogeneous GNNs was set to 8. The baseline models are trained for 300 epochs, and the model with the lowest validation loss is used for testing. For the metapath-based heterogeneous graph embedding models, the metapath settings follow the specifications outlined in their papers. All downstream tasks were conducted ten times, and we report the average values of evaluation metrics.

D. Node Classification (RQ1)

Node classification was performed by applying support vector machines on embedding vectors of labeled nodes.

As shown in Table III, HypHGT achieves the best performance across all datasets (i.e., IMDB, DBLP and ACM) in both Macro-F1 and Mirco-F1 scores, outperforming all baselines. These results demonstrate the effectiveness of our proposed model HypHGT in learning both local and global structural information within heterogeneous graphs. We discuss three main empirical findings based on the results.

First, the comparison between HypHGT and Hypformer highlights the importance of learning heterogeneous relational information within heterogeneous graphs. While Hypformer demonstrates strong performance as a hyperbolic transformer model, it is designed for homogeneous graphs and therefore lacks the ability to learn semantic information inherent in multiple relation types within heterogeneous graphs. In contrast, HypHGT extends this transformer architecture to heterogeneous graphs by introducing relation-specific attention mechanism and distinct hyperbolic space for each relation type. This allows HypHGT to effectively learn relation-dependent semantic representations, leading to significant improvements.

Second, the comparison between Euclidean heterogeneous models (i.e., category iii) and hyperbolic heterogeneous models (i.e., category iv) demonstrates distinct ability to learn hierarchical structures within heterogeneous graphs. Since the Euclidean space expands polynomially, Euclidean heterogeneous models such as GTN, HGT struggle to represent hierarchical structures that are observed in heterogeneous graphs. In contrast, hyperbolic heterogeneous models use the hyperbolic space as an embedding space, which expands exponentially and thus enables a more effective representation of hierarchical structures.

Finally, compared to previous hyperbolic heterogeneous models that rely heavily on tangent space, HypHGT performs attention mechanisms entirely within the hyperbolic space. This fully hyperbolic space operations minimizes mapping distortions and enables more stable and efficient heterogeneous graphs representation learning.

E. Ablation Study (RQ2)

We compose three variants of HypHGT to validate the effectiveness of each component of HypHGT. Specifically, 1) **HypHGT_{EUC}** replaces the hyperbolic space with Euclidean space as the embedding space, 2) **HypHGT_{w/o GNN}** removes

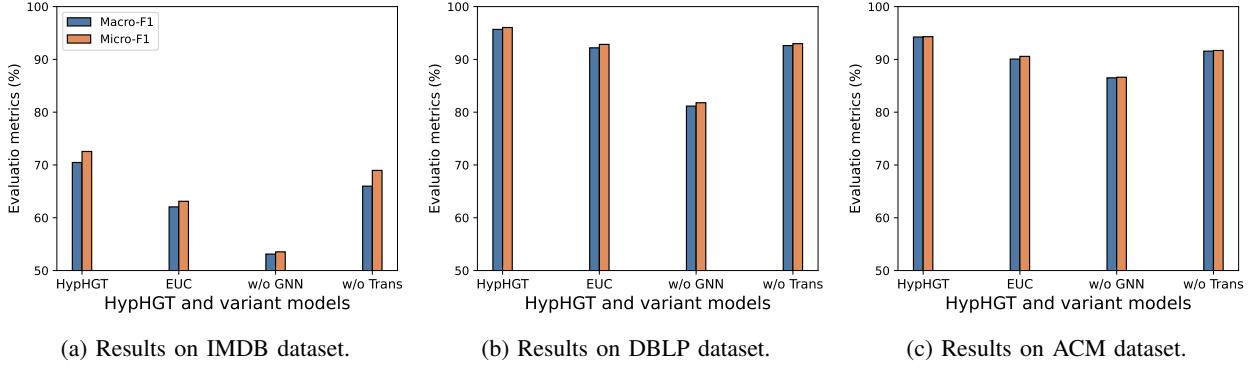


Fig. 4: Results of the ablation study.

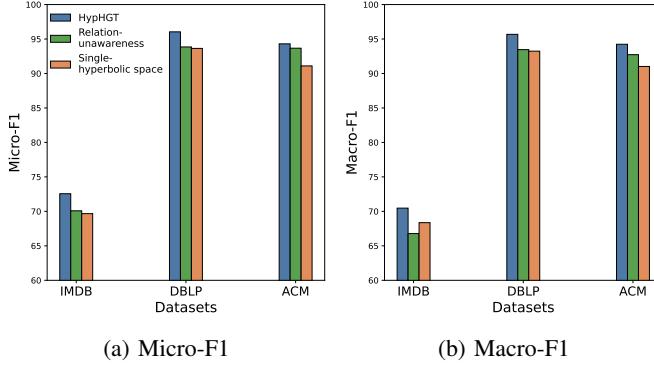


Fig. 5: Node classification accuracy according to the configuration of hyperbolic spaces.

heterogeneous GNN layers, and 3) **HypHGT_{w/o Trans}** removes hyperbolic heterogeneous graph transformer layers. Figure 4 represents the results of the ablation study.

First, when comparing HypHGT with **HypHGT_{EUC}**, the results demonstrate that the hyperbolic space is more effective in capturing the hierarchical structures within heterogeneous graphs. Second, the comparison between HypHGT and **HypHGT_{w/o GNN}** demonstrates that, since graphs are inherently composed of local relations between entities, using the hyperbolic heterogeneous graph transformer alone fails to capture local neighborhood information, which is a critical limitation. Consequently, the **HypHGT_{w/o GNN}** variant shows the most significant performance decrease. Finally, when comparing HypHGT with **HypHGT_{w/o Trans}**, although **HypHGT_{w/o Trans}** achieves competitive performance, it still struggles to capture the global hierarchical structures. This limitation arises because the graph convolution operations in GNNs are primarily localized within neighborhoods, making it difficult to capture long-range dependencies in heterogeneous graphs. In contrast, HypHGT overcomes this limitation by integrating the hyperbolic heterogeneous graph transformer with heterogeneous GNNs, enabling the model to capture both global structural information and local neighborhood dependencies effectively.

F. Hyperbolic Space Analysis (RQ3)

First, to analyze the effectiveness of using multiple hyperbolic spaces for each relation type, we consider two variants. Figure 5 represents the results of the hyperbolic space analysis and the details of each variant are as follows:

- **Relation unawareness** is the variant that uses the input and output hyperbolic spaces as HypHGT. However, this model learns all relation-specific information within a single hyperbolic space rather than multiple relation-specific hyperbolic spaces.
- **Single hyperbolic space** is the variant that performs all operations for obtaining embeddings within a single hyperbolic space, without distinguishing between different relation-specific and input/output hyperbolic spaces.

Note that unlike the above variants, HypHGT uses both input/output hyperbolic spaces and relation-specific hyperbolic spaces for each relation type. As shown in Figure 5, the results demonstrate the effectiveness of relation-specific hyperbolic spaces of HypHGT across all datasets. Both in Micro-F1 and Macro-F1 variants show significant performance decrease. This results demonstrate that learning multiple relational distributions into a single hyperbolic space limits the ability to represent hierarchical structures corresponding to specific relation type effectively. In contrast, HypHGT, which learns relation-aware representations through multiple relation-specific hyperbolic spaces, effectively preserves the diversity of hierarchical structures corresponding to each relation type.

Second, Figure 6 illustrates how the learnable curvatures of relation-specific hyperbolic spaces converge to their optimal values on the IMDB and DBLP datasets during HypHGT training. The number of training epochs is set to 50, and the vertical red dashed lines indicate the local optimal values achieved within 50 epochs. Figure 6(b), (c) and Figure 6(e), (f) show the link-degree distributions of each relation corresponding their underlying structures in IMDB and DBLP datasets, respectively. In the IMDB dataset, as shown in Figure 6(b) and (c), both Movie-Actor (M-A) and Movie-Director (M-D) relations shows similar power-law degree distributions, indicating comparable hierarchical structures. Accordingly, as shown in Figure 6(a), the curvatures of M-A and M-D relations

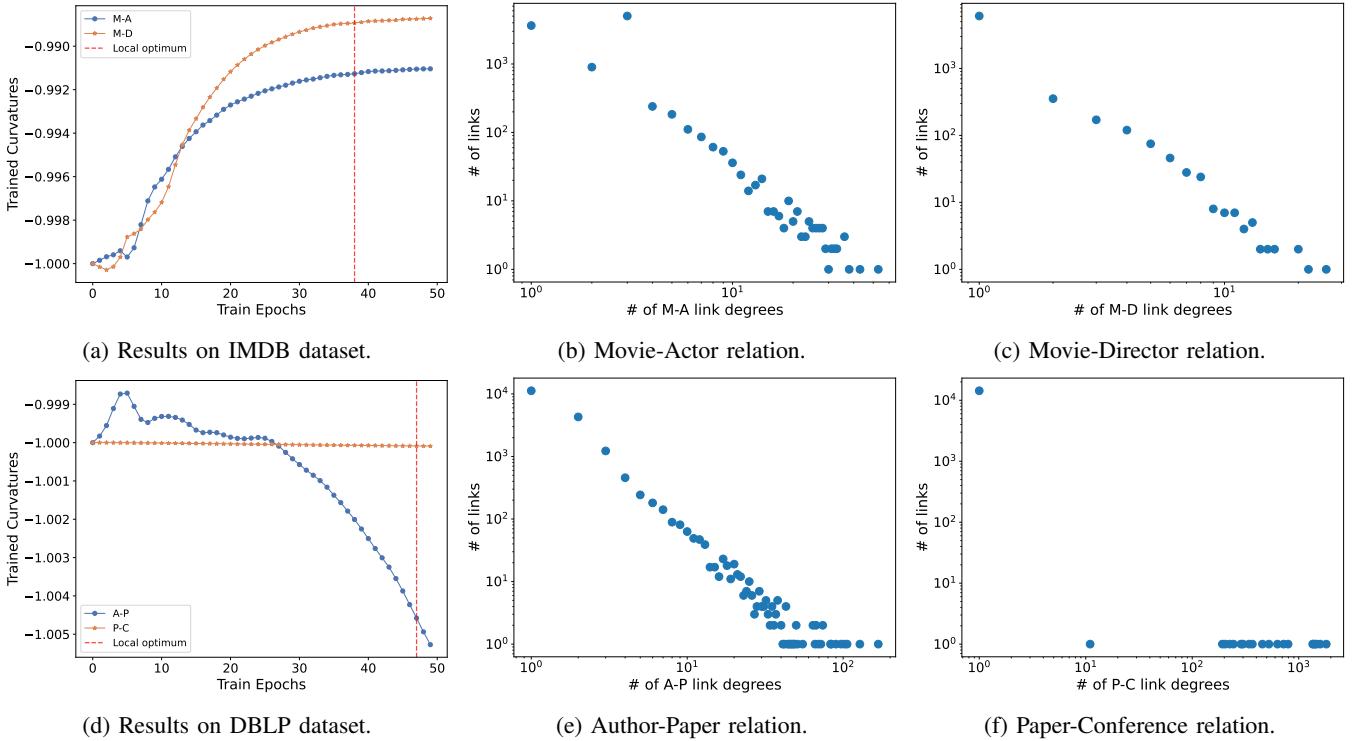


Fig. 6: Curvature variations of relation-specific hyperbolic spaces over train epochs on the IMDB and DBLP dataset.

converge to nearly similar values, demonstrating that HypHGT learns the curvature of each relation-specific hyperbolic space in accordance with its hierarchical structure.

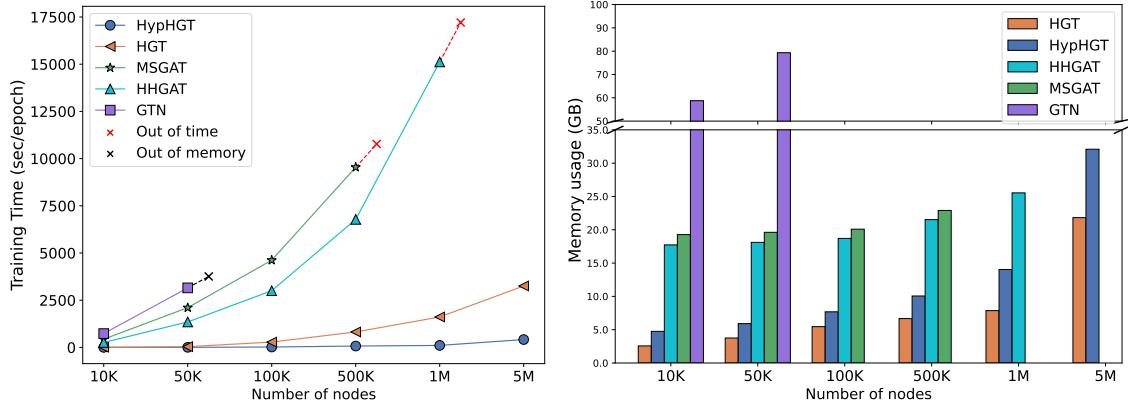
In contrast, in the DBLP dataset, the Author-Paper (A-P) relation (Figure 6(e)) shows a power-law degree distribution, where as the Paper-Conference relation (Figure 6(f)) shows a flatter distribution with weak hierarchical properties. Consistent with this observation, as shown in Figure 6(d), the curvature of the A-P relation decreases substantially to capture its hierarchical structures, while the curvature of P-C relation almost unchanged from its initial curvature. This result demonstrate that HypHGT adaptively adjusts the curvature of relation-specific hyperbolic space. These results highlight that the learnable curvature in HypHGT is closely aligned with the structural properties of each relation rather than being uniform across all relations, and demonstrate that, through the proposed relation-specific hyperbolic spaces, HypHGT can effectively learn relation-specific representations that capture the distinct structural properties of each relation type.

G. Scalability and Complexity Analysis (RQ4)

To analyze the scalability of HypHGT, we generate a synthetic heterogeneous graph using the Barabasi-Albert model [47], which captures the scale-free property commonly observed in real-world networks. First, we generate a power-law distributed homogeneous graph with the Brabasi-Albert model. Then, to give heterogeneity, we randomly assign node types to all nodes within this homogeneous graph, resulting in a heterogeneous graph. Finally, the generated synthetic

heterogeneous graph consists of three node types {A,B,C} in a ratio of 6:3:1 and two link types {AB, AC}. The number of nodes is varied from 10K to 5M, allowing us to construct heterogeneous graphs of different scales and evaluate the time and memory consumption of the model under increasing graph scales. We set the batch size to 512 for training on large-scale synthetic heterogeneous graphs.

Figures 7(a) and 7(b) illustrate the scalability analysis of HypHGT in terms of time and memory consumption, respectively, with respect to number of nodes in the synthetic heterogeneous graphs. In Figure 7(a), HypHGT shows near-linear growth in training time as the graph size increases from 10K to 5M nodes, while other baselines such as GTN, HHGAT, and MSGAT suffer from exponential increases in computation time, with several failing to complete training due to out-of-time or out-of-memory issues. In contrast, HGT, which was proposed to learn web-scale heterogeneous graphs, is able to scale effectively even as the graph size increases. Figure 7(b) further confirms that HypHGT maintains consistently low memory usage across all scales, while MSGAT and GTN consume a large amount of memory with increasing graph size. These empirical results are well aligned with the theoretical analysis in Section IV-F: the near-linear training time validates the overall $O(N)$ complexity of HypHGT induced by the linear hyperbolic attention mechanism. Moreover, as shown in Figure 8, HypHGT achieves high accuracy with significantly lower training time and memory consumption compared to other baselines, demonstrating its efficiency in learning real-world heterogeneous graphs.



(a) Time consumption for varying number of nodes. (b) Memory consumption for varying number of nodes.

Fig. 7: Scalability analysis on synthetic heterogeneous graphs.

H. Hyperparameter Sensitivity Analysis

We investigate the impact of hyperparameters used in HypHGT and report the performance on node classification using the IMDB dataset. As shown in Figure 9(a), the performance increases as the number of hyperbolic heterogeneous graph transformer heads increases, reaching its peak at eight heads. However, when the number exceeds eight, the performance decreases. Similarly, in Figure 9(b), the performance improves as the number of heterogeneous GNN heads increases, peaking at eight heads. Beyond eight, the performance decreases. Next, as shown in Figure 9(c) and (d), increasing the number of hyperbolic heterogeneous graph transformer layers and heterogeneous GNNs layers enhances performance up to three. However, when the number of each layer increases beyond three, the performance starts to decrease. Figure 9(e) shows that, as the dimension of node embedding increases, the performance of HypHGT improves, reaching its best performance when the dimension is 64. However, beyond this peak, the performance decreases steadily. Figure 9(f) shows that a λ value of 0.5 results in the optimal performance on the IMDB dataset. λ controls the balance between the representations learned by the hyperbolic heterogeneous graph transformer and the heterogeneous GNNs. When λ decreases, HypHGT relies more on the representations from the heterogeneous GNNs to obtain the final node embeddings. Conversely, as λ increases, it relies more on the representations from the hyperbolic heterogeneous graph transformer. Notably, when λ is set to 1, HypHGT depends solely on the hyperbolic heterogeneous graph transformer to obtain the node embeddings. Similar to the observation in the ablation study, this setting (i.e., $\lambda = 1$) leads to a significant performance decrease, as it fails to capture the local neighbor information.

VI. CONCLUSION

In this paper, we proposed Hyperbolic Heterogeneous Graph Transformer (HypHGT), a novel model for efficient and effective heterogeneous graph representation learning in the

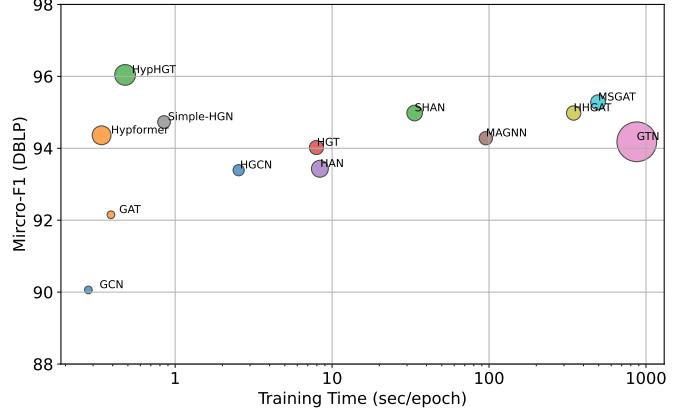


Fig. 8: Time and memory comparison for baselines on DBLP dataset. The area of the circles represents the relative memory consumption.

hyperbolic space. Unlike previous hyperbolic heterogeneous embedding models that rely on tangent space and predefined metapath structures, HypHGT performs hyperbolic attention mechanism entirely within relation-specific hyperbolic spaces and avoids the requirement for dataset-dependent metapath definitions. Specifically, we propose a linear hyperbolic heterogeneous attention mechanism that enables relation-aware representation learning in multiple hyperbolic spaces, significantly reducing time and memory consumption compared to previous hyperbolic heterogeneous graph embedding models. Furthermore, by integrating heterogeneous GNN layers with the hyperbolic transformer layers, HypHGT effectively captures both global hierarchical structures and neighbor information, leading to semantically rich node representations.

Comprehensive experiments on real-world and synthetic heterogeneous graphs demonstrated that HypHGT consistently outperforms state-of-the-art baselines in node classification tasks while maintaining superior efficiency. Ablation and hyperbolic space analysis confirmed that relation-specific hyper-

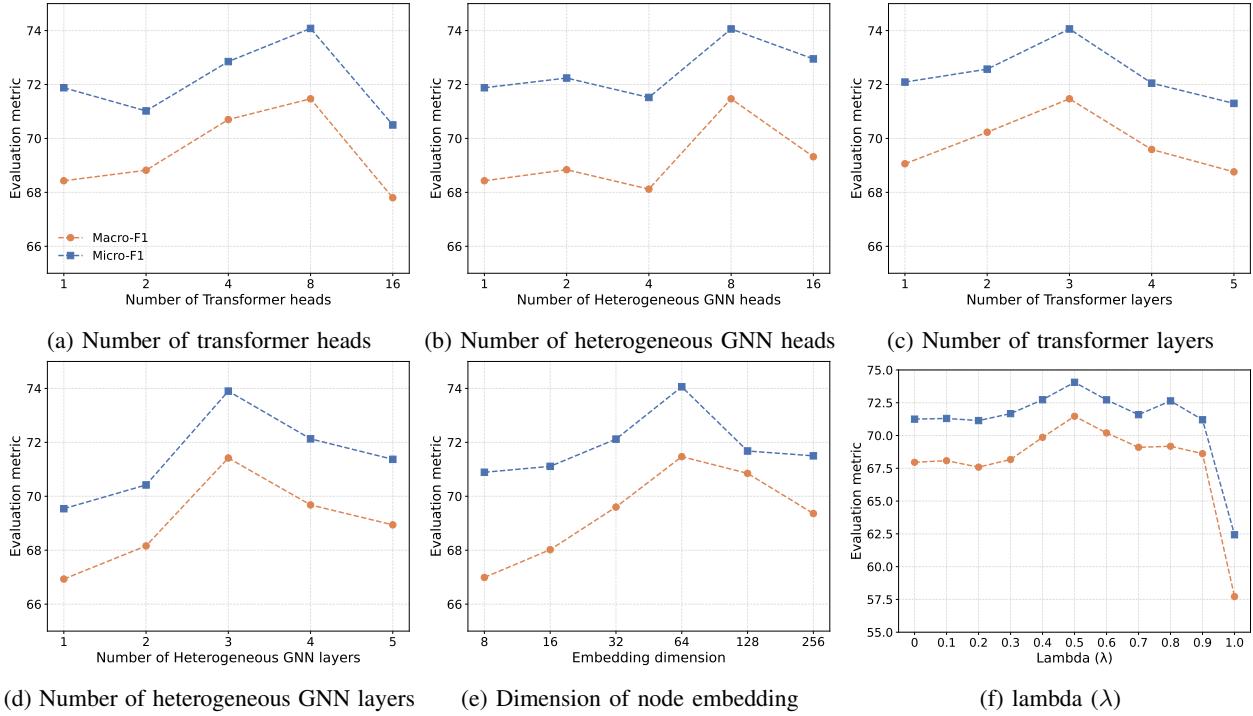


Fig. 9: Hyperparameter sensitivity of HypHGT.

bolic spaces and joint learning of local and global information are key to achieving this improvement.

Potential avenues of future research include 1) extending HypHGT to dynamic heterogeneous graphs and its application to cross-domain scenarios where relational distributions change over time and 2) investigating curvature-adaptive training strategies for improving stability and interpretability of hyperbolic representations in heterogeneous graph learning.

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