**Exercise 4.67**

If the joint density function of X and Y is given by:

Find the expected value of . Additionally, determine whether X and Y are independent.

**Solution**

**Part A**

We start by breaking the above equation for g(X,Y) into two parts:

The expected values for both components can be found by integrating f(x,y) multiplied by g(X,Y)

Utilizing Wolfram Alpha to solve, we get:

Combing these two we get the expected value:

**Part B**

Variables are independent if:

Utilizing Wolfram Alpha, we determine both g(x) and h(y)

Multiplying these two together we get:

This does not equal the original equation f(x,y), therefore X and Y are not independent.

**Exercise 4.76**

Seventy new jobs are opening up at an automobile manufacturing plant, and 1000 applicants show up for the 70 positions. To select the best 70 from among the applicants, the company gives a test that covers mechanical skill, manual dexterity, and mathematical ability. The mean grade on this test turns out to be 60, and the scores have a standard deviation of 6. Can a person who scores 84 count on getting one of the jobs?

**Solution**

Chebyshev’s theorem states:

A person scoring an 84 has a score above the mean, because of this, we can isolate one part of the above equation in order to solve for k:

Utilizing this value for k, we can insert values into the original equation:

We can also expand the above to include all scores below 84:

From here, we take the complement of this probability to get:

To determine how many applicants have a score of 84 or above, we multiple this number by the number of applicants:

From this, we can say that at most 63 candidates have a score greater than or equal to 84. As there are 70 open positions, the applicant can count on getting one of the jobs.

**Summary**

In these exercises we determined the expected value for a function g(X,Y) given the original function f(x,y). We then determined, based off the original f(x,y) equation, whether the variables X and Y were independent, and found that they were not. In the second exercise, we utilized Chebyshev’s theorem to determine whether a prospective applicant would receive a job offer based on their test scores. After finding the expected percentage of applicants that would score equal to or better than the applicant and what percentage of applicants would be accepted, we determined that the applicant would be receiving the job offer.