**Exercise 5.55**

The probability that a student pilot passes the written test for a private pilot’s license is 0.7. Find the probability that a given student will pass the test:

1. On the third try
2. Before the fourth try

**Solution**

**Part A**

Part a can be solved by utilizing the geometric distribution, where the probability of an event occurring for the first time on the xth try is given by the formula:

Where n is the total number of occurrences, p is the probability of success, and q is the probability of failure, or 1-p. For this problem, x=3, p=0.7, and q =0.3. Plugging into the equation we get:

Therefore, the probability that a given student will first pass the test on their third attempt is 0.063.

**Part B**

Part b can be solved by summing geometric probabilities. Specifically, the probability that the student passes before the fourth try is equal to the probability that they pass on the first time, plus the probability that they pass on the second time, plus the probability that they pass on the third time. In equation form, this equals:

Utilizing the formula for the geometric distribution, this can be expressed as:

Therefore, the probability that a given student will pass before their fourth try is 0.973.

**Exercise 6.11**

A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters:

1. What fraction of the cups will contain more than 224 milliliters?
2. What is the probability that a cup contains between 191 and 209 milliliters?
3. How many cups will probably overflow if 230-milliliter cups are used for the next 1000 trials?
4. Below what value do we get the smallest 25% of the drinks?

**Solution**

**Part A**

The Z-score for the given problem can be calculated using the following formula:

Plugging the numbers for this problem in yields:

The probability of the Z-score being greater than this number can be found via a Z-table:

Therefore, the probability that any given cup will contain more than 224 milliliters is 0.0548.

**Part B**

The first step is to find the Z-scores of the two given values:

From this, we can use a Z-table to find the requested probability:

Therefore, the probability that a cup will contain between 191 and 209 milliliters is 0.4514.

**Part C**

We must first determine the probability that a cup will be filled with more than 230 milliliters. The Z-score for this is:

Which gives the formula:

To determine the expected value, we multiply the probability with the number of trials. Which gives:

Therefore, as there are no partial cups, approximately 23 cups will overflow out of every 1000.

**Part D**

Finding the smallest 25% of values can be done by finding the z-score which is closest to 0.25. This z-value is -0.67. Reverse engineering the X value from the z-score can be done by the formula:

Plugging the values in we get:

Therefore, values smaller than 189.95 correspond to the smallest 25% of values

**Exercise 8.25**

The average like of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution find:

1. The probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years
2. The value of x to the right of which 15% of the means computed from random samples of size 9 would fall

**Solution**

**Part A**

First, we must calculate the sample standard deviation utilizing the following formula:

Plugging in the values for this problem we get:

With this, we can calculate the Z-score using the equation:

For this problem we get the following Z-scores:

Utilizing a Z-score table we get:

Therefore, the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years is 0.6898

**Part B**

First, we much find the Z-score to which 15% of the means computed from random samples of size 9 would fall. This corresponds to a table value which is closest to 0.85, and the Z-score closest to this value is Z=1.04. From here, we use the following formula to denormalize the Z-score:

Therefore, the value of x to the right of which 15% of the means computed from random samples of size 9 would fall is 7.35.

**Exercise 8.54**

Construct a QQ-plot of these data, which represent the lifetimes, in hours, of fifty 40-watt, 110-volt internally frosted incandescent lamps taken from forced life tests:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 919 | 1196 | 785 | 1126 | 936 | 918 |
| 1156 | 920 | 948 | 1067 | 1092 | 1162 |
| 1170 | 929 | 950 | 905 | 972 | 1035 |
| 1045 | 855 | 1195 | 1195 | 1340 | 1122 |
| 938 | 970 | 1237 | 956 | 1102 | 1157 |
| 978 | 832 | 1009 | 1157 | 1151 | 1009 |
| 765 | 958 | 902 | 1022 | 1333 | 811 |
| 1217 | 1085 | 896 | 958 | 1311 | 1037 |
| 702 | 923 |  |  |  |  |

**Solution**

**Summary**

These exercises explored finding the probability of specific events which are represented by differing distributions. The first exercise involved finding the probability of a specific event and a range of events using a geometric distribution. The next exercise required utilizing the normal distribution, specifically finding the Z-scores associated with a specific event and using that to determine the probability of a specific event, a range of events, and what value represents a specific percentage of the potential values. The exercises from Ch 8 involved calculating the probability of the distributions associated with a number of samples, rather than the population as a whole. Finally, the last exercise involved creating a QQ plot of the provided data.