**Exercise 11.20**

Test the hypothesis that = 10 in Exercise 11.8 on page 399 against the alternative that < 10. Use a 0.05 level of signiﬁcance.

Exercise 11.8: A mathematics placement test is given to all entering freshmen at a small college. A student who receives a grade below 35 is denied admission to the regular mathematics course and placed in a remedial class. The placement test scores and the ﬁnal grades for 20 students who took the regular course were recorded.

**Solution**

To start with, we’ll state the hypotheses:

Next, we need to find using the following equations:

Using the Excel sheet shown below, was found to be 0.4711 and was found to be 32.5059.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Placement Test (X) | Course Grade (Y) | xi-xBar | (xi-xBar)^2 | yi-yBar | (xi-xBar)\* (yi-yBar) |  |  |  |
|  | 50 | 53 | -5.5 | 30.25 | -5.65 | 31.075 |  | n | 20 |
|  | 35 | 41 | -20.5 | 420.25 | -17.65 | 361.825 |  |  |  |
|  | 35 | 61 | -20.5 | 420.25 | 2.35 | -48.175 |  | B1 | 0.4711 |
|  | 40 | 56 | -15.5 | 240.25 | -2.65 | 41.075 |  | B0 | 32.506 |
|  | 55 | 68 | -0.5 | 0.25 | 9.35 | -4.675 |  |  |  |
|  | 65 | 36 | 9.5 | 90.25 | -22.65 | -215.175 |  |  |  |
|  | 35 | 11 | -20.5 | 420.25 | -47.65 | 976.825 |  |  |  |
|  | 60 | 70 | 4.5 | 20.25 | 11.35 | 51.075 |  |  |  |
|  | 90 | 79 | 34.5 | 1190.25 | 20.35 | 702.075 |  |  |  |
|  | 35 | 59 | -20.5 | 420.25 | 0.35 | -7.175 |  |  |  |
|  | 90 | 54 | 34.5 | 1190.25 | -4.65 | -160.425 |  |  |  |
|  | 80 | 91 | 24.5 | 600.25 | 32.35 | 792.575 |  |  |  |
|  | 60 | 48 | 4.5 | 20.25 | -10.65 | -47.925 |  |  |  |
|  | 60 | 71 | 4.5 | 20.25 | 12.35 | 55.575 |  |  |  |
|  | 60 | 71 | 4.5 | 20.25 | 12.35 | 55.575 |  |  |  |
|  | 40 | 47 | -15.5 | 240.25 | -11.65 | 180.575 |  |  |  |
|  | 55 | 53 | -0.5 | 0.25 | -5.65 | 2.825 |  |  |  |
|  | 50 | 68 | -5.5 | 30.25 | 9.35 | -51.425 |  |  |  |
|  | 65 | 57 | 9.5 | 90.25 | -1.65 | -15.675 |  |  |  |
|  | 50 | 79 | -5.5 | 30.25 | 20.35 | -111.925 |  |  |  |
| Avg | 55.5 | 58.65 |  |  |  |  |  |  |  |

Next, utilizing another Excel formula, we find that the critical region for a one-sided alternative hypothesis, at a 0.05 level of significance, and with 18 degrees of freedom is 1.7341. Finally, we compute the test statistic for the data provided using the following formula:

To solve this, we need to solve for , which can be done using the following equation:

Using Excel once again, we find that equals 261.6222. Plugging the square root of this into the equation above for t, we get:

The computed test statistic of 1.7807 is greater than the critical region statistic of 1.7341 and has a p-value of 0.04592. Therefore, we reject the null hypothesis and claim that is likely less than 10.

**Exercise 11.23**

With reference to Exercise 11.6 on page 399, use the value of found in Exercise 11.18(a) to compute:

(a) a 95% conﬁdence interval for the mean shear resistance when x = 24.5.

(b) a 95% prediction interval for a single predicted value of the shear resistance when x = 24.5.

**Solution**

Using Excel and the same formulas utilized in exercise 11.20 to calculate , , and , we find that using the Excel sheet shown below:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Normal Stress, X | Shear Resistance, Y | xi-xBar | (xi-xBar)^2 | yi-yBar | (xi-xBar)\* (yi-yBar) |  |  |  |
|  | 26.8 | 26.5 | 0.8333 | 0.69444 | 1.7333 | 1.444444 |  | n | 12 |
|  | 25.4 | 27.3 | -0.567 | 0.32111 | 2.5333 | -1.43556 |  |  |  |
|  | 28.9 | 24.2 | 2.9333 | 8.60444 | -0.567 | -1.66222 |  | B1 | -0.68608 |
|  | 23.6 | 27.1 | -2.367 | 5.60111 | 2.3333 | -5.52222 |  | B0 | 42.5818 |
|  | 27.7 | 23.6 | 1.7333 | 3.00444 | -1.167 | -2.02222 |  |  |  |
|  | 23.9 | 25.9 | -2.067 | 4.27111 | 1.1333 | -2.34222 |  |  |  |
|  | 24.7 | 26.3 | -1.267 | 1.60444 | 1.5333 | -1.94222 |  |  |  |
|  | 28.1 | 22.5 | 2.1333 | 4.55111 | -2.267 | -4.83556 |  |  |  |
|  | 26.9 | 21.7 | 0.9333 | 0.87111 | -3.067 | -2.86222 |  |  |  |
|  | 27.4 | 21.4 | 1.4333 | 2.05444 | -3.367 | -4.82556 |  |  |  |
|  | 22.6 | 25.8 | -3.367 | 11.3344 | 1.0333 | -3.47889 |  |  |  |
|  | 25.6 | 24.9 | -0.367 | 0.13444 | 0.1333 | -0.04889 |  |  |  |
| Avg | 25.96667 | 24.76666667 |  |  |  |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| yi-yHat | (yi-yHat)^2 |  |  |  |
| 2.305064 | 5.3133213 |  | s^2 | 2.688452 |
| 2.144556 | 4.5991217 |  | s | 1.63965 |
| 1.445826 | 2.0904135 |  |  |  |
| 0.709617 | 0.503557 |  |  |  |
| 0.022534 | 0.0005078 |  |  |  |
| -0.28456 | 0.080974 |  |  |  |
| 0.664302 | 0.4412976 |  |  |  |
| -0.80304 | 0.644866 |  |  |  |
| -2.42633 | 5.8870676 |  |  |  |
| -2.38329 | 5.6800686 |  |  |  |
| -1.27646 | 1.6293493 |  |  |  |
| -0.11823 | 0.0139779 |  |  |  |

**Part A**

The formula for a confidence interval at a specific x value is given by the following formula:

The t statistic for a 95% confidence interval with 10 degrees of freedom is 2.2281. To predict at the given value for x, we use the standard linear regression formula:

Plugging this and the other values found earlier into the equation we get:

Therefore, the 95% confidence interval for is .

**Part B**

The formula for a prediction interval at a specific x value is given by the following formula:

Utilizing the same numbers as Part A, we get:

Therefore, the 95% confidence interval for is .

**Summary**

In the first exercise, we performed inference on a liner regression of a set of data and ultimately rejected the null hypothesis in favor of the alternative. In the second exercise, we created 95% confidence and prediction intervals at a specific predictor value.