mitiq

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Mitiq

A Python toolkit for implementing error mitigation on quantum computers.

1.1 Features

Mitiq performs error mitigation protocols on quantum circuits using zero-noise extrapolation.

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1.3 Installation

To install locally use:

```
pip install -e .
```

To install for development use:

```
pip install -e .[development]
```

Note that this will install our testing environment that depends on `qiskit` and `pyquil`.

1.4 Use

A Getting Started tutorial can be found in the Documentation.

1.5 Documentation

`Mitiq` documentation is found under `mitiq/docs`. A pdf with the documentation updated to the latest release can be found here.

1.6 Development and Testing

Ensure that you have installed the development environment. Then you can run tests with `pytest`.

1.7 Contributing

You can contribute to `mitiq` code by raising an issue reporting a bug or proposing new feature, using the labels to organize it. You can open a pull request by pushing changes from a local branch, explaining the bug fix or new feature. You can use `mitiq.about()` to document your dependencies and work environment.

To contribute to the documentation, read the instructions in the `mitiq/docs` folder.

1.8 Authors

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1.9 License

GNU GPL v.3.0.

CHAPTER 2

Users Guide

2.1 Overview of mitiq

Welcome to the *mitiq* Users Guide.

2.1.1 What is mitiq for?

Today's quantum computers have a lot of noise. This is a problem for quantum programmers everywhere. *Mitiq* is an open source Python library currently under development by Unitary Fund. It helps solve this problem by compiling your programs to be more robust to noise.

Mitiq helps you do more quantum programming with less quantum compute.

Today's *mitiq* library is based around the zero-noise extrapolation technique. These references [1][2] give background on the technique. The implementation in mitiq is an optimized, extensible framework for zero-noise extrapolation. In the future other error-mitigating techniques will be added to *mitiq*.

Mitiq is a framework agnostic library with a long term vision to be useful for quantum programmers using any quantum programming framework and any quantum backend. Today we support *cirq* and *qiskit* inputs and backends.

Check out more in our getting started section.

2.2 Getting Started

Improving the performance of your quantum programs is only a few lines of code away.

This getting started shows examples using cirq cirq and qiskit. We'll first test `mitiq` by running against the noisy simulator built into `cirq`. The qiskit example work similarly as you will see in *Qiskit Mitigation*.

2.2.1 Error Mitigation with Zero-Noise Extrapolation

We define some functions that make it simpler to simulate noise in `cirq`. These don't have to do with `mitiq` directly.

```
import numpy as np
from cirq import Circuit, depolarize
from cirq import LineQubit, X, DensityMatrixSimulator

SIMULATOR = DensityMatrixSimulator()
# 0.1% depolarizing noise
NOISE = 0.001

def noisy_simulation(circ: Circuit, shots=None) -> float:
    """ Simulates a circuit with depolarizing noise at level NOISE.
    Args:
        circ: The quantum program as a cirq object.
        shots: This unused parameter is needed to match mitiq's expected type
               signature for an executor function.

    Returns:
        The observable's measurements as as
        tuple (expectation value, variance).
    """
    circuit = circ.with_noise(depolarize(p=NOISE))
    rho = SIMULATOR.simulate(circuit).final_density_matrix
    # define the computational basis observable
    obs = np.diag([1, 0])
    expectation = np.real(np.trace(rho @ obs))
    return expectation
```

Now we can look at our example. We'll test single qubit circuits with even numbers of X gates. As there are an even number of X gates, they should all evaluate to an expectation of 1 in the computational basis if there was no noise.

```
from cirq import Circuit, LineQubit, X

qbit = LineQubit(0)
circ = Circuit(X(qbit) for _ in range(80))
unmitigated = noisy_simulation(circ)
exact = 1
print(f"Error in simulation is {exact - unmitigated:.{3}}")
```

```
Error in simulation is 0.0506
```

This shows the impact the noise has had. Let's use `mitiq` to improve this performance.

```
from mitiq import execute_with_zne

mitigated = execute_with_zne(circ, noisy_simulation)
print(f"Error in simulation is {exact - mitigated:.{3}}")
```

```
Error in simulation is 0.000519
```

```
print(f"Mitigation provides a {(exact - unmitigated) / (exact - mitigated):.{3}}␣
↪factor of improvement.")
```

```
Mitigation provides a 97.6 factor of improvement.
```

The variance in the mitigated expectation value is now stored in `var`.

You can also use `mitiq` to wrap your backend execution function into an error-mitigated version.

```
from mitiq import mitigate_executor

run_mitigated = mitigate_executor(noisy_simulation)
mitigated = run_mitigated(circ)
print(round(mitigated,5))
```

```
0.99948
```

The default implementation uses Richardson extrapolation to extrapolate the expectation value to the zero noise limit [1]. `Mitiq` comes equipped with other extrapolation methods as well. Different methods of extrapolation are packaged into `Factory` objects. It is easy to try different ones.

```
from mitiq import execute_with_zne
from mitiq.factories import LinearFactory

fac = LinearFactory(scale_factors=[1.0, 2.0, 2.5])
linear = execute_with_zne(circ, noisy_simulation, fac=fac)
print(f"Mitigated error with the linear method is {exact - linear:.{3}}")
```

```
Mitigated error with the linear method is 0.00638
```

You can read more about the `Factory` objects that are built into `mitiq` and how to create your own *here*.

Another key step in zero-noise extrapolation is to choose how your circuit is transformed to scale the noise. You can read more about the noise scaling methods built into `mitiq` and how to create your own *here*.

2.2.2 Qiskit Mitigation

`Mitiq` is designed to be agnostic to the stack that you are using. Thus for `qiskit` things work in the same manner as before. Since we are now using `qiskit`, we want to run the error mitigated programs on a qiskit backend. Let's define the new backend that accepts `qiskit` circuits. In this case it is a simulator, but you could also use a QPU.

```
import qiskit
from qiskit import QuantumCircuit

# Noise simulation packages
from qiskit.providers.aer.noise import NoiseModel
from qiskit.providers.aer.noise.errors.standard_errors import depolarizing_error

# 0.1% depolarizing noise
NOISE = 0.001

QISKIT_SIMULATOR = qiskit.Aer.get_backend("qasm_simulator")

def qs_noisy_simulation(circuit: QuantumCircuit, shots: int = 4096) -> float:
    """Runs the quantum circuit with a depolarizing channel noise model at
    level NOISE.

    Args:
        circuit (qiskit.QuantumCircuit): Ideal quantum circuit.
        shots (int): Number of shots to run the circuit
                     on the back-end.
```

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```
    Returns:
        expval: expected values.
    """
    # initialize a qiskit noise model
    noise_model = NoiseModel()

    # we assume a depolarizing error for each
    # gate of the standard IBM basis
    noise_model.add_all_qubit_quantum_error(depolarizing_error(NOISE, 1), ["u1", "u2",
↪ "u3"])

    # execution of the experiment
    job = qiskit.execute(
        circuit,
        backend=QISKIT_SIMULATOR,
        basis_gates=["u1", "u2", "u3"],
        # we want all gates to be actually applied,
        # so we skip any circuit optimization
        optimization_level=0,
        noise_model=noise_model,
        shots=shots
    )
    results = job.result()
    counts = results.get_counts()
    expval = counts["0"] / shots
    return expval
```

We can then use this backend for our mitigation.

```
from qiskit import QuantumCircuit
from mitiq import execute_with_zne

circ = QuantumCircuit(1, 1)
for __ in range(120):
    _ = circ.x(0)
_ = circ.measure(0, 0)

unmitigated = qs_noisy_simulation(circ)
mitigated = execute_with_zne(circ, qs_noisy_simulation)
exact = 1
# The mitigation should improve the result.
print(abs(exact - mitigated) < abs(exact - unmitigated))
```

```
True
```

Note that we don't need to even redefine factories for different stacks. Once you have a `Factory` it can be used with different front and backends.

2.3 Unitary Folding

At the gate level, noise is amplified by mapping gates (or groups of gates) G to

$$G \mapsto GG^\dagger G.$$

This makes the circuit longer (adding more noise) while keeping its effect unchanged (because $G^\dagger = G^{-1}$ for unitary gates). We refer to this process as *unitary folding*. If G is a subset of the gates in a circuit, we call it *local folding*. If G is the entire circuit, we call it *global folding*.

In `mitiq`, folding functions input a circuit and a *stretch* (or *stretch factor*), i.e., a floating point value which corresponds to (approximately) how much the length of the circuit is scaled. The minimum stretch is one (which corresponds to folding no gates), and the maximum stretch is three (which corresponds to folding all gates).

2.3.1 Local folding methods

For local folding, there is a degree of freedom for which gates to fold first. As such, `mititq` defines several local folding methods.

We introduce three folding functions:

1. `mitiq.folding.fold_gates_from_left`
2. `mitiq.folding.fold_gates_from_right`
3. `mitiq.folding.fold_gates_at_random`

The `mitiq` function `fold_gates_from_left` will fold gates from the left (or start) of the circuit until the desired stretch factor is reached.

```
>>> import cirq
>>> from mitiq.folding import fold_gates_from_left

# Get a circuit to fold
>>> qreg = cirq.LineQubit.range(2)
>>> circ = cirq.Circuit(cirq.ops.H.on(qreg[0]), cirq.ops.CNOT.on(qreg[0], qreg[1]))
>>> print("Original circuit:", circ, sep="\n")
Original circuit:
0: ───H───@───
          │
1: ───────X───

# Fold the circuit
>>> folded = fold_gates_from_left(circ, stretch=2.)
>>> print("Folded circuit:", folded, sep="\n")
Folded circuit:
0: ───H───H───H───@───
                  │
1: ───────────────X───
```

In this example, we see that the folded circuit has the first (Hadamard) gate folded.

Note: `mitiq` folding functions do not modify the input circuit.

Because input circuits are not modified, we can reuse this circuit for the next example. In the following code, we use the `fold_gates_from_right` function on the same input circuit.

```
>>> from mitiq.folding import fold_gates_from_right

# Fold the circuit
>>> folded = fold_gates_from_right(circ, stretch=2.)
>>> print("Folded circuit:", folded, sep="\n")
Folded circuit:
0: ───H───@───@───@───
          │   │   │
1: ───────X───X───X───
```

We see the second (CNOT) gate in the circuit is folded, as expected when we start folding from the right (or end) of the circuit instead of the left (start).

Finally, we mention `fold_gates_at_random` which folds gates according to the following rules.

1. Gates are selected at random and folded until the input stretch factor is reached.
2. No gate is folded more than once.
3. "Virtual gates" (i.e., gates appearing from folding) are never folded.

2.3.2 Any supported circuits can be folded

Any program types supported by `mitiq` can be folded. The interface for all folding functions is the same. In the following example, we fold a Qiskit circuit.

Note: This example assumes you have Qiskit installed. `mitiq` can interface with Qiskit, but Qiskit is not a core `mitiq` requirement and is not installed by default.

```
>>> import qiskit
>>> from mitiq.folding import fold_gates_from_left

# Get a circuit to fold
>>> qreg = qiskit.QuantumRegister(2)
>>> circ = qiskit.QuantumCircuit(qreg)
>>> _ = circ.h(qreg[0])
>>> _ = circ.cnot(qreg[0], qreg[1])
>>> # print("Original circuit:", circ, sep="\n")
```

This code (when the print statement is uncommented) should display something like:

```
Original circuit:
                 ┌───┐
q0_0: |0>┤ H ├──■──
         └───┘┌─┴─┐
q0_1: |0>─────┤ X ├
              └───┘
```

We can now fold this circuit as follows.

> # Fold the circuit >>> folded = fold_gates_from_left(circ, stretch=2.) >>> # print("Folded circuit:", folded, sep="n")

This code (when the print statement is uncommented) should display something like:

```
Folded circuit:
              ┌───────────────────────────────────────────────────┐
q_0: |0> H ├ Ry(pi/4) ├ Rx(-pi) ├ Ry(-pi/4) ├ H ├───
         └──┘└─────────┘└────────┘└──────────┘└──┘
q_1: |0>──────────────────────────────────────────────── X ├
                                                         └──
```

By default, the folded circuit has the same type as the input circuit. To return an internal `mitiq` representation of the folded circuit (a Cirq circuit), one can use the keyword argument `return_mitiq=True`.

Note: Compared to the previous example which input a Cirq circuit, we see that this folded circuit has more gates. In particular, the inverse Hadamard gate is expressed differently (but equivalently) as a product of three rotations. This behavior occurs because circuits are first converted to `mitiq`'s internal representation (Cirq circuits), then folded, then converted back to the input circuit type. Because different circuits decompose gates differently, some gates (or their inverses) may be expressed differently (but equivalently) across different circuits.

2.3.3 Global folding

As mentioned, global folding methods fold the entire circuit instead of individual gates. An example using the same Cirq circuit above is shown below.

```
>>> import cirq
>>> from mitiq.folding import fold_global

# Get a circuit to fold
>>> qreg = cirq.LineQubit.range(2)
>>> circ = cirq.Circuit(cirq.ops.H.on(qreg[0]), cirq.ops.CNOT.on(qreg[0], qreg[1]))
>>> print("Original circuit:", circ, sep="\n")
Original circuit:
0: ───H───@───
          │
1: ───────X───

# Fold the circuit
>>> folded = fold_global(circ, stretch=3.)
>>> print("Folded circuit:", folded, sep="\n")
Folded circuit:
0: ───H───@───@───H───H───@───
          │   │           │
1: ───────X───X───────────X───
```

Notice that this circuit is still logically equivalent to the input circuit, but the global folding strategy folds the entire circuit until the input stretch factor is reached.

2.3.4 Folding with larger stretches

The three local folding methods introduced require that the stretch factor be between one and three (inclusive). To fold circuits with larger stretch factors, the function `mitiq.folding.fold_local` can be used. This function inputs a circuit, an arbitrary stretch factor, and a local folding method, as in the following example.

```
>>> import cirq
>>> from mitiq.folding import fold_local, fold_gates_from_left

# Get a circuit to fold
>>> qreg = cirq.LineQubit.range(2)
>>> circ = cirq.Circuit(cirq.ops.H.on(qreg[0]), cirq.ops.CNOT.on(qreg[0], qreg[1]))
>>> print("Original circuit:", circ, sep="\n")
Original circuit:
0: ───H───@───
          │
1: ───────X───

# Fold the circuit
>>> folded = fold_local(circ, stretch=5., fold_method=fold_gates_from_left)
>>> print("Folded circuit:", folded, sep="\n")
Folded circuit:
0: ───H───H───H───H───H───H───H───@───@───@───
                                  │   │   │
1: ───────────────────────────────X───X───X───
```

2.3.5 Local folding with a custom strategy

The `fold_local` method from the previous example can input custom folding functions. The signature of this function must be as follows.

```
import cirq

def my_custom_folding_function(circuit: cirq.Circuit, stretch: float) -> cirq.Circuit:
    # Implements the custom folding strategy
    return folded_circuit
```

This function can then be used with `fold_local` as in the previous example via

```
# Variables circ and stretch are a circuit to fold and a stretch factor, respectively
folded = fold_local(circ, stretch, fold_method=my_custom_folding_function)
```

2.4 Factory Objects

Factories are important elements of the `mitiq` library.

The abstract class `Factory` is a high-level representation of a generic error mitigation method. A factory is not just hardware-agnostic, it is even *quantum-agnostic*, in the sense that it only deals with classical data: the classical input and the classical output of a noisy computation.

Specific classes derived from `Factory`, like `LinearFactory`, `RichardsonFactory`, etc., represent different zero-noise extrapolation methods.

The main tasks of a factory are:

1. Record the result of the computation executed at the chosen noise level;
2. Determine the noise scale factor at which the next computation should be run;
3. Given the history of noise scale factors (`self.instack`) and results (`self.outstack`), evaluate the associated zero-noise extrapolation.

The structure of the `Factory` class is adaptive by construction, since the choice of the next noise level can depend on the history of `self.instack` and `self.outstack`.

The abstract class of a non-adaptive extrapolation method is `BatchedFactory`. The main feature of `BatchedFactory` is that all the noise scale factors are determined *a priori* by the initialization argument `scale_factors`. All non-adaptive methods are derived from `BatchedFactory`.

2.4.1 Example: basic usage of a factory

To make an example, let us assume that the result of our quantum computation is an expectation value which has a linear dependance on the noise. Since our aim is to understand the usage of a factory, instead of actually running quantum experiments, we simply simulate an effective classical model which returns the expectation value as a function of the noise scale factor.

```
def noise_to_expval(scale_factor: float) -> float:
    """A simple linear model for the expectation value."""
    ZERO_NOISE_LIMIT = 0.5
    NOISE_ERROR = 0.7
    return ZERO_NOISE_LIMIT + NOISE_ERROR * scale_factor
```

In this case the zero-noise limit is `0.5` and we would like to deduce it by evaluating the function only for values of `scale_factor` which are larger than or equal to 1.

Note: For implementing zero-noise extrapolation, it is not necessary to know the details of the noise model. It is also not necessary to control the absolute strength of the noise acting on the physical system. The only key assumption is that we can artificially scale the noise with respect to its normal level by a dimensionless `scale_factor`. A practical approach for scaling the noise is discussed in the *Unitary Folding* section.

In this example, we plan to measure the expectation value at 3 different noise scale factors: `SCALE_FACTORS = [1.0, 2.0, 3.0]`.

To get the zero-noise limit, we are going to use a `LinearFactory` object, run it until convergence (in this case until 3 expectation values are measured and saved) and eventually perform the zero-noise extrapolation.

```
from mitiq.factories import LinearFactory

# Some fixed noise scale factors
SCALE_FACTORS = [1.0, 2.0, 3.0]

# Instantiate a LinearFactory object
fac = LinearFactory(SCALE_FACTORS)

# Run the factory until convergence
while not fac.is_converged():
    # Get the next noise scale factor from the factory
    next_scale_factor = fac.next()
    # Evaluate the expectation value
    expval = noise_to_expval(next_scale_factor)
    # Save the noise scale factor and the result into the factory
```

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```
    fac.push(next_scale_factor, expval)

# Evaluate the zero-noise extrapolation.
zn_limit = fac.reduce()
print(f"{zn_limit:.3}")
```

```
0.5
```

In the previous code block we used the main methods of a typical `Factory` object:

- **self.next** to get the next noise scale factor;
- **self.push** to save data into the factory;
- **self.is_converged** to know if enough data has been pushed;
- **self.reduce** to get the zero-noise extrapolation.

Since our idealized model `noise_to_expval` is linear and noiseless, the extrapolation will exactly match the true zero-noise limit `0.5`:

```
print(f"The zero-noise extrapolation is: {zn_limit:.3}")
```

```
The zero-noise extrapolation is: 0.5
```

Note: In a real scenario, the quantum expectation value can be determined only up to some statistical uncertainty (due to a finite number of measurement shots). This makes the zero-noise extrapolation less trivial. Moreover the expectation value could depend non-linearly on the noise level. In this case factories with higher extrapolation *order* (`PolyFactory`, `RichardsonFactory`, etc.) could be more appropriate.

The `run_factory` function

Running a factory until convergence is a typical step of the zero-noise extrapolation workflow. For this reason, in `mitiq.zne` there is a function which can be used for this task: `run_factory`. The previous example can be simplified to the following equivalent code:

```
from mitiq.factories import LinearFactory
from mitiq.zne import run_factory

# Some fixed noise scale factors
SCALE_FACTORS = [1.0, 2.0, 3.0]
# Instantiate a LinearFactory object
fac = LinearFactory(SCALE_FACTORS)
# Run the factory until convergence
run_factory(fac, noise_to_expval)
# Evaluate the zero-noise extrapolation.
zn_limit = fac.reduce()
print(f"The zero-noise extrapolation is: {zn_limit:.3}")
```

```
The zero-noise extrapolation is: 0.5
```

2.4.2 Built-in factories

All the built-in factories of `mitiq` can be found in the submodule `mitiq.factories`.

mitiq.factories.LinearFactory	Factory object implementing a zero-noise extrapolation algorithm based on a linear fit.
mitiq.factories.RichardsonFactory	Factory object implementing Richardson's extrapolation.
mitiq.factories.PolyFactory	Factory object implementing a zero-noise extrapolation algorithm based on a polynomial fit.
mitiq.factories.ExpFactory	Factory object implementing a zero-noise extrapolation algorithm assuming an exponential ansatz y(x) = a + b * exp(-c * x), with c > 0.
mitiq.factories.PolyExpFactory	Factory object implementing a zero-noise extrapolation algorithm assuming an (almost) exponential ansatz with a non linear exponent, i.e.:
mitiq.factories.AdaExpFactory	Factory object implementing an adaptive zero-noise extrapolation algorithm assuming an exponential ansatz y(x) = a + b * exp(-c * x), with c > 0.

2.4.3 Defining a custom Factory

If necessary, the user can modify an existing extrapolation method by subclassing the corresponding factory.

A new adaptive extrapolation method can be derived from the abstract class `Factory`. In this case its core methods must be implemented: `self.next`, `self.push`, `self.is_converged`, and `self.reduce`. Moreover `self.__init__` can also be overridden if necessary.

A new non-adaptive method can instead be derived from the `BatchedFactory` class. In this case it is usually sufficient to override only `self.__init__` and `self.reduce`, which are responsible for the initialization and for the final zero-noise extrapolation, respectively.

2.4.4 Example: a simple custom factory

Assume that, from physical considerations, we know that the ideal expectation value (measured by some quantum circuit) must always be within two limits: `min_expval` and `max_expval`. For example, this is a typical situation whenever the measured observable has a bounded spectrum.

We can define a linear non-adaptive factory which takes into account this information and clips the result if it falls outside its physical domain.

```
from typing import Iterable
from mitiq.factories import BatchedFactory
import numpy as np

class MyFactory(BatchedFactory):
    """Factory object implementing a linear extrapolation taking
    into account that the expectation value must be within a given
    interval. If the zero-noise extrapolation falls outside the
    interval, its value is clipped.
    """

    def __init__(
```

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```
        self,
        scale_factors: Iterable[float],
        min_expval: float,
        max_expval: float,
    ) -> None:
    """
    Args:
        scale_factors: The noise scale factors at which
                       expectation values should be measured.
        min_expval: The lower bound for the expectation value.
        min_expval: The upper bound for the expectation value.
    """
    super(MyFactory, self).__init__(scale_factors)
    self.min_expval = min_expval
    self.max_expval = max_expval

def reduce(self) -> float:
    """
    Fits a line to the data with a least squared method.
    Extrapolates and, if necessary, clips.

    Returns:
        The clipped extrapolation to the zero-noise limit.
    """
    # Fit a line and get the intercept
    _, intercept = np.polyfit(self.instack, self.outstack, 1)

    # Return the clipped zero-noise extrapolation.
    return np.clip(intercept, self.min_expval, self.max_expval)
```

This custom factory can be used in exactly the same way as we have shown in the previous section. By simply replacing `LinearFactory` with `MyFactory` in all the previous code snippets, the new extrapolation method will be applied.

2.5 About Error Mitigation

This is intended as a primer on quantum error mitigation, providing a collection of up-to-date resources from the academic literature, as well as other external links framing this topic in the open-source software ecosystem.

- *Noise in quantum devices*
- *What is quantum error mitigation*
- *Related fields*
- *Why is quantum error mitigation important*
- *External References*

2.5.1 Noise in quantum devices

A series of issues arise when someone wants to perform a calculation on a quantum computer.

This is due to the fact that quantum computers are devices that are embedded in an environment and interact with it. This means that stored information can be corrupted, or that, during calculations, the protocols are not faithful.

Errors occur for a series of reasons in quantum computers and the microscopic description at the physical level can vary broadly, depending on the quantum computing platform that is used, as well as the computing architecture.

For example, superconducting-circuit-based quantum computers have chips that are prone to cross-talk noise, while qubits encoded in trapped ions need to be shuttled with electromagnetic pulses, and solid-state artificial atoms, including quantum dots, are heavily affected by inhomogeneous broadening [3].

More in general, quantum computing devices can be studied in the framework of open quantum systems [4][5][6][7], that is, systems that exchange energy and information with the surrounding environment.

The qubit-environment exchange can be controlled, and this feature is actually fundamental to extract information and process it. When this interaction is not controlled, and at the fundamental level it cannot be completely suppressed, noise eventually kicks in, thus introducing errors that are disruptive for the *fidelity* of the information-processing protocols.

2.5.2 What is quantum error mitigation

Quantum error mitigation refers to a series of modern techniques aimed at reducing (*mitigating*) the errors that occur in quantum computing algorithms.

Unlike software bugs affecting code in usual computers, the errors which we attempt to reduce with mitigation are due to the hardware.

Quantum error mitigation techniques try to *reduce* the impact of noise in quantum computations. They generally do not completely remove it.

Alternative nomenclature refers to error mitigation as (approximate) error suppression or approximate quantum error correction, but it is worth noting that it is different from error correction.

Among the ideas that have been developed so far for quantum error mitigation, a leading candidate is zero-noise extrapolation.

Zero-noise extrapolation

The key idea behind zero-noise extrapolation is that it is possible to make some general assumptions on the kind of noise that affects, with error, the results of a quantum computation.

In an ideal device, the time evolution is unitary, and as such it is modeled in the intermediate representation of a quantum circuit,

to

$|\psi\rangle(t) =$

$U(t)|\psi\rangle = e^{-i\int_0^t H(t')dt'/\hbar}|\psi\rangle,$ (2.1)

=

$U(t)|\psi\rangle = e^{-i\int_0^t H(t')dt'/\hbar}|\psi\rangle,$

where $|\psi\rangle$ is the initial state of the system (e.g., the qubits involved in the operation) and $U(t)$ the unitary time evolution set by a time-dependent Hamiltonian, H(t).

In the simplest scenario for the system-environment interaction, it is still possible to describe the time evolution in terms of operators acting on the system only, at the cost of losing the unitarity of the evolution.

The first required condition to develop such framework, is that the system interacts more weakly with the environment than within its own sub-constituents. This allows to proceed with a perturbative approach to solve the problem, with a coupling constant λ quantifying the magnitude of the first-order expansion terms.

In this case, it is possible to write the time evolution of the density matrix associated to the state, $\hat{\rho} = |\psi\rangle\langle\psi|$, as

to

$\frac{\partial}{\partial t}\hat{\rho} =$

$\frac{i}{\hbar}[H(t), \hat{\rho}] + \lambda\mathcal{L}[\hat{\rho}],$ (2.1)

=

$\frac{i}{\hbar}[H(t), \hat{\rho}] + \lambda\mathcal{L}[\hat{\rho}],$

where $mathcalL$ is a super-operator acting on the Hilbert space.

The subsequent most straightforward set of sensible approximations includes assuming that at time zero the system and environment are not entangled, that the environment is memoryless, and that there is a dominant scale of times set by the interactions, wich allows to cut off high-frequency perturbations.

These approximations -- called the Born, Markov, and Rotating-Wave approximations, respectively -- lead to a so-called Lindblad form of the *dissipation*, i.e. to a special structure of the system-environment interaction that can be represented with a linear superoperator that always admits the Lindblad form

$$to$$

$$\mathcal{L}[\hat{\rho}] =$$

$$\mathcal{L}\hat{\rho} = \sum_{i=1}^{N^2-1} \gamma_i \left(A_i \hat{\rho} A_i^\dagger - \frac{1}{2}(A_i^\dagger A_i \hat{\rho} + \hat{\rho} A_i^\dagger A_i) \right), (2.1)$$

=

$$\mathcal{L}\hat{\rho} = \sum_{i=1}^{N^2-1} \gamma_i \left(A_i \hat{\rho} A_i^\dagger - \frac{1}{2}(A_i^\dagger A_i \hat{\rho} + \hat{\rho} A_i^\dagger A_i) \right),$$

where γ_i are constants that set the strenghts of the dissipation mechanisms defined by the jump operators, A_i.

The crucial idea behind zero-noise extrapolation is that, while some minimum strength of noise is unavoidable in the system, it is still possible to *increase* it to a value $\lambda' = c\lambda$, with $c > 1$, so that it is then possible to extrapolate the zero-noise limit.

This is done in practice by running a quantum circuit (simulation) and calculating a given expectation variable, $\langle X \rangle_\lambda$, then re-running the calculation (which is indeed a time evolution) for $\langle X \rangle_{\lambda'}$, and then extracting $\langle X \rangle_0$. The extraction for $\langle X \rangle_0$ can occur with several statistical fitting models, which can be linear or non-linear. These methods are contained in the `mitiq.factories` and `mitiq.zne` modules.

In experiments, zero-noise extrapolation can be performed with pulse stretching as a means to introduce a difference between the effective time that a gate is affected by decoherence during its execution on hardware in terms of time-resolved pulses.

Unitary folding

A way to stretch time with respect to noise-related processes is obtained by inserting identity gates. These gates can be decomposed in terms of gates already present in the circuit, and their transpose. This technique, referred to as *unitary folding*, is present in the `mitiq` toolchain.

Other error mitigation techniques

Other examples of error mitigation techniques include injecting noisy gates for randomized compiling and probabilistic error cancellation, or the use of subspace reductions and symmetries. A collection of references on this cutting-edge implementations can be found in the *Research articles* subsection.

2.5.3 Related fields

Quantum error mitigation is connected to quantum error correction and quantum optimal control, two fields of study that also aim at reducing the impact of errors in quantum information processing in quantum computers. While these are fluid boundaries, it can be useful to point out some differences among these two well-established fields and the emerging field of quantum error mitigation.

It is fair to say that even the terminology of "quantum error mitigation" or "error mitigation" has only recently coalesced (from ~2015 onward), while even in the previous decade similar concepts or techniques were scattered across these and other fields. Suggestions for additional references are welcome.

Quantum error correction

Quantum error correction is different from quantum error mitigation, as it introduces a series of techniques that generally aim at completely *removing* the impact of errors on quantum computations. In particular, if errors occurs below a certain threshold, the robustness of the quantum computation can be preserved, and fault tolerance is reached.

The main issue of quantum error correction techniques are that generally they require a large overhead in terms of additional qubits on top of those required for the quantum computation. Current quantum computing devices have been able to demonstrate quantum error correction only with a very small number of qubits.

What is now referred quantum error mitigation is generally a series of techniques that stemmed as more practical quantum error correction solutions [8].

Quantum optimal control

Optimal control theory is a very versatile set of techniques that can be applied for many scopes. It entails many fields, and it is generally based on a feedback loop between an agent and a target system. Optimal control is applied to several quantum technologies, including in the pulse shaping of gate design in quantum circuits calibration against noisy devices [9].

A key difference between some quantum error mitigation techniques and quantum optimal control is that the former can be implemented in some instances with post-processing techniques, while the latter relies on an active feedback loop.

An example of a specific application of optimal control to quantum dynamics that can be seen as a quantum error mitigation technique, is in dynamical decoupling [10]. This technique employs fast control pulses to effectively decouple a system from its environment, with techniques pioneered in the nuclear magnetic resonance community.

2.5.4 Why is quantum error mitigation important

The noisy intermediate scale quantum computing (NISQ) era is characterized by short or medium-depth circuits and noise affecting operations, state preparation, and measurement [11].

Current short-depth quantum circuits are noisy, and at the same time it is not possible to implement quantum error correcting codes on them due to the needed qubit number and circuit depth required by these codes.

Error mitigation offers the prospects of writing more compact quantum circuits that can estimate observables with more precision, i.e. increase the performance of quantum computers.

By implementing quantum optics tools (such as the modeling noise and open quantum systems) [4][5][6][7], standard as well as cutting-edge statistics and inference techniques, and tweaking them for the needs of the quantum computing community, `mitiq` aims at providing the most comprehensive toolchain for error mitigation.

2.5.5 External References

Here is a list of useful external resources on quantum error mitigation, including software tools that provide the possibility of studying quantum circuits.

Research articles

A list of research articles academic resources on error mitigation:

- **On zero-noise extrapolation:**
  - Theory, Y. Li and S. Benjamin, *Phys. Rev. X*, 2017 [12] and K. Temme *et al.*, *Phys. Rev. Lett.*, 2017 [1]
  - Experiment on superconducting circuit chip, A. Kandala *et al.*, *Nature*, 2019 [2]
- **On randomization methods:**
  - Randomized compiling with twirling gates, J. Wallman *et al.*, *Phys. Rev. A*, 2016 [13]
  - Porbabilistic error correction, K. Temme *et al.*, *Phys. Rev. Lett.*, 2017 [1]
  - Practical proposal, S. Endo *et al.*, *Phys. Rev. X*, 2018 [14]
  - Experiment on trapped ions, S. Zhang *et al.*, *Nature Comm.* 2020 [15]
  - Experiment with gate set tomography on a supeconducting circuit device, J. Sun *et al.*, 2019 arXiv [16]
- **On subspace expansion:**
  - By hybrid quantum-classical hierarchy introduction, J. McClean *et al.*, *Phys. Rev. A*, 2017 [17]
  - By symmetry verification, X. Bonet-Monroig *et al.*, *Phys. Rev. A*, 2018 [18]
  - With a stabilizer-like method, S. McArdle *et al.*, *Phys. Rev. Lett.*, 2019, [19]
  - Exploiting molecular symmetries, J. McClean *et al.*, *Nat. Comm.*, 2020 [20]
  - Experiment on a superconducting circuit device, R. Sagastizabal *et al.*, *Phys. Rev. A*, 2019 [21]
- **On other techniques such as:**
  - Approximate error-correcting codes in the generalized amplitude-damping channels, C. Cafaro *et al.*, *Phys. Rev. A*, 2014 [22]:
  - Extending the variational quantum eigensolver (VQE) to excited states, R. M. Parrish *et al.*, *Phys. Rev. Lett.*, 2017 [23]
  - Quantum imaginary time evolution, M. Motta *et al.*, *Nat. Phys.*, 2020 [24]
  - Error mitigation for analog quantum simulation, J. Sun *et al.*, 2020, arXiv [16]
- For an extensive introduction: S. Endo, *Hybrid quantum-classical algorithms and error mitigation*, PhD Thesis, 2019, Oxford University (Link).

Software

Here is a (non-comprehensive) list of open-source software libraries related to quantum computing, noisy quantum dynamics and error mitigation:

- **IBM Q**'s Qiskit provides a stack for quantum computing simulation and execution on real devices from the cloud. In particular, `qiskit.Aer` contains the `NoiseModel` object, integrated with `mitiq` tools. Qiskit's OpenPulse provides pulse-level control of qubit operations in some of the superconducting circuit devices. `mitiq` is integrated with `qiskit`, in the *`qiskit_utils`* and *`conversions`* modules.
- **Goole AI Quantum**'s Cirq offers quantum simulation of quantum circuits.

The `cirq.Circuit` object is integrated in `mitiq` algorithms as the default circuit.

- **Rigetti Computing**'s PyQuil is a library for quantum programming. Rigetti's stack offers the execution of quantum circuits on superconducting circuits devices from the cloud, as well as their simulation on a quantum virtual machine (QVM), integrated with `mitiq` tools in the *`pyquil_utils`* module.
- QuTiP, the quantum toolbox in Python, contains a quantum information processing module that allows to simulate quantum circuits, their implementation on devices, as well as the simulation of pulse-level control and time-dependent density matrix evolution with the `qutip.Qobj` object and the `Processor` object in the `qutip.qip` module.
- Krotov is a package implementing Krotov method for optimal control interfacing with QuTiP for noisy density-matrix quantum evolution.
- Pennylane is a hardware-agnostic library that brings together machine learning and quantum circuits.
- PyGSTi allows to characterize quantum circuits by implementing techniques such as gate set tomography (GST) and randomized benchmarking.

This is just a selection of open-source projects related to quantum error mitigation. A more comprehensinve collection of software on quantum computing can be found here and on Unitary Fund's list of supported projects.

CHAPTER 3

API-doc

This is the top level module from which functions and classes of Mitiq can be directly imported.

`mitiq.version()`
Returns the Mitiq version number.

3.1 About

Command line output of information on Mitiq and dependencies.

`mitiq.about.about()`
About box for Mitiq. Gives version numbers for Mitiq, NumPy, SciPy, Cirq, PyQuil, Qiskit.

3.2 Factories

Contains all the main classes corresponding to different zero-noise extrapolation methods.

class `mitiq.factories.AdaExpFactory`(*steps: int, scale_factor: float = 2, asymptote: Optional[float] = None*)
Factory object implementing an adaptive zero-noise extrapolation algorithm assuming an exponential ansatz y(x) = a + b * exp(-c * x), with c > 0.

The noise scale factors are are chosen adaptively at each step, depending on the history of collected results.

If the asymptotic value (y(x->inf) = a) is known, a linear fit with respect to z(x) := log[sing(b) (y(x) - a)] is used. Otherwise, a non-linear fit of y(x) is performed.

is_converged() → bool
Returns True if all the needed expectation values have been computed, else False.

next() → float
Returns the next noise level to execute a circuit at.

reduce() → float
Returns the zero-noise limit, assuming an exponential ansatz: y(x) = a + b * exp(-c * x), with c > 0.

class mitiq.factories.**BatchedFactory**(*scale_factors: Iterable[float]*)
Abstract class of a non-adaptive Factory.

This is initialized with a given batch of "scale_factors". The "self.next" method trivially iterates over the elements of "scale_factors" in a non-adaptive way. Convergence is achieved when all the correpsonding expectation values have been measured.

Specific (non-adaptive) zero-noise extrapolation algorithms can be derived from this class by overriding the "self.reduce" and (if necessary) the "__init__" method.

is_converged() → bool
Returns True if all needed expectation values have been computed, else False.

next() → float
Returns the next noise level to execute a circuit at.

class mitiq.factories.**ExpFactory**(*scale_factors: Iterable[float], asymptote: Optional[float] = None*)
Factory object implementing a zero-noise extrapolation algorithm assuming an exponential ansatz y(x) = a + b * exp(-c * x), with c > 0.

If the asymptotic value (y(x->inf) = a) is known, a linear fit with respect to z(x) := log[sing(b) (y(x) - a)] is used. Otherwise, a non-linear fit of y(x) is performed.

reduce() → float
Returns the zero-noise limit, assuming an exponential ansatz: y(x) = a + b * exp(-c * x), with c > 0.

class mitiq.factories.**Factory**
Abstract class designed to adaptively produce a new noise scaling parameter based on a historical stack of previous noise scale parameters ("self.instack") and previously estimated expectation values ("self.outstack").

Specific zero-noise extrapolation algorithms, adaptive or non-adaptive, are derived from this class. A Factory object is not supposed to directly perform any quantum computation, only the classical results of quantum experiments are processed by it.

abstract is_converged() → bool
Returns True if all needed expectation values have been computed, else False.

abstract next() → float
Returns the next noise level to execute a circuit at.

push(*instack_val: float*, *outstack_val: float*) → None
Appends "instack_val" to "self.instack" and "outstack_val" to "self.outstack". Each time a new expectation value is computed this method should be used to update the internal state of the Factory.

abstract reduce() → float
Returns the extrapolation to the zero-noise limit.

reset() → None
Resets the instack and outstack of the Factory to empty values.

class mitiq.factories.**LinearFactory**(*scale_factors: Iterable[float]*)
Factory object implementing a zero-noise extrapolation algorithm based on a linear fit.

Example

```
>>> NOISE_LEVELS = [1.0, 2.0, 3.0]
>>> fac = LinearFactory(NOISE_LEVELS)
```

reduce() → float
Determines, with a least squared method, the line of best fit associated to the data points. The intercept is returned.

class mitiq.factories.**PolyExpFactory**(*scale_factors: Iterable[float], order: int, asymptote: Optional[float] = None*)
Factory object implementing a zero-noise extrapolation algorithm assuming an (almost) exponential ansatz with a non linear exponent, i.e.:

y(x) = a + s * exp(z(x)), where z(x) is a polynomial of a given order.

The parameter "s" is a sign variable which can be either 1 or -1, corresponding to decreasing and increasing exponentials, respectively. The parameter "s" is automatically deduced from the data.

If the asymptotic value (y(x->inf) = a) is known, a linear fit with respect to z(x) := log[s(y(x) - a)] is used. Otherwise, a non-linear fit of y(x) is performed.

reduce() → float
Returns the zero-noise limit, assuming an exponential ansatz: y(x) = a + s * exp(z(x)), where z(x) is a polynomial of a given order. The parameter "s" is a sign variable which can be either 1 or -1, corresponding to decreasing and increasing exponentials, respectively. The parameter "s" is automatically deduced from the data. It is also assumed that z(x-->inf)=-inf, such that y(x-->inf)-->a.

static static_reduce(*instack: List[float], outstack: List[float], asymptote: Optional[float], order: int, eps: float = 1e-09*) → Tuple[float, List[float]]
Determines the zero-noise limit, assuming an exponential ansatz: y(x) = a + s * exp(z(x)), where z(x) is a polynomial of a given order.

The parameter "s" is a sign variable which can be either 1 or -1, corresponding to decreasing and increasing exponentials, respectively. The parameter "s" is automatically deduced from the data.

It is also assumed that z(x-->inf)=-inf, such that y(x-->inf)-->a.

If asymptote is None, the ansatz y(x) is fitted with a non-linear optimization. Otherwise, a linear fit with respect to z(x) := log(sign * (y(x) - asymptote)) is performed.

This static method is equivalent to the "self.reduce" method of PolyExpFactory, but can be called also by other factories which are related to PolyExpFactory, e.g., ExpFactory, AdaExpFactory.

Parameters

- **instack** -- x data values.
- **outstack** -- y data values.
- **asymptote** -- y(x->inf).
- **order** -- Extrapolation order.
- **eps** -- Epsilon to regularize log(sign (instack - asymptote)) when the argument is to close to zero or negative.

Returns

Where "znl" is the zero-noise-limit and "params" are the optimal fitting parameters.

Return type (znl, params)

class mitiq.factories.**PolyFactory**(*scale_factors: Iterable[float], order: int*)
Factory object implementing a zero-noise extrapolation algorithm based on a polynomial fit.

Note: RichardsonFactory and LinearFactory are special cases of PolyFactory.

reduce() → float
Determines with a least squared method, the polynomial of degree equal to "self.order" which optimally fits the input data. The zero-noise limit is returned.

static static_reduce(*instack: List[float], outstack: List[float], order: int*) → float
Determines with a least squared method, the polynomial of degree equal to 'order' which optimally fits the input data. The zero-noise limit is returned.

This static method is equivalent to the "self.reduce" method of PolyFactory, but can be called also by other factories which are particular cases of PolyFactory, e.g., LinearFactory and RichardsonFactory.

class mitiq.factories.**RichardsonFactory**(*scale_factors: Iterable[float]*)
Factory object implementing Richardson's extrapolation.

reduce() → float
Returns the Richardson's extrapolation to the zero-noise limit.

Testing of zero-noise extrapolation methods (factories) with classically generated data.

mitiq.tests.test_factories.**f_exp_down**(*x: float, err: float = 0.0001*) → float
Exponential decay.

mitiq.tests.test_factories.**f_exp_up**(*x: float, err: float = 0.0001*) → float
Exponential growth.

mitiq.tests.test_factories.**f_lin**(*x: float, err: float = 0.0001*) → float
Linear function.

mitiq.tests.test_factories.**f_non_lin**(*x: float, err: float = 0.0001*) → float
Non-linear function.

mitiq.tests.test_factories.**f_poly_exp_down**(*x: float, err: float = 0.0001*) → float
Poly-exponential decay.

mitiq.tests.test_factories.**f_poly_exp_up**(*x: float, err: float = 0.0001*) → float
Poly-exponential growth.

mitiq.tests.test_factories.**test_ada_exp_factory_no_asympt**(*test_f: Callable[[float], float]*)
Test of the adaptive exponential extrapolator.

mitiq.tests.test_factories.**test_ada_exp_factory_no_asympt_more_steps**(*test_f: Callable[[float], float]*)
Test of the adaptive exponential extrapolator.

mitiq.tests.test_factories.**test_ada_exp_factory_with_asympt**(*test_f: Callable[[float], float]*)
Test of the adaptive exponential extrapolator.

mitiq.tests.test_factories.**test_ada_exp_factory_with_asympt_more_steps**(*test_f: Callable[[float], float]*)
Test of the adaptive exponential extrapolator.

mitiq.tests.test_factories.**test_exp_factory_no_asympt**(*test_f: Callable[[float], float]*)
Test of exponential extrapolator.

mitiq.tests.test_factories.**test_exp_factory_with_asympt**(*test_f: Callable[[float], float]*)
Test of exponential extrapolator.

mitiq.tests.test_factories.**test_linear_extr**()
Test of linear extrapolator.

mitiq.tests.test_factories.**test_poly_exp_factory_no_asympt**(*test_f: Callable[[float], float]*)
Test of (almost) exponential extrapolator.

mitiq.tests.test_factories.**test_poly_exp_factory_with_asympt**(*test_f: Callable[[float], float]*)
Test of (almost) exponential extrapolator.

mitiq.tests.test_factories.**test_poly_extr**()
Test of polynomial extrapolator.

mitiq.tests.test_factories.**test_richardson_extr**(*test_f: Callable[[float], float]*)
Test of the Richardson's extrapolator.

3.3 Folding

Functions for folding gates in valid mitiq circuits.

Public functions work for any circuit types supported by mitiq. Private functions work only for iternal mitiq circuit representations.

exception mitiq.folding.**UnsupportedCircuitError**

mitiq.folding.**convert_from_mitiq**(*circuit: cirq.circuits.circuit.Circuit, conversion_type: str*) → Optional[cirq.circuits.circuit.Circuit]
Converts a mitiq circuit to a type specificed by the conversion type.

Parameters

- **circuit** -- Mitiq circuit to convert.
- **conversion_type** -- String specifier for the converted circuit type.

mitiq.folding.**convert_to_mitiq**(*circuit: Optional[cirq.circuits.circuit.Circuit]*) → Tuple[cirq.circuits.circuit.Circuit, str]
Converts any valid input circuit to a mitiq circuit.

Parameters **circuit** -- Any quantum circuit object supported by mitiq. See mitiq.SUPPORTED_PROGRAM_TYPES.

Raises *UnsupportedCircuitError* -- If the input circuit is not supported.

Returns Mitiq circuit equivalent to input circuit. input_circuit_type: Type of input circuit represented by a string.

Return type circuit

mitiq.folding.**converter**(*fold_method: Callable*) → Callable
Decorator for handling conversions.

Unit tests for folding Cirq circuits.

`mitiq.tests.test_folding.`**`test_convert_to_from_mitiq_qiskit`**`()`
Basic test for converting a Qiskit circuit to a Cirq circuit.

`mitiq.tests.test_folding.`**`test_fold_at_random_with_qiskit_circuits`**`()`
Tests folding at random with Qiskit circuits.

`mitiq.tests.test_folding.`**`test_fold_from_left_bad_stretch`**`()`
Tests that a ValueError is raised for an invalid stretch factor.

`mitiq.tests.test_folding.`**`test_fold_from_left_no_stretch`**`()`
Unit test for folding gates from left for a stretch factor of one.

`mitiq.tests.test_folding.`**`test_fold_from_left_three_qubits`**`()`
Unit test for folding gates from left to stretch a circuit.

`mitiq.tests.test_folding.`**`test_fold_from_left_with_qiskit_circuits`**`()`
Tests folding from left with Qiskit circuits.

`mitiq.tests.test_folding.`**`test_fold_from_left_with_terminal_measurements_max_stretch`**`()`
Tests folding from left with terminal measurements.

`mitiq.tests.test_folding.`**`test_fold_from_left_with_terminal_measurements_min_stretch`**`()`
Tests folding from left with terminal measurements.

`mitiq.tests.test_folding.`**`test_fold_from_right_basic`**`()`
Tests folding gates from the right for a two-qubit circuit.

`mitiq.tests.test_folding.`**`test_fold_from_right_max_stretch`**`()`
Tests that fold from right = fold from left with maximum stretch.

`mitiq.tests.test_folding.`**`test_fold_from_right_with_qiskit_circuits`**`()`
Tests folding from right with Qiskit circuits.

`mitiq.tests.test_folding.`**`test_fold_from_right_with_terminal_measurements_max_stretch`**`()`
Tests folding from left with terminal measurements.

`mitiq.tests.test_folding.`**`test_fold_from_right_with_terminal_measurements_min_stretch`**`()`
Tests folding from left with terminal measurements.

`mitiq.tests.test_folding.`**`test_fold_gate_at_index_in_moment_bad_moment`**`()`
Tests local folding with a moment index not in the input circuit.

`mitiq.tests.test_folding.`**`test_fold_gate_at_index_in_moment_empty_circuit`**`()`
Tests local folding with a moment, index with an empty circuit.

`mitiq.tests.test_folding.`**`test_fold_gate_at_index_in_moment_one_qubit`**`()`
Tests local folding with a moment, index for a one qubit circuit.

`mitiq.tests.test_folding.`**`test_fold_gate_at_index_in_moment_two_qubit_gates`**`()`
Tests local folding with a moment, index for a two qubit circuit with two qubit gates.

`mitiq.tests.test_folding.`**`test_fold_gate_at_index_in_moment_two_qubits`**`()`
Tests local folding with a moment, index for a two qubit circuit with single qubit gates.

`mitiq.tests.test_folding.`**`test_fold_gates`**`()`
Test folding gates at specified indices within specified moments.

`mitiq.tests.test_folding.`**`test_fold_gates_at_random_no_stretch`**`()`
Tests folded circuit is identical for a stretch factor of one.

`mitiq.tests.test_folding.`**`test_fold_gates_at_random_seed_one_qubit`**`()`
Test for folding gates at random on a one qubit circuit with a seed for repeated behavior.

mitiq.tests.test_folding.**test_fold_gates_in_moment_multi_qubit_gates**()
Tests folding gates at given indices within a moment.

mitiq.tests.test_folding.**test_fold_gates_in_moment_single_qubit_gates**()
Tests folding gates at given indices within a moment.

mitiq.tests.test_folding.**test_fold_global_with_qiskit_circuits**()
Tests fold_local with input Qiskit circuits.

mitiq.tests.test_folding.**test_fold_local_big_stretch_from_left**()
Test for local folding with stretch > 3.

mitiq.tests.test_folding.**test_fold_local_small_stretch_from_left**()
Test for local folding with stretch < 3.

mitiq.tests.test_folding.**test_fold_local_stretch_three_from_left**()
Test for local folding with stretch > 3.

mitiq.tests.test_folding.**test_fold_local_with_qiskit_circuits**()
Tests fold_local with input Qiskit circuits.

mitiq.tests.test_folding.**test_fold_moments**()
Tests folding moments in a circuit.

mitiq.tests.test_folding.**test_fold_random_bad_stretch**()
Tests that an error is raised when a bad stretch is provided.

mitiq.tests.test_folding.**test_fold_random_max_stretch**()
Tests that folding at random with max stretch folds all gates on a multi-qubit circuit.

mitiq.tests.test_folding.**test_fold_random_min_stretch**()
Tests that folding at random with min stretch returns a copy of the input circuit.

mitiq.tests.test_folding.**test_fold_random_no_repeats**()
Tests folding at random to ensure that no gates are folded twice and folded gates are not folded again.

mitiq.tests.test_folding.**test_fold_random_with_terminal_measurements_max_stretch**()
Tests folding from left with terminal measurements.

mitiq.tests.test_folding.**test_fold_random_with_terminal_measurements_min_stretch**()
Tests folding from left with terminal measurements.

mitiq.tests.test_folding.**test_fold_with_intermediate_measurements_raises_error**(*fold_method*)
Tests folding from left with intermediate measurements.

mitiq.tests.test_folding.**test_global_fold_min_stretch**()
Tests that global fold with stretch = 1 is the same circuit.

mitiq.tests.test_folding.**test_global_fold_min_stretch_with_terminal_measurements**()
Tests that global fold with stretch = 1 is the same circuit.

mitiq.tests.test_folding.**test_global_fold_raises_error_intermediate_measurements**()
Tests than an error is raised when trying to globally fold a circuit with intermediate measurements.

mitiq.tests.test_folding.**test_global_fold_stretch_factor_eight_terminal_measurements**()
Tests global folding with a stretch factor not a multiple of three so that local folding is also called.

mitiq.tests.test_folding.**test_global_fold_stretch_factor_nine_with_terminal_measurements**()
Tests global folding with the stretch as a factor of 9 for a circuit with terminal measurements.

mitiq.tests.test_folding.**test_global_fold_stretch_factor_of_three**()
Tests global folding with the stretch as a factor of 3.

`mitiq.tests.test_folding.`**`test_global_fold_stretch_factor_of_three_with_terminal_measurement`**
Tests global folding with the stretch as a factor of 3 for a circuit with terminal measurements.

`mitiq.tests.test_folding.`**`test_is_measurement`**`()`
Tests for checking if operations are measurements.

`mitiq.tests.test_folding.`**`test_pop_measurements_and_add_measurements`**`()`
Tests popping measurements from a circuit..

`mitiq.tests.test_folding.`**`test_update_moment_indices`**`()`
Tests indices of moments are properly updated.

3.4 Matrices

`mitiq.matrices.`**`npI = array([[1, 0], [0, 1]])`**
Defines the identity matrix in SU(2) algebra as a (2,2) *np.array*.

`mitiq.matrices.`**`npX = array([[0, 1], [1, 0]])`**
Defines the sigma_x Pauli matrix in SU(2) algebra as a (2,2) *np.array*.

`mitiq.matrices.`**`npY = array([[ 0.+0.j, -0.-1.j], [ 0.+1.j, 0.+0.j]])`**
Defines the sigma_y Pauli matrix in SU(2) algebra as a (2,2) *np.array*.

`mitiq.matrices.`**`npZ = array([[ 1, 0], [ 0, -1]])`**
Defines the sigma_z Pauli matrix in SU(2) algebra as a (2,2) *np.array*.

3.5 PyQuil Utils

`mitiq.mitiq_pyquil.pyquil_utils.`**`add_depolarizing_noise`**`(`*pq: pyquil.Program*, *noise: float*`)` → pyquil.Program
Returns a quantum program with depolarizing channel noise.

Parameters

- **`pq`** -- Quantum program as `Program`.
- **`noise`** -- Noise constant for depolarizing channel.

Returns Expected value.

Return type expval

`mitiq.mitiq_pyquil.pyquil_utils.`**`random_identity_circuit`**`(`*depth=None*`)`
Returns a single-qubit identity circuit based on Pauli gates.

`mitiq.mitiq_pyquil.pyquil_utils.`**`run_program`**`(`*pq: pyquil.Program*, *shots: int = 500*`)` → float
Returns the expected value of a circuit run several times.

Parameters

- **`pq`** -- Quantum circuit as `Program`.
- **`shots`** -- (Default: 500) Number of shots the circuit is run.

Returns Quantum program with added noise.

Return type pq

`mitiq.mitiq_pyquil.pyquil_utils.`**`run_with_noise`**(*circuit: pyquil.Program*, *noise: float*, *shots: int*) → float

Returns the expected value of a circuit run several times with noise.

Parameters

- **`circuit`** -- Quantum circuit as `Program`.
- **`noise`** -- Noise constant for depolarizing channel.
- **`shots`** -- Number of shots the circuit is run.

Returns Expected value.

Return type expval

`mitiq.mitiq_pyquil.pyquil_utils.`**`scale_noise`**(*pq: pyquil.Program*, *param: float*) → pyquil.Program

Returns a circuit rescaled by the depolarizing noise parameter.

Parameters

- **`pq`** -- Quantum circuit as `Program`.
- **`param`** -- noise scaling.

Returns Quantum program with added noise.

Tests for zne.py with PyQuil backend.

3.6 Qiskit Utils

Functions to convert from Mitiq's internal circuit representation to Qiskit representations.

Unit tests for circuit conversions between Mitiq circuits and Qiskit circuits.

`mitiq.mitiq_qiskit.tests.test_conversions.`**`test_bell_state_to_from_circuits`**()

Tests cirq.Circuit --> qiskit.QuantumCircuit --> cirq.Circuit with a Bell state circuit.

`mitiq.mitiq_qiskit.tests.test_conversions.`**`test_bell_state_to_from_qasm`**()

Tests cirq.Circuit --> QASM string --> cirq.Circuit with a Bell state circuit.

`mitiq.mitiq_qiskit.tests.test_conversions.`**`test_random_circuit_to_from_circuits`**()

Tests cirq.Circuit --> qiskit.QuantumCircuit --> cirq.Circuit with a random one-qubit circuit.

`mitiq.mitiq_qiskit.tests.test_conversions.`**`test_random_circuit_to_from_qasm`**()

Tests cirq.Circuit --> QASM string --> cirq.Circuit with a random one-qubit circuit.

`mitiq.mitiq_qiskit.qiskit_utils.`**`measure`**(*circuit*, *qid*) → qiskit.circuit.quantumcircuit.QuantumCircuit

Apply the measure method on the first qubit of a quantum circuit given a classical register.

Parameters

- **`circuit`** -- Quantum circuit.
- **`qid`** -- classical register.

Returns circuit after the measurement.

Return type circuit

`mitiq.mitiq_qiskit.qiskit_utils.`**`random_identity_circuit`**(*depth=None*)

Returns a single-qubit identity circuit based on Pauli gates.

Parameters **depth** (*int*) -- depth of the quantum circuit.

Returns quantum circuit as a `qiskit.QuantumCircuit` object.

Return type circuit

`mitiq.mitiq_qiskit.qiskit_utils.`**run_program**(*pq: qiskit.circuit.quantumcircuit.QuantumCircuit*, *shots: int = 100*) → float

Runs a quantum program.

Parameters

- **pq** -- Quantum circuit.
- **shots** (*int*) -- Number of shots to run the circuit on the back-end.

Returns expected value.

Return type expval

`mitiq.mitiq_qiskit.qiskit_utils.`**run_with_noise**(*circuit: qiskit.circuit.quantumcircuit.QuantumCircuit*, *noise: float*, *shots: int*) → float

Runs the quantum circuit with a depolarizing channel noise model.

Parameters

- **circuit** (*qiskit.QuantumCircuit*) -- Ideal quantum circuit.
- **noise** (*float*) -- Noise constant going into *depolarizing_error*.
- **shots** (*int*) -- Number of shots to run the circuit on the back-end.

Returns expected values.

Return type expval

`mitiq.mitiq_qiskit.qiskit_utils.`**scale_noise**(*pq: qiskit.circuit.quantumcircuit.QuantumCircuit*, *param: float*) → qiskit.circuit.quantumcircuit.QuantumCircuit

Scales the noise in a quantum circuit of the factor *param*.

Parameters

- **pq** -- Quantum circuit.
- **noise** (*float*) -- Noise constant going into *depolarizing_error*.
- **shots** (*int*) -- Number of shots to run the circuit on the back-end.

Returns quantum circuit as a `qiskit.QuantumCircuit` object.

Return type pq

Tests for zne.py with Qiskit backend.

`mitiq.mitiq_qiskit.tests.test_zne.`**basic_executor**(*qp: Optional[cirq.circuits.circuit.Circuit]*, *shots: int = 500*) → float

Runs a program.

Args: qp: quantum program. shots: number of executions of the program.

Returns A float.

`mitiq.mitiq_qiskit.tests.test_zne.`**test_execute_with_zne**()

Tests a random identity circuit execution with zero-noise extrapolation.

mitiq.mitiq_qiskit.tests.test_zne.**test_mitigate_executor**()
Tests a random identity circuit executor.

mitiq.mitiq_qiskit.tests.test_zne.**test_qrun_factory**()
Tests qrun of a Richardson Factory.

mitiq.mitiq_qiskit.tests.test_zne.**test_zne_decorator**()
Tests a zne decorator.

3.7 Utils

Utility functions.

mitiq.utils.**random_circuit**(*depth: int*, *seed: Optional[int] = None*) → cirq.circuits.circuit.Circuit
Returns a random single-qubit circuit with Pauli gates.

Parameters

- **depth** -- Number of gates in the circuit.
- **seed** -- Seed for the random number generator.

Returns the randomized quantum circuit as a `cirq.Circuit`.

Return type circuit

Unit test for utility functions.

3.8 Zero Noise Extrapolation

Zero-noise extrapolation tools.

mitiq.zne.**execute_with_zne**(*qp: Optional[cirq.circuits.circuit.Circuit]*, *executor: Callable[[Optional[cirq.circuits.circuit.Circuit]], float]*, *fac: mitiq.factories.Factory = None*, *scale_noise: Callable[[Optional[cirq.circuits.circuit.Circuit], float], Optional[cirq.circuits.circuit.Circuit]] = None*) → float
Takes as input a quantum circuit and returns the associated expectation value evaluated with error mitigation.

Parameters

- **qp** -- Quantum circuit to execute with error mitigation.
- **executor** -- Function executing a circuit and producing an expect. value (without error mitigation).
- **fac** -- Factory object determining the zero-noise extrapolation algorithm. If not specified, LinearFactory([1.0, 2.0]) will be used.
- **scale_noise** -- Function for scaling the noise of a quantum circuit. If not specified, a default method will be used.

mitiq.zne.**mitigate_executor**(*executor: Callable[[Optional[cirq.circuits.circuit.Circuit]], float]*, *fac: mitiq.factories.Factory = None*, *scale_noise: Callable[[Optional[cirq.circuits.circuit.Circuit], float], Optional[cirq.circuits.circuit.Circuit]] = None*) → Callable[[Optional[cirq.circuits.circuit.Circuit]], float]
Returns an error-mitigated version of the input "executor". Takes as input a generic function ("executor"), defined by the user, that executes a circuit with an arbitrary backend and produces an expectation value.

Returns an error-mitigated version of the input "executor", having the same signature and automatically performing ZNE at each call.

Parameters

- **executor** -- Function executing a circuit and returning an exp. value.
- **fac** -- Factory object determining the zero-noise extrapolation algorithm. If not specified, LinearFactory([1.0, 2.0]) is used.
- **scale_noise** -- Function for scaling the noise of a quantum circuit. If not specified, a default method is used.

`mitiq.zne.`**`qrun_factory`**(*fac: mitiq.factories.Factory, qp: Optional[cirq.circuits.circuit.Circuit], executor: Callable[[Optional[cirq.circuits.circuit.Circuit]], float], scale_noise: Callable[[Optional[cirq.circuits.circuit.Circuit], float], Optional[cirq.circuits.circuit.Circuit]]*) → None

Runs the factory until convergence executing quantum circuits. Accepts different noise levels.

Parameters

- **fac** -- Factory object to run until convergence.
- **qp** -- Circuit to mitigate.
- **executor** -- Function executing a circuit; returns an expectation value.
- **scale_noise** -- Function that scales the noise level of a quantum circuit.

`mitiq.zne.`**`run_factory`**(*fac: mitiq.factories.Factory, noise_to_expval: Callable[[float], float], max_iterations: int = 100*) → None

Runs a factory until convergence (or iterations reach "max_iterations").

Parameters

- **fac** -- Instance of Factory object to be run.
- **noise_to_expval** -- Function mapping noise scale to expectation vales.
- **max_iterations** -- Maximum number of iterations (optional). Default: 100.

`mitiq.zne.`**`zne_decorator`**(*fac: mitiq.factories.Factory = None, scale_noise: Callable[[Optional[cirq.circuits.circuit.Circuit], float], Optional[cirq.circuits.circuit.Circuit]] = None*) → Callable[[Optional[cirq.circuits.circuit.Circuit]], float]

Decorator which automatically adds error mitigation to any circuit-executor function defined by the user.

It is supposed to be applied to any function which executes a quantum circuit with an arbitrary backend and produces an expectation value.

Parameters

- **fac** -- Factory object determining the zero-noise extrapolation algorithm. If not specified, LinearFactory([1.0, 2.0]) will be used.
- **scale_noise** -- Function for scaling the noise of a quantum circuit. If not specified, a default method will be used.

CHAPTER 4

Contributors Guide

This is the Contributors guide for the documentation of Mitiq, a Python toolkit for implementing error mitigation on quantum computers.

4.1 Requirements

The documentation is generated with Sphinx.

```
pip install -U sphinx m2r sphinxcontrib-bibtex pybtex
```

`m2r` allows to include `.md` files, besides `.rst`, in the documentation; `sphinxcontrib-bibtex` allows to include citations in a `.bib` file and `pybtex` allows to customize how they are rendered, e.g., APS-style.

You can check that Sphinx is installed with `sphinx-build --version`.

4.2 How to Update the Documentation

4.2.1 The configuration file

- Since the documentation is already created, you need not to generate a configuration file from scratch (this is done with `sphinx-quickstart`). Meta-data, extentions and other custom specifications are accounted for in the `conf.py` file.

4.2.2 Add features in the conf.py file

- To add specific feature to the documentation, extensions can be include. For example to add classes and functions to the API doc, make sure that autodoc extension is enabled in the `conf.py` file, and for tests the `doctest` one,

```
extensions = ['sphinx.ext.autodoc','sphinx.ext.doctest']
```

4.2.3 Update the guide with a tree of restructured text files

You need not to modify the `docs/build` folder, as it is automatically generated. You will modify only the `docs/source` files.

The documentation is divided into a **guide**, whose content needs to be written from scratch, and an **API-doc** part, which can be partly automatically generated.

- To add information in the guide, it is possible to include new information as a restructured text (`.rst`) or markdown (`.md`) file.

The main file is `index.rst`. It includes a `guide.rst` and an `apidoc.rst` file, as well as other files. Like in LaTeX, each file can include other files. Make sure they are included in the table of contents

```
.. toctree::
   :maxdepth: 2
   :caption: Contents:

   changelog.rst
```

4.2.4 You can include markdown files in the guide

- Information to the guide can also be added from markdown (`.md`) files, since `m2r` (`pip install --upgrade m2r`) is installed and added to the `conf.py` file (`extensions = ['m2r']`). Just add the `.md` file to the toctree.

To include `.md` files outside of the documentation `source` directory, you can add in `source` an `.rst` file to the toctree that contains inside it the

`.. mdinclude:: ../file.md` command, where `file.md` is the one to be added.

4.2.5 Automatically add information to the API doc

- New modules, classes and functions can be added by listing them in the appropriate `.rst` file (such as `autodoc.rst` or a child), e.g.,

```
Factories
---------
.. automodule:: mitiq.factories
   :members:
```

will add all elements of the `mitiq.factories` module. One can hand-pick classes and functions to add, to comment them, as well as exclude them.

4.2.6 Build the documentation locally

- To build the documentation, from `bash`, move to the `docs` folder and run .. code-block:: bash

  make html

  this generates the `docs/build` folder. This folder is not kept track of in the github repository, as `docs/build` is present in the `.gitignore` file.

The `html`, `latex` and `pdf` files will be automatically created in the `docs/build` folder with similar commands.

Note that `make html` reads the `make.bat` file in the `docs/` folder; this was generated by `sphinx-quickstart`, used to generate the documentation in the first place. The makefile automatically runs a script that implements the explicit command `sphinx-build -b html source build`.

````
### Create the pdf
- To create the latex files and output a pdf, run

```bash
make latexpdf
````

4.3 How to Test the Documentation Examples

There are several ways to check that the documentation examples work. Currently, `mitiq` is testing them with the `doctest` extension of `sphinx`. This is set in the `conf.py` file and is executed with

```
make doctest
```

This tests the code examples in the guide and ".rst" files, as well as testing the docstrings, since these are imported with the `autodoc` extension.

When writing a new example, you can use different directives in the rst file to include code blocks. One of them is

```
.. code-block:: python

   1+1        # simple example
```

In order to make sure that the block is parsed with `make doctest`, use the `testcode` directive. This can be used in pair with `testoutput`, if something is printed, and, eventually `testsetup`, to import modules or set up variables in an invisible block. An example is:

```
.. testcode:: python

   1+1        # simple example
```

with no output and

```
.. testcode:: python

   print(1+1)        # explicitly print

.. testoutput:: python

   2        # match the print message
```

The use of `testsetup` allows blocks that do not render:
```

```
.. testsetup:: python

   import numpy as np  # this block is not rendered in the html or pdf

.. testcode:: python

   np.array(2)

.. testoutput:: python

   array(2)
```

There is also the `doctest` directive, which allows to include interactive Python blocks. These need to be given this way:

```
  .. doctest:: python

     >>> import numpy as np
     >>> print(np.array(2))
     array(2)

  Notice that no space is left between the last input and the output.

A way to test docstrings without installing sphinx is with `\ ``pytest`` +
``doctest`` <http://doc.pytest.org/en/latest/doctest.html>`_\ :
```

```
pytest --doctest-glob='*.rst'
```

or alternatively

```
pytest --doctest-modules
```

However, this only checks `doctest` blocks, and does not recognize `testcode` blocks. Moreover, it does not parse the `conf.py` file nor uses sphinx. A way to include testing of `testcode` and `testoutput` blocks is with the ``*pytest-sphinx*` <https://github.com/thisch/pytest-sphinx>`_ plugin. Once installed,

```
pip install pytest-sphinx
```

it will show up as a plugin, just like `pytest-coverage` and others, simply calling

```
pytest --doctest-glob='*.rst'
```

The `pytest-sphinx` plugin does not support `testsetup` directives.

In order to skip a test, if this is problematic, one can use the `SKIP` and `IGNORE` keywords, adding them as comments next to the relevant line or block:

```
>>> something_that_raises()  # doctest: +IGNORE
```

One can also use various `doctest` features by configuring them in the `docs/pytest.ini` file.

4.4 How to Make a New Release of the Documentation

4.4.1 Work in an environment

- Create a conda environment for the documentation .. code-block:: bash

  conda create -n mitiqenv conda activate mitiqenv

4.4.2 Create a new branch

- Create a branch in `git` for the documentation with the release number up to minor (e.g., 0.0.2--->00X) .. code-block:: bash

  (mitiqenv) git checkout -b mitiq00X

4.4.3 Create the html and pdf file and save it in the `docs/pdf` folder

- To create the html structure .. code-block:: bash

  make html

  and for the pdf, .. code-block:: bash

  make latexpdf

  Since the `docs/build` folder is not kept track of, copy the pdf file with the documentation from `docs/build/latex` to the `docs/pdf` folder, naming it according to the release version with major and minor. Make a copy named `Mitiq-latest-release.pdf` in the same folder.

4.5 Additional information

Here are some notes on how to build docs.

Here is a cheat sheet for restructed text formatting, e.g. syntax for links etc.

CHAPTER 5

Change Log

5.1 Version 0.1.0 (Date)

- **Initial release.**

CHAPTER 6

References

CHAPTER 7

Indices and tables

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Python Module Index

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