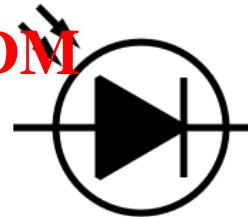
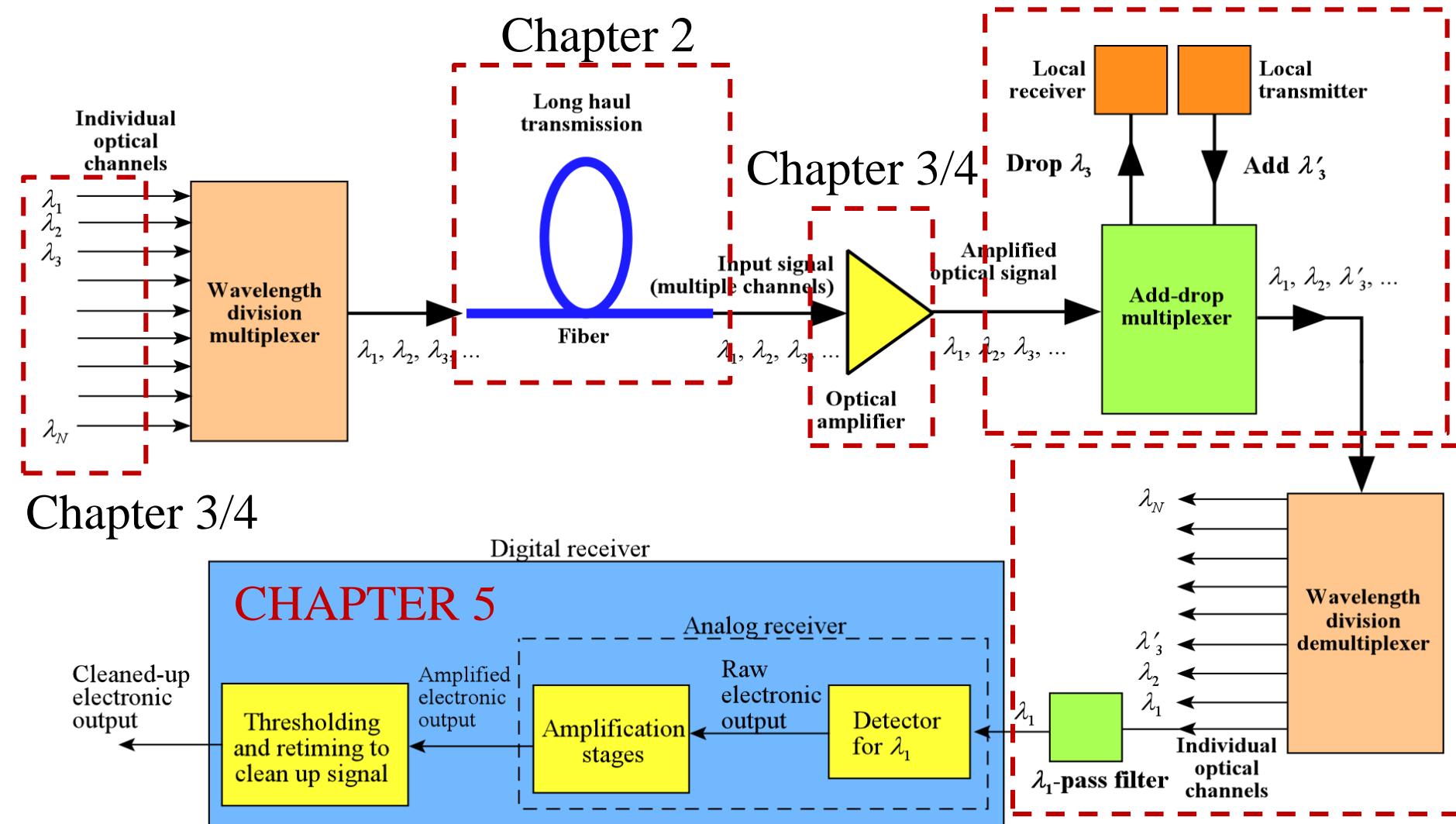


# WAVELENGTH DIVISION MULTIPLEXING: WDM

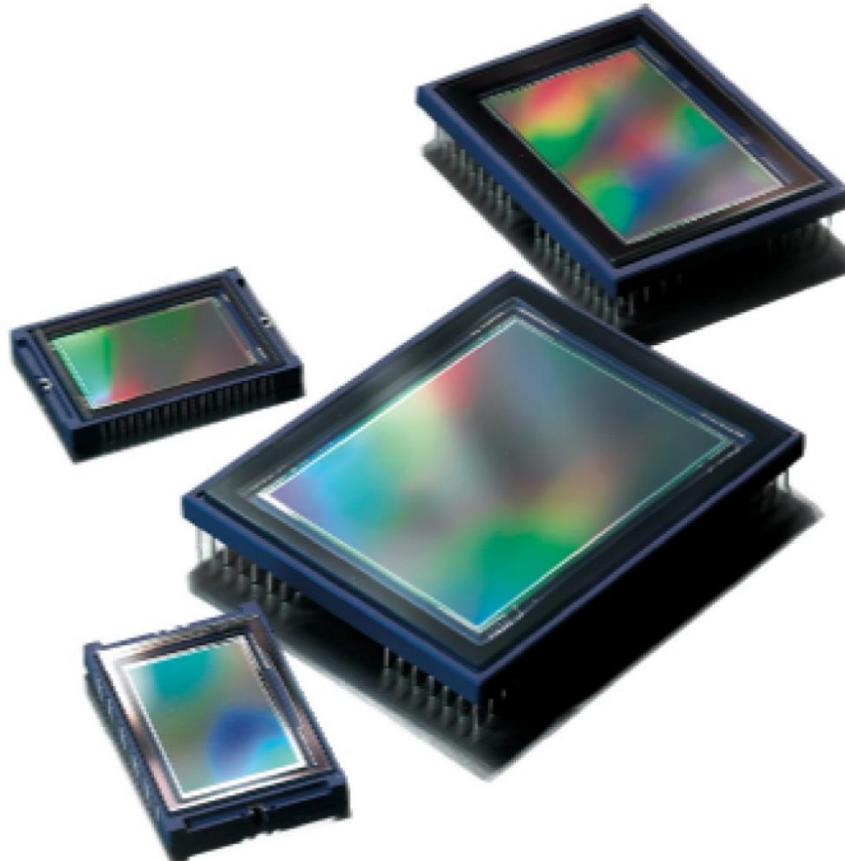
# Chapter 5 Photodetectors and Image Sensors



The inventors of the CCD (charge coupled device) image sensor at AT&T Bell Labs: Willard Boyle (left) and George Smith (right). The CCD was invented in 1969, the first CCD solid state camera was demonstrated in 1970, and a broadcast quality TV camera by 1975. (W. S. Boyle and G. E. Smith, "Charge Coupled Semiconductor Devices", *Bell Systems Technical Journal*, 49, 587, 1970. (Courtesy of Alcatel-Lucent Bell Labs.)

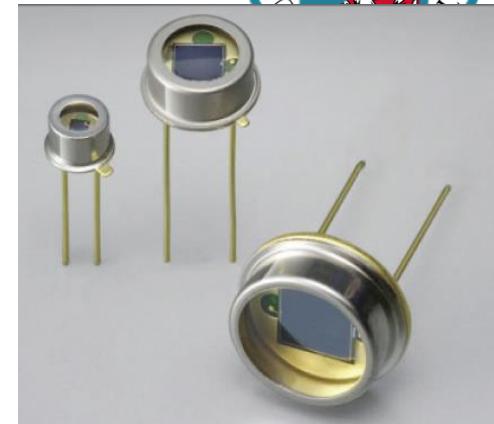
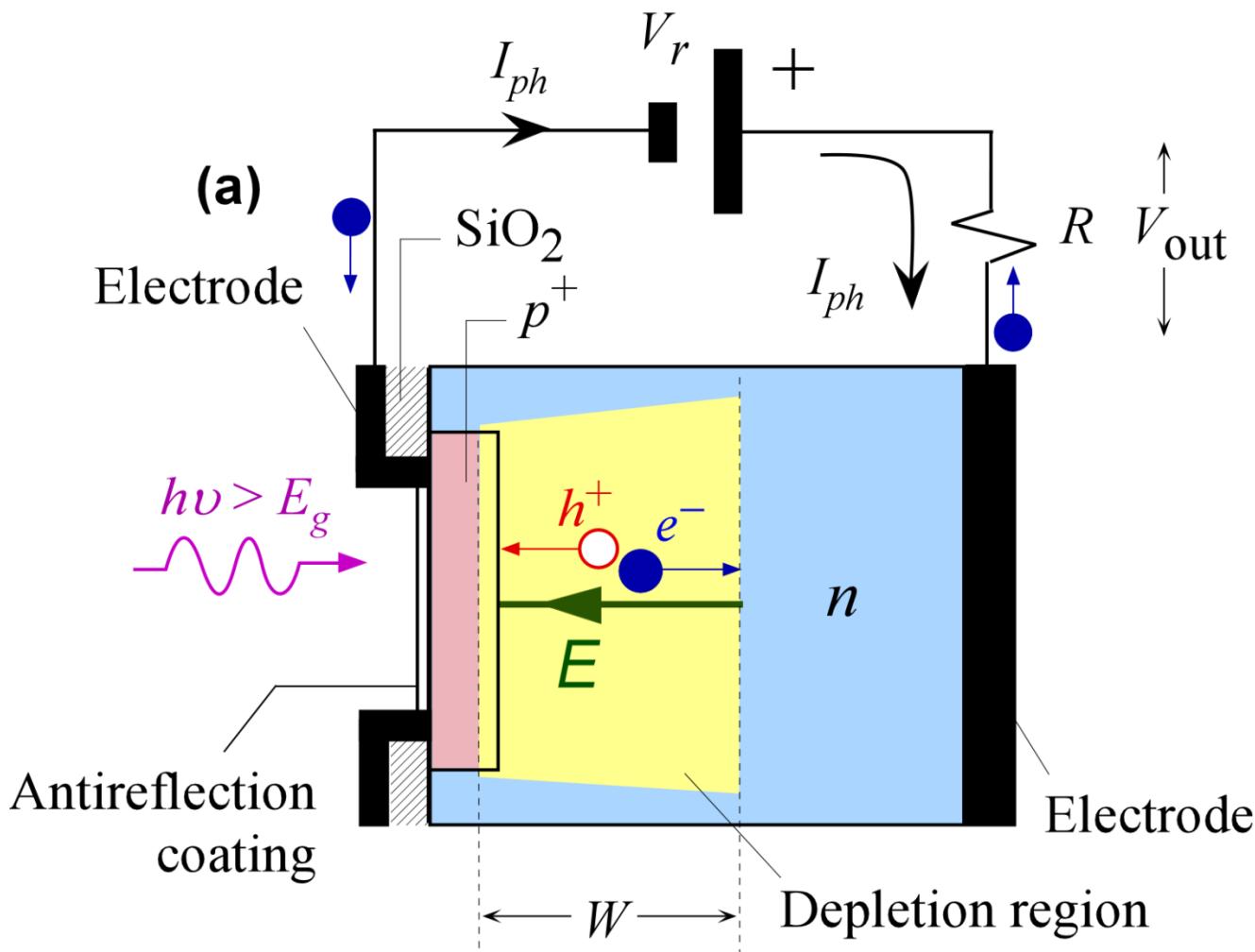
# Chapter 5

## Photodetectors and Image Sensors



Top, courtesy of Voxel Inc. Bottom, courtesy of Hamamatsu. Right, courtesy of Teledyne-DALSA

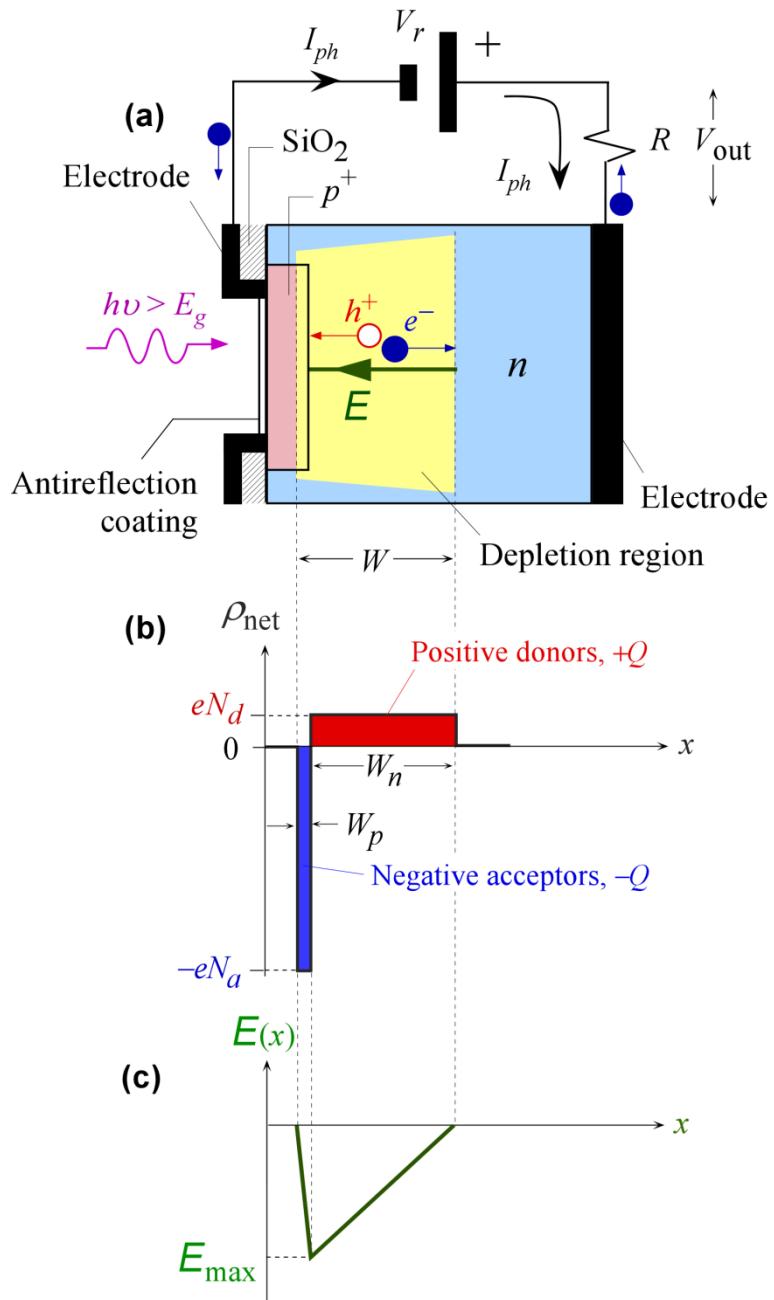
# *pn* Junction Photodiode



Courtesy of Hamamatsu  
(Used with permission)

A schematic diagram of a reverse biased *pn* junction photodiode.

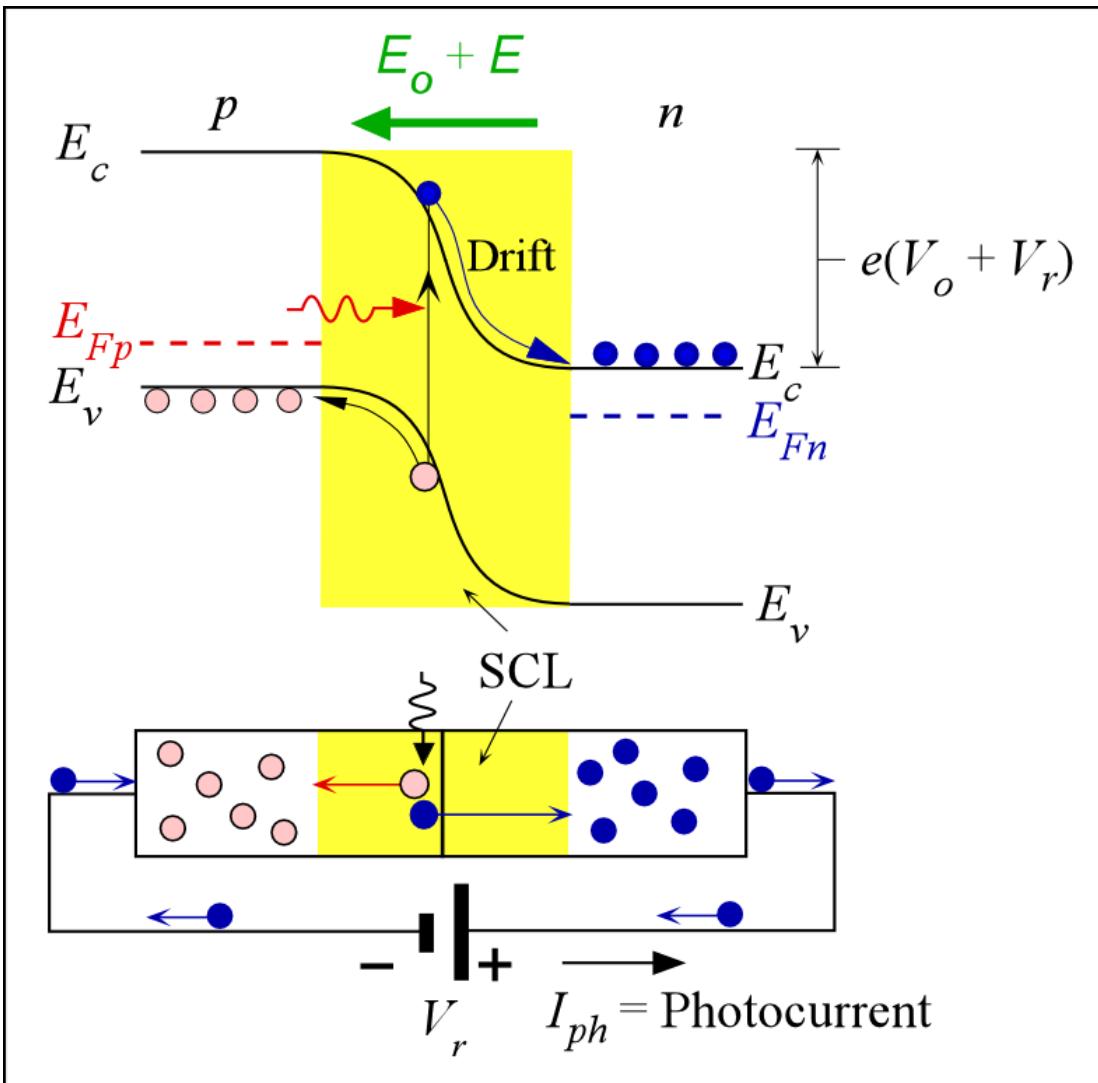
# *pn* Junction Photodiode



(a) A schematic diagram of a reverse biased *pn* junction photodiode. (b) Net space charge across the diode in the depletion region.  $N_d$  and  $N_a$  are the donor and acceptor concentrations in the *p* and *n* sides. (c) The field in the depletion region.

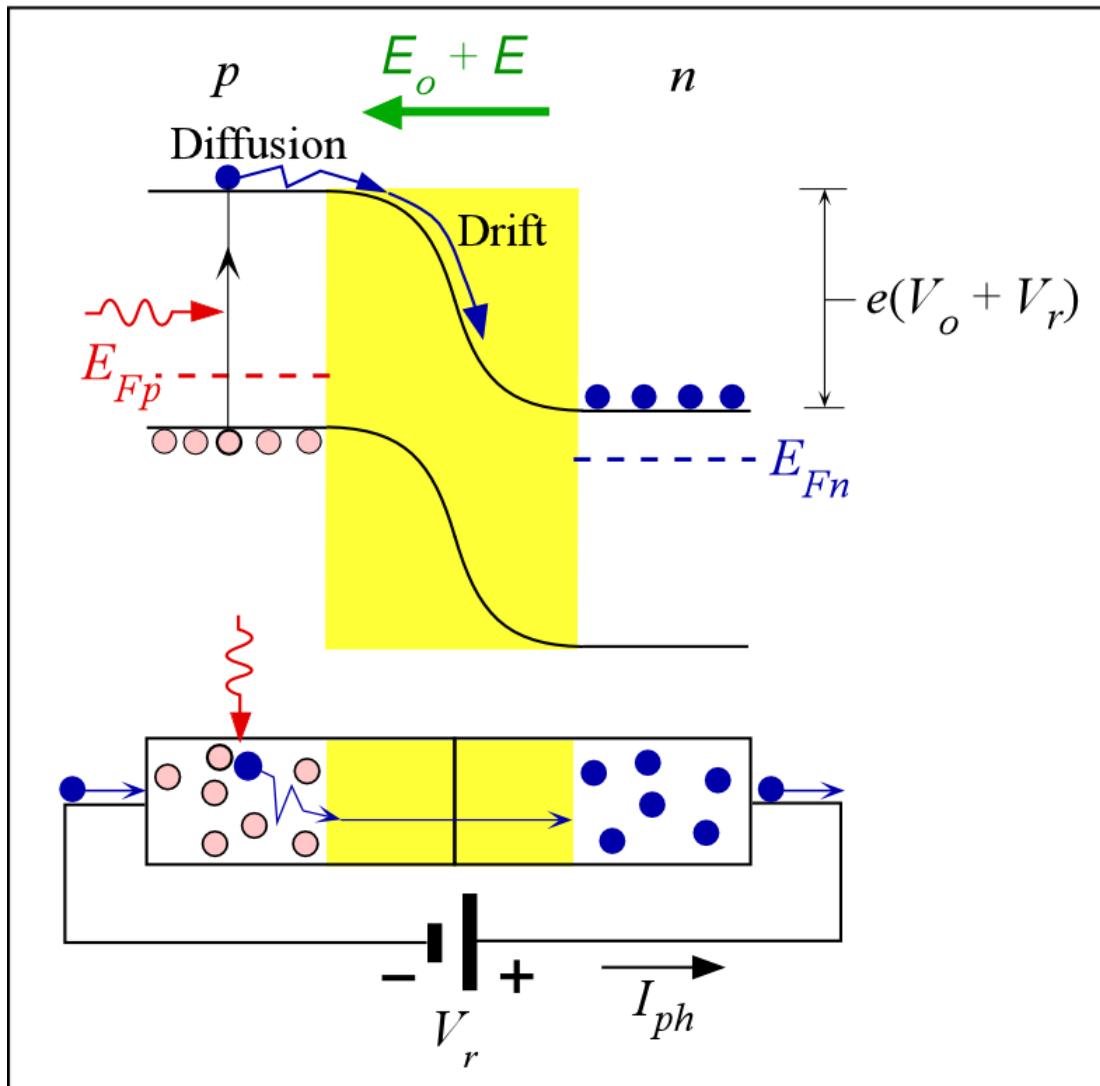
(Note: Depletion region shape in (a) is schematic only.)

# *pn* Junction Photodiode ENERGY BANDS



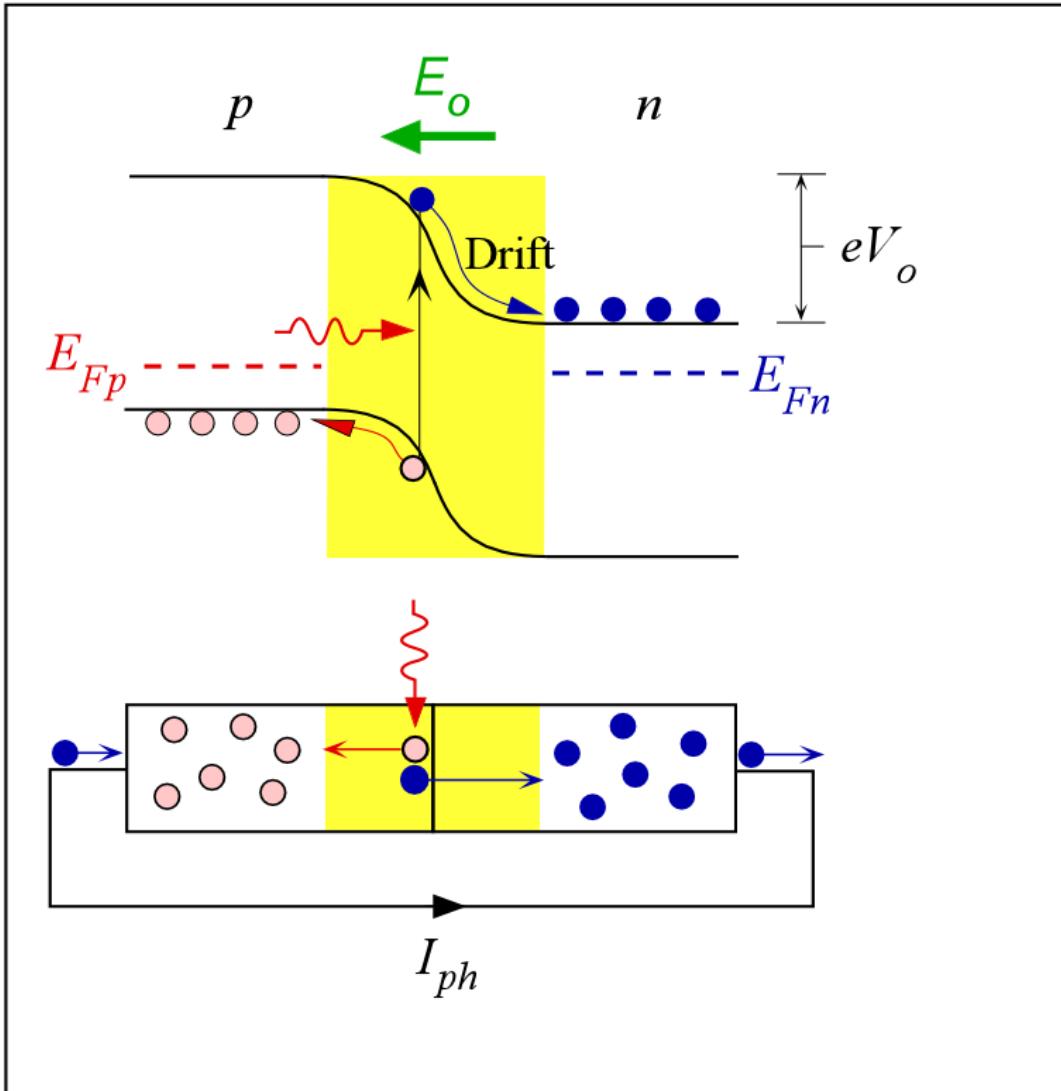
A reverse biased *pn* junction. Photogeneration inside the SCL generates an electron and a hole. Both fall their respective energy hills (electron along  $E_c$  and hole along  $E_h$ ) *i.e.* they drift, and cause a photocurrent  $I_{ph}$  in the external circuit.

# *pn* Junction Photodiode



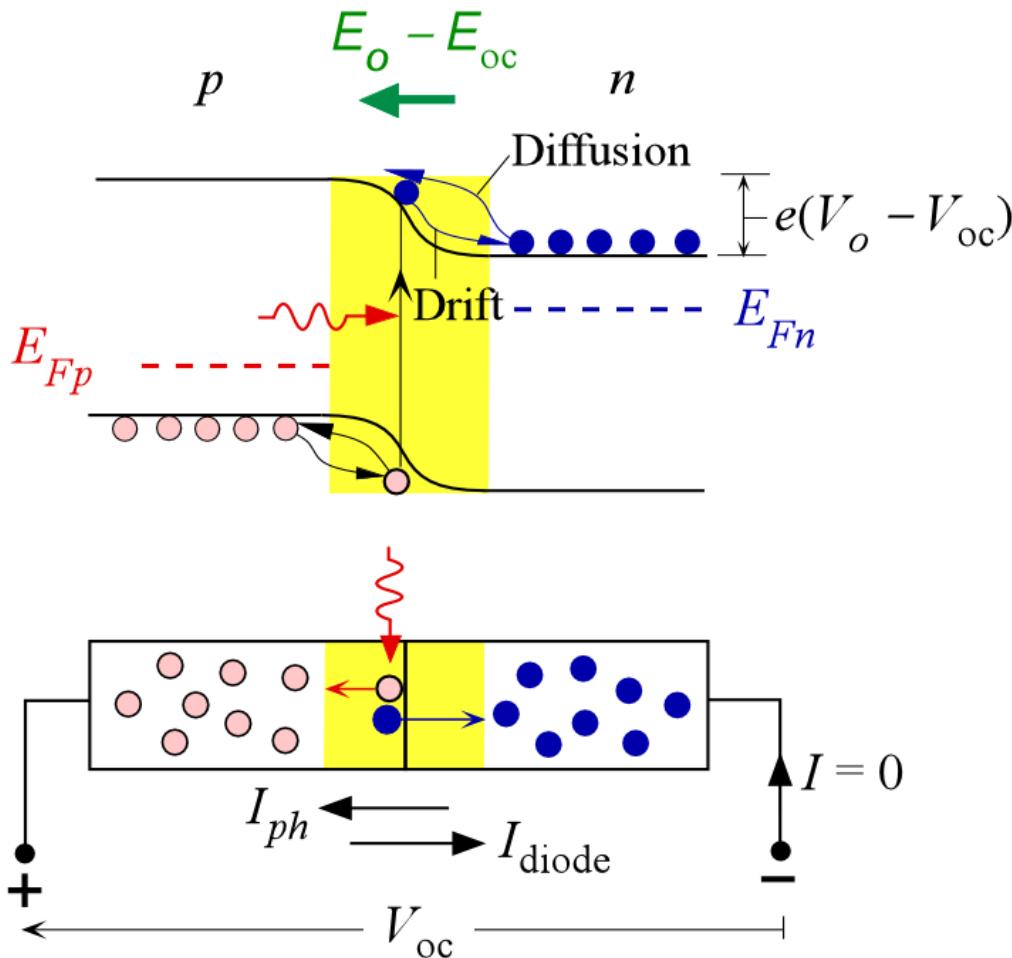
Photogeneration occurs in the neutral region. The electron has to diffuse to the depletion layer and then roll down the energy hill *i.e.* drift across the SCL.

# *pn* Junction Photodiode (photovoltaic mode)



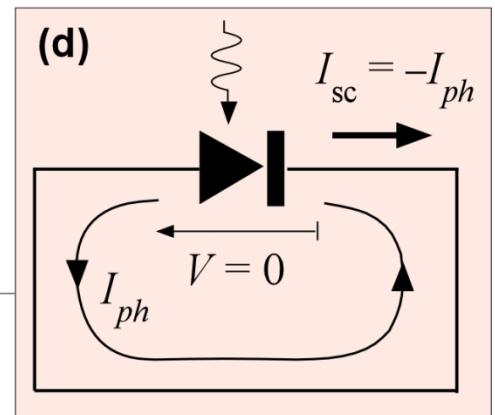
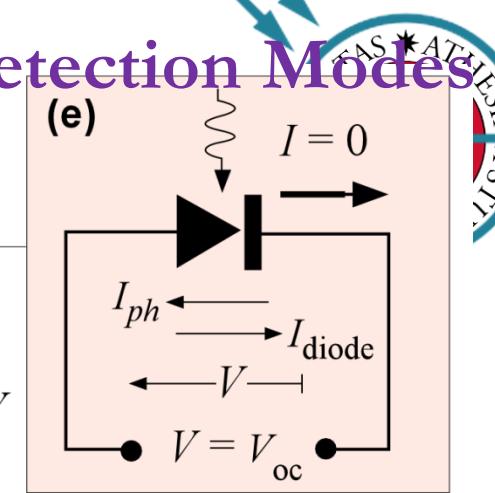
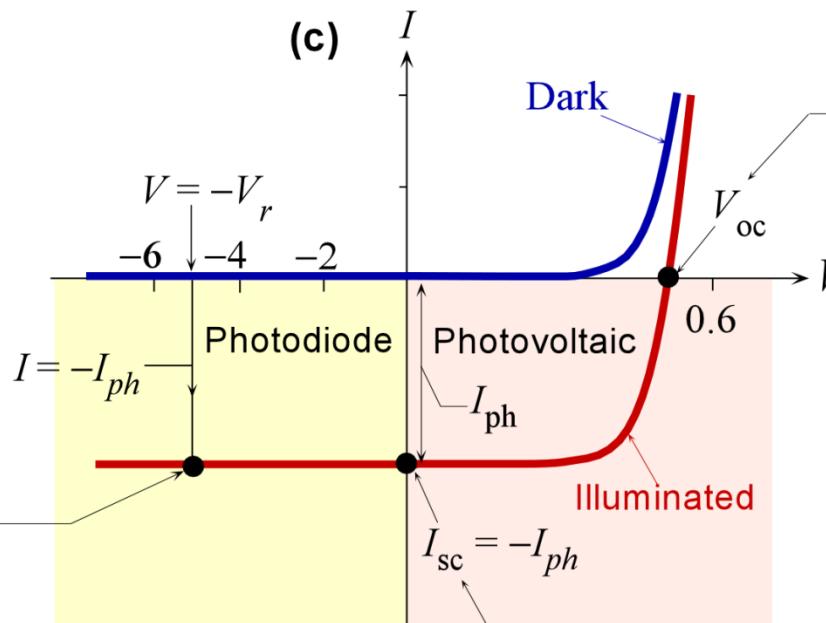
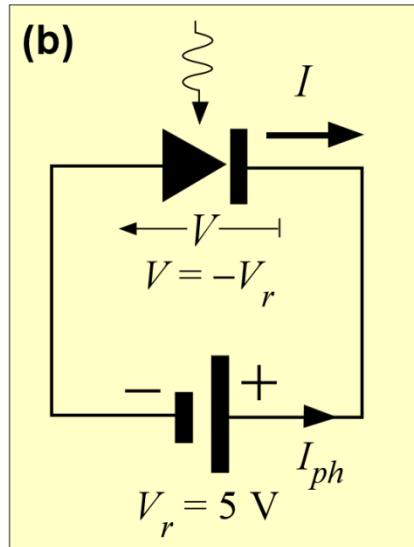
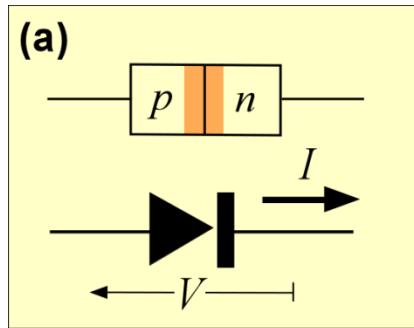
A shorted *pn* junction. The photogenerated electron and hole in the SCL roll down their energy hills, *i.e.* drift across the SCL, and cause a current  $I_{ph}$  in the external circuit.

# *pn* Junction Photodiode



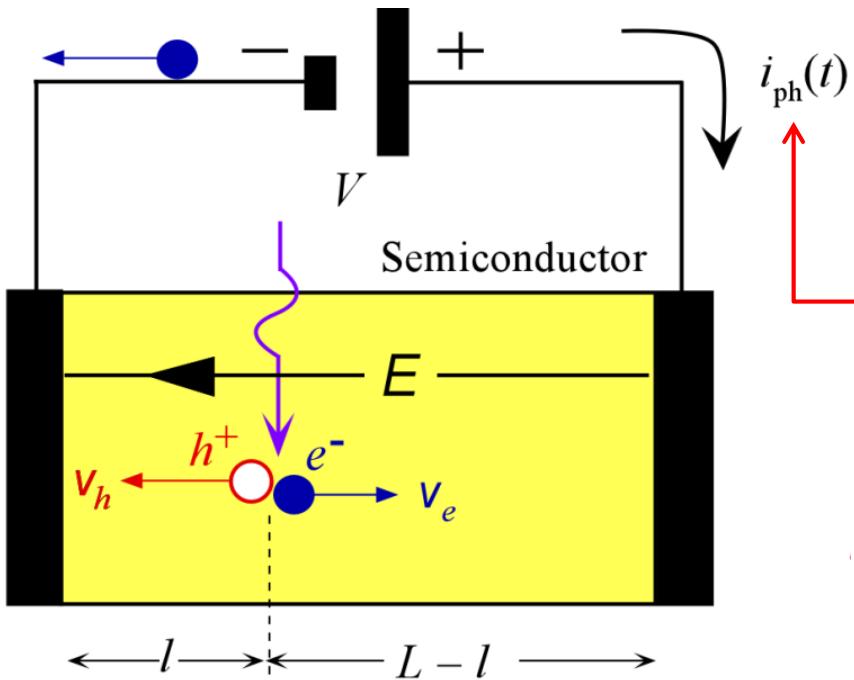
The *pn* junction in open circuit. The photogenerated electron and hole roll down their energy hills (drift) but there is a voltage  $V_{oc}$  across the diode that causes them the diffuse back so that the net current is zero.

# Photodetection Modes



- (a) The sign convention for the voltage  $V$  and current  $I$  for a *pn* junction.
- (b) If the *pn* junction is reverse biased by  $V_r = 5\text{ V}$ , then  $V = -V_r = -5\text{ V}$ . Under illumination, the *pn* junction current  $I = -I_{ph}$  and is negative.
- (c) The  $I$ - $V$  characteristics of a *pn* junction in the dark and under illumination.
- (d) A short circuit *pn* junction under illumination. The voltage  $V = 0$  but there is a short circuit current so that  $I = I_{sc} = -I_{ph}$ .
- (e) An open circuit *pn* junction under illumination generates an open circuit voltage  $V_{oc}$ .

# Shockley-Ramo Theorem



An electron and hole pair (EHP) is photogenerated at  $x = l$ . The electron and the hole drift in opposite directions with drift velocities  $v_h$  and  $v_e$ .

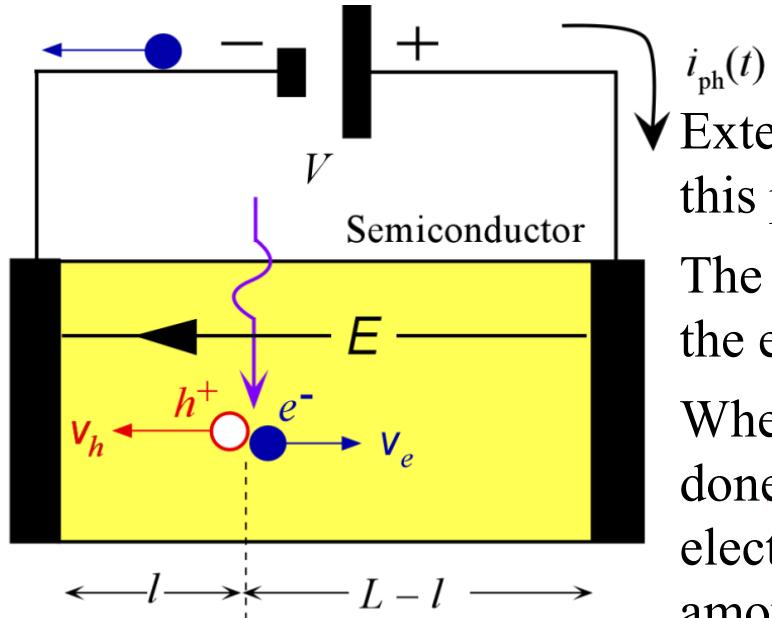
## Question

What is the induced current  $i_{\text{ph}}(t)$ ?

This is a simplified version of the more general treatment that examines the induced current on an electrode due to the motion of an electron. Its origins lie in tube-electronics in which engineers were interested in calculating how much current would flow into various electrodes of a vacuum tube as the electrons in the tube drifted.

See W. Shockley, *J. Appl. Phys.*, **9**, 635, 1938 and S. Ramo, *Proc. IRE* **27**, 584, 1939.

# Shockley-Ramo Theorem



External photocurrent due to the motion of this photogenerated electron is  $i_e(t)$ .

The electron is acted on by the force  $eE$  of the electric field.

When it moves a distance  $dx$ , work must be done by the external circuit. In time  $dt$ , the electron drifts a distance  $dx$  and does an amount of work  $eEdx$

Work done  $eEdx$  is provided by the battery in time  $dt$

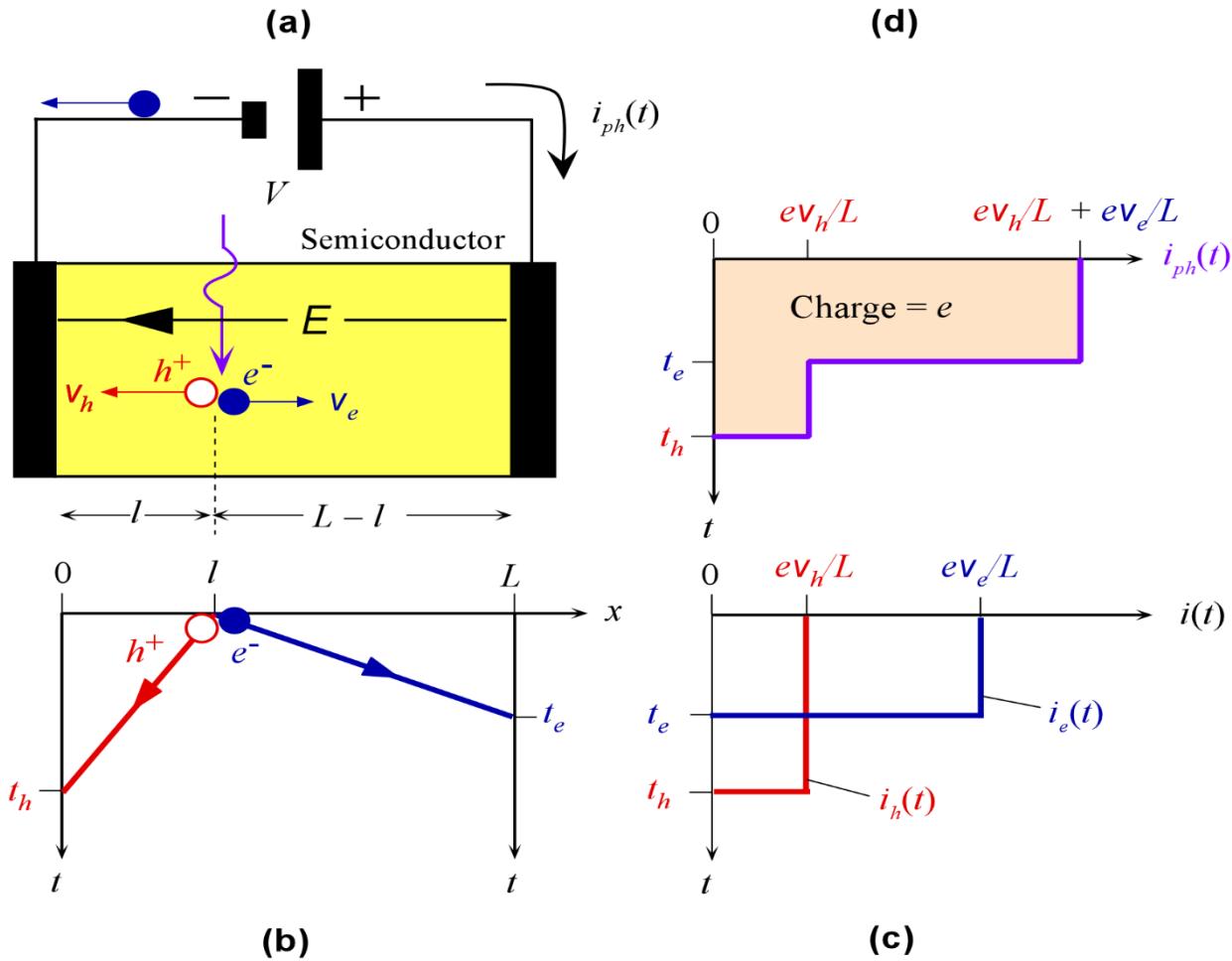
Electrical energy provided by the battery in time  $dt = Vi_e(t)dt$

Thus,  $eEdx = Vi_e(t)dt$ . In time  $dt$ , the electron drift a distance  $dx = v_e dt$

$$eEdx = Vi_e(t)dt \rightarrow e(V/L)(v_e dt) = Vi_e(t)dt \rightarrow i_e(t) = \frac{eV_e}{L}$$

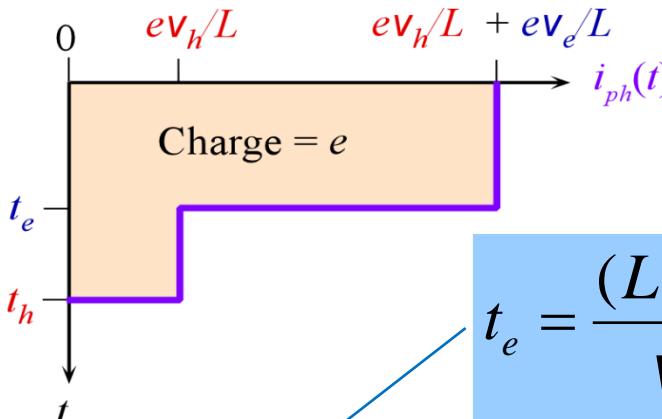
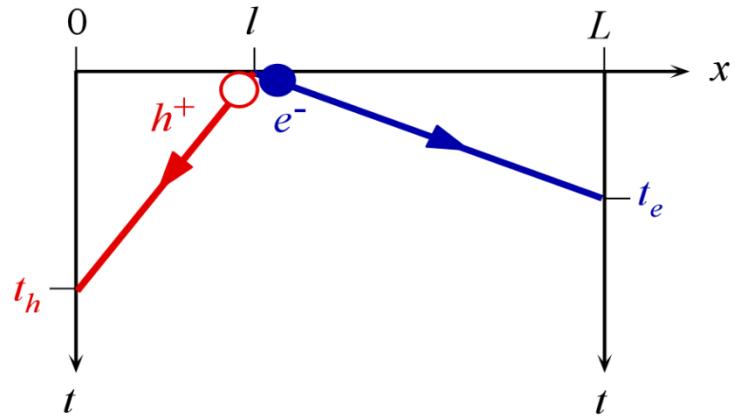
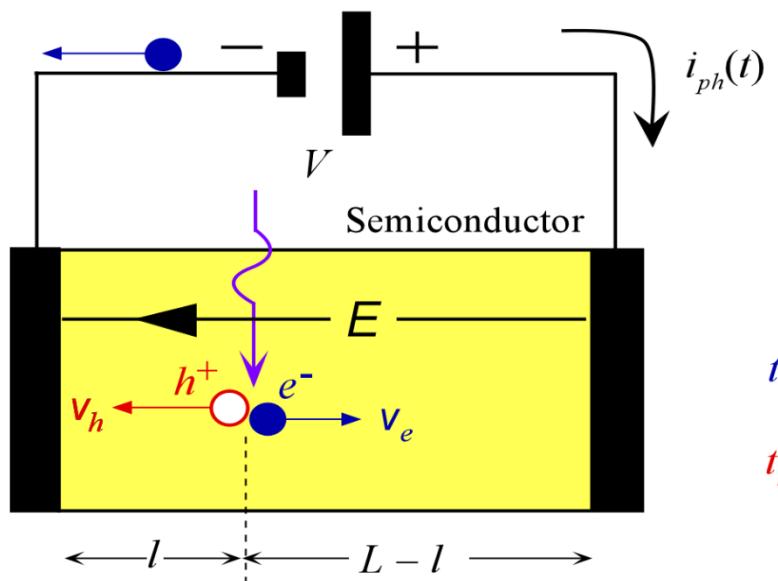
$i_e$  flows while the electron is drifting, for a time  $t_e = (L - l)/v_e$

# Shockley-Ramo Theorem



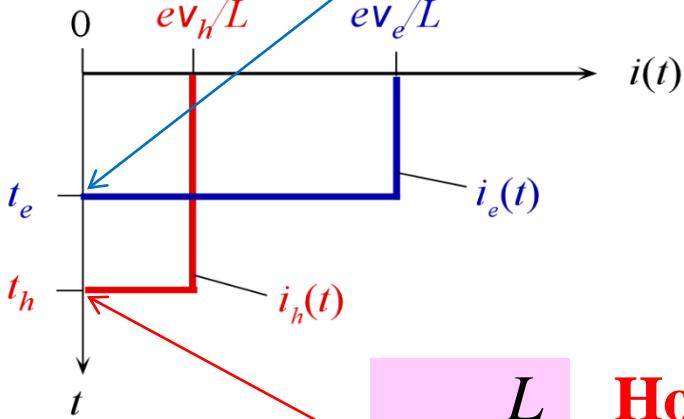
(a) An electron and hole pair (EHP) is photogenerated at  $x = l$ . The electron and the hole drift in opposite directions with drift velocities  $v_h$  and  $v_e$ . (b) The electron arrives at time  $t_e = (L - l)/v_e$  and the hole arrives at time  $t_h = l/v_h$ . (c) As the electron and hole drift, each generates an external photocurrent shown as  $i_e(t)$  and  $i_h(t)$ . (d) The total photocurrent is the sum of hole and electron.

# Shockley-Ramo Theorem



$$t_e = \frac{(L - l)}{v_e}$$

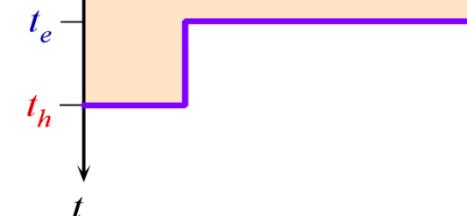
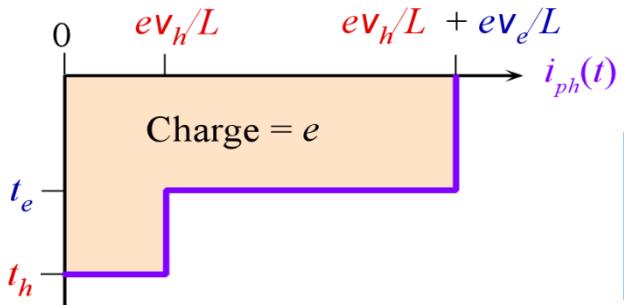
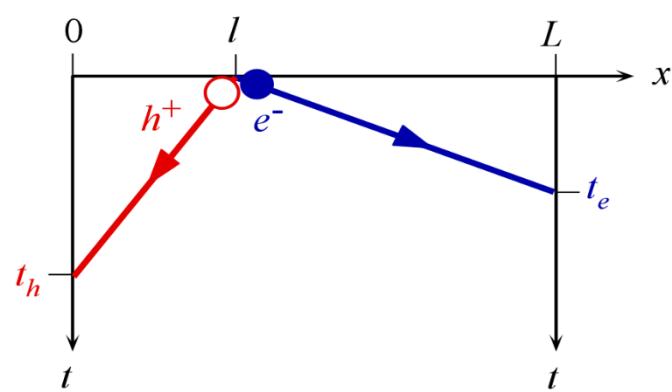
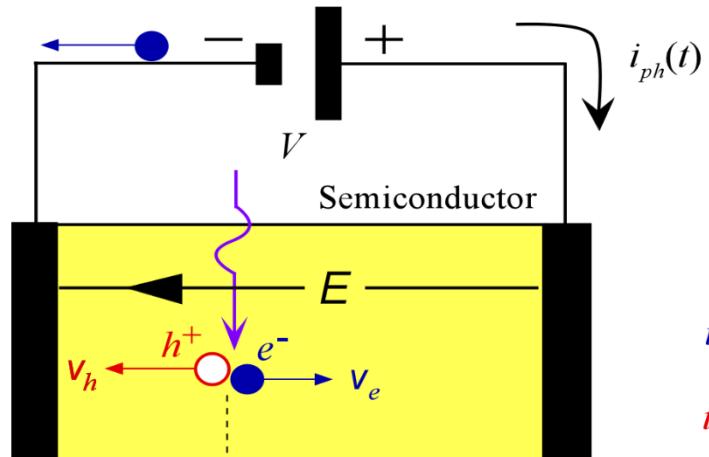
**Electron transit times**



$$t_h = \frac{L}{v_h}$$

**Hole transit time**

# Shockley-Ramo Theorem



$$i_e(t) = \frac{eV_e}{L} \quad t < t_e$$

$$i_h(t) = \frac{eV_h}{L} \quad t < t_h$$

$$Q_{\text{collected}} = \int_0^{t_e} i_e(t) dt + \int_0^{t_h} i_h(t) dt = e$$

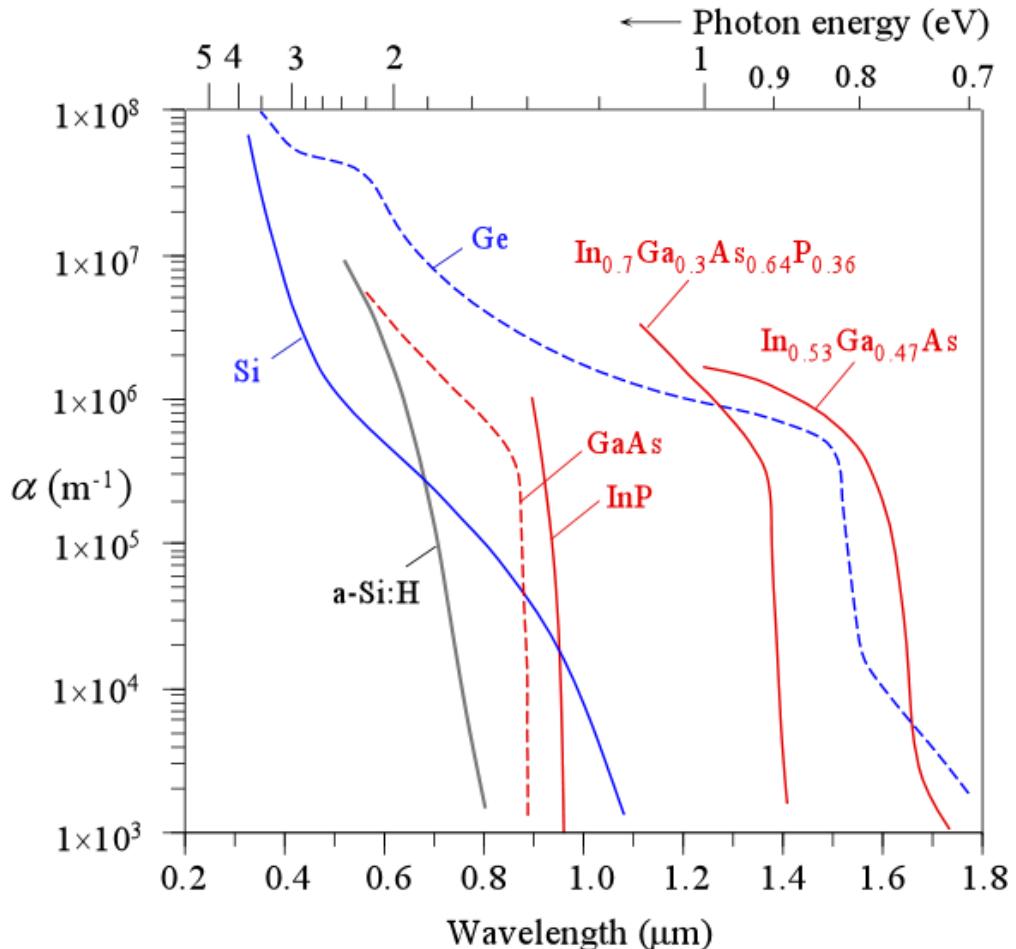
**Total collected charge =  $e$**

# Absorption Coefficient $\alpha$

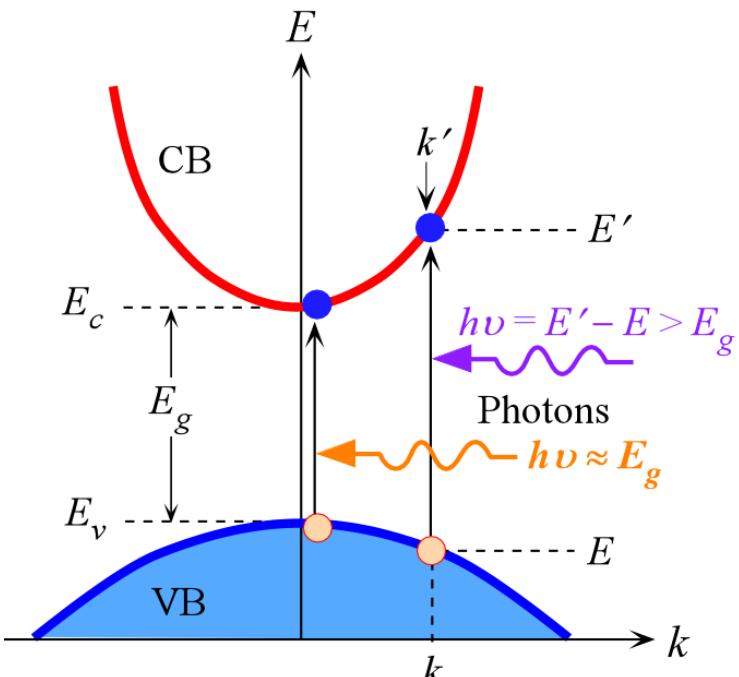


$$I(x) = I_0 \exp(-\alpha x)$$

$\delta = 1/\alpha = \text{penetration or absorption depth}$

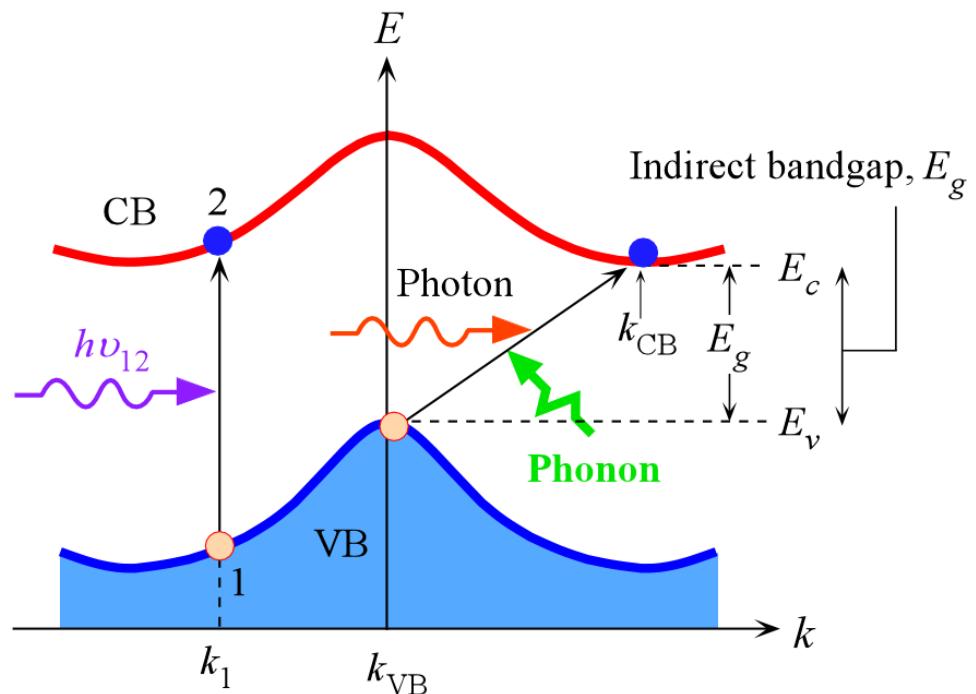


# Absorption and Direct and Indirect Transitions



GaAs (Direct bandgap)

**(a)**



Si (Indirect bandgap)

**(b)**

(a) Photon absorption in a direct bandgap semiconductor. (b) Photon absorption in an indirect bandgap semiconductor (VB, valence band; CB, conduction band)

# Absorption and the Bandgap



## Absorption cutoff wavelength

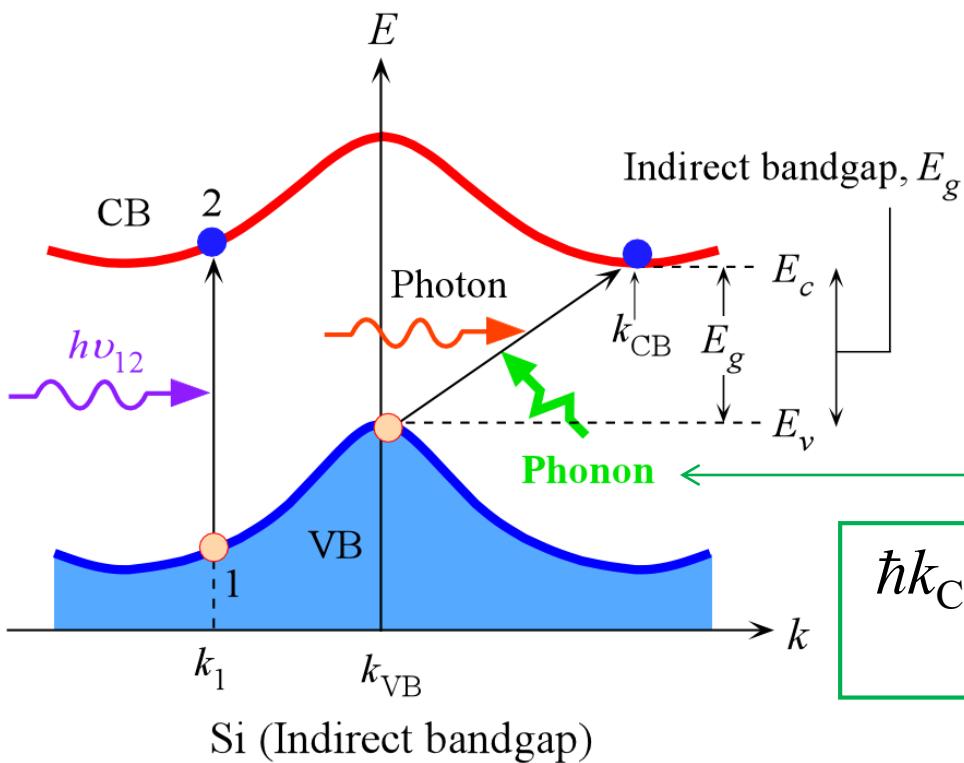
$$\lambda_g (\mu\text{m}) \approx \frac{1.24}{E_g (\text{eV})}$$

Wavelength in  
microns  
(micrometers)

Bandgap in eV

Wavelengths greater than roughly  $\lambda_g$  are not absorbed (by band-to band transitions)

# Indirect Bandgap Semiconductors



Photon energy absorbed

$$h\nu = E_g \pm \hbar\vartheta$$

Phonon frequency

$$\hbar k_{CB} - \hbar k_{VB} = \text{Phonon momentum} \\ = \hbar K$$

Photon energy absorbed,  $h\nu = E_g \pm \hbar\vartheta$

Phonon energy, small e.g. less than 0.1 eV

# Semiconductors

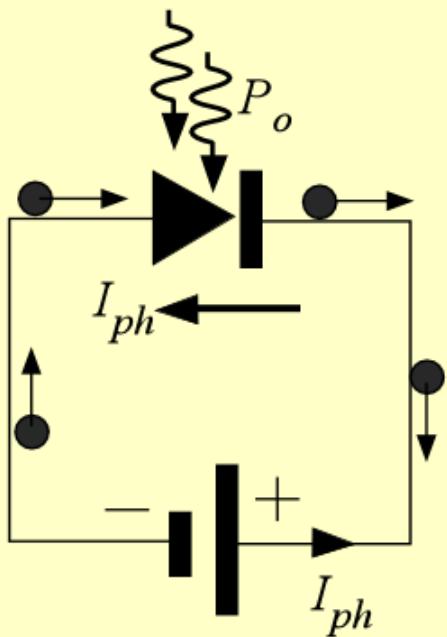


Band gap energy  $E_g$  at 300 K, cut-off wavelength  $\lambda_g$  and type of bandgap (D = Direct and I = Indirect) for some photodetector materials

Semiconductor	$E_g$ (eV)	$\lambda_g$ (eV)	Type
InP	1.35	0.91	D
GaAs <sub>0.88</sub> Sb <sub>0.12</sub>	1.15	1.08	D
Si	1.12	1.11	I
In <sub>0.7</sub> Ga <sub>0.3</sub> As <sub>0.64</sub> P <sub>0.36</sub>	0.89	1.4	D
In <sub>0.53</sub> Ga <sub>0.47</sub> As	0.75	1.65	D
Ge	0.66	1.87	I
InAs	0.35	3.5	D
InSb	0.18	7	D



# External quantum efficiency (QE) $\eta_e$ of the detector

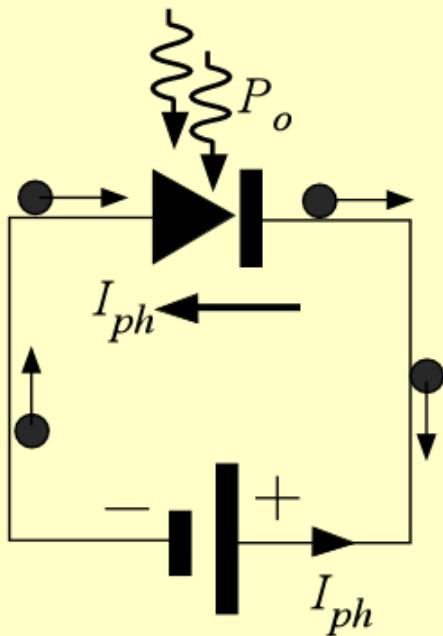


$\eta_e = \frac{\text{Number of free EHP generated and collected}}{\text{Number of incident photons}}$

$$\eta_e = \frac{I_{ph} / e}{P_o / h\nu}$$



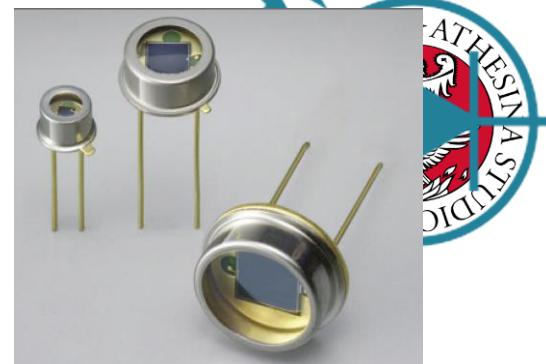
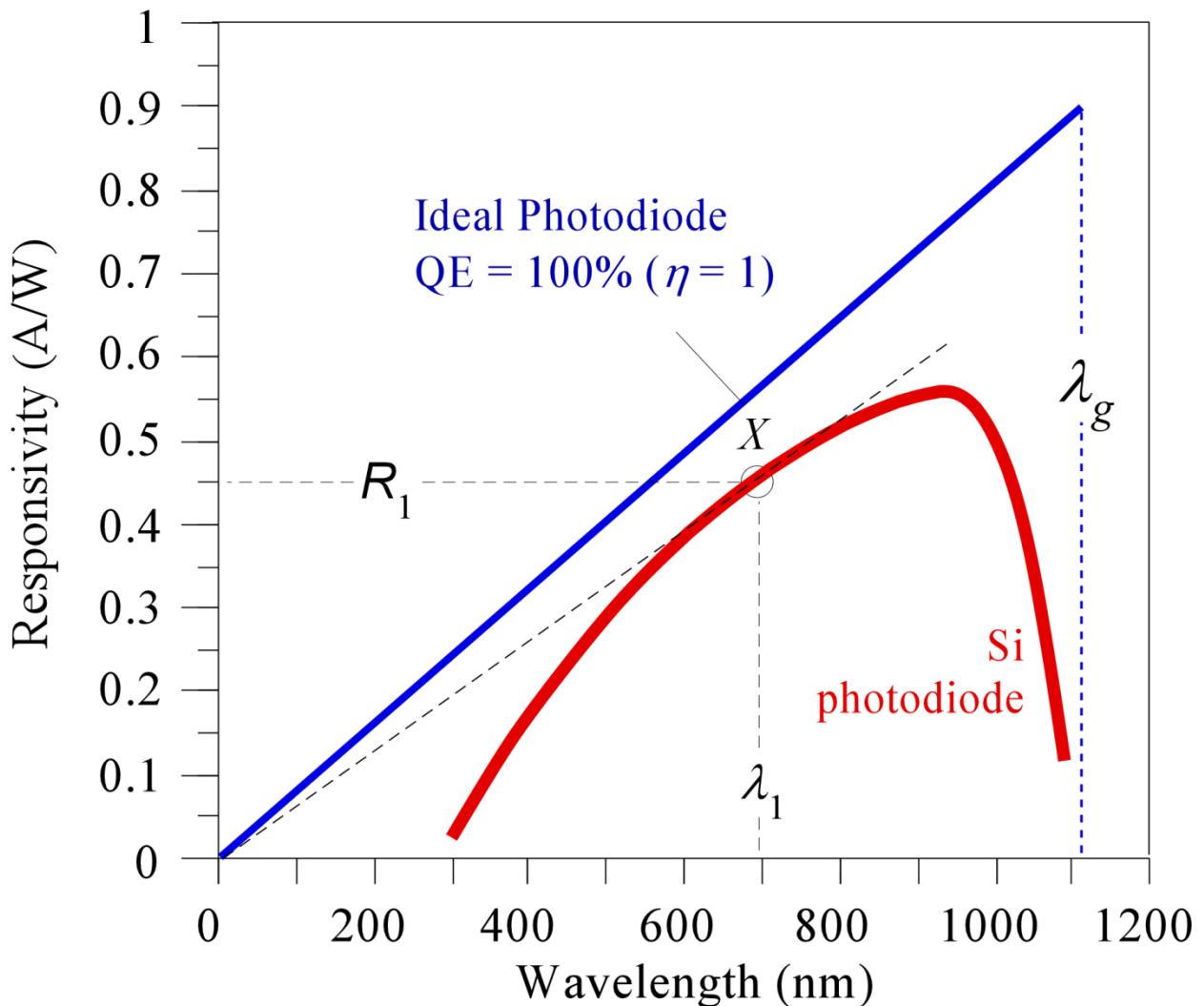
# Responsivity $R$



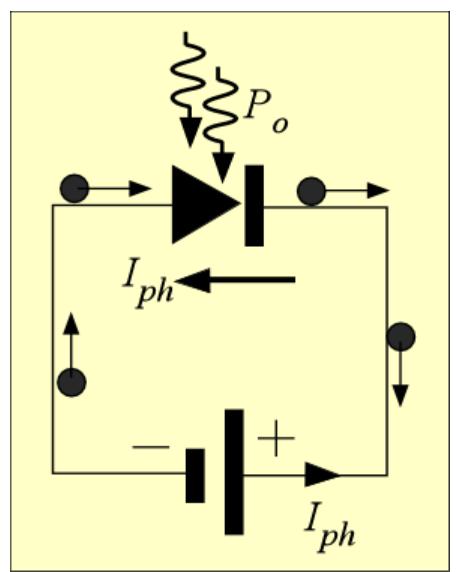
$$R = \frac{\text{Photocurrent (A)}}{\text{Incident Optical Power (W)}} = \frac{I_{ph}}{P_o}$$

$$R = \eta_e \frac{e}{h\nu} = \eta_e \frac{e\lambda}{hc}$$

# Responsivity $R$

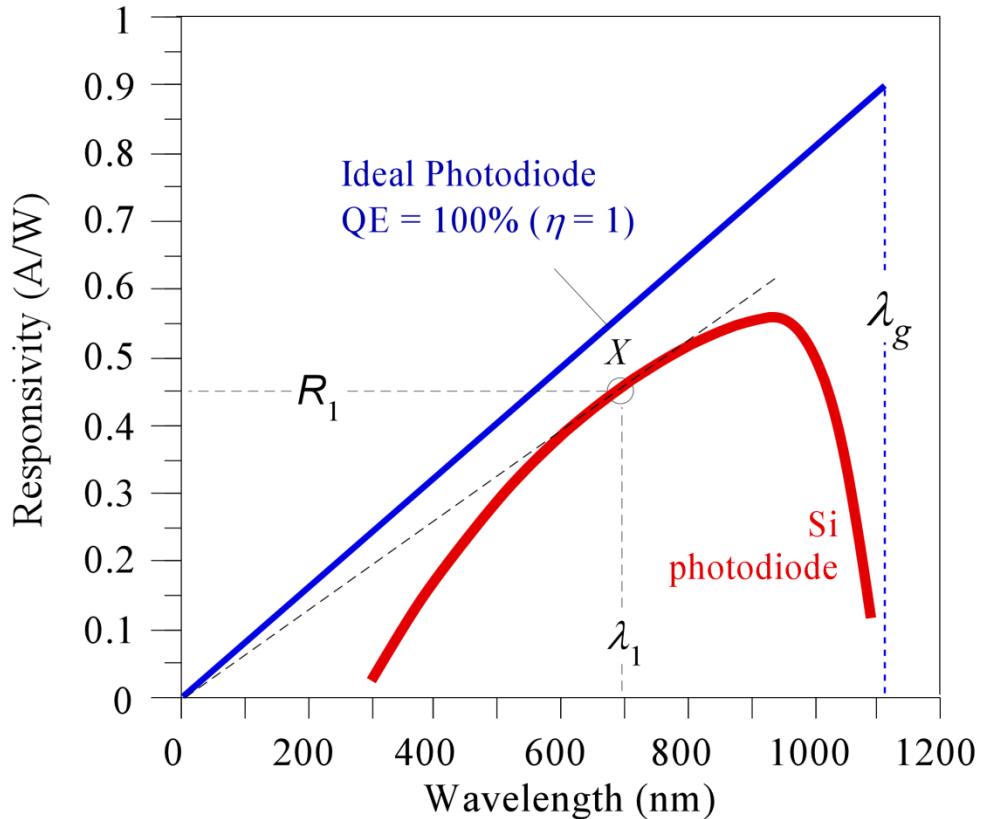


Si photodiodes of various sizes (S1336 series).  
(Courtesy of Hamamatsu)





# Responsivity $R$



Responsivity ( $R$ ) vs. wavelength ( $\lambda$ ) for an ideal photodiode with  $QE = 100\%$  ( $\eta_e = 1$ ) and for a typical inexpensive commercial Si photodiode. The exact shape of the responsivity curve depends on the device structure.

**The line through the origin that is a tangent to the responsivity curve at  $X$ , identifies operation at  $\lambda_1$  with maximum QE**

# EXAMPLE: Quantum efficiency and responsivity

Consider the photodiode shown in Figure 5.7. What is the QE at peak responsivity? What is the QE at 450 nm (blue)? If the photosensitive device area is  $1 \text{ mm}^2$ , what would be the light intensity corresponding to a photocurrent of 10 nA at the peak responsivity?



## Solution

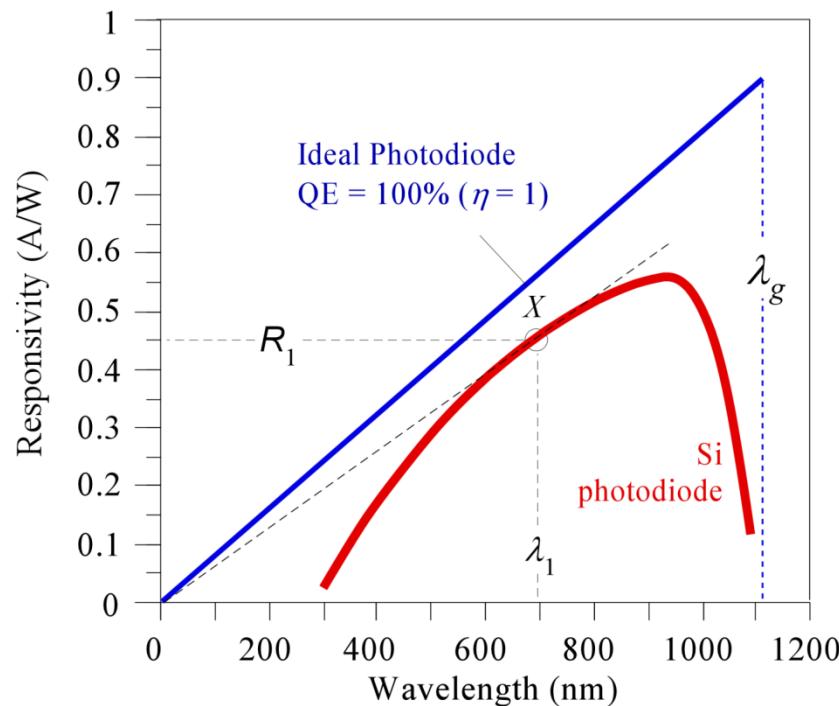
The peak responsivity in Figure 5.7 occurs at about  $\lambda \approx 940 \text{ nm}$  where  $R \approx 0.56 \text{ A W}^{-1}$ . Thus, from Eq. (5.4.4), that is  $R = \eta_e e \lambda / hc$ , we have

$$0.56 \text{ AW}^{-1} = \eta_e \frac{(1.6 \times 10^{-19} \text{ C})(940 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})}$$

i.e.  $\eta_e = 0.74$  or **74%**

We can repeat the calculation for  $\lambda = 450 \text{ nm}$ , where  $R \approx 0.24 \text{ AW}^{-1}$ , which gives  $\eta_e = 0.66$  or 66%.

From the definition of responsivity,  $R = I_{ph}/P_o$ , we have  $0.56 \text{ AW}^{-1} = (10 \times 10^{-9} \text{ A})/P_o$ , i.e.  $P_o = 1.8 \times 10^{-8} \text{ W}$  or 18 nW. Since the area is  $1 \text{ mm}^2$  the intensity must be  $18 \text{ nW mm}^{-2}$ .



# EXAMPLE: Maximum quantum efficiency

Show that a photodiode has maximum QE when

$$\frac{dR}{d\lambda} = \frac{R}{\lambda}$$

(5.4.5)

that is, when the tangent  $X$  at  $\lambda_1$  in Figure 5.7 passes through the origin ( $R = 0, \lambda = 0$ ). Hence determine the wavelengths where the QE is maximum for the Si photodiode in Figure 5.7

## Solution

From Eq. (5.4.4) the QE is given by

$$\eta_e = \frac{hcR(\lambda)}{e\lambda} \quad (5.4.6)$$

where  $R(\lambda)$  depends on  $\lambda$  and there is also  $\lambda$  in the denominator. We can differentiate Eq. (5.4.6) with respect to  $\lambda$  and then set to zero to find the maximum point  $X$ . Thus

$$\frac{d\eta_e}{d\lambda} = \frac{hc}{e\lambda} \frac{dR}{d\lambda} - \frac{hcR}{e} \left( \frac{1}{\lambda^2} \right) = 0$$

which leads to Eq. (5.4.5). Equation (5.4.5) represents a line through the origin that is a tangent to the  $R$  vs  $\lambda$  curve. This tangential point is  $X$  in Figure 5.7, where  $\lambda_1 = 700$  nm and  $R_1 = 0.45 \text{ AW}^{-1}$ . Then, using Eq. (5.4.6), the maximum QE is

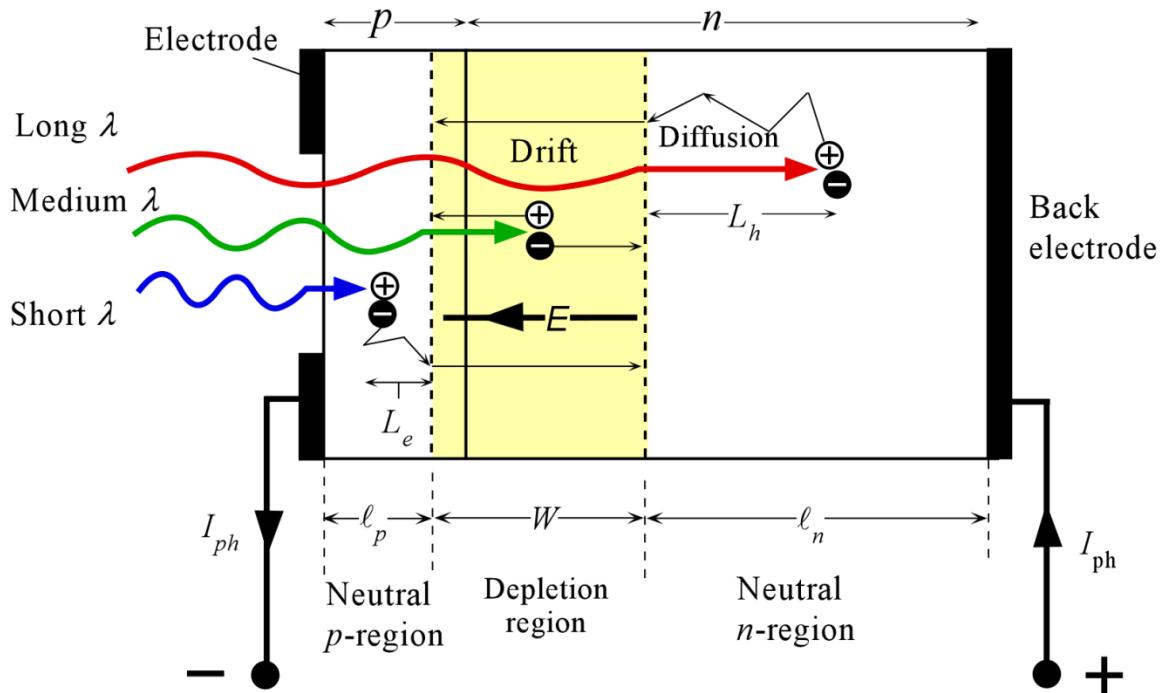
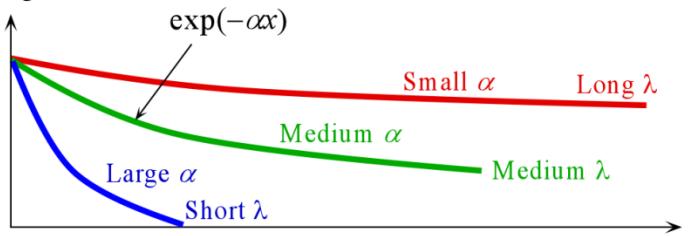
$$\begin{aligned}\eta_e &= (6.626 \times 10^{-34} \text{ J s}) (3 \times 10^8 \text{ m s}^{-1}) (0.45 \text{ A W}^{-1}) / (1.6 \times 10^{-19} \text{ C}) (700 \times 10^{-9} \text{ m}) \\ &= \mathbf{0.80 \text{ or } 80\%}\end{aligned}$$

# External Quantum Efficiency and Responsivity



Schematic photogeneration profiles

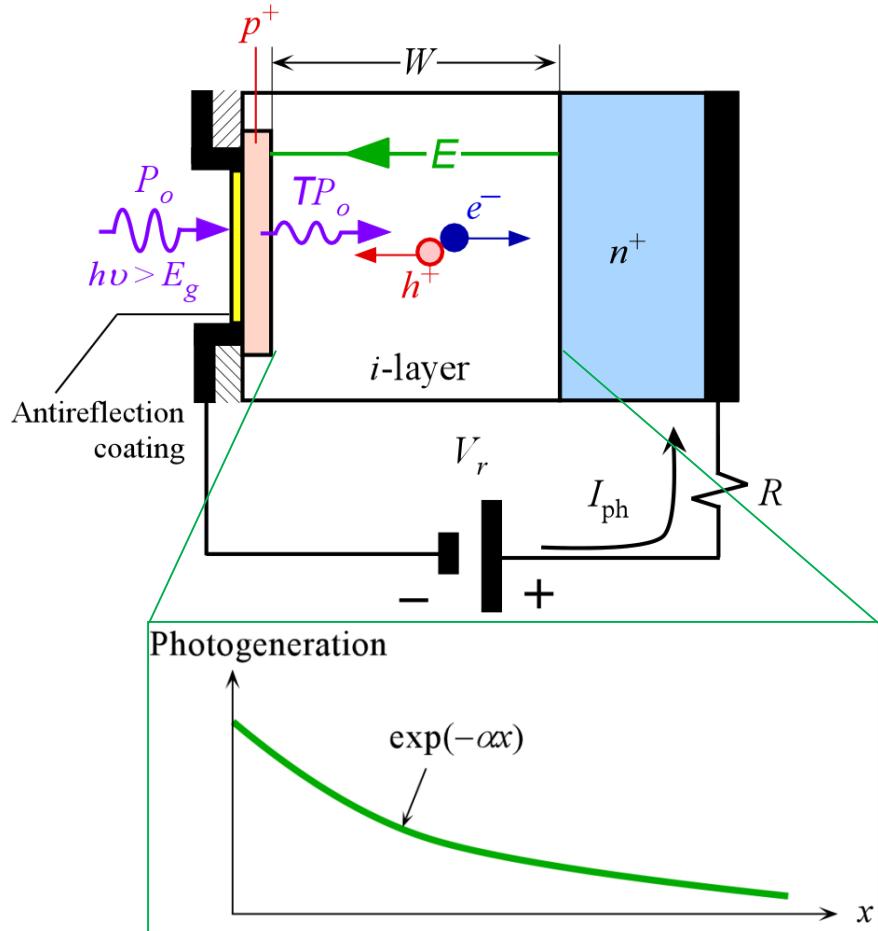
Photogeneration



Different contributions to the photocurrent  $I_{ph}$ . Photogeneration profiles corresponding to short, medium and long wavelengths are also shown.

# Internal Quantum Efficiency $\eta_i$

$$\eta_i = \text{Internal Quantum Efficiency} = \frac{\text{Number of EHP photogenerated}}{\text{Number of absorbed photons}}$$



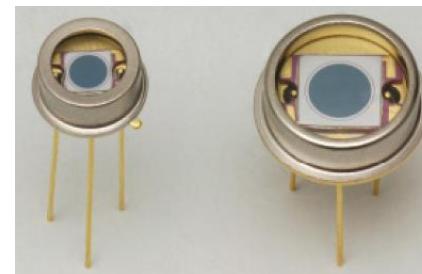
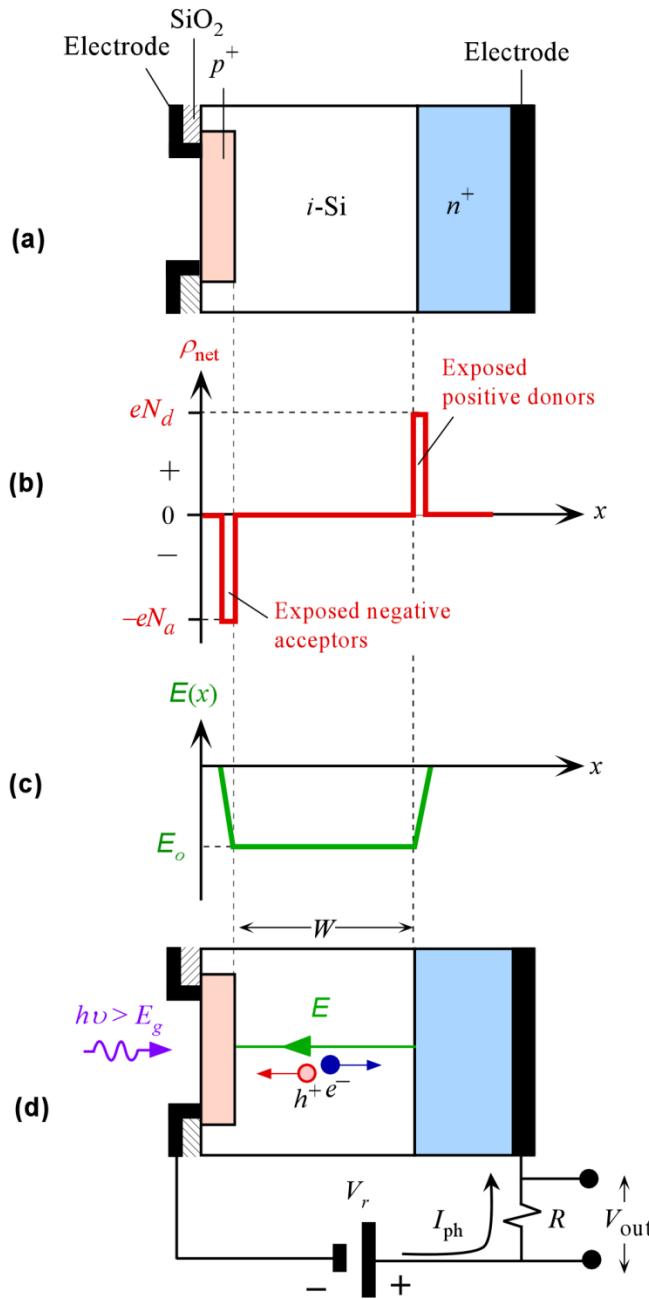
Assuming  $\ell_p$  is very thin, and assuming  $W \gg L_h$

$$I_{ph} \approx \frac{e \eta_i T P_o(0)}{h\nu} [1 - \exp(-\alpha W)]$$

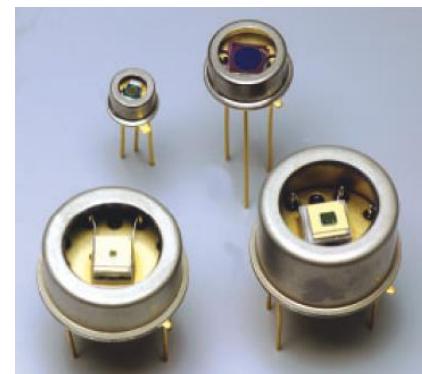
$T$  = Transmission coefficient of AR coating  
 $\alpha$  = Absorption coefficient



# pin Photodiode



Si pin

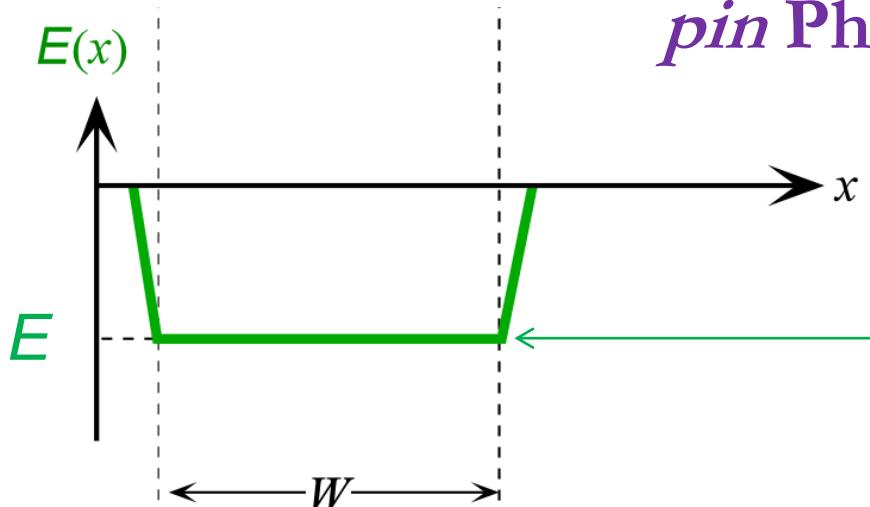


InGaAs pin

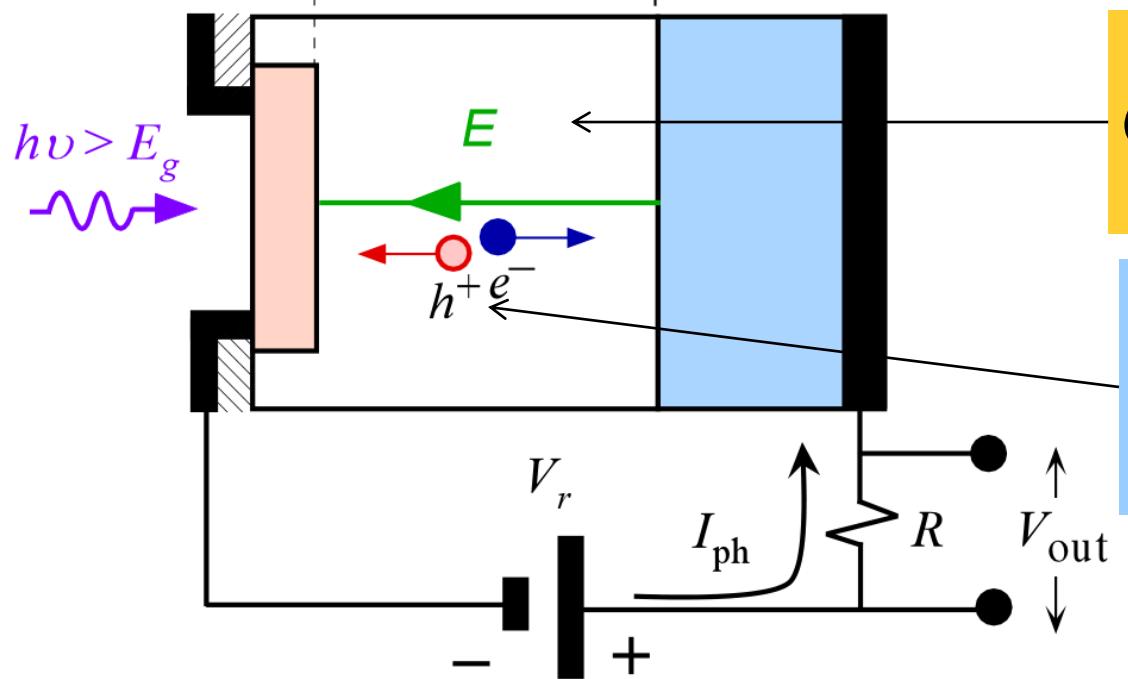
Courtesy of Hamamatsu



# pin Photodiode



$$E = E_o + \frac{V_r}{W} \approx \frac{V_r}{W}$$

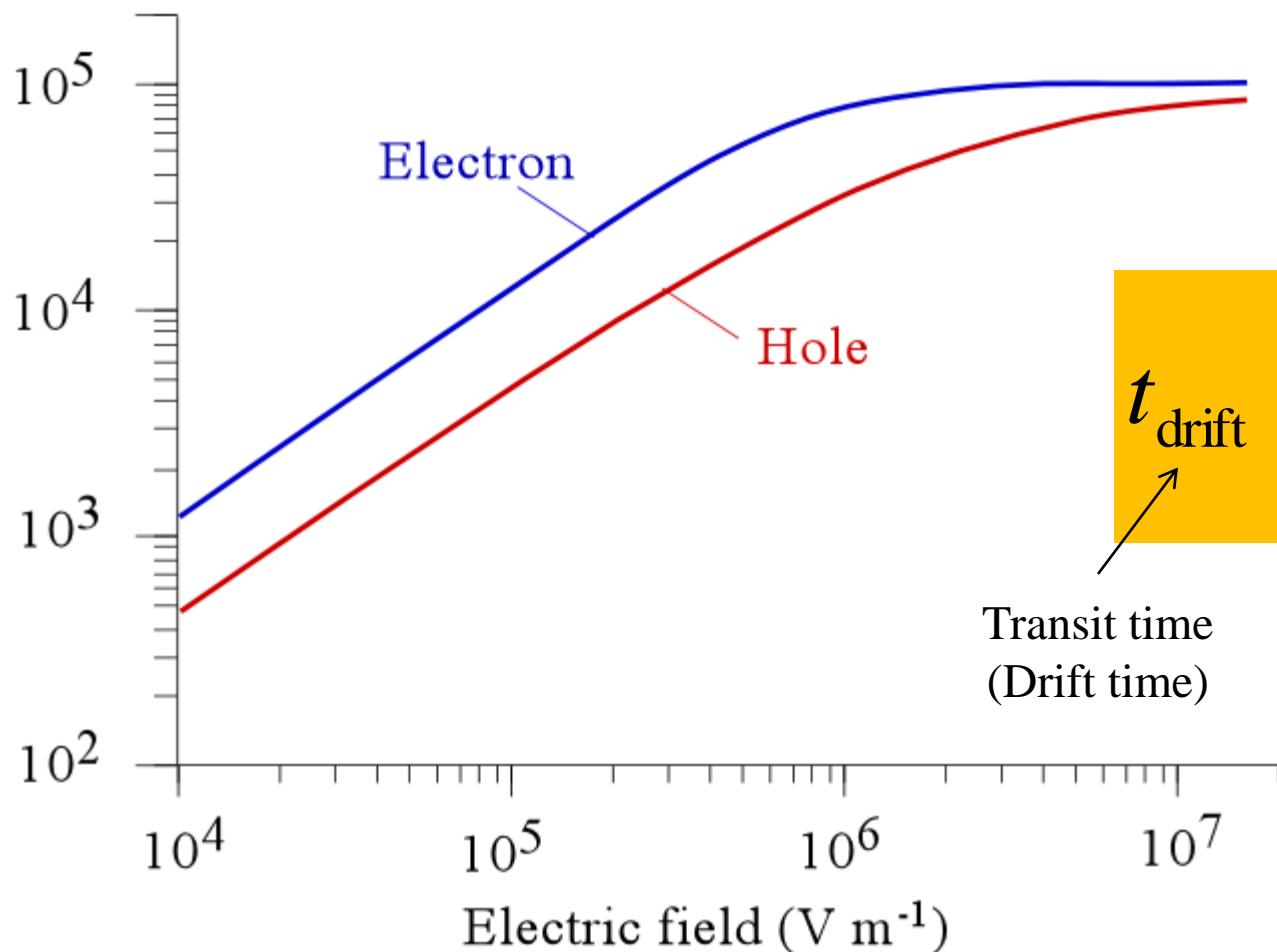


$$C_{dep} = \frac{\epsilon_o \epsilon_r A}{W}$$

$$t_{drift} = \frac{W}{V_d}$$

# *p*n Photodiode

Drift velocity ( $\text{m s}^{-1}$ )



Width of *i*-region  
(Depletion region)

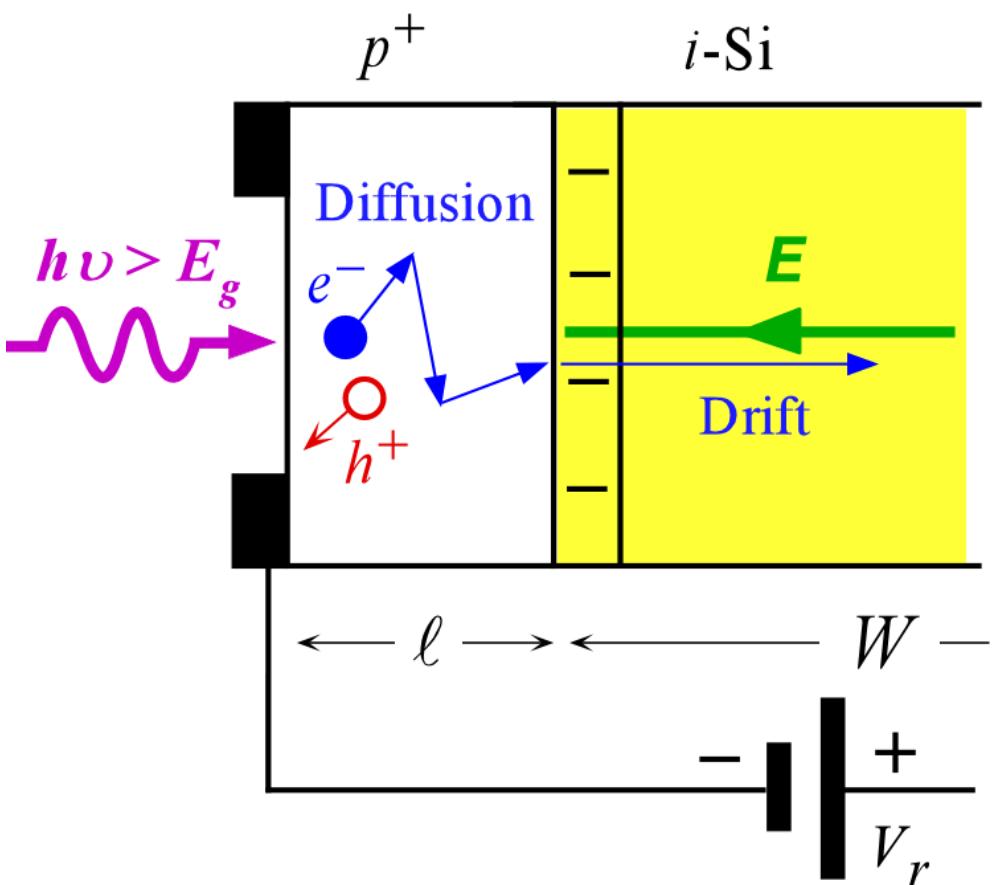
$$W = \frac{V_d}{v_d}$$

Transit time  
(Drift time)

Drift velocity vs. electric field for holes and electrons in Si.



## pin Photodiode Speed

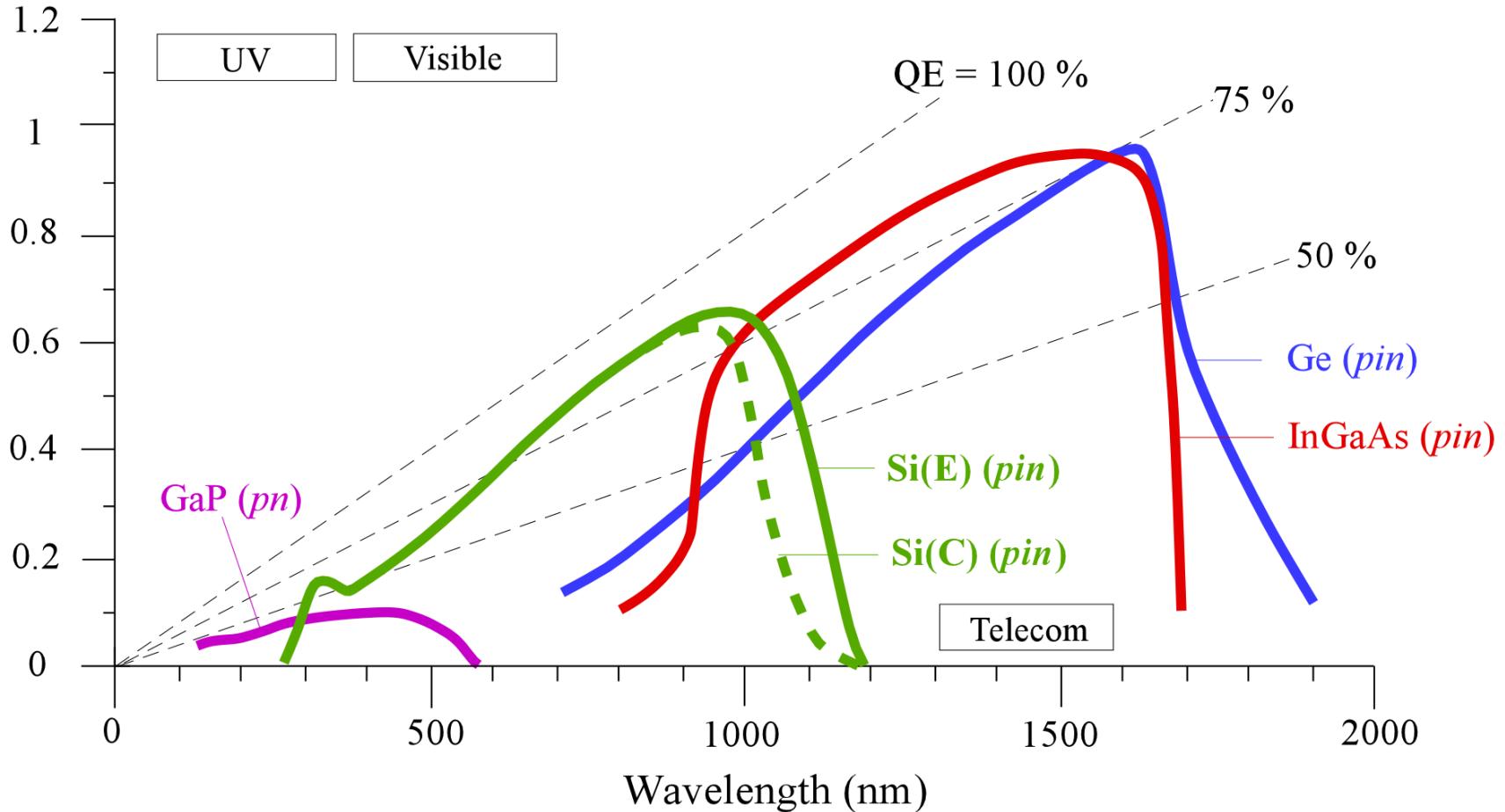


A reverse biased *pin* photodiode is illuminated with a short wavelength light pulse that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across.

In time  $t$ , an electron, on average, diffuses a distance  $\ell$  given by

$$\ell = (2D_e t)^{1/2}$$

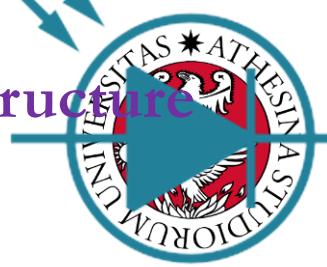
Electron diffusion coefficient

$R$  (A/W)*pin* Photodiode

The responsivity of Si, InGaAs and Ge *pin* type photodiodes. The *pn* junction GaP detector is used for UV detection. GaP (Thorlabs, FGAP71), Si(E), IR enhanced Si (Hamamatsu S11499), Si(C), conventional Si with UV enhancement, InGaAs (Hamamatsu, G8376), and Ge (Thorlabs, FDG03).

The dashed lines represent the responsivity due to  $\text{QE} = 100\%$ ,  $75\%$  and  $50\%$ .

# Responsivity $R$ depends on the device structure



Responsivity (A/W)

0.6

0.5

0.4

0.3

0.2

0.1

0

Wavelength (nm)

A has UV response

A

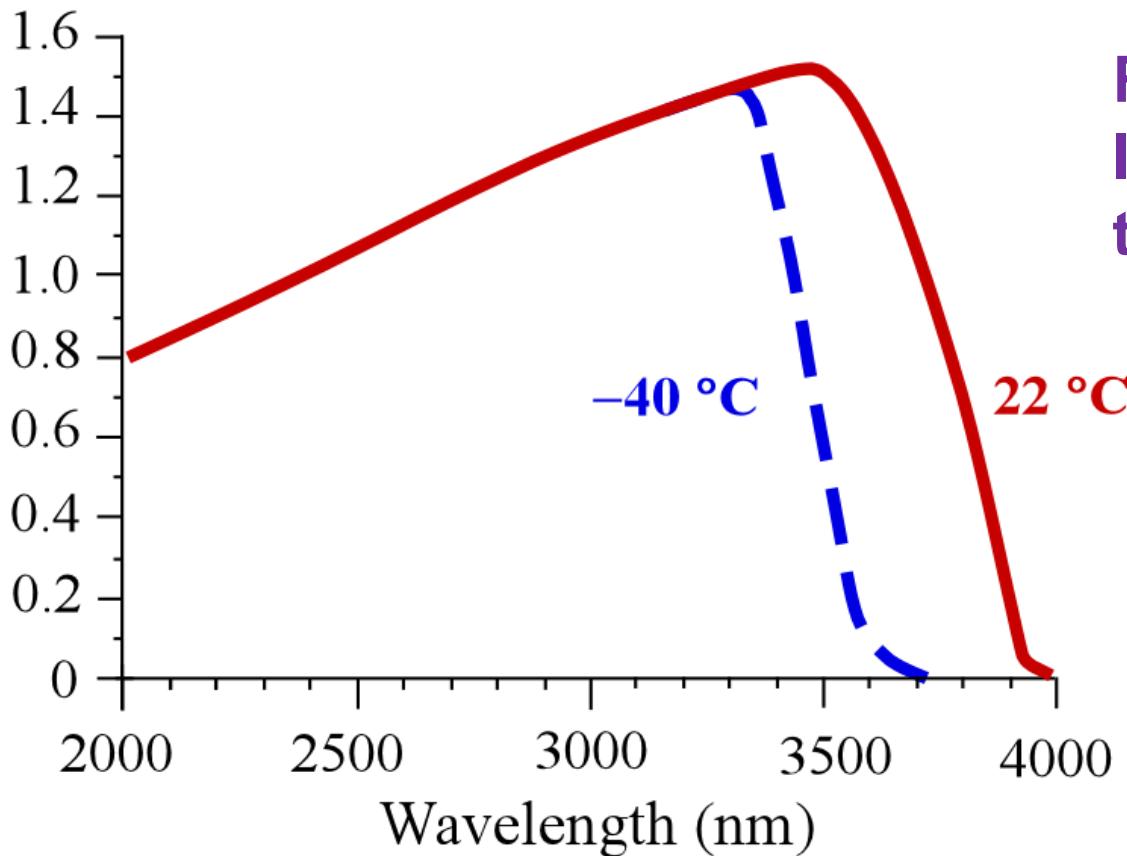
B

Two Si *pin* photodiodes with different device structures.

Responsivity  $R$  depends on the temperature



Responsivity (A/W)



**Responsivity of an  
InAs photodiode at  
two temperatures**



## EXAMPLE: Responsivity of a *pin* photodiode

A Si *pin* photodiode has an active light receiving area of diameter 0.4 mm. When radiation of wavelength 700 nm (red light) and intensity  $0.1 \text{ mW cm}^{-2}$  is incident, it generates a photocurrent of 56.6 nA. What is the responsivity and external QE of the photodiode at 700 nm?

### Solution

The incident light intensity  $I = 0.1 \text{ mW cm}^{-2}$  means that the incident power for conversion is

$$P_o = AI = [\pi(0.02 \text{ cm})^2](0.1 \times 10^{-3} \text{ W cm}^{-2}) = 1.26 \times 10^{-7} \text{ W or } 0.126 \mu\text{W}.$$

The responsivity is

$$R = I_{ph}/P_o = (56.6 \times 10^{-9} \text{ A})/(1.26 \times 10^{-7} \text{ W}) = 0.45 \text{ A W}^{-1}$$

The QE can be found from

$$\eta = R \frac{hc}{e\lambda} = (0.45 \text{ A W}^{-1}) \frac{(6.62 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})}{(1.6 \times 10^{-19} \text{ C})(700 \times 10^{-9} \text{ m})} = 0.80 = 80 \%$$

# EXAMPLE: Operation and speed of a *pin* photodiode

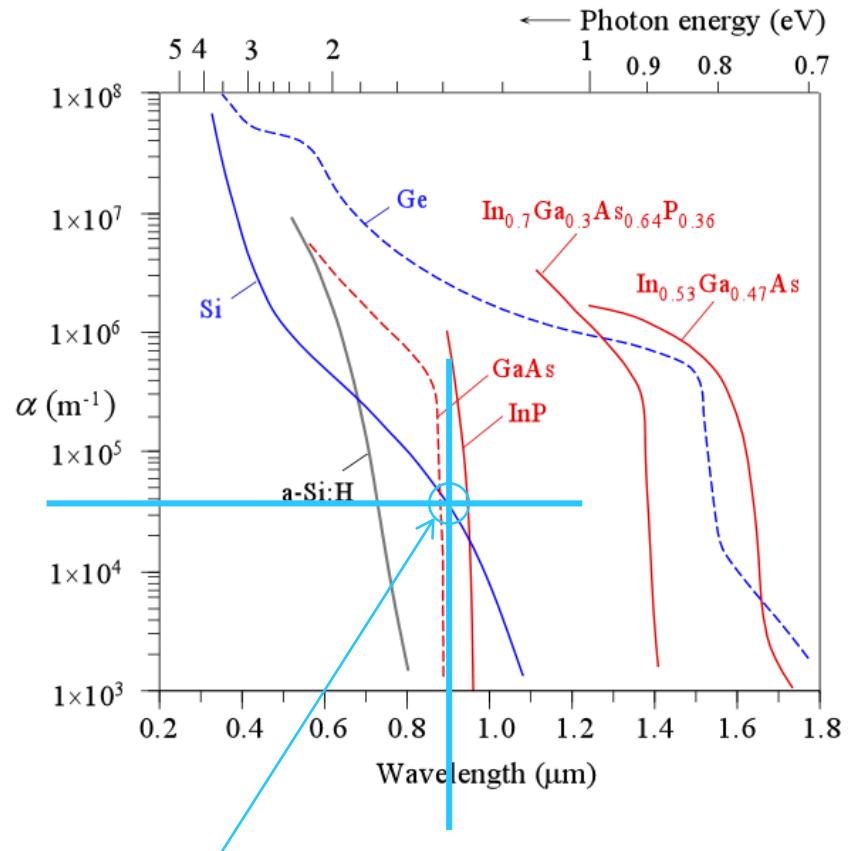
A Si *pin* photodiode has an *i*-Si layer of width 20  $\mu\text{m}$ . The  $p^+$ -layer on the illumination side is very thin (0.1  $\mu\text{m}$ ). The *pin* is reverse biased by a voltage of 100 V and then illuminated with a very short optical pulse of wavelength 900 nm. What is the duration of the photocurrent if absorption occurs over the whole *i*-Si layer?

## Solution

From Figure 5.5 , the absorption coefficient at 900 nm is  $\sim 3 \times 10^4 \text{ m}^{-1}$  so that the absorption depth is  $\sim 33 \mu\text{m}$ .

We can assume that absorption and hence photogeneration occurs over the entire width  $W$  of the *i*-Si layer. The field in the *i*-Si layer is

$$\begin{aligned} E &\approx V_r / W \\ &= (100 \text{ V}) / (20 \times 10^{-6} \text{ m}) \\ &= 5 \times 10^6 \text{ V m}^{-1} \end{aligned}$$



Note: The absorption coefficient is between  $3 \times 10^4 \text{ m}^{-1}$  and  $4 \times 10^4 \text{ m}^{-1}$

# EXAMPLE: Operation and speed of a *pin* photodiode

## Solution (continued)



At this field the electron drift velocity  $v_e$  is very near its saturation at  $10^5 \text{ m s}^{-1}$ , whereas the hole drift velocity  $v_h \approx 7 \times 10^4 \text{ m s}^{-1}$  as shown in Figure 5.10. Holes are slightly slower than the electrons. The transit time  $t_h$  of holes across the *i*-Si layer is

$$\begin{aligned} t_h &= W/v_h = (20 \times 10^{-6} \text{ m})/(7 \times 10^4 \text{ m s}^{-1}) \\ &= 2.86 \times 10^{-10} \text{ s or } \mathbf{0.29 \text{ ns}} \end{aligned}$$

This is the response time of the *pin* as determined by the transit time of the slowest carriers, holes, across the *i*-Si layer. To improve the response time, the width of the *i*-Si layer has to be narrowed but this decreases the quantity of photons absorbed and hence reduces the responsivity. There is therefore a trade off between speed and responsivity.

# EXAMPLE : Photocarrier Diffusion in a *pin* photodiode

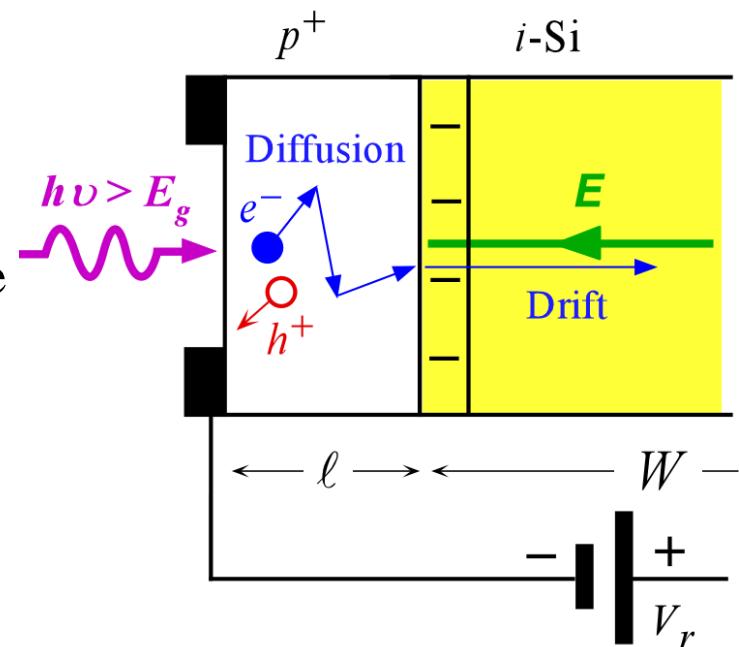
A reverse biased *pin* photodiode is illuminated with a short wavelength light pulse that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across by the field in this region. What is the speed of response of this photodiode if the *i-Si* layer is 20  $\mu\text{m}$  and the  $p^+$ -layer is 1  $\mu\text{m}$  and the applied voltage is 60 V? The diffusion coefficient ( $D_e$ ) of electrons in the heavily doped  $p^+$ -region is approximately  $3 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ .

## Solution

There is no electric field in the  $p^+$ -side outside the depletion region as shown in Figure 5.12 . The photogenerated electrons have to make it across to the  $n^+$ -side to give rise to a photocurrent. In the  $p^+$ -side, the electrons move by diffusion. In time  $t$ , an electron, on average, diffuses a distance  $\ell$  given by

$$\ell = [2D_e t]^{1/2}$$

The *diffusion time*  $t_{\text{diff}}$  is the time it takes for an electron to diffuse across the  $p^+$ -side (of length  $\ell$  ) to reach the depletion layer and is given by



# EXAMPLE: Photocarrier Diffusion in a *pin* photodiode

## Solution (continued)



$$t_{\text{diff}} = \ell^2 / (2D_e) = (1 \times 10^{-6} \text{ m})^2 / [2(3 \times 10^{-4} \text{ m}^2 \text{ s}^{-1})] = 1.67 \times 10^{-9} \text{ s or } 1.67 \text{ ns.}$$

On the other hand, once the electron reaches the depletion region, it becomes drifted across the width  $W$  of the *i*-Si layer at the saturation drift velocity since the electric field here is  $E = V_r / W = 60 \text{ V} / 20 \mu\text{m} = 3 \times 10^6 \text{ V m}^{-1}$ ; and at this field the electron drift velocity  $v_e$  saturates at  $10^5 \text{ m s}^{-1}$ . The drift time across the *i*-Si layer is

$$t_{\text{drift}} = W / v_e = (20 \times 10^{-6} \text{ m}) / (1 \times 10^5 \text{ m s}^{-1}) = 2.0 \times 10^{-10} \text{ s or } 0.2 \text{ ns.}$$

Thus, the response time of the *pin* to a pulse of short wavelength radiation that is absorbed near the surface is very roughly  $t_{\text{diff}} + t_{\text{drift}}$  or 1.87 ns. Notice that the diffusion of the electron is much slower than its drift. In a proper analysis, we have to consider the **diffusion and drift of many carriers, and we have to average ( $t_{\text{diff}} + t_{\text{drift}}$ ) for all the electrons.**



## EXAMPLE: Steady state photocurrent in the *pin* photodiode

Consider a pin photodiode that is reverse biased and illuminated, as in Figure 5.9 and operating under steady state conditions.

Assume that the photogeneration takes place inside the depletion layer of width  $W$ , and the neutral *p*-side is very narrow.

If the incident optical power on the semiconductor is  $P_o(0)$ , then  $TP_o(0)$  will be transmitted, where  $T$  is the transmission coefficient.

At a distance  $x$  from the surface, the optical power  $P_o(x) = TP_o(0)\exp(-\alpha x)$ .

In a small volume  $\delta x$  at  $x$ , the absorbed radiation power (by the definition of  $\alpha$ ) is  $\alpha P_o(x)\delta x$ , and the number of photons absorbed per second is  $\alpha P_o(x)\delta x/h\nu$ .

Of these absorbed photons, only a fraction  $\eta_i$  will photogenerate EHPs, where  $\eta_i$  is the **internal quantum efficiency** IQE.

Thus,  $\eta_i \alpha P_o(x)\delta x/h\nu$  number of EHPs will be generated per second.

## EXAMPLE: Steady state photocurrent in the *pin* photodiode



We assume these will drift through the depletion region and thereby contribute to the photocurrent. The current contribution  $\delta I_{ph}$  from absorption and photogeneration at  $x$  within the SCL will thus be

$$\delta I_{ph} = \frac{e \eta_i \alpha P_o(x) \delta x}{h\nu} = \frac{e \eta_i \alpha T P_o(0)}{h\nu} \exp(-\alpha x) \delta x$$

We can integrate this from  $x = 0$  (assuming  $\ell_p$  is very thin) to the end of  $x = W$ , and assuming  $W \gg L_h$  to find

$$I_{ph} \approx \frac{e \eta_i T P_o(0)}{h\nu} [1 - \exp(-\alpha W)]$$

*Steady state photocurrent pin photodiode*      (5.5.4)

where the approximate sign embeds the many assumptions we made in deriving Eq. (5.5.4). Consider a *pin* photodiode without an AR coating so that  $T = 0.68$ . Assume  $\eta_i = 1$ . The SCL width is 20  $\mu\text{m}$ . If the device is to be used at 900 nm, what would be the photocurrent if the incident radiation power is 100 nW? What is the responsivity? Find the photocurrent and the responsivity if a perfect AR coating is used. What is the primary limiting factor? What is the responsivity if  $W = 40 \mu\text{m}$ ?

$$I_{ph} = \frac{e \eta_i T P_o}{h\nu} \{ \exp[-\alpha(\ell_p - L_e)] - \exp[-\alpha(\ell_p + W + L_h)] \}$$

# EXAMPLE: Steady state photocurrent in the *pin* photodiode

## Solution (continued)

From Figure 5.5, at  $\lambda = 900 \text{ nm}$ ,  $\alpha \approx 3 \times 10^4 \text{ m}^{-1}$ . Further for  $\lambda = 0.90 \mu\text{m}$ , the photon energy  $h\nu = 1.24 / 0.90 = 1.38 \text{ eV}$ . Given  $P_o(0) = 100 \text{ nW}$ , we have

$$I_{ph} \approx \frac{(1.6 \times 10^{-19})(1)(0.68)(100 \times 10^{-9})}{(1.38 \times 1.6 \times 10^{-19})} [1 - \exp(-3 \times 10^4 \times 20 \times 10^{-6})] = 22 \text{ nA}$$

and the responsivity  $R = 22 \text{ nA} / 100 \text{ nW} = 0.22 \text{ A W}^{-1}$ , which is on the low-side.

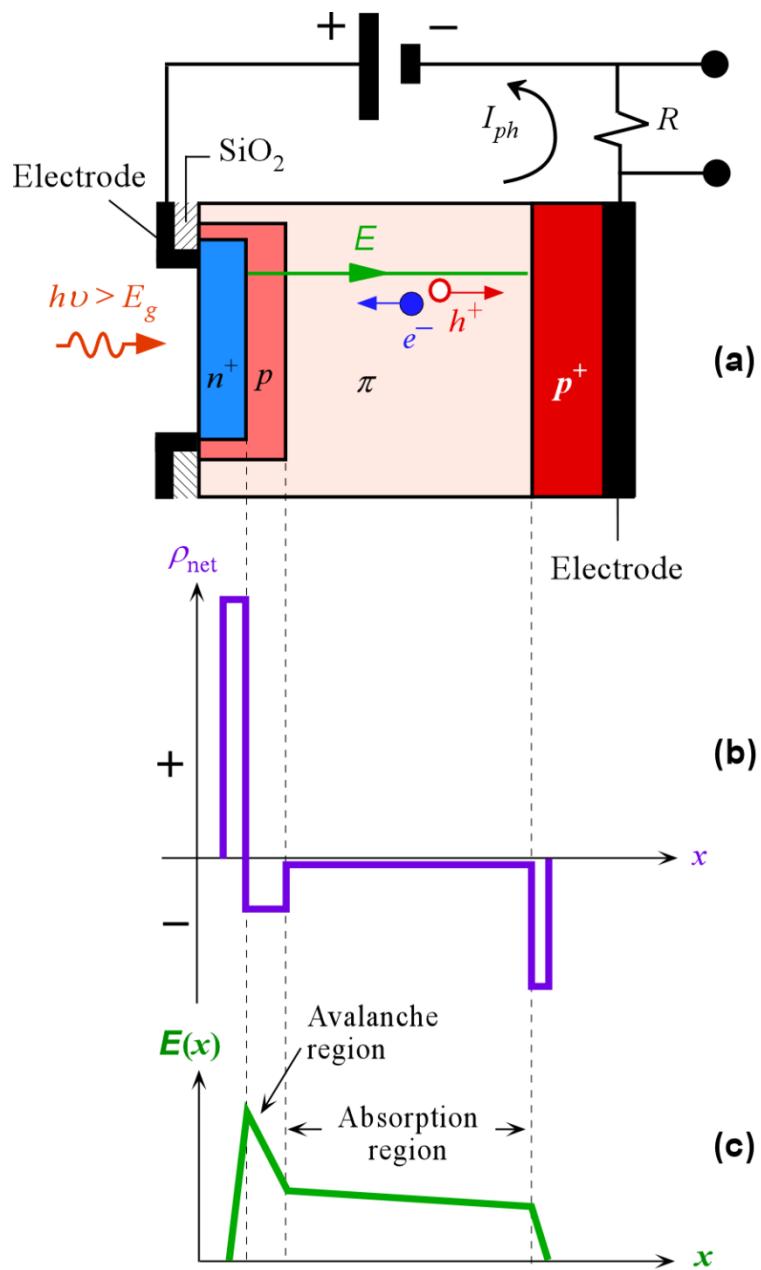
Consider next, a perfect AR coating so that  $T = 1$ , and using Eq. (5.5.4) again, we find  $I_{ph} = 32.7 \text{ nA}$  and  $R = 0.33 \text{ A W}^{-1}$ , a significant improvement.

The factor  $[1 - \exp(-\alpha W)]$  is only 0.451, and can be significantly improved by making the SCL thicker. Setting  $W = 40 \mu\text{m}$ , gives  $[1 - \exp(-\alpha W)] = 0.70$  and  $R = 0.51$ , which is close to values for commercial devices.

The maximum theoretical photocurrent would be obtained by setting  $\exp(-\alpha W) \approx 0$ ,  $T = 1$ ,  $\eta_i = 1$ , which gives  $I_{ph} = 73 \text{ nA}$  and  $R = 0.73 \text{ A W}^{-1}$ .



# Avalanche Photodiode

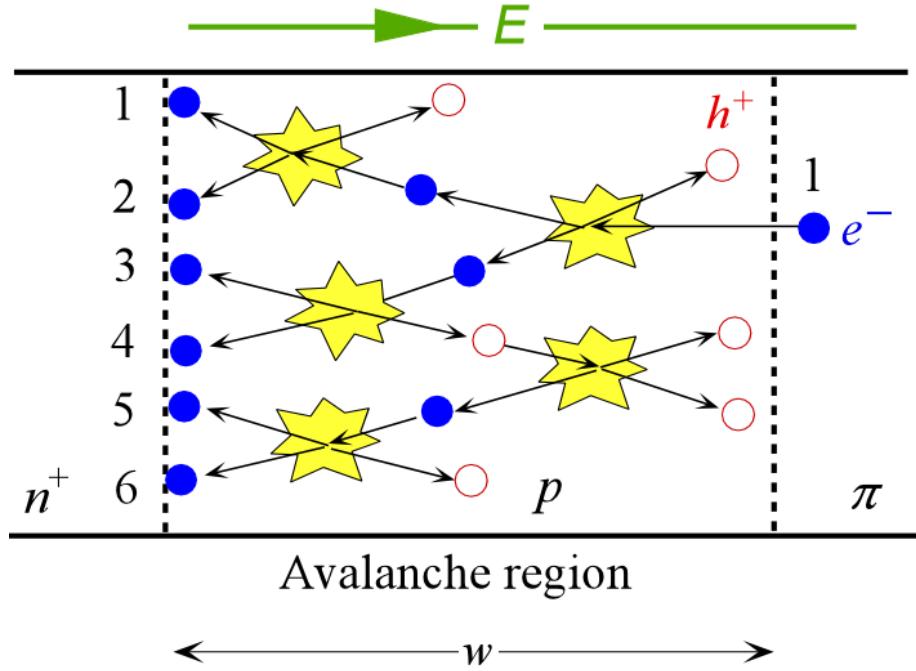


- (a) A schematic illustration of the structure of an avalanche photodiode (APD) biased for avalanche gain.
- (b) The net space charge density across the photodiode.
- (c) The field across the diode and the identification of absorption and multiplication regions.

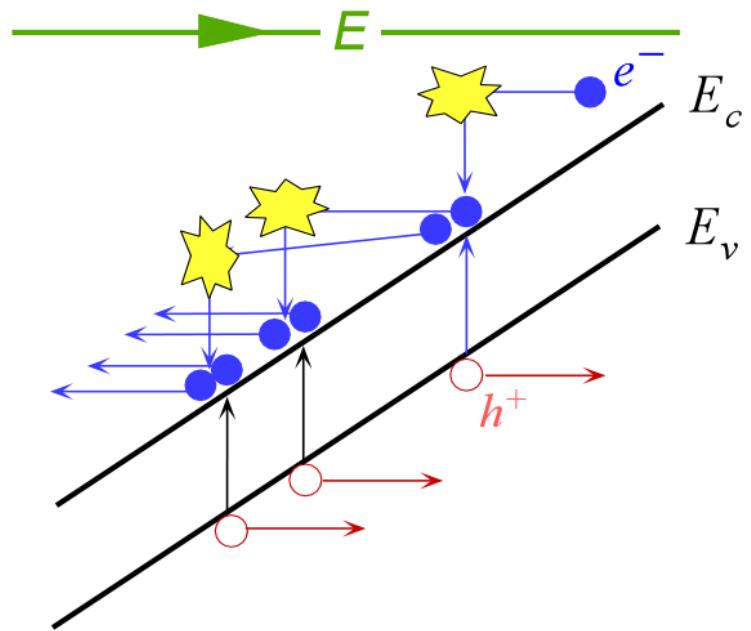
# Avalanche Photodiode



(a)

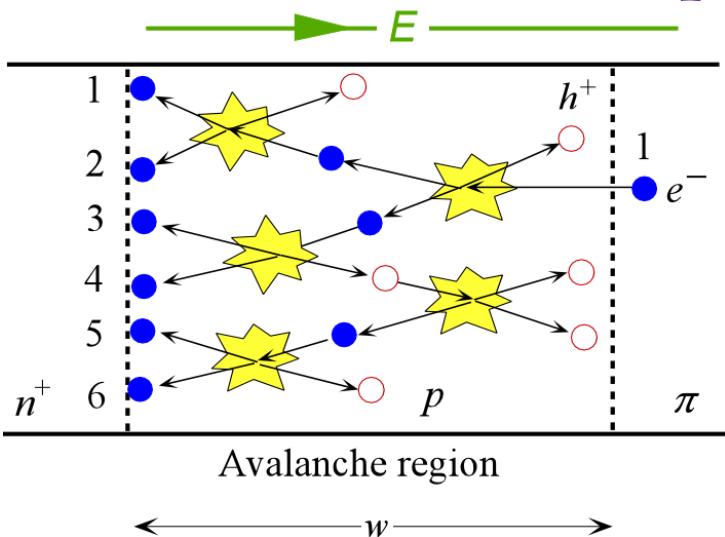


(b)



(a) A pictorial view of impact ionization processes releasing EHPs and the resulting avalanche multiplication. (b) Impact of an energetic conduction electron with crystal vibrations transfers the electron's kinetic energy to a valence electron and thereby excites it to the conduction band

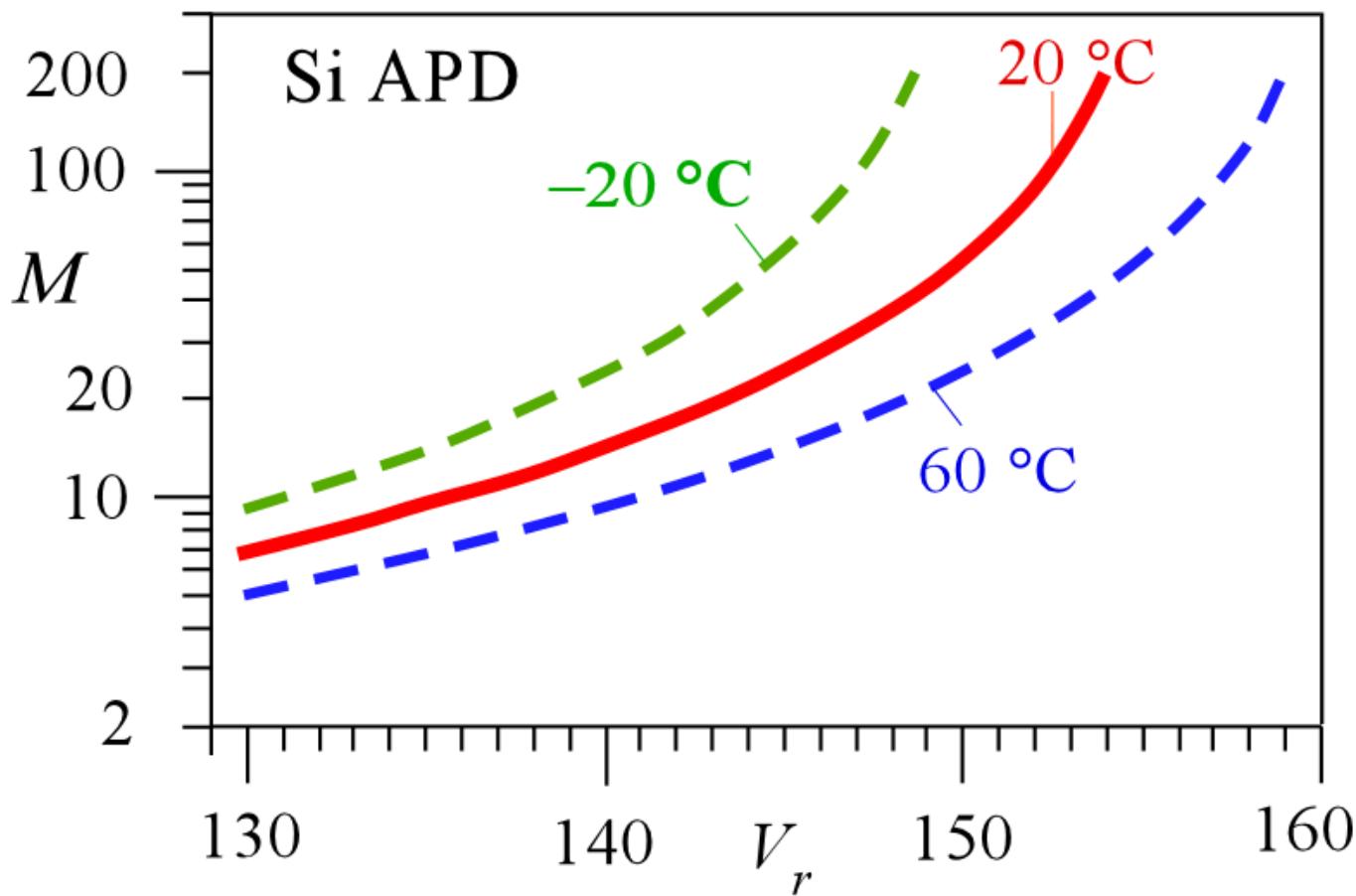
# Avalanche Photodiode Gain or Multiplication $M$



$$M = \frac{\text{Multiplied photocurrent}}{\text{Primary unmultiplied photocurrent}} = \frac{I_{ph}}{I_{pho}}$$

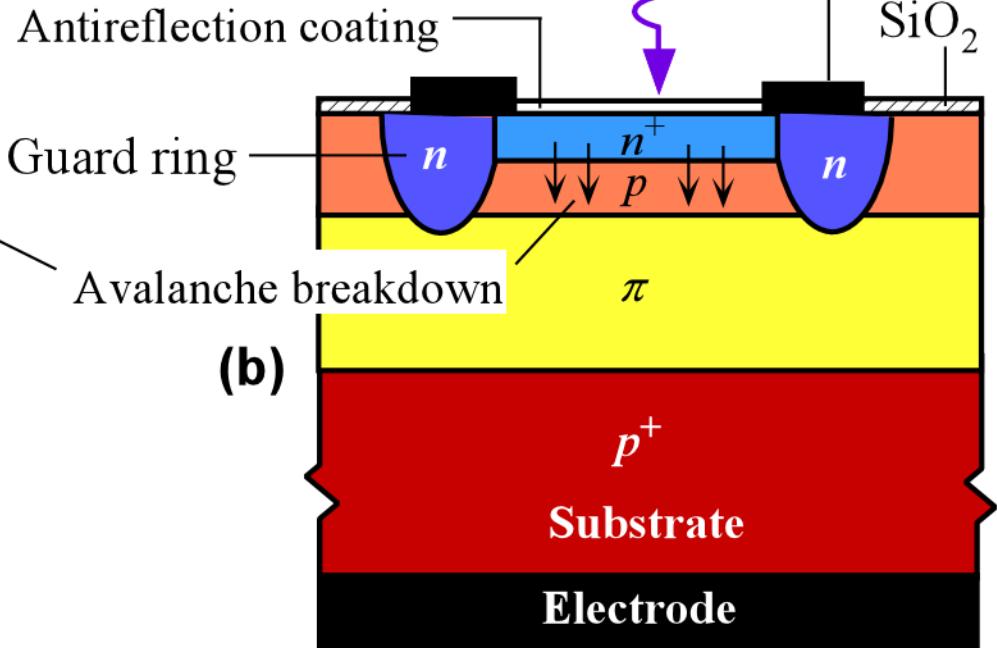
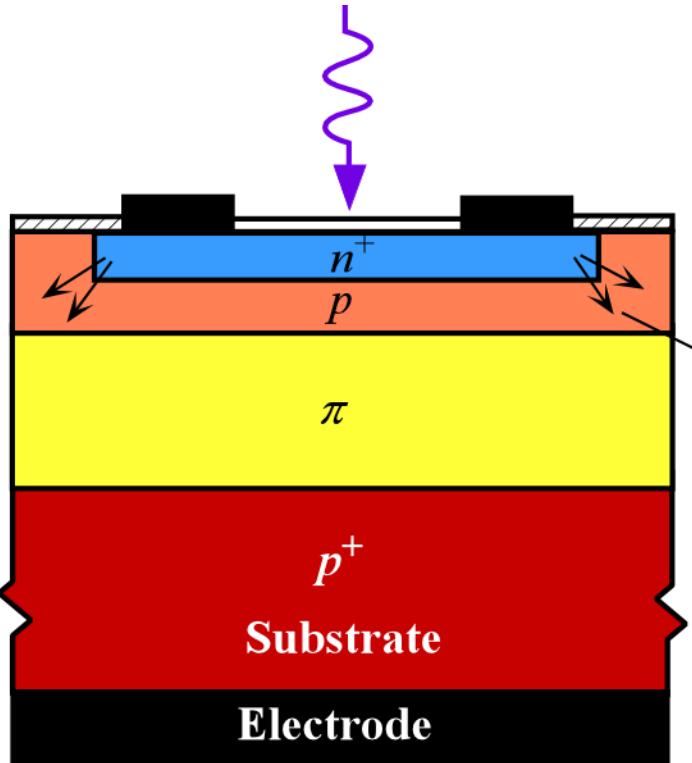
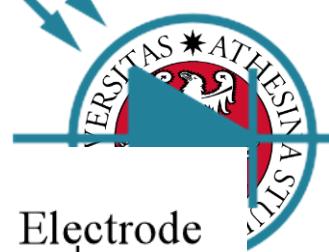
$$M = \frac{1}{1 - \left( \frac{V_r}{V_{br}} \right)^m}$$

# Avalanche Photodiode



Typical multiplication (gain)  $M$  vs. reverse bias characteristics for a typical commercial Si APD, and the effect of temperature. ( $M$  measured for a photocurrent generated at 650 nm of illumination)

# Avalanche Photodiode



(a) A Si APD structure without a guard ring. (b) A schematic illustration of the structure of a more practical Si APD. Note:  $\text{SiO}_2$  is silicon dioxide and serves as an insulating passivation layer.

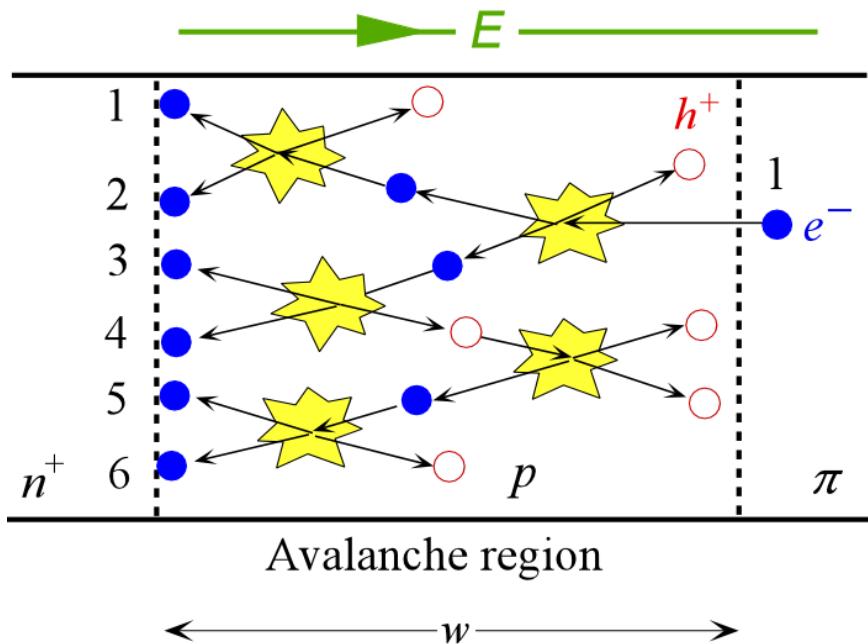
# Photodiode Comparison



Photodiode	$\lambda$ range	$\lambda_{\text{peak}}$	R at $\lambda_{\text{peak}}$	Gain	$I_d$ For 1 mm <sup>2</sup>	Features
GaP <i>pin</i>	nm 150–550	nm 450	A/W 0.1	<1	1 nm 0.005–0.1 nA	UV detection <sup>a</sup>
GaAsP <i>pn</i>	150–750	500–720	0.2–0.4	<1		UV to visible, covering the human eye, low $I_d$ .
GaAs <i>pin</i>	570–870	850	0.5–0.5	<1	0.1–1 nA	High speed and low $I_d$
Si <i>pn</i>	200–1100	600–900	0.5–0.6	<1	0.005–0.1 nA	Inexpensive, general purpose, low $I_d$
Si <i>pin</i>	300–1100	800–1000	0.5–0.6	<1	0.1–1 nA	Faster than <i>pn</i>
Si APD	400–1100	800–900	0.4–0.6 <sup>b</sup>	10–10 <sup>3</sup>	1–10 nA <sup>c</sup>	High gains and fast
Ge <i>pin</i>	700–1800	1500–1580	0.4–0.7	<1	0.1–1 $\mu$ A	IR detection, fast.
Ge APD	700–1700	1500–1580	0.4–0.8 <sup>b</sup>	10–20	1–10 $\mu$ A <sup>c</sup>	IR detection, fast
InGaAs <i>pin</i>	800–1700	1500–1600	0.7–1	<1	1–50 nA	Telecom, high speed, low $I_d$
InGaAs APD	800–1700	1500–1600	0.7–0.95 <sup>b</sup>	10–20	0.05–10 $\mu$ A <sup>c</sup>	Telecom, high speed and gain.
InAs <i>pn</i>	2–3.6 $\mu$ m	3.0–3.5 $\mu$ m	1–1.5	<1	>100 $\mu$ A	Photovoltaic mode. Normally cooled
InSb <i>pn</i>	4–5.5 $\mu$ m	5 $\mu$ m	3	<1	Large	Photovoltaic mode. Normally cooled

NOTE: <sup>a</sup>FGAP71 (Thorlabs); <sup>a</sup>At  $M = 1$ ; <sup>c</sup>At operating multiplication.

# Avalanche Photodiode Gain or Multiplication $M$



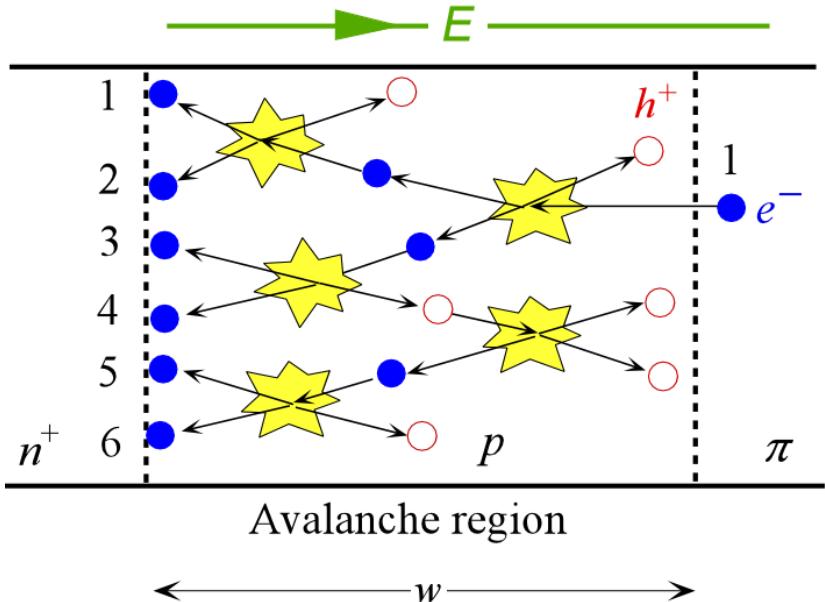
## Ionization coefficient ratio

$$k = \frac{\text{Ionization coefficient for holes}}{\text{Ionization coefficient for electrons}} = \frac{\alpha_h}{\alpha_e}$$

$$\alpha_e = A \exp(-B/E)$$

Chyweth's law

# Avalanche Photodiode Gain or Multiplication $M$



Electrons only

$$M = \exp(\alpha_e w)$$

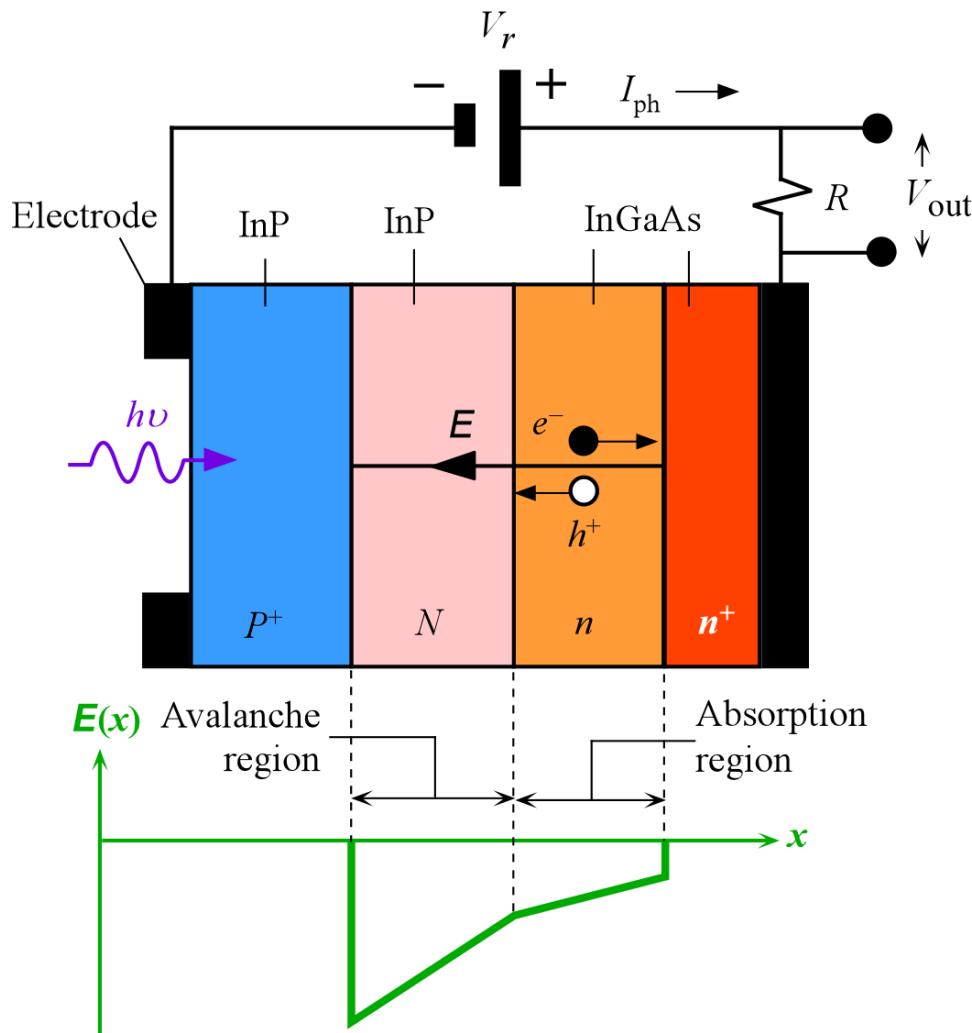
Ionization coefficient

Electrons and holes

$$M = \frac{1 - k}{\exp[-(1 - k)\alpha_e w] - k}$$

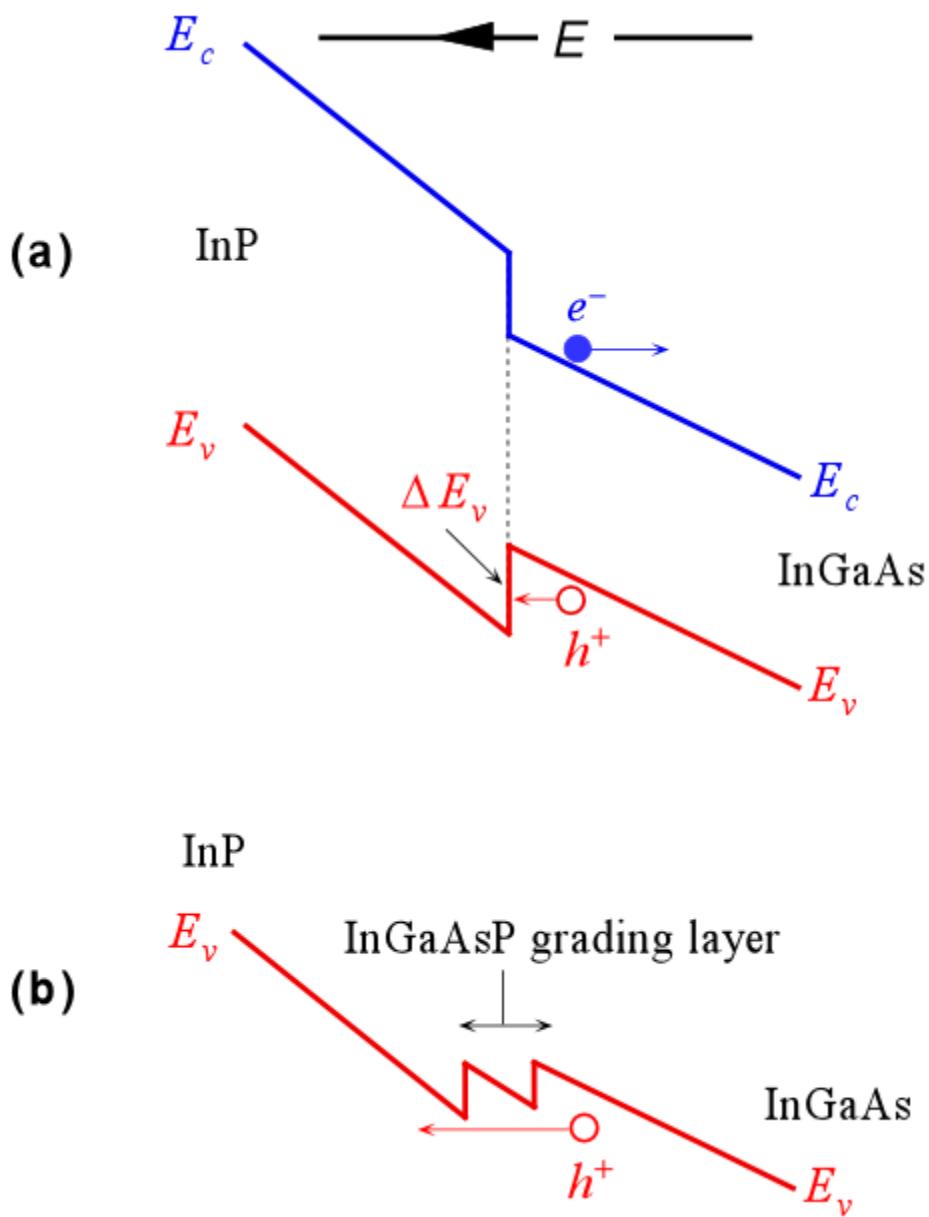
$$k = \alpha_h / \alpha_e$$

# Heterojunction Photodiodes: SAM



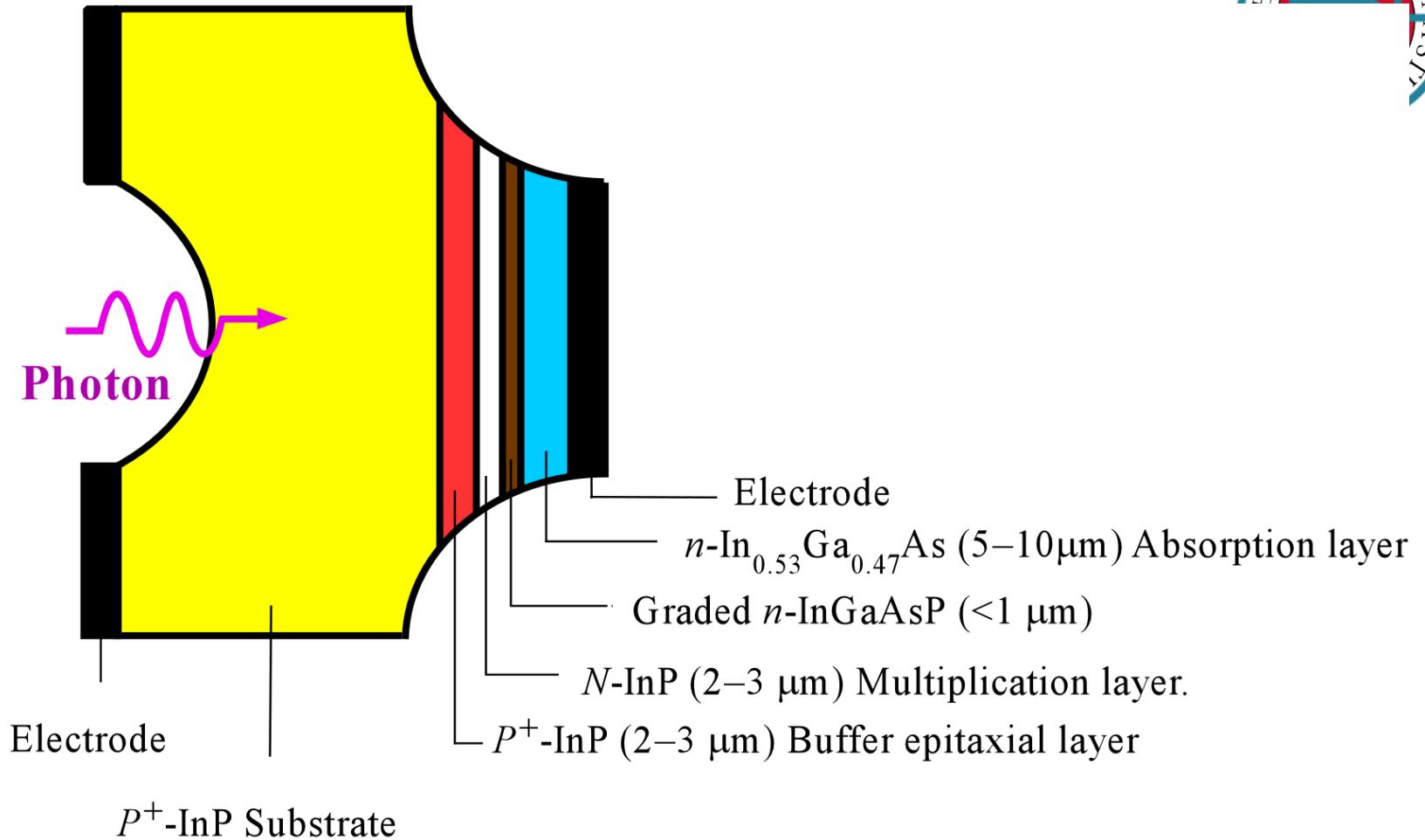
Simplified schematic diagram of a **separate absorption and multiplication** (SAM) APD using a heterostructure based on InGaAs-InP.  $P$  and  $N$  refer to  $p$  and  $n$ -type wider-bandgap semiconductor.

# Heterojunction Photodiodes: SAM



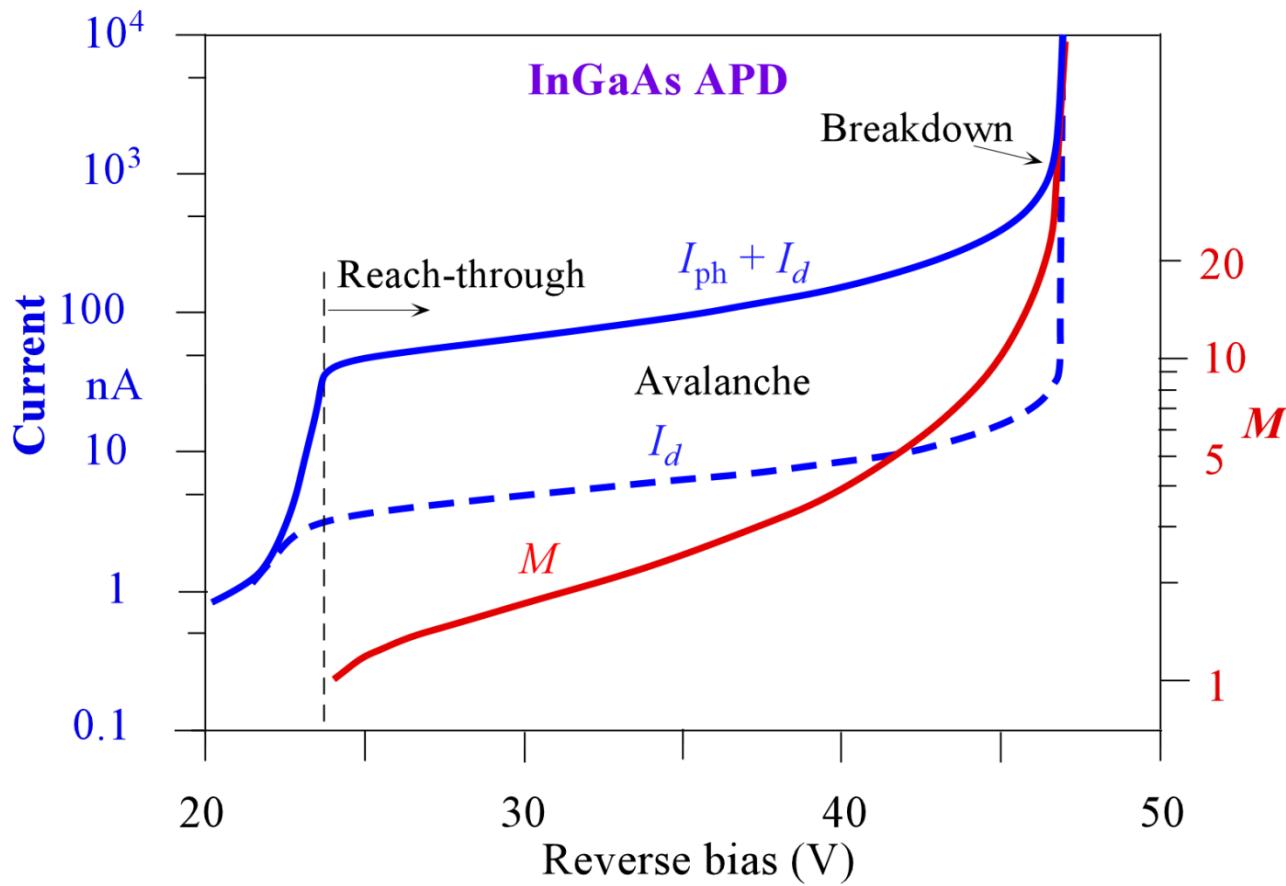
- (a) Energy band diagrams for a SAM detector with a step junction between InP and InGaAs. There is a valence band step  $\Delta E_v$  from InGaAs to InP that slows hole entry into the InP layer.
- (b) An interposing grading layer (InGaAsP) with an intermediate bandgap breaks  $\Delta E_v$  and makes it easier for the hole to pass to the InP layer for a detector with a graded junction between InP and InGaAs. This is the SAGM structure.

# Heterojunction Photodiodes: SAM



Simplified schematic diagram of a more practical mesa-etched  
SAGM layered APD

# APD Characteristics



Typical current and gain ( $M$ ) vs. reverse bias voltage for a commercial InGaAs reach-through APD.  $I_d$  and  $I_{ph}$  are the dark current and photocurrent respectively. The input optical power is  $\sim 100$  nW. The gain  $M$  is 1 when the diode has attained reach-through and then increases with the applied voltage. (The data extracted selectively from Voxtel Catalog, Voxtel, Beaverton, OR 97006)

# EXAMPLE: InGaAs APD Responsivity

An InGaAs APD has a quantum efficiency (QE,  $\eta_e$ ) of 60 % at 1.55  $\mu\text{m}$  in the absence of multiplication ( $M = 1$ ). It is biased to operate with a multiplication of 12. Calculate the photocurrent if the incident optical power is 20 nW. What is the responsivity when the multiplication is 12?

## Solution

The responsivity at  $M = 1$  in terms of the quantum efficiency is

$$R = \eta_e \frac{e\lambda}{hc} = (0.6) \frac{(1.6 \times 10^{-19} \text{ C})(1550 \times 10^{-9} \text{ m})}{(6.626 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})} = \mathbf{0.75 \text{ A W}^{-1}}$$

If  $I_{pho}$  is the primary photocurrent (unmultiplied) and  $P_o$  is the incident optical power then by definition,  $R = I_{pho}/P_o$  so that

$$\begin{aligned} I_{pho} &= RP_o \\ &= (0.75 \text{ A W}^{-1})(20 \times 10^{-9} \text{ W}) \\ &= 1.5 \times 10^{-8} \text{ A or } 15 \text{ nA.} \end{aligned}$$

The photocurrent  $I_{ph}$  in the APD will be  $I_{pho}$  multiplied by  $M$ ,

$$\begin{aligned} I_{ph} &= MI_{pho} \\ &= (12)(1.5 \times 10^{-8} \text{ A}) \\ &= 1.80 \times 10^{-7} \text{ A or } 180 \text{ nA.} \end{aligned}$$

The responsivity at  $M = 12$  is

$$R' = I_{ph}/P_o = M R = (12) / (0.75) = \mathbf{9.0 \text{ A W}^{-1}}$$

# EXAMPLE: Silicon APD

A Si APD has a QE of 70 % at 830 nm in the absence of multiplication, that is  $M = 1$ . The APD is biased to operate with a multiplication of 100. If the incident optical power is 10 nW what is the photocurrent?

## Solution

The unmultiplied responsivity is given by,

$$R = \eta_e \frac{e\lambda}{hc} = (0.70) \frac{(1.6 \times 10^{-19} \text{ C})(830 \times 10^{-9} \text{ m})}{(6.626 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})} = 0.47 \text{ A W}^{-1}$$

The unmultiplied primary photocurrent from the definition of  $R$  is

$$I_{pho} = RP_o = (0.47 \text{ A W}^{-1})(10 \times 10^{-9} \text{ W}) = 4.7 \text{ nA}$$

The multiplied photocurrent is

$$I_{ph} = MI_{pho} = (100)(4.67 \text{ nA}) = \mathbf{470 \text{ nA or } 0.47 \mu\text{A}}$$

# EXAMPLE: Avalanche multiplication in Si APDs

The electron and hole ionization coefficients  $\alpha_e$  and  $\alpha_h$  in silicon are approximately given by Eq. (5.6.4) with  $A \approx 0.740 \times 10^6 \text{ cm}^{-1}$ ,  $B \approx 1.16 \times 10^6 \text{ V cm}^{-1}$  for electrons ( $\alpha_e$ ) and  $A \approx 0.725 \times 10^6 \text{ cm}^{-1}$  and  $B \approx 2.2 \times 10^6 \text{ V cm}^{-1}$  for holes ( $\alpha_h$ ). Suppose that the width  $w$  of the avalanche region is  $0.5 \mu\text{m}$ . Find the multiplication gain  $M$  when the applied field in this region reaches  $4.00 \times 10^5 \text{ V cm}^{-1}$ ,  $4.30 \times 10^5 \text{ V cm}^{-1}$  and  $4.38 \times 10^5 \text{ V cm}^{-1}$ . What is your conclusion?

## Solution

At the field of  $E = 4.00 \times 10^5 \text{ V cm}^{-1}$ , from Eq. (5.6.4)

$$\begin{aligned}\alpha_e &= A \exp(-B/E) \\ &= (0.74 \times 10^6 \text{ cm}^{-1}) \exp[-(1.16 \times 10^6 \text{ V cm}^{-1})/(4.00 \times 10^5 \text{ V cm}^{-1})] \\ &= 4.07 \times 10^4 \text{ cm}^{-1}.\end{aligned}$$

Similarly using Eq. (5.6.4) for holes,  $\alpha_h = 2.96 \times 10^3 \text{ cm}^{-1}$ . Thus  $k = \alpha_h/\alpha_e = 0.073$ . Using this  $k$  and  $\alpha_e$  above in Eq. (5.6.6) with  $w = 0.5 \times 10^{-4} \text{ cm}$ ,

$$M = \frac{1 - 0.073}{\exp[-(1 - 0.073)(4.07 \times 10^4 \text{ cm})(0.5 \times 10^{-4} \text{ cm}^{-1})] - 0.073} = 11.8$$

Note that if we had only electron avalanche without holes ionizing, then the multiplication would be

$$M_e = \exp(\alpha_e w) = \exp[(4.07 \times 10^4 \text{ cm}^{-1})(0.5 \times 10^{-4} \text{ cm})] = 7.65$$

# EXAMPLE: Avalanche multiplication in Si APDs

## Solution (continued)



We can now repeat the calculations for  $E = 4.30 \times 10^5 \text{ V cm}^{-1}$  and again for  $E = 4.38 \times 10^5 \text{ V cm}^{-1}$ . The results are summarized in Table 5.3 for both  $M$  and  $M_e$ . **Notice how quickly  $M$  builds up with the field and how a very small change at high fields causes an enormous change in  $M$  that eventually leads to a breakdown.** ( $M$  running away to infinity as  $V_r$  increases.) Notice also that in the presence of only electron-initiated ionization,  $M_e$  simply increases without a sharp run-away to breakdown.

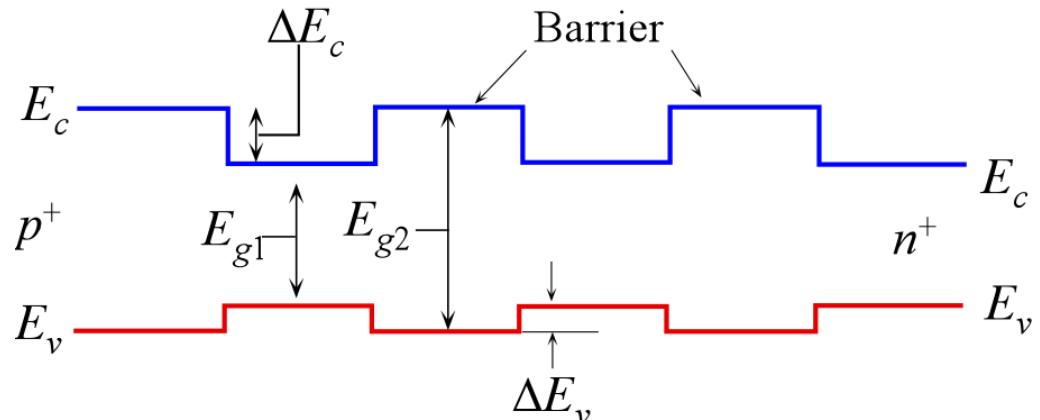
$E$ (V cm $^{-1}$ )	$a_e$ (cm $^{-1}$ )	$a_h$ (cm $^{-1}$ )	$k$	$M$	$M_e$	Comment
$4.00 \times 10^5$	$4.07 \times 10^4$	$2.96 \times 10^3$	0.073	11.8	7.65	$M$ and $M_e$ not too different at low $E$
$4.30 \times 10^5$	$4.98 \times 10^4$	$4.35 \times 10^3$	0.087	57.2	12.1	7.5% increase in $E$ , large difference between $M$ and $M_e$
$4.38 \times 10^5$	$5.24 \times 10^4$	$4.77 \times 10^3$	0.091	647	13.7	1.9% increase in $E$

# Superlattice APD

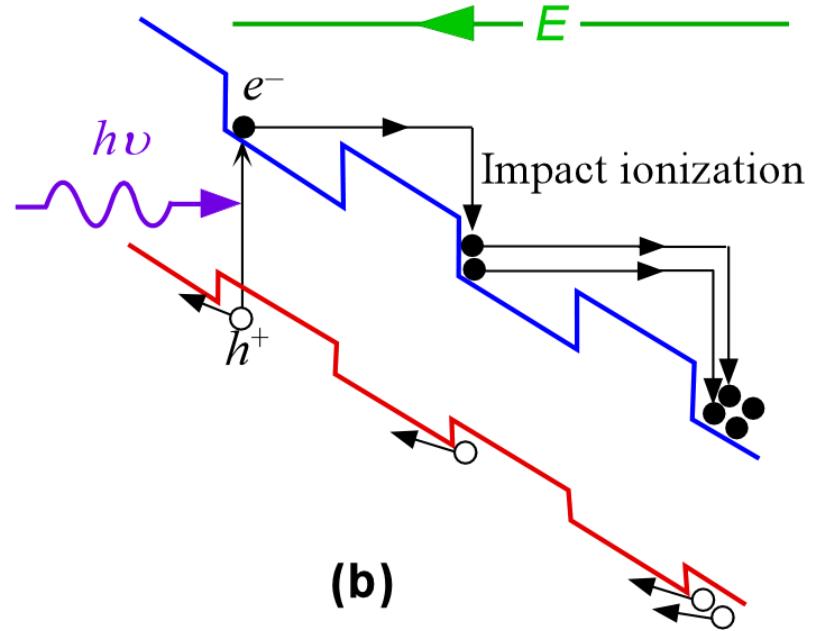
## Multiple Quantum Well Detectors



Superlattice APD



(a)



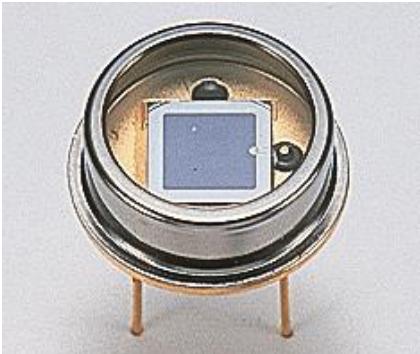
(b)

- (a) Energy band diagram of a MQW superlattice APD.  
(b) Energy band diagram with an applied field and impact ionization.

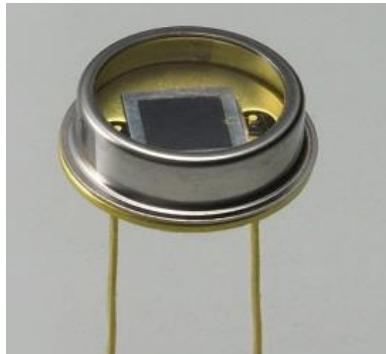
# Schottky Junction Photodiodes



Schottky junction type metal-semiconductor-metal (MSM) type photodetectors. (Courtesy of Hamamatsu)



GaAsP Schottky junction photodiode for 190-680 nm detection, from UV to red (Courtesy of Hamamatsu)

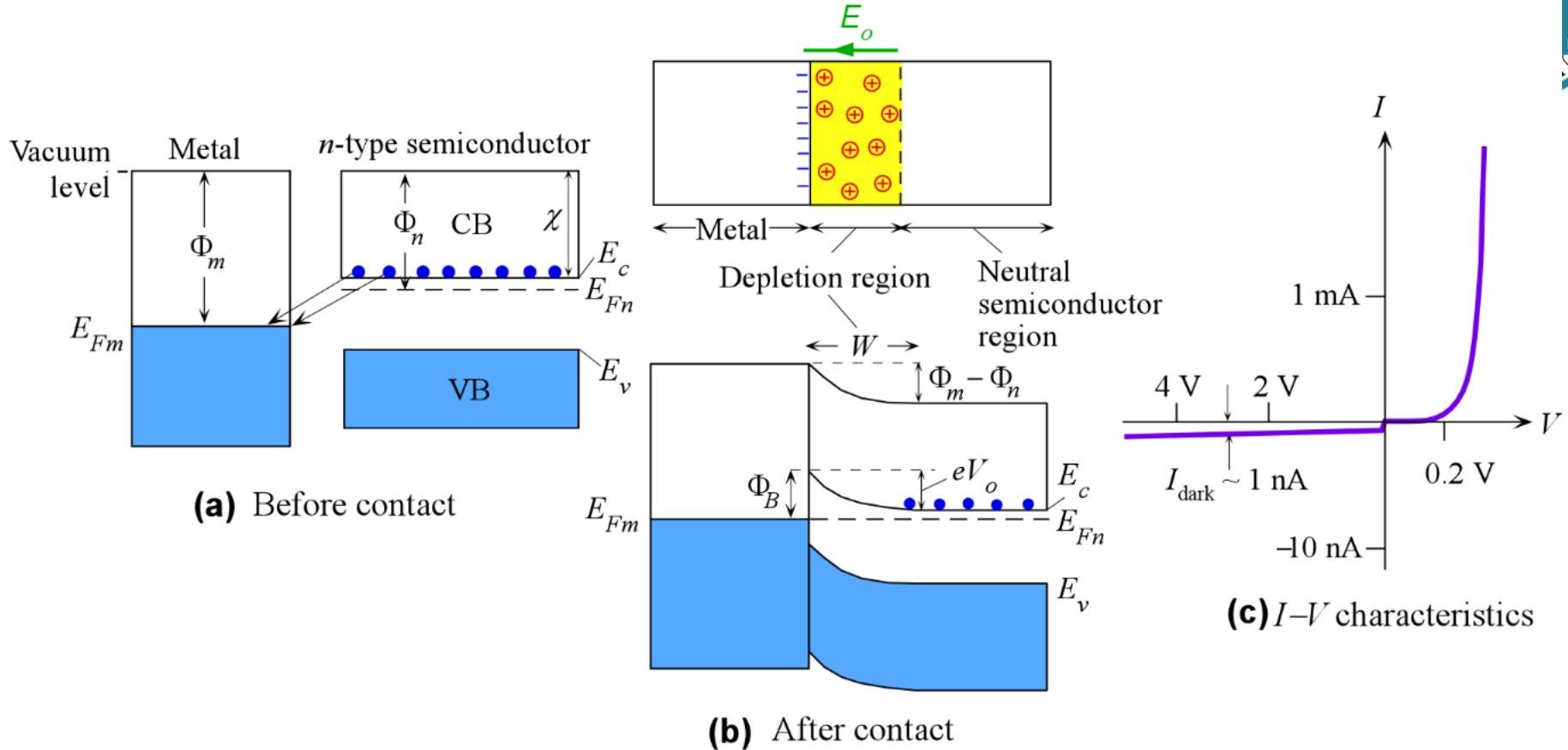


GaP Schottky junction photodiode for 190 nm to 550 nm detection. (Courtesy of Hamamatsu)



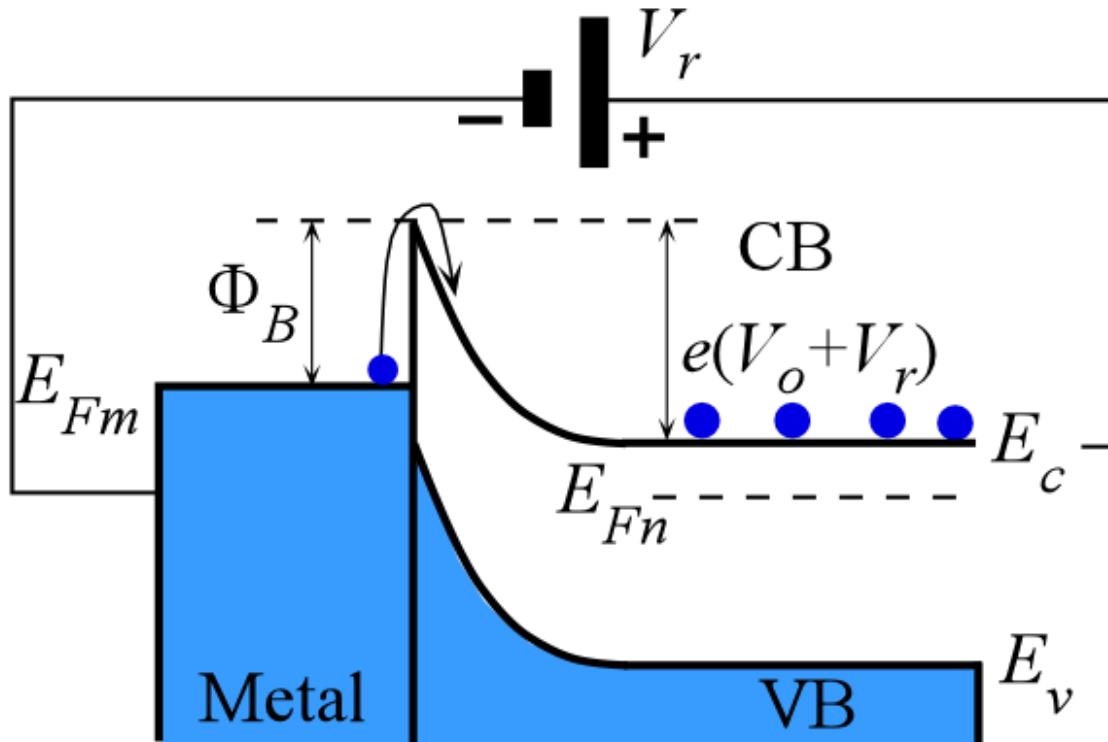
AlGaN Schottky junction photodiode for UV detection (Courtesy of sglux, Germany)

# Schottky Junction



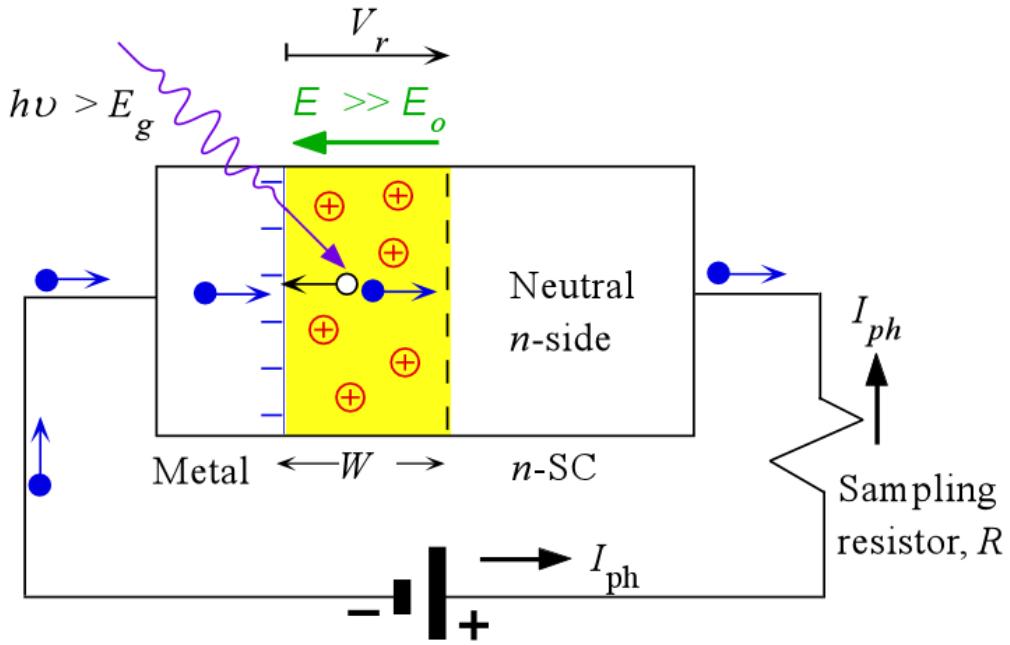
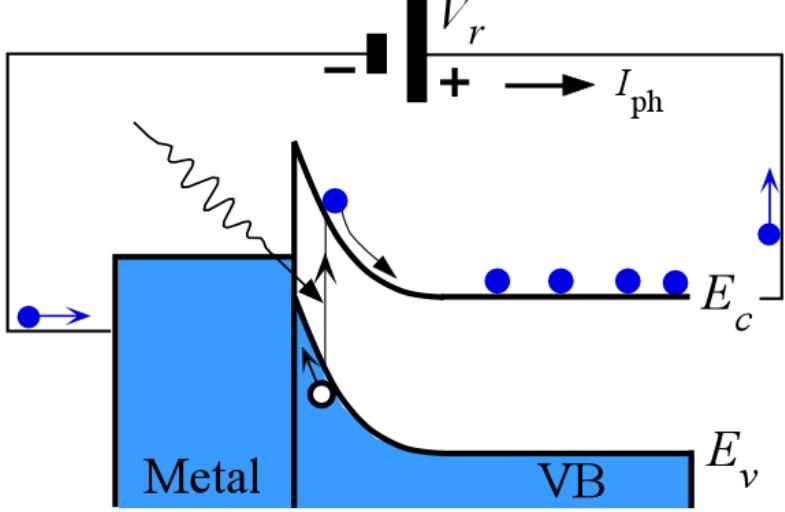
(a) Metal and an *n*-type semiconductor before contact. The metal work function  $\Phi_m$  is greater than that of the *n*-type semiconductor (b) A Schottky junction forms between the metal and the semiconductor. There is a depletion region in the semiconductor next to the metal and a built-in field  $E_o$  (c) Typical  $I$  vs.  $V$  characteristics of a Schottky contact device.

# Schottky Junction



Reverse biased Schottky junction and the dark current due to the injection of electrons from the metal into the semiconductor over the barrier  $\Phi_B$ .

# Schottky Junction



LEFT: Photogeneration in the depletion region and the resulting photocurrent. RIGHT: The Schottky junction photodetector

# Schottky Junction Photodiodes

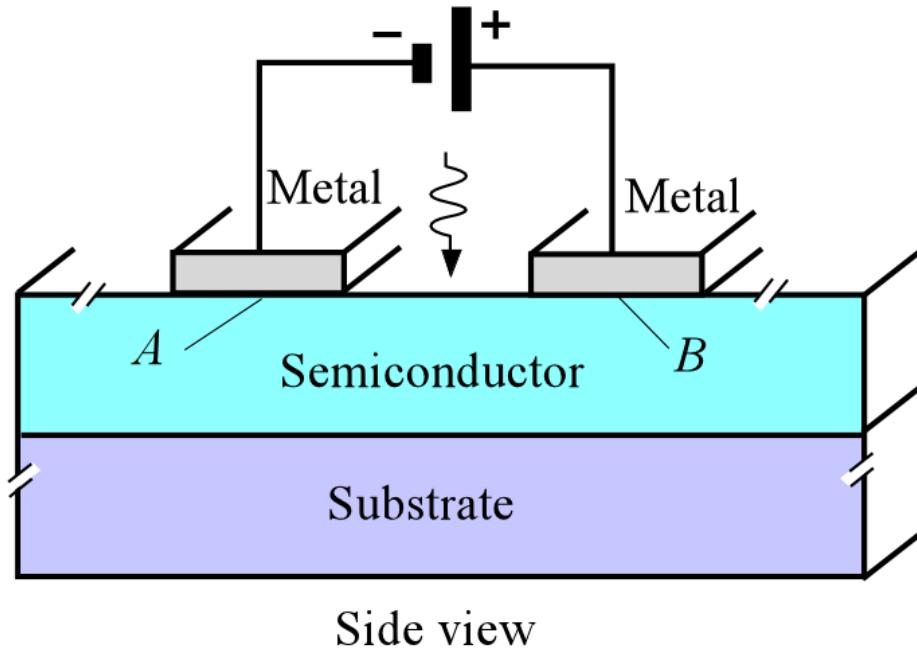


Schottky junction based photodetectors and some of their features.  $\tau_R$  and  $\tau_F$  are the rise and fall times of the output of the photodetector for an optical pulse input. The rise and fall times represent the times required for the output to rise from 10% to 90% of its final steady state value and to fall from 90% to 10% of its value before the optical pulse is turned off.

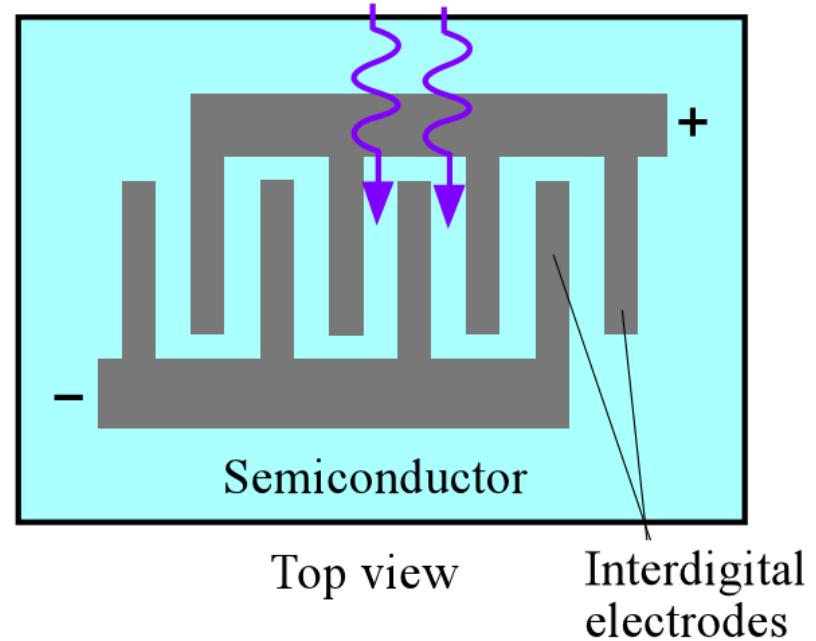
Schottky junction	$\lambda$ range nm	$R_{\text{peak}}$ (at peak) A/W	$J_{\text{dark}}$ per mm <sup>2</sup>	Features with typical values
GaAsP	190–680	0.18 (610 nm)	5 pA	UV to red, $\tau_R = 3.5 \mu\text{s}$ . (G1126 series <sup>a</sup> )
GaP	190–550	0.12 (440 nm)	5 pA	UV to green, $\tau_R = 5 \mu\text{s}$ . (G1961 <sup>a</sup> )
AlGaN	220–375	0.13 (350 nm)	1 pA	Measurement of UV; blind to visible light. (AG38S <sup>b</sup> )
GaAs	320–900	0.2 (830 nm)	~ 1 nA	Wide bandwidth > 10 GHz, $\tau_R < 30 \text{ ps}$ . (UPD-30-VSG-P <sup>c</sup> )
InGaAs MSM	850–1650	0.4 (1300 nm)	5 $\mu\text{A}$	Optical high speed measurements, $\tau_R = 80 \text{ ps}$ , $\tau_F = 160 \text{ ps}$ . (G7096 <sup>a</sup> )
GaAs MSM	450–870	0.3 (850 nm)	0.1 nA	Optical high speed measurements, $\tau_R = 30 \text{ ps}$ , $\tau_F = 30 \text{ ps}$ . (G4176 <sup>a</sup> )

<sup>a</sup>Hamamatsu (Japan); <sup>b</sup>sglux (Germany); <sup>c</sup>Alphalas

# Schottky Junction Photodiodes



Side view

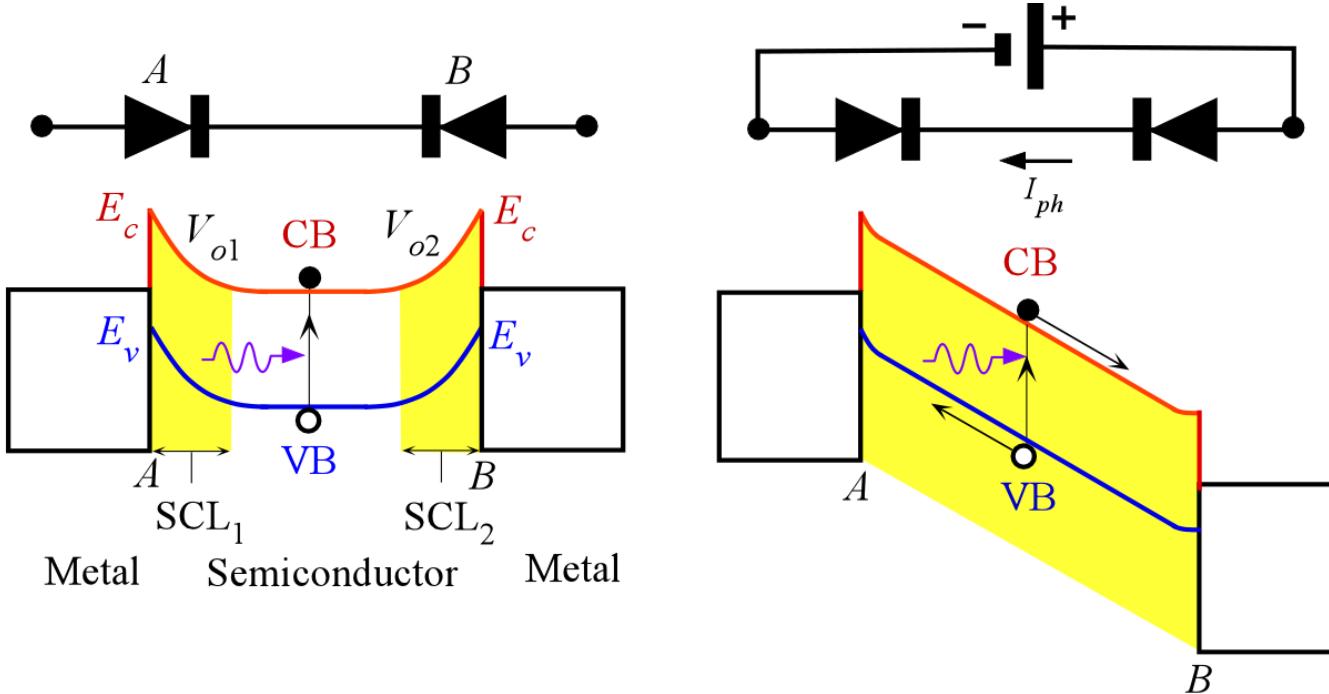


Top view

Interdigital  
electrodes

LEFT: The metal electrodes are on the surface of the semiconductor crystal (which is grown on a suitable substrate).  
RIGHT: The electrodes are configured to be interdigital and on the surface of the crystal.

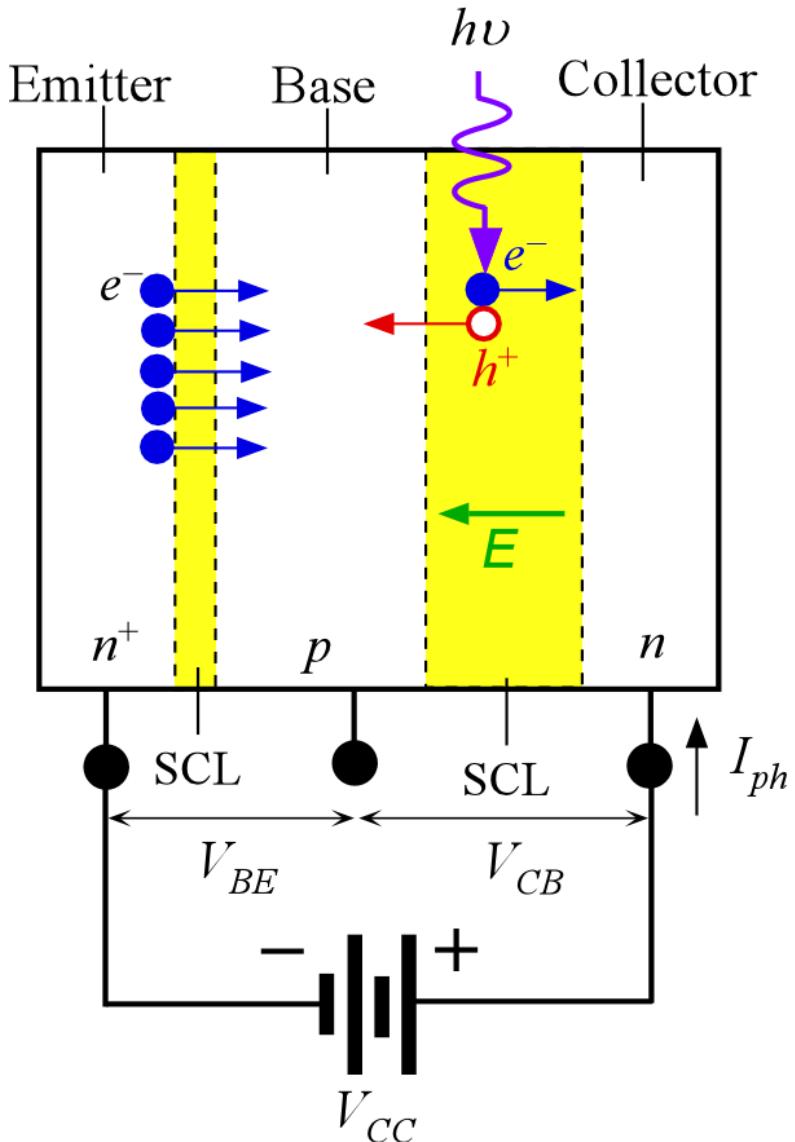
# Schottky Junction Photodiodes



LEFT: Two neighboring Schottky junctions are connected end-to-end, but in opposite directions as shown for A and B. The energy band diagram without any bias is symmetrical. The grey areas represent the SCL<sub>1</sub> and SCL<sub>2</sub> at A and B. RIGHT: Under a sufficiently large bias, the SCL<sub>1</sub> from A extends and meets that from B so that the whole semiconductor between the electrodes is depleted. There is a large field in this region, and the photogenerated EHPs become separated and then drifted, which results in a photocurrent.



# Phototransistor



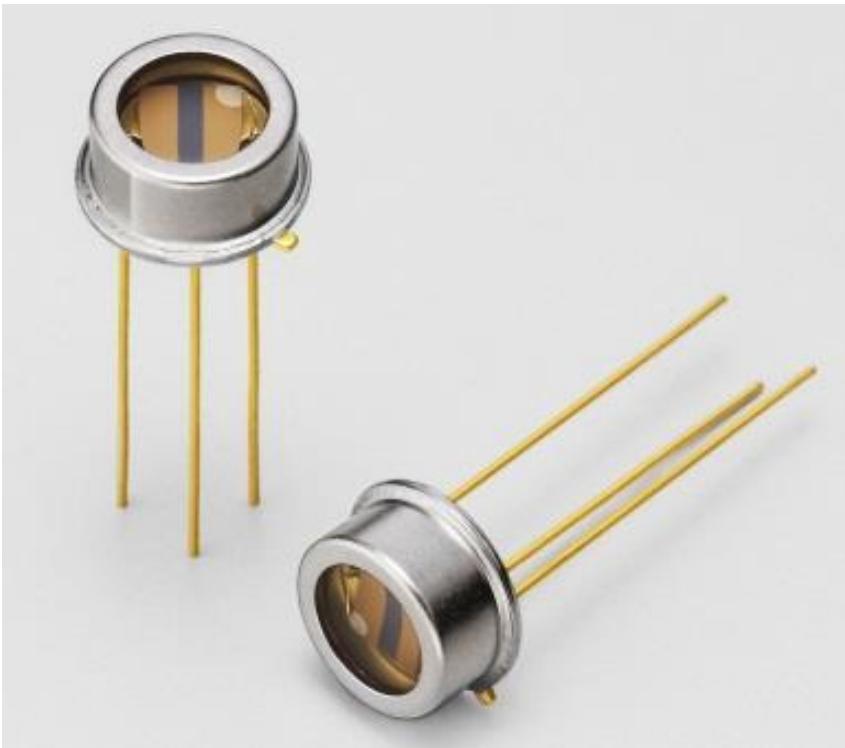
Transistor action

$$I_E \propto \exp(eV_{BE}/k_B T)$$

Gain

$$I_{ph} \approx \beta I_{pho}$$

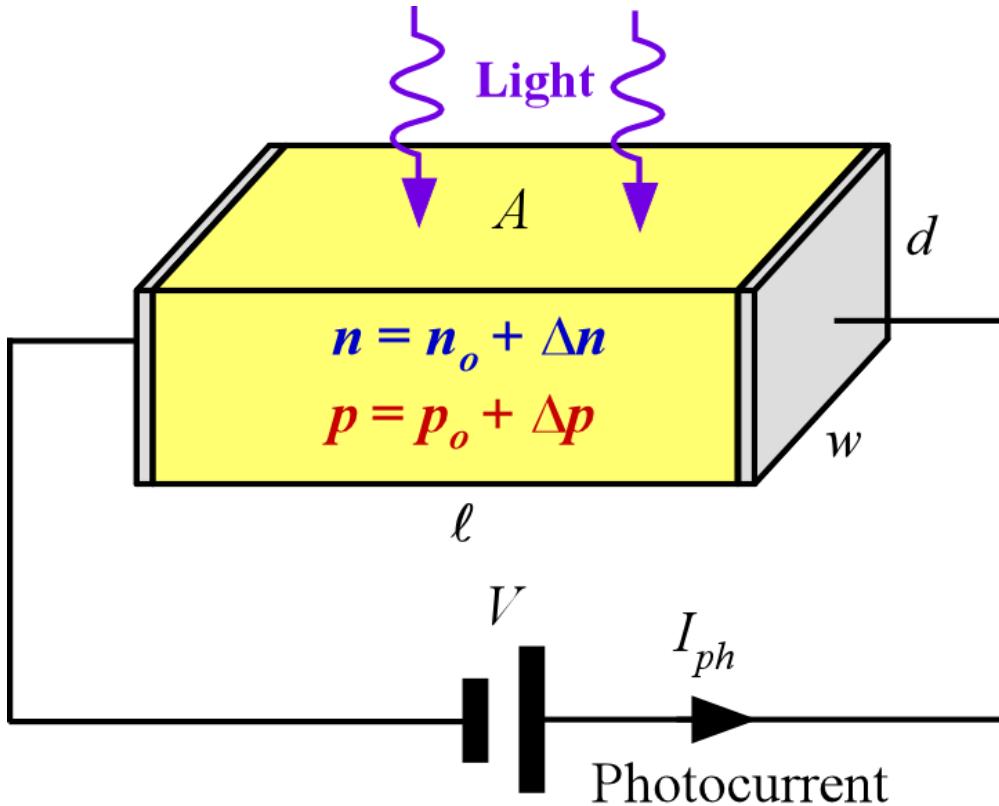
# Photoconductive Detectors



PbS (lead sulfide) photoconductive detectors for the detection of IR radiation up to  $2.9 \mu\text{m}$ . They are typically used in such applications as radiation thermometers, flame monitors, water content and food ingredient analyzers, spectrophotometers etc.. (P9217 series)

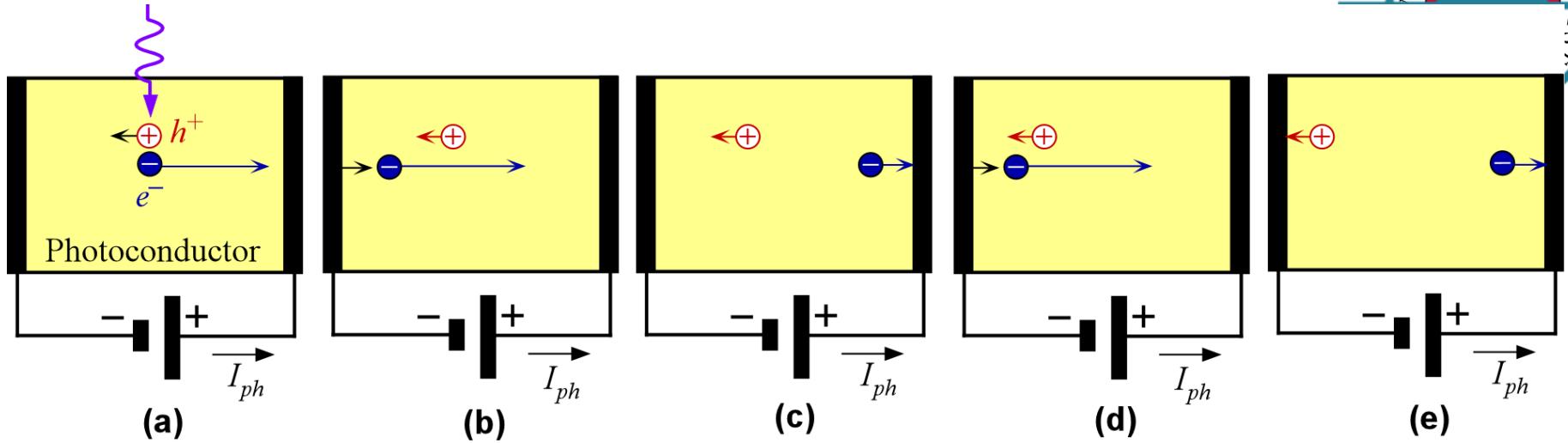
(Courtesy of Hamamatsu.)

# Photoconductive Detectors



A semiconductor slab of length  $\ell$ , width  $w$  and depth  $d$   
is illuminated with light of wavelength  $\lambda$

# Photoconductive Detectors

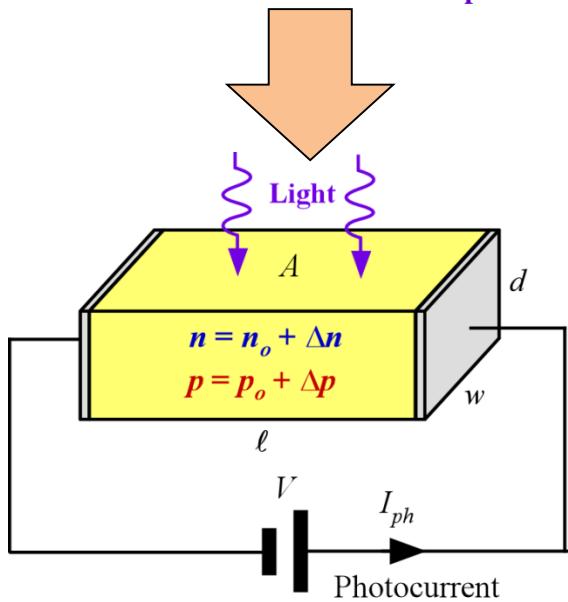


A photoconductor with ohmic contacts (contacts not limiting carrier entry) can exhibit gain. As the slow hole drifts through the photoconductors, many fast electrons enter and drift through the photoconductor because, at any instant, the photoconductor must be neutral. Electrons drift faster which means as one leaves, another must enter.

# Photoconductivity $\Delta\sigma$ and Photocurrent Density $J_{ph}$



**Photon flux =  $\Phi_{ph}$**



## Photogeneration rate

$$g_{ph} = \frac{\eta_i A \Phi_{ph}}{Ad} = \frac{\eta_i \left( \frac{I}{hv} \right)}{d} = \frac{\eta_i I \lambda}{hcd}$$

$\eta_i$  = Internal quantum efficiency

## Steady state illumination

$$\frac{d\Delta n}{dt} = g_{ph} - \frac{\Delta n}{\tau} = 0$$

**Photoconductivity**  $\Delta\sigma = e\mu_e \Delta n + e\mu_h \Delta p = e\Delta n(\mu_e + \mu_h)$

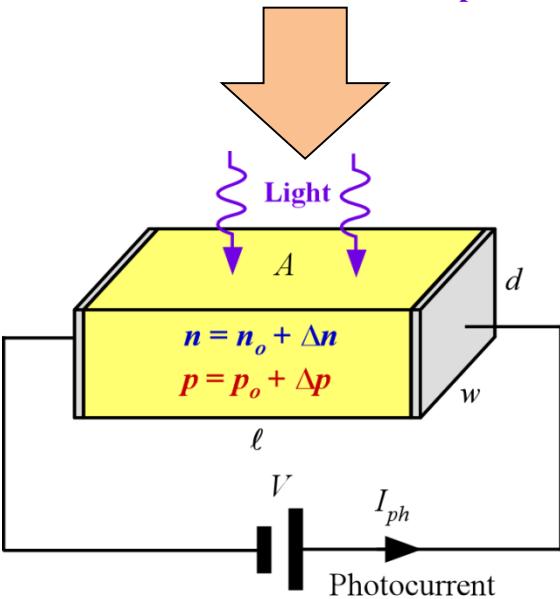
$$\Delta\sigma = \frac{e\eta_i I \lambda \tau (\mu_e + \mu_h)}{hcd}$$

$$J_{ph} = \Delta\sigma \frac{V}{\ell} = \Delta\sigma E$$

# Photoconductive Gain



Photon flux =  $\Phi_{ph}$



$$\text{Rate of electron flow} = \frac{I_{ph}}{e} = \frac{wdJ_{ph}}{e} = \frac{\eta_i l w \lambda \tau (\mu_e + \mu_h) E}{hc}$$

$$\text{Rate of electron generation} = (\text{Volume})g_{ph} = (wd\ell)g_{ph} = w\ell \frac{\eta_i l \lambda}{hc}$$

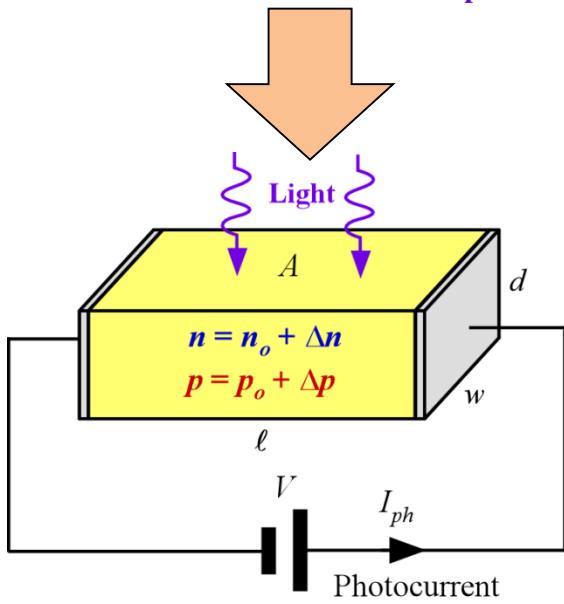
**Photoconductive gain  $G$**

$$G = \frac{\text{Rate of electronflow in external circuit}}{\text{Rate of electron generation by light absorption}} = \frac{\tau(\mu_e + \mu_h)E}{\ell}$$

# Photoconductive Gain



**Photon flux =  $\Phi_{ph}$**



$$G = \frac{\text{Rate of electronflow in external circuit}}{\text{Rate of electrongeneration by light absorption}} = \frac{\tau(\mu_e + \mu_h)E}{\ell}$$

Electron and hole transit times (time to cross the semiconductor) are

$$t_e = \ell / (\mu_e E)$$

Electron

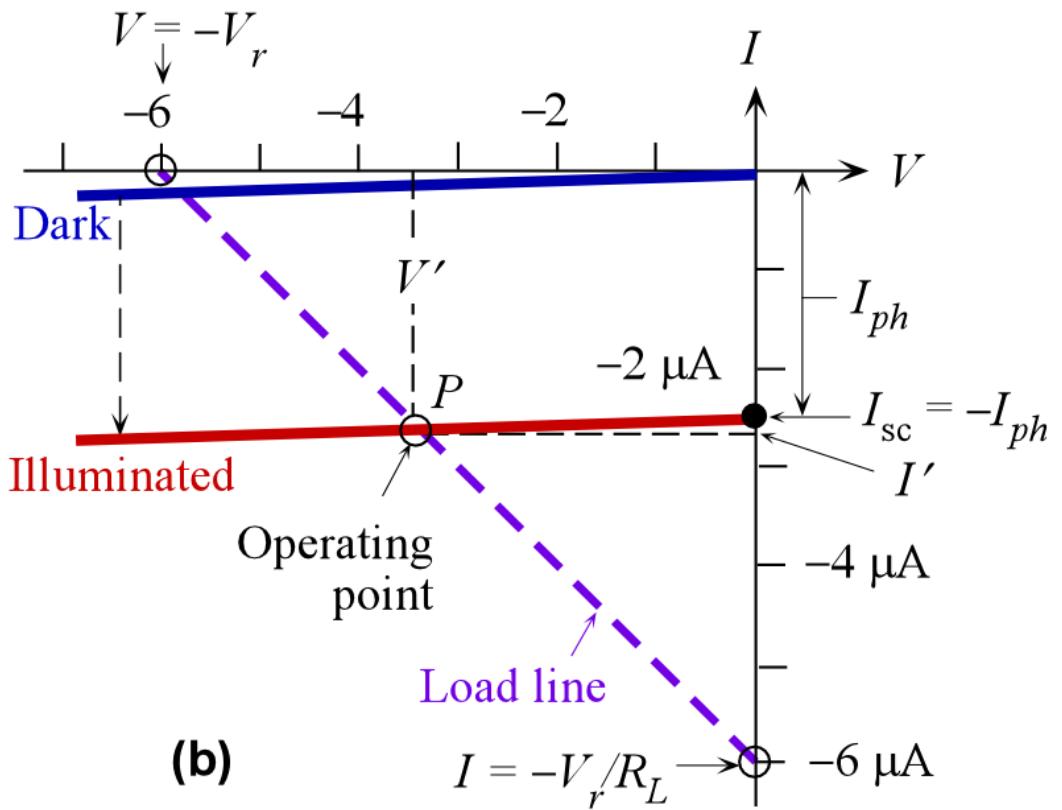
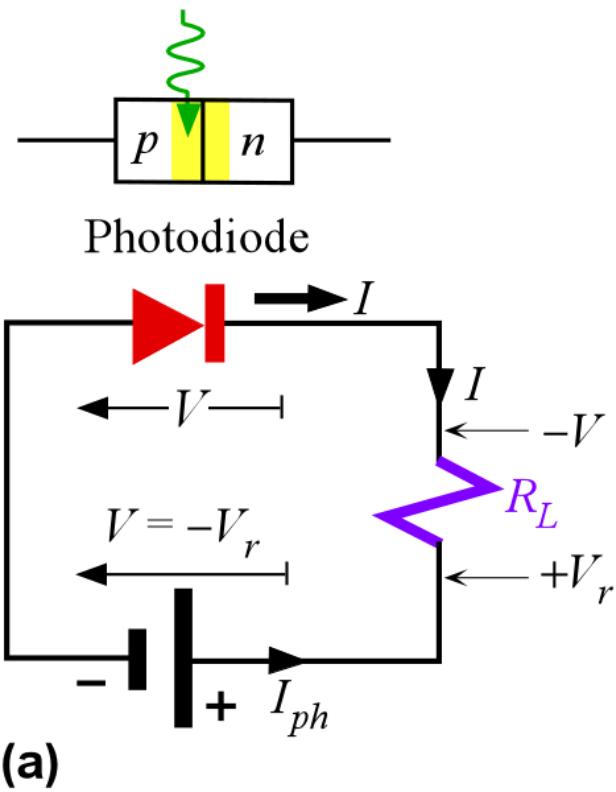
$$t_h = \ell / (\mu_h E)$$

Hole

## Photoconductive gain $G$

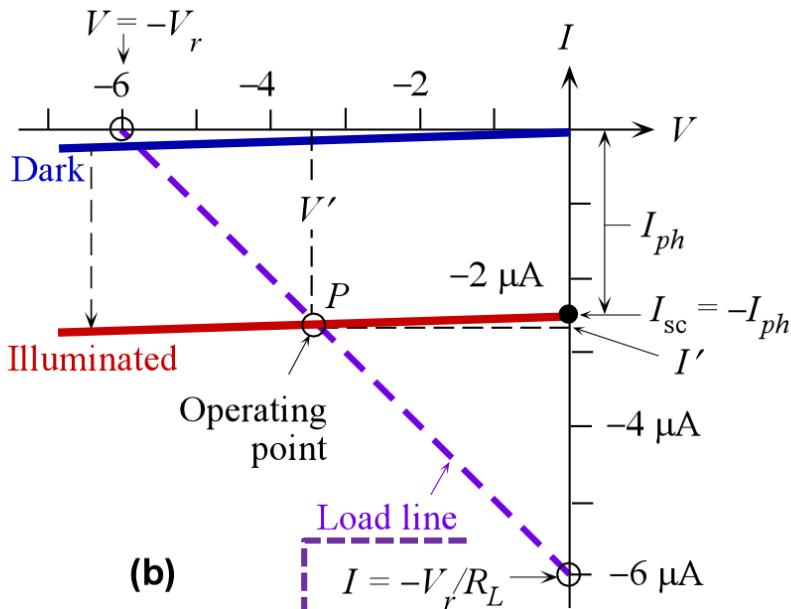
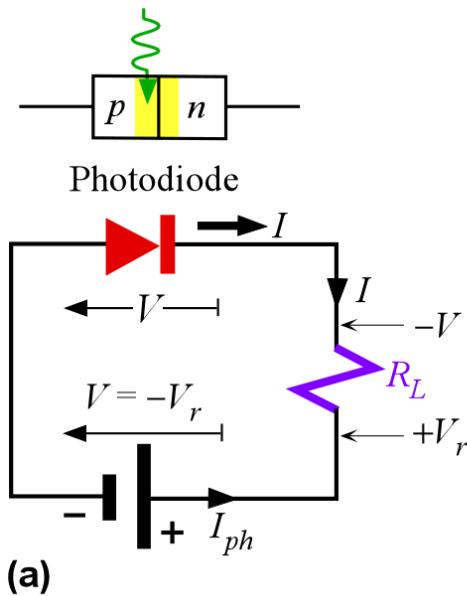
$$G = \frac{\tau}{t_e} + \frac{\tau}{t_h} = \frac{\tau}{t_e} \left( 1 + \frac{\mu_h}{\mu_e} \right)$$

# Basic Photodiode Circuits



(a) The photodiode is reverse biased through  $R_L$  and illuminated. Definitions of positive  $I$  and  $V$  are shown as if the photodiode were forward biased. (b)  $I$ - $V$  characteristics of the photodiode with positive  $I$  and  $V$  definitions in (a). The load line represents the behavior of the load  $R$ . The operating point is  $P$  where the current and voltage are  $I'$  and  $V'$ .

# Basic Photodiode Circuits: The Load Line



$P$  is the operating point

$$V' = -3.5 \text{ V}$$

$$I' = -2.5 \mu\text{A}$$

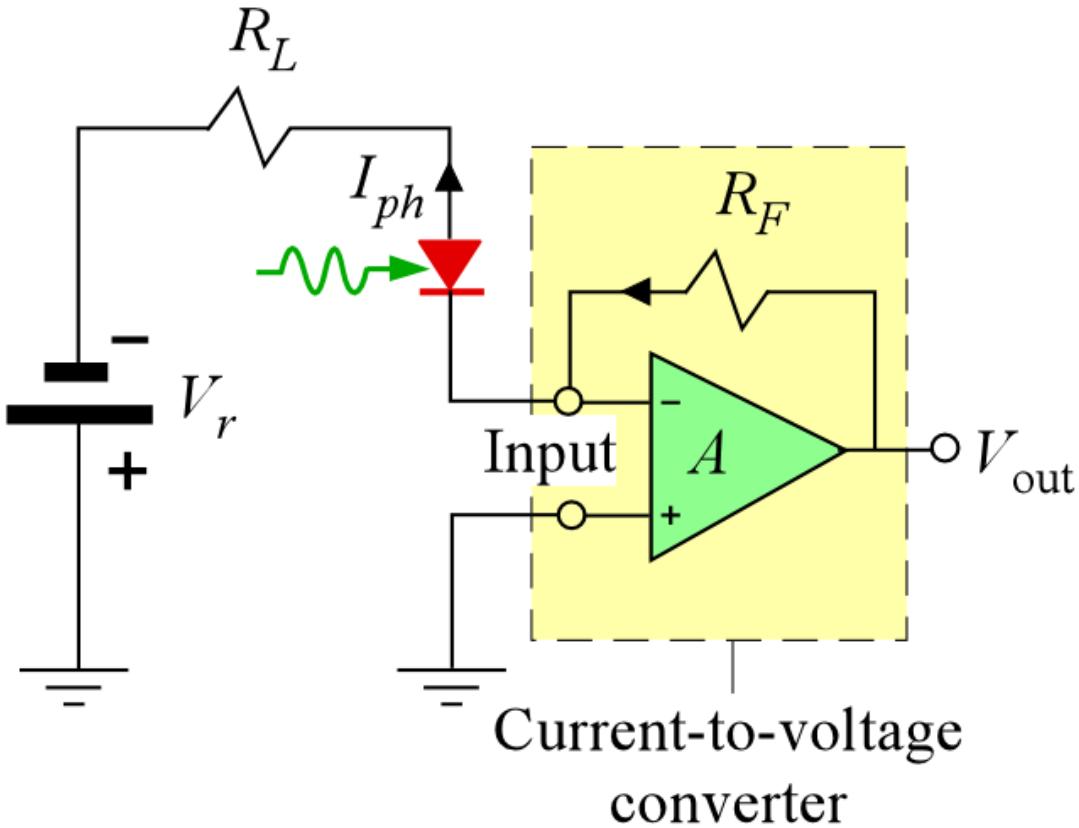
$$I' \approx I_{ph}$$

The current through  $R_L$  is

$$I = -(V + V_r) / R_L$$

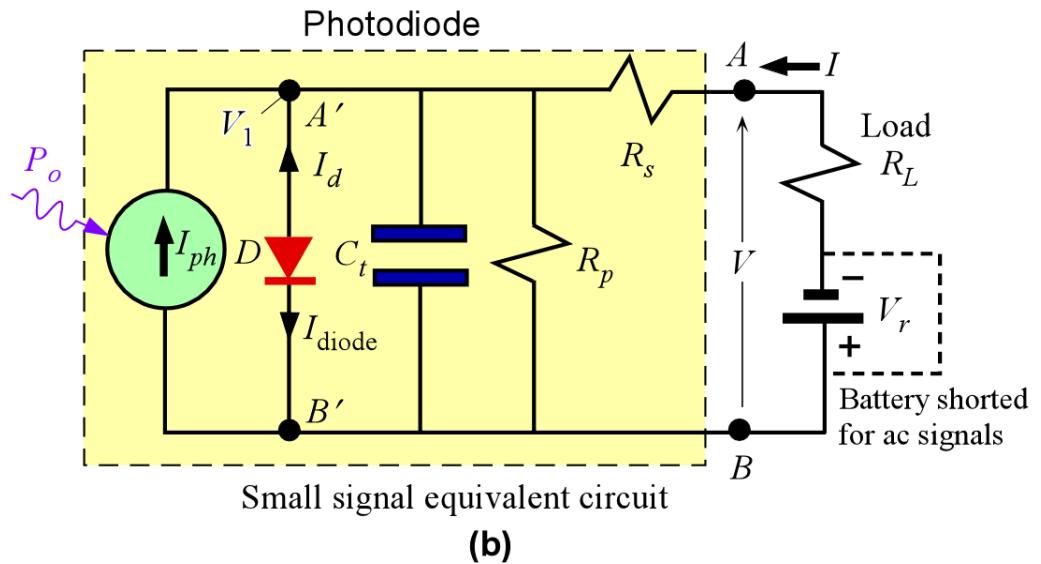
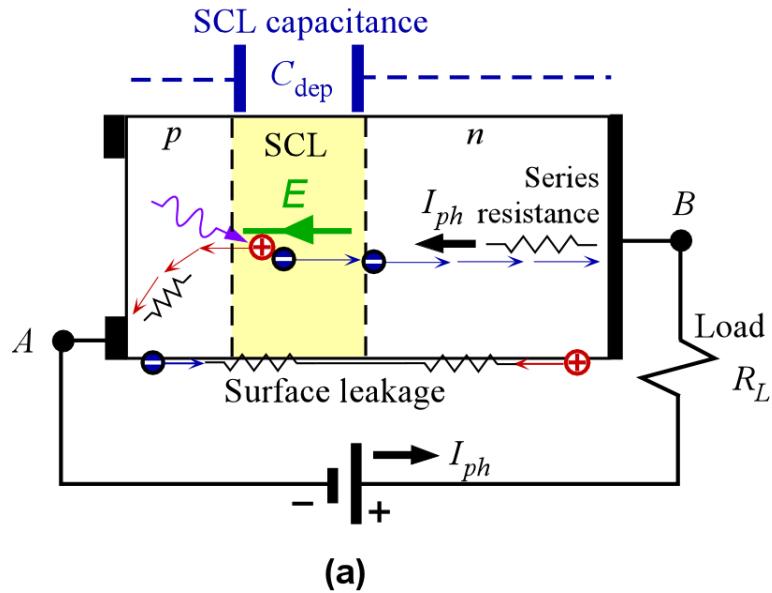
This is the **load line** shown in the figure.  $P$  is the intersection of the load line with the photodiode  $I$  vs.  $V$  curve and is the operating point.

# Basic Photodiode Circuits



A simple circuit for the measurement of the photocurrent  $I_{ph}$  by using a current-voltage converter or a transimpedance amplifier. The reverse bias  $V_r$  is a positive number. Note that biasing circuit for the op amp is not shown.

# Photodiode Equivalent Circuit



(a) A real photodiode has series and parallel resistances  $R_s$  and  $R_p$  and a SCL capacitance  $C_{dep}$ .  $A$  and  $C$  represent anode and cathode terminals. (b) The equivalent circuit of a photodiodes. For ac (or transient) signals, the battery can be shorted since ac signals will simply pass through the battery.

# Reverse Biased Photodiode Equivalent Circuit

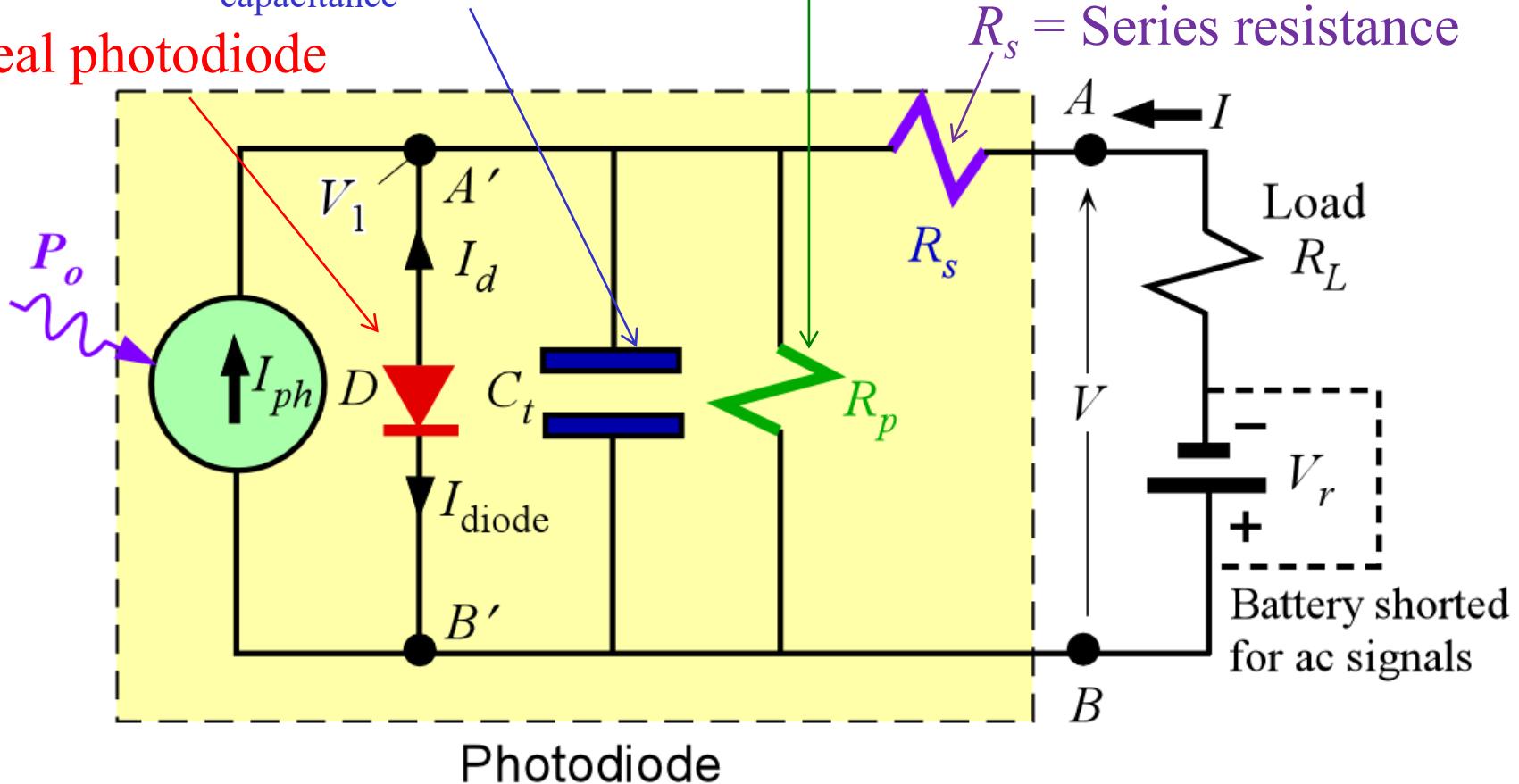


Total capacitance = ideal photodiode SCL capacitance + terminal capacitance

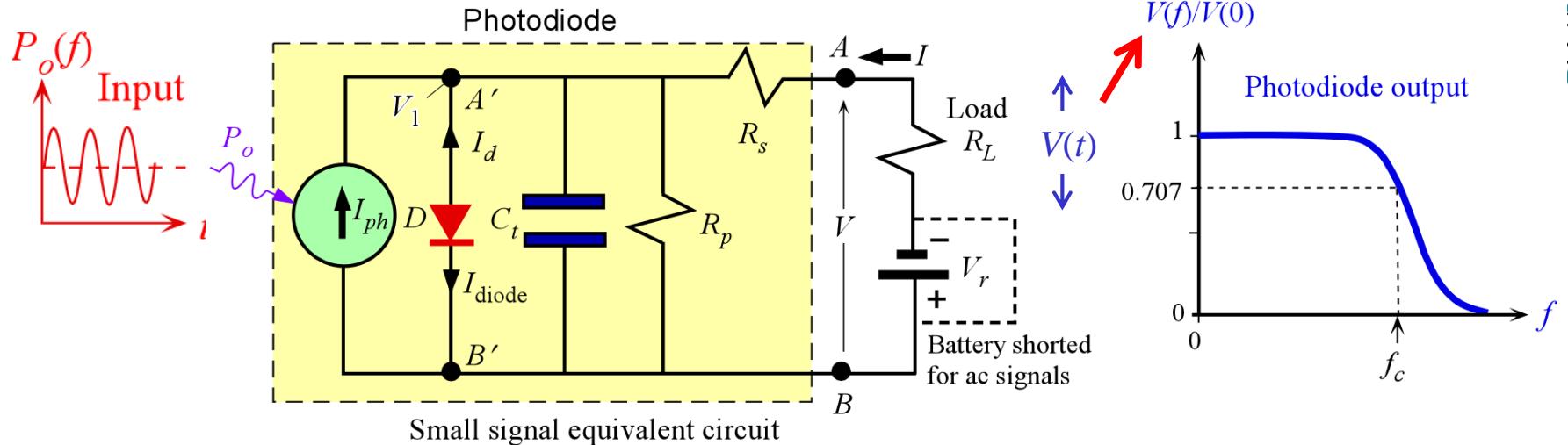
$R_p$  = Shunt (parallel) resistance

$R_s$  = Series resistance

Ideal photodiode



# Cutoff Frequency $f_c$



**The cutoff frequency or the bandwidth of the PD**

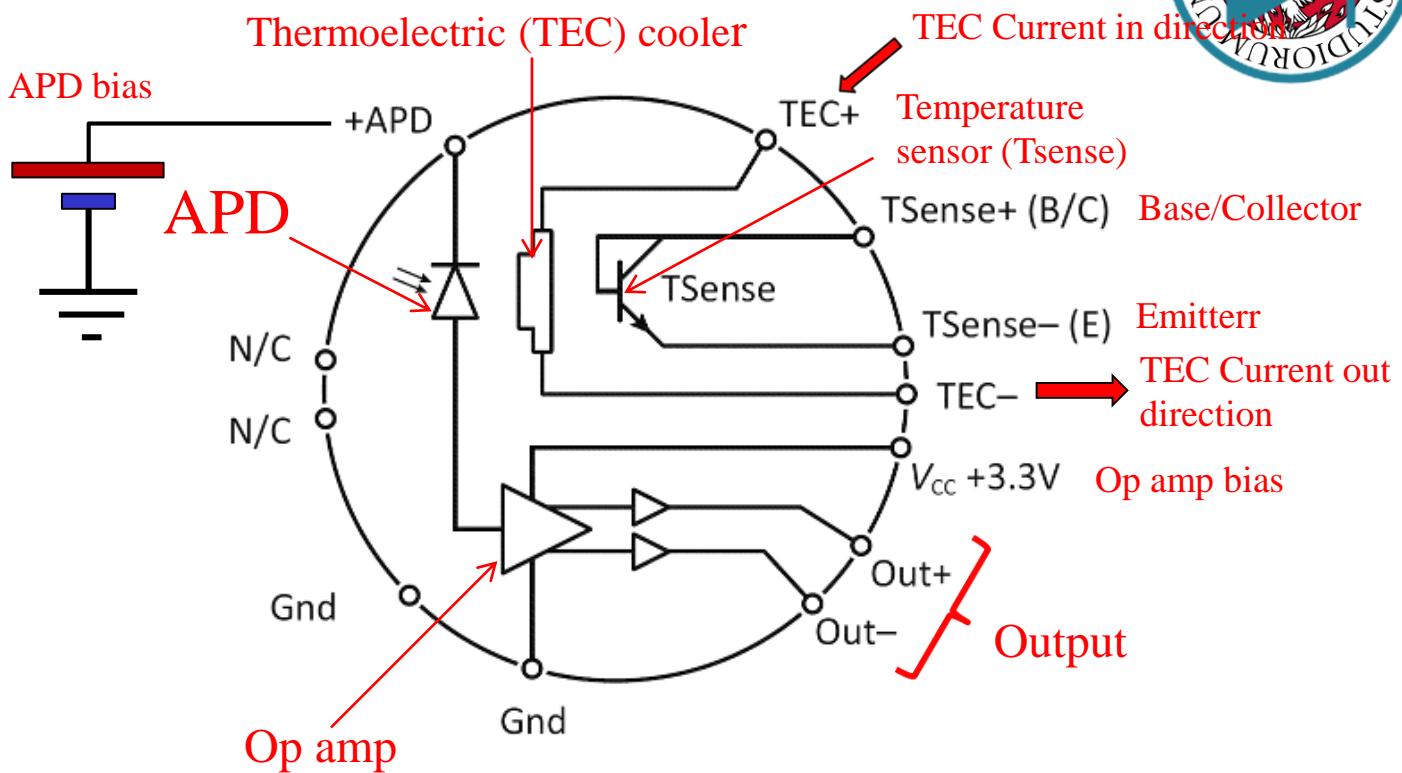
$$f_c = \frac{1}{2\pi R_{eq} C_t} \approx \frac{1}{2\pi(R_s + R_L)C_t} \approx \frac{1}{2\pi R_L C_t}$$

$R_{eq}$  is equivalent resistance and represents  $(R_s + R_L)$  in parallel with  $R_p$

## Assumption

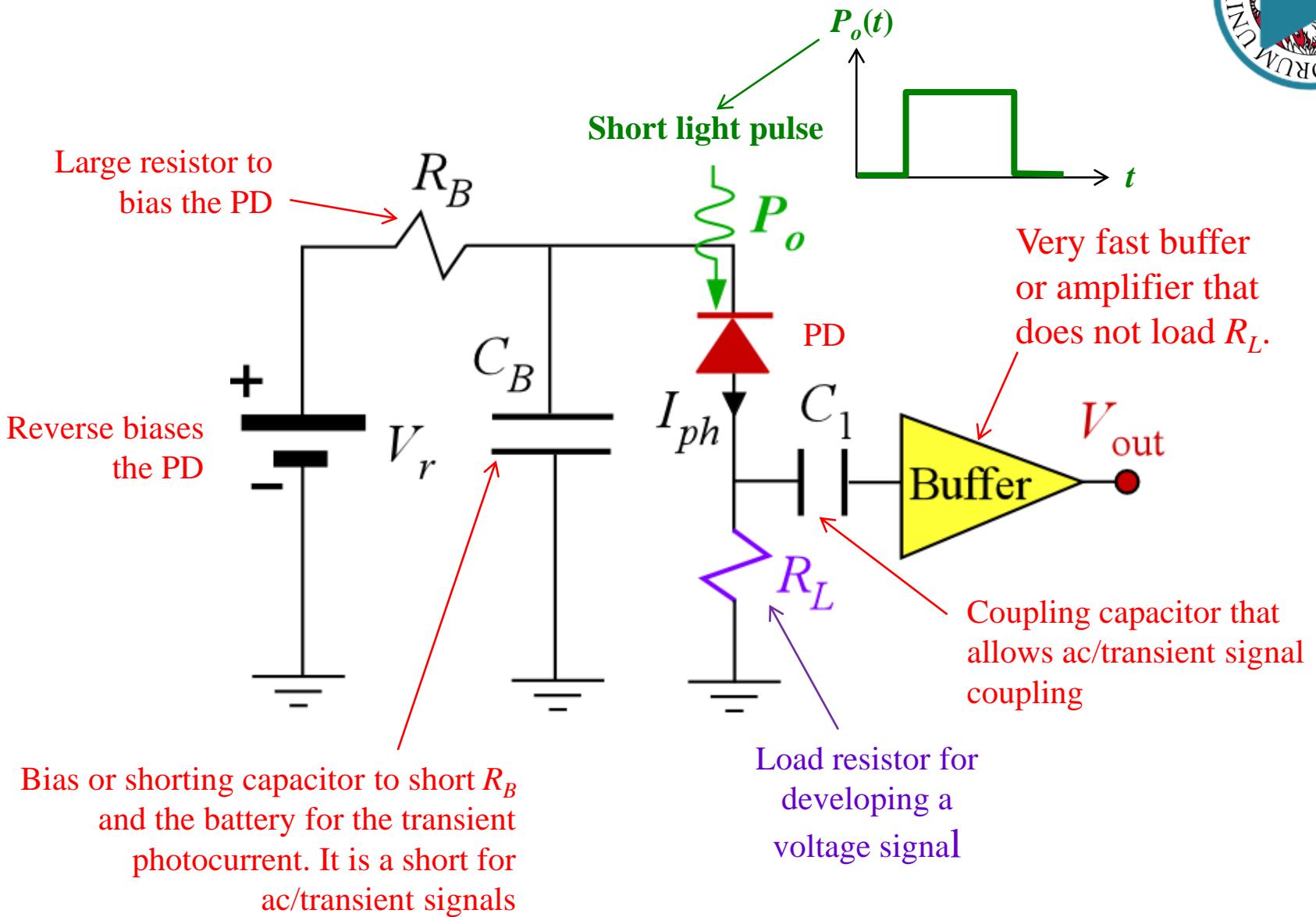
Drift time of carriers is much less than  $1/f_c$ .  
Response is not limited by drift and diffusion times of carriers within the device.

# A Commercial Photoreceiver



A photoreceiver that has an InGaAs APD and peripheral electronics (ICs) to achieve high gain and high sensitivity. There is also a thermoelectric cooler (TEC) and a temperature sensor (TSense). Courtesy of Voxel Inc ([www.voxtel-inc.com](http://www.voxtel-inc.com))

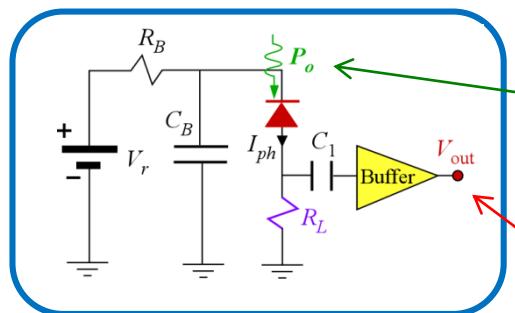
# Pulsed Excitation





# Pulsed Excitation

The Experiment



$P_o$

Input pulse

$t_{on}$

$t_{off}$

$V_{out}$

100%  
90%

10%

$t_{on}$

$t_{off}$

Output

Assume: The buffer is extremely fast and does not limit the response

$\tau_R$

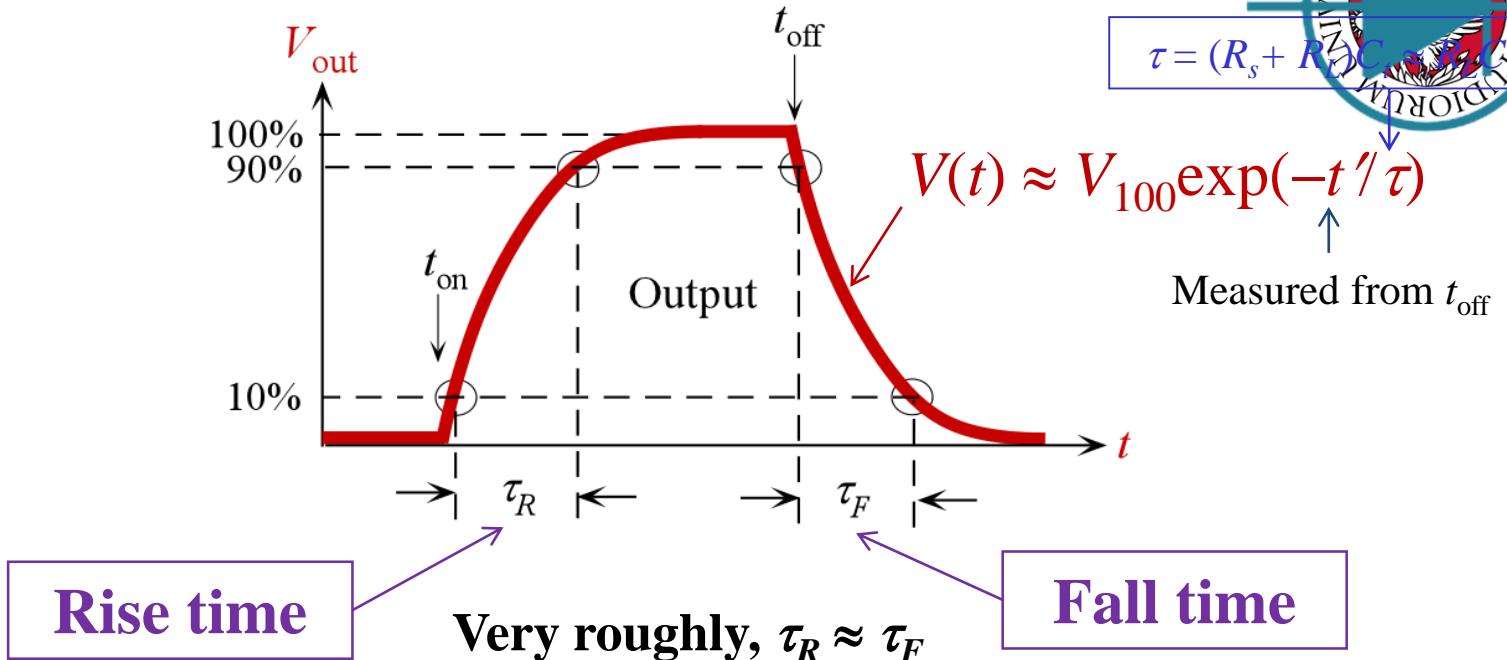
$\tau_F$

Rise time

Fall time

Are these related to  $f_c$ ?

# Rise and Fall Times, and Bandwidth



$$\tau_F = 2.2\tau$$

$$\tau = (R_s + R_L)C_t \approx R_L C_t$$

$$f_c \approx \frac{1}{2\pi R_L C_t} = \frac{1}{2\pi\tau} = \frac{0.35}{\tau_F} = \frac{350 \text{ MHz}}{\tau_F (\text{ns})}$$

# Pulsed Excitation

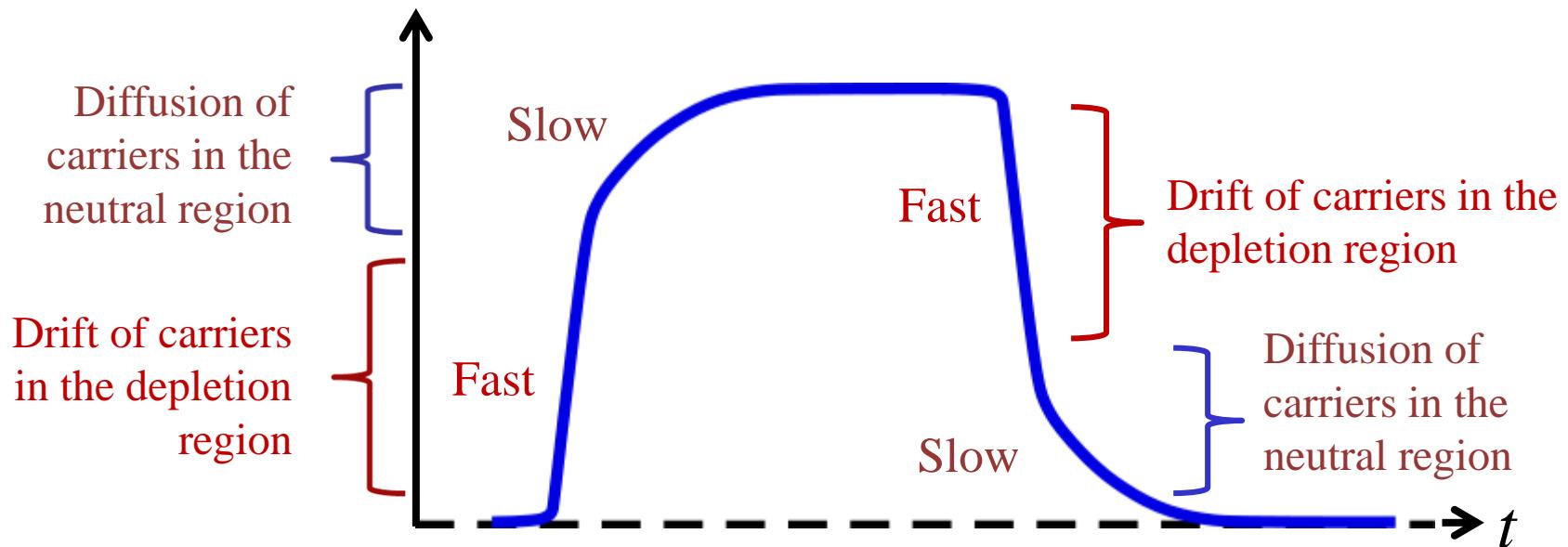
## Non- $R_L C_t$ response



Response due to the diffusion and drift of photogenerated carriers

Assume  $R_s + R_L$  is very small so that  $(R_s + R_L)C_t$  is negligible

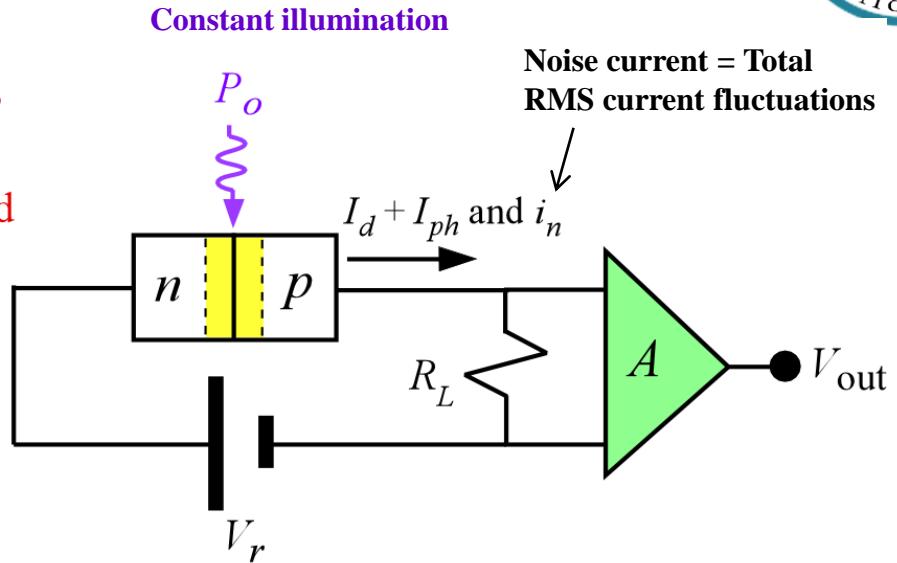
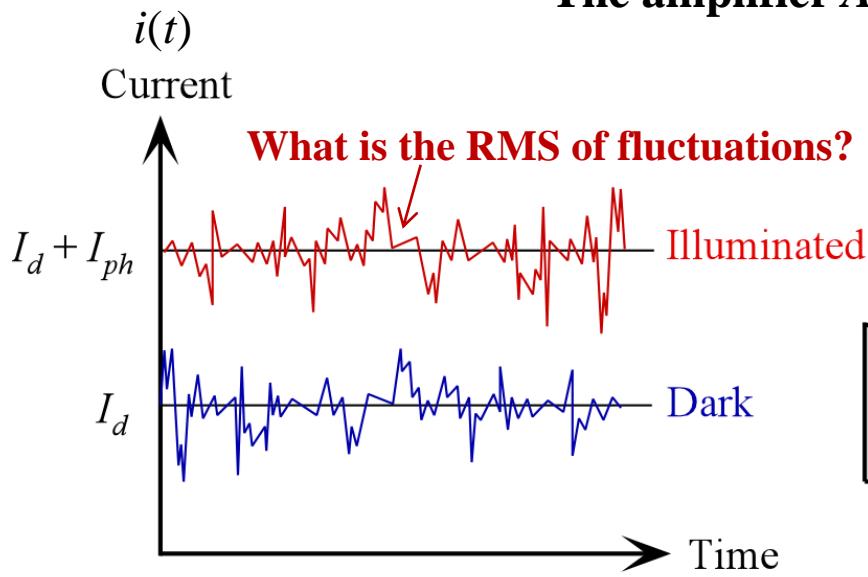
### Photocurrent



# Noise in Photodiodes



Consider a receiver with a photodiode and a sampling resistor  $R_L$   
The amplifier  $A$  is assumed noiseless



Consider **constant illumination  $P_o$**

Total current without noise = Dark current ( $I_d$ ) + Photocurrent ( $I_{ph}$ ) = “Constant”

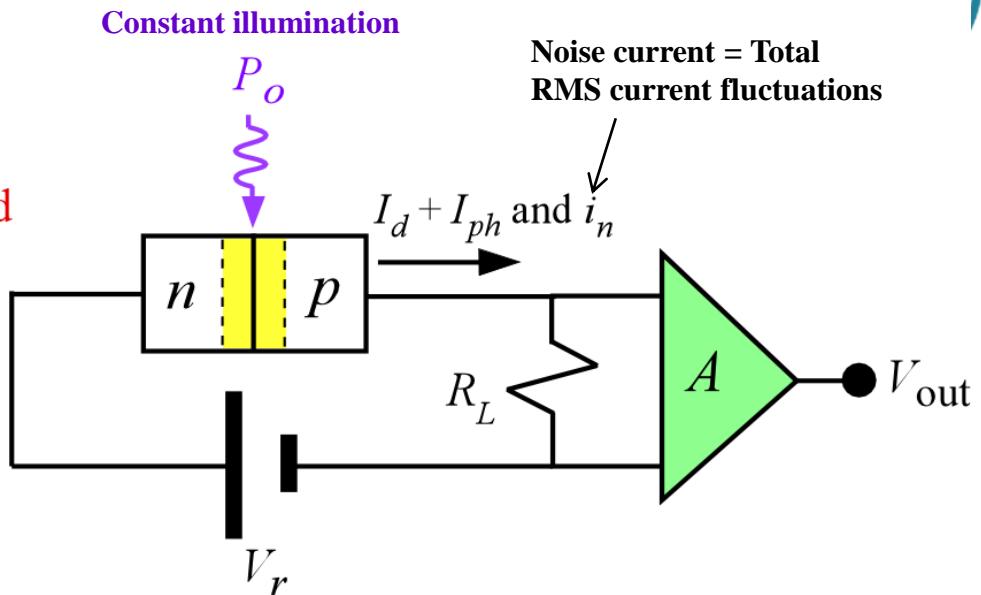
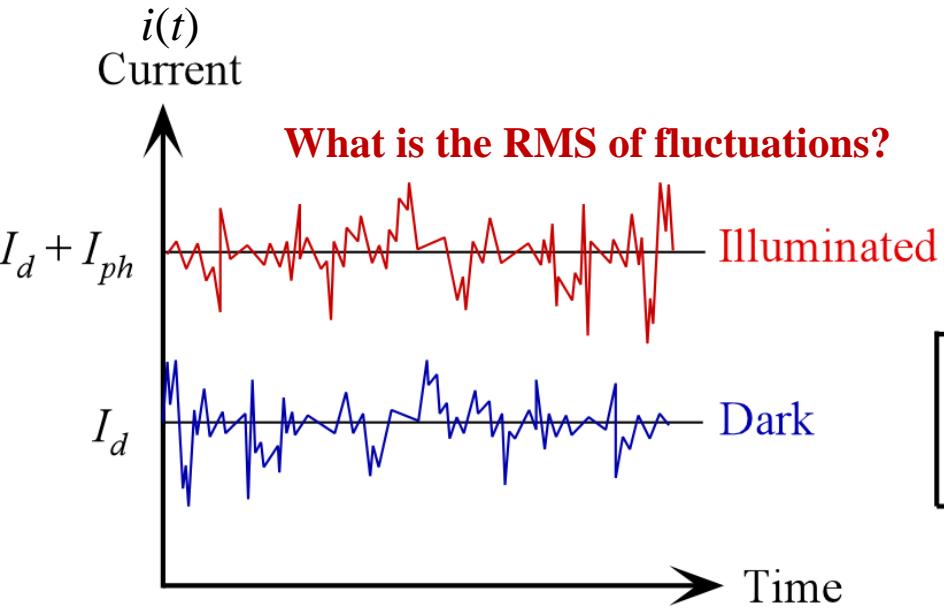
Observed Current = Dark current + Photocurrent **and Fluctuations (Noise)**

What is this “Noise” ?

We can represent the “noise current”  
by the **RMS of fluctuations**

$$\text{RMS of fluctuations} = \sqrt{i(t)^2}$$

# Noise in Photodiodes



The dark current has **shot noise** or fluctuations about  $I_d$ ,

$$i_{n\text{-dark}} = (2eI_d B)^{1/2}$$

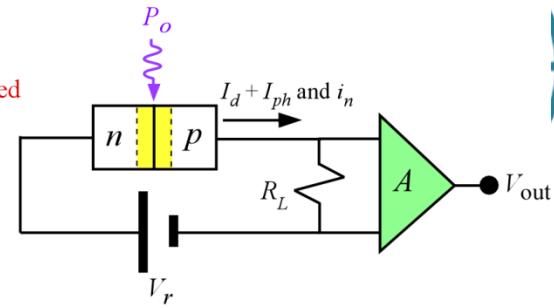
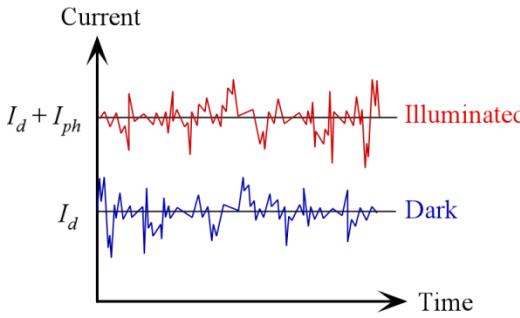
$B$  = Bandwidth

**Quantum noise** is due to the photon nature of light and its effects are the same as **shot noise**. Photocurrent has quantum noise or shot noise

$$i_{n\text{-quantum}} = (2eI_{ph} B)^{1/2}$$

# Noise in Photodiodes

Total shot noise current,  $i_n$



$$\dot{i}_n^2 = \dot{i}_{n-\text{dark}}^2 + \dot{i}_{n-\text{quantum}}^2$$

$$\dot{i}_n = [2e(I_d + I_{ph})B]^{1/2}$$

We can conceptually view the photodetector current as

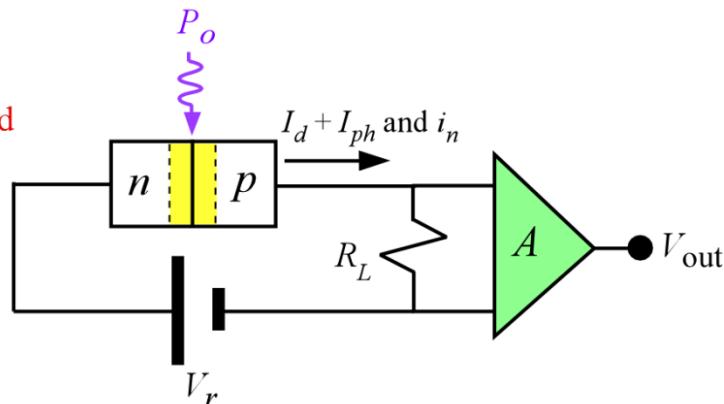
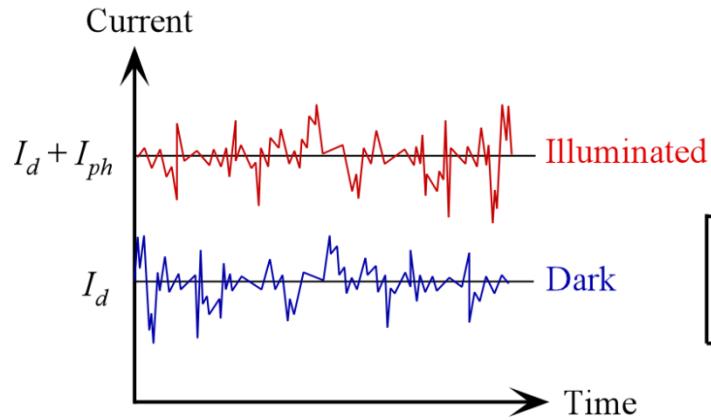
$$I_d + I_{ph} + \dot{i}_n$$

This flows through a load resistor  $R_L$  and voltage across  $R_L$  is amplified by  $A$  to give  $V_{out}$

The noise voltage (RMS) due to shot noise in PD =  $i_n R_L A$



# Noise in Photodiodes



Total current flowing into  $R_L$  has three components:

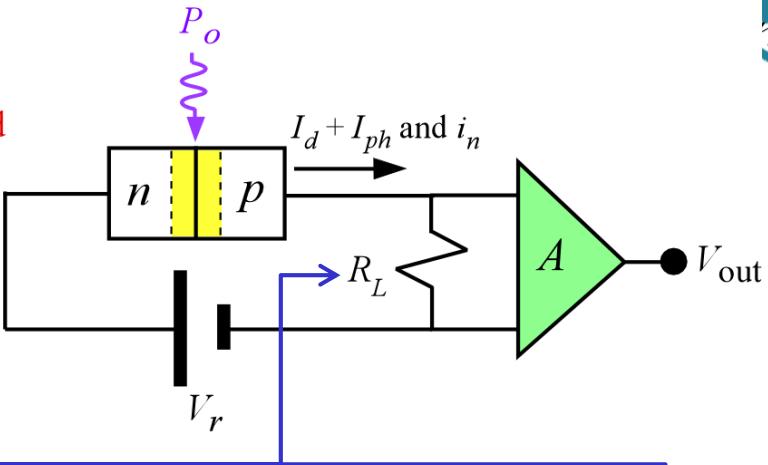
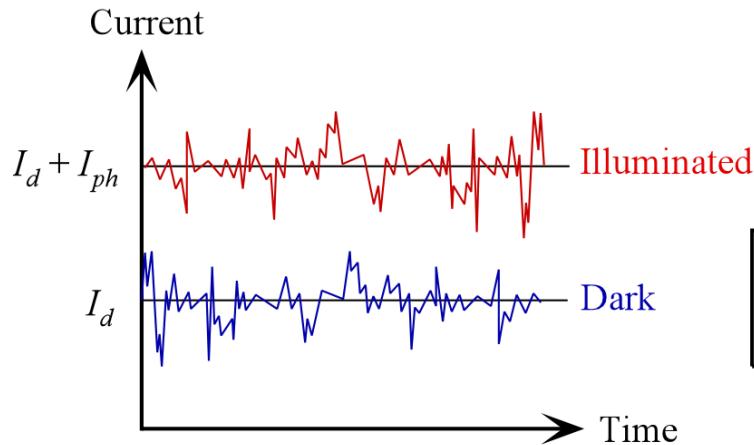
$I_d$  = Dark current. In principle, we can subtract this or block it with a capacitor if  $I_{ph}$  is an ac (transient) signal.

$I_{ph}$  = Photocurrent. This is the signal. We need this. It could be a steady or varying (ac or transient) signal.

$i_n$  = Total shot noise. Due to shot noise from  $I_d$  and  $I_{ph}$ . We cannot eliminate this.



# Noise in Photodiodes



The resistor  $R_L$  exhibits **thermal noise (Johnson noise)**

**Power in thermal fluctuations in  $R_L = 4k_B T B$**

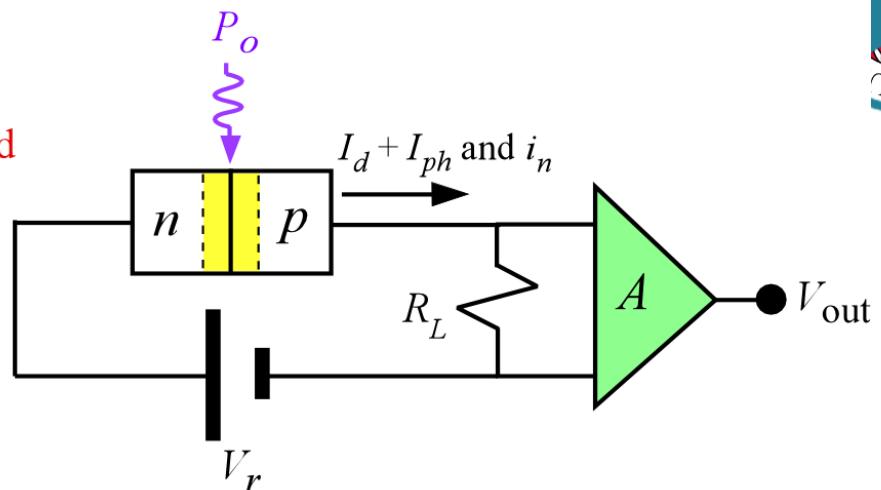
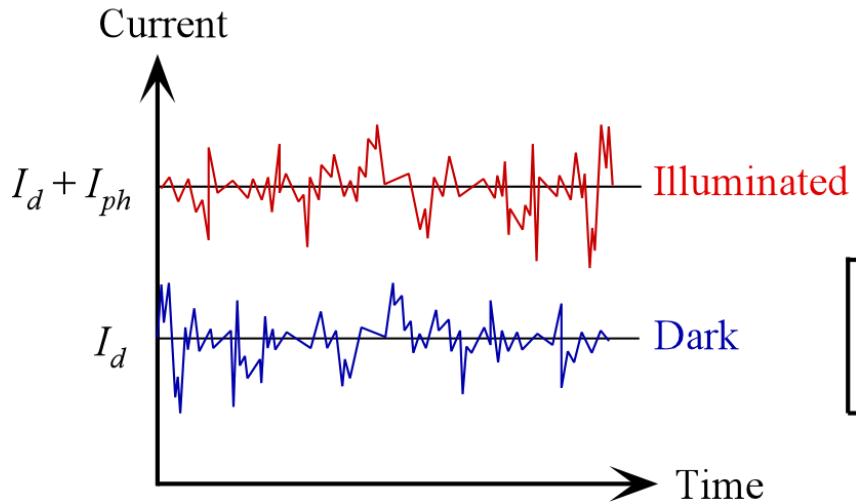
$$\sqrt{i^2}$$

$$\therefore R_L \overline{i^2} = 4k_B T B \quad i = \text{Current in } R_L$$

$$i_{\text{th}} = \text{Thermal noise current from } R_L = \left[ \frac{4k_B T B}{R_L} \right]^{1/2}$$



# Summary of Noise in PD and $R_L$



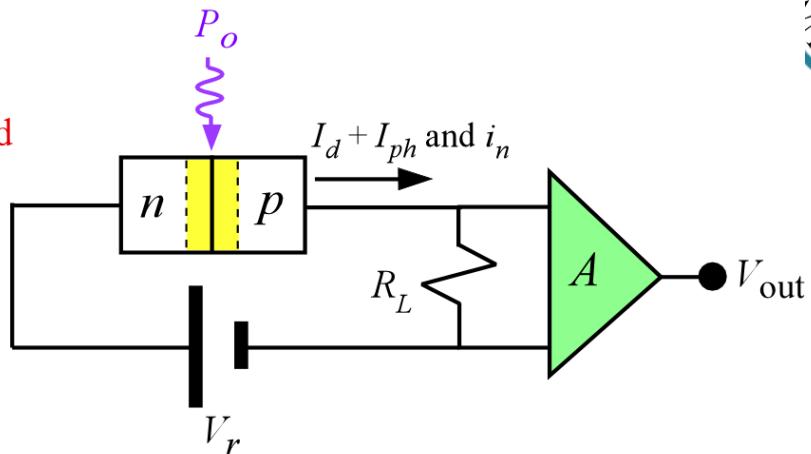
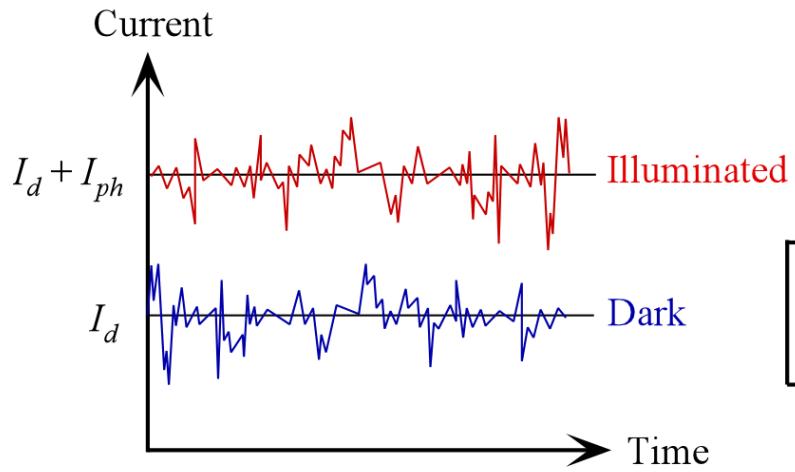
$$\text{Power in shot noise in PD} = i_n^2 R_L = [2e(I_d + I_{ph})B]R_L$$

$$\text{Power in thermal fluctuations in } R_L = 4k_B T B$$

Important Note: Total noise is always found by first summing the average powers involved in individual fluctuations *e.g. power in shot noise + power in thermal noise*

**Noise in the amplifier  $A$  must also be included**  
**See advanced textbooks**

# Signal to Noise Ratio

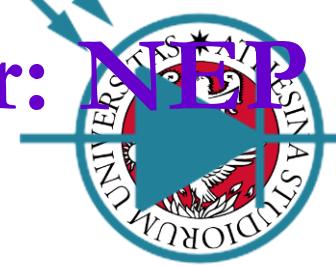


$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

$$\text{SNR} = \frac{I_{ph}^2 R_L}{i_n^2 R_L + 4k_B T B} = \frac{I_{ph}^2}{[2e(I_d + I_{ph})B] + \frac{4k_B T B}{R_L}}$$

Important Note: Total noise is always found by first summing the average powers involved in individual fluctuations *e.g. power in shot noise + power in thermal noise*

# Noise Equivalent Power: NEP



## Definition

$$\text{NEP} = \frac{\text{Input power for SNR} = 1}{\sqrt{\text{Bandwidth}}} = \frac{P_1}{B^{1/2}}$$

**NEP is defined as the required optical input power to achieve a SNR of 1 within a bandwidth of 1 Hz**

$$\text{NEP} = \frac{P_1}{B^{1/2}} = \frac{1}{R} [2e(I_d + I_{ph})]^{1/2}$$

Units for NEP are  $\text{W Hz}^{-1/2}$



# Detectivity, $D$

## Definition

$$\text{Detectivity} = \frac{1}{\text{NEP}}$$

## Specific detectivity $D^*$

$$D^* = \frac{A^{1/2}}{\text{NEP}}$$

Units for  $D^*$  are  $\text{cm Hz}^{-1/2} \text{ W}^{-1}$ , or Jones

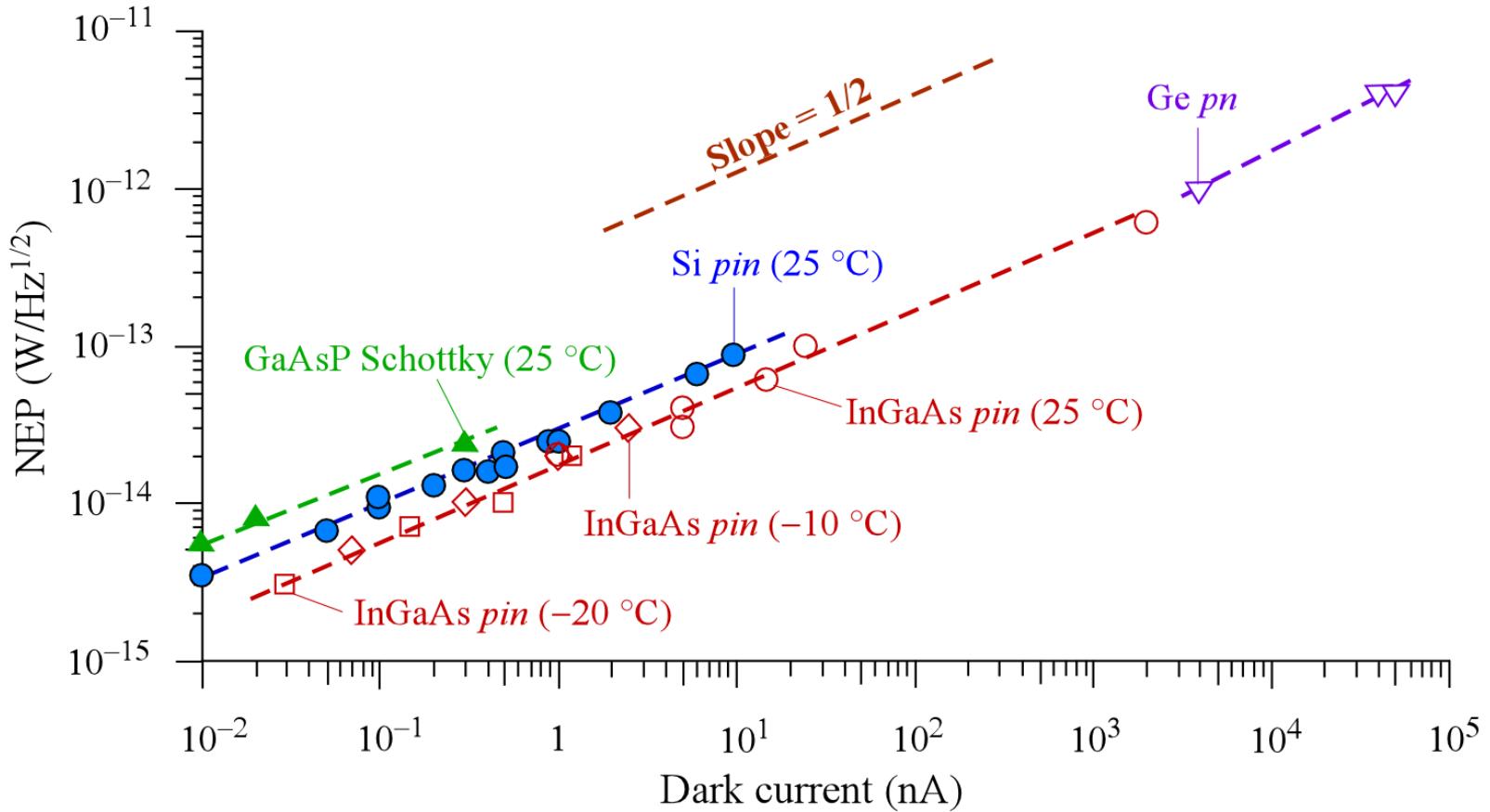
# NEP and Detectivity of Photodetectors

Typical noise characteristics of a few selected commercial photodetectors. PC means a photoconductive detector, whose photoconductivity is used to detect light. For PC detectors, what is important is the dark resistance  $R_d$ , which depends on the temperature.

Photodiode	GaP Schottky	Si <i>pin</i>	Ge <i>pin</i>	InGaAs <i>pin</i>	PbS (PC) -10°C	PbSe (PC) -10 °C	InSb (PC) -10°C
$\lambda_{\text{peak}} \text{ (μm)}$	<b>0.44</b>	<b>0.96</b>	<b>1.5</b>	<b>1.55</b>	<b>2.4</b>	<b>4.1</b>	<b>5.5</b>
$I_d$ or $R_d$	<b>10 pA</b>	<b>0.4 nA</b>	<b>3 μA</b>	<b>5 nA</b>	<b>0.1–1 MΩ</b>	<b>0.1–1 MΩ</b>	<b>1–10 kΩ</b>
NEP W Hz <sup>-1/2</sup>	<b><math>5.4 \times 10^{-15}</math></b>	<b><math>1.6 \times 10^{-14}</math></b>	<b><math>1 \times 10^{-12}</math></b>	<b><math>4 \times 10^{-14}</math></b>	-	-	
$D^* \text{ cm Hz}^{1/2}/ \text{W}$	<b><math>1 \times 10^{13}</math></b>	<b><math>1 \times 10^{12}</math></b>	<b><math>1 \times 10^{11}</math></b>	<b><math>5 \times 10^{12}</math></b>	<b><math>1 \times 10^9</math></b>	<b><math>5 \times 10^9</math></b>	<b><math>1 \times 10^9</math></b>

$$\text{NEP} = \frac{P_1}{B^{1/2}} = \frac{1}{R} [2e(I_d + I_{ph})]^{1/2}$$

# NEP and Dark Current



The dependence of NEP ( $\text{W Hz}^{1/2}$ ) on the photodetector dark current  $I_d$  for Si and InGaAs *pin*, Ge *pn* junction, and GaP Schottky photodiodes. Dashed lines indicate observed trends. Filled circle, Si *pin*; open circle, InGaAs *pin* at 25 °C, open diamond at -10 °C, open square, -20 °C; inverted triangle, Ge *pn*; triangle, GaAsP Schottky. (Data extracted from datasheets of 35 commercial photodiodes)

# Noise in Avalanche Photodiode (APD)



Ideally the shot noise is simply multiplied so that we should expect

$$i_{n\text{-APD}} = Mi_n = M[2e(I_{do} + I_{pho})B]^{1/2}$$

$$i_{n\text{-APD}} = [2e(I_{do} + I_{pho})M^2B]^{1/2}$$

But, we observe excess noise above this shot noise

## Avalanche Noise

$$i_{n\text{-APD}} = [2e(I_{do} + I_{pho})M^2FB]^{1/2}$$

Excess Noise Factor

# Noise in Avalanche Photodiode (APD)



APDs exhibit excess avalanche noise due to the randomness of the impact ionization process in the multiplication region.

Some carriers travel far and some short distances within this zone before they cause impact ionization

## Excess Avalanche Noise Factor $F$

$$i_{n\text{-APD}} = [2e(I_{do} + I_{pho})M^2FB]^{1/2}$$

### Excess Noise Factor

$F \approx M^x$  where  $x$  is an index that depends on the semiconductor, the APD structure and the type of carrier that initiates the avalanche (electron or hole)

For Si APDs,  $x$  is 0.3–0.5 whereas for Ge and III-V (such as InGaAs) alloys it is 0.7–1

# EXAMPLE: Noise of an ideal photodetector

Consider an ideal photodiode with  $\eta_e = 1$  (QE = 100%) and no dark current,  $I_d = 0$ . Show that the minimum optical power required for a signal to noise ratio (SNR) of 1 is

$$P_1 = \frac{2hc}{\lambda} B \quad (5.12.9)$$

Calculate the minimum optical power for a SNR = 1 for an ideal photodetector operating at 1300 nm with a bandwidth of 1 GHz? What is the corresponding photocurrent?

## Solution

We need the incident optical power  $P_1$  that makes the photocurrent  $I_{ph}$  equal to the noise current  $i_n$ , so that  $\text{SNR} = 1$ . The photocurrent (signal) is equal to the noise current when

$$I_{ph} = i_n = [2e(I_d + I_{ph})B]^{1/2} = [2eI_{ph}B]^{1/2}$$

since  $I_d = 0$ . Solving the above,  $I_{ph} = 2eB$

From Eqs. (5.4.3) and (5.4.4), the photocurrent  $I_{ph}$  and the incident optical power  $P_1$  are related by

$$I_{ph} = \frac{\eta_e e P_1 \lambda}{hc} = 2eB$$

Thus,  $P_1 = \frac{2hc}{\eta_e \lambda} B$

# EXAMPLE: Noise of an ideal photodetector

## Solution (continued)



For an ideal photodetector,  $\eta_e = 1$  which leads to Eq. (5.12.9). We note that for a bandwidth of 1Hz, NEP is numerically equal to  $P_1$  or  $\text{NEP} = 2hc/\lambda$ .

For an ideal photodetector operating at 1.3  $\mu\text{m}$  and at 1 GHz,

$$\begin{aligned} P_1 &= 2hcB/\eta_e\lambda \\ &= 2(6.63 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})(10^9 \text{ Hz}) / (1)(1.3 \times 10^{-6} \text{ m}) \\ &= 3.1 \times 10^{-10} \text{ W or } \mathbf{0.31 \text{ nW}.} \end{aligned}$$

This is the minimum signal for a  $\text{SNR} = 1$ . The noise current is due to quantum noise. The corresponding photocurrent is

$$I_{ph} = 2eB = 2(1.6 \times 10^{-19} \text{ C})(10^9 \text{ Hz}) = 3.2 \times 10^{-10} \text{ A or } \mathbf{0.32 \text{ nA}.}$$

Alternatively we can calculate  $I_{ph}$  from  $I_{ph} = \eta_e e P_1 \lambda / hc$  with  $\eta_e = 1$ .



## EXAMPLE: NEP of a Si *pin* photodiode

A Si *pin* photodiode has a quoted NEP of  $1 \times 10^{-13} \text{ W Hz}^{-1/2}$ . What is the optical signal power it needs for a signal to noise ratio (SNR) of 1 if the bandwidth of operation is 1GHz?

### Solution

By definition, NEP is that optical power per square root of bandwidth which generates a photocurrent equal to the noise current in the detector.

$$\mathbf{NEP = P_1/B^{1/2}}$$

Thus,

$$\begin{aligned} P_1 &= \mathbf{NEPB^{1/2}} \\ &= (10^{-13} \text{ W Hz}^{-1/2})(10^9 \text{ Hz})^{1/2} \\ &= 3.16 \times 10^{-9} \text{ W or } \mathbf{3.16 \text{ nW}} \end{aligned}$$

# EXAMPLE: SNR of a receiver

Consider an InGaAs *pin* photodiode used in a receiver circuit as in Figure 5.31 with a load resistor of  $10\text{ k}\Omega$ . The photodiode has a dark current of  $2\text{ nA}$ . The bandwidth of the photodiode and the amplifier together is  $1\text{ MHz}$ . Assuming that the amplifier is noiseless, calculate the SNR when the incident optical power generates a mean photocurrent of  $5\text{ nA}$  (corresponding to an incident optical power of about  $6\text{ nW}$  since  $R$  is about  $0.8\text{--}0.9\text{ nA/nW}$  at the peak wavelength of  $1550\text{ nm}$ ).

## Solution

The noise generated comes from the photodetector as shot noise and from  $R_L$  as thermal noise. The mean thermal noise power in the load resistor  $R_L$  is  $4k_B TB$ . If  $I_{ph}$  is the photocurrent and  $i_n$  is the shot noise in the photodetector then

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{I_{ph}^2 R_L}{i_n^2 R_L + 4k_B TB} = \frac{I_{ph}^2}{\left[ 2e(I_d + I_{ph})B \right] + 4k_B TB / R_L}$$

The term  $4k_B TB/R_L$  in the denominator represents the mean square of the thermal noise current in the resistor. We can evaluate the magnitude of each noise current by substituting,  $I_{ph} = 5\text{ nA}$ ,  $I_d = 2\text{ nA}$ ,  $B = 1\text{ MHz}$ ,  $R_L = 10^4\text{ }\Omega$ ,  $T = 300\text{ K}$ .

# EXAMPLE: SNR of a receiver

## Solution (continued)



Shot noise current from the detector =  $[2e(I_d + I_{ph})B]^{1/2} = 0.047 \text{ nA}$

$$= 1.29 \text{ nA}$$

Thus, the noise contribution from  $R_L$  is greater than that from the photodiode. The SNR is

$$\text{SNR} = \frac{(5 \times 10^{-9} \text{ A})^2}{(0.047 \times 10^{-9} \text{ A})^2 + (1.29 \times 10^{-9} \text{ A})^2} = 15.0$$

Generally SNR is quoted in decibels. We need  $10\log(\text{SNR})$ , or  $10\log(15.0)$  i.e., 11.8 dB. Clearly, **the load resistance has a dramatic effect on the overall noise performance.**

# EXAMPLE: Noise in an APD

Consider an InGaAs APD with  $x \approx 0.7$  which is biased to operate at  $M = 10$ . The unmultiplied dark current is 10 nA and bandwidth is 700 MHz.

- (a) What is the APD noise current per square root of bandwidth?
- (b) What is the APD noise current for a bandwidth of 700 MHz?
- (c) If the responsivity (at  $M = 1$ ) is  $0.8 \text{ A W}^{-1}$  what is the minimum optical power for a SNR of 10?

## Solution

- (a) In the absence of any photocurrent, the noise in the APD comes from the dark current. If the unmultiplied dark current is  $I_{do}$  then the noise current (rms) is

$$i_{n\text{-dark}} = [2eI_{do}M^{2+x}B]^{1/2}$$

Thus,

$$\begin{aligned}\frac{i_{n\text{-dark}}}{\sqrt{B}} &= \sqrt{2eI_{do}M^{2+x}} = \sqrt{2(1.6 \times 10^{-19} \text{ C})(10 \times 10^{-9} \text{ A})(10)^{2+0.7}} \\ &= 1.27 \times 10^{-12} \text{ A Hz}^{-1/2} \text{ or } \mathbf{1.27 \text{ pA Hz}^{-1/2}.}\end{aligned}$$

- (b) In a bandwidth  $B$  of 700 MHz, the noise current is

$$\begin{aligned}i_{n\text{-dark}} &= (700 \times 10^6 \text{ Hz})^{1/2}(1.27 \text{ pA Hz}^{-1/2}) \\ &= 3.35 \times 10^{-8} \text{ A or } \mathbf{33.5 \text{ nA}.}\end{aligned}$$

# EXAMPLE: Noise in an APD

## Solution (continued)



- (c) The SNR with a primary photocurrent  $I_{pho}$  in the APD is

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{M^2 I_{pho}^2}{[2e(I_{do} + I_{pho})M^{2+x}B]}$$

Rearranging to obtain  $I_{pho}$  we get,

$$(M^2)I_{pho}^2 - [2eM^{2+x}B(\text{SNR})]I_{pho} - [2eM^{2+x}B(\text{SNR})I_{do}] = 0$$

This is a quadratic equation in  $I_{pho}$  with defined coefficients since  $M$ ,  $x$ ,  $B$ ,  $I_{do}$  and SNR are given. Solving this quadratic with a  $\text{SNR} = 10$  for  $I_{pho}$  we find

$$I_{pho} \approx 1.76 \times 10^{-8} \text{ A or } 17.6 \text{ nA}$$

While it may seem odd that  $I_{pho}$  is less than the dark noise current (33.5 nA) itself, the actual photocurrent  $I_{ph}$  however is 176 nA, since it is multiplied by  $M$ . Further the total noise current,  $i_{n\text{-APD}} = [2e(I_{do} + I_{pho})M^{2+x}B]^{1/2}$  is 55.7 nA so that one can easily check that  $\text{SNR} = I_{ph}^2 / i_{n\text{-APD}}^2$  is indeed 10.

By the definition of responsivity,  $R = I_{pho}/P_o$ , we find,

$$P_o = I_{pho}/R = (1.76 \times 10^{-8} \text{ A})/(0.8 \text{ A W}^{-1}) = 2.2 \times 10^{-8} \text{ W or } 22 \text{ nW}$$

# CCD Image Sensor

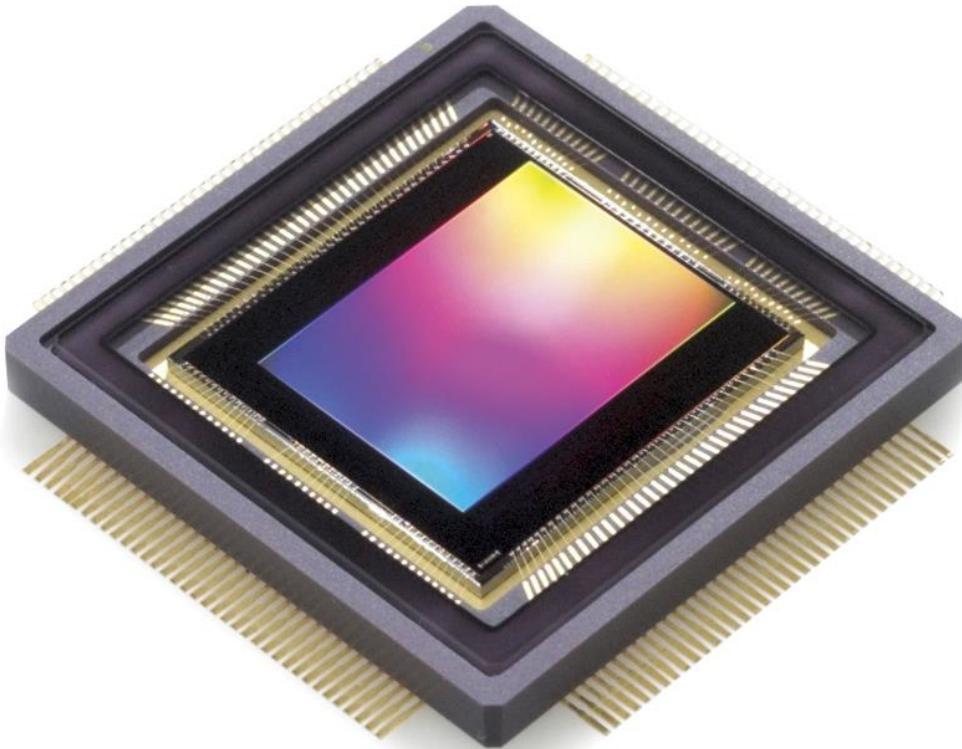


The inventors of the CCD (charge coupled device) image sensor at AT&T Bell Labs: Willard Boyle (left) and George Smith (right). The CCD was invented in 1969, the first CCD solid state camera was demonstrated in 1970, and a broadcast quality TV camera by 1975. (W. S. Boyle and G. E. Smith,

"Charge Coupled Semiconductor Devices", *Bell Systems Technical Journal*, 49, 587, 1970. (Courtesy of Alcatel-Lucent Bell Labs.)

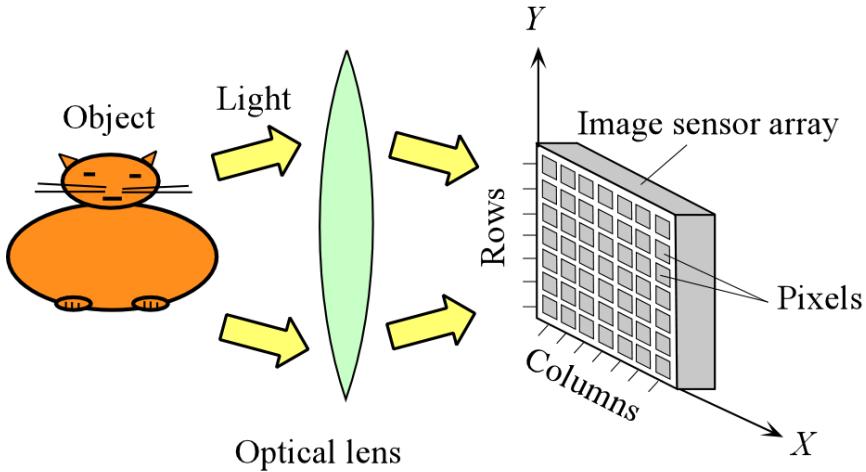
A CCD image sensor. The FTF6040C is a full-frame color CCD image sensor designed for professional digital photography, scientific and industrial applications with 24 megapixels and a wide dynamic range. Chip imaging area is  $36 \times 24 \text{ mm}^2$ , and pixel size is  $6 \mu\text{m} \times 6 \mu\text{m}$ . (Courtesy of Teledyne-DALSA)

# CMOS Image Sensor

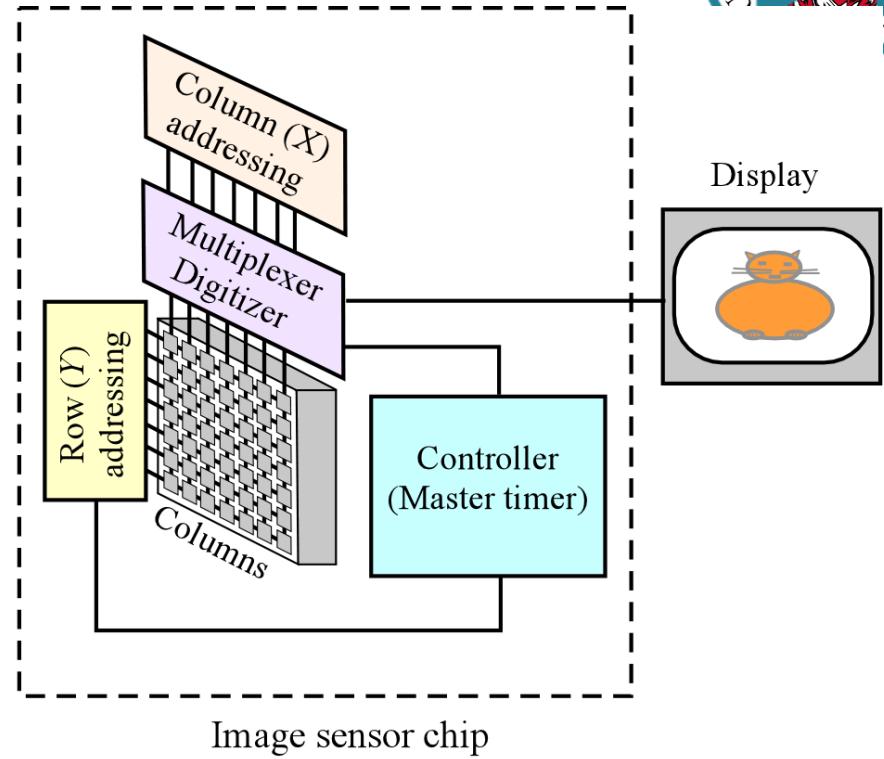


4 Megapixel CMOS image sensor  
(Courtesy of Teledyne-DALSA)

# Image Sensors



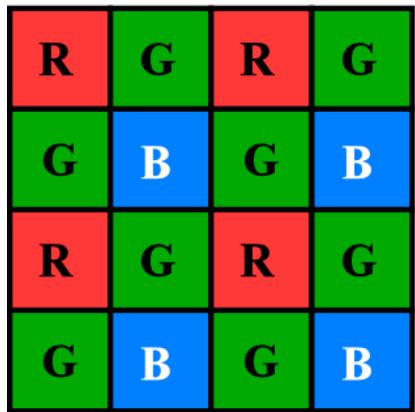
(a)



(b)

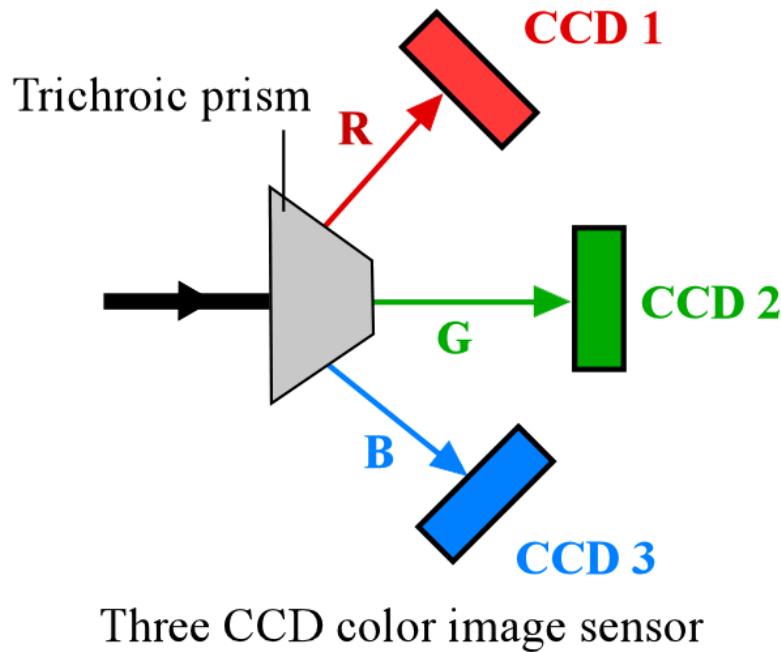
**(a) The basic image sensing operation using an array of photosensitive pixels. (b) The image sensor chip that incorporates the auxiliary electronics that run the sensor array (CMOS technology)**

# Image Sensors



Bayer filtering

(a)

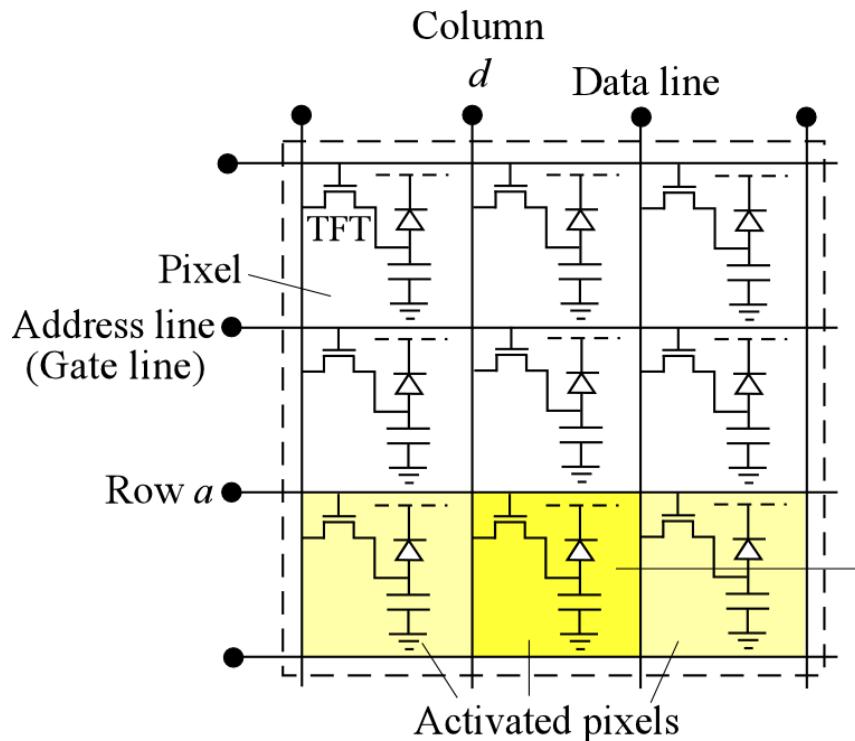


Three CCD color image sensor

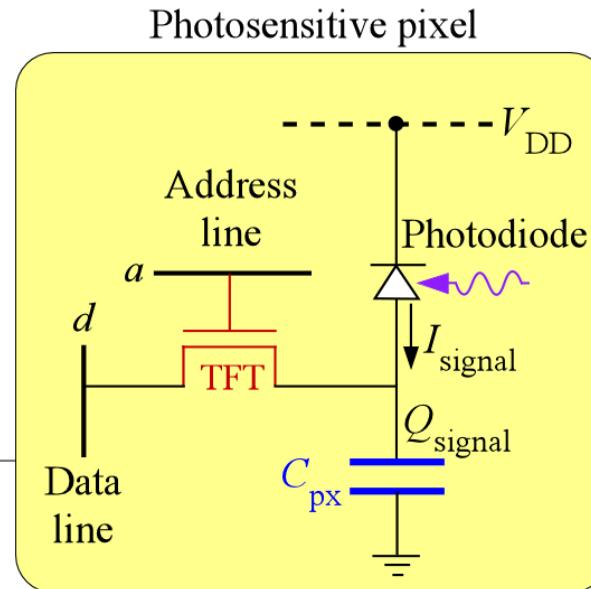
(b)

Color imaging by using two different techniques. Bayer filtering uses red (R), green (G) and blue (B) filters on three pixels for capturing the R, G and B information. 3CCD uses a trichroic prism to separate the image colors into red, green and blue and uses three CCD chips for each.

# Active Matrix Readout



(a)

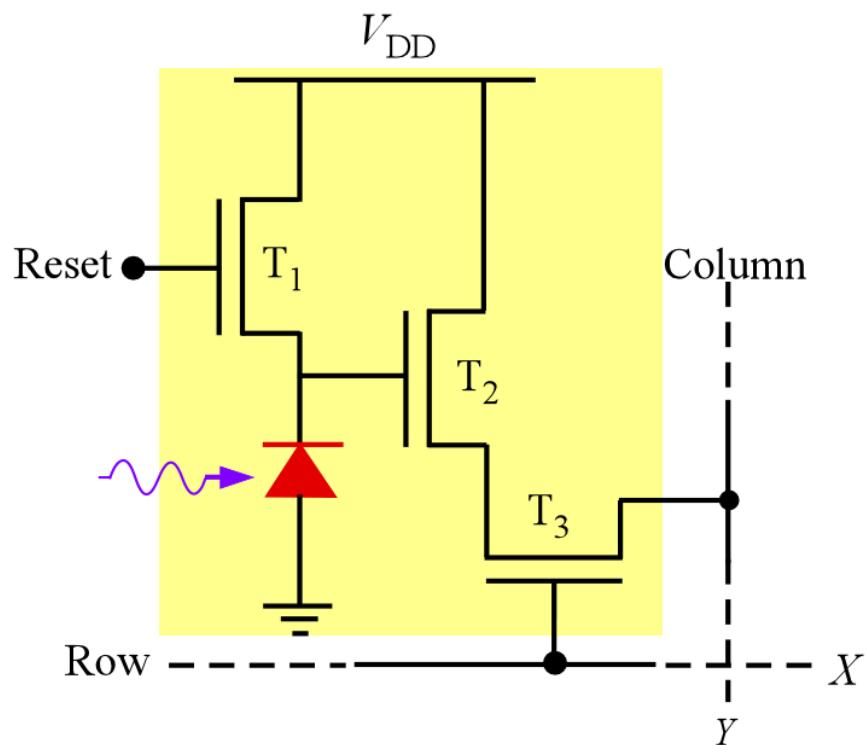


(b)

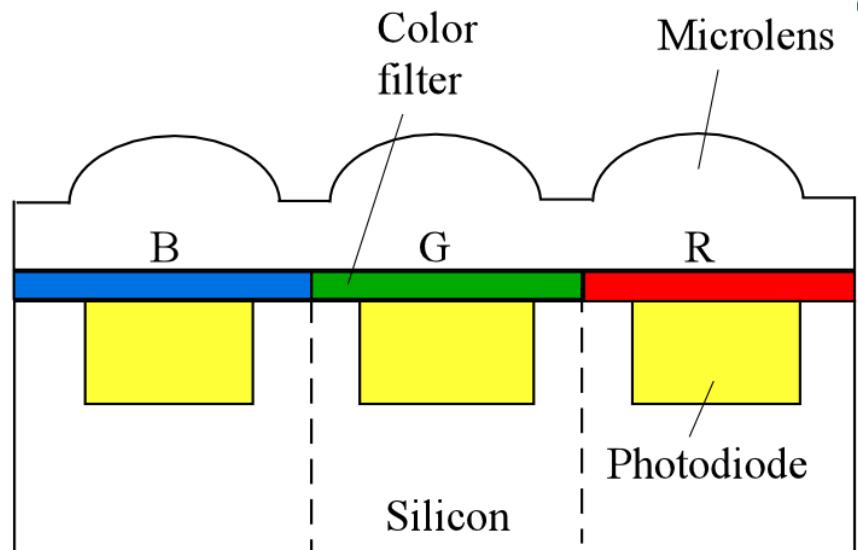
(a) An active matrix array (AMA)

(b) A basic photosensitive pixel structure for detecting the photons arriving at the pixel defined by row  $a$  and column  $d$ .

# CMOS Image Sensor

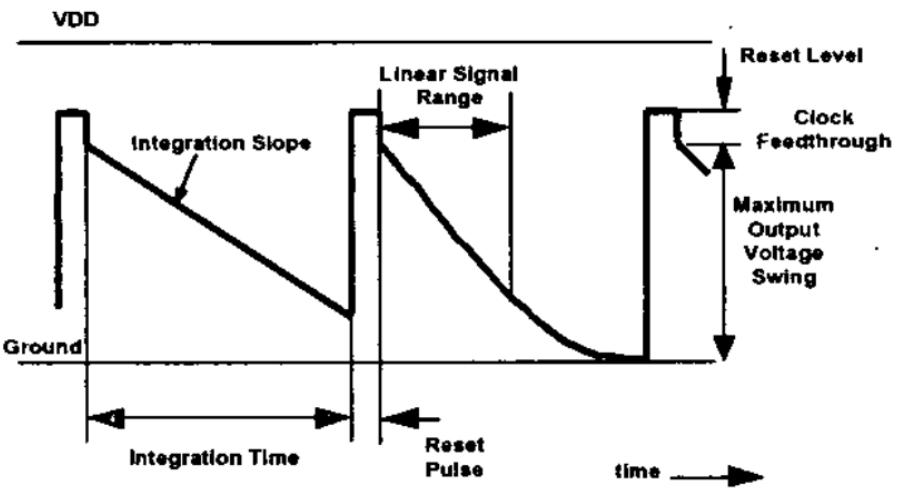
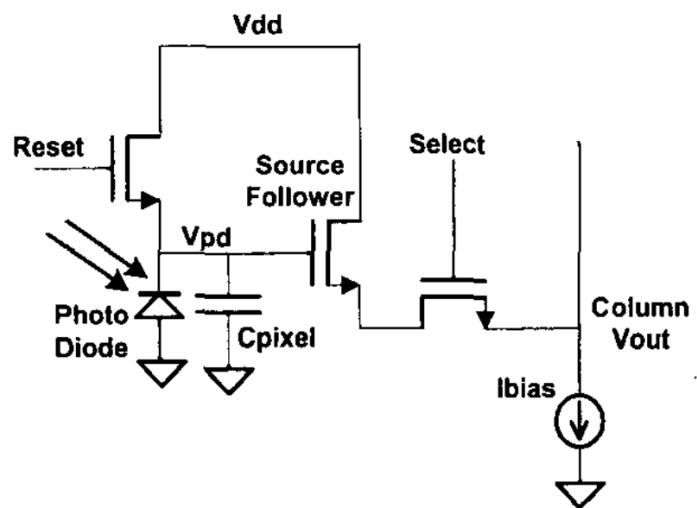


(a)



(b)

(a) The pixel architecture in a CMOS image sensor. (b) A cross section of a CMOS imager with microlenses and color filters (B = blue, G = green, R = red) for color imaging

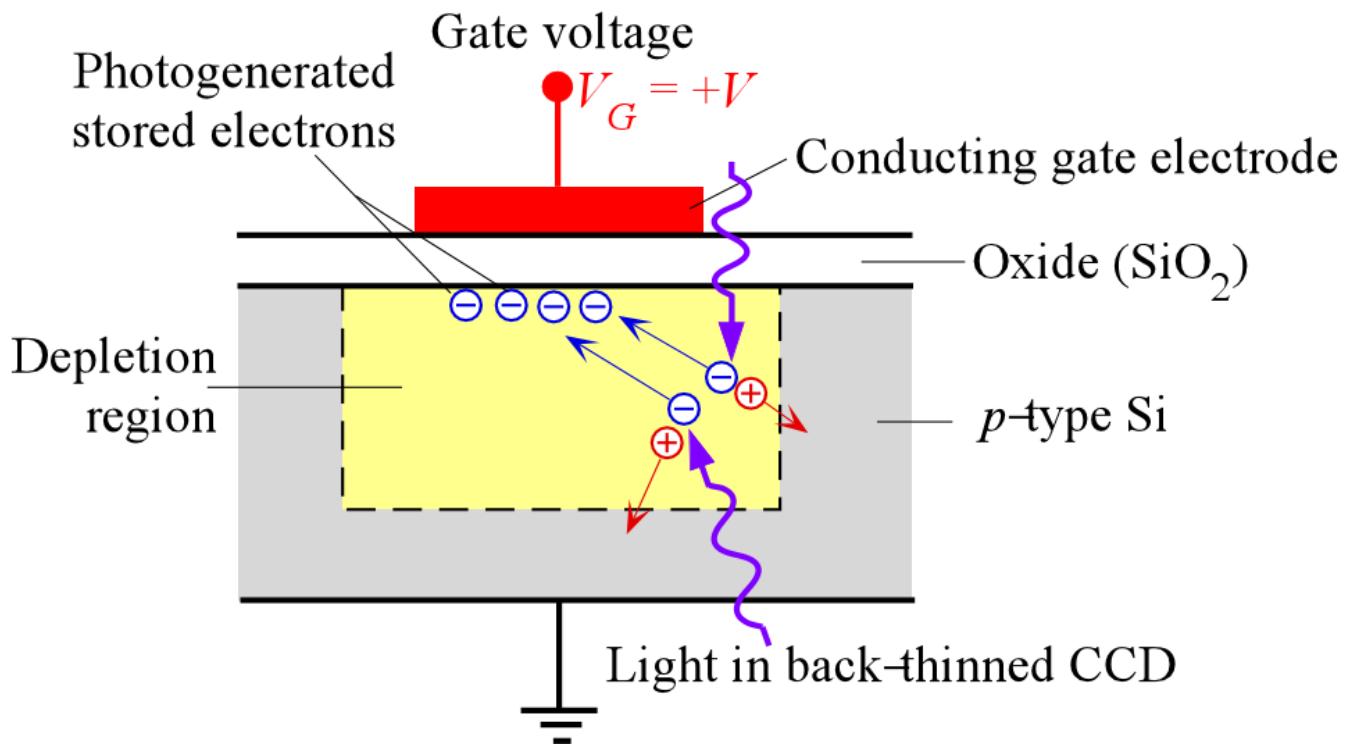


# CCD Image Sensor



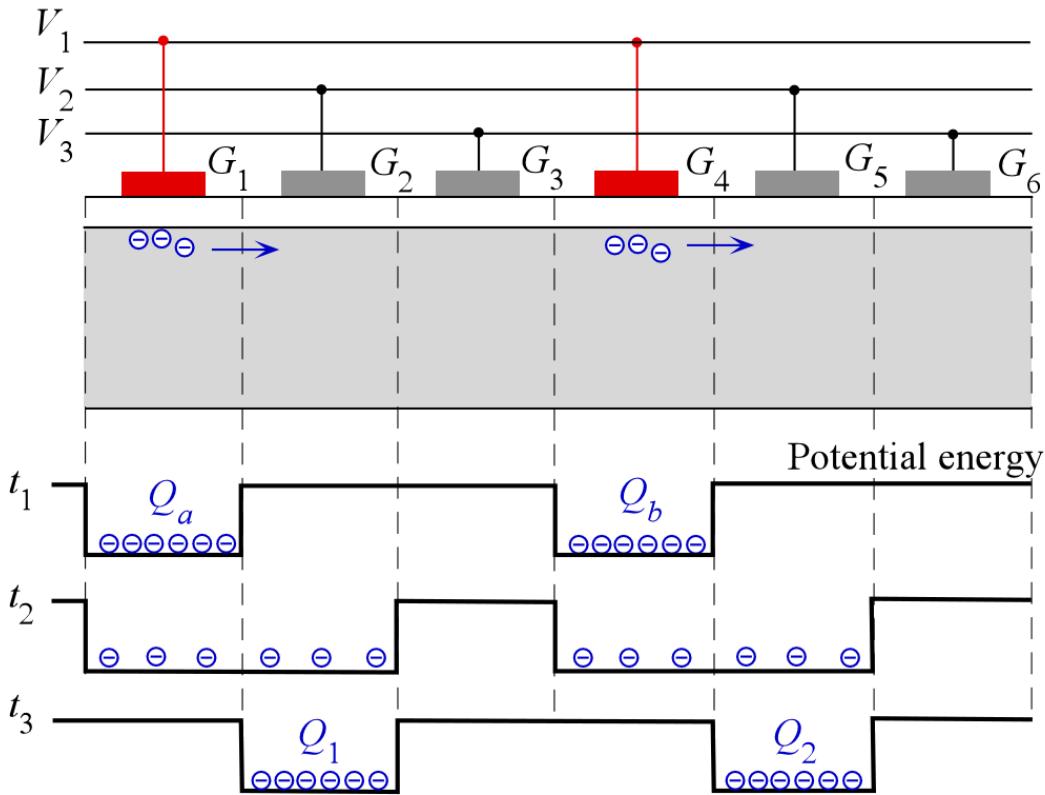
(Courtesy of Teledyne-DALSA)

# CCD Image Sensor



One element of a CCD imaging sensor, which is a MOS (metal-oxide-semiconductor) device

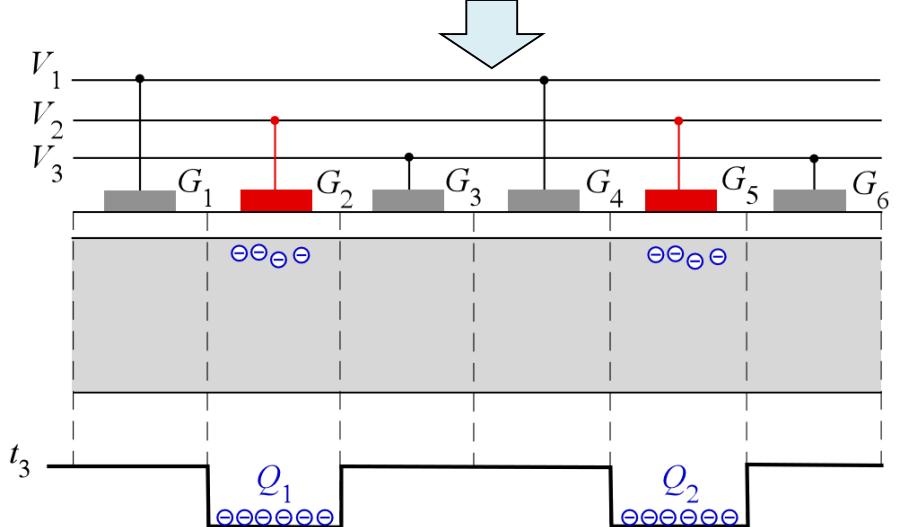
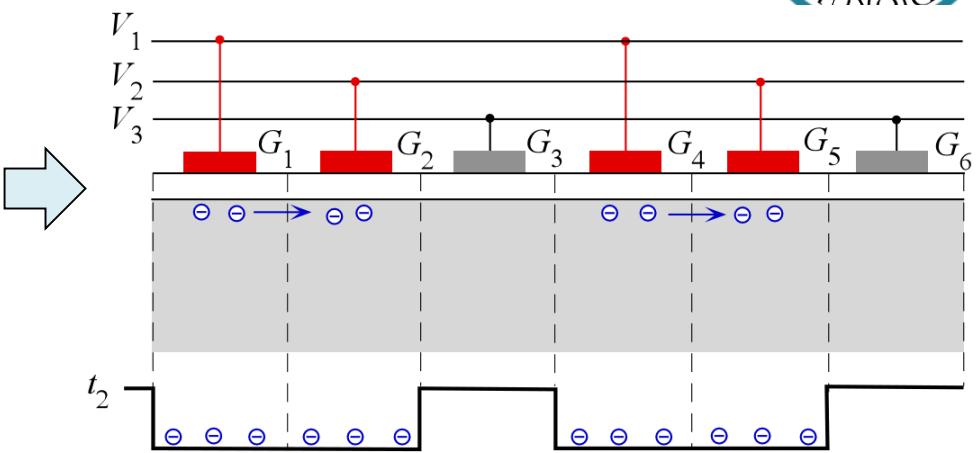
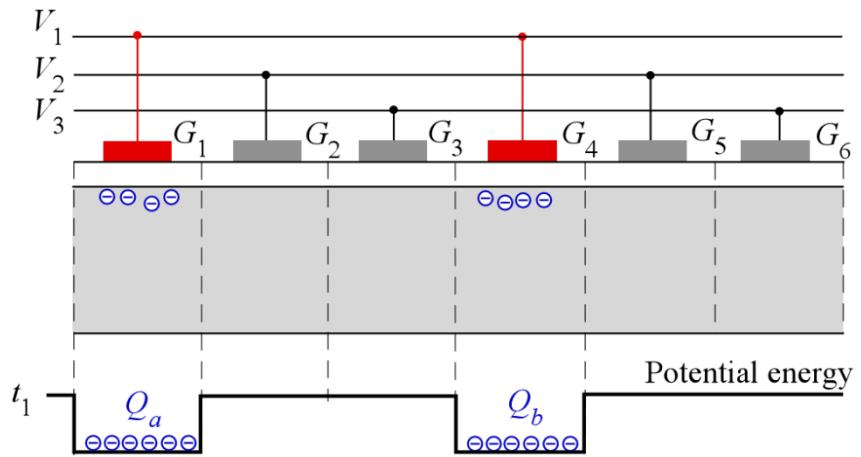
# CCD Image Sensor



Time	$V_1$	$V_2$	$V_3$
$t_1$	+V	0	0
$t_2$	+V	+V	0
$t_3$	0	+V	0

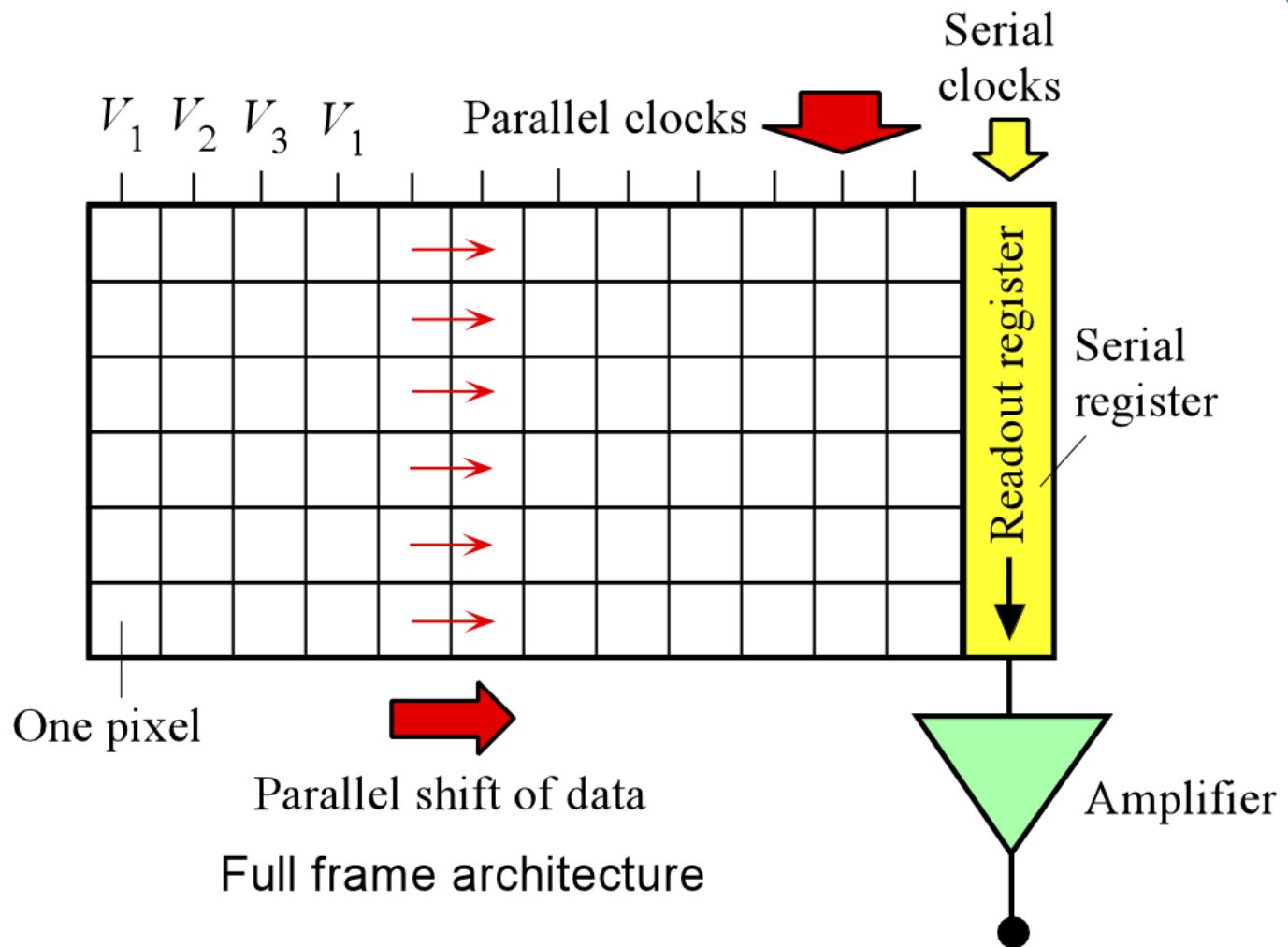
Transfer of charge from one well to another by clocking the gate voltages. The table shows the gate voltage sequences in a three phase CCD. (Schematic only)

# CCD Image Sensor



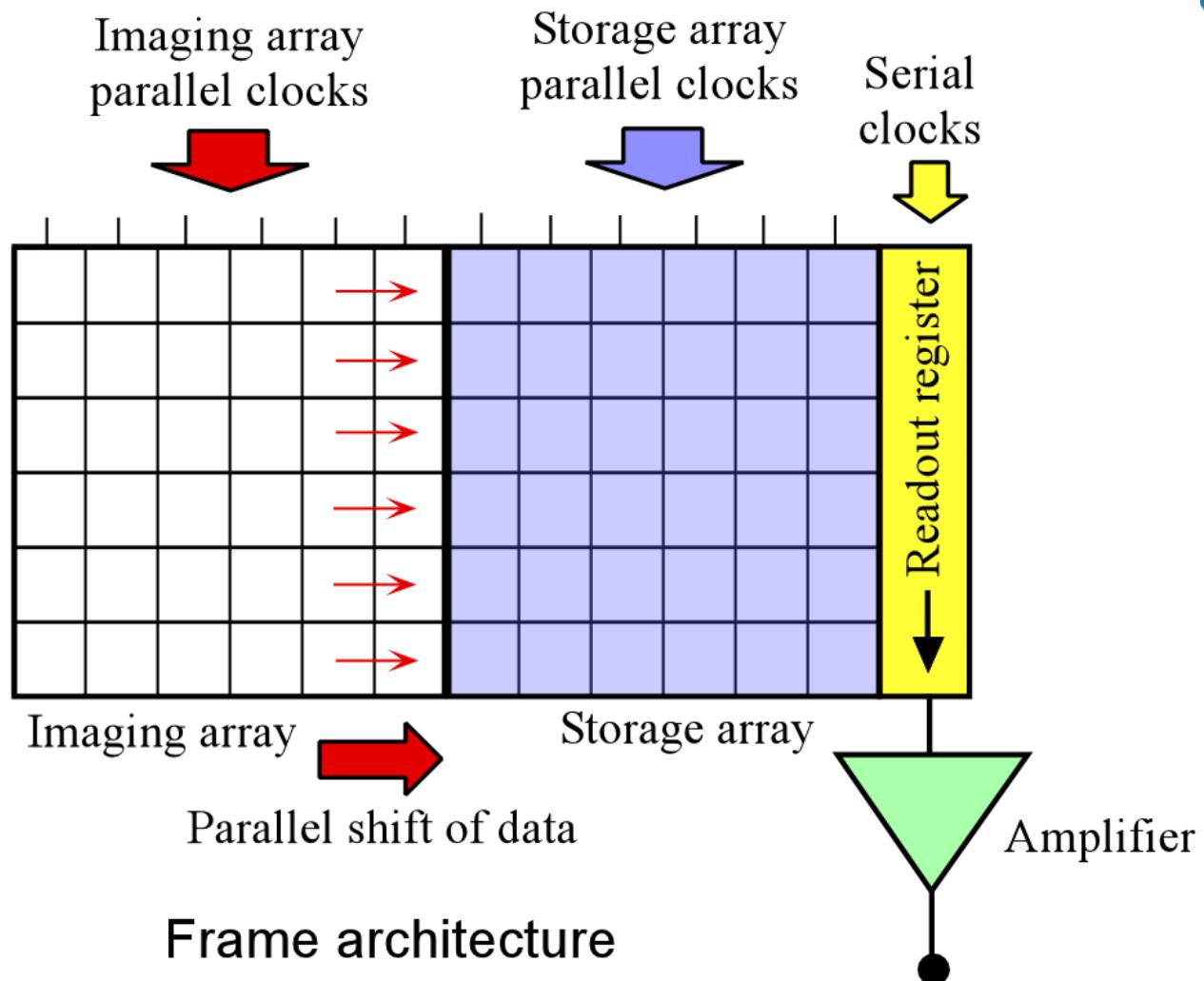
Time	$V_1$	$V_2$	$V_3$
$t_1$	$+V$	0	0
$t_2$	$+V$	$+V$	0
$t_3$	0	$+V$	0

# CCD Image Sensor

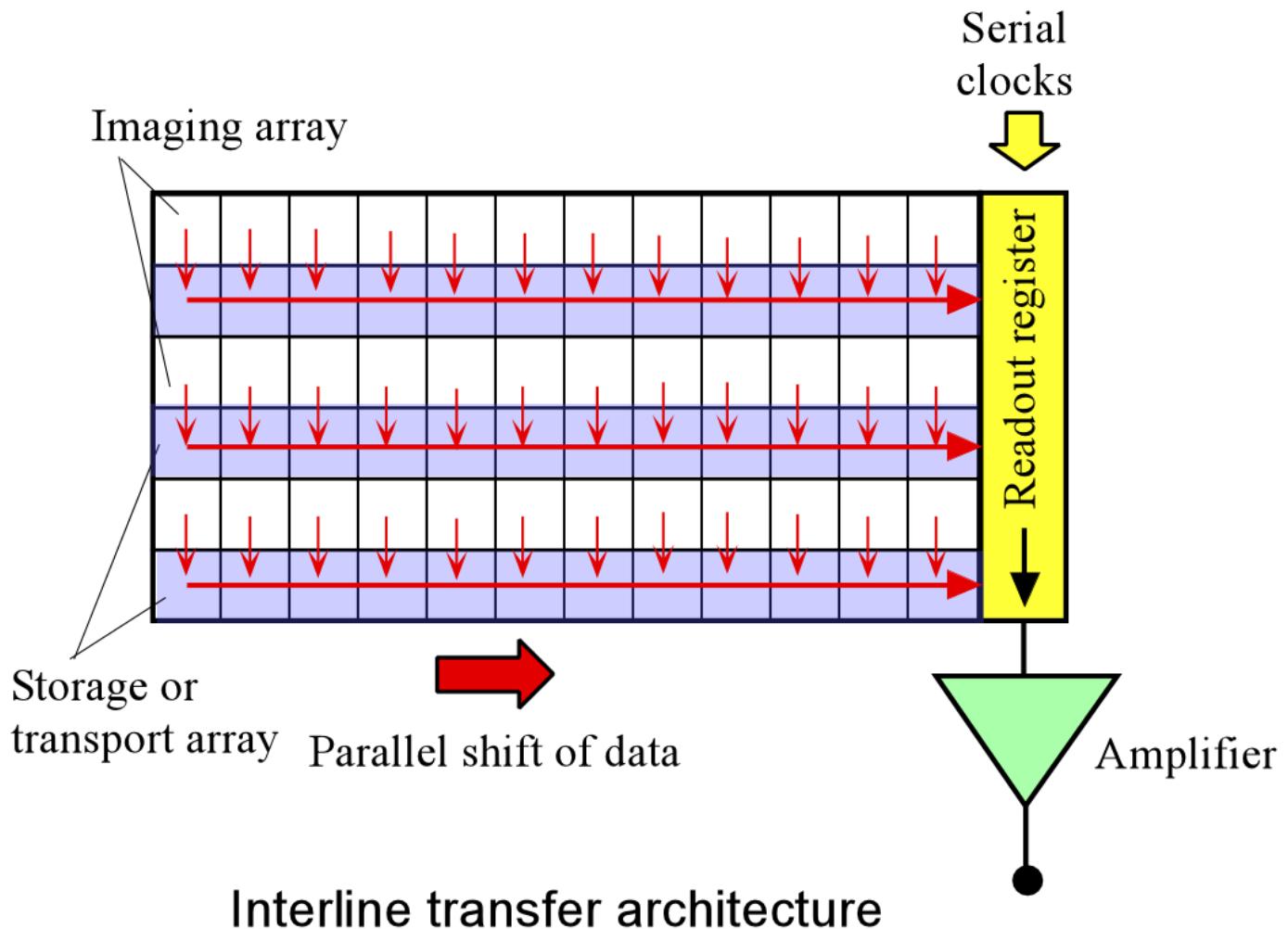




# CCD Image Sensor



# CCD Image Sensor

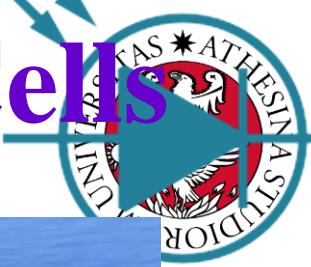




# CCD

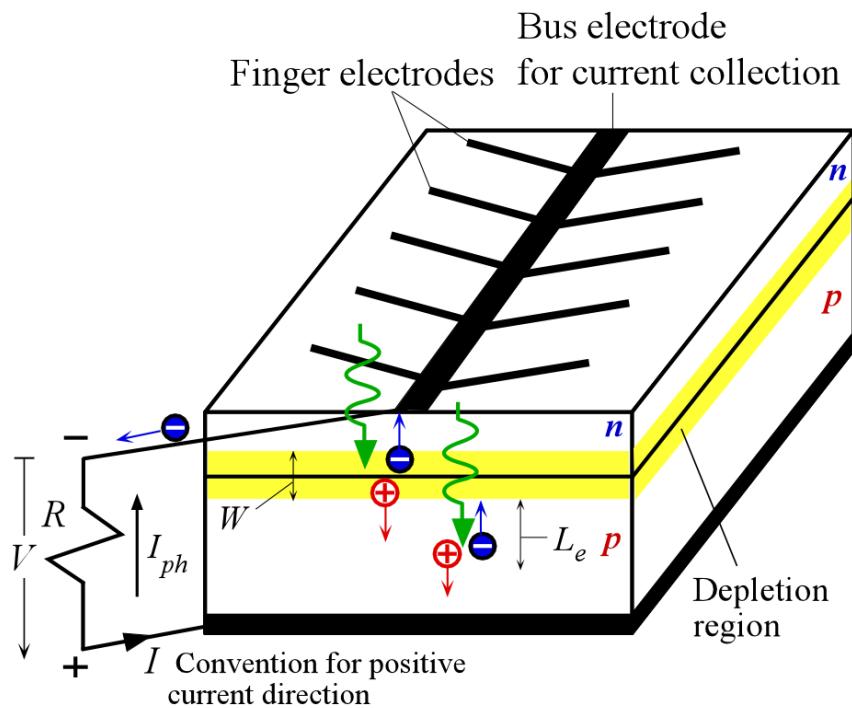
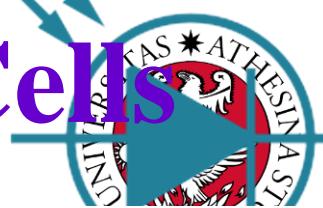
- Advantages
  - Higher sensitivity and lower noise due to enhanced surface use (higher fill factor)
  - Fewer defective pixels due to the simpler structure
  - Better image homogeneity thanks to the central A/D converter
- Disadvantages
  - Slower readout, as only one central A/D converter digitalises
  - No direct pixel access, like in case of the CMOS sensor, as the CCD sensor must be read out serially
  - More complex camera layout due to required additional electronics leads to larger and more expensive cameras
  - Higher energy consumption of the entire camera
  - More smearing and blooming effects when overexposing compared to a CMOS sensor

# Photovoltaic Devices: Solar Cells

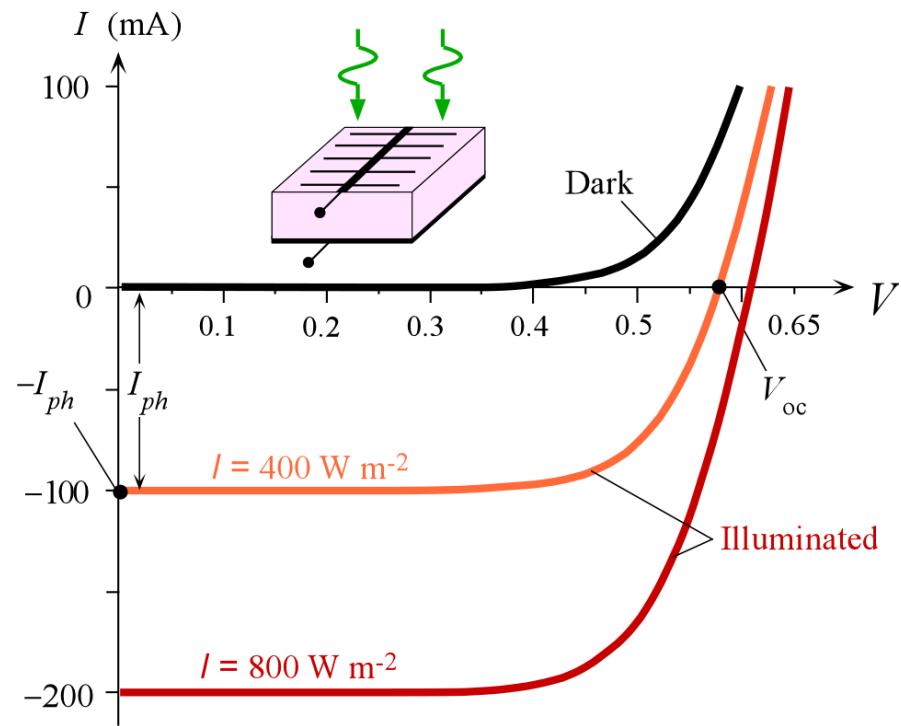


An experimental solar cell aircraft called Helios flying over the coast of Hawaii  
(Courtesy of NASA Dryden Research Centre)

# Photovoltaic Devices: Solar Cells



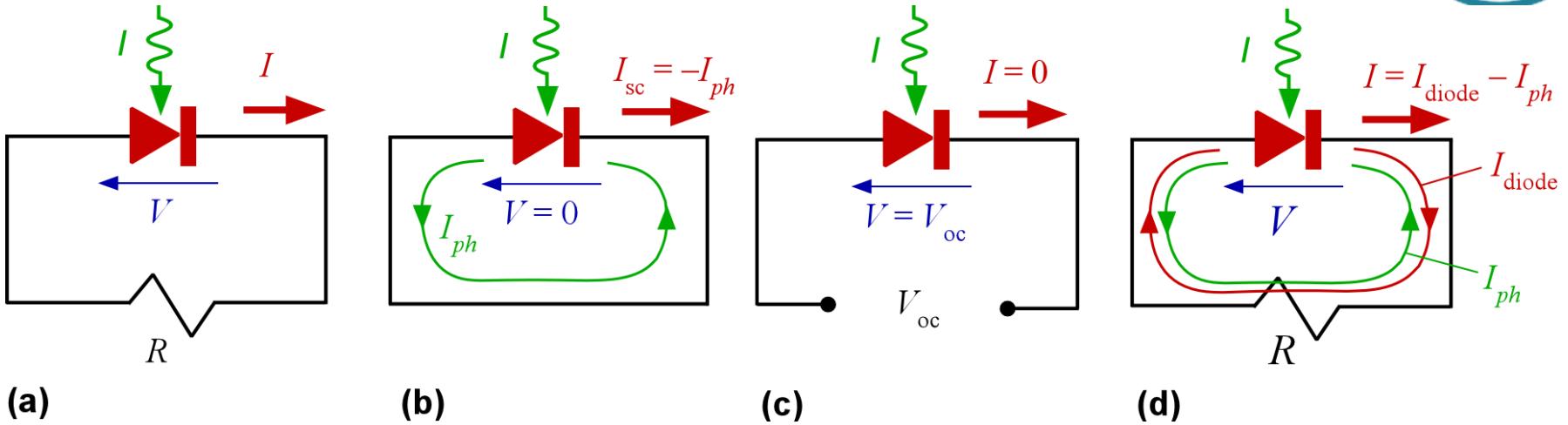
(a)



(b)

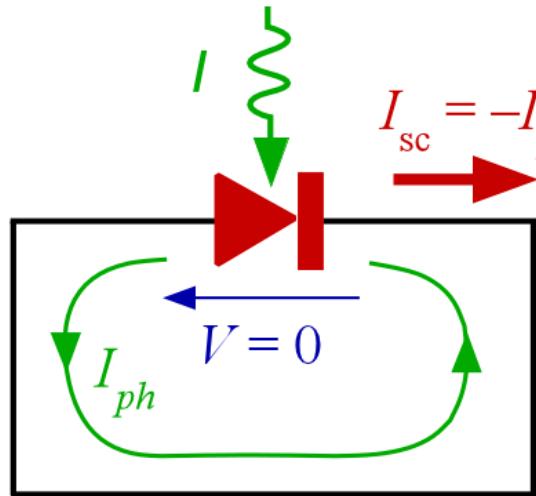
- (a) Typical *pn* junction solar cell. Finger electrodes on the surface of a solar cell reduce the series resistance
- (b)  $I$ - $V$  characteristics in the dark and under illumination at intensities corresponding to 400 and  $800 \text{ W m}^{-2}$

# Photovoltaic Devices: Solar Cells



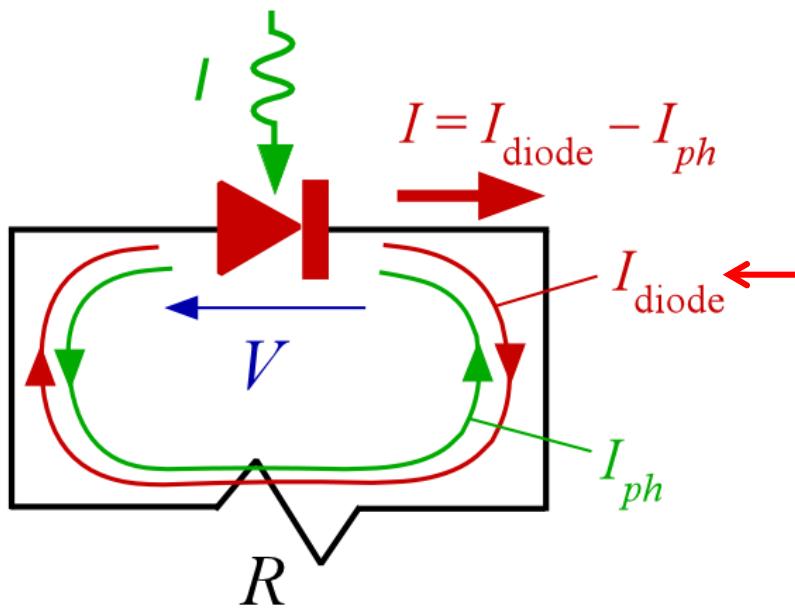
- (a) The solar cell connected to an external load  $R$  and the convention for the definitions of positive voltage and positive current. (b) The solar cell in short circuit. The magnitude of the external current  $I_{sc}$  is the photocurrent  $I_{ph}$ . (c) Under open circuit conditions there is a voltage  $V_{oc}$  at the output terminals. (d) The solar cell driving an external load  $R$ . There is a voltage  $V$  and current  $I$  in the circuit.

# Photovoltaic Devices: Solar Cells



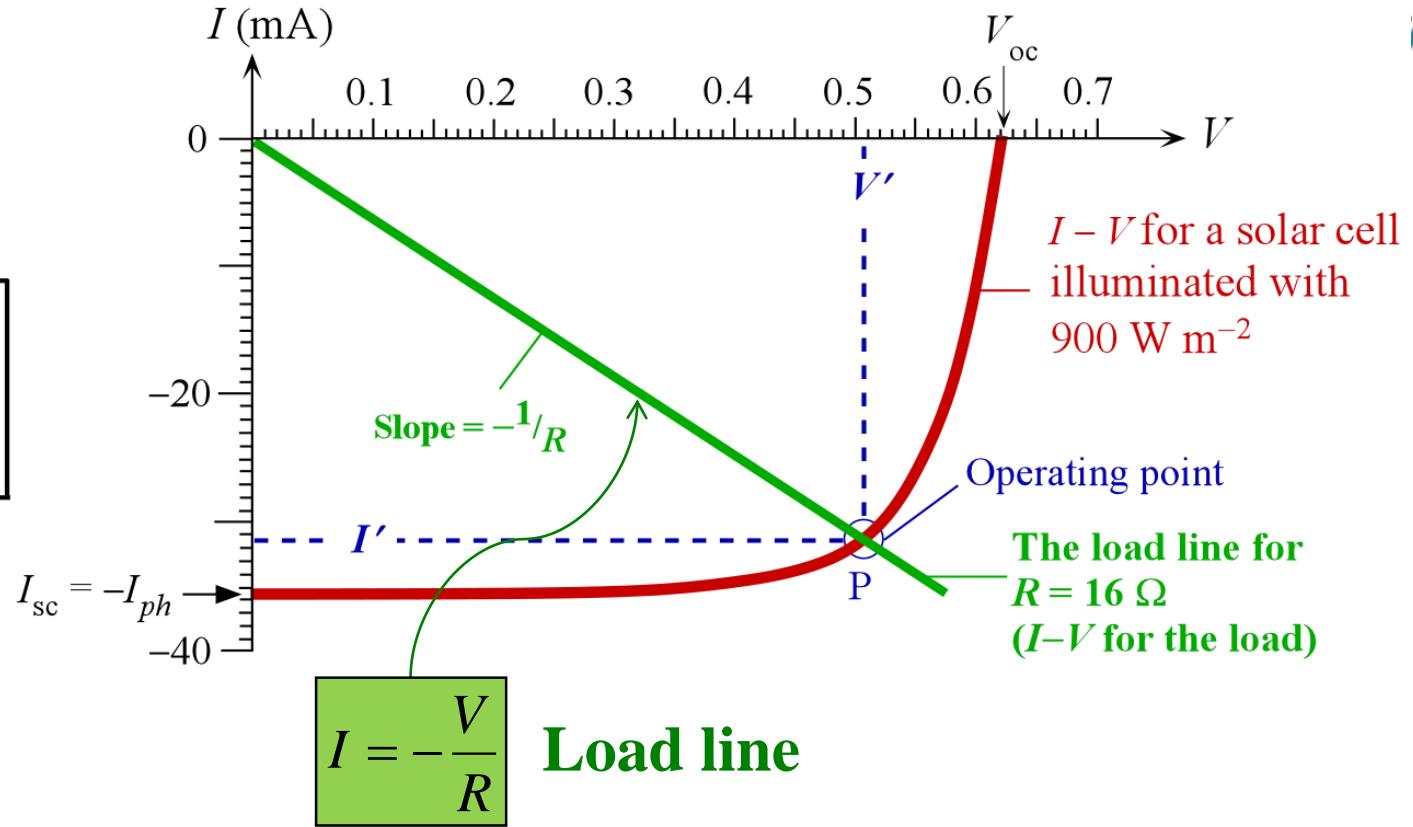
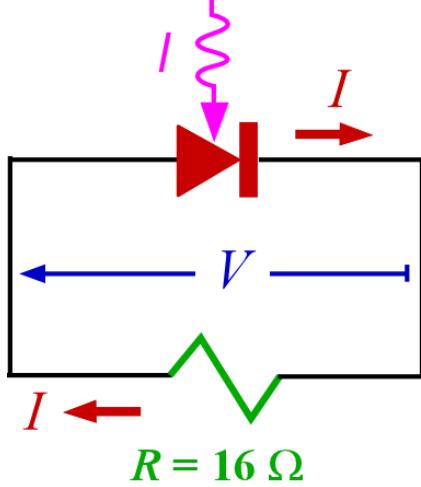
$$I_{ph} = K I$$

Device  
dependent  
constant



$$I_{diode} = I_o \left[ \exp\left(\frac{eV}{\eta k_B T}\right) - 1 \right]$$

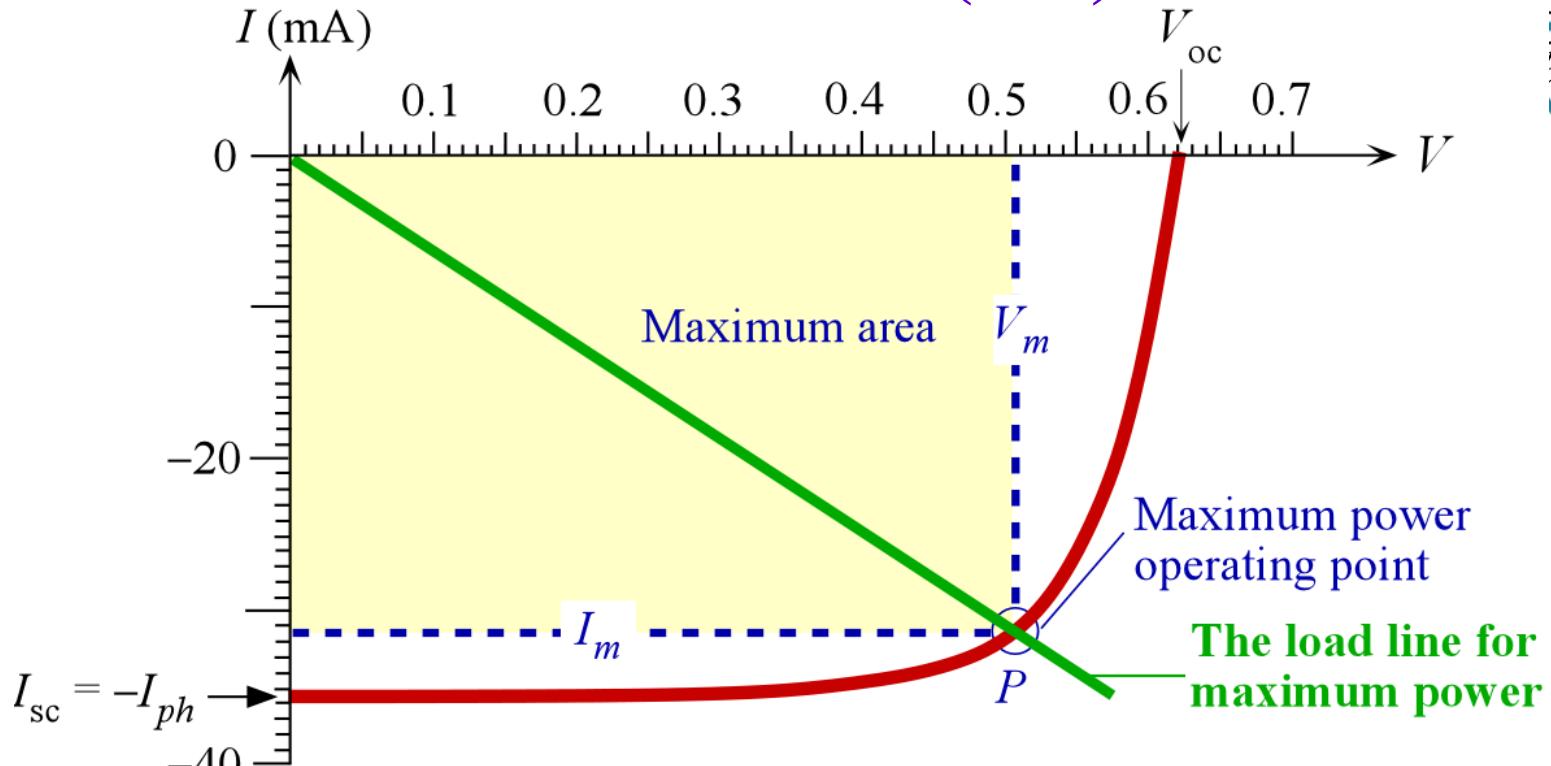
# Photovoltaic Devices: Solar Cells



LEFT: A solar cell driving a load  $R$  and the definitions of positive current  $I$  and voltage  $V$ . RIGHT: The load line construction for finding the operating point when a load  $R_L = 16 \Omega$  is connected across the solar cell.



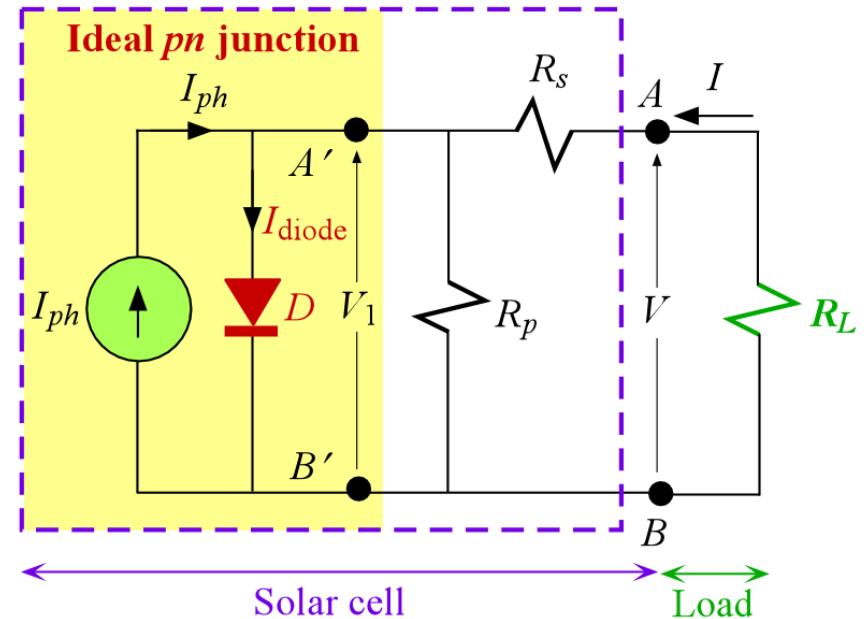
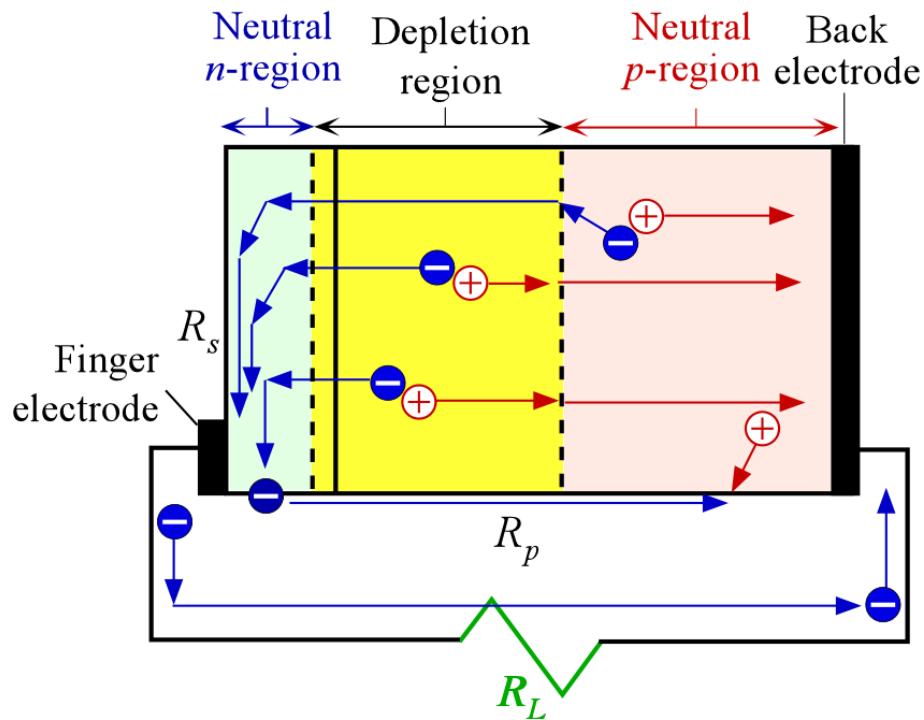
# Fill Factor (FF)



The load line is such that the area  $I_m V_m$  is maximum

$$\text{Fill Factor} = \text{FF} = \frac{I_m V_m}{I_{sc} V_{oc}}$$

# Equivalent Circuit



LEFT: Series and shunt resistances and various fates of photogenerated EHPs.  
 RIGHT: A simple equivalent circuit diagram for a solar cell. The current  $I$  into  $A$ , and  $V$  across  $A$  and  $B$ , follow the convention for positive current and voltage.  $A'$  and  $B'$  represent the ideal diode terminals.

# Equivalent Circuit

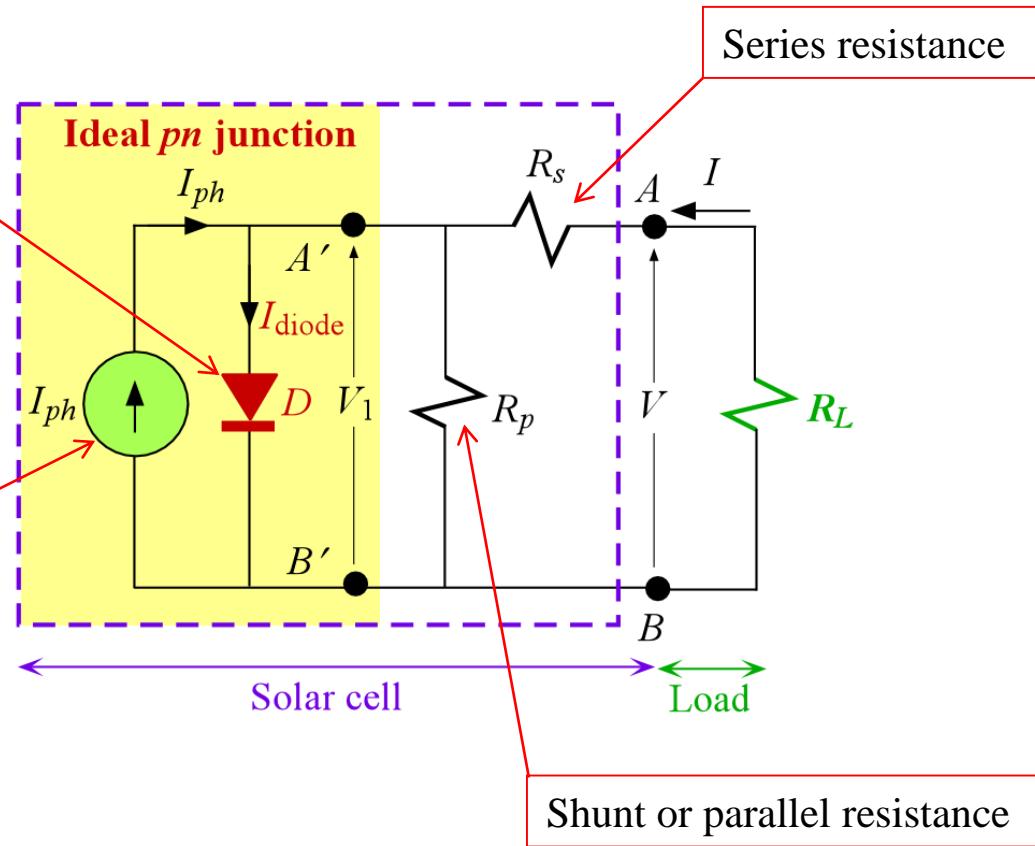


Ideal diode current

$$I_{\text{diode}} = I_o \left[ \exp\left(\frac{eV}{\eta k_B T}\right) - 1 \right]$$

$$I_{ph} = Kl$$

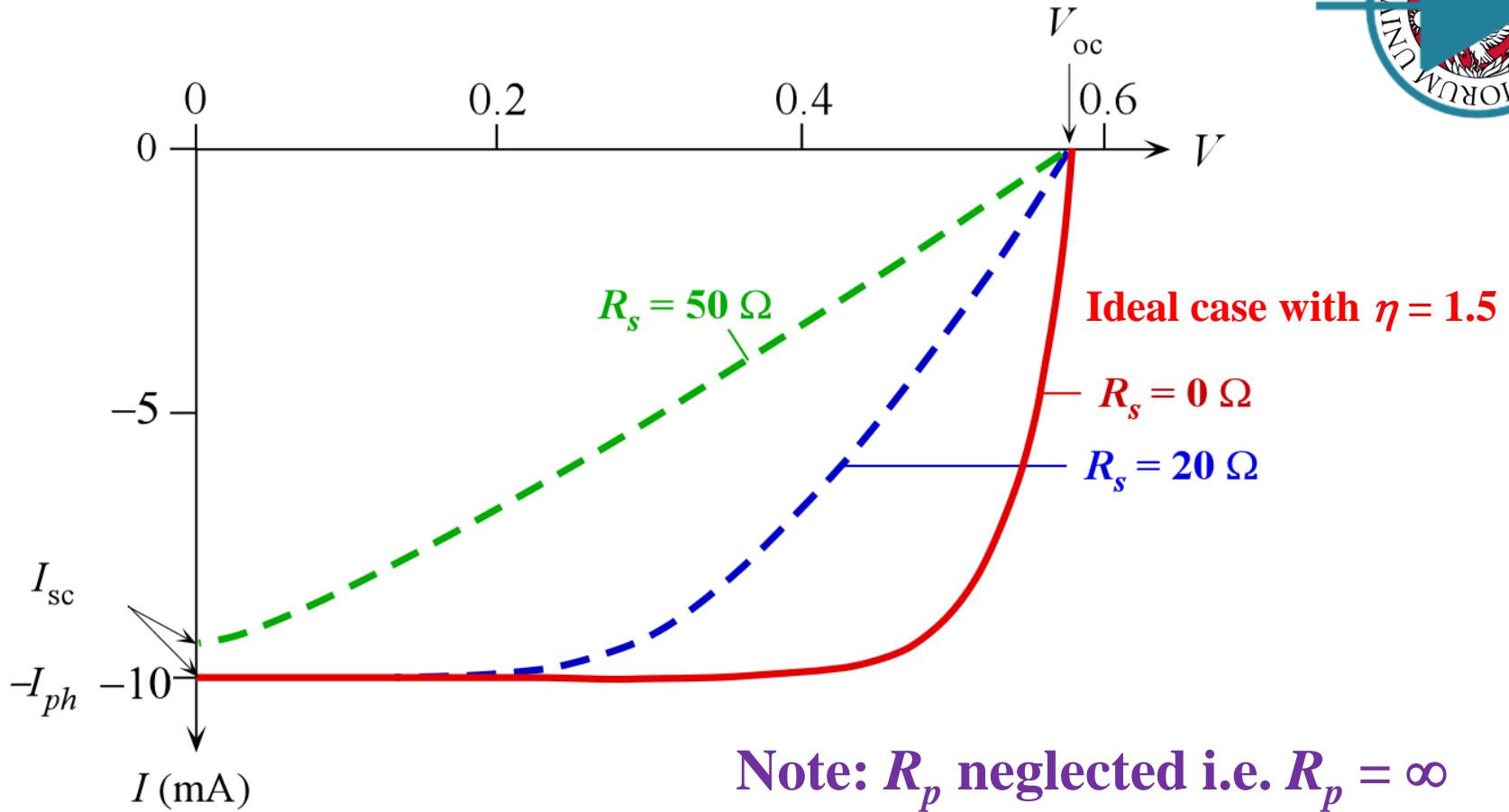
Current generator



External solar cell terminals are  $A$  and  $B$

$A'$  and  $B'$  are the internal terminals for the **ideal diode**.

# The Effect of Series Resistance



The effect of series resistance  $R_s$  on the  $I$ - $V$  characteristics. This example is a Si  $pn$  junction solar cell with  $\eta = 1.5$  and  $I_o = 3 \times 10^{-6}$  mA. The light intensity is such that it generates  $I_{ph} = 10$  mA.

# Solar Cell Efficiencies

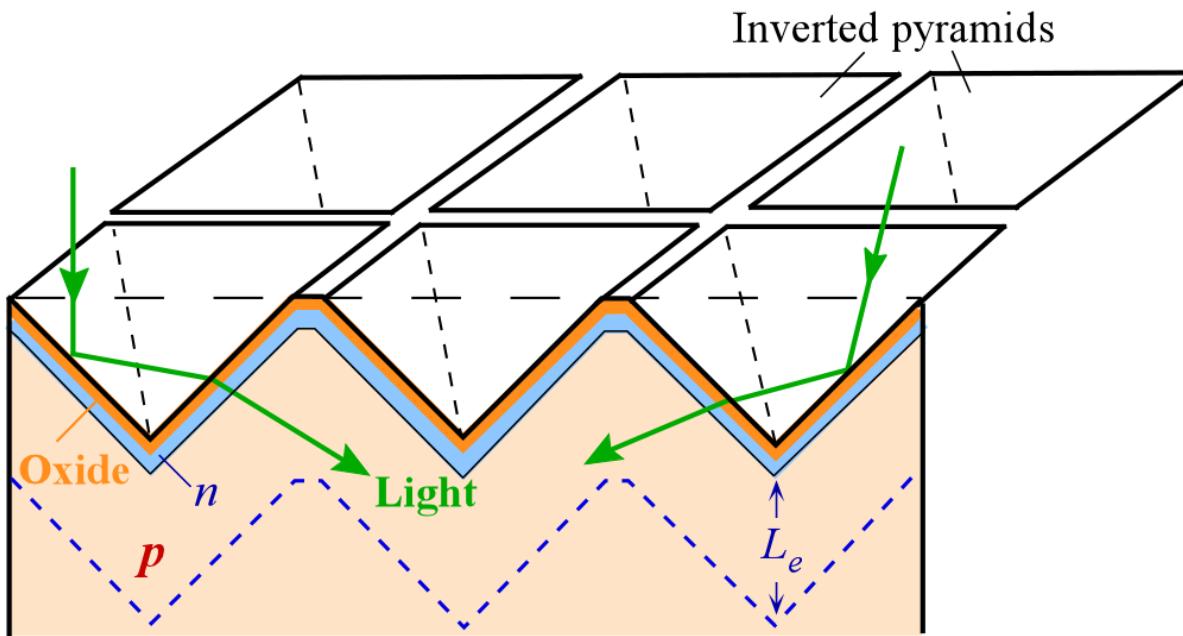


Semiconductor	$V_{oc}$	$ J_{sc} $	FF	Efficiency	$E_g$
	V	mA cm <sup>-2</sup>	%	%	eV
Si, single crystal (PERL)	0.707	42.7	82.8	25.0	1.11
Si, polycrystalline	0.664	38.0	80.9	20.4	1.11
Amorphous Si:H (pin)	0.886	16.75	67.0	10.1	~1.7
GaAs, single crystal	1.030	29.8	86.0	26.4	1.42
GaAs, thin film	1.107	29.6	84.1	27.6	1.42
InP, single crystal	0.878	29.5	85.4	22.1	1.35
GalnP/GaAs Tandem	2.488	14.22	85.6	30.3	1.95/1.42

GalnP/GaAs/GaTandem cells of solar cells, and reported efficiencies under 1.95/1.42/0.56

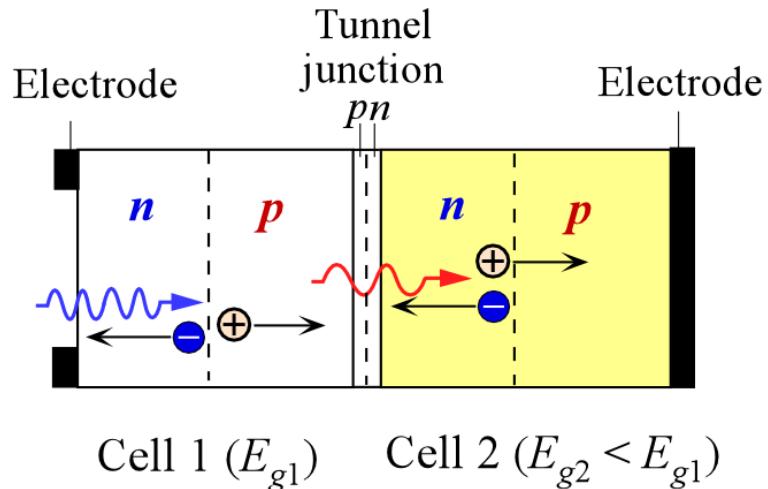
solar spectrum (1000 W/m<sup>2</sup>) at 25°C. (Data have been extracted from M. A. Green, K. Emery, Y. Hishikawa, W. Warta, *Progress in Photovoltaics: Research and Applications*, **18**, 346, 2010 and **19**, 84, 2011. The original tables in the latter have extensive confirmed data on a number of important solar cells and modules with references. In addition, the original tables also list the uncertainties and errors involved in the reported efficiency values.)

# High Efficiency Solar Cells



Inverted pyramid textured surface substantially reduces reflection losses and increases absorption probability in the device

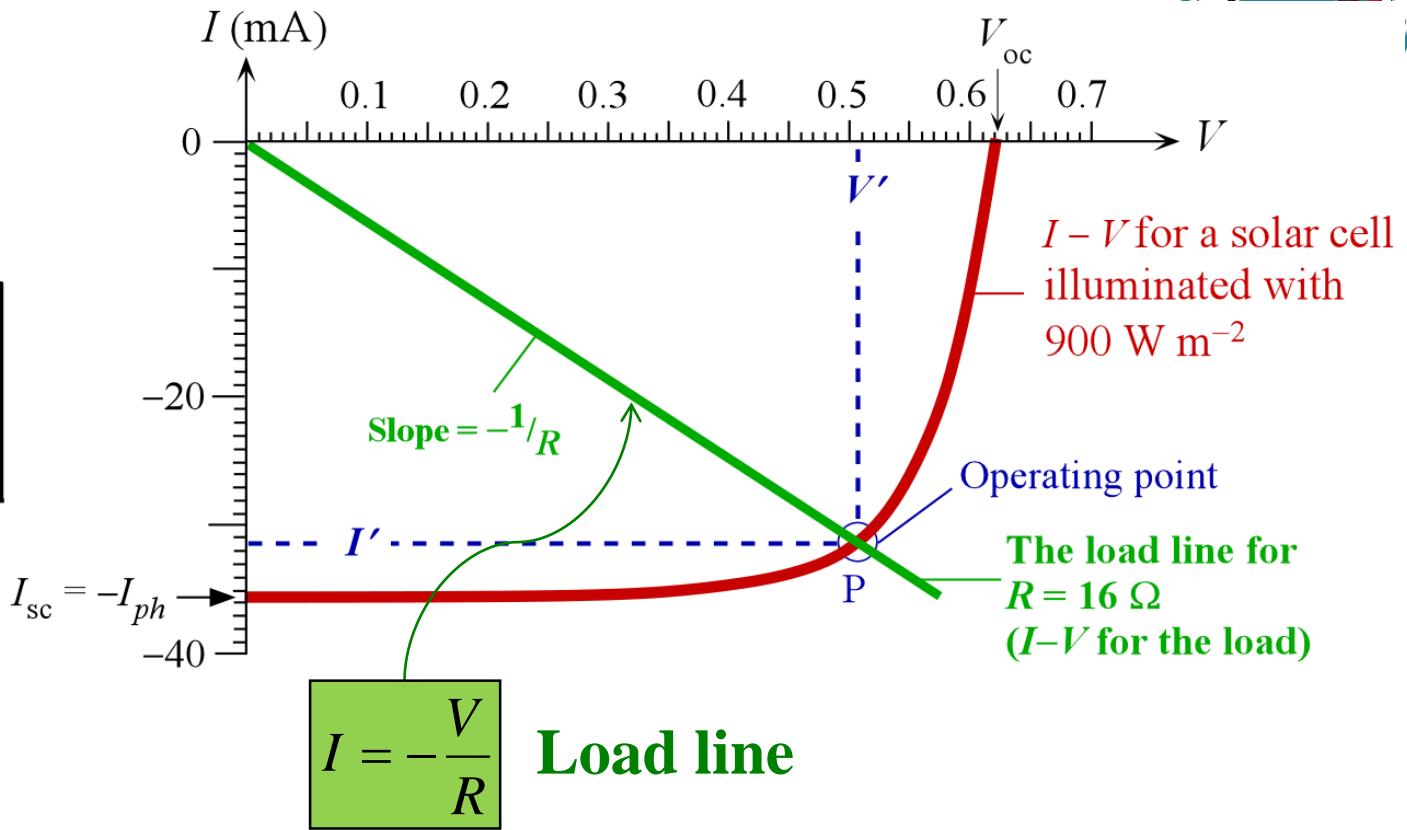
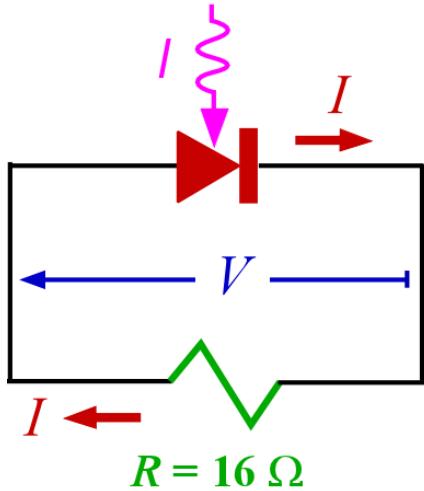
# Tandem or Multijunction Solar Cells



LEFT: A heterojunction solar cell that can absorb both high and low energy photons and generate a photocurrent. RIGHT: A tandem solar cell.

Semiconductor	$V_{oc}$ V	$ J_{sc} $ $\text{mA cm}^{-2}$	FF %	Efficiency %	$E_g$ eV
GalnP/GaAs Tandem	2.488	14.22	85.6	30.3	1.95/1.42
GalnP/GaAs/Ge Tandem	2.622	14.37	85.0	32.0	1.95/1.42/0.66

# EXAMPLE: Solar cell driving a load



LEFT: A solar cell driving a load  $R$  and the definitions of positive current  $I$  and voltage  $V$ . RIGHT: The load line construction for finding the operating point when a load  $R_L = 16 \Omega$  is connected across the solar cell.

## EXAMPLE: Solar cell driving a load

Consider the solar cell driving a  $16\text{-}\Omega$  resistive load as in Figure 5.42 (b). Suppose that the cell has an area of  $1\text{ cm} \times 1\text{ cm}$  and is illuminated with light of intensity  $900\text{ W m}^{-2}$  as in the figure. What are the current and voltage in the circuit? What is the power delivered to the load? What is the efficiency of the solar cell in this circuit? If you assume it is operating close to the maximum deliverable power, what is the FF?

### Solution

The  $I$ - $V$  characteristic of the load is the load line as described by Eq. (5.14.4),  $I = -V/R$  with  $R = 16\text{ }\Omega$ . This line is drawn in Figure 5.42 (b) with a slope  $1/(16\text{ }\Omega)$ . It cuts the  $I$ - $V$  characteristics of the solar cell at  $I' \approx -31.5\text{ mA}$  and  $V' \approx 0.505\text{ V}$  which are the current and voltage in the photovoltaic circuit of Figure 5.42 (b). In fact, from Eq. (5.14.4),  $V'/I'$  gives  $-16\text{ }\Omega$  as expected. The power delivered to the load is

$$P_{\text{out}} = |IV'| = (31.5 \times 10^{-3}\text{ A})(0.505\text{ V}) = \mathbf{0.0159\text{ W or }15.9\text{ mW}}$$

This is not necessarily the maximum power available from the solar cell. The input sun-light power is

$$\begin{aligned} P_{\text{in}} &= (\text{Light Intensity})(\text{Surface Area}) = (900\text{ W m}^{-2})(0.01\text{ m})^2 \\ &= \mathbf{0.090\text{ W}} \end{aligned}$$

## EXAMPLE: Solar cell driving a load

### Solution (continued)

The efficiency is

$$\begin{aligned}\text{Efficiency} &= 100 \times (P_{\text{out}} / P_{\text{in}}) = 100 (15.9 \text{ mW} / 90 \text{ mW}) \\ &= \mathbf{17.7\%}\end{aligned}$$

This will increase if the load is adjusted to extract the maximum power from the solar cell but the increase will be small as the rectangular area  $IV'$  in Figure 5.42 in it is already close to the maximum. Assuming that  $|IV'|$  is roughly the maximum power available (maximum area for the rectangle  $IV'$ ), then  $I_m \approx I' \approx -31.5 \text{ mA}$  and  $V_m \approx V' \approx 0.505 \text{ V}$ . For the solar cell in Figure 5.42 (b),  $I_{\text{sc}} = -35.5 \text{ mA}$  and  $V_{\text{oc}} = 0.62 \text{ V}$ . Then,

$$\begin{aligned}\text{FF} &= I_m V_m / I_{\text{sc}} V_{\text{oc}} \approx (-31.5 \text{ mA})(0.505 \text{ V}) / (-35.5 \text{ mA})(0.62 \text{ V}) \\ &= \mathbf{0.72 \text{ or } 72\%}\end{aligned}$$

# EXAMPLE: Open circuit voltage and short circuit current

A solar cell under an illumination of  $500 \text{ W m}^{-2}$  has a short circuit current  $I_{sc}$  of  $-16 \text{ mA}$  and an open circuit output voltage  $V_{oc}$ , of  $0.50 \text{ V}$ . What are the short circuit current and open circuit voltages when the light intensity is doubled? Assume  $\eta = 1$ .

## Solution

The general  $I$ - $V$  characteristics under illumination is given by Eq. (5.14.3). The short circuit current corresponds to the photocurrent so that, from Eq. (5.14.2), at double the intensity the photocurrent is

$$I_{ph2} = \left( \frac{I_2}{I_1} \right) I_{ph1} = (16 \text{ mA})(1000/500) = 32 \text{ mA}$$

Setting  $I = 0$  for open circuit we can obtain the open circuit voltage  $V_{oc}$ ,

$$I = -I_{ph} + I_o [\exp(eV_{oc}/\eta k_B T) - 1] = 0$$

Assuming that  $V_{oc} \gg \eta k_B T/e$ , rearranging the above equation we can find  $V_{oc}$

$$V_{oc} = \frac{\eta k_B T}{e} \ln \left( \frac{I_{ph}}{I_o} \right) \quad \text{Open circuit output voltage} \quad (5.14.6)$$

# EXAMPLE: Open circuit voltage and short circuit current

## Solution (continued)

In Eq. (5.14.6), the photocurrent,  $I_{ph}$ , depends on the light intensity  $I$  via,  $I_{ph} = KI$ . At a given temperature, then the change in  $V_{oc}$  is

$$V_{oc2} - V_{oc1} = \frac{\eta k_B T}{e} \ln\left(\frac{I_{ph2}}{I_{ph1}}\right) = \frac{\eta k_B T}{e} \ln\left(\frac{I_2}{I_1}\right)$$

Assuming  $\eta = 1$ , the new open circuit voltage is

$$V_{oc2} = V_{oc1} + \frac{\eta k_B T}{e} \ln\left(\frac{I_2}{I_1}\right) = 0.50 \text{ V} + (1)(0.0259 \text{ V})\ln(2) \approx \mathbf{0.52 \text{ V}}$$

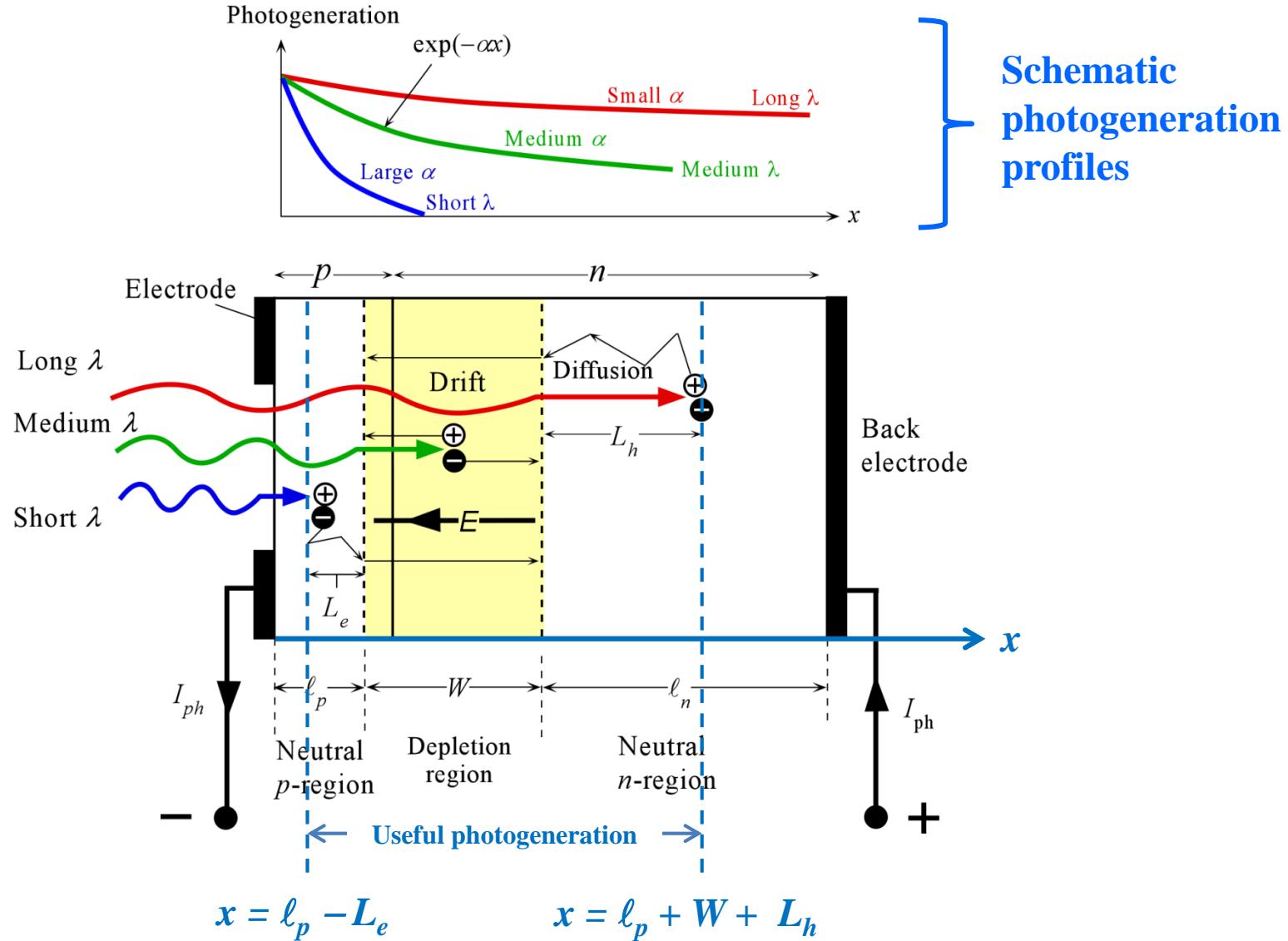
**NOTE: This is a ~4% increase in  $V_{oc}$  compared with the 100% increase in illumination and the short circuit current.**



# Slides on Questions and Problems

## Additional Slides

# Photocurrent and Responsivity Depend on the Wavelength

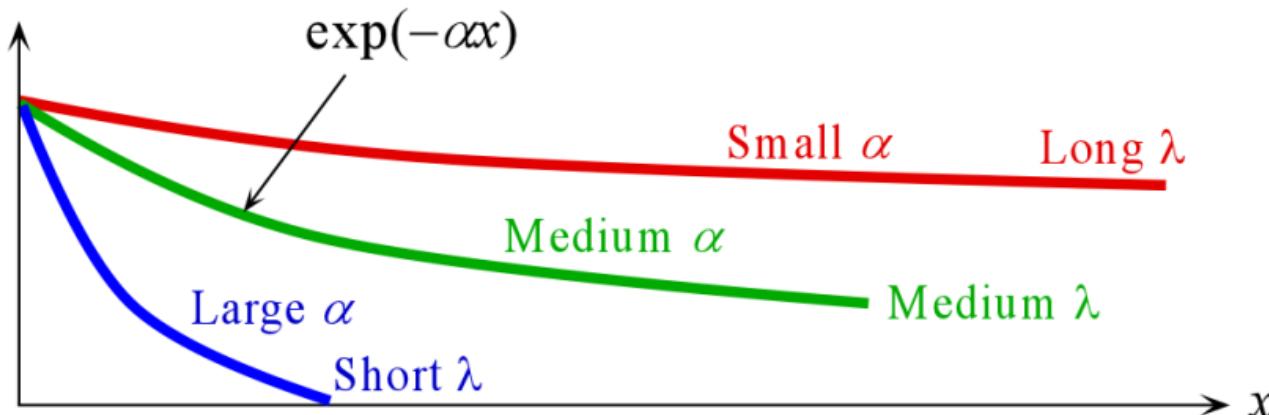


Different contributions to the photocurrent  $I_{ph}$ . Photogeneration profiles corresponding to short, medium and long wavelengths are also shown.

# Photocurrent and Responsivity Depend on the Wavelength

## $\alpha$ depends on $\lambda$

Photogeneration



$$\frac{dN_{\text{ph}}}{dt} = \frac{TP_o}{h\nu} \exp(-\alpha x)$$

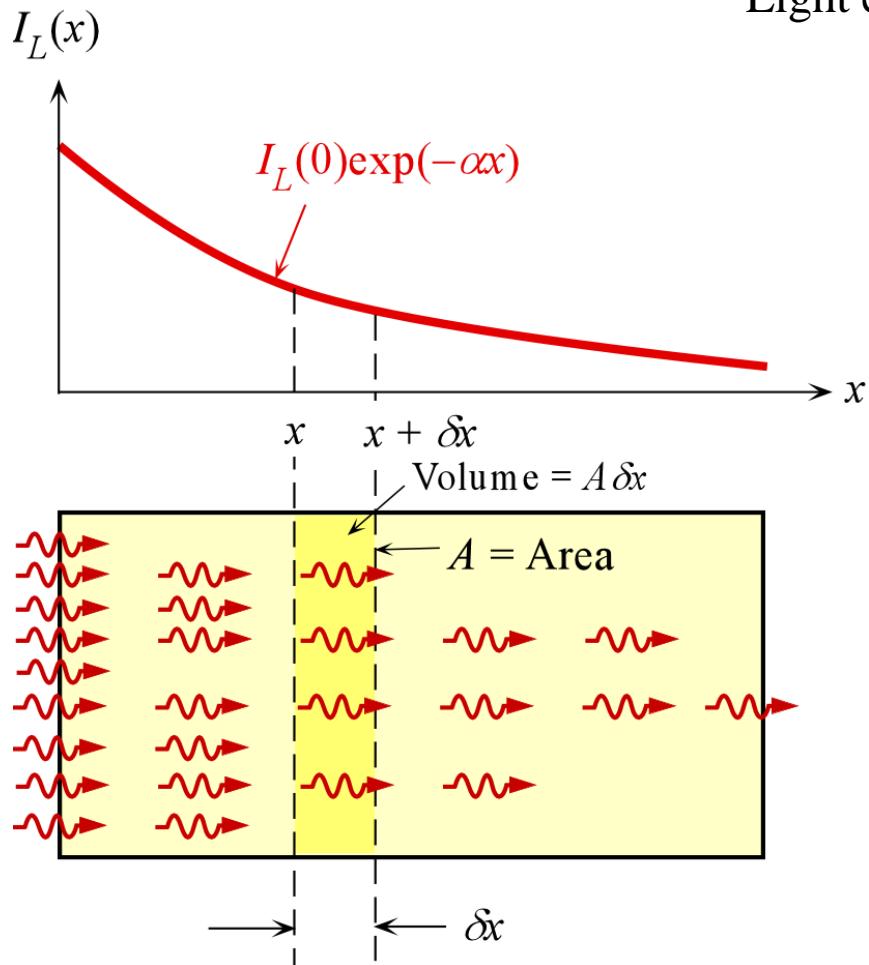
Transmittance      Incident optical power  
Photon energy

Photon flux entering the  
semiconductor at  $x = 0$

# $\alpha$ depends on $\lambda$

$$I_L(x) = I_L(0) \exp(-\alpha x)$$

Light energy



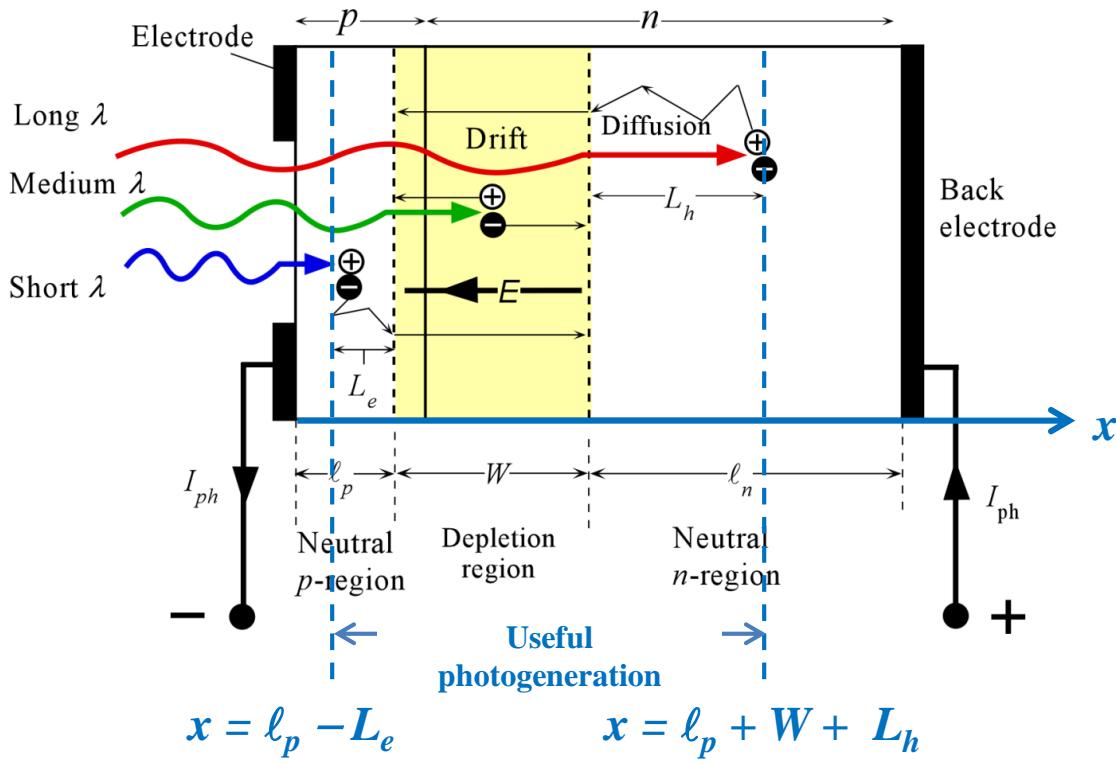
Number of **absorbed photons per unit second** in the volume  $A \delta x$  at  $x$  is

$$\begin{aligned}\delta N'_{\text{ph}} &= \frac{-A \delta I_L}{h\nu} \\ &= \frac{\alpha A I_L(x) \delta x}{h\nu} \\ &= \delta x \frac{\alpha A I_L(0)}{h\nu} \exp(-\alpha x)\end{aligned}$$

Let

$\eta_i$  = quantum efficiency

$d\delta N_{\text{EHP}}/dt$  = number of EHPs photogenerated per second in  $A \delta x$ .



## Rate of EHP generation in a volume $A\delta x$ at $x$

$$\frac{dN_{EHP}}{dt} = \delta x \frac{\eta_i \alpha A I_L(0)}{h\nu} \exp(-\alpha x)$$

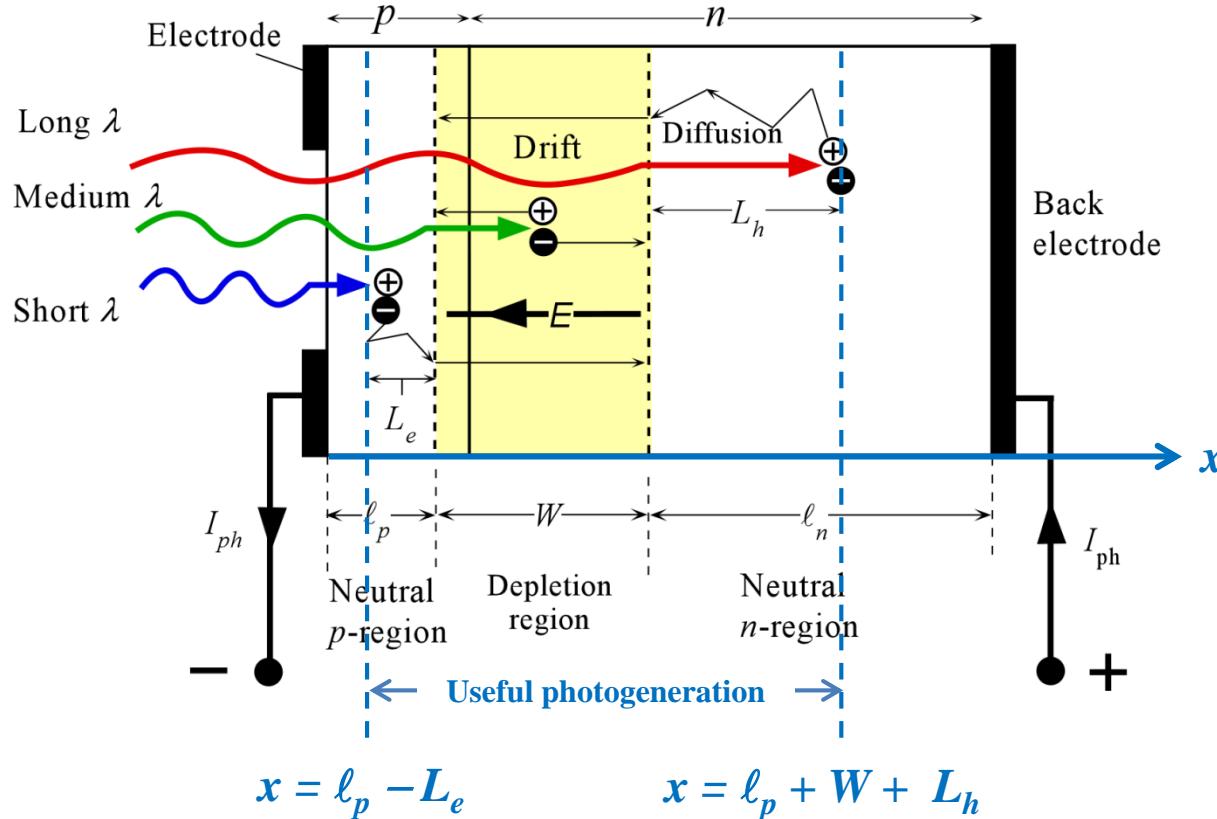
Useful (collectable) EHPs are photogenerated in the volume

from  $x = \ell_p - L_e$

to  $x = \ell_p + W + L_h$

$$\frac{dN_{EHP}}{dt} = \frac{\eta_i \alpha A I_L(0)}{h\nu} \int_{x=\ell_p - L_e}^{x=\ell_p + W + L_h} \exp(-ax) dx$$

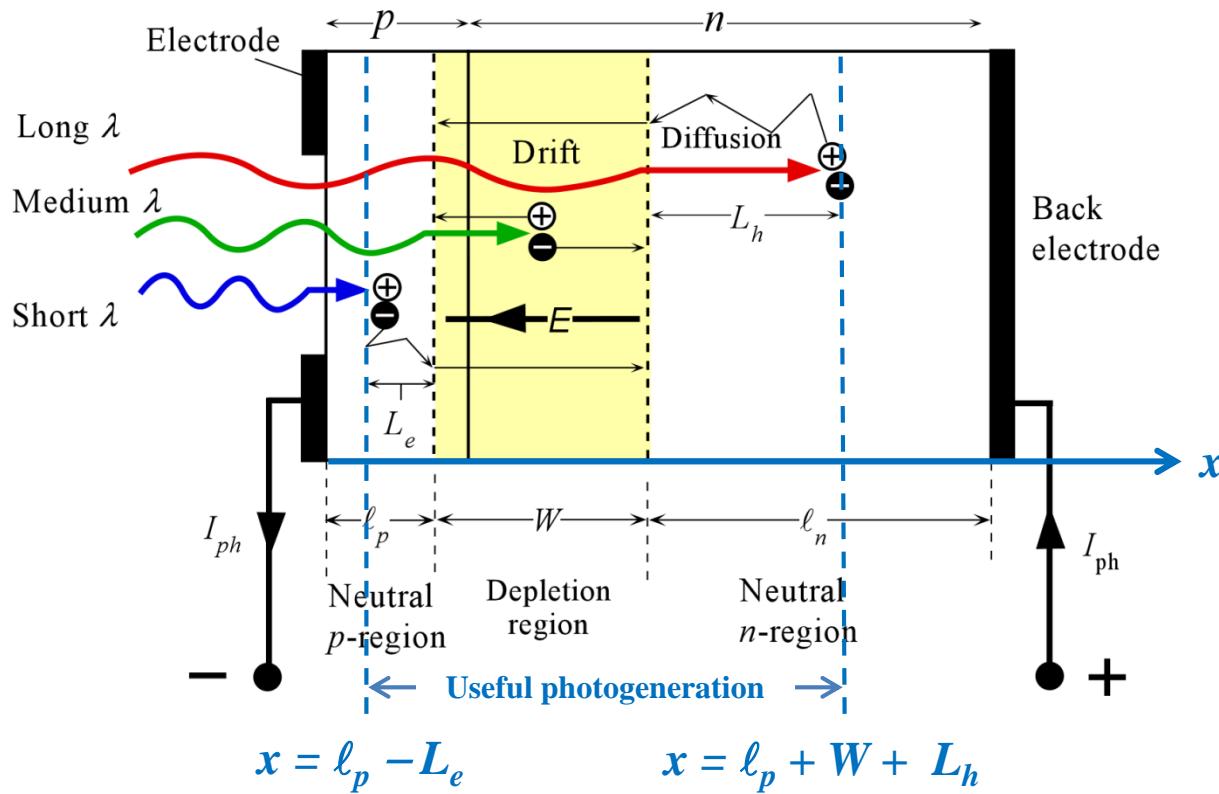
# Photocurrent and Responsivity Depend on the Wavelength



The number of EHP photogenerated per second is in the useful photogeneration region is

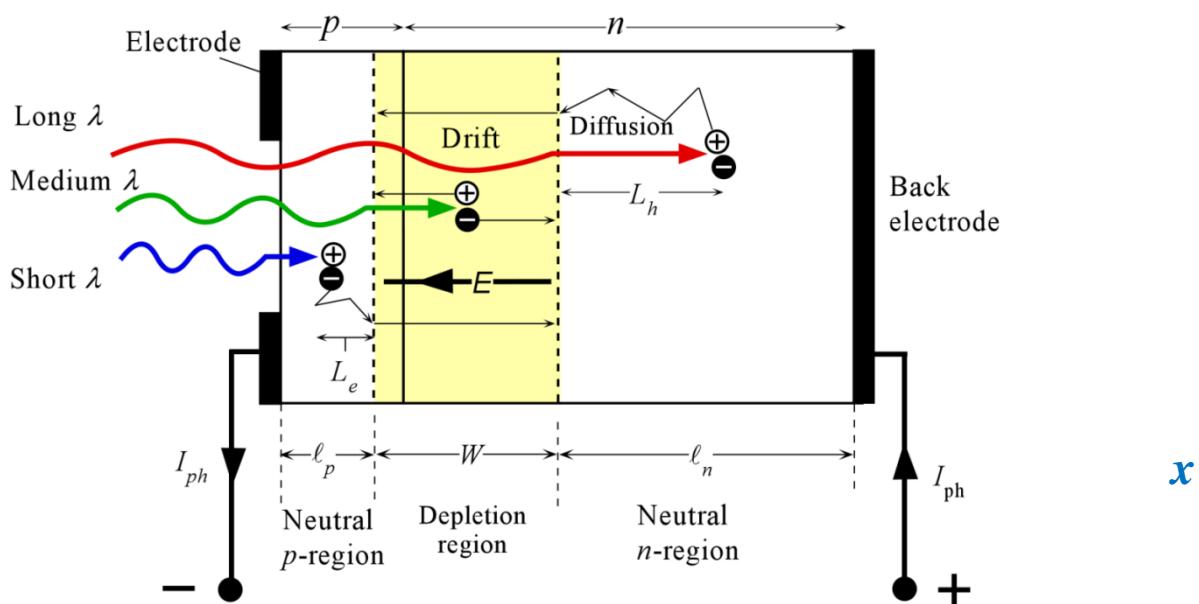
$$\frac{dN_{\text{EHP}}}{dt} = \frac{\eta_i A I_L(0)}{h\nu} [\exp(-\alpha(\ell_p - L_e)) - \exp(-\alpha(\ell_p + W + L_h))]$$

# Photocurrent and Responsivity Depend on the Wavelength



$I_L(0)$  is the light intensity inside the semiconductor at  $x = 0$ . If  $P_o$  is the **incident optical power**,  $T P_o$  will be transmitted and  $I_L(0) = T P_o / A$ . Thus,

$$I_{ph} = \frac{e \eta_i T P_o}{h \nu} \{ \exp[-\alpha(\ell_p - L_e)] - \exp[-\alpha(\ell_p + W + L_h)] \}$$



$$I_{ph} = \frac{e \eta_i T P_o}{h \nu} \{ \exp[-\alpha(\ell_p - L_e)] - \exp[-\alpha(\ell_p + W + L_h)] \}$$

**Table 5.7** Properties of a  $p^+n$  and  $pin$  photodiode.

Assume  $\eta_i = 1$ ,  $T = 1$  and  $P_o = 1 \mu\text{W}$ .

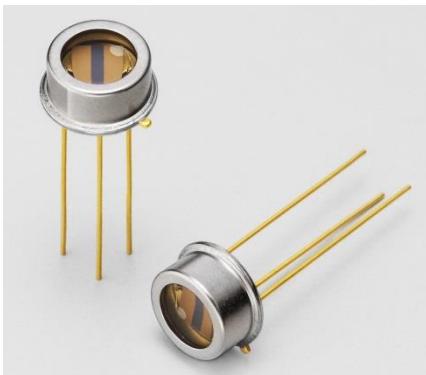
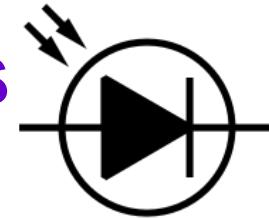
Detector	$\ell_p$	$L_e$	$W$	$L_h$	$\alpha$ (at 800 nm)	$I_{ph}$
$p^+n$	0.5 $\mu\text{m}$	0.1 $\mu\text{m}$	1 $\mu\text{m}$	10 $\mu\text{m}$	$1 \times 10^5 \text{ m}^{-1}$	<b>0.42 <math>\mu\text{A}</math></b>
$pin$	0.5 $\mu\text{m}$	0.1 $\mu\text{m}$	30 $\mu\text{m}$	0.1 $\mu\text{m}$	$1 \times 10^5 \text{ m}^{-1}$	<b>0.59 <math>\mu\text{A}</math></b>

Generally  $pin$  has a better responsivity at longer wavelengths. See Problem 5.4

Note: Page 430, second equation is printed with a typographical error. Delete  $L_e$  in the argument of the second exponential so that the equation matches the above equation.

# Photoconductive (PC) Detectors

Used for radiation measurement at long wavelengths



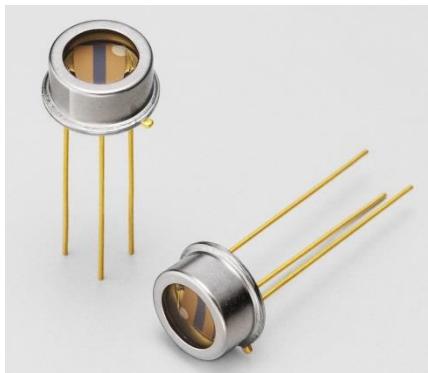
PbS photoconductive  
detectors  
(Courtesy of  
Hamamatsu)

Photodiode	PbS (PC)	PbSe (PC)	InSb (PC)
$\lambda_{\text{peak}} (\mu\text{m})$	2.4	4.1	5.5
$I_d$ or $R_d$	0.1–1 $\text{M}\Omega$	0.1–1 $\text{M}\Omega$	1–10 $\text{k}\Omega$
$\text{NEP W Hz}^{-1/2}$	-	-	
$D^* \text{ cm Hz}^{1/2}/\text{W}$	$1\times 10^9$	$5\times 10^9$	$1\times 10^9$

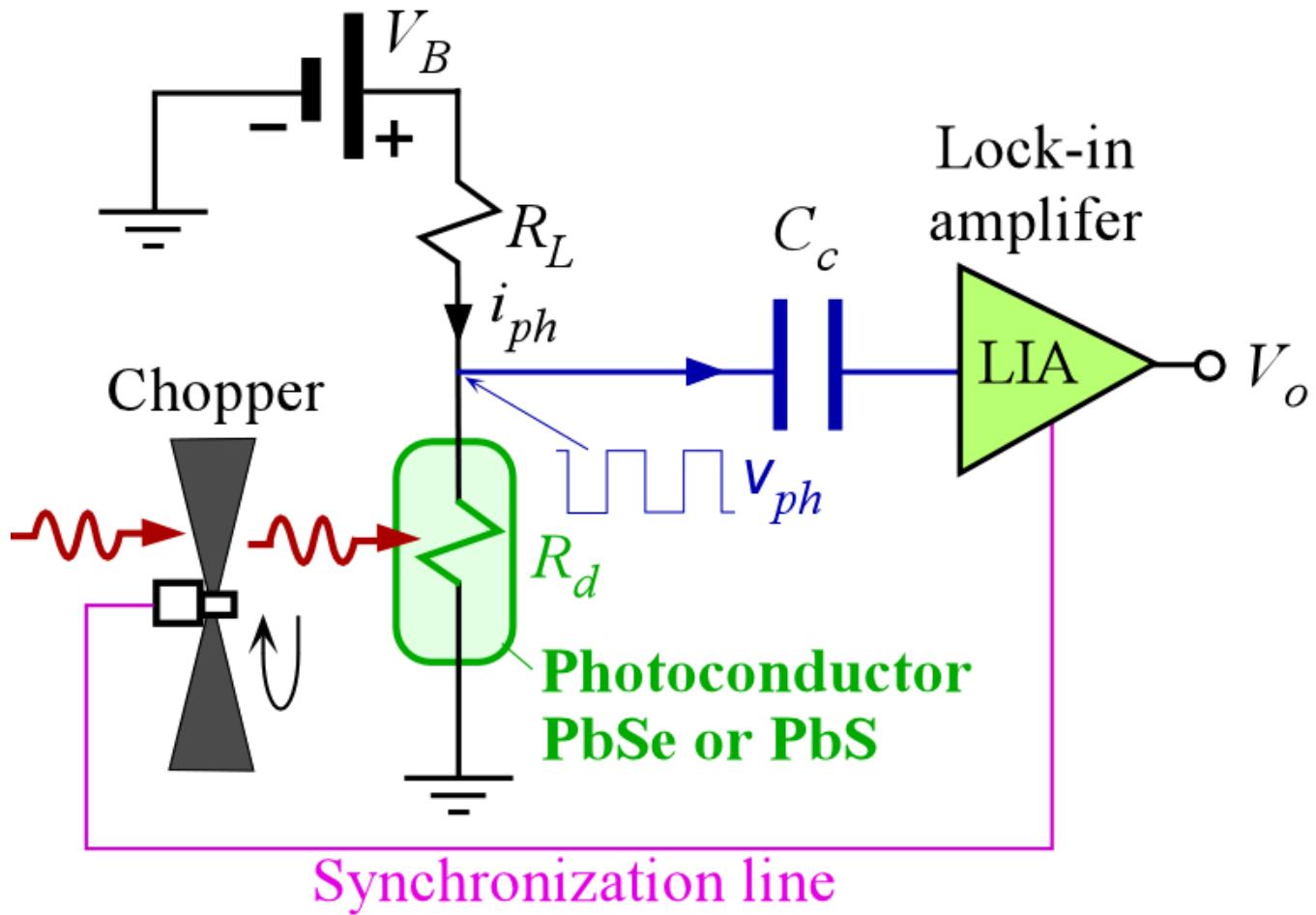
# Photoconductive (PC) Detectors



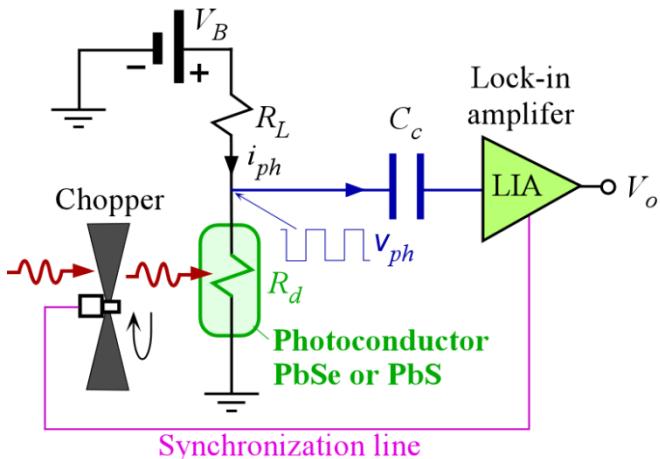
Used for radiation measurement at long wavelengths



PbS photoconductive  
detectors  
(Courtesy of  
Hamamatsu)



# Photoconductive (PC) Detectors



The basic principle in photoconductive detectors is the change in the resistance of the semiconductor upon exposure to light.

The photoconductor (PC) has a dark resistance  $R_d$  and is biased by  $V_B$  through a load  $R_L$ .

A chopper (either mechanical or electronic) chops the light at a frequency  $f_c$ . The resistance of the PC changes periodically at the chopper frequency.

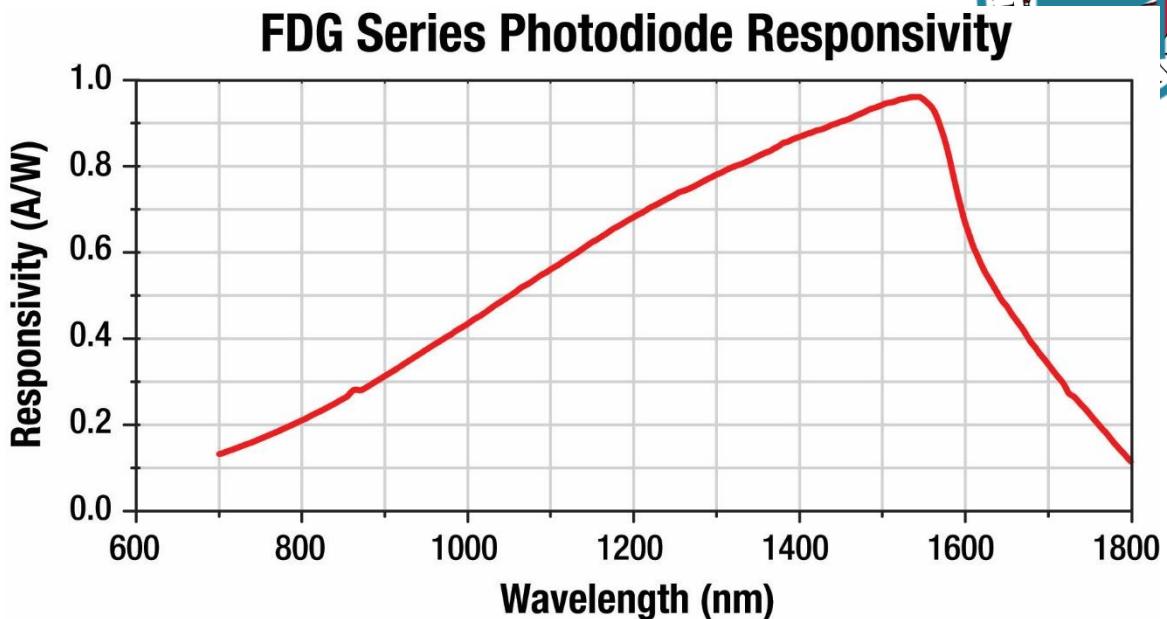
The periodic change in  $R_d$  causes a period change in the current. This periodic change is the photocurrent  $i_{ph}(t)$  signal and is an ac-type signal at the frequency  $f_c$ .

The photocurrent generates a varying voltage signal  $v_{ph}(t)$  across  $R_d$ , which can be coupled through a coupling capacitor into a lock-in amplifier LIA. This amplifier is synchronized with the chopper and only amplifies the signal if it is in phase with the chopper. Its output is a dc signal that represents the magnitude of  $v_{ph}$  that is in phase with the chopped light.

# Ge pin Photodiode



FDG03 Ge Photodiode  
(Courtesy of Thorlabs)



Responsivity of FDG03 Ge Photodiode  
(Courtesy of Thorlabs)

Peak responsivity = 1550 nm

Useful wavelength range = 700 – 1800 nm

NEP (at 1550 nm) =  $1 \times 10^{-12} \text{ W Hz}^{-1/2}$

Dark current = 4  $\mu\text{A}$

Advantage: High damage threshold

Disadvantage: High dark current

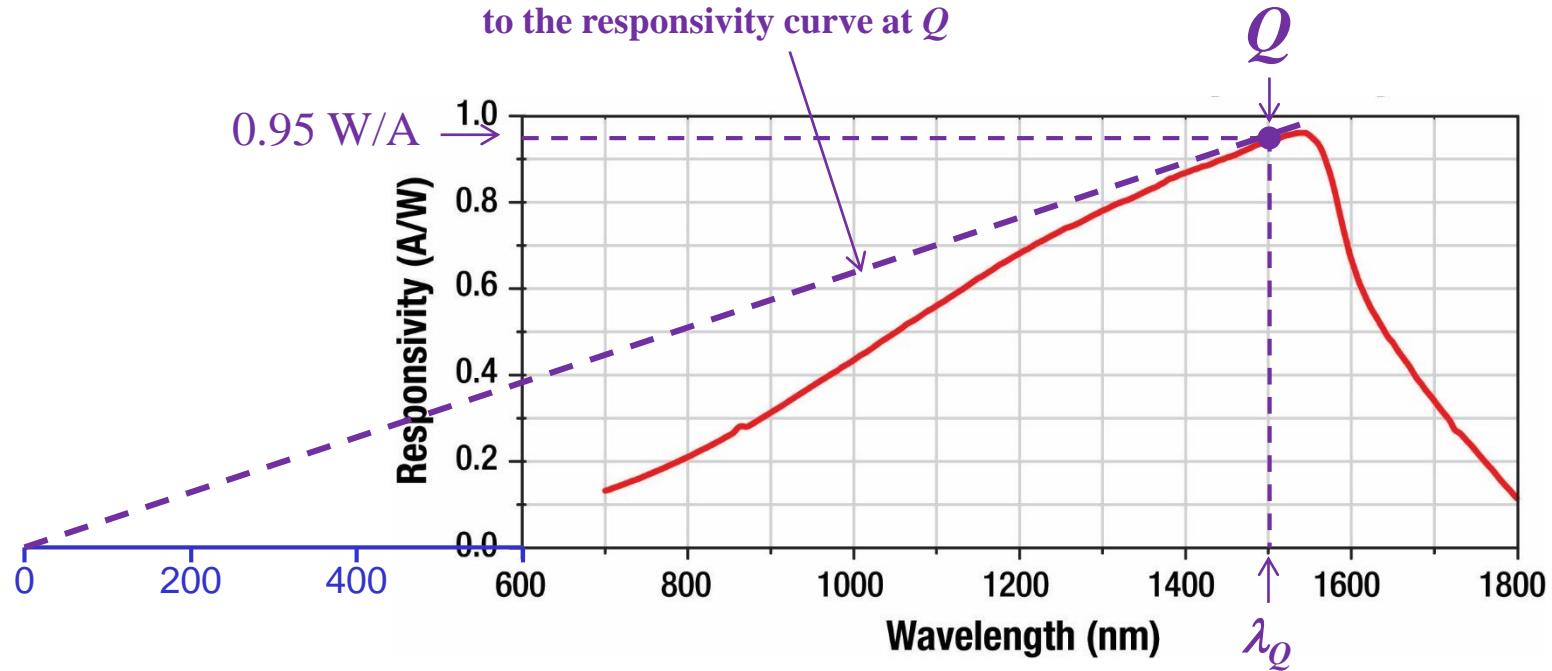
Active area diameter = 3.0 mm

Capacitance (maximum) = 4.0 nF

# Ge pin Photodiode



Line through the origin and tangent to the responsivity curve at  $Q$



Maximum QE occurs at  $Q$  when  $\lambda = \lambda_Q = 1500$  nm and  $R = 0.95$  W/A

$$\eta_e(\text{Maximum}) = \frac{hcR(\lambda_Q)}{e\lambda_Q}$$

$$\begin{aligned}\eta_e &= (6.626 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})(0.95 \text{ A W}^{-1}) \\ &/ (1.6 \times 10^{-19} \text{ C})(1500 \times 10^{-9} \text{ m}) = 0.79 \text{ or } 79\%\end{aligned}$$