A polychromator is an instrument in which a diffraction grating separates out different wavelengths (different λ_{Bs}) of light, which are then made incident on a CCD (charge coupled device) optical image sensor array (a pixelated photodetector). As the wavelength λ_{B1} shifts, the corresponding light beam becomes displaced, and moves along the pixels of the CCD sensor, which are read out from the CCD into a computer for data analysis and interpretation (done by software). Note that there is usually another FBG that generates a reference wavelength λ_R as shown in Figure 2.50.

EXAMPLE 2.15.1 Fiber Bragg grating at 1550 nm

A silica fiber-based FBG is required to operate at 1550 nm. What should be the periodicity of the grating Λ? If the amplitude of index variation Δn is 10^{-4} and total length of the FBG is 10 mm, what is the reflectance at the Bragg wavelength and the spectral width of the reflected light? Assume that the effective refractive index $\bar{n}$ is 1.450.

Solution

We can use $\lambda_B = 2\bar{n}\Lambda$, that is, $1550\,\text{nm} = 2(1.450)(\Lambda)$ so that the grating periodicity is

$$\Lambda = 534.5\ \text{nm}$$

The coupling coefficient κ is given in Eq. (2.15.2)

$$\kappa = \pi\Delta n/\lambda = \pi(10^{-4})/(1.550 \times 10^{-9}\,\text{m}) = 202.7\ \text{m}^{-1}$$

Thus, $\kappa L = 2.027$, so the FBG is a strong grating. The reflectance is

$$R = \tanh^2(\kappa L) = \tanh^2(2.027) = 0.933 \quad \text{or} \quad 93.3\%$$

The spectral width is given by

$$\Delta\lambda_{\text{strong}} = \frac{4\kappa\lambda_B^2}{\pi\bar{n}} = \frac{4(202.7)(1.55 \times 10^{-6})^2}{\pi(1.450)} = 0.428\,\text{nm}$$

The reflectance and spectral width values are approximate inasmuch as Eqs. (2.15.2) and (2.15.3) contain a number of assumptions.

Questions and Problems

2.1 Symmetric dielectric slab waveguide Consider two rays such as 1 and 2′ interfering at point P in Figure 2.4. Both are moving with the same incidence angle but have different κ_m wave vectors just before point P. In addition, there is a phase difference between the two due to the different paths taken to reach point P. We can represent the two waves as $E_1(y, z, t) = E_0\cos(\omega t - \kappa_m y - \beta_m z + \delta)$ and $E_2(y, z, t) = E_0\cos(\omega t + \kappa_m y - \beta_m z)$ where the $\kappa_m y$ terms have opposite signs indicating that the waves are traveling in opposite directions. δ has been used to indicate that the waves have a phase difference and travel different optical paths to reach point P. We also know that $\kappa_m = k_1\cos\theta_m$ and $\beta_m = k_1\sin\theta_m$, and obviously have the waveguide condition already incorporated into them through θ_m. Show that the superposition of E_1 and E_2 at P is given by

$$E(y, z, t) = 2E_0\cos\left(\kappa_m y - \frac{1}{2}\delta\right)\cos\left(\omega t - \beta_m z + \frac{1}{2}\delta\right)$$

What do the two cosine terms represent?

The planar waveguide is symmetric, which means that the intensity, E^2, must be either maximum (even m) or minimum (odd m) at the center of the guide. Choose suitable δ values and plot the relative magnitude of the electric field across the guide for $m = 0$, 1, and 2 for the following symmetric dielectric planar guide: $n_1 = 1.4550$, $n_2 = 1.4400$, $a = 10\,\mu\text{m}$, $\lambda = 1.5\,\mu\text{m}$ (free space), the first three modes have $\theta_1 = 88.84°$, $\theta_2 = 87.67°$, $\theta_3 = 86.51°$. Scale the field values so that the maximum field is unity for $m = 0$ at the center of the guide. [Alternatively, you can choose δ so that intensity (E^2) is the same at the boundaries at $y = a$ and $y = -a$; it would give the same distribution.]

2.2 Standing waves inside the core of a symmetric slab waveguide Consider a symmetric planar dielectric waveguide. Upward and downward traveling waves inside the core of the planar waveguide set up a standing wave along y. The standing wave can only exist if the wave can be replicated after it has traveled along the y-direction over one round trip. Put differently, a wave starting at A in Figure 2.51 and traveling toward the upper face will travel along y, be reflected at B, travel down, become reflected again at A, and then it would be traveling in the same direction as it started. At this point, it must have an identical phase to its starting phase so that it can replicate and not destroy itself. Given that the wave vector along y is κ_m, derive the waveguide condition.

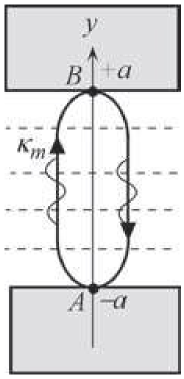


FIGURE 2.51 Upward and downward traveling waves along y set up a standing wave. The condition for setting up a standing wave is that the wave must be identical, and able to replicate itself, after one round trip along y.

2.3 Dielectric slab waveguide

(a) Consider the two parallel rays 1 and 2 in Figure 2.52. Show that when they meet at C at a distance y above the guide center, the phase difference is

$$\Phi_m = k_1 2(a - y)\cos\theta_m - \phi_m$$

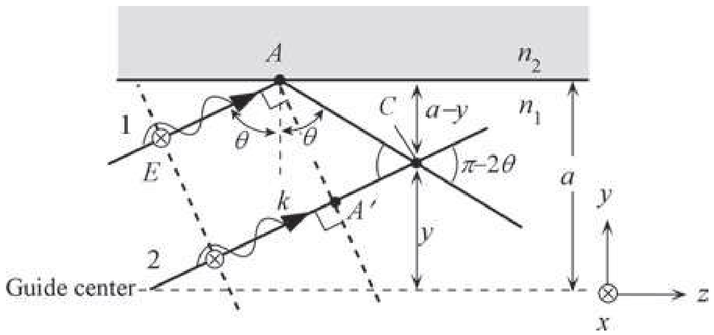


FIGURE 2.52 Rays 1 and 2 are initially in phase as they belong to the same wavefront. Ray 1 experiences total internal reflection at A. 1 and 2 interfere at C. There is a phase difference between the two waves. (For simplicity θ_m is shown as θ.)

(b) Using the waveguide condition, show that

$$\Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

(c) The two waves interfering at C can be most simply and conveniently represented as

$$E(y) = A\cos(\omega t) + A\cos\left[\omega t + \Phi_m(y)\right]$$

Hence find the amplitude of the field variation along y, across the guide. What is your conclusion?

2.4 Slab waveguide Consider an asymmetric planar waveguide with $n_1 = 2.35$ and $n_2 = 2.25$. The width of the waveguide is 2.5 μm, operating at 1300 nm. Calculate the number of TE and TM modes guided by this waveguide. Find out the range of wavelengths for which this waveguide will remain single-mode.

2.5 TE and TM Modes in dielectric slab waveguide Consider a planar dielectric guide with a core thickness of 15 μm, $n_1 = 1.45$, $n_2 = 1.43$, and light wavelength of 1.30 μm. Given the waveguide condition, and the expressions for phase changes ϕ and ϕ' in TIR for the TE and TM modes, respectively,

$$\tan\left(\tfrac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} \quad \text{and} \quad \tan\left(\tfrac{1}{2}\phi'_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\left(\frac{n_2}{n_1}\right)^2\cos\theta_m}$$

using a graphical solution find the angle θ for the fundamental TE and TM modes and compare their propagation constants along the guide.

2.6 Group velocity We can calculate the group velocity of a given mode as a function of frequency using a convenient math software package. It is assumed that the math software package can carry out symbolic algebra such as partial differentiation (the author used LiveMath by MathMonkeys, though others can also be used). The propagation constant of a given mode is $\beta = k_1 \sin\theta$, where β and θ imply β_m and θ_m. The objective is to express β and ω in terms of θ. Since $k_1 = n_1\omega/c$, the waveguide condition is

$$\tan\left(a\frac{\beta}{\sin\theta}\cos\theta - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta - (n_2/n_1)^2\right]^{1/2}}{\cos\theta}$$

so that

$$\beta \approx \frac{\tan\theta}{a}\left[\arctan\left(\sec\theta\sqrt{\sin^2\theta - (n_2/n_1)^2}\right) + m(\pi/2)\right] = F_m(\theta) \tag{P2.1}$$

where $F_m(\theta)$ is a function of θ at a given m. The frequency ω is given by

$$\omega = \frac{c\beta}{n_1\sin\theta} = \frac{c}{n_1\sin\theta}F_m(\theta) \tag{P2.2}$$

Both β and ω are now a function of θ in Eqs. (P2.1) and (P2.2). Then the group velocity is found by differentiating Eqs. (P2.1) and (P2.2) with respect to θ, that is,

$$v_g = \frac{d\omega}{d\beta} = \left[\frac{d\omega}{d\theta}\right] \times \left[\frac{d\theta}{d\beta}\right] = \frac{c}{n_1}\left[\frac{F'_m(\theta)}{\sin\theta} - \frac{\cos\theta}{\sin^2\theta}F_m(\theta)\right] \times \left[\frac{1}{F'_m(\theta)}\right]$$

that is,

Group velocity, planar waveguide

$$v_g = \frac{c}{n_1\sin\theta}\left[1 - \cot\theta\frac{F_m(\theta)}{F'_m(\theta)}\right] \tag{P2.3}$$

where $F'_m = dF_m/d\theta$ is found by differentiating the second term of Eq. (P2.1). For a given m value, Eqs. (P2.2) and (P2.3) can be plotted parametrically, that is, for each θ value we can calculate ω and v_g and plot v_g vs. ω. Figure 2.11 shows an example for a guide with the characteristics as given in the figure caption. Using a convenient math software package, or by other means, obtain a ω vs. β_m graph for this waveguide and obtain v_g. If ω vs. β_m is a straight line, point out its significance. Plot v_g vs. ω. Discuss intermodal dispersion.

2.7 Dielectric slab waveguide Consider a dielectric slab waveguide that has a thin GaAs layer of thickness 0.25 μm between two AlGaAs layers. The refractive index of GaAs is 3.6 and that of the AlGaAs layers is 3.4. What is the cutoff wavelength beyond which only a single mode can propagate in the waveguide, assuming that the refractive index does not vary greatly with the wavelength? If a radiation of wavelength 860 nm (corresponding to bandgap radiation) is propagating in the GaAs layer, what is the penetration of the evanescent wave into the AlGaAs layers? What is the mode field width (MFW) of this radiation? Point out the effect of change of radiation wavelength (λ) on the MFW

2.8 Dielectric slab waveguide Consider a slab dielectric waveguide that has a core thickness ($2a$) of 20 μm, $n_1 = 3.00$, $n_2 = 1.50$. Solution of the waveguide condition in Eq. (2.1.9) gives the mode angles θ_0 and θ_1 for the TE_0 and TE_1 modes for selected wavelengths as summarized in Table 2.7. For each wavelength calculate ω and β_m and then plot ω vs. β_m. On the same plot show the lines with slopes c/n_1 and c/n_2. Compare your plot with the dispersion diagram in Figure 2.10.

TABLE 2.7 **The solution of the waveguide condition for $a = 10\ \mu m$, $n_1 = 3.00$, $n_2 = 1.50$ gives the incidence angles θ_0 and θ_1 for modes 0 and 1 at the wavelengths shown**

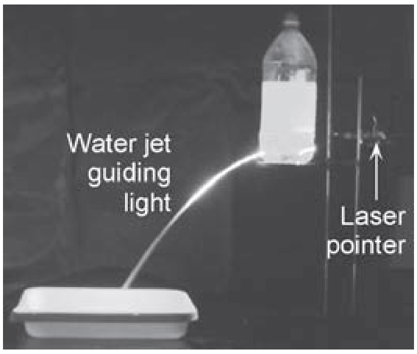
λ (μm)	15	20	25	30	40	45	50	70	100	150	200
$\theta_0°$	77.8	74.52	71.5	68.7	63.9	61.7	59.74	53.2	46.4	39.9	36.45
$\theta_1°$	65.2	58.15	51.6	45.5	35.5	32.02	30.17	–	–	–	–

2.9 **Dielectric slab waveguide** Consider a planar dielectric waveguide with a core thickness of 10 μm, $n_1 = 1.4446$, $n_2 = 1.4440$. Calculate the V-number, the mode angle θ_m for $m = 0$ (use a graphical solution, if necessary), penetration depth, and mode field width, MFW $= 2a + 2\delta$, for light wavelengths of 1.0 μm and 1.5 μm. What is your conclusion? Compare your MFW calculation with $2w_o = 2a(V + 1)/V$. The mode angle θ_0 is given as $\theta_0 = 88.85°$ for $\lambda = 1$ μm and $\theta_0 = 88.72°$ for $\lambda = 1.5$ μm for the fundamental mode $m = 0$.

2.10 **A multimode fiber** Consider a multimode fiber with a core diameter of 60 μm, core refractive index of 1.47, and a cladding refractive index of 1.45, both at 870 nm. Consider operating this fiber at $\lambda = 870$ nm. (a) Calculate the numerical aperture. (b) Find out the normalized core–cladding index difference. (c) Calculate the V-number for the fiber and estimate the number of guided modes. (d) Calculate the wavelength beyond which the fiber becomes single-mode. (e) Calculate the modal dispersion $\Delta\tau$ and hence estimate the bit rate × distance product.

2.11 **A water jet guiding light** One of the early demonstrations of the way in which light can be guided along a higher refractive index medium by total internal reflection involved illuminating the starting point of a water jet as it comes out from a water tank. The refractive index of water is 1.330. Consider a water jet of diameter 3 mm that is illuminated by green light of wavelength 560 nm. What is the V-number, numerical aperture, total acceptance angle of the jet? How many modes are there? What is the cutoff wavelength? The diameter of the jet increases (slowly) as the jet flows away from the original spout. However, the light is still guided. Why?

Light guided along a thin water jet. A small hole is made in a plastic soda drink bottle full of water to generate a thin water jet. When the hole is illuminated with a laser beam (from a green laser pointer), the light is guided by total internal reflections along the jet to the tray. Water with air bubbles (produced by shaking the bottle) was used to increase the visibility of light. Air bubbles scatter light and make the guided light visible. First such demonstration has been attributed to Jean-Daniel Colladon, a Swiss scientist, who demonstrated a water jet guiding light in 1841.



2.12 Results:
V = 2.48 > 2.405
so the fiber is multimode
(3 mode)
Lambda = 1.3416 um
NA = 0.17117
Alpha=9.9°
Dm = 8 ps...
Dw = -5 ps…
BL = 0.017Gb/s * km

2.12 **Single-mode fiber** Consider a fiber with an SiO_2-13.5%GeO_2 (*i.e.* 86.5%SiO_2-13.5%GeO_2%) core of diameter of 6 μm and refractive index of 1.47 and a cladding refractive index of 1.46, both refractive indices at 1300 nm, where the fiber is to be operated using a laser source with a half maximum width of 2 nm. (a) Calculate the V-number for this fiber. (b) What is the maximum allowed diameter of the core that maintains operations in single-mode? (c) Calculate the wavelength below which the fiber becomes multimode. (d) Calculate the numerical aperture. (e) Calculate the maximum acceptance angle. (f) Obtain the material dispersion and waveguide dispersion and hence estimate the bit rate × distance product ($B \times L$) of the fiber.

2.13 **Single-mode fiber** Consider a step-index fiber with a core of diameter of 8 μm and refractive index of 1.45 at 1.55 μm and a normalized refractive index difference of 0.25% where the fiber is to be operated using a laser source with a half-maximum width of 3 nm. At 1.55 μm, the material and waveguide dispersion coefficients of this fiber are approximately given by $D_m = 12$ ps nm^{-1} km^{-1} and $D_w = -6$ ps nm^{-1} km^{-1} (a) Calculate the V-number for the fiber. Is this a single-mode fiber? (b) Calculate the wavelength below which the fiber becomes multimode. (c) Calculate the numerical aperture. (d) Calculate the maximum total acceptance angle. (e) Calculate the material, waveguide, and chromatic dispersion per kilometer of fiber. (f) Estimate the bit rate × distance product ($B \times L$) of this fiber. (g) What is the mode field diameter? (h) Find out optical and electrical bandwidth for a 10 km link length of this fiber.

2.14 Normalized propagation constant *b* Consider a weakly guiding step-index fiber in which $(n_1 - n_2)/n_1$ is very small. Show that

(a)
$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{(\beta/k) - n_2}{n_1 - n_2}$$

Note: Since Δ is very small, $n_2/n_1 \approx 1$ can be assumed were convenient. The first equation can be rearranged as

$$\beta/k = \left[n_2^2 + b(n_1^2 - n_2^2)\right]^{1/2} = n_2^2(1 + x)^{1/2}; \quad x = b(n_1^2 - n_2^2)/n_2^2$$

where x is small. Taylor's expansion in x to the first linear term would then provide a linear relationship between β and b.

(b) Plot a graph between V value vs. b of the step-index fiber for fundamental and first higher order modes.

2.15 Group velocity of the fundamental mode Reconsider Example 2.3.4, which has a single-mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 μm, operating at 1.5 μm. Use the equation

$$b \approx \frac{(\beta/k) - n_2}{n_1 - n_2}; \quad \beta = n_2 k\left[1 + b\Delta\right]$$

to recalculate the propagation constant β. Change the operating wavelength to 1.3 μm, and then recalculate the new propagation constant β. Then determine the group velocity v_g of the fundamental mode at 1.55 μm, and the group delay τ_g over 2 km of fiber.

2.16 A single-mode fiber design The Sellmeier dispersion equation provides n vs. λ for pure SiO_2 and SiO_2-13.5 mol.%GeO_2 in Table 1.2 in Chapter 1. The refractive index increases linearly with the addition of GeO_2 to SiO_2 from 0 to 13.5 mol.%. A single-mode step-index fiber is required to have the following properties: NA = 0.15, core diameter of 8 μm, and a cladding of pure silica, and operate at 1.3 μm and 1.55 μm. What should the core composition be at these wavelengths?

2.17 Material dispersion If N_g is the group refractive index of the core material of a step fiber, then the propagation time (group delay time) of the fundamental mode is

$$\tau = L/v_g = LN_{g1}/c.$$

Since N_g will depend on the wavelength, show that the material dispersion coefficient D_m is given approximately by

$$D_m = \frac{d\tau}{Ld\lambda} \approx \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

Using the Sellmeier equation and the constants in Table 1.2 in Chapter 1, evaluate the material dispersion at $\lambda = 1.3$ and 1.55 μm for pure silica (SiO_2) and SiO_2-13.5%GeO_2 glass.

2.18 Waveguide dispersion Waveguide dispersion arises as a result of the dependence of the propagation constant on the V-number, which depends on the wavelength. It is present even when the refractive index is constant—no material dispersion. Let us suppose that n_1 and n_2 are wavelength (or k) independent. Suppose that β is the propagation constant of mode lm and $k = 2\pi/\lambda$ in which λ is the free-space wavelength. Then the normalized propagation constant b and propagation constant β are related by

$$\beta = n_2 k(1 + b\Delta) \tag{P2.4}$$

The group velocity is defined and given by

$$v_g = \frac{d\omega}{d\beta} = c\frac{dk}{d\beta}$$

Show that the propagation time, or the group delay time, τ of the mode is

$$\tau = \frac{L}{v_g} = \frac{Ln_2}{c} + \frac{Ln_2\Delta}{c}\frac{d(kb)}{dk} \tag{P2.5}$$

Given the definition of V

$$V = ka\left[n_1^2 - n_2^2\right]^{1/2} \approx kan_2(2\Delta)^{1/2} \tag{P2.6}$$

and

$$\frac{d(Vb)}{dV} = \frac{d}{dV}\left[bkan_2(2\Delta)^{1/2}\right] = an_2(2\Delta)^{1/2}\frac{d}{dV}(bk) \tag{P2.7}$$

Show that

$$\frac{d\tau}{d\lambda} = -\frac{Ln_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} \quad \textbf{(P2.8)}$$

and that the waveguide dispersion coefficient is

Waveguide dispersion coefficient

$$D_w = \frac{d\tau}{Ld\lambda} = -\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} \quad \textbf{(P2.9)}$$

Figure 2.53 shows the dependence of $V\left[d^2(Vb)/dV^2\right]$ on the V-number. In the range $1.5 < V < 2.4$,

$$V\frac{d^2(Vb)}{dV^2} \approx \frac{1.984}{V^2}$$

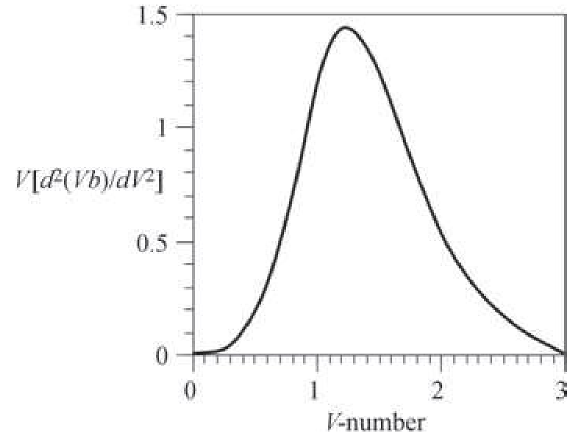


FIGURE 2.53 $d^2(Vb)/dV^2$ vs. V-number for a step-index fiber. (*Source*: Data extracted from W. A. Gambling *et al.*, *The Radio and Electronics Engineer*, *51*, 313, 1981.)

Show that,

$$D_w \approx -\frac{n_2\Delta}{c\lambda}\frac{1.984}{V^2} = -\frac{(n_1 - n_2)}{c\lambda}\frac{1.984}{V^2} \quad \textbf{(P2.10)}$$

which simplifies to

$$D_w \approx -\frac{1.984}{c(2\pi a)^2 2n_2}\lambda \quad \textbf{(P2.11)}$$

that is,

Waveguide dispersion coefficient

$$D_w(\text{ps nm}^{-1}\text{ km}^{-1}) \approx -\frac{83.76\,\lambda(\mu\text{m})}{\left[a(\mu\text{m})\right]^2 n_2} \quad \textbf{(P2.12)}$$

Consider a fiber with a core of diameter of 8 μm and refractive index 1.468 and a cladding refractive index of 1.464, both refractive indices at 1300 nm. Suppose that a 1.3-μm laser diode with a spectral linewidth of 2 nm is used to provide the input light pulses. Estimate the waveguide dispersion per kilometer of fiber using Eqs. (P2.9) and (P2.11).

2.19 Profile dispersion Total dispersion in a single-mode, step-index fiber is primarily due to material dispersion and waveguide dispersion. However, there is an additional dispersion mechanism called *profile dispersion* that arises from the propagation constant β of the fundamental mode also depending on the refractive index difference Δ. Consider a light source with a range of wavelengths $\delta\lambda$ coupled into a step-index fiber. We can view this as a change $\delta\lambda$ in the input wavelength λ. Suppose that n_1, n_2, hence Δ depends on the wavelength λ. The propagation time, or the group delay time, τ_g *per unit length* is

$$\tau_g = 1/v_g = d\beta/d\omega = (1/c)(d\beta/dk) \quad \textbf{(P2.13)}$$

where k is the free-space propagation constant $(2\pi/\lambda)$, and we used $d\omega = cdk$. Since β depends on n_1, Δ, and V, consider τ_g as a function of n_1, Δ (thus n_2), and V. A change $\delta\lambda$ in λ will change each of these quantities. Using the partial differential chain rule

$$\frac{\delta\tau_g}{\delta\lambda} = \frac{\partial\tau_g}{\partial n_1}\frac{\partial n_1}{\partial\lambda} + \frac{\partial\tau_g}{\partial V}\frac{\partial V}{\partial\lambda} + \frac{\partial\tau_g}{\partial\Delta}\frac{\partial\Delta}{\partial\lambda} \tag{P2.14}$$

The mathematics turns out to be complicated, but the statement in Eq. (P2.14) is equivalent to

$$\begin{aligned}\text{Total dispersion} = {} & \text{Material dispersion (due to } \partial n_1/\partial\lambda) \\ & + \text{Waveguide dispersion (due to } \partial V/\partial\lambda) + \text{Profile dispersion (due to } \partial\Delta/\partial\lambda)\end{aligned}$$

in which the last term is due to Δ depending on λ; although small, this is not zero. Even the statement in Eq. (P2.14) above is oversimplified but nonetheless provides an insight into the problem. The total intramode (chromatic) dispersion coefficient D_{ch} is then given by

$$D_{ch} = D_m + D_w + D_p \tag{P2.15}$$

in which D_m, D_w, and D_p are material, waveguide, and profile dispersion coefficients, respectively. The waveguide dispersion is given by Eqs. (P2.11) and (P2.12), and the *profile dispersion coefficient* is (very) approximately,[30]

$$D_p \approx -\frac{N_{g1}}{c}\left(V\frac{d^2(Vb)}{dV^2}\right)\left(\frac{d\Delta}{d\lambda}\right) \tag{P2.16}$$

Profile dispersion coefficient

in which b is the normalized propagation constant and $Vd^2(Vb)/dV^2$ vs. V is shown in Figure 2.53. We can also use $Vd^2(Vb)/dV^2 \approx 1.984/V^2$.

Consider a fiber with a core of diameter of 8 μm. The refractive and group indices of the core and cladding at $\lambda = 1.55\,\mu\text{m}$ are $n_1 = 1.4500$, $n_2 = 1.4444$, $N_{g1} = 1.4680$, $N_{g2} = 1.4628$, and $d\Delta/d\lambda = 232\,\text{m}^{-1}$. Estimate the waveguide and profile dispersion per km of fiber per nm of input light linewidth at this wavelength. (*Note*: The values given are approximate and for a fiber with silica cladding and 3.6% germania-doped core.)

2.20 Dispersion at zero dispersion coefficient Since the spread in the group delay $\Delta\tau$ along a fiber depends on the $\Delta\lambda$, the linewidth of the source, we can expand $\Delta\tau$ as a Taylor series in $\Delta\lambda$. Consider the expansion at $\lambda = \lambda_0$ where $D_{ch} = 0$. The first term with $\Delta\lambda$ would have $d\Delta\tau/d\lambda$ as a coefficient, that is D_{ch}, and at λ_0 this will be zero; but not the second term with $(\Delta\lambda)^2$ that has a differential, $d^2\Delta\tau/d\lambda^2$ or $dD_{ch}/d\lambda$. Thus, the dispersion at λ_0 would be controlled by the slope S_0 of D_{ch} vs. λ curve at λ_0. Show that the chromatic dispersion at λ_0 is

$$\Delta\tau = \frac{L}{2}S_0(\Delta\lambda)^2$$

Chromatic dispersion at λ_0

A single-mode fiber has a zero dispersion at $\lambda_0 = 1310$ nm, dispersion slope $S_0 = 0.090$ ps nm^{-2} km. What is the dispersion for a laser with $\Delta\lambda = 1.5$ nm? What would control the dispersion in practice?

2.21 Polarization mode dispersion A fiber manufacturer specifies a maximum value of 0.025 ps km$^{-1/2}$ for the polarization mode dispersion (PMD) in its single-mode fiber. What would be the dispersion, maximum bit rate, and the optical bandwidth for this fiber over an optical link that is 400 km long if the only dispersion mechanism was PMD?

2.22 Polarization mode dispersion Consider a particular single-mode fiber (ITU-T G.652 compliant) that has a chromatic dispersion of 18 ps nm^{-1} km^{-1}. The chromatic dispersion is zero at 1300 nm, and the dispersion slope is 0.092 ps nm^{-2} km^{-1}. The PMD coefficient is 0.05 ps km$^{-1/2}$. Calculate the total dispersion over 180 km if the fiber is operated at 1300 nm and the source is a laser diode with a linewidth $\Delta\lambda = 1$ nm. What should be the linewidth of the laser source so that over 180 km the chromatic dispersion is the same as PMD?

2.23 Dispersion compensation Calculate the total dispersion and the overall net dispersion coefficient when a 1000-km transmission fiber with chromatic dispersion $D_{ch} = +20$ ps nm^{-1} km^{-1} is spliced to a compensating fiber that is 100 km long and has $D_{ch} = -100$ ps nm^{-1} km^{-1}. Assume that the input light spectral width is 1 nm.

2.24 Cladding diameter A comparison of two step-index fibers, one SMF and the other MMF, shows that the SMF has a core diameter of 9 μm but a cladding diameter of 125 μm, while the MMF has a core diameter of 100 μm but a cladding diameter of 125 μm. Discuss why the manufacturer has chosen those values.

[30] J. Gowar, *Optical Communications Systems*, 2nd Edition (Prentice Hall, Pearson Education, 1993), Ch. 8 has the derivation of this equation.

2.25 Graded index fiber Consider an optimal graded index fiber with a core diameter of 20 μm and a refractive index of 1.475 at the center of the core and a cladding refractive index of 1.453. What are its NA at the fiber axis, and its effective NA? Calculate the V value and the number of modes at 1300 nm operation for this fiber. Suppose that the fiber is coupled to a laser diode emitter at 1300 nm and a spectral linewidth (FWHM) of 2 nm. The material dispersion coefficient at this wavelength is about -5 ps nm^{-1} km^{-1}. Calculate the total dispersion and estimate the bit rate $\times$ distance product of the fiber. How does this compare with the performance of a multimode fiber with same core radius, and n_1 and n_2? What would the total dispersion and maximum bit rate be if an LED source of spectral width (FWHM) $\Delta\tau_{1/2} \approx 60$ nm is used?

2.26 Graded index fiber Consider a graded index fiber with a core diameter of 62.5 μm and a refractive index of 1.47 at the center of the core and a cladding refractive index of 1.45 at a wavelength 1550 nm. Suppose that we use a laser diode emitter with a spectral FWHM linewidth of 2 nm to transmit along this fiber at a wavelength of 1550 nm. Calculate the total dispersion and estimate the bit rate $\times$ distance product of the fiber. The material dispersion coefficient D_m at 1550 nm is -20 ps nm^{-1} km^{-1}. How does this compare with the performance of a multimode fiber with the same core radius, and n_1 and n_2?

2.27 Graded index fiber A standard graded index fiber from a particular fiber manufacturer has a core diameter of 62.5 μm, cladding diameter of 125 μm, a NA of 0.300. The core refractive index n_1 is 1.455. The manufacturer quotes minimum optical bandwidth $\times$ distance values of 400 MHz · km at 1300 nm and 800 MHz · km at 1550 nm. Assume that a laser is to be used with this fiber and the laser linewidth $\Delta\lambda = 2$ nm. What are the corresponding dispersion values? What type of dispersion do you think dominates? Is the graded index fiber assumed to have the ideal optimum index profile? (State your assumptions.) What is the optical link distance for operation at 2 Gbs^{-1} at 850 nm and 1550 nm?

2.28 Graded index fiber and optimum dispersion The graded index fiber theory and equations tend to be quite complicated. If γ is the profile index then the rms *intermodal dispersion* is given by[31]

$$\sigma = \frac{Ln_1\Delta}{2c}\left(\frac{\gamma}{\gamma+1}\right)\left(\frac{\gamma+2}{3\gamma+2}\right)^{1/2} \times \left[c_1^2 + \frac{4c_1c_2(\gamma+1)\Delta}{2\gamma+1} + \frac{16c_2^2\Delta^2(\gamma+1)^2}{(5\gamma+2)(3\gamma+2)}\right]^{1/2} \quad \textbf{(P2.17)}$$

where c_1 and c_2 are given by

$$c_1 = \frac{\gamma - 2 - \delta}{\gamma + 2}; \quad c_2 = \frac{3\gamma - 2 - 2\delta}{2(\gamma + 2)}; \quad \delta = -2\left(\frac{n_1}{N_{g1}}\right)\left(\frac{\lambda}{\Delta}\right)\left(\frac{d\Delta}{d\lambda}\right) \quad \textbf{(P2.18)}$$

where δ is a small unitless parameter that represents the change in Δ with λ. The optimum profile coefficient γ_o is

$$\gamma_o = 2 + \delta - \Delta\frac{(4+\delta)(3+\delta)}{(5+2\delta)} \quad \textbf{(P2.19)}$$

Consider a graded index fiber for use at 850 nm, with $n_1 = 1.475$, $N_{g1} = 1.489$, $\Delta = 0.015$, $d\Delta/d\lambda = -683$ m^{-1}. Plot σ in ps km^{-1} vs. γ from $\gamma = 1.8$ to 2.4 and find the minimum. (Consider plotting σ on a logarithmic axis.) Compare the minimum σ and the optimum γ, with the relevant expressions in Section 2.8. Find the percentage change in γ for a 10× increase in σ. What is your conclusion?

2.29 GRIN rod lenses Figure 2.32 shows graded index rod lenses. (a) How would you represent Figure 2.32 (a) using two conventional converging lenses. What are O and O'? (b) How would you represent Figure 2.32 (b) using a conventional converging lens. What is O'? (c) Sketch ray paths for a GRIN rod with a pitch between 0.25P and 0.5P starting from O at the face center. Where is O'? (d) What use is 0.23P GRIN rod lens in Figure 2.32 (c)?

2.30 Photonic Crystal Fibers (PCFs) Explain the lightwave guidance mechanism in solid and hollow core PCFs. Why are PCFs called endlessly single-mode fibers?

2.31 Attenuation Average power launched into a 10 km length of optical fiber is 100 μW and the average optical power at the fiber output is 2 μW. (a) Find the signal attenuation in dB assuming there are no splices. (b) Find the signal attenuation per km for this fiber. (c) Find the overall signal attenuation for a 10 km optical link using the same fiber with splices at 1 km intervals, with splice loss of 1 dB.

[31] R. Olshansky and D. Keck, *Appl. Opt.*, *15*, 483, 1976.

2.32 Cut-back method of attenuation measurement Cut-back method is a destructive measurement technique for determining the attenuation α of a fiber. The first part of the experiment involves measuring the optical power P_{far} coming out from the fiber at the far end as shown in Figure 2.54. Then, in the second part, keeping everything the same, the fiber is cut close to the launch or the source end. The output power P_{near} is measured at the near end from the shortcut fiber. The attenuation is then given by

$$\alpha = (-10/L)\log(P_{\text{far}}/P_{\text{near}})$$

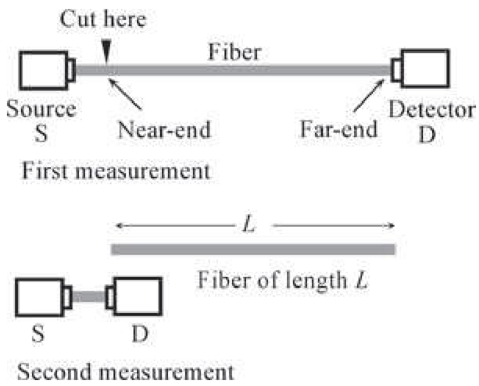


FIGURE 2.54 Illustration of the cut-back method for measuring the fiber attenuation. S is an optical source and D is a photodetector.

in which L is the separation of the measurement points, the length of the cut fiber, and α is in dB per unit length. The output P_{near} from the shortcut fiber in the second measurement is actually the input into the fiber under test in the first experiment. Usually a mode scrambler (mode stripper) is used for multimode fibers before the input. The power output from a particular fiber is measured to be 13 nW. Then, 10 km of fiber is cut out and the power output is measured again and found to be 43 nW. What is the attenuation of the fiber?

2.33 Intrinsic losses

(a) Consider a silica-based standard single-mode fiber with a NA of 0.15. What is its attenuation at 1550 nm? How does this compare with the attenuation quotes for the Corning SMF@28e$^+$, 0.16–0.19 dB km^{-1}at 1550 nm?

(b) Consider a step index fiber $n_1 = 1.47$ and $n_2 = 1.46$, core diameter = 50 μm. Corresponding to the maximum value of the incident angle for which light rays will be guided through this fiber, calculate the number of reflections and the loss that would take place in one kilometre length of the fiber assuming 0.01% loss of power at each reflection.

2.34 Scattering losses and fictive temperature Rayleigh scattering process decreases with wavelength, and as mentioned in Chapter 1, it is inversely proportional to λ^4. The expression for the attenuation α_R in a single component glass such as silica due to Rayleigh scattering is approximately given by two sets of different equations in the literature[32]

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_T k_B T_f \quad \text{and} \quad \alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2 - 1)^2 \beta_T k_B T_f$$

in which λ is the free-space wavelength, n is the refractive index at the wavelength of interest, β_T is the isothermal compressibility (at T_f) of the glass, k_B is the Boltzmann constant, and T_f is a quantity called the *fictive temperature* (or the glass transition temperature) at which the liquid structure during the cooling of the fiber is frozen to become the glass structure. Fiber is drawn at high temperatures. As the fiber cools, eventually the temperature drops sufficiently for the atomic motions to be so sluggish that the structure becomes essentially "frozen-in" and remains like this even at room temperature. Thus, T_f marks the temperature below which the liquid structure is frozen and hence the density fluctuations are also frozen into the glass structure. Use these two equations and calculate the attenuation in dB km^{-1} due to Rayleigh scattering at $\lambda = 1.55$ μm given that pure silica (SiO_2) has the following properties: $T_f \approx 1180$°C; $\beta_T \approx 7 \times 10^{-11}$ m^2 N^{-1} (at high temperatures); $n = 1.5$ at 1.5 μm; $p = 0.28$. The lowest reported attenuation around this wavelength is about 0.14 dB km^{-1}. What is your conclusion? (Note that α_B above is in Nepers per meter.)

2.35 Bending loss Bending losses always increase with the mode field diameter. Since the MFD increases for decreasing V, $2w \approx 2 \times 2.6a/V$, smaller V fibers have higher bending losses. How does the bending loss

[32]For example, R. Olshansky, *Rev. Mod. Phys.*, *51*, 341, 1979.

α vs. radius of curvature R behavior look like on a semilogarithmic plot [as in Figure 2.39 (a)] for two values of the V-number V_1 and V_2 if $V_2 > V_1$? It is found that for a single-mode fiber with a cutoff wavelength $\lambda_c = 1180$ nm, operating at 1300 nm, the microbending loss reaches 1 dB m^{-1} when the radius of curvature of the bend is roughly 6 mm for $\Delta = 0.00825$, 12 mm for $\Delta = 0.00550$, and 35 mm for $\Delta = 0.00275$. Explain these findings.

2.36 Bend loss reduced fibers Consider the bend loss measurements listed in Table 2.8 for four different types of fiber. The trench fibers have a trench placed in the cladding where the refractive index is lowered as shown in Figure 2.39. The nanoengineered fiber is shown in Figure 2.55. There is a ring of region in the cladding in which there are gas-filled nanoscale voids. (They are introduced during fabrication.) A void in the ring has a circular cross-section but has a length along the fiber that can be a few meters. These voids occupy a volume in the ring that is only 1–10%. Plot the bending loss semilogarithmically (α on a log scale and R on a linear scale) and fit the data to $\alpha_{\text{micobend}} = A\exp(-R/R_c)$ and find A and R_c. What is your conclusion? Suppose that we set our maximum acceptable bending loss to 0.1 dB turn^{-1} in installation (the present goal is to bring the bending loss to below 0.1 dB turn^{-1}). What are the allowed radii of curvature for each turn?

TABLE 2.8 Bend radius R in mm, α in dB turn^{-1}. Data over 1.55–1.65 μm range

Standard SMF[a] 1550 nm		Trench Fiber 1[b] 1650 nm		Trench Fiber 2[c] 1625 nm		Nanoengineered Fiber[a] 1550 nm	
R (mm)	α (dB turn^{-1})	R (mm)	α (dB turn^{-1})	R (mm)	α (dB turn^{-1})	R (mm)	α (dB turn^{-1})
5.0	15.0	7.50	0.354	5.0	0.178	5.0	0.031
7.0	4.00	10.0	0.135	7.5	0.0619	7.5	0.0081
10.0	0.611	15.0	0.020	10.0	0.0162	10.0	0.0030
12.5	0.124			15.0	0.00092	15.0	0.00018
16.0	0.0105						
17.5	0.0040						

(*Source:* Data used from a number of sources: [a]M. -J. Li *et al.*, *J. Light Wave Technol.*, *27*, 376, 2009; [b]K. Himeno *et al.*, *J. Light Wave Technol.*, *23*, 3494, 2005; [c]L. -A. de Montmorillon, *et al.* "Bend-Optimized G.652D Compatible Trench-Assisted Single-Mode Fibers," *Proceedings of the 55th IWCS/Focus*, pp. 342–347, November, 2006.)

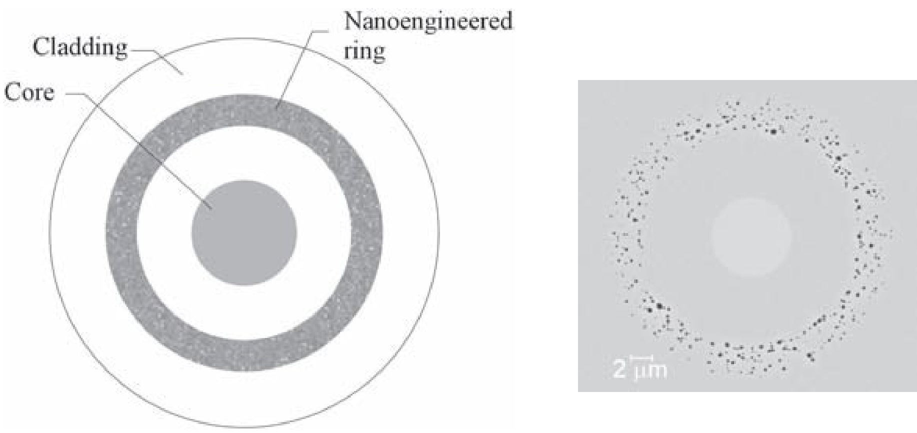


FIGURE 2.55 Left: The basic structure of bend-insensitive fiber with a nanoengineered ring in the cladding. Right: An SEM picture of the cross-section of a nanoengineered fiber with reduced bending losses. *(Courtesy of Ming-Jun Li, Corning Inc. For more information see U.S. Patent 8,055.110, 2011.)*

2.37 Microbending loss Microbending loss α_B depends on the fiber characteristics and wavelength. We will calculate α_B approximately given various fiber parameters using the single-mode fiber microbending loss equation (D. Marcuse, *J. Op. Soc. Am.*, *66*, 216, 1976)

$$\alpha_B \approx \frac{\pi^{1/2}\kappa^2}{2\gamma^{3/2}V^2\left[K_1(\gamma a)\right]^2} R^{-1/2} \exp\left(-\frac{2\gamma^3}{3\beta^2}R\right)$$

where R is the bend radius of curvature; a is the fiber radius; β is the propagation constant, determined by b, normalized propagation constant, which is related to V, $\beta = n_2 k\left[1 + b\Delta\right]$; $k = 2\pi/\lambda$ is the free-space wave vector; $\gamma = [\beta^2 - n_2^2 k^2]^{1/2}$; $\kappa = [n_1^2 k^2 - \beta^2]^{1/2}$; and $K_1(x)$ is a first-order modified Bessel function, available in math software packages. The normalized propagation constant b can be found from $b = (1.1428 - 0.996V^{-1})^2$. Consider a single-mode fiber with $n_1 = 1.450$, $n_2 = 1.446$, $2a$ (diameter) $= 3.9\ \mu$m. Plot α_B vs. R for $\lambda = 633$ nm and 790 nm from $R = 2$ mm to 15 mm. Figure 2.56 shows the experimental results on an SMF that has the same properties as the fiber above. While the data in Figure 2.56 are for a 10 cm fiber, the equation above gives the attenuation per meter. We need to divide the calculated values by 10 to compare the two sets of data. What is your conclusion? (Note that while the data in Figure 2.56 are for a 10 cm fiber, the equation above gives the attenuation per meter. We need to divide the calculated attenuation values by 10 to compare the two sets of data. You might wish to compare your calculations with the experiments of A. J. Harris and P. F. Castle, *IEEE J. Light Wave Technol., LT4*, 34, 1986).

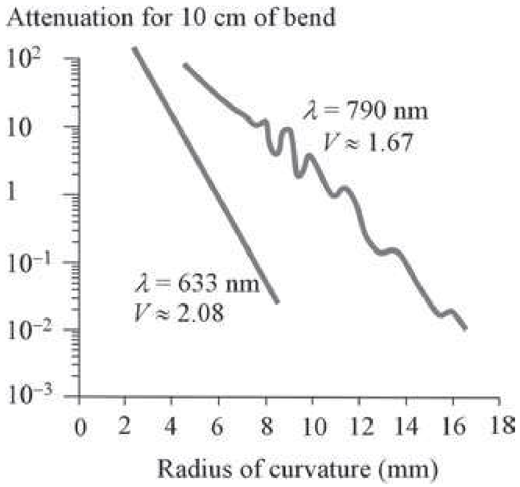


FIGURE 2.56 Measured microbending loss (attenuation) for a 10-cm fiber bent by different amounts of radius of curvature R. Single-mode fiber with a core diameter of 3.9 μm, cladding radius of 48 μm, $\Delta = 0.00275$, NA ≈ 0.10, $V \approx 1.67$ and 2.08. (*Source:* Data extracted from A. J. Harris and P. F. Castle, "Bend Loss Measurements on High Aperture Single-Mode Fibers as a Function of Wavelength and Bend Radius," *IEEE J. Light Wave Technol.*, *LT14*, 34, 1986, and replotted with a smoothed curve; [see original article for the discussion of peaks in α_B vs. R at 790 nm]).

2.38 Fiber Bragg Grating (FBG) A silica fiber-based FBG is required to operate at 850 nm. What should be the periodicity of the grating Λ? If the amplitude of the index variation Δn is 2×10^{-5} and total length of the FBG is 5 mm, what are the maximum reflectance at the Bragg wavelength and the bandwidth of the FBG? Assume that the effective refractive index $\bar{n}$ is 1.460. What are the reflectance and the bandwidth if Δn is 2×10^{-4}?

Fiber Bragg Grating (FBG)–based optical strain sensor for monitoring strains in civil structures, for example, bridges, dams, buildings, and tunnels. The fiber is mounted on a steel carrier and maintained stretched. *(Courtesy of Micron Optics.)*

2.39 Fiber Bragg Grating sensor array Consider a FBG strain sensor array embedded in a silica fiber that is used to measure strain at various locations on an object. Two neighboring sensors have grating periodicities of $\Lambda_1 = 534.5$ nm and $\Lambda_2 = 539.7$ nm. The effective refractive index is 1.450 and the photoelastic coefficient is 0.22. What is the maximum strain that can be measured? What would be the main problem with this sensor array? What is the strain at fracture if the fiber fractures roughly at an applied stress of 700 MPa and the elastic modulus is 70 GPa? What is your conclusion?

Fiber Bragg Grating (FBG)–based optical temperature sensors for use from −200°C to 250°C. FBGs are mounted in different packages: top left, all dielectric; top right, stainless steel; and bottom, copper. The sensors operate over 1510–1590 nm. *(Courtesy of Lake Shore Cryotronics Inc.)*