EXAMPLE 1.1.1 A diverging laser beam

Consider a He-Ne laser beam at 633 nm with a spot size of 1 mm. Assuming a Gaussian beam, what is the divergence of the beam? What are the Rayleigh range and the beam width at 25 m?

Solution

Using Eq. (1.1.7), we find

$$2\theta = \frac{4\lambda}{\pi(2w_o)} = \frac{4(633 \times 10^{-9}\ \mathrm{m})}{\pi(1 \times 10^{-3}\ \mathrm{m})} = 8.06 \times 10^{-4}\ \mathrm{rad} = 0.046^\circ$$

The Rayleigh range is

$$z_o = \frac{\pi w_o^2}{\lambda} = \frac{\pi\left[(1 \times 10^{-3}\ \mathrm{m})/2\right]^2}{(633 \times 10^{-9}\ \mathrm{m})} = 1.24\ \mathrm{m}$$

The beam width at a distance of 25 m is

$$2w = 2w_o[1 + (z/z_o)^2]^{1/2} = (1 \times 10^{-3}\ \mathrm{m})\{1 + [(25\ \mathrm{m})/(1.24\ \mathrm{m})]^2\}^{1/2}$$

$$= 0.0202\ \mathrm{m} \quad \text{or} \quad 20\ \mathrm{mm}.$$

$$\boxed{2w = 2w_o\left[1+\left(\frac{z}{z_o}\right)^2\right]^{1/2} \approx 2w_o\frac{z}{z_o} = (1\,\mathrm{mm})\frac{25\,\mathrm{m}}{1.24\,\mathrm{m}} = 20\,\mathrm{mm}}$$

Example

EXAMPLE 1.2.1 Sellmeier equation and diamond

Using the Sellmeier coefficients for diamond in Table 1.2, calculate its refractive index at 610 nm (red light) and compare with the experimental quoted value of 2.415 to three decimal places.

Solution

The Sellmeier dispersion relation for diamond is

$$n^2 = 1 + \frac{0.3306\lambda^2}{\lambda^2 - 175\ \text{nm}^2} + \frac{4.3356\lambda^2}{\lambda^2 - 106\ \text{nm}^2}$$

$$n^2 = 1 + \frac{0.3306(610\ \text{nm})^2}{(610\ \text{nm})^2 - (175\ \text{nm})^2} + \frac{4.3356(610\ \text{nm})^2}{(610\ \text{nm})^2 - (106\ \text{nm})^2} = 5.8308$$

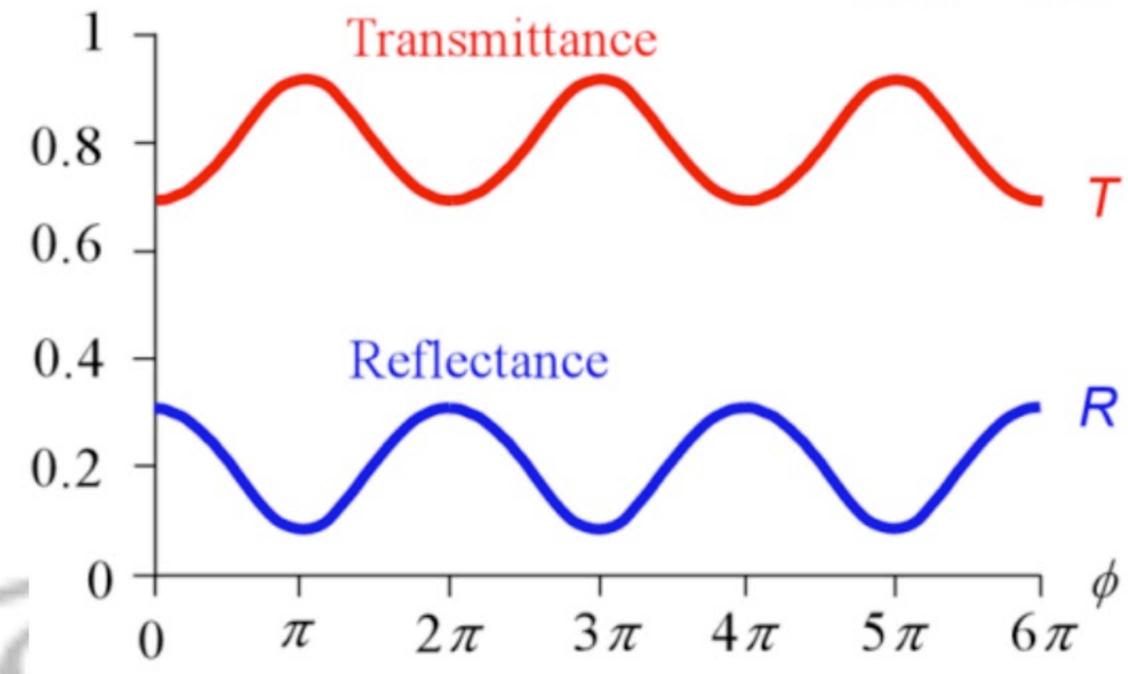
So that

$$n = 2.4147$$

which is 2.415 to three decimal places and matches the experimental value.

Example: Thin Film Optics

Consider a semiconductor device with $n_3 = 3.5$ that has been coated with a transparent optical film (a dielectric film) with $n_2 = 2.5$, $n_1 = 1$ (air). If the film thickness is 160 nm, find the minimum and maximum reflectances and transmittances and their corresponding wavelengths in the visible range. (Assume normal incidence.)



Solution: We have $n_1 < n_2 < n_3$. R_{min} occurs at $\phi = \pi$ or odd multiple of π, and maximum reflectance R_{max} at $\phi = 2\pi$ or an integer multiple of 2π.

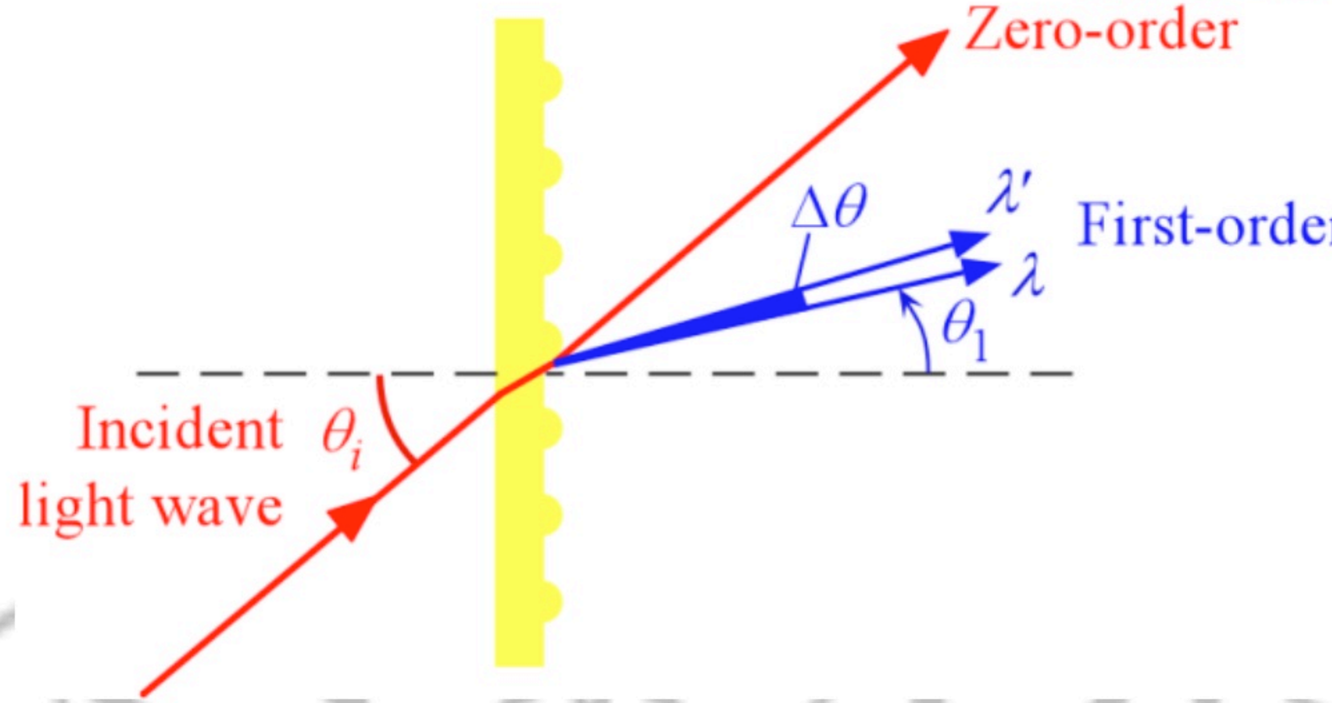
$$R_{min} = \left(\frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3}\right)^2 = \left(\frac{2.5^2 - (1)(3.5)}{2.5^2 + (1)(3.5)}\right)^2 = 0.080 \text{ or } 8.0\%$$

$$T_{max} = 1 - R_{min} = 0.92 \text{ or } 92\%$$

$$R_{max} = \left(\frac{n_3 - n_1}{n_3 + n_1}\right)^2 = \left(\frac{3.5 - 1}{3.5 + 1}\right)^2 = 0.31 \text{ or } 31\%$$

$$T_{min} = 1 - R_{max} = 0.69 \text{ or } 69\%$$

Example on Wavelength Separation



$\theta_i = 45°$. Periodicity $= d = 3$ μm

$$d(\sin\theta_m - \sin\theta_i) = m\lambda.$$

Substitute $d = 3$ μm, $\lambda = 1.550$ μm, $\theta_i = 45°$,

and calculate the diffraction angle θ_m for $m = -1$

$$(3\ \mu\text{m})[\sin\theta_1 - \sin(45°)] = (-1)(1.550\ \mu\text{m})$$

$\therefore \quad \boldsymbol{\theta_1 = 10.978°}$

A $\lambda' = 1.540$ μm, examining the same order, $m = -1$, we find $\boldsymbol{\theta_1' = 11.173°}$

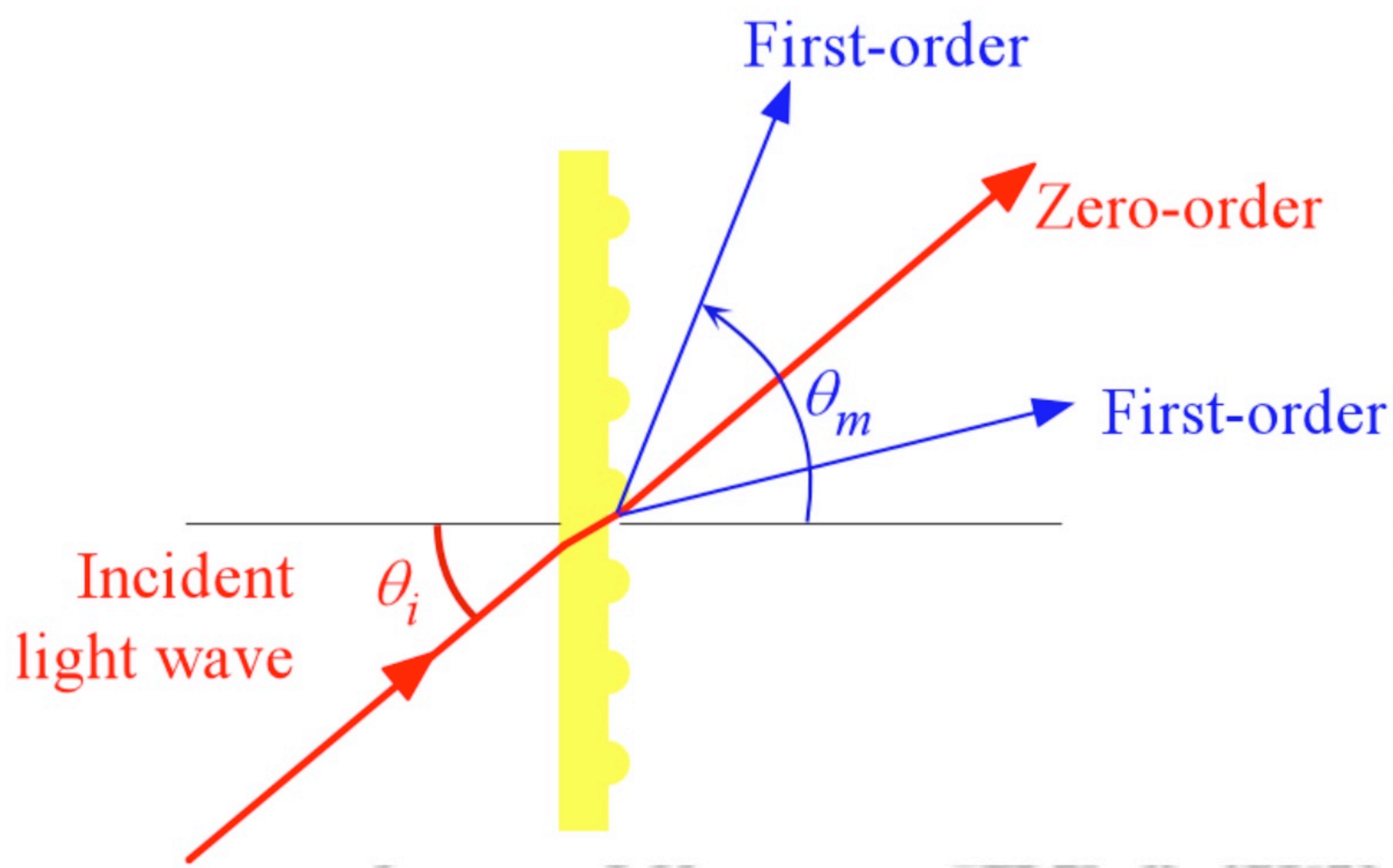
$\therefore \quad \boldsymbol{\Delta\theta_1 = 11.173° - 10.978° = 0.20°}$

Note, $m = 1$ gives a complex angle and should be neglected.

The problem is that the optical power in the zero-order beam is wasted. The zero-order does not separate wavelengths. Volume Phase Gratings (VPN) overcome this problem.

Example on Wavelength Separation by Diffraction

A transmission diffraction grating has a periodicity of 3 μm. The angle of incidence is 30° with respect to the normal to the diffraction grating. What is the angular separation of the two wavelength component s at 1550 nm and 1540 nm, separated by 10 nm?



$$d(\sin\theta_m - \sin\theta_i) = m\lambda$$

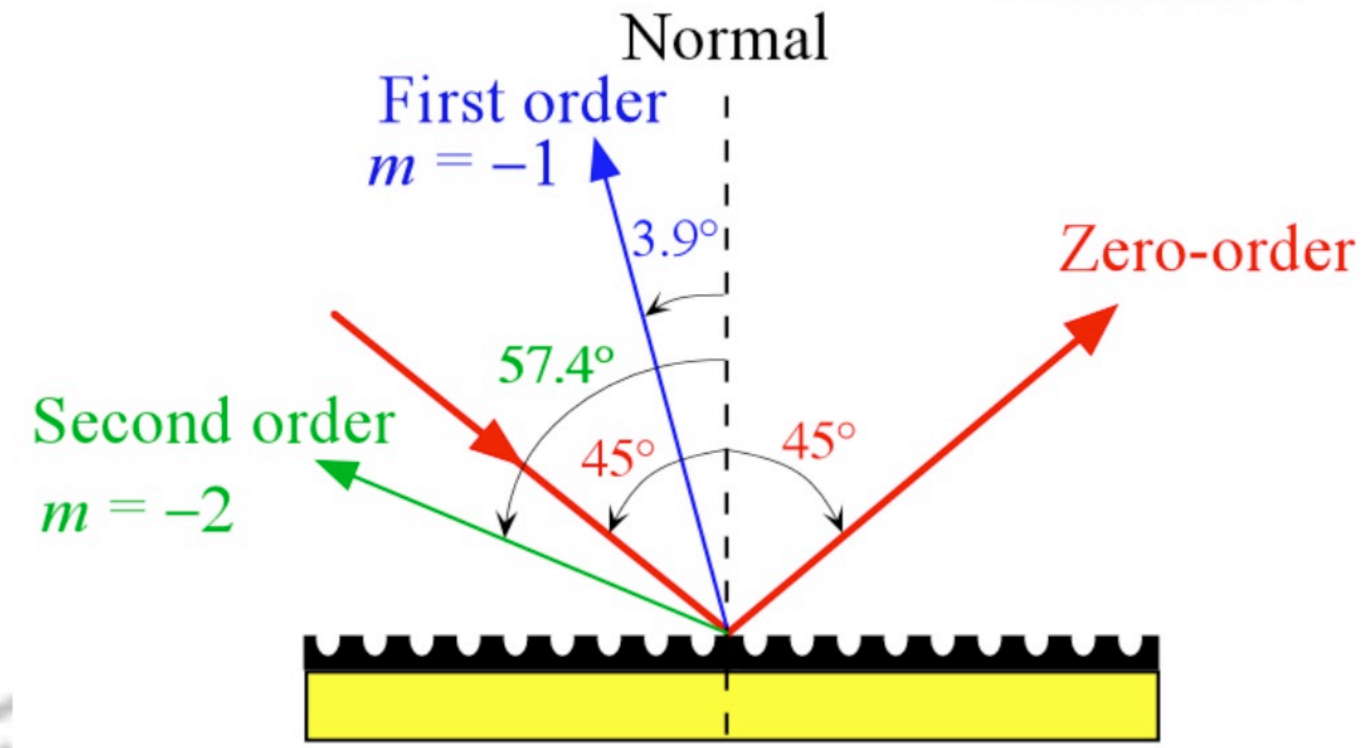
Suppose that we reduce d to 2 μm

Recalculating the above we find

$\theta_m = -3.9°$ for $m = -1$

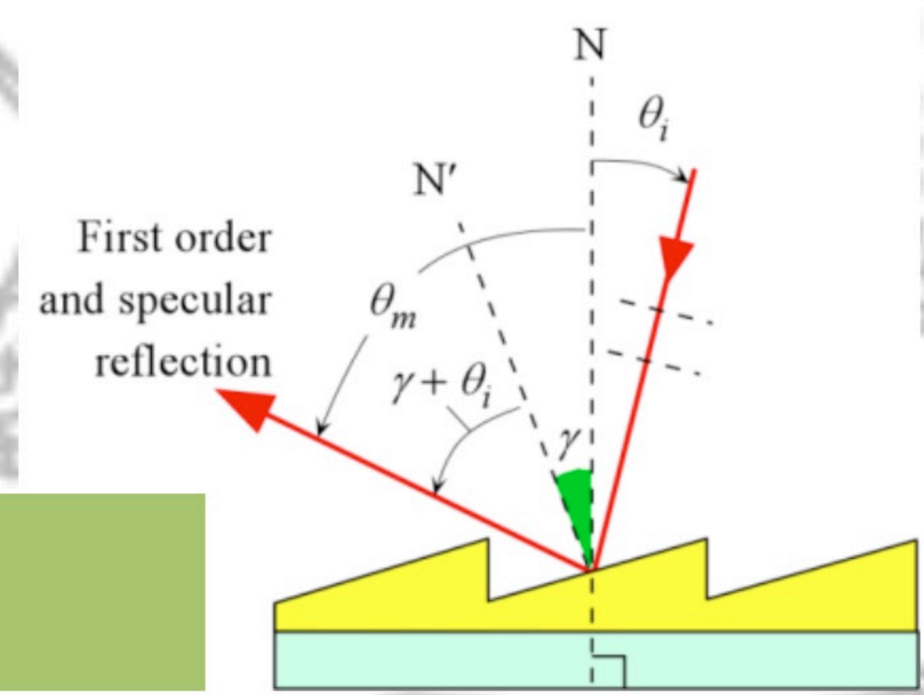
and imaginary for $m = +1$.

Further, for $m = -2$, there is a second order diffraction beam at $\theta_m = -57.4°$.



If we increase the angle of incidence, for example, $\theta_i = 85°$ on the first grating, the diffraction angle for $m = -1$ increases from 33.5° to 57.2° and the other diffraction peak ($m = 1$) disappears

Example: A reflection grating



The secular reflection from the grooved surface coincides with the m_{th} order diffraction when

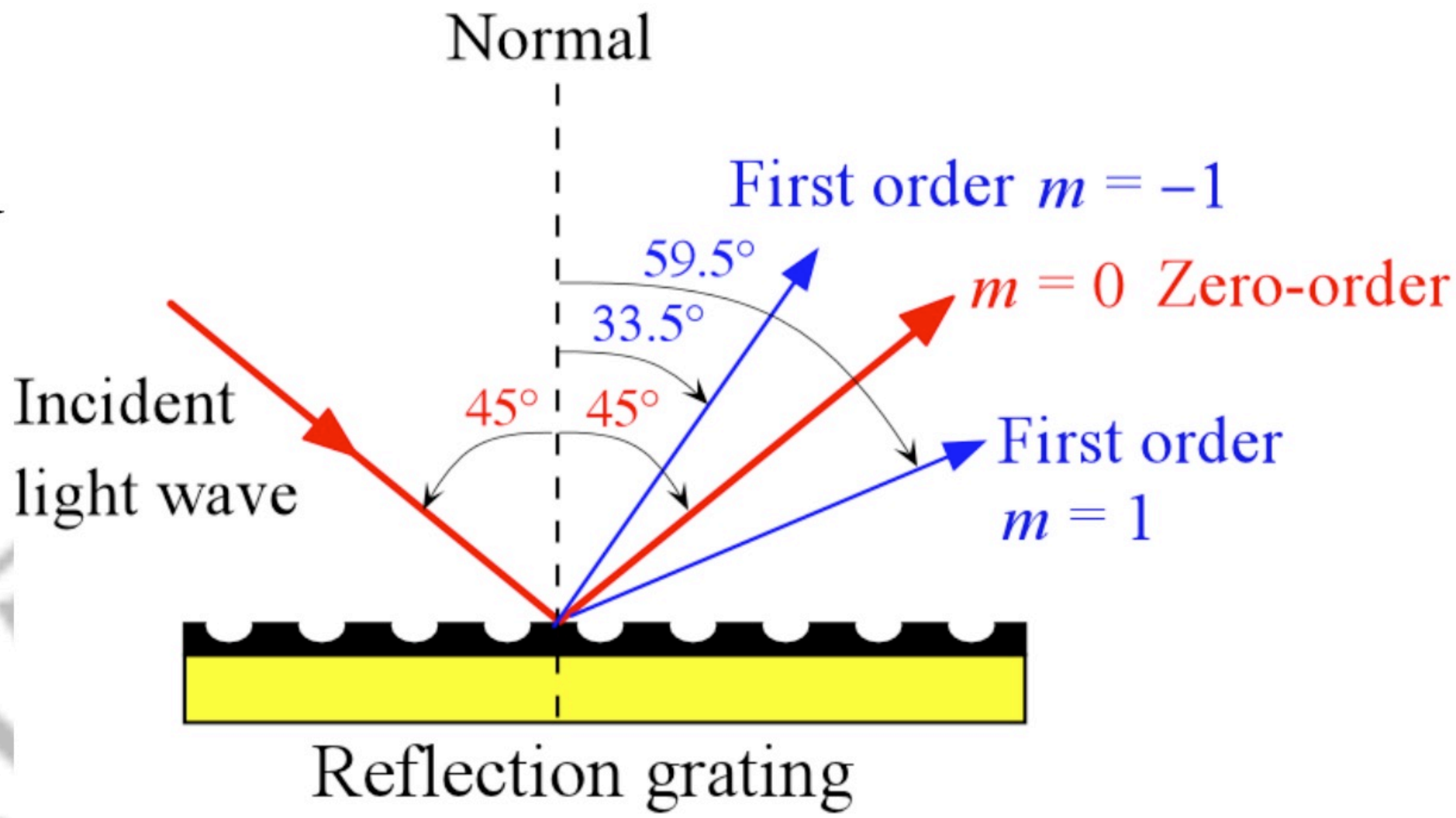
$$2\gamma = \theta_m - \theta_i$$

$$\therefore \quad \gamma = (1/2)(\theta_m - \theta_i) = (1/2)(59.6° - 45°) = 7.3°$$

Diffraction Grating

Example: A reflection grating

Consider a reflection grating with a period d that is 10 μm. Find the diffracted beams if a collimated light wave of wavelength 1550 nm is incident on the grating at an angle of 45° to its normal. What should be the blazing angle γ if we were to use the blazed grating with the same periodicity? What happens to the diffracted beams if the periodicity is reduced to 2 μm?



Solution: Put, $m = 0$ to find the zero-order diffraction, $\theta_0 = 45°$, as expected.
The general Bragg diffraction condition is

$$d(\sin\theta_m - \sin\theta_i) = m\lambda.$$

$$\therefore\ (10\ \mu\text{m})(\sin\theta_m - \sin(45°)) = (+1)(1.55\ \mu\text{m})$$

$$\therefore\ (10\ \mu\text{m})(\sin\theta_m - \sin(45°)) = (-1)(1.55\ \mu\text{m})$$

Solving these two equations, we find

$\theta_m = 59.6°$ for $m = 1$ and $\theta_m = 33.5°$ for $m = -1$

Example: Semiconductor Optical Cavity

Solution: Continued

Spectral width of a mode in wavelength is

$$\delta\lambda_m = \left|\delta\left(\frac{c}{\upsilon_m}\right)\right| = \left|-\frac{c}{\upsilon_m^2}\right|\delta\upsilon_m = \frac{(3\times10^8)}{(3.33\times10^{14})^2}(5.03\times10^{10}) = 0.136\text{ nm}$$

The Q-factor is

$$Q = mF = (1374)(29.8) = 4.1\times10^4$$

Example: Semiconductor Optical Cavity

Solution: Continued

Mode number m corresponding to 1310 nm is

$$m = \frac{2nL}{\lambda} = \frac{2(3.6)(250 \times 10^{-6})}{(1310 \times 10^{-9})} = 1374.05$$

which must be an integer (1374) so that the actual mode wavelength is

$$\lambda_m = \frac{2nL}{m} = \frac{2(3.6)(250 \times 10^{-6})}{(1374)} = 1310.04 \text{ nm}$$

For all practical purposes the mode wavelength is 1310 nm

Mode frequency is

$$\upsilon_m = \frac{c}{\lambda_m} = \frac{(3 \times 10^{8})}{(1310 \times 10^{-9})} = 2.3 \times 10^{14} \text{ Hz}$$

Consider a Fabry-Perot optical cavity of a semiconductor material of length 250 microns with mirrors, each with a reflectance of 0.90. Calculate the cavity mode nearest to 1310 nm. Calculate the separation of the modes, finesse, the spectral width of each mode, and the *Q*-factor. Take $n = 3.6$ for the semiconductor medium.

Solution

Given, $L = 250\times10^{-6}$ m, $n = 3.6$, $R = 0.90$

$$\Delta\upsilon_m = \upsilon_f = c/2nL = \text{Separation of modes} = 1.67\times10^{11}\ \text{Hz}$$

$$F = \frac{\pi R^{1/2}}{1-R} = \frac{\pi 0.9^{1/2}}{1-0.9} = 29.8$$

$$\delta\upsilon_m = \frac{\upsilon_f}{F} = \frac{1.67\times10^{11}}{29.8} = 5.59\,\text{GHz}$$

Solution: Continued

Example: An Optical Resonator in Air

$$\upsilon_f = c/2L = \text{separation of modes}$$

$$= (3\times10^8) \,/\, [2(100\times10^{-6})] = 1.5\times10^{12}\ \text{Hz.}$$

$$F = \frac{\pi R^{1/2}}{1-R} = \frac{\pi 0.90^{1/2}}{1-0.90} = 29.8$$

$$\delta\upsilon_m = \frac{\upsilon_f}{F} = \frac{1.5\times10^{12}}{29.8} = 50.3\,\text{GHz}$$

$$\delta\lambda_m = \left|\delta\left(\frac{c}{\upsilon_m}\right)\right| = \left|-\frac{c}{\upsilon_m^2}\right|\delta\upsilon_m = \frac{(3\times10^8)}{(3.33\times10^{14})^2}(5.03\times10^{10}) = 0.136\ \text{nm}$$

The Q-factor is

$$Q = mF = (222)(29.8) = 6.6\times10^3$$

Consider a Fabry-Perot optical cavity in air of length 100 microns with mirrors that have a reflectance of 0.90. Calculate the cavity mode nearest to the wavelength 900 nm, and corresponding wavelength. Calculate the separation of the modes, the finesse, the spectral width of each mode and the *Q*-factor

Solution

Find the mode number m corresponding to 900 nm and then take the integer

$$m = \frac{2L}{\lambda} = \frac{2(100\times10^{-6})}{(900\times10^{-9})} = 222.2 \quad \lambda_m = \frac{2L}{m} = \frac{2(100\times10^{-6})}{(222)} = 900.9 \text{ nm}$$

Thus, $m = 222$ (must be an integer)

$$\lambda_m = 900.90 \text{ nm} \approx 900 \text{ nm (very close)}$$

The frequency corresponding to λ_m is

$$\upsilon_m = c/\lambda_m = (3\times10^8)/(900.9\times10^{-9}) = 3.33\times10^{14}\text{ Hz}$$

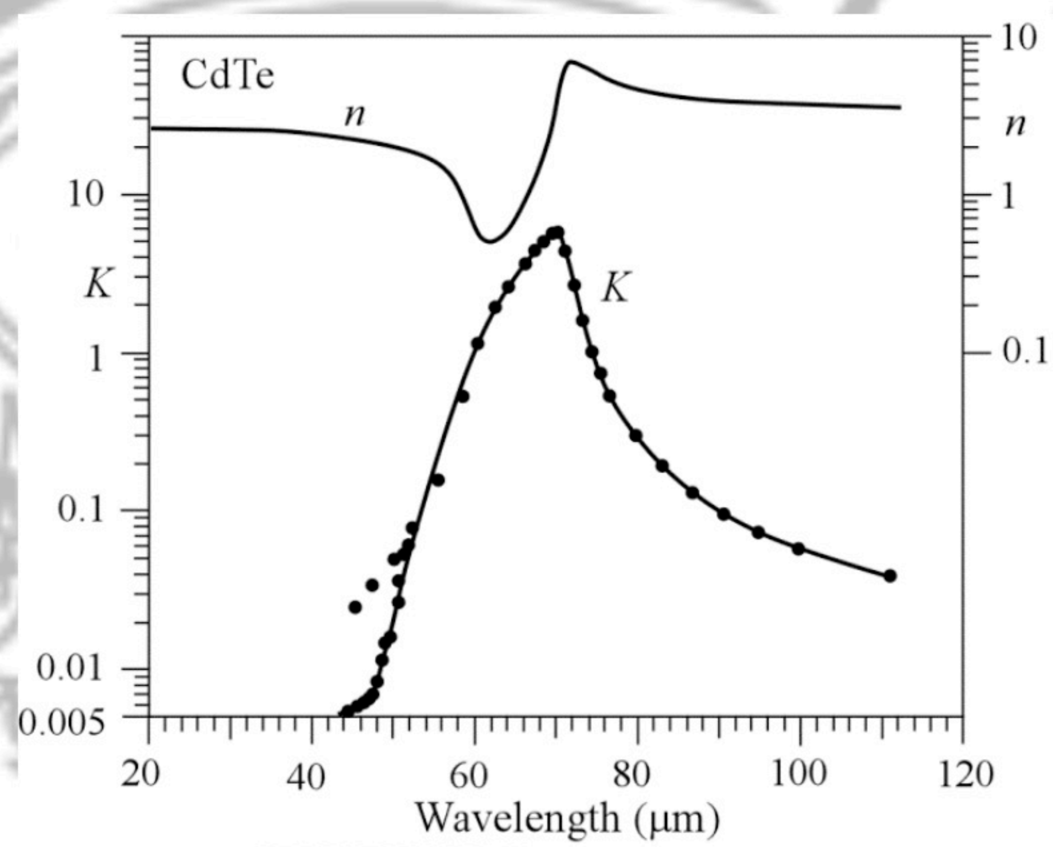
Solution continued: At the Reststrahlen peak, $\lambda \approx 70$ μm, $K \approx 6$, and $n \approx 4$, so that

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2} = \frac{(4-1)^2 + 6^2}{(4+1)^2 + 6^2} \approx 0.74 \text{ or } 74\%$$

At $\lambda = 50$ μm, $K \approx 0.02$, and $n \approx 2$. Repeating the above calculations we get

$$\boldsymbol{\alpha = 5.0 \times 10^3\ \mathrm{m}^{-1}}$$

$$\boldsymbol{R = 0.11 \text{ or } 11\ \%}$$



There is a sharp increase in the reflectance from 11 to 72% as we approach the Reststrahlen peak

Example: Complex Refractive Index for CdTe

Calculate the absorption coefficient α and the reflectance R of CdTe at the Reststrahlen peak, and also at 50 μm. What is your conclusion?

Solution: At the Reststrahlen peak, $\lambda \approx 70$ μm, $K \approx 6$, and $n \approx 4$. The free-space propagation constant is

$$k_o = 2\pi/\lambda = 2\pi/(70\times10^{-6}\text{ m}) = 9.0\times10^{4}\text{ m}^{-1}$$

The absorption coefficient α is $2k$,

$$\alpha = 2k'' = 2k_o K = 2(9.0\times10^{4}\text{ m}^{-1})(6) = \mathbf{1.08\times10^{6}\ m^{-1}}$$

which corresponds to an **absorption depth** $1/\alpha$ of about 0.93 micron.

Solution

We can only compare bandwidths $\Delta\lambda$ for "infinite" stacks (those with $R \approx$ 100%) For the TiO_2-SiO_2 stack

$$\Delta\lambda \approx \lambda_o (4/\pi) \arcsin\left(\frac{n_2 - n_1}{n_2 + n_1}\right)$$

$$\Delta\lambda \approx (850 \text{ nm})(4/\pi) \arcsin\left(\frac{2.49 - 1.55}{2.49 + 1.55}\right) = 254 \, \text{nm}$$

For the Ta_2O_5-SiO_2 infinite stack, we get $\Delta\lambda$ =74.8 nm

As expected $\Delta\lambda$ is narrower for the smaller contrast stack

Solution

$N = 12$. For 12 pairs of layers, the maximum reflectance R_{12} is

$$R_{12} = \left[\frac{(1.78)^{2(12)} - (1/1.47)(1.55)^{2(12)}}{(1.78)^{2(12)} + (1/1.47)(1.55)^{2(12)}}\right]^2 = 0.906 \text{ or } 90.6\%$$

Now use TiO_2 for the high-n layer with $n_1 = n_H = 2.49$,

$\boldsymbol{R_4 = 94.0\%}$ and $\boldsymbol{R_{12} = 100\%}$ (to two decimal places).

The refractive index contrast is **important**. For the TiO_2-SiO_2 stack we only need 4 double layers to get roughly the same reflectance as from 12 pairs of layers of Ta_2O_5-SiO_2. If we interchange n_H and n_L in the 12-pair stack, *i.e.* $n_1 = n_L$ and $n_2 = n_H$, the Ta_2O_5-SiO_2 reflectance falls to 80.8% but the TiO_2-SiO_2 stack is unaffected since it is already reflecting nearly all the light.

Example: Dielectric Mirror

A dielectric mirror has quarter wave layers consisting of Ta_2O_5 with $n_H = 1.78$ and SiO_2 with $n_L = 1.55$ both at 850 nm, the central wavelength at which the mirror reflects light. The substrate is Pyrex glass with an index $n_s = 1.47$ and the outside medium is air with $n_0 = 1$. Calculate the maximum reflectance of the mirror when the number N of double layers is 4 and 12. What would happen if you use TiO_2 with $n_H = 2.49$, instead of Ta_2O_5? Consider the $N = 12$ mirror. What is the bandwidth and what happens to the reflectance if you interchange the high and low index layers? Suppose we use a Si wafer as the substrate, what happens to the maximum reflectance?

Solution

$n_0 = 1$ for air, $n_1 = n_H = 1.78$, $n_2 = n_L = 1.55$, $n_3 = n_s = 1.47$, $N = 4$. For 4 pairs of layers, the maximum reflectance R_4 is

$$R_4 = \left[\frac{(1.78)^{2(4)} - (1/1.47)(1.55)^{2(4)}}{(1.78)^{2(4)} + (1/1.47)(1.55)^{2(4)}}\right]^2 = 0.4 \text{ or } 40\%$$

Example: Antireflection coatings on solar cells

When light is incident on the surface of a semiconductor it becomes partially reflected. Partial reflection is an important energy loss in solar cells.

The refractive index of Si is about 3.5 at wavelengths around 700 - 800 nm. Reflectance with n_1(air) = 1 and n_2(Si) ≈ 3.5 is

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1 - 3.5}{1 + 3.5}\right)^2 = 0.309$$

i.e.

$$\alpha_2 = \frac{2\pi(1.440)}{(1300\times10^{-9}\ \text{m})}\left[\left(\frac{1.460}{1.440}\right)^2 \sin^2(87^\circ)-1\right]^{1/2}$$

$= 1.10\times10^6\ \text{m}^{-1}$.

The penetration depth is,

$\delta = 1/\alpha_2 = 1/(1.104\times10^6\ \text{m}) = 9.06\times10^{-7}$ m, or **0.906 μm**

For 90°, repeating the calculation, $\alpha_2 = 1.164\times10^6\ \text{m}^{-1}$, so that

$\delta = 1/\alpha_2 =$ **0.859 μm**

The penetration is greater for smaller incidence angles

$$E_{t,\perp}(y,t) \propto E_{to,\perp}\exp(-\alpha_2 y)$$

The field strength drops to e^{-1} when $y = 1/\alpha_2 = \delta$, which is called the penetration depth. The attenuation constant α_2 is

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

so that

$$\tan(^1/_2\phi_{//} + {}^1/_2\pi) = (n_1/n_2)^2\tan(\phi_\perp/2) =$$

$$(1.460/1.440)^2\tan(^1/_2 143°)$$

which gives $\phi_{//} = 143.95° - 180°$ or $-36.05°$

Repeat with $\theta_i = 90°$ to find $\phi_\perp = 180°$ and $\phi_{//} = 0°$.

Note that as long as $\theta_i > \theta_c$, the magnitude of the reflection coefficients are unity. Only the phase changes.

$= 2.989 = \tan[{}^1/_2(143.0°)]$

$$\tan\left(\tfrac{1}{2}\phi_\perp\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i} = \frac{\left[\sin^2(87°) - \left(\dfrac{1.440}{1.460}\right)^2\right]^{1/2}}{\cos(87°)}$$

so that the phase change $\phi_\perp = 143°$.

For the $E_{r,//}$ component, the phase change is

$$\tan\left(\tfrac{1}{2}\phi_{//} + \tfrac{1}{2}\pi\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{n^2\cos\theta_i} = \frac{1}{n^2}\tan\left(\tfrac{1}{2}\phi_\perp\right)$$

Solution

(a) The critical angle θ_c for TIR is given by

$\sin\theta_c = n_2/n_1 = 1.440/1.460$ so that $\theta_c = 80.51°$

(b) Since the incidence angle $\theta_i > \theta_c$ there is a phase shift in the reflected wave. The phase change in $E_{r,\perp}$ is given by $\phi_\perp$.

Using $n_1 = 1.460$, $n_2 = 1.440$ and $\theta_i = 87°$,

Example: Reflection of light from a less dense medium (internal reflection)

A ray of light which is traveling in a glass medium of refractive index $n_1 = 1.460$ becomes incident on a less dense glass medium of refractive index $n_2 = 1.440$. The free space wavelength (λ) of the light ray is 1300 nm.

(a) What should be the minimum incidence angle for TIR?

(b) What is the phase change in the reflected wave when $\theta_i = 87°$ and when $\theta_i = 90°$?

(c) What is the penetration depth of the evanescent wave into medium 2 when $\theta_i = 87°$ and when $\theta_i = 90°$?

Reflection and Transmission – An Example

To find the transmittance for each polarization, we need the refraction angle θ_t. From Snell's law, $n_1 \sin\theta_i = n_t \sin\theta_t$ *i.e.* $(1)\sin(56.31°) = (1.5)\sin\theta_t$, we find **$\theta_t = 33.69°$**.

$$T_{//} = \frac{n_2 \left|E_{to,//}\right|^2}{n_1 \left|E_{io,//}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{//}\right|^2$$

$$T_{\perp} = \frac{n_2 \left|E_{to,\perp}\right|^2}{n_1 \left|E_{io,\perp}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{\perp}\right|^2$$

$$T_{//} = \left[\frac{(1.5)\cos(33.69^{\circ})}{(1)\cos(56.31^{\circ})}\right](0.667)^2 = 1$$

$$T_{\perp} = \left[\frac{(1.5)\cos(33.69^{\circ})}{(1)\cos(56.31^{\circ})}\right](0.615)^2 = 0.852$$

Clearly, light with polarization parallel to the plane of incidence has greater intensity.

If we were to reflect light from a glass plate, keeping the angle of incidence at 56.3°, then the reflected light will be polarized with an electric field component perpendicular to the plane of incidence. The transmitted light will have the field greater in the plane of incidence, that is, it will be partially polarized. By using a stack of glass plates one can increase the polarization of the transmitted light. (This type of *pile-of-plates polarizer* was invented by Dominique F.J. **Arago** in 1812.)

Reflection and Transmission – An Example

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_\perp = \frac{2\cos(56.31^\circ)}{\cos(56.31^\circ) + [1.5^2 - \sin^2(56.31^\circ)]^{1/2}} = 0.615$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_{//} = \frac{2(1.5)\cos(56.31^\circ)}{(1.5)^2\cos(56.31^\circ) + [1.5^2 - \sin^2(56.31^\circ)]^{1/2}} = 0.667$$

Notice that $r_{//} + nt_{//} = 1$ and $r_\perp + 1 = t_\perp$, as we expect.

Reflection and Transmission – An Example

Question A light beam traveling in air is incident on a glass plate of refractive index 1.50 . What is the Brester or polarization angle? What are the relative intensities of the reflected and transmitted light for the polarization perpendicular and parallel to the plane of incidence at the Brestwer angle of incidence?

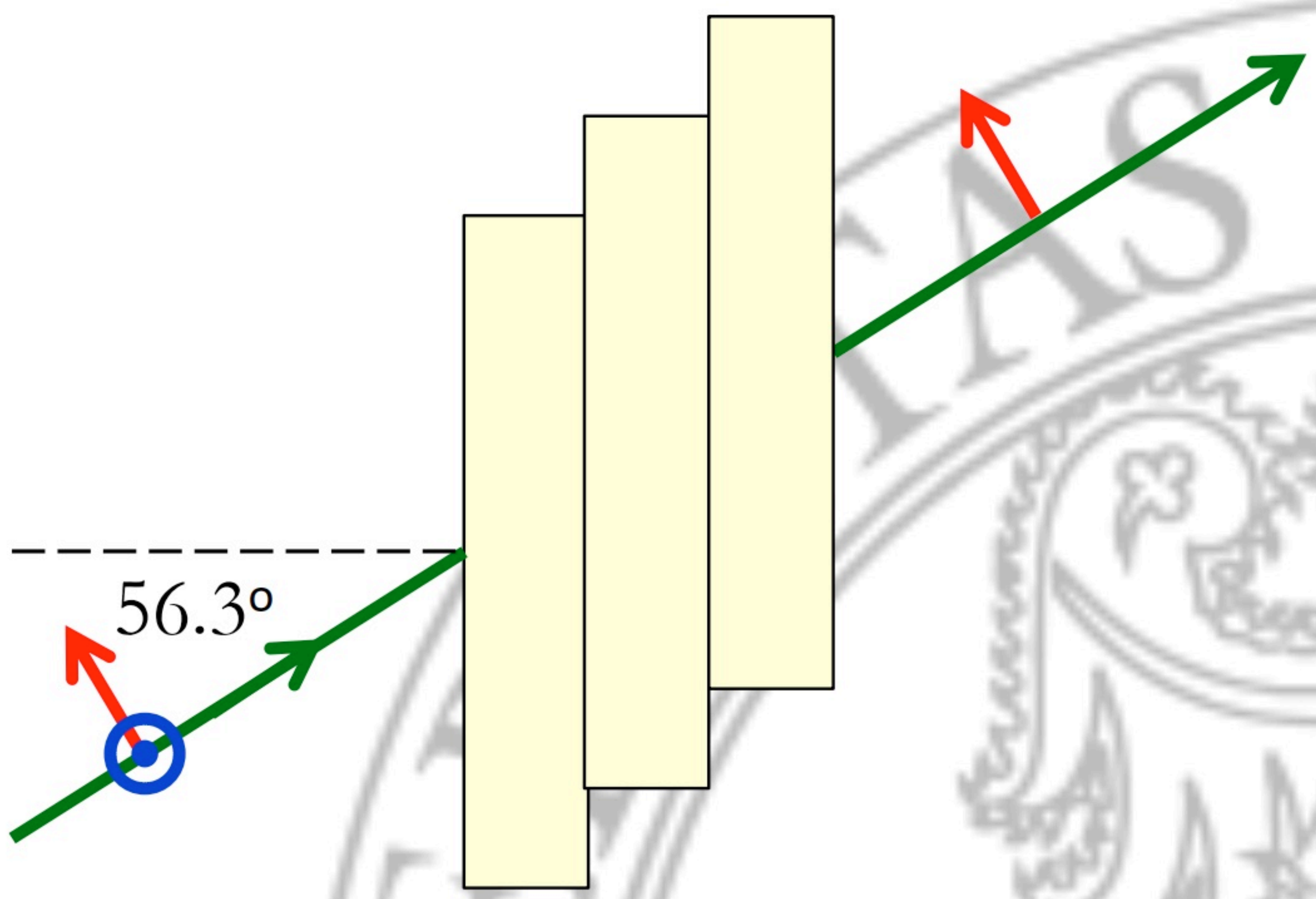
Solution Light is traveling in air and is incident on the glass surface at the polarization angle θ_p. Here $n_1 = 1$, $n_2 = 1.5$ and $\tan\theta_p = (n_2/n_1) = 1.5$ so that $\theta_p = 56.31°$. We now have to use Fresnel's equations to find the reflected and transmitted amplitudes. For the perpendicular polarization

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - [n^2 - \sin^2\theta_i]^{1/2}}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$r_\perp = \frac{\cos(56.31°) - [1.5^2 - \sin^2(56.31°)]^{1/2}}{\cos(56.31°) + [1.5^2 - \sin^2(56.31°)]^{1/2}} = -0.385$$

On the other hand, $r_{//} = 0$. The reflectances $R_\perp = |r_\perp|^2 = 0.148$ and $R_{//} = |r_{//}|^2 = 0$ so that $R = 0.074$, and has no parallel polarization in the plane of incidence. Notice the negative sign in $r_\perp$, which indicates a phase change of π.

(c) Light is traveling in air and is incident on the glass surface at the polarization angle. Here $n_1 = 1$, $n_2 = 1.5$ and $\tan\theta_p = (n_2/n_1) = 1.5$ so that $\theta_p = 56.3°$.



This type of *pile-of-plates polarizer* was invented by Dominique F.J. Arago in 1812

(b) The light travels in glass and becomes partially reflected at the glass-air interface which corresponds to internal reflection. $n_1 = 1.5$ and $n_2 = 1$. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$

There is no phase shift. The reflectance is again 0.04 or 4%. In both cases **(a)** and **(b)** the amount of reflected light is the same.

Solution

(a) The light travels in air and becomes partially reflected at the surface of the glass which corresponds to external reflection. Thus $n_1 = 1$ and $n_2 = 1.5$. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1-1.5}{1+1.5} = -0.2$$

This is negative which means that there is a 180° phase shift. The reflectance (R), which gives the fractional reflected power, is

$$R = r_{//}^2 = 0.04 \text{ or } 4\%.$$

Example: Reflection at normal incidence. Internal and external reflection

Consider the reflection of light at normal incidence on a boundary between a glass medium of refractive index 1.5 and air of refractive index 1.

(a) If light is traveling from air to glass, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

(b) If light is traveling from glass to air, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

(c) What is the polarization angle in the external reflection in **a** above? How would you make a polaroid from this?

Example: Lateral Displacement (Continued)

Solution (Continued)

$$\frac{d}{L} = \sin\theta_i \left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}} \right]$$

$L = 10$ mm

$\theta_i = 45°$

$n = 1.600$

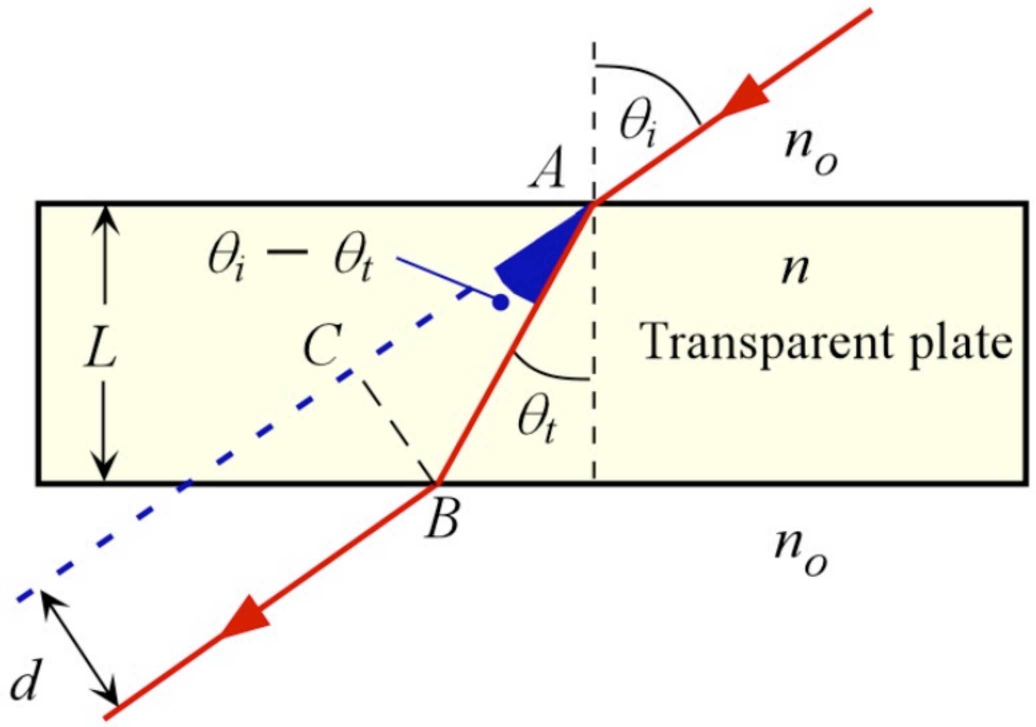
$n_o = 1$

$\boxed{\boldsymbol{d = 3.587\ \text{mm}}}$

Example: Lateral Displacement (Continued)

Solution (Continued)

Expand $\sin(\theta_i - \theta_t)$ and eliminate $\sin\theta_t$ and $\sin\theta_t$



$$d = L\left[\frac{\sin(\theta_i - \theta_t)}{\cos\theta_t}\right]$$

$$\sin(\theta_i - \theta_t) = \sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

Snell's law $n\sin\theta_t = n_o\sin\theta_i$

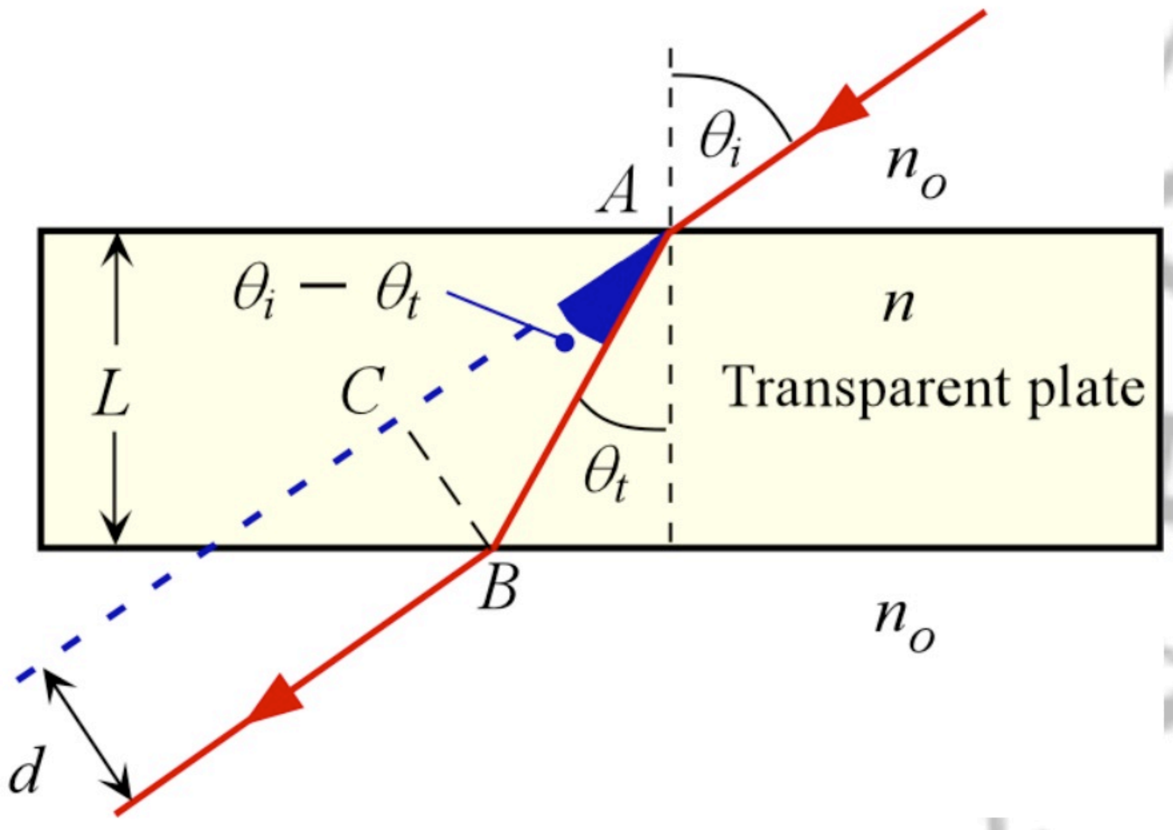
$$\frac{d}{L} = \sin\theta_i\left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}}\right]$$

Example: Lateral Displacement

Lateral displacement of light, or, **beam displacement,** occurs when a beam if light passes obliquely through a plate of transparent material, such as a glass plate. When a light beam is incident on a plate of transparent material of refractive index *n*, it emerges from the other side traveling parallel to the incident light but displaced from it by a distance *d*, called *lateral displacement*. Find the displacement *d* in terms of the incidence angle the plate thickness *L*. What is *d* for a glass of $n = 1.600$, $L = 10$ mm if the incidence angle is 45°

Solution

The displacement $d = BC = AB\sin(\theta_i - \theta_t)$. Further, $L/AB = \cos\theta_t$ so that combining these two equation we find



$$d = L\left[\frac{\sin(\theta_i - \theta_t)}{\cos\theta_t}\right]$$

Gaussian Beam: example

$$I(z,0) = I_{\max} = I_o \frac{w_o^2}{w^2} = I_o \frac{z_o^2}{z^2}$$

The Rayleigh range z_o was calculated previously, but we can recalculate

$$z_o = \pi w_o^2/\lambda = \pi(0.5\times10^{-3}\ \mathrm{m})^2/(633\times10^{-9}\ \mathrm{m}) = 1.24\ \mathrm{m}.$$

The beam width at 25 m is

$$2w = 2w_o[1 + (z/z_o)]^{1/2} = 20\ \mathrm{mm}$$

The irradiance at the beam axis is

$$I_{\mathrm{axis}} = I_o \frac{z_o^2}{z^2} = (1.273\ \mathrm{W\,cm^{-2}})\frac{(1.24\ \mathrm{m})^2}{(25\ \mathrm{m})^2} = 3.14\ \mathrm{mW\,cm^{-2}}$$

Gaussian Beam: example

I_o = Maximum irradiance at the center $r = 0$ at the waist $\longrightarrow$ $$I_o = \frac{P_o}{\frac{1}{2}\pi w_o^2}$$

Example 1.4.2 Power and irradiance of a Gaussian beam

Consider a 5 mW HeNe laser that is operating at 633 nm, and has a spot size that is 1 mm. Find the maximum irradiance of the beam and the axial (maximum) irradiance at 25 m from the laser.

Solution

The 5 mW rating refers to the total optical power P_o available, and 633 nm is the free space output wavelength λ. Apply

$$P_o = I_o(\tfrac{1}{2}\pi w_o^2)$$

$$\therefore \quad 5\times10^{-3}\ \text{W} = I_o[\tfrac{1}{2}\pi(0.5\times10^{-3}\ \text{m})^2]$$

$$I_o = 1.273\ \text{W cm}^{-2}$$

Example

EXAMPLE 1.2.2 Cauchy equation and diamond

Using the Cauchy coefficients for diamond in Table 1.2, calculate the refractive index at 610 nm.

Solution

At $\lambda = 610$ nm, the photon energy is

$$h\upsilon = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\,\mathrm{J\,s})(2.998 \times 10^{8}\,\mathrm{m\,s^{-1}})}{(610 \times 10^{-9}\,\mathrm{m})} \times \frac{1}{1.602 \times 10^{-19}\,\mathrm{J\,eV^{-1}}} = 2.0325\,\mathrm{eV}$$

Using the Cauchy dispersion relation for diamond with coefficients from Table 1.2,

$$\begin{aligned} n &= n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4 \\ &= (-1.07 \times 10^{-5})(2.0325)^{-2} + 2.378 + (8.01 \times 10^{-3})(2.0325)^2 \\ &\quad + (1.04 \times 10^{-4})(2.0325)^4 \\ &= 2.4140 \end{aligned}$$

which is slightly different than the value calculated in Example 1.2.1; one reason for the discrepancy is due to the Cauchy coefficients quoted in Table 1.2 being applicable over a wider wavelength range at the expense of some accuracy. Although both dispersion relations have four parameters, A_1, A_2, λ_1, λ_2 for Sellmeier and n_{-2}, n_0, n_2, n_4 for Cauchy, the functional forms are different.