Chapter 2 Dielectric Waveguides and Optical Fibers

Charles Kao, Nobel Laureate (2009)
Courtesy of the Chinese University of Hong Kong

Dielectric Waveguides and Optical Fibers

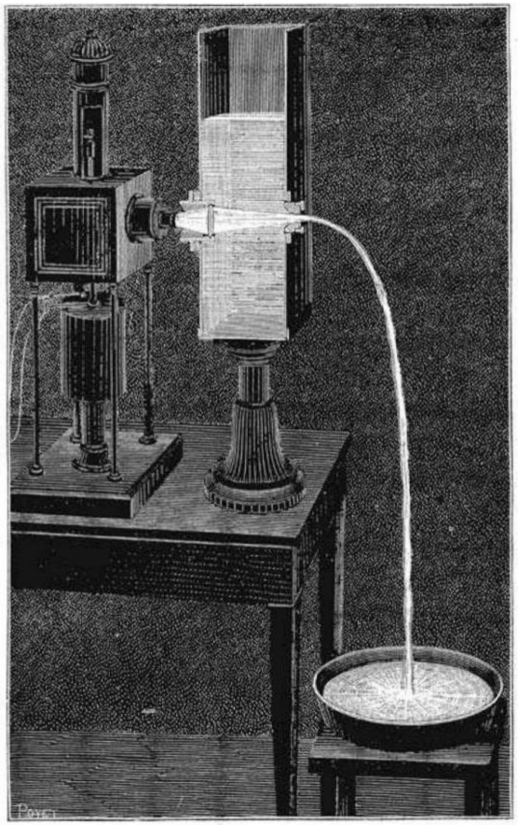
"The introduction **of optical fiber systems will revolutionize the communications network**. The low-transmission loss and the large bandwidth capability of the fiber systems allow signals to be transmitted for establishing communications contacts over large distances with few or no provisions of intermediate amplification."

[Charles K. Kao (one of the pioneers of glass fibers for optical communications) *Optical Fiber Systems: Technology, Design, and Applications* (McGraw-Hill Book Company, New York, USA, 1982), p. 1]

Courtesy of the Chinese University of Hong Kong

Charles Kao and his colleagues carried out the early experiments on optical fibers at the Standard Telecommunications Laboratories Ltd (the research center of Standard Telephones and Cables) at Harlow in the United Kingdom, during the 1960s. He shared the Nobel Prize in 2009 in Physics with Willard Boyle and George Smith for "groundbreaking achievements concerning the transmission of light in fibers for optical communication." In a milestone paper with George Hockam published in the IEE Proceedings in 1966 they predicted that the intrinsic losses of glass optical fibers could be much lower than 20 dB/km, which would allow their use in long distance telecommunications. Today, optical fibers are used not only in telecommunications but also in various other technologies such as instrumentation and sensing.

Jean-Daniel Colladon and the Light Guiding in a Water Jet



1841

Light is guided along a water jet as demonstrated by Jean-Daniel Colladon. This illustration was published in *La Nature*, *Revue des Sciences*, in 1884 (p. 325). His first demonstration was around 1841. (*Comptes Rendes*, 15, 800-802, Oct. 24, 1842). A similar demonstration was done by John Tyndall for the Royal Institution in London in his 1854 lecture. Apparently, Michael Faraday had originally suggested the experiment to John Tyndall though Faraday himself probably learned about it either from another earlier demonstration or through Jean-Daniel Colladon's publication. Although John Tyndall is often credited with the original discovery of a water-jet guiding light, Tyndall, himself, does not make that claim but neither does he attribute it to someone else.

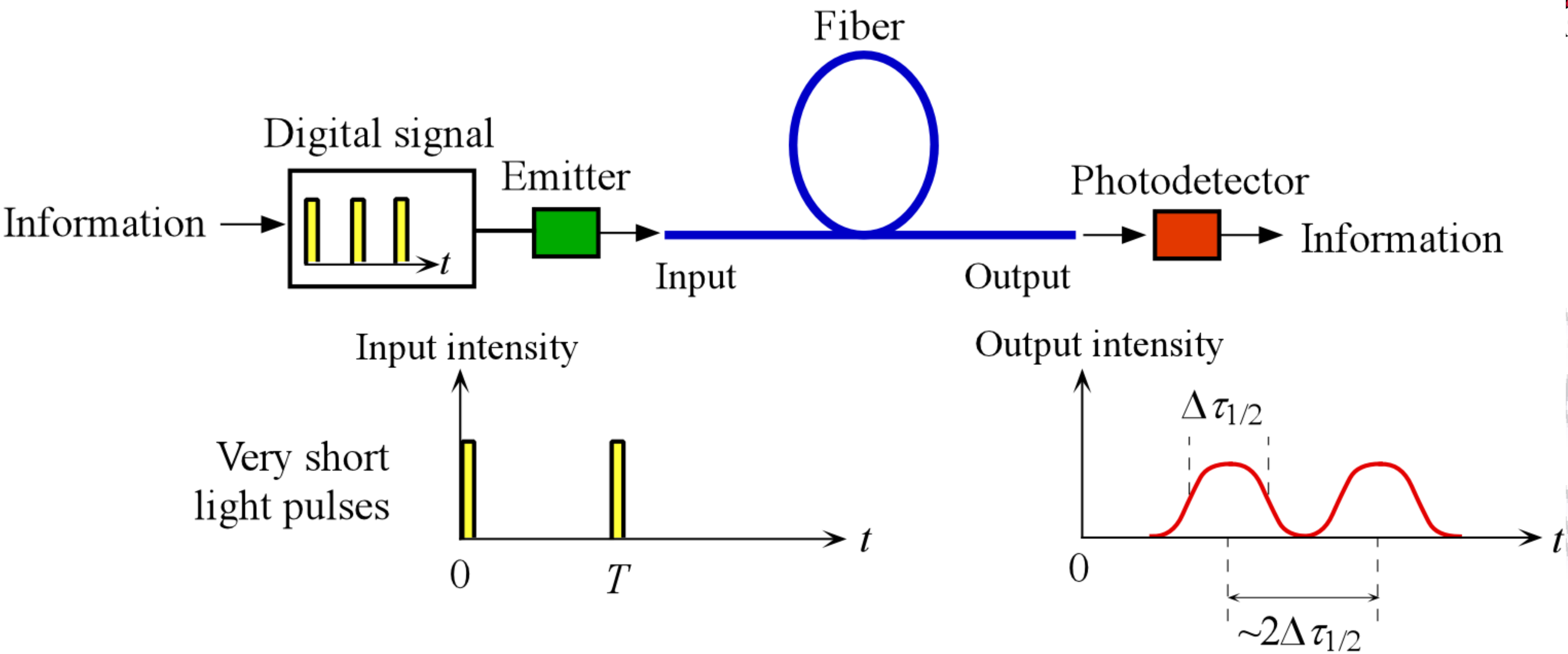
(The fountain, courtesy of Conservatoire Numérique des Arts et Métiers, France; Colladon's portrait, courtesy of Musée d'histoire des sciences, Genève, Switzerland.)

Reference: Jeff Hecht, "Illuminating the Origin of Light Guiding," *Optics & Photonics News*, **10**, 26, 1999 and his wonderful book *The City of Light* (Oxford University Press, 2004) describe the evolution of the optical fiber from the water jet experiments of Colladon and Tyndall to modern fibers with historical facts and references.

Narinder Singh Kapany

Narinder Singh Kapany was born in Punjab in India, studied at the Agra University and then obtained his PhD from the Imperial College of Science and Technology, University of London in 1955. He held a number of key-positions in both academia and industry, including a Regents Professor at the University of California, Berkeley, the University of California, Santa Cruz (UCSC), the Director of the Center for Innovation and Entrepreneurial Development at UCSC. He made significant contributions to optical glass fibers starting in 1950s, and essentially coined the term fiber optics in the 1960s. His book Fibre Optics: Principles and Applications, published in 1967, was the first in optical fibers. (Courtesy of Dr. Narinder S. Kapany)

A Century and Half Later

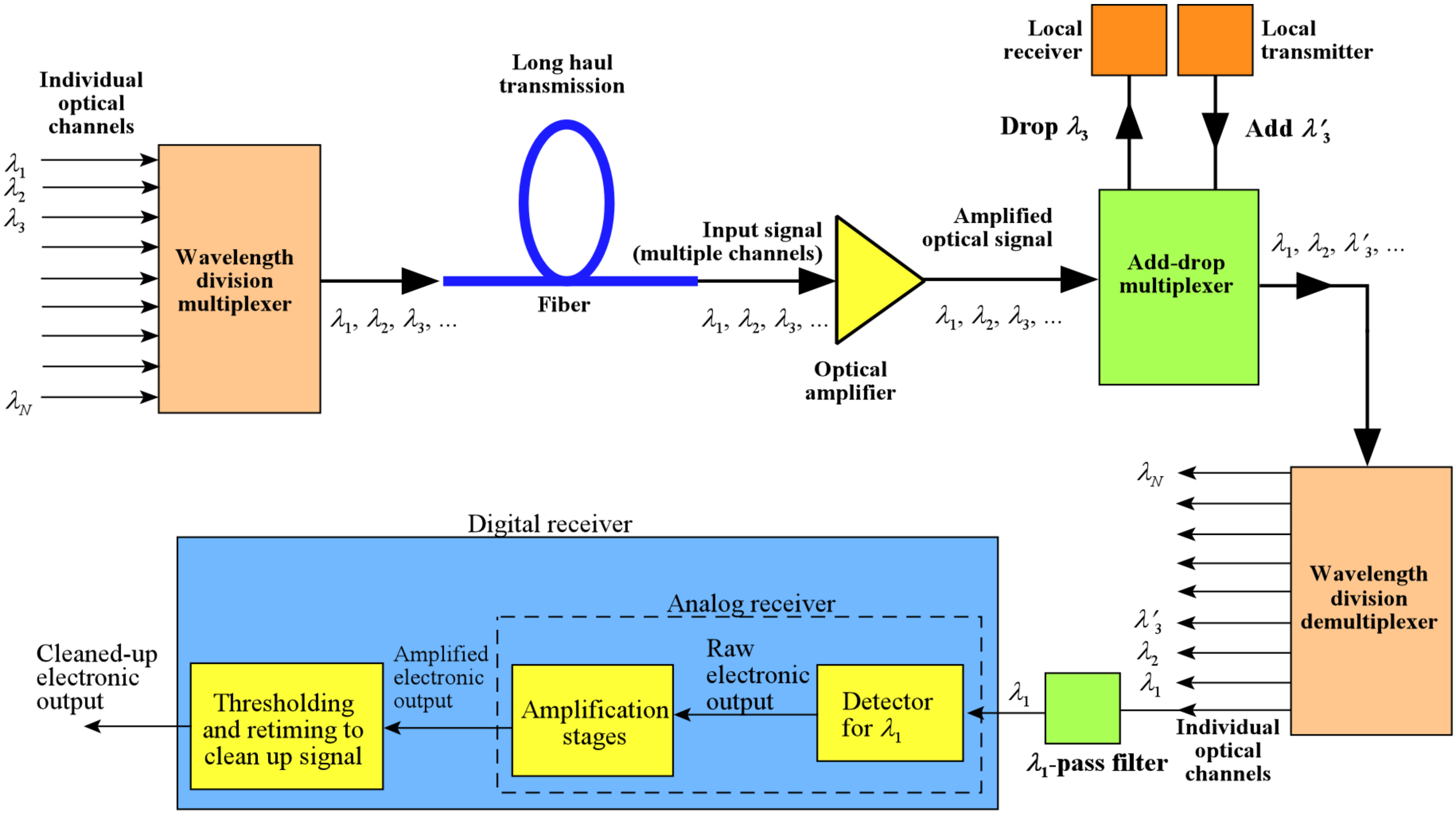


Light has replaced copper in communications. Photons have replaced electrons.

Will "Photonics Engineering" replace Electronics Engineering?

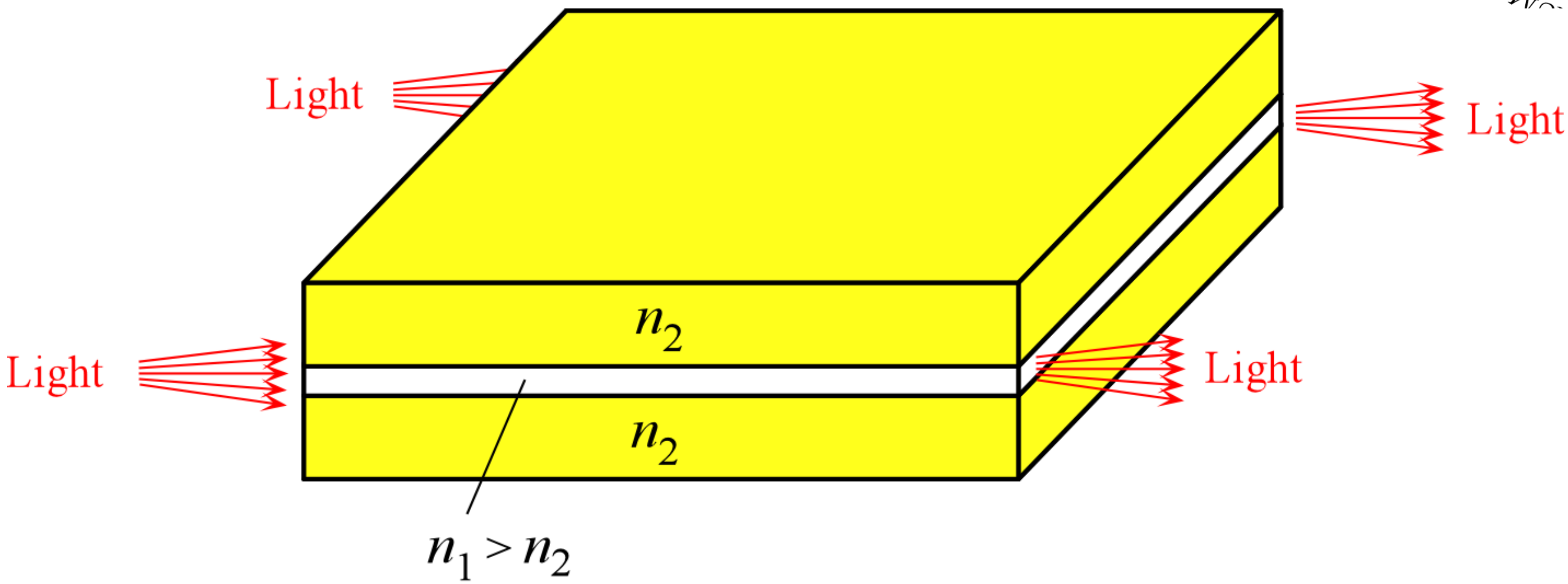
➔ It's already a new job…

WAVELENGTH DIVISION MULTIPLEXING: WDM



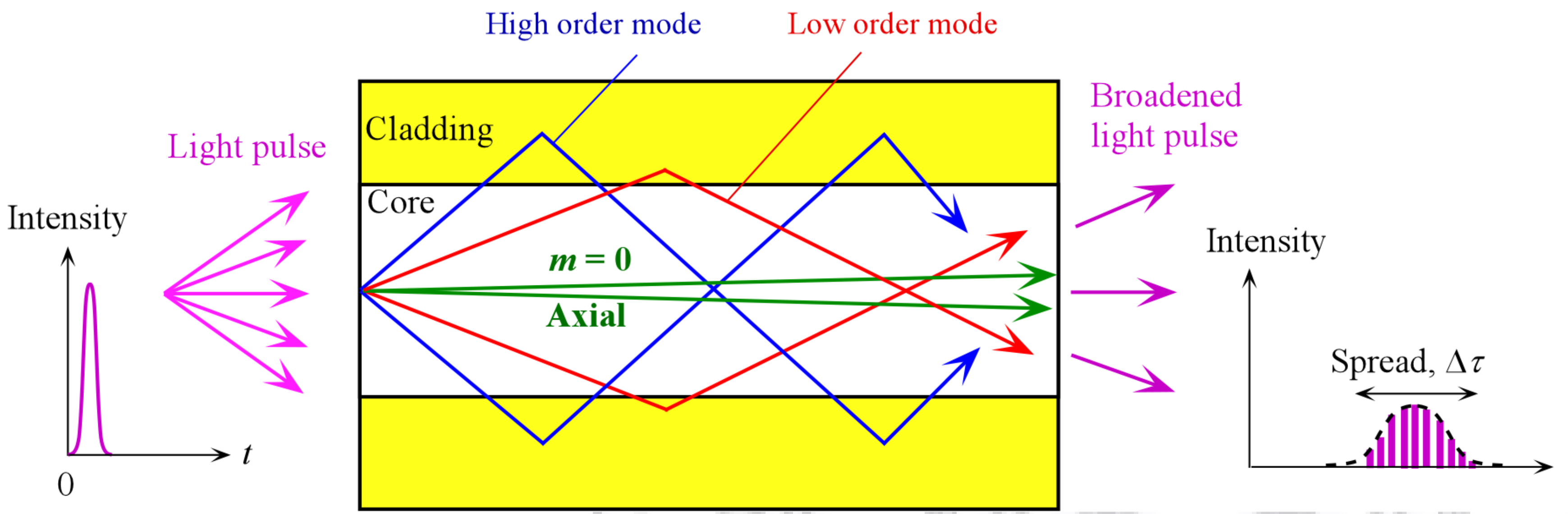
S.O. Kasap, *Optoelectronics and Photonics: Principles and Practices*, Second Edition, © 2013 Pearson Education

Planar Optical Waveguide



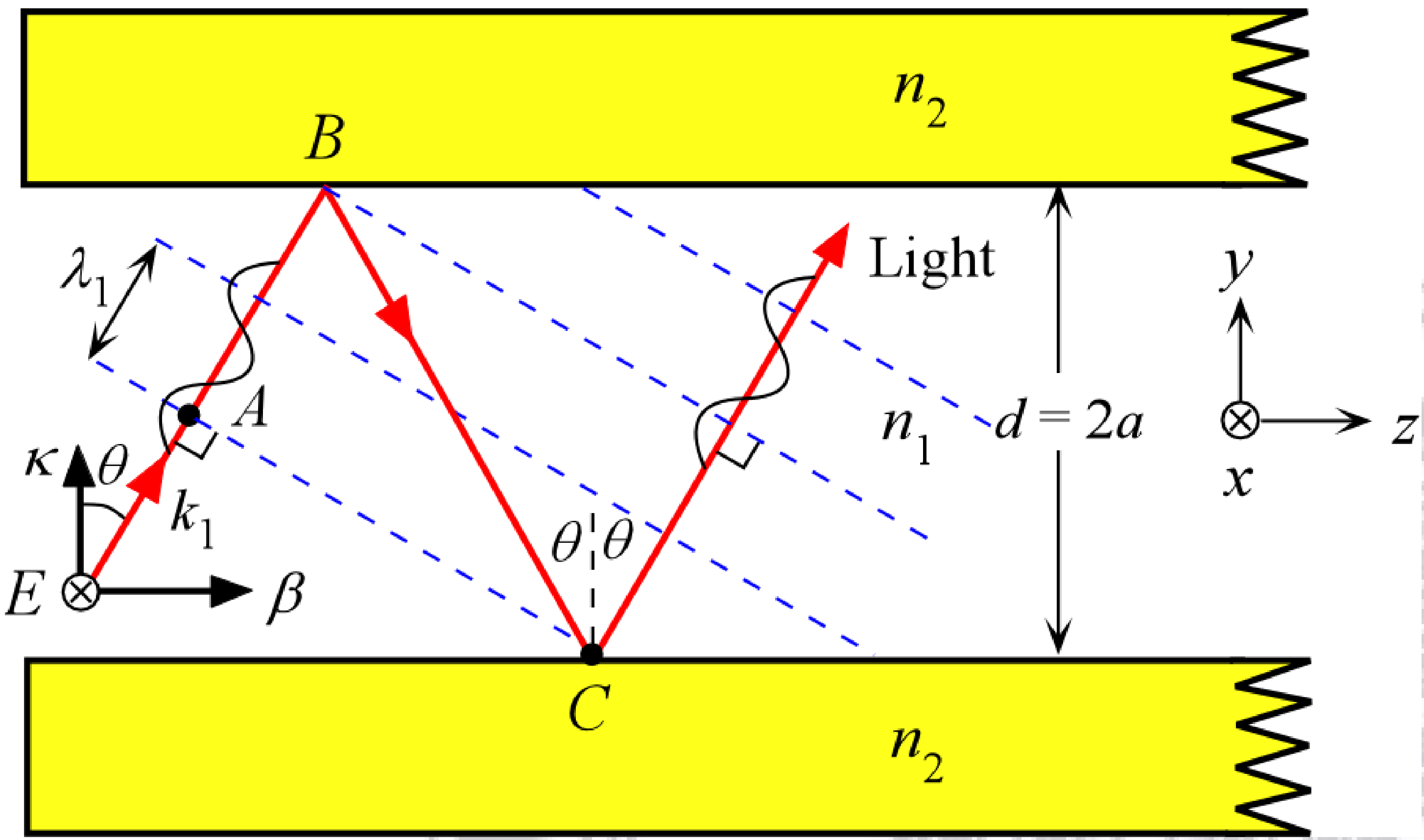
A planar dielectric waveguide has a central rectangular region of higher refractive index n_1 than the surrounding region which has a refractive index n_2. It is assumed that the waveguide is **infinitely wide** and the central region is of thickness $2a$. It is illuminated at one end by a nearly monochromatic light source.

Optical Waveguide



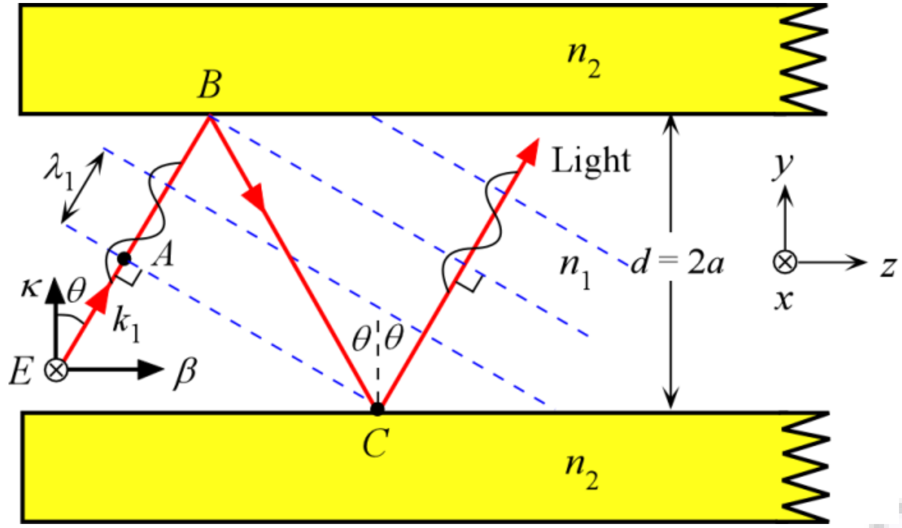
Light waves zigzag along the guide. Is that really what happens?

Waves Inside the Core



A light ray traveling in the guide must interfere constructively with itself to propagate successfully. Otherwise destructive interference will destroy the wave. E is parallel to x. (λ_1 and k_1 are the wavelength and the propagation constant inside the core medium n_1 *i.e.* $\lambda_1 = \lambda/n_1$.)

Waveguide Condition and Modes



$$k_1 = kn_1 = 2\pi n_1/\lambda,$$

$$\Delta\phi(AC) = k_1(AB + BC) - 2\phi = m(2\pi)$$

$$BC = d/\cos\theta \text{ and } AB = BC\cos(2\theta)$$

$$AB + BC = BC\cos(2\theta) + BC = BC[(2\cos^2\theta - 1) + 1] = 2d\cos\theta$$

$$\mathbf{k_1[2d\cos\theta] - 2\phi = m(2\pi)}$$

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m* = 0, 1, 2, 3 *etc
Integer

"Mode number"

Waveguide condition

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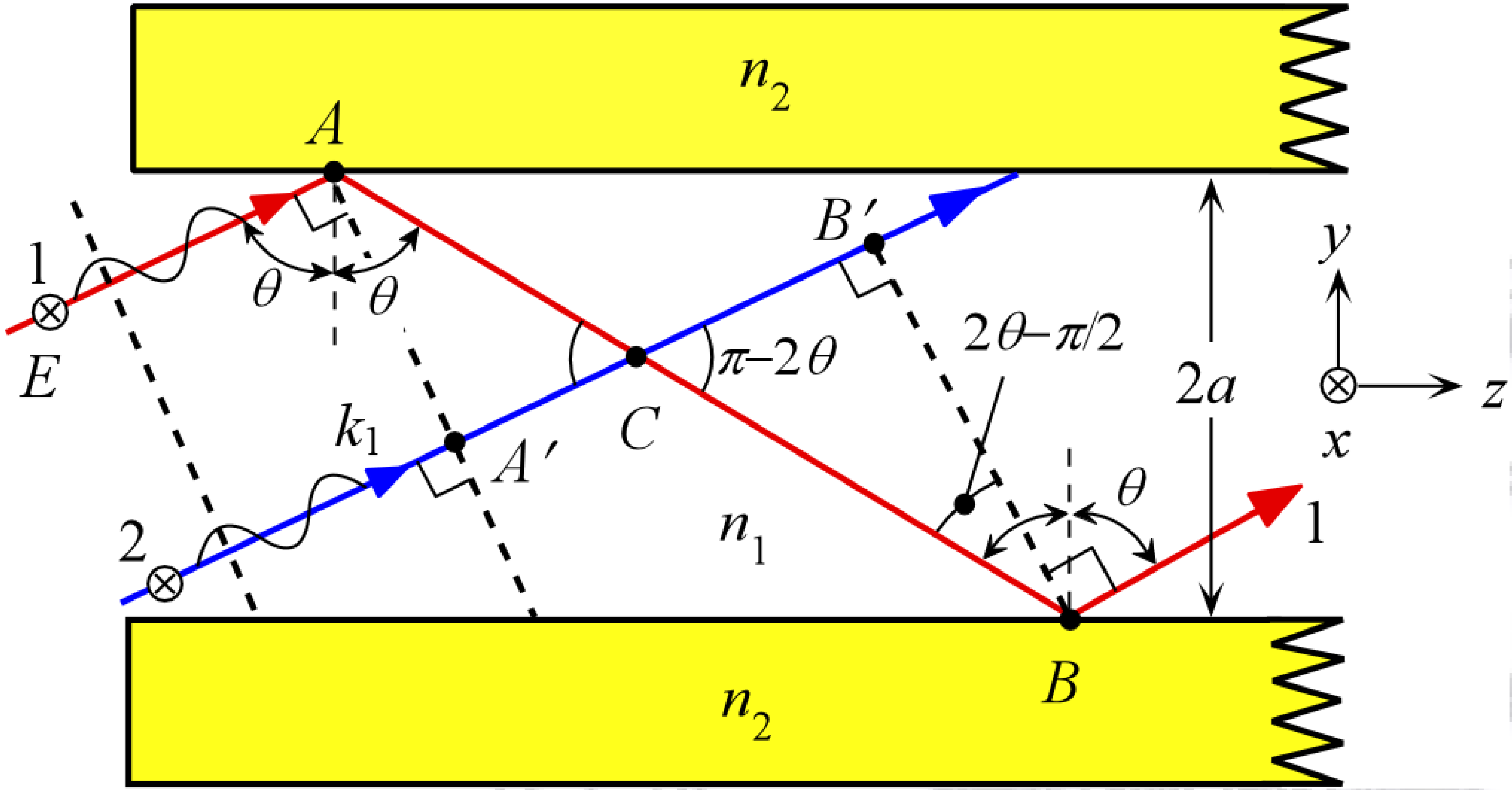
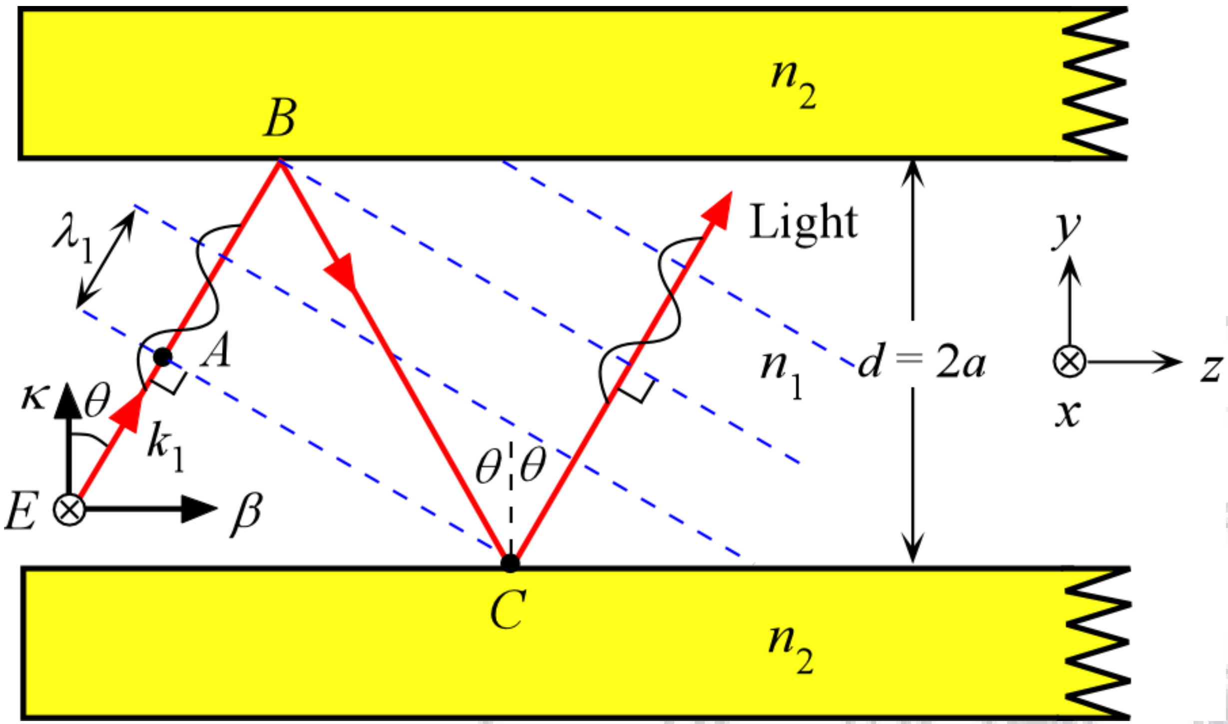
Waves Inside the Core



Two arbitrary waves 1 and 2 that are initially in phase must remain in phase after reflections. Otherwise the two will interfere destructively and cancel each other.



$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

Propagation constant along the guide

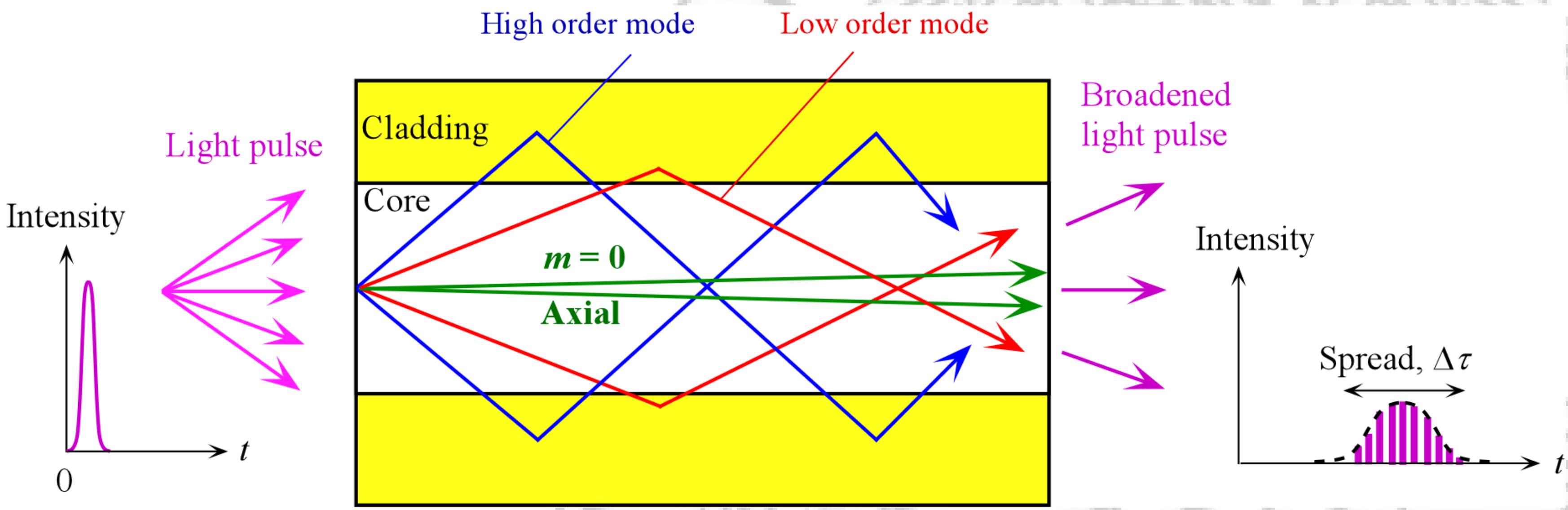
$$\kappa_m = k_1 \cos\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\cos\theta_m$$

Transverse Propagation constant

Waveguide Condition and Waveguide Modes

To get a propagating wave along a guide you must have constructive interference. All these rays interfere with each other. Only certain angles are allowed . Each allowed angle represents a **mode** of propagation.

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$



Waveguide Condition

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m **= integer,** $\boldsymbol{n_1}$ **= core refractive index,** $\boldsymbol{\theta_m}$ **is the incidence angle, 2*a* is the core thickness.**

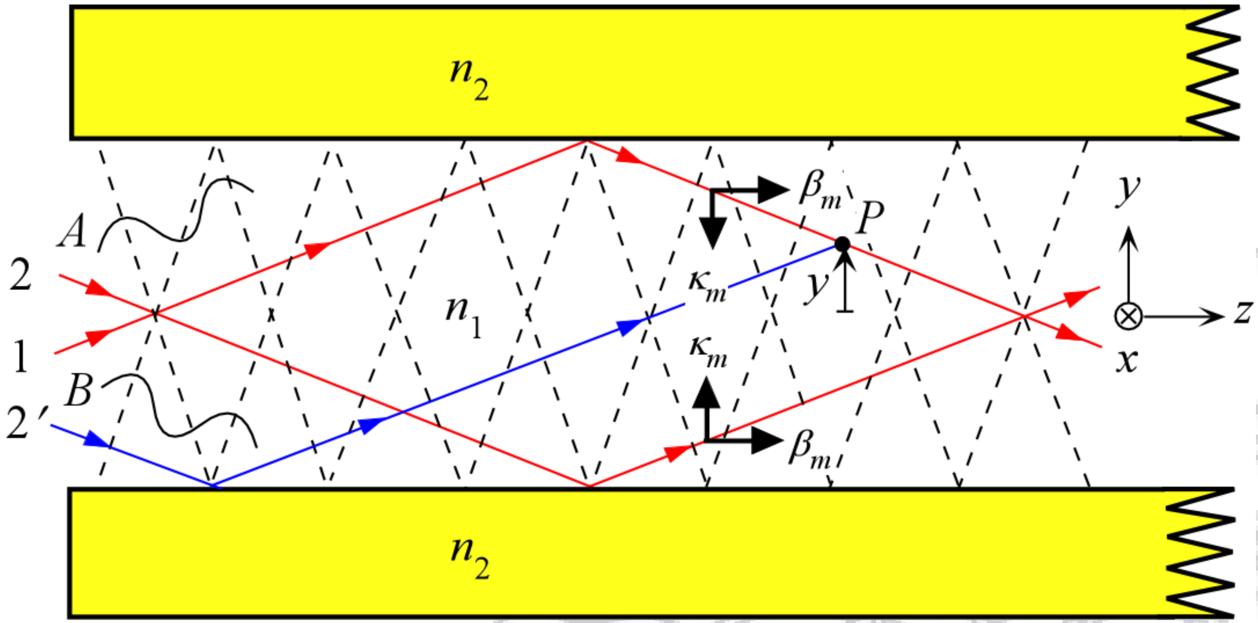
Minimum $\boldsymbol{\theta_m}$ **and maximum *m* must still satisfy TIR.**

There are only a finite number of modes.

Propagation along the guide for a mode *m* is

$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

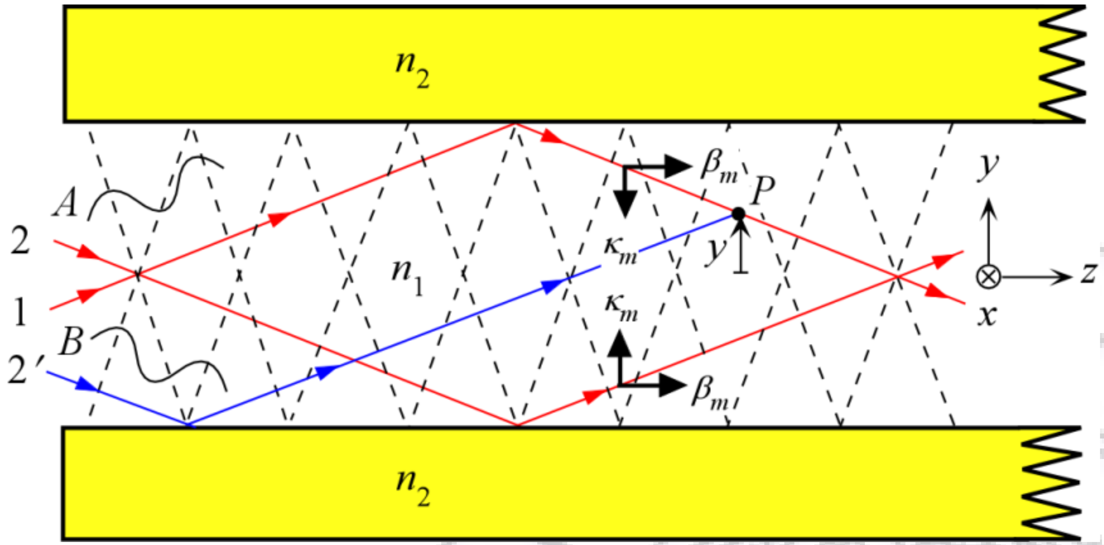
Waveguide Condition and Modes



To get a propagating wave along a guide you must have constructive interference. All these rays interfere with each other. Only certain angles are allowed . Each allowed angle represents a **mode** of propagation.

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

Modes in a Planar Waveguide



We can identify upward (A) and downward (B) traveling waves in the guide which interfere to set up a standing wave along y and a wave that is propagating along z. Rays 2 and 2′ belong to the same wave front but 2′ becomes reflected before 2. The interference of 1 and 2′ determines the field at a height y from the guide center. The field $E(y, z, t)$ at P can be written as

$$E(y,z,t) = \underbrace{E_m(y)}_{\text{Field pattern along } y}\underbrace{\cos(\omega t - \beta_m z)}_{\text{Traveling wave along } z}$$

Modes in a Planar Waveguide: Summary

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m = integer, n_1 = core refractive index, θ_m is the incidence angle, $2a$ is the core thickness.

$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

$$E(y,z,t) = \underbrace{E_m(y)}_{\text{Field pattern along } y}\underbrace{\cos(\omega t - \beta_m z)}_{\text{Traveling wave along } z}$$

Traveling wave along z

Field pattern along y

Vertical profile along y

We consider point C

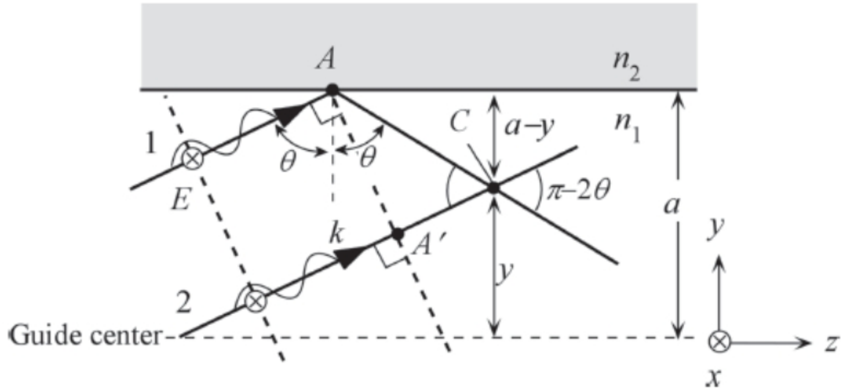
(a) From the geometry we have the following:

$$(a-y)/AC = \cos\theta$$

and $$A'C/AC = \cos(\pi - 2\theta)$$

The phase difference between the waves meeting at C is

$$\Phi = kAC - \phi - kA'C = k_1AC - k_1AC\cos(\pi - 2\theta) - \phi$$



$$\Phi_m = k_1AC[1 - \cos(\pi - 2\theta)] - \phi = k_1AC[1 + \cos(2\theta)] - \phi$$
$$= k_1[(a-y)/\cos\theta][1 + 2\cos^2\theta - 1] - \phi$$
$$= k_1[(a-y)/\cos\theta][2\cos^2\theta] - \phi$$
$$= 2k_1(a-y)\cos\theta - \phi$$

$$\Phi_m = 2k_1(a-y)\cos\theta_m - \phi_m = 2k_1(a-y)\frac{m\pi + \phi_m}{k_1(2a)} - \phi_m$$

$$\Phi_m = \Phi(y) = m\pi - \frac{y}{a}(m\pi + \phi_m) \quad \Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

Given, $\left[\frac{2\pi(2a)n_1}{\lambda}\right]\cos\theta_m$ -

$$\cos\theta_m = \frac{\lambda(m\pi + \phi_m)}{2\pi n_1(2a)}$$

The two waves interfering at C are out phase by Φ,

$$E(y) = A\cos(\omega t) + A\cos[\omega t + \Phi_m(y)]$$

where A is an arbitrary amplitude. Thus,

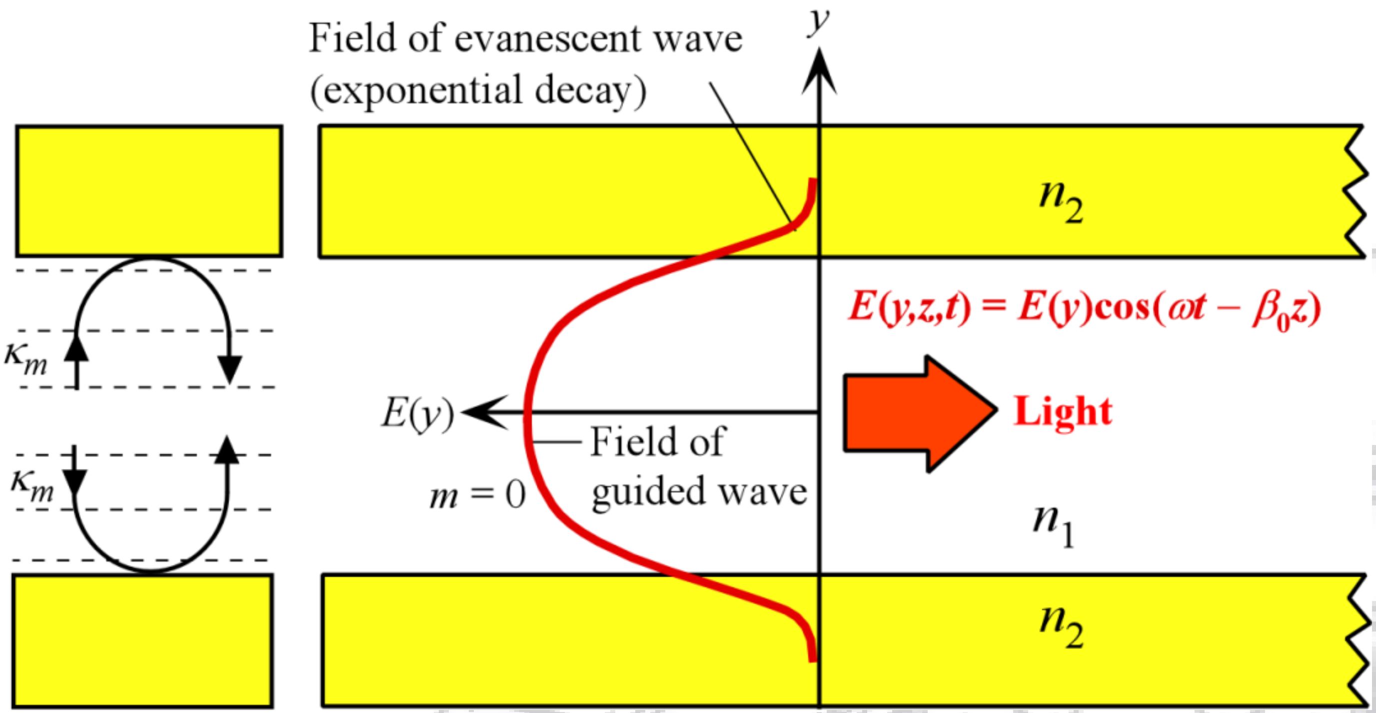
$$E = 2A\cos[\omega t + \tfrac{1}{2}\Phi_m(y)]\cos[\tfrac{1}{2}\Phi_m(y)]$$

or $$E = \{2A\cos[\tfrac{1}{2}\Phi_m(y)]\}\cos(\omega t + \Phi') = E_o\cos(\omega t + \Phi')$$

in which $\Phi' = \Phi_m/2$, and $\cos(\omega t + \Phi')$ is the time dependent part that represents the wave phenomenon, and the curly brackets contain the effective amplitude. Thus, the amplitude E_o is

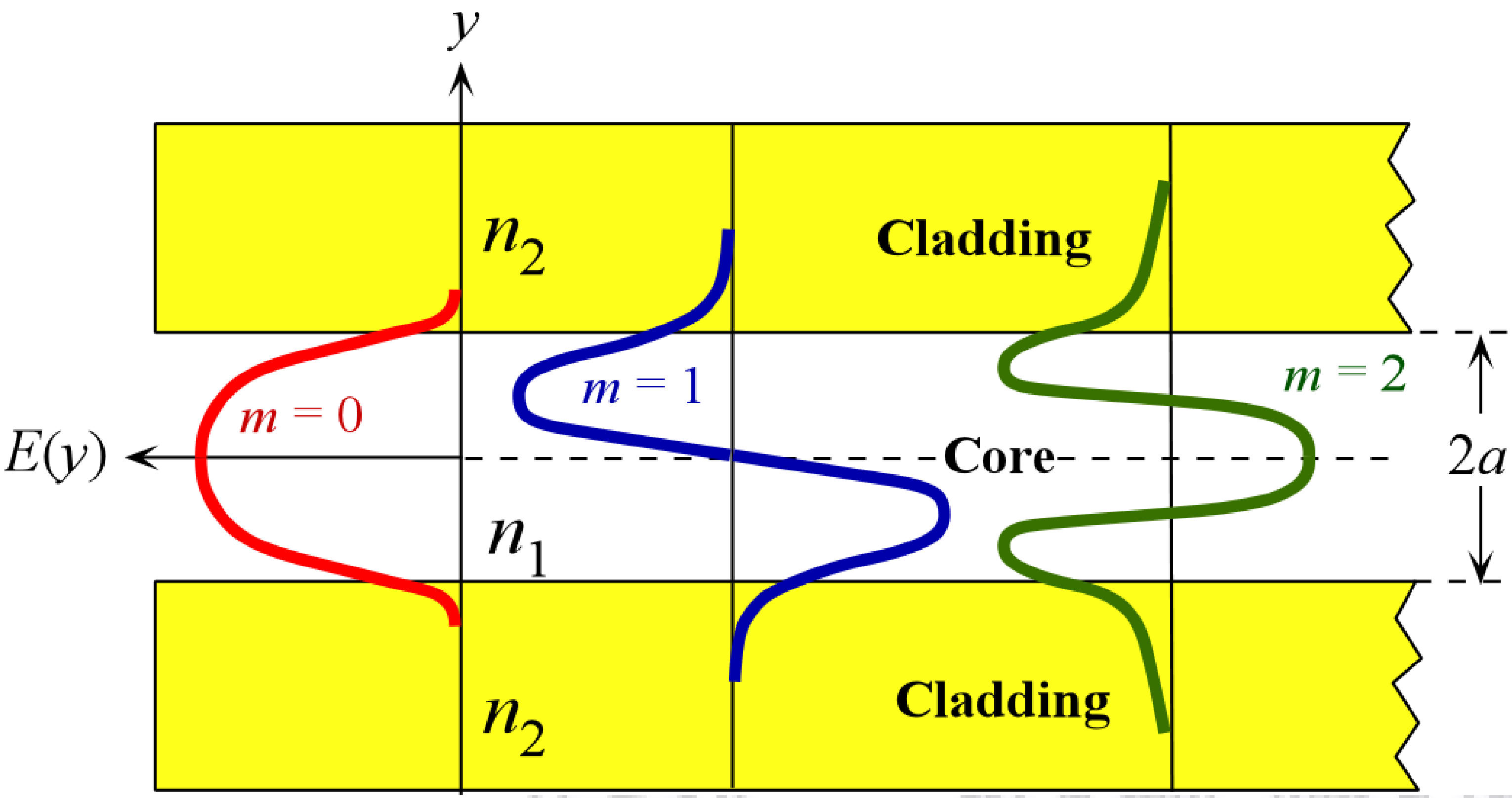
$$E_o = 2A\cos\left[\frac{m\pi}{2} - \frac{y}{2a}(m\pi + \phi_m)\right]$$

Mode Field Pattern



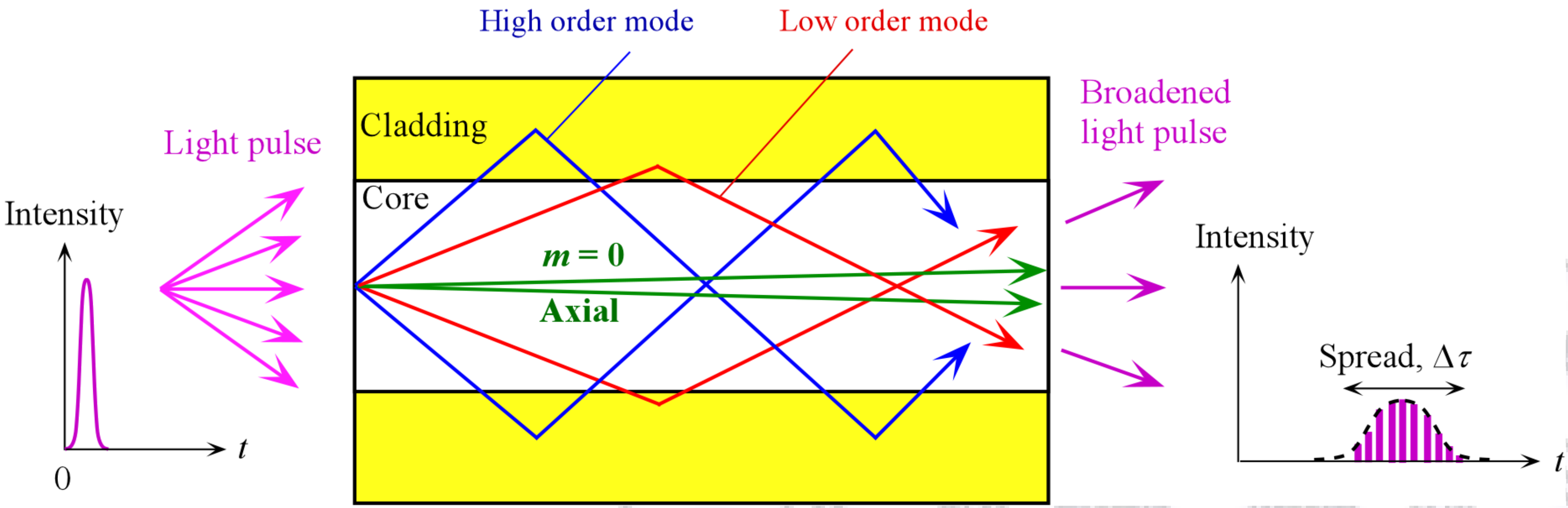
Left: The upward and downward traveling waves have equal but opposite wavevectors κ_m and interfere to set up a standing electric field pattern across the guide. Right: The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the highest θ. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide

Modes in a Planar Waveguide



The electric field patterns of the first three modes (m = 0, 1, 2) traveling wave along the guide. Notice different extents of field penetration into the cladding (by continuity we know that we have an exponential tail). More on that in a few slides

Intermode (Intermodal or Modal) Dispersion



Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

V-Number

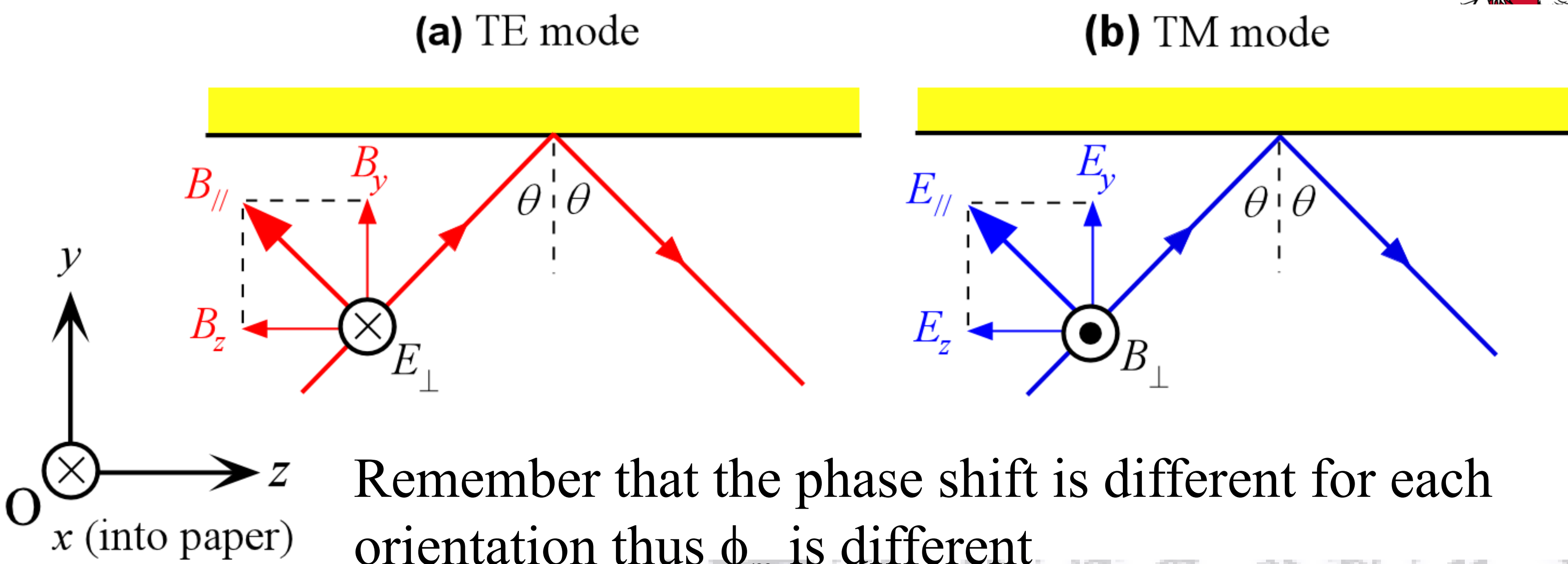
All waveguides are characterized by a parameter called the ***V*-number** or **normalized frequency**

$$\boxed{V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}}$$

$V < \pi/2$, $m = 0$ is the only possibility and only the fundamental mode ($m = 0$) propagates along the dielectric slab waveguide: a **single mode** planar waveguide.

$\lambda = \lambda_c$ for $V = \pi/2$ is the **cut-off wavelength**, and above this wavelength, only one-mode, the fundamental mode will propagate.

TE and TM Modes



Remember that the phase shift is different for each orientation thus ϕ_m is different

$E_\perp$ is along x, so that $E_\perp = E_x$

$B_\perp$ is along $-x$, so that $B_\perp = -B_x$

Possible modes can be classified in terms of (a) transverse electric field (TE) and (b) transverse magnetic field (TM). Plane of incidence is the paper.

Example on Waveguide Modes

Consider a planar dielectric guide with a core thickness 20 μm, n_1 = 1.455, n_2 = 1.440, light wavelength of 900 nm. Find the modes

TIR phase change ϕ_m for TE mode

$$\tan\left(\tfrac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$$

TE mode

Waveguide condition

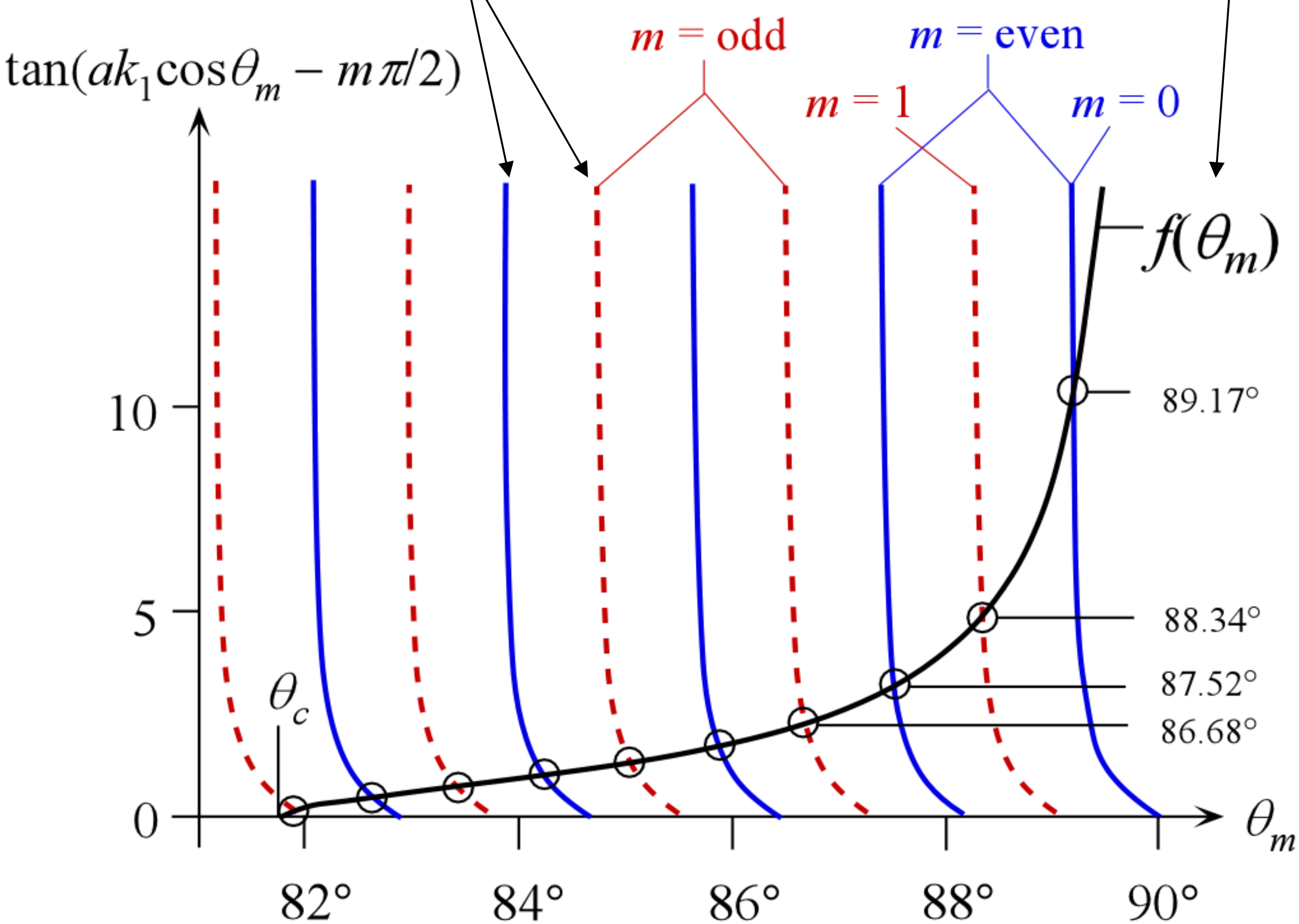
$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

Waveguide condition

$$\phi_m = 2ak_1\cos\theta_m - m\pi$$

$$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} = f(\theta_m)$$

TE mode



$$\frac{1}{\delta_m} = \alpha_m = \frac{2\pi n_2 \left[\left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_m - 1 \right]^{1/2}}{\lambda}$$

Mode m, incidence angle θ_m and penetration δ_m for a planar dielectric waveguide with $d = 2a = 20$ μm, $n_1 = 1.455$, $n_2 = 1.440$ ($\lambda = 900$ nm)

m	0	1	2	3	4	5	6	7	8	9
θ_m	89.2°	88.3°	87.5°	86.7°	85.9°	85.0°	84.2°	83.4°	82.6°	81.9°
δ_m (μm)	0.691	0.702	0.722	0.751	0.793	0.866	0.970	1.15	1.57	3.83
n_{eff}	1,4548	1,4544	1,4536	1,4525	1,4512	1,4495	1,4476	1,4454	1,4429	**1,4404**

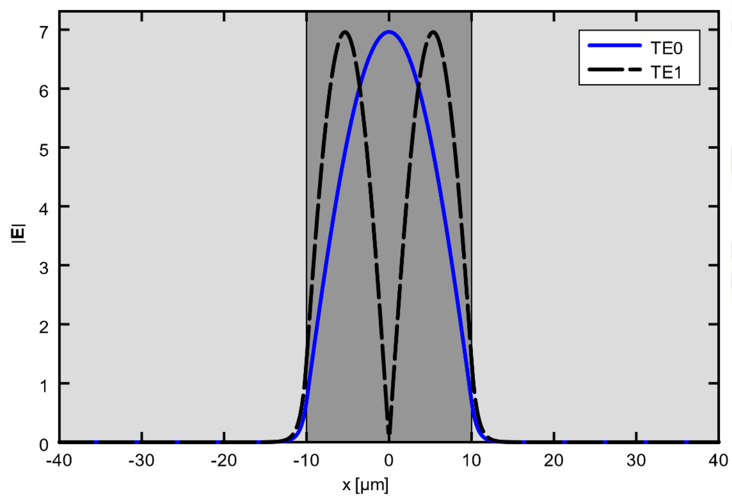
$m = 0$
Fundamental mode

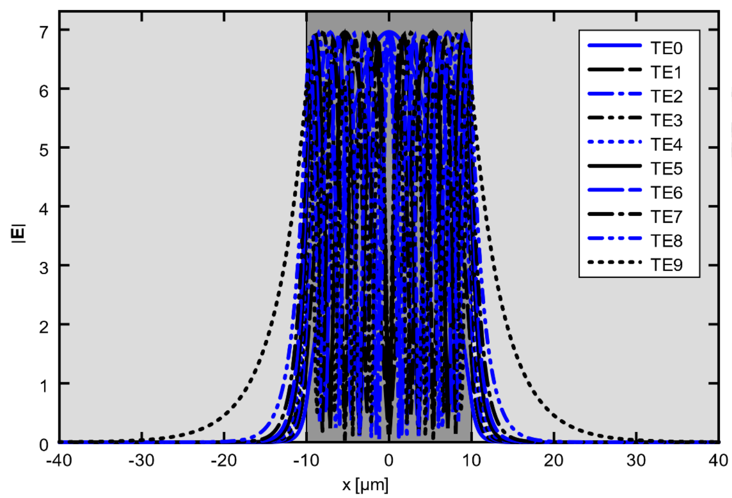
Highest mode

Critical angle $\theta_c = \arcsin(n_2/n_1) = 81.77°$

Mode profiles

- https://www.computational-photonics.eu/oms.html





- Effective index is an indication of the confinement of the mode

Number of Modes M

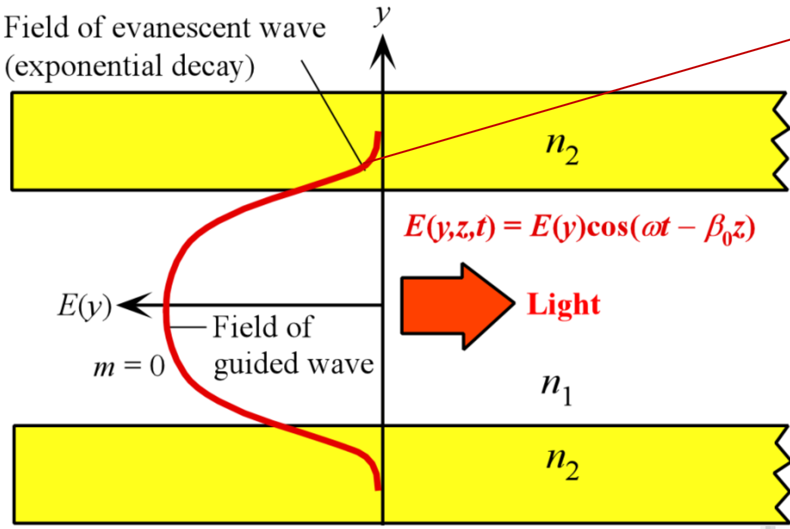
Waveguide condition $$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

One mode when $V < \pi/2$

Multimode when $V > \pi/2$

$$M = \mathrm{Int}\left(\frac{2V}{\pi}\right) + 1$$

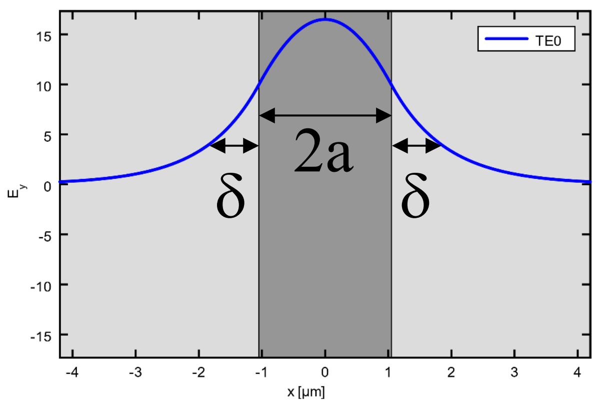
Mode Field Width (MFW) $2w_o$



$$E_{cladding}(y') = E_{cladding}(0)\exp(-\alpha_{cladding}\, y')$$

$$\alpha_{cladding} = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

$$\approx \frac{2\pi}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{V}{a}$$

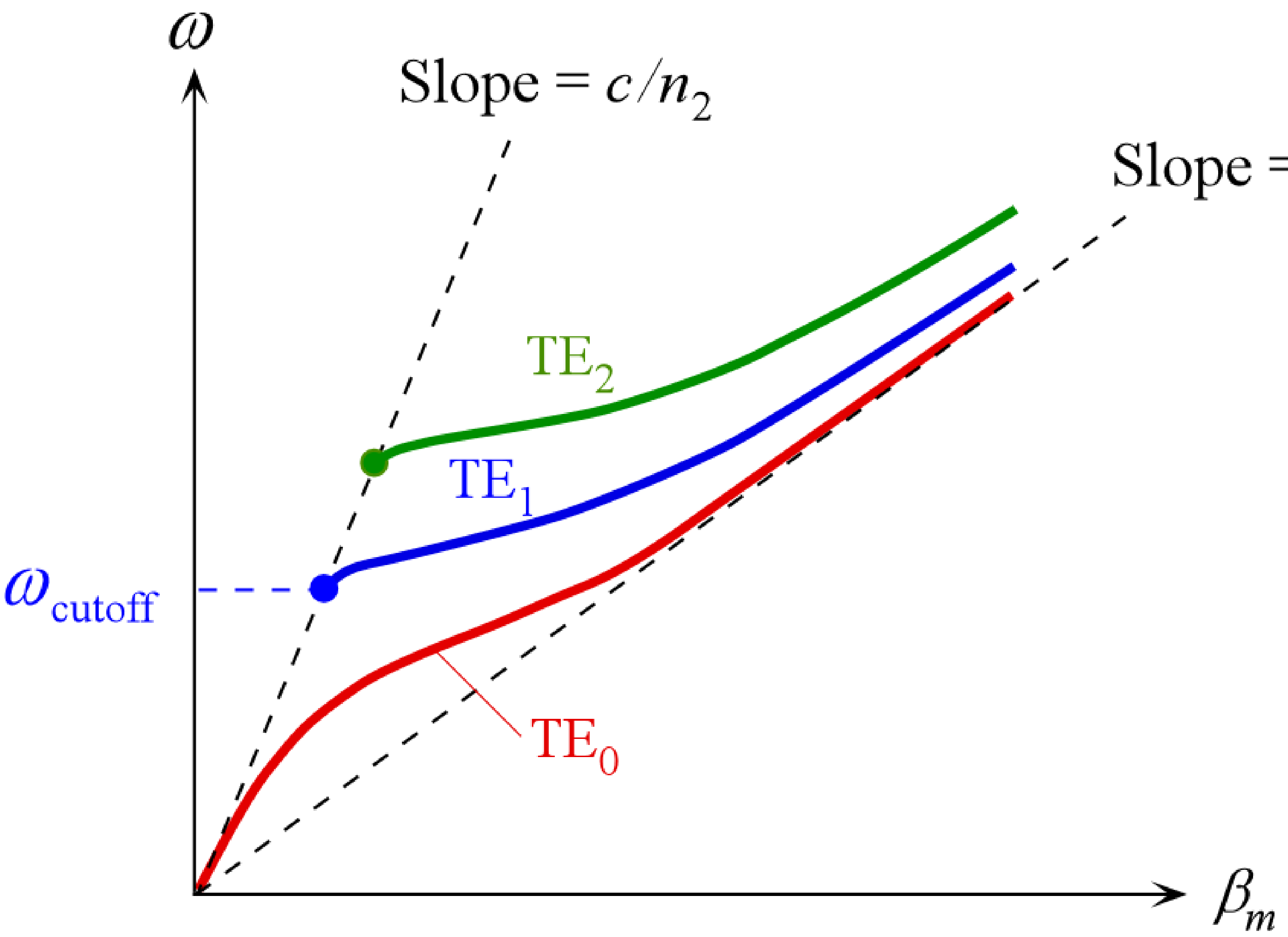


$$2w_0 \approx 2a + 2\delta = 2a\frac{(V+1)}{V}$$

Mode Field Width $2w_o$

Note: The MFW definition here is semiquantitive. A more rigorous approach needs to consider the optical power in the mode and how much of this penetrates the cladding. See optical fibers section.

Waveguide Dispersion Curve

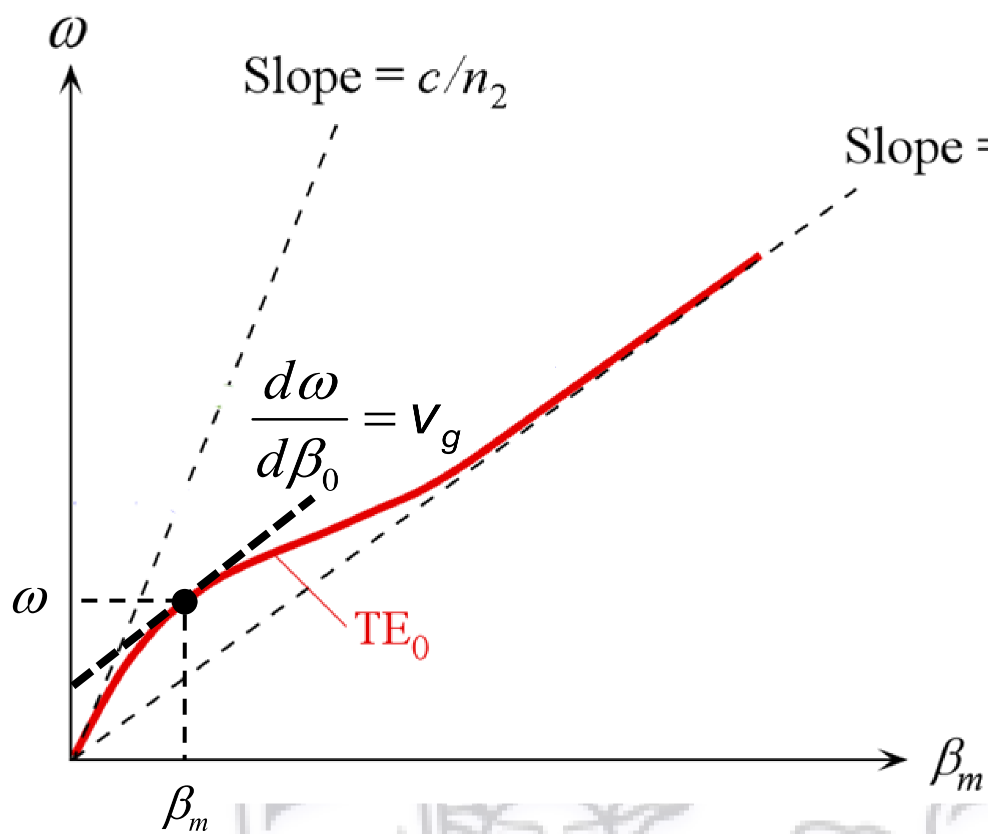


The slope of ω vs. β is the group velocity v_g

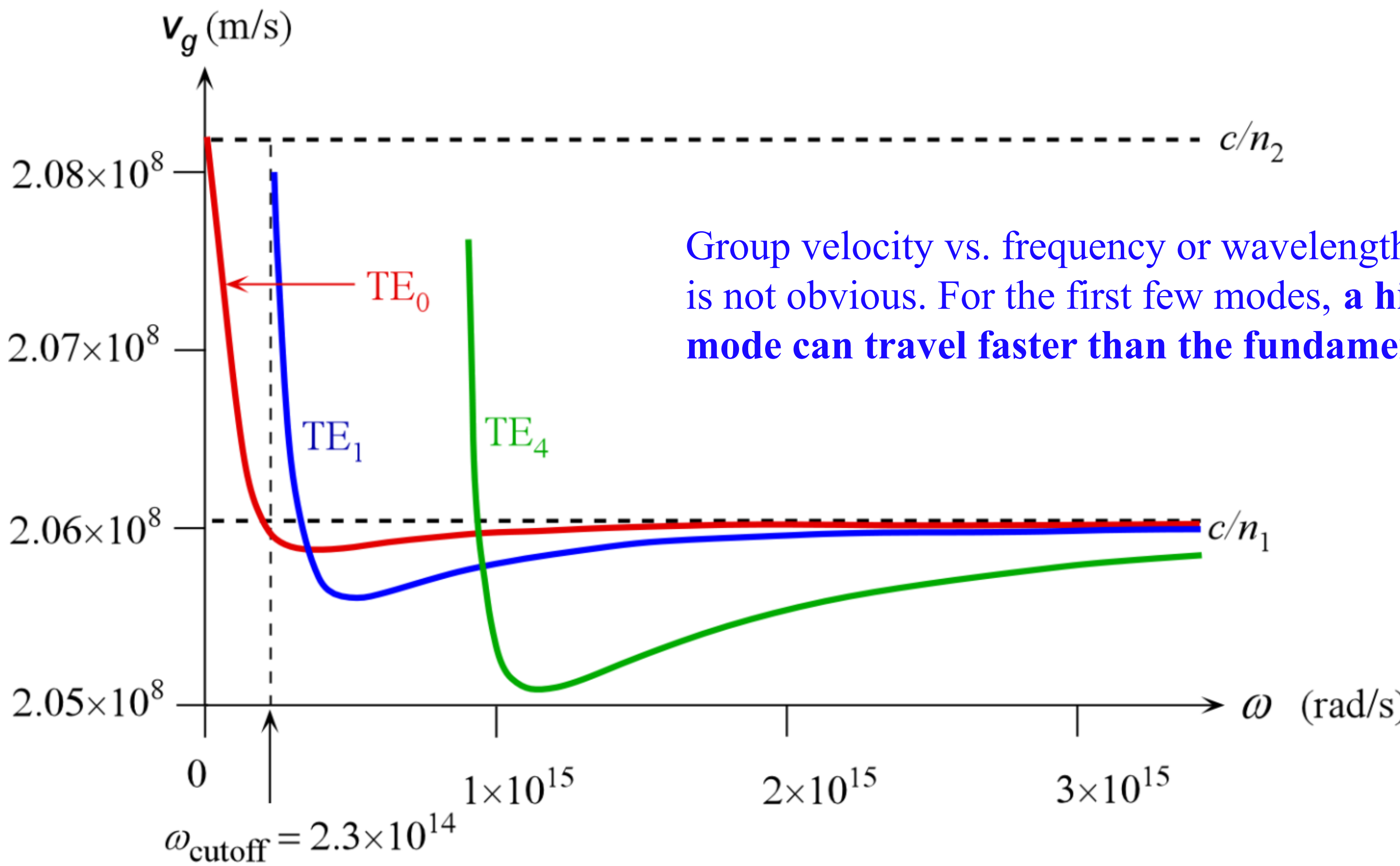
Waveguide Dispersion Curve

Slope = Group Velocity

The slope of ω vs. β is the group velocity v_g



Mode Group Velocities from Dispersion Diagram

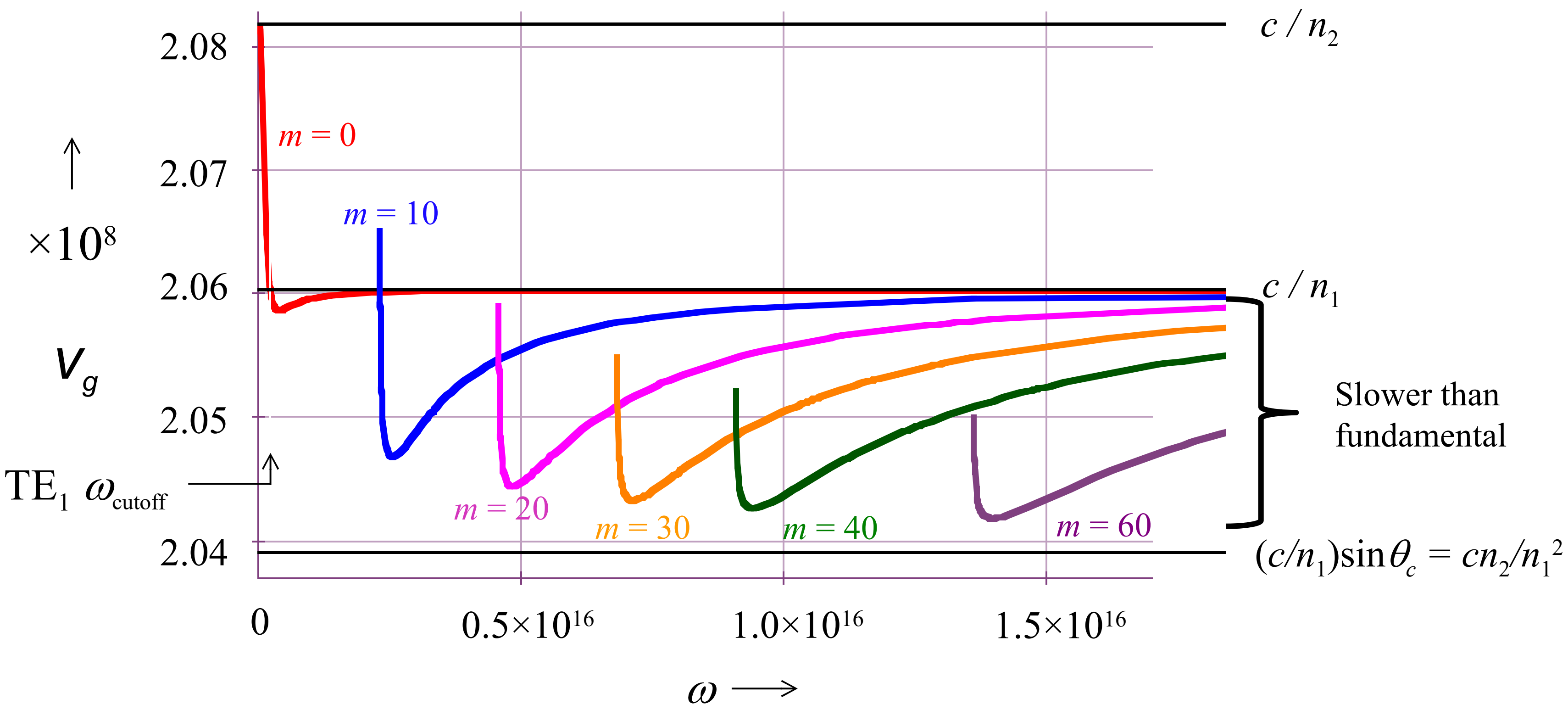


Group velocity vs. frequency or wavelength behavior is not obvious. For the first few modes, **a higher mode can travel faster than the fundamental**.

The group velocity v_g vs. ω for a planar dielectric guide with a core thickness ($2a$) = 20 μm, n_1 = 1.455, n_2 = 1.440. TE_0, TE_1 and TE_4

Not in the textbook

A Planar Dielectric Waveguide with Many Modes



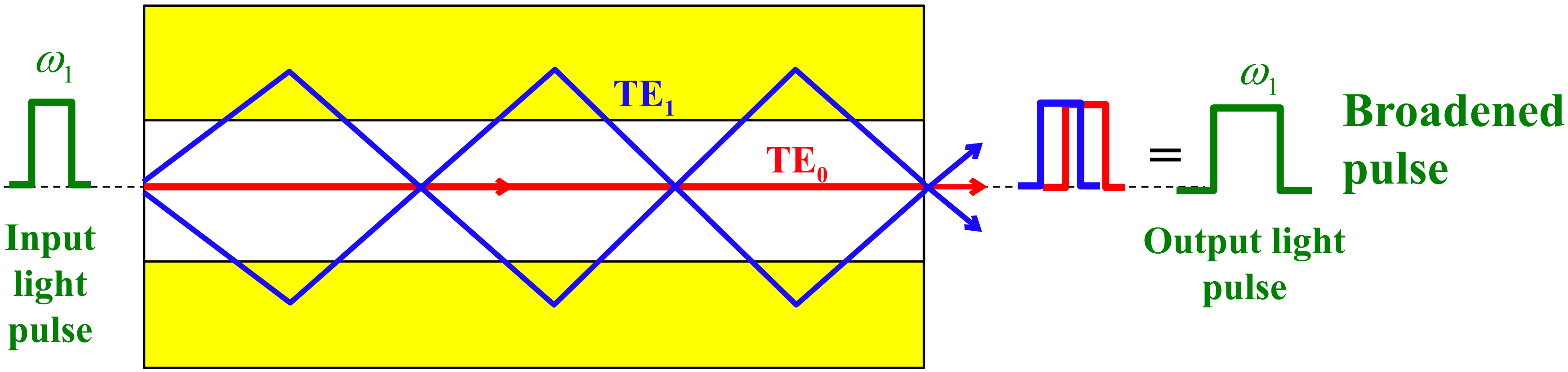
The group velocity v_g vs. ω for a planar dielectric guide

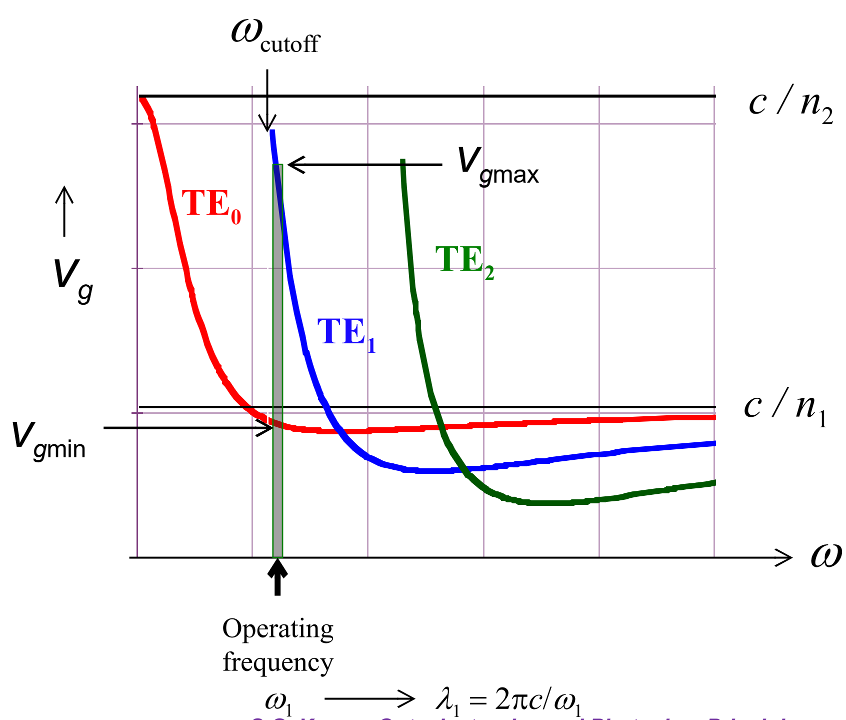
Core thickness ($2a$) = 20 μm, n_1 = 1.455, n_2 = 1.440

[Calculations by the author]

Not in the textbook

Dispersion in the Planar Dielectric Waveguide with TE_0 and TE_1 (Near cut-off)





$$v_{gmax} \approx c/n_2$$

$$v_{gmin} \approx c/n_1$$

$$\Delta\tau = \frac{L}{v_{gmin}} - \frac{L}{v_{gmax}}$$

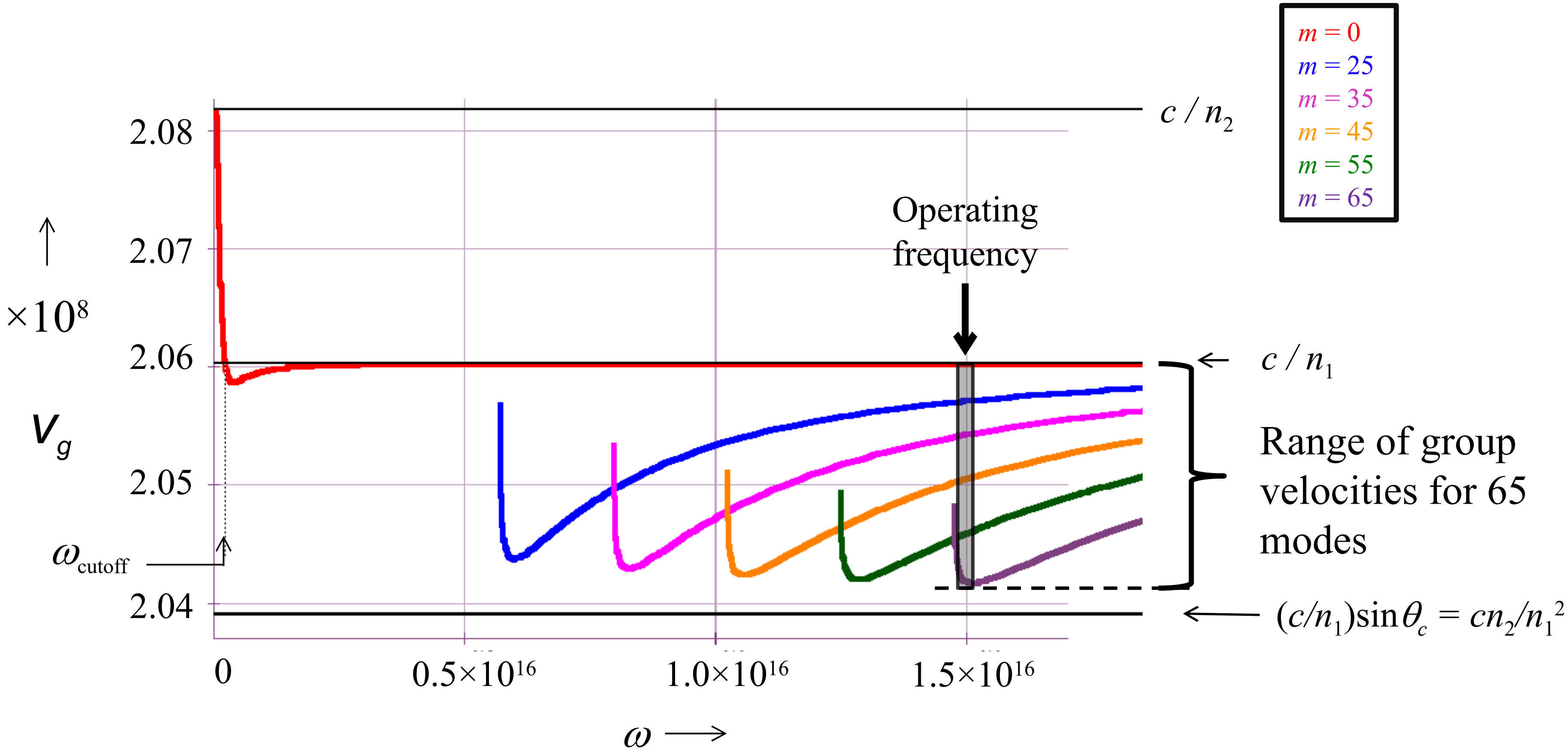
Spread in arrival times

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

Dispersion

Not in the textbook

A Planar Dielectric Waveguide with Many Modes

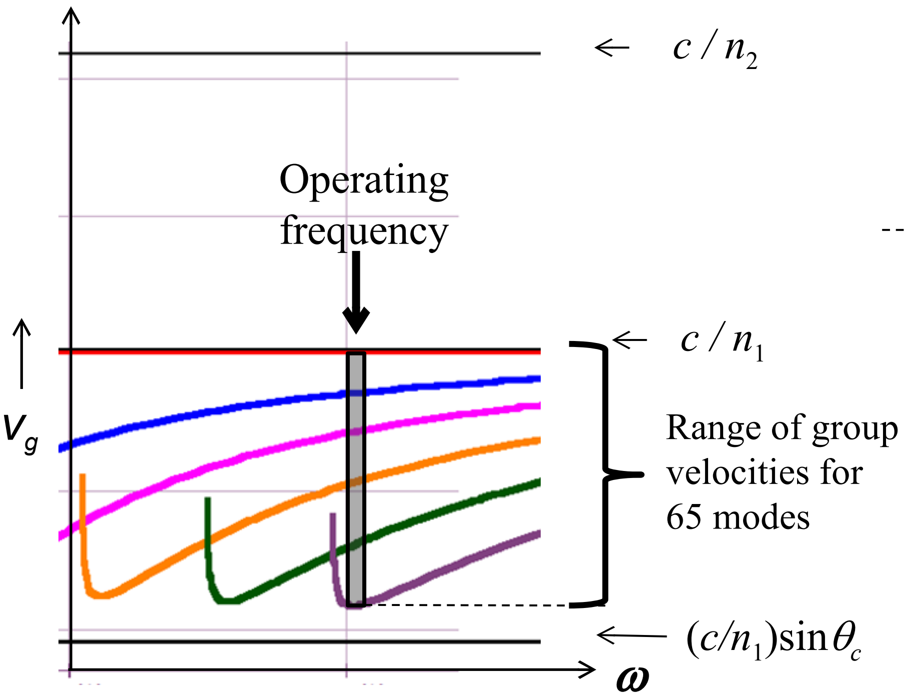


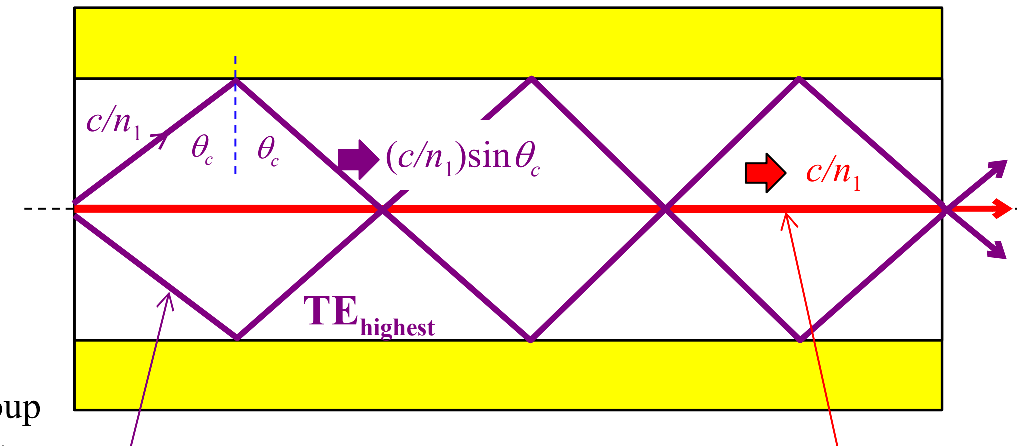
Multimode operation in which many modes propagate with different group velocities

v_g vs. ω for a planar dielectric guide with a core thickness $(2a)$ = 20 µm, n_1 = 1.455, n_2 = 1.440
[Calculations by the author]

Not in the textbook

Dispersion in the Planar Dielectric Waveguide with Many Modes Far from Cutoff





$$v_{g\min} \approx \frac{c}{n_1}\sin\theta_c = \frac{c}{n_1}\left(\frac{n_2}{n_1}\right)$$

$$v_{g\max} \approx \frac{c}{n_1}$$

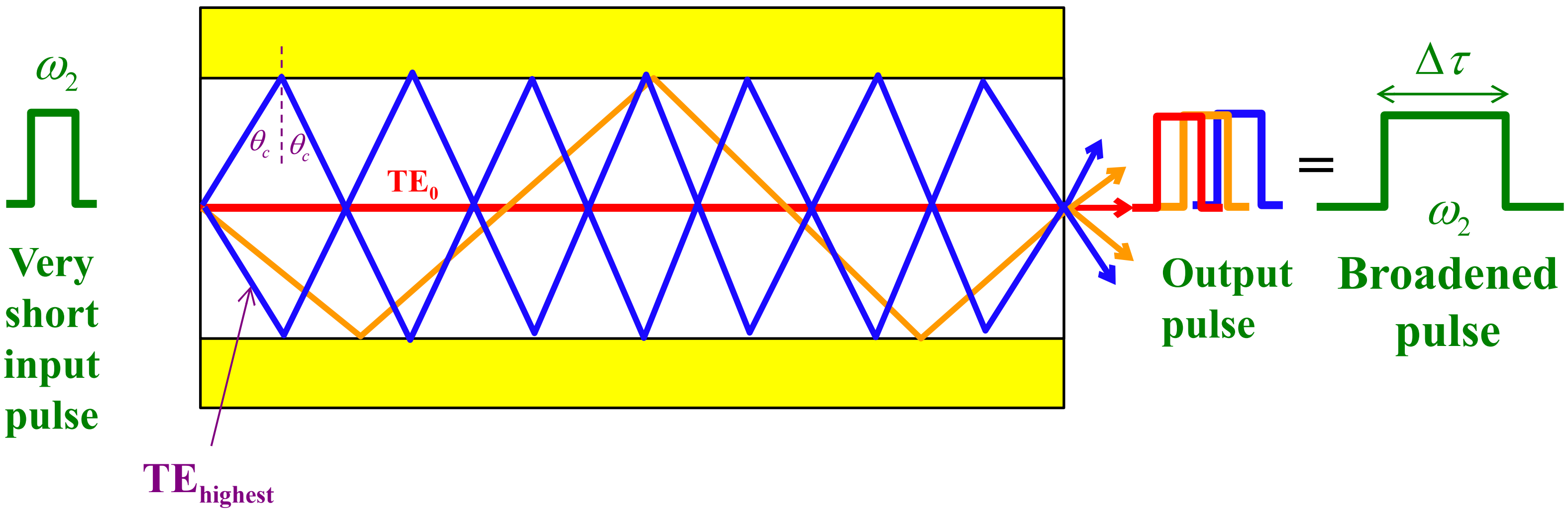
$$\frac{\Delta\tau}{L} = \frac{1}{v_{g\min}} - \frac{1}{v_{g\max}} \Rightarrow \frac{\Delta\tau}{L} = \frac{n_1^2}{cn_2} - \frac{n_1}{c} = \frac{1}{c}\left[\frac{(n_1 - n_2)n_1}{n_2}\right] \approx \frac{(n_1 - n_2)}{c}$$

$$\boxed{\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}}$$

(Since n_1 and n_2 are only slightly different.)

Not in the textbook

Dispersion in the Planar Dielectric Waveguide
Many Modes



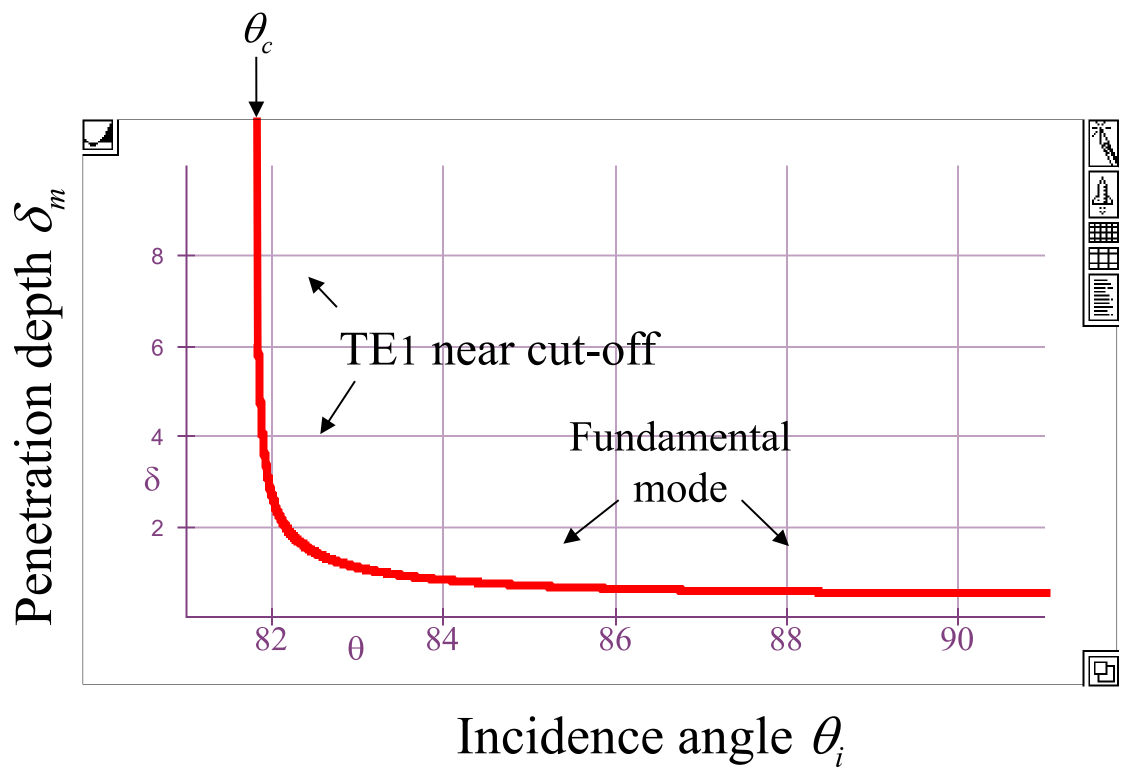
$$\frac{\Delta\tau}{L} = \frac{1}{v_{g\min}} - \frac{1}{v_{g\max}} \Rightarrow \frac{\Delta\tau}{L} \approx \frac{(n_1 - n_2)}{c}\left(\frac{n_1}{n_2}\right)$$

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

(Since $n_1/n_2 \approx 1$)

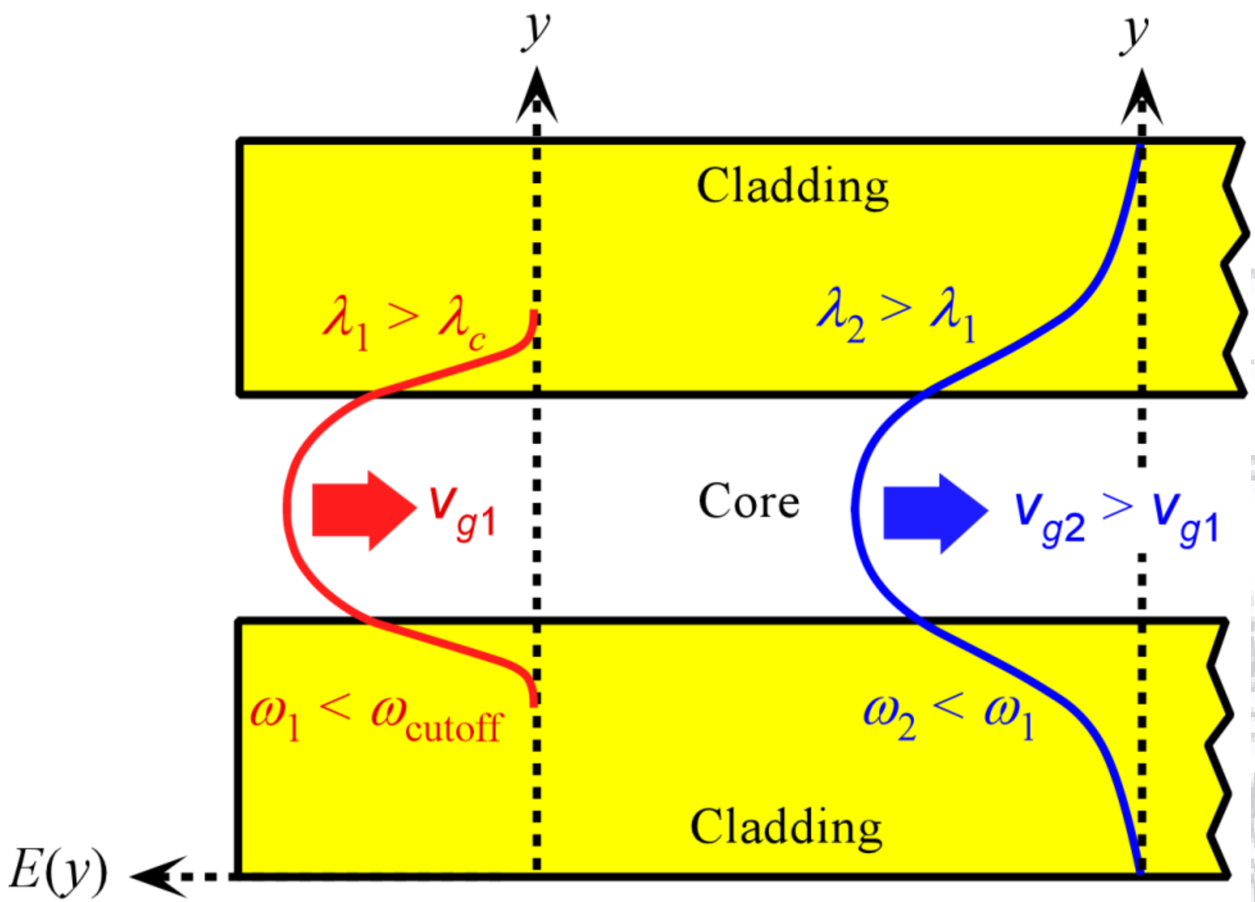
Not in the textbook

How can a higher mode such as TE_1 or TE_2 travel faster than the fundamental near cut-off?



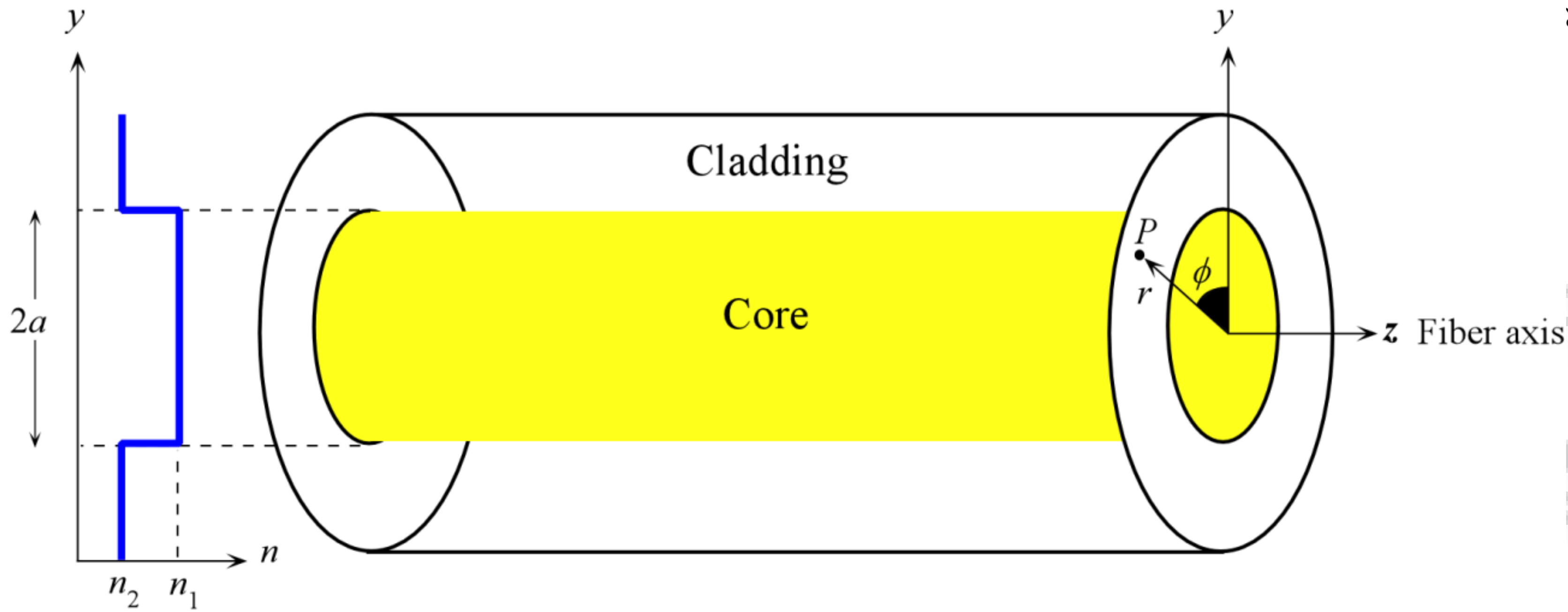
The mode TE_1 penetrates into the cladding where its velocity is higher than in the core. If penetration is large, as near cut-off, TE_1 group velocity along the guide can exceed that of TE_0.

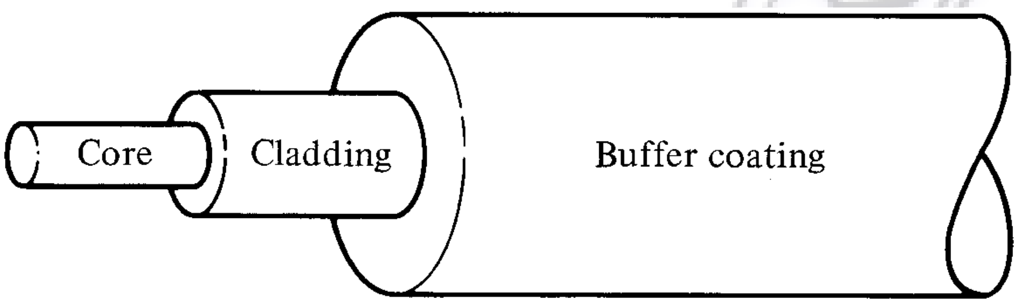
Group Velocity and Wavelength: Fundamental Mode



The electric field of TE_0 mode extends more into the cladding as the wavelength increases. As more of the field is carried by the cladding, the group velocity increases.

Optical Fibers

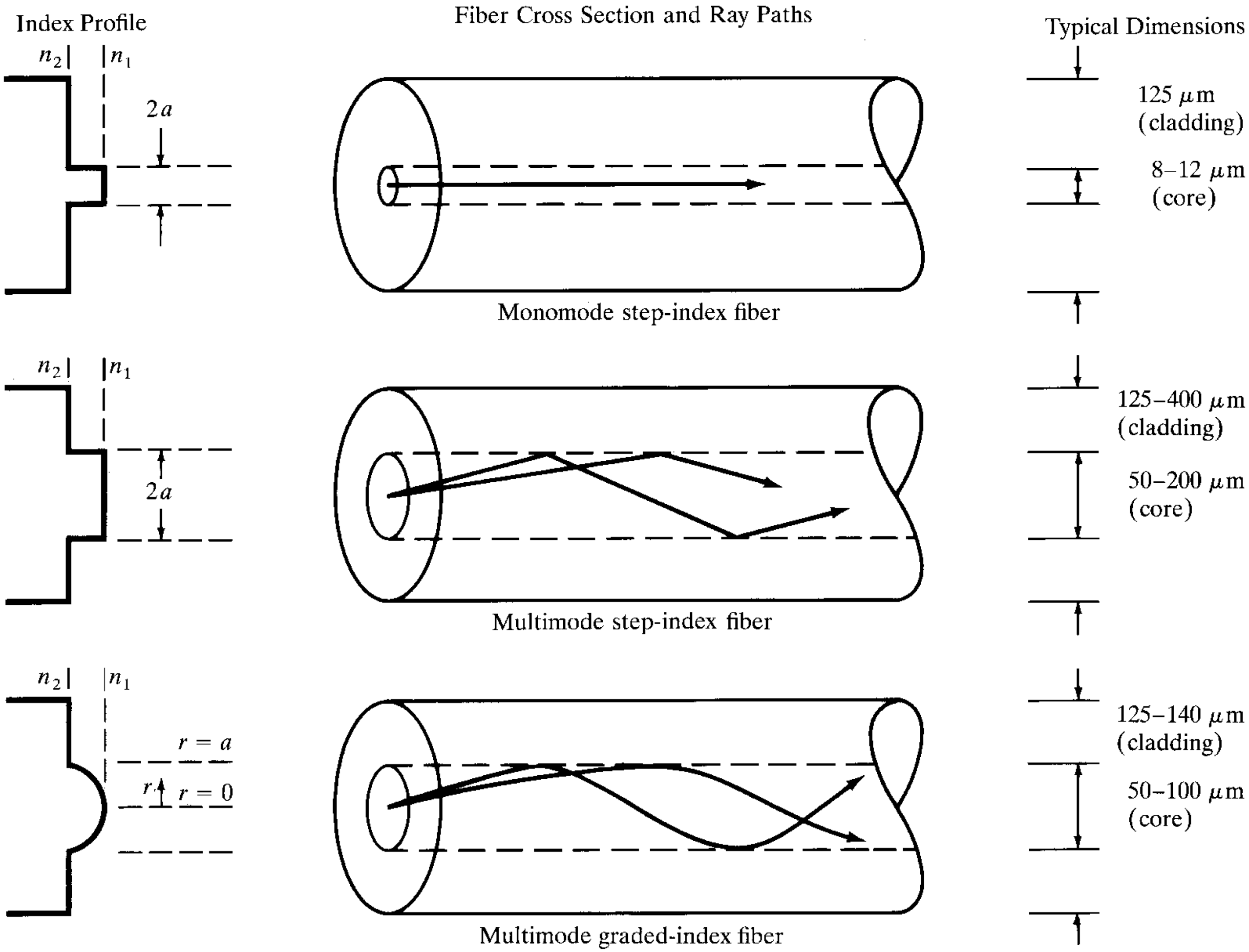




The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry.

The coordinates r, ϕ, z are used to represent any point P in the fiber. Cladding is normally much thicker than shown.

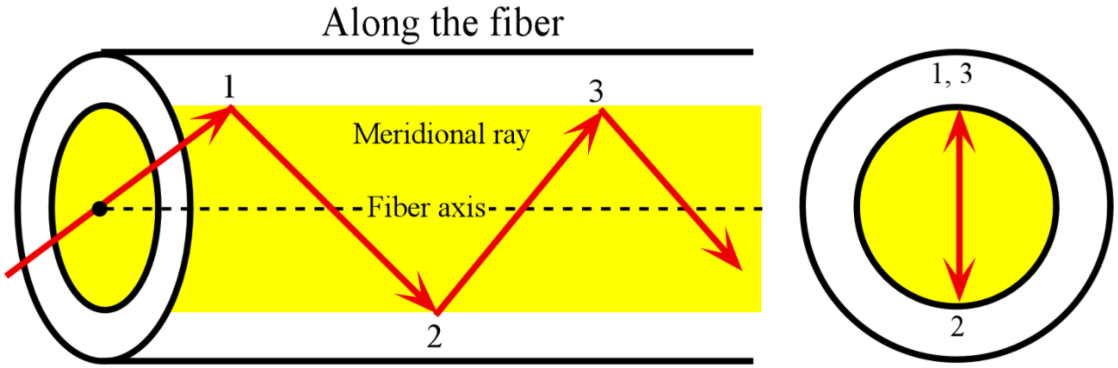
Different Structures of Optical Fiber



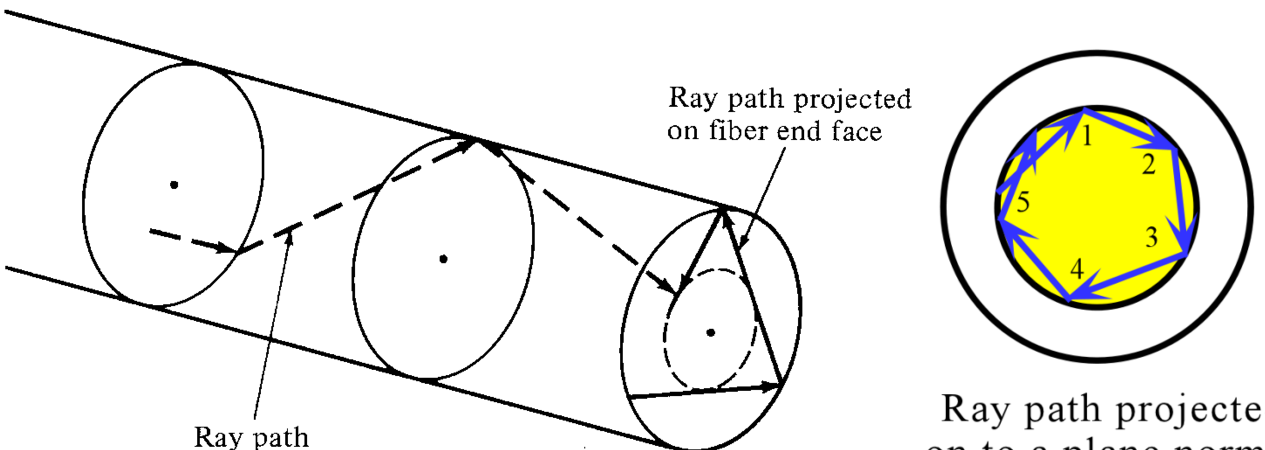
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Meridional ray enters the fiber through the fiber axis and hence also crosses the fiber axis on each reflection as it zigzags down the fiber. It travels in a plane that contains the fiber axis.

Skew ray enters the fiber off the fiber axis and zigzags down the fiber without crossing the axis



(a) A meridional ray always crosses the fiber axis.



(b) A skew ray does not have to cross the fiber axis. It zigzags around the fiber axis.

Ray path projected on to a plane normal to fiber axis

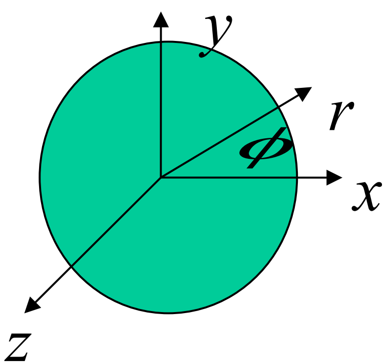
Linearly Polarized Modes LP_{lm}

Weakly guiding modes in fibers

$\Delta=(n_1-n_2)/n_1 \ll 1$ weakly guiding fibers ➔ almost plane wave behaviour since all the mode are almost axial

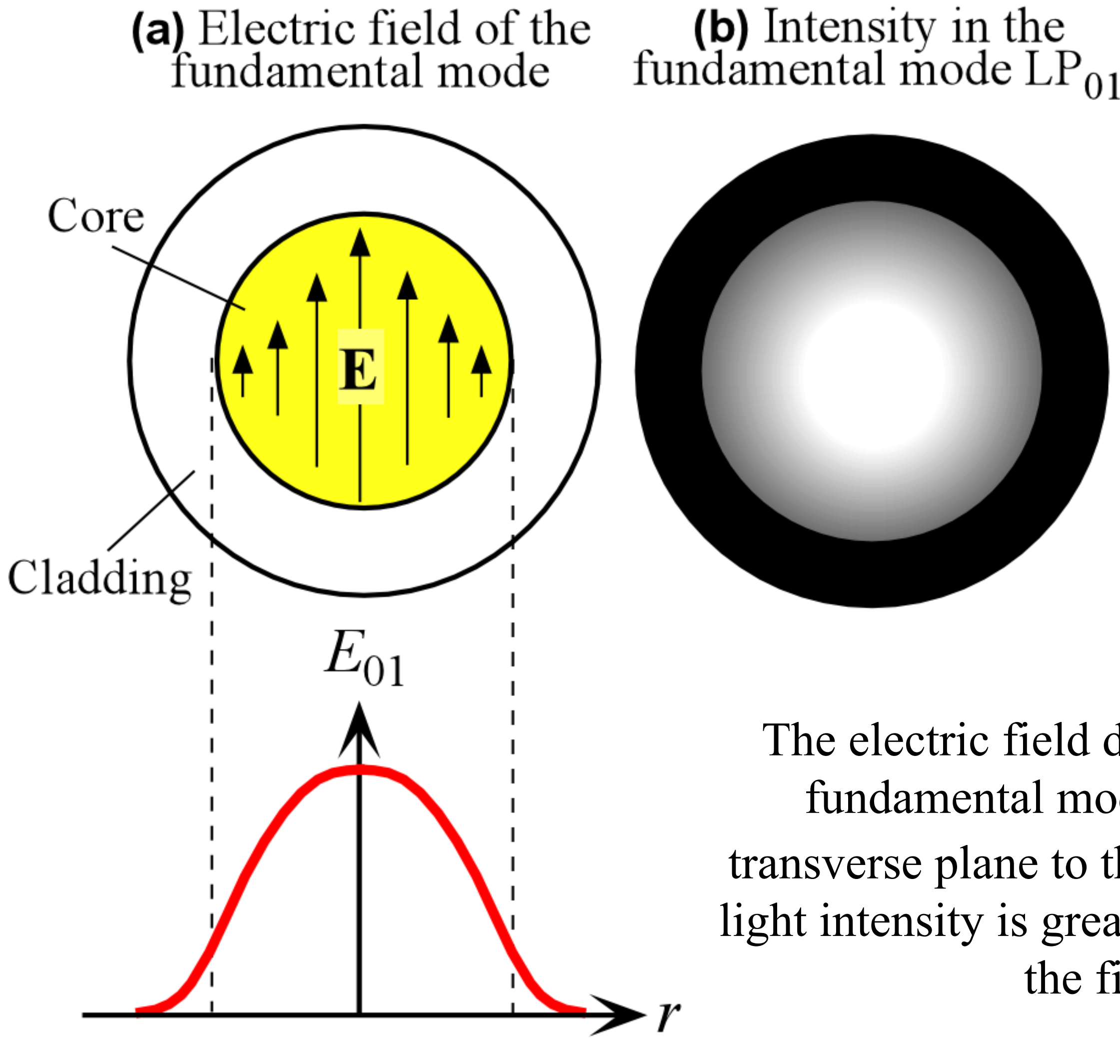
$$\sin\theta_c = \frac{n_2}{n_1}$$

$$E_{\mathrm{LP}} = \underbrace{E_{lm}(r,\phi)}_{\text{Field Pattern}}\ \underbrace{\exp j(\omega t - \beta_{lm} z)}_{\text{Traveling wave}}$$



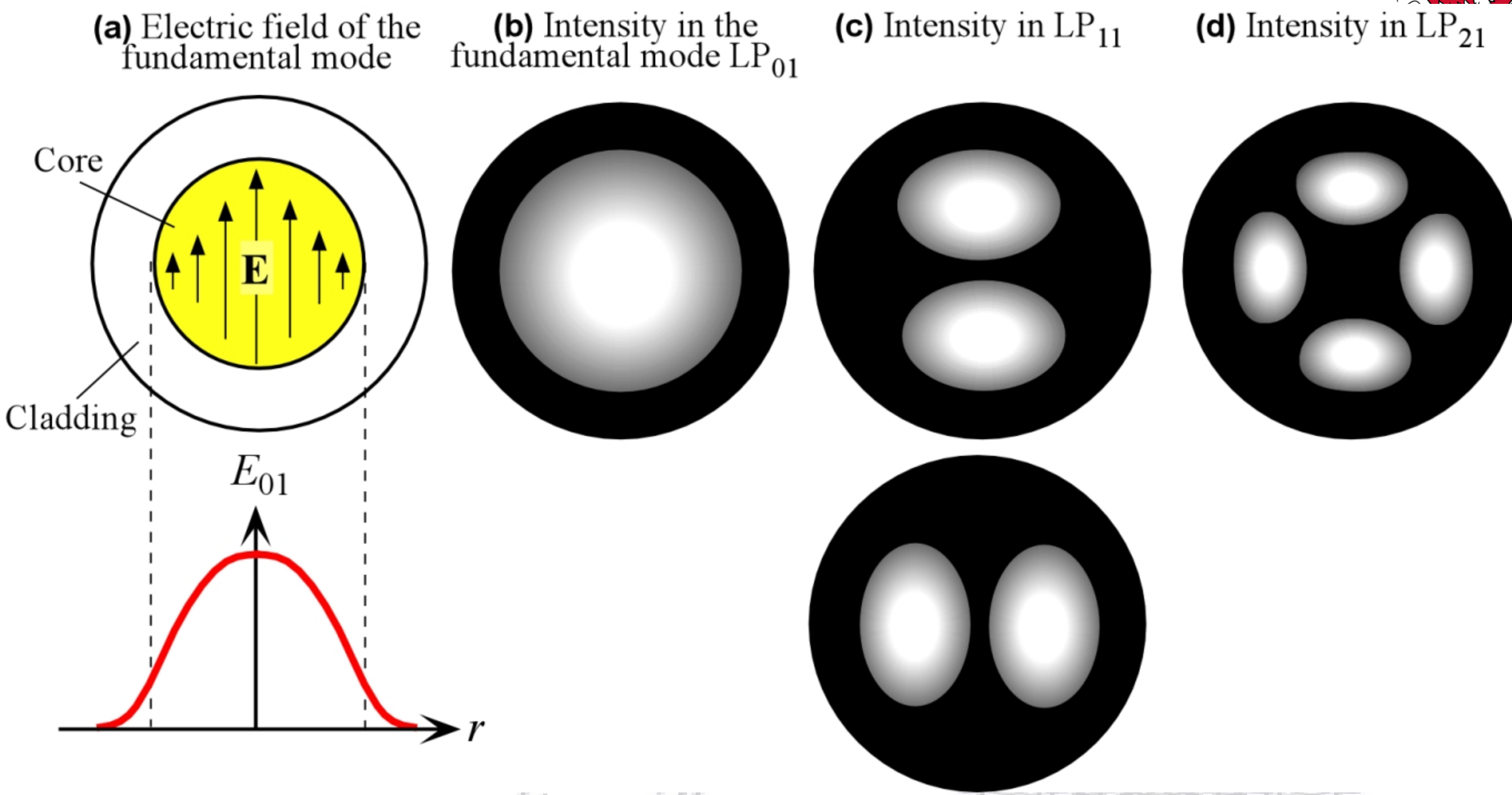
E and B are 90° to each other and z

Fundamental Mode is the LP_{01} mode: $l = 0$ and $m = 1$



The electric field distribution of the fundamental mode, LP_{01}, in the transverse plane to the fiber axis z. The light intensity is greatest at the center of the fiber

Very close to gaussian beam



The electric field distribution of the fundamental mode in the transverse plane to the fiber axis z. The light intensity is greatest at the center of the fiber. Intensity patterns in LP_{01}, LP_{11} and LP_{21} modes. (a) The field in the fundamental mode. (b)-(d) Indicative light intensity distributions in three modes, LP_{01}, LP_{11} and LP_{21}.

LP_{lm}

$$E_{\mathrm{LP}} = E_{lm}(r,\phi)\ \exp j(\omega t - \beta_{lm} z)$$

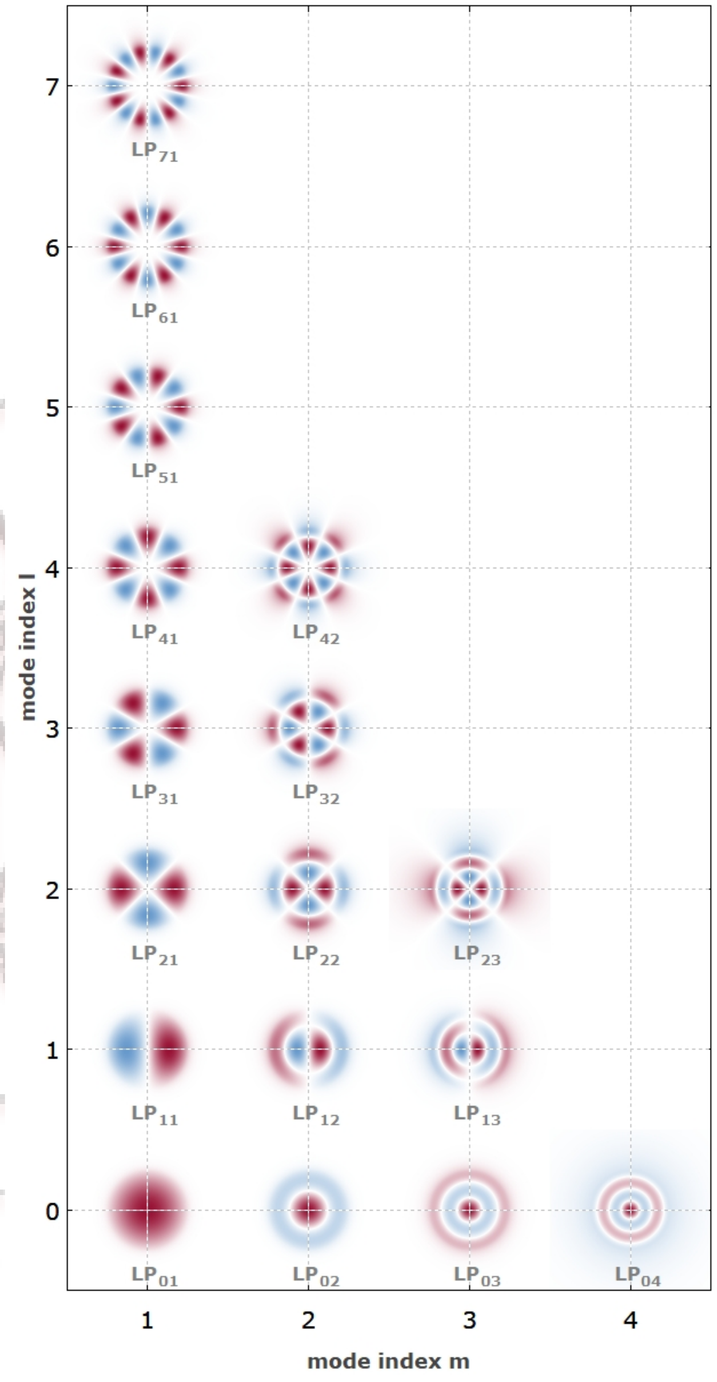
***m* = number of maxima along *r* starting from the core center. Determines the reflection angle θ**

2*l* = number of maxima around a circumference

***l* - radial mode number**
***l* - extent of helical propagation, i.e. the amount of skew ray contribution to the mode.**

LP modes

- Different modes are supported



Electric field amplitude profiles for all the guided modes of a step-index fiber. The two colors indicate different signs of the electric field values.

Optical Fiber Parameters

$n = (n_1 + n_2)/2$ = **average refractive index**

Δ = **normalized index difference**

$\Delta = (n_1 - n_2)/n_1 \approx (n_1^2 - n_2^2)/2n_1^2$

V-number $$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} \approx \frac{2\pi a n_1}{\lambda}(2\Delta)^{1/2}$$

$V < 2.405$ only 1 mode exists. Fundamental mode

$V < 2.405$ or $\lambda > \lambda_c$ Single mode fiber

$V > 2.405$ Multimode fiber

Number of modes $$M \approx \frac{V^2}{2}$$

Modes in an Optical Fiber

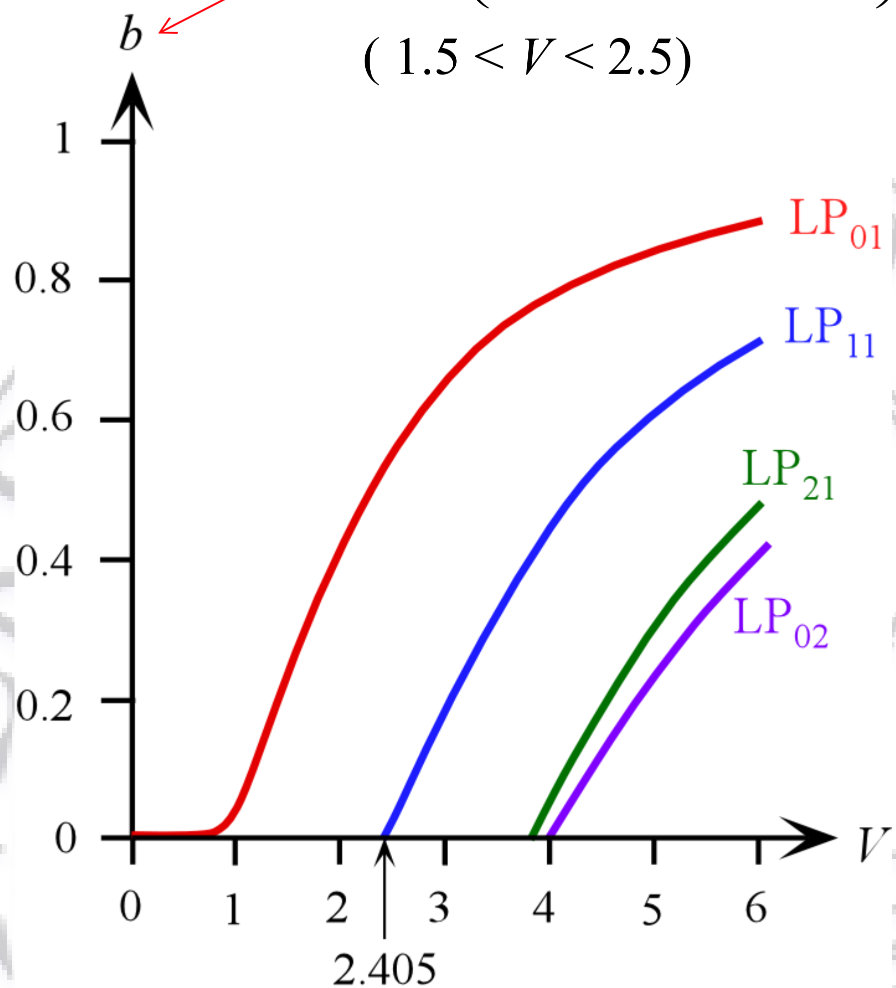
Normalized propagation constant

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2$$

$(1.5 < V < 2.5)$

$$k = 2\pi/\lambda$$



Normalized propagation constant b vs. V-number for a step-index fiber for various LP modes

Modes in an Optical Fiber

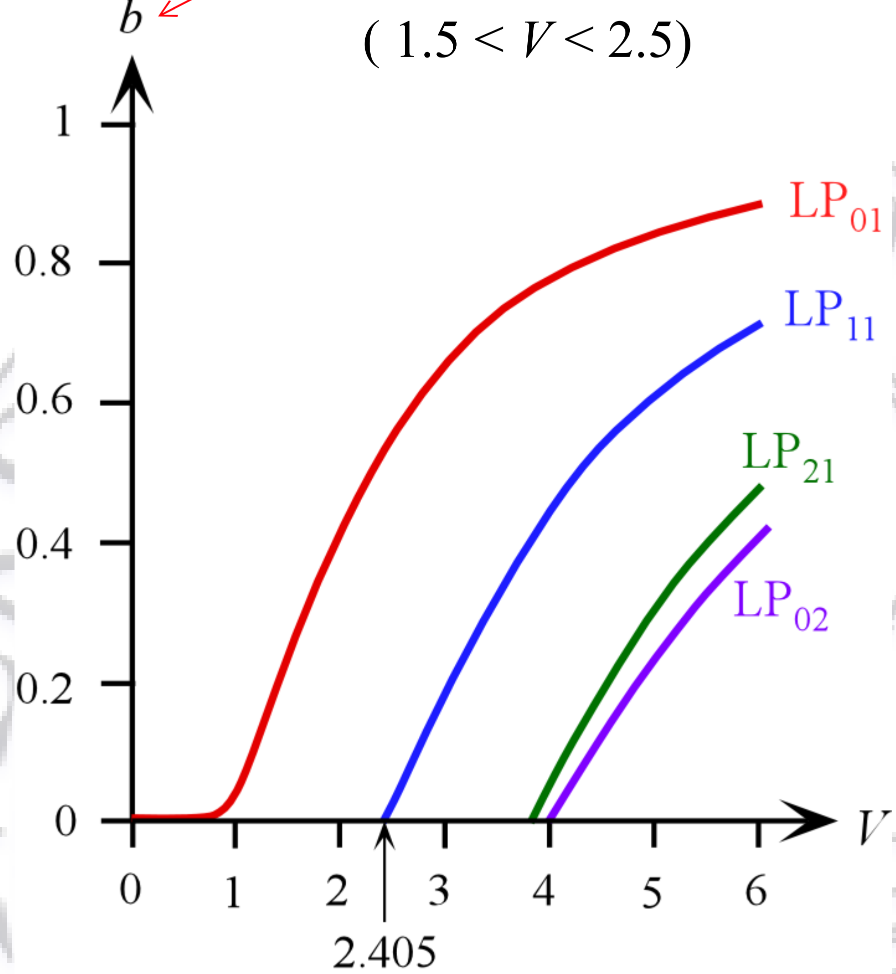
Normalized propagation constant can also be defined as

$$b = \frac{(\beta / k) - n_2}{n_1 - n_2}$$

$$k = 2\pi/\lambda$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2$$

$$(1.5 < V < 2.5)$$



Normalized propagation constant b vs. V-number for a step-index fiber for various LP modes

Group Velocity and Group Delay

Consider a single mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 μm, operating at 1.5 μm. *What are the group velocity and group delay at this wavelength*?

$$b = \frac{(\beta / k) - n_2}{n_1 - n_2} \Longrightarrow \beta = n_2 k[1 + b\Delta]$$

$k = 2\pi/\lambda = 4{,}188{,}790\ \mathrm{m}^{-1}$ and $\omega = 2\pi c/\lambda = 1.255757\times10^{15}\ \mathrm{rad\ s}^{-1}$

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = 1.910088$

$1.5 < V < 2.5$ $$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2$$

$b = 0.3860859$, and $\beta = 6.044796\times10^6\ \mathrm{m}^{-1}$.

Increase wavelength by 0.1% and recalculate. Values in the table

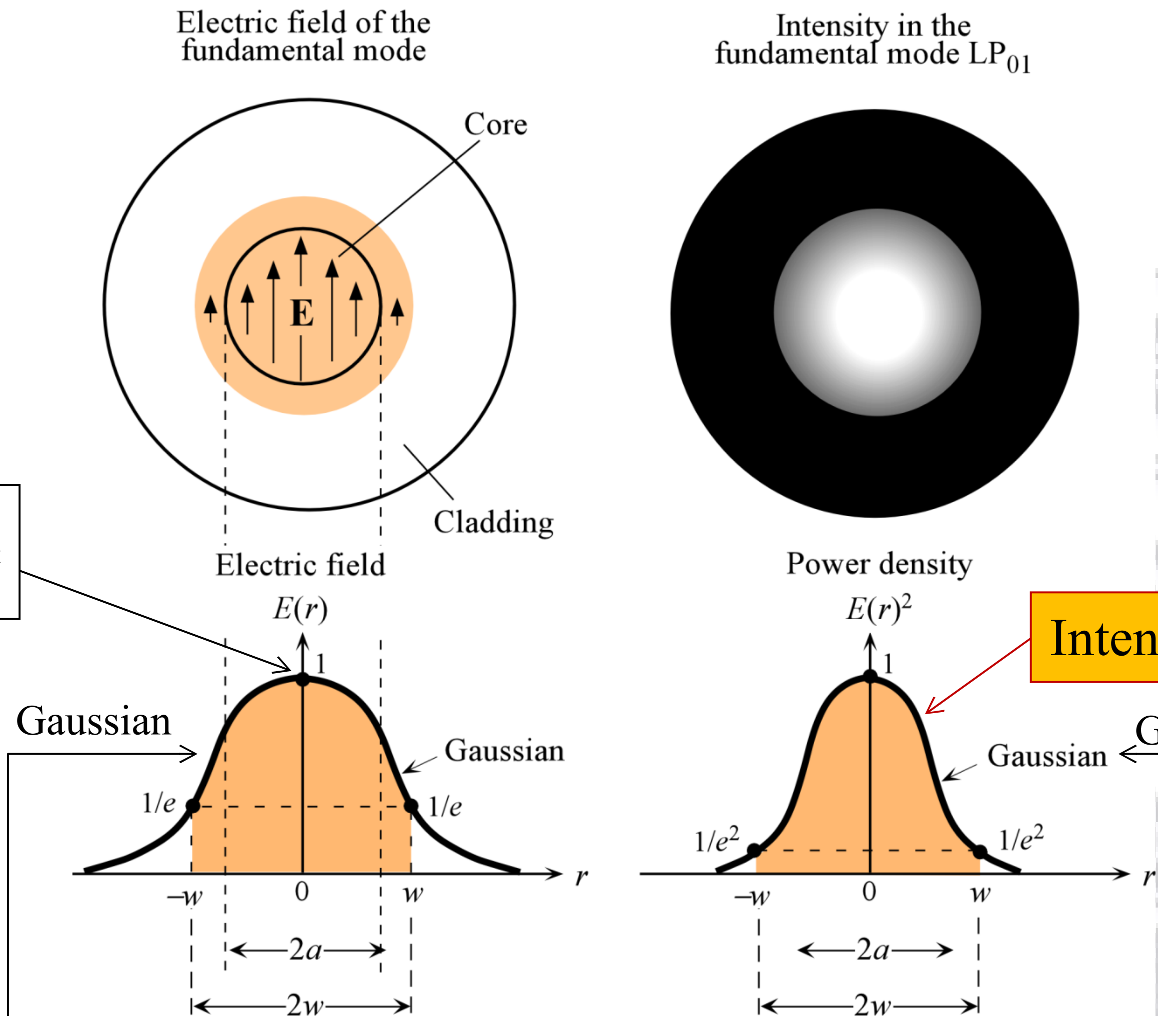
Group Velocity and Group Delay

Calculation →	***V***	***k* (m^{-1})**	**ω (rad s^{-1})**	***b***	**β (m^{-1})**
λ = 1.500000 μm	1.910088	4188790	1.255757×10^{15}	0.3860859	6.044818×10^{6}
λ' = 1.50150 μm	1.908180	4184606	1.254503×10^{15}	0.3854382	6.038757×10^{6}

$$v_g = \frac{d\omega}{d\beta} = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.254503 - 1.255757)\times10^{15}}{(6.038757 - 6.044818)\times10^{6}} \approx 2.07\times10^{8}\ \mathrm{m\,s^{-1}}$$

The group delay τ_g over 1 km is 4.83 μs

Mode Field Diameter (2w)



Note: Maximum set arbitrarily to 1

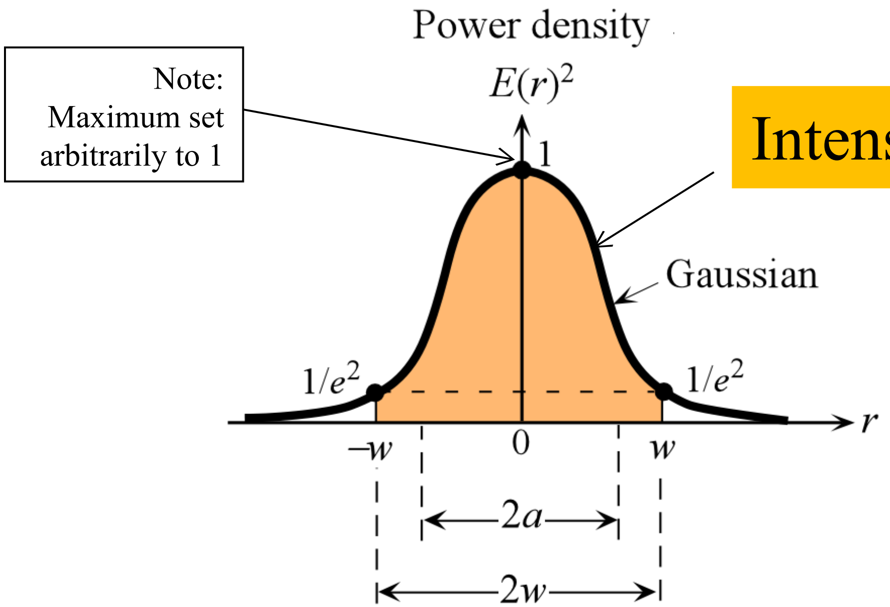
Intensity $\propto v_g \times E(r)^2$

$$E(r) = E(0)\exp[-(r/w)^2]$$

$$E(r)^2 = E(0)^2\exp[-2(r/w)^2]$$

Mode Field Diameter

$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$



Intensity $\propto v_g \times E(r)^2$

$$2w = \text{Mode Field Diameter (MFD)}$$

Marcuse MFD Equation

$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})$$

$0.8 < V < 2.5$

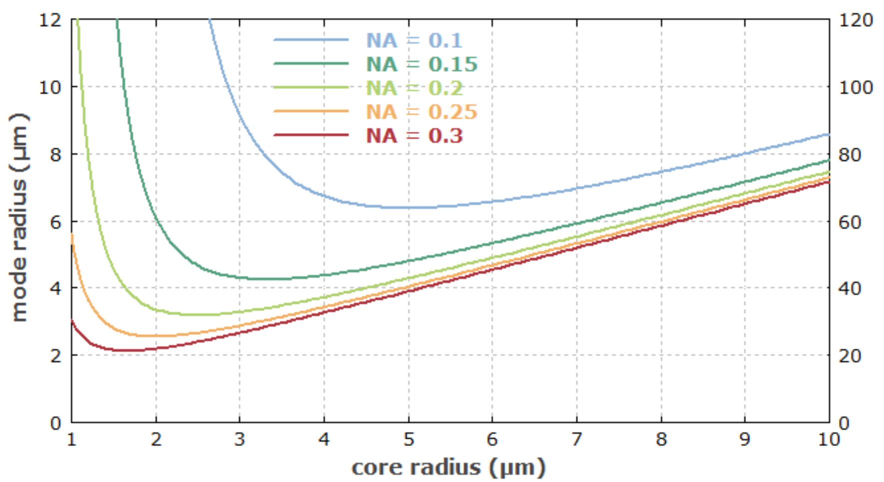
$$2w \approx (2a)(2.6/V)$$

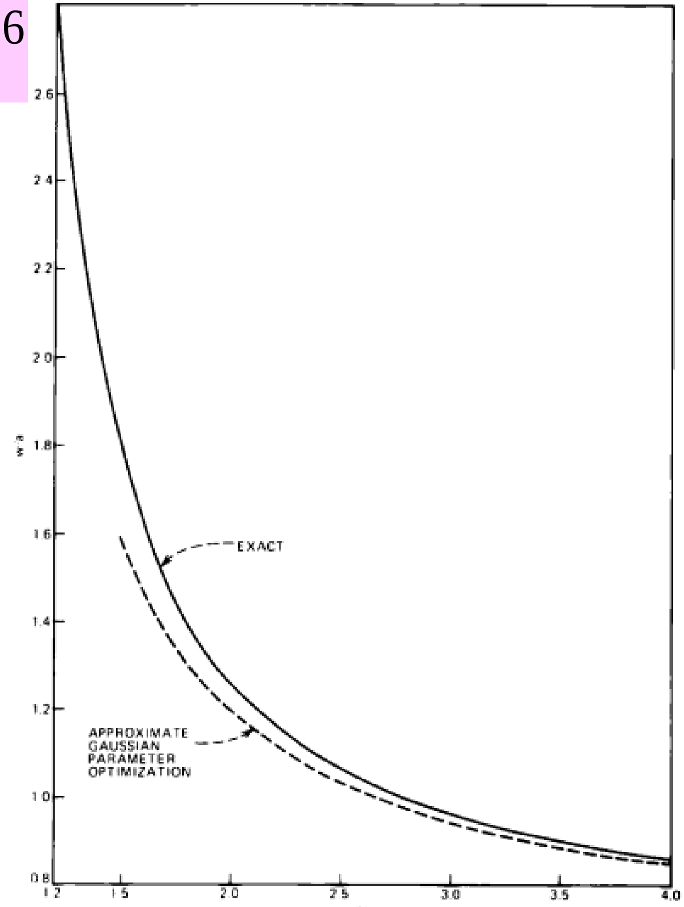
$1.6 < V < 2.4$

Marcuse equation

$$w/a = 0.65 + 1.619\,V^{-3/2} + 2.879\,V^{-6}$$

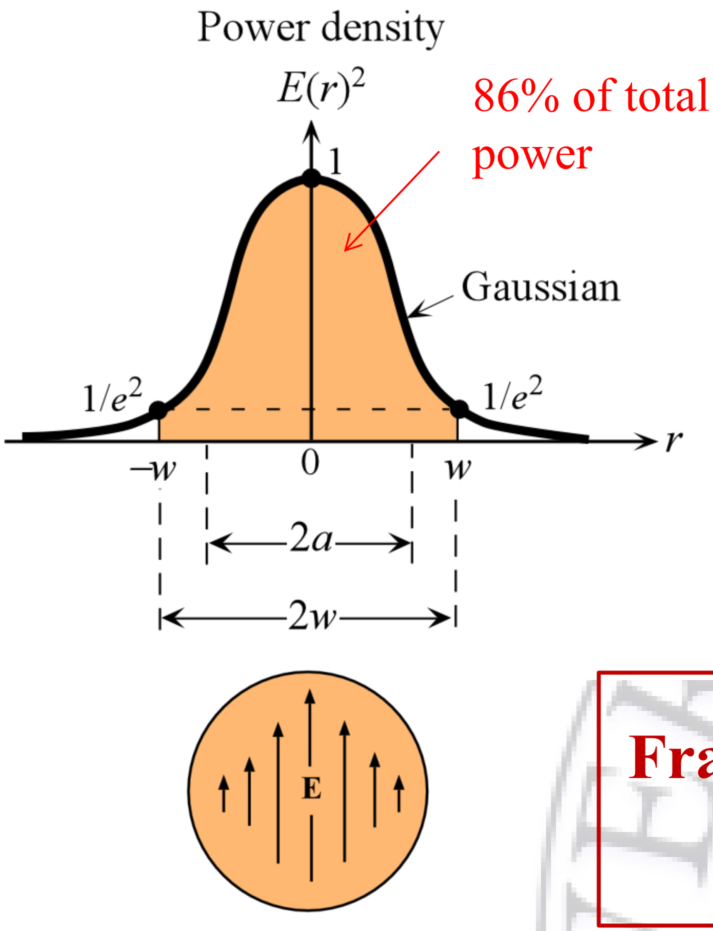
This is accurate within 1% for V between 1.5, and 2.5, the range of highest practical interest.



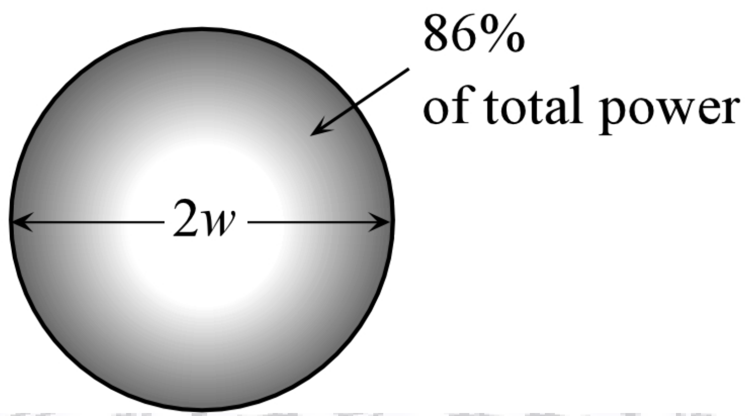


LP_{01} mode radius (D4σ value) of step-index fibers with different numerical apertures as a function of the core

$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$



Mode Field Diameter (2*w*)



Fraction of optical power within MFD = 86 %

This is the same as the fraction of optical power within a radius *w* from the axis of a Gaussian beam (See Chapter 1)

Example: A multimode fiber

Calculate the number of allowed modes in a multimode step index fiber which has a core of refractive index of 1.468 and diameter 100 μm, and a cladding of refractive index of 1.447 if the source wavelength is 850 nm.

Solution

Substitute, $a = 50$ μm, $\lambda = 0.850$ μm, $n_1 = 1.468$, $n_2 = 1.447$ into the expression for the V-number,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = (2\pi 50/0.850)(1.468^2 - 1.447^2)^{1/2}$$

$$= 91.44.$$

Since $V >> 2.405$, the number of modes is

$$M \approx V^2/2 = (91.44)^2/2 = 4181$$

which is large.

Example: A single mode fiber

What should be the core radius of a single mode fiber which has a core of $n_1 = 1.4680$, cladding of $n_2 = 1.447$ and it is to be used with a source wavelength of 1.3 μm?

Solution

For single mode propagation, $V \leq 2.405$. We need,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

or

$$[2\pi a/(1.3\ \mu\text{m})](1.468^2 - 1.447^2)^{1/2} \leq 2.405$$

which gives $\boldsymbol{a \leq 2.01\ \mu\text{m}}$.

Rather thin for easy coupling of the fiber to a light source or to another fiber; a is comparable to λ which means that the geometric ray picture, strictly, cannot be used to describe light propagation.

Example: Single mode cut-off wavelength

Calculate the cut-off wavelength for single mode operation for a fiber that has a core with diameter of 8.2 μm, a refractive index of 1.4532, and a cladding of refractive index of 1.4485. What is the V-number and the mode field diameter (MFD) for operation at $\lambda = 1.31$ μm?

Solution

For single mode operation,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

Substituting for a, n_1 and n_2 and rearranging we get,

$$\lambda > [2\pi(4.1\ \mu\text{m})(1.4532^2 - 1.4485^2)^{1/2}]/2.405 = 1.251\ \mu\text{m}$$

Wavelengths shorter than 1.251 μm give multimode propagation.

At $\lambda = 1.31$ μm,

$$V = 2\pi[(4.1\ \mu\text{m})/(1.31\ \mu\text{m})](1.4532^2 - 1.4485^2)^{1/2} = 2.30$$

Mode field diameter MFD

Solution (continued)

Mode field diameter MFD from the Marcuse Equation is

$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})$$
$$= 2(4.1)[0.65 + 1.62(2.30)^{-3/2} + 2.88(2.30)^{-6}]$$

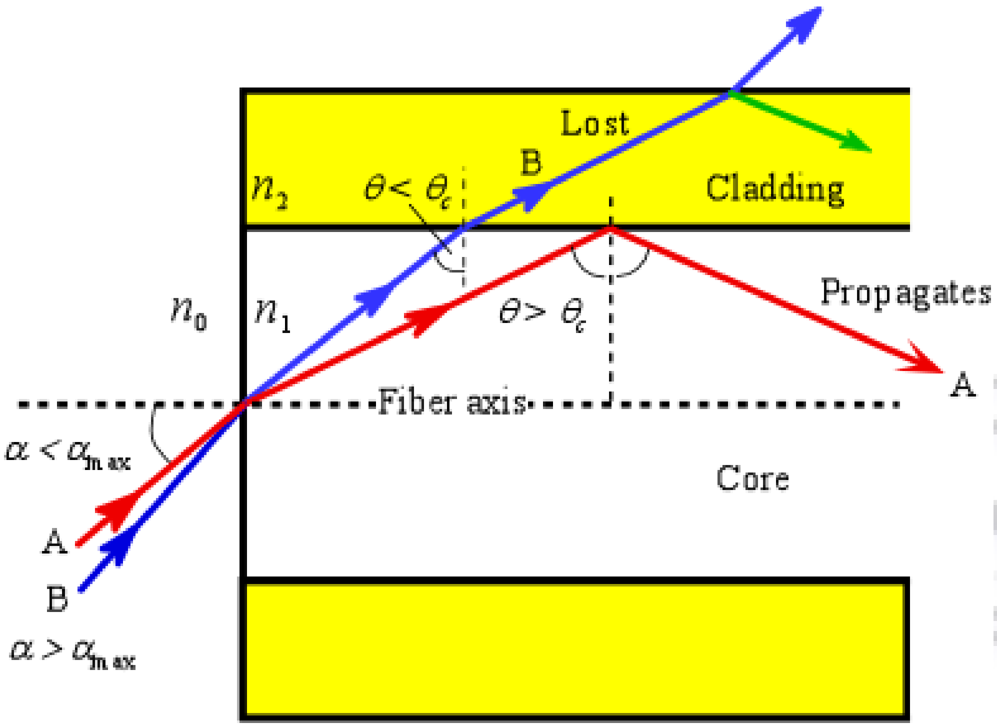
$2w = 9.30$ μm 86% of total power is within this diameter

$2w = (2a)(2.6/V) = 2(4.1)(2.6/2.30) = 9.28$ μm

$2w = 2a[(V+1)/V] = 11.8$ μm

This is for a planar waveguide, and the definition is different than that for an optical fiber

Numerical Aperture NA



Maximum acceptance angle α_{max} is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha = \alpha_{max}$ then $\theta = \theta_c$. Rays with $\alpha > \alpha_{max}$ (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

$$\mathrm{NA} = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\sin\alpha_{max} = \frac{\left(n_1^2 - n_2^2\right)^{1/2}}{n_o} = \frac{\mathrm{NA}}{n_o}$$

$$V = \frac{2\pi a}{\lambda}\mathrm{NA}$$

$2\alpha_{max} =$ **total acceptance angle**

NA is an important factor in light launching designs into the optical fiber.

Example: A multimode fiber and total acceptance angle

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

Normalized refractive index
$\Delta = (n_1 - n_2) / n_1 = \mathbf{0.0135 \text{ or } 1.35\%}$

Solution

The numerical aperture is

$$NA = (n_1^2 - n_2^2)^{1/2} = (1.480^2 - 1.460^2)^{1/2} = \mathbf{0.2425 \text{ or } 24.3\%}$$

From, $\sin\alpha_{max} = NA/n_o = 0.2425/1$

Acceptance angle $\alpha_{max} = \mathbf{14°}$

Total acceptance angle $2\alpha_{max} = \mathbf{28°}$

V-number in terms of the numerical aperture can be written as,

$$V = (2\pi a/\lambda)NA = [(2\pi 50\ \mu m)/(0.85\ \mu m)](0.2425) = 89.62$$

The number of modes, $M \approx V^2/2 = \mathbf{4016}$

Example: A single mode fiber and cut-off wavelength

A typical single mode optical fiber has a core of diameter 8 μm and a refractive index of 1.460. The normalized index difference is 0.3%. The cladding diameter is 125 μm. Calculate the numerical aperture and the total acceptance angle of the fiber. What is the single mode cut-off frequency λ_c of the fiber?

Solution

The numerical aperture

$$NA = (n_1^2 - n_2^2)^{1/2} = [(n_1 + n_2)(n_1 - n_2)]^{1/2}$$

Substituting $(n_1 - n_2) = n_1\Delta$ and $(n_1 + n_2) \approx 2n_1$, we get

$$NA \approx [(2n_1)(n_1\Delta)]^{1/2} = n_1(2\Delta)^{1/2} = 1.46(2\times0.003)^{1/2} = \mathbf{0.113 \text{ or } 11.3\ \%}$$

The acceptance angle is given by

$$\sin\alpha_{max} = NA/n_o = 0.113/1 \text{ or } \alpha_{max} = \mathbf{6.5°}, \text{ and } 2\alpha_{max} = \mathbf{13°}$$

The condition for single mode propagation is $V \leq 2.405$. At cut-off,

$$V = (2\pi a/\lambda_c)(n_1^2 - n_2^2)^{1/2} = 2.405$$

$$\therefore\ \lambda_c = [2\pi a NA]/2.405 = [(2\pi)(4\ \mu m)(0.113)]/2.405 = \mathbf{1.18\ \mu m}$$

Wavelengths shorter than 1.18 μm will result in multimode operation.

Dispersion = Spread of Information

- Intermode (Intermodal) Dispersion: Multimode fibers only
- Material Dispersion

  Group velocity depends on N_g and hence on λ
- Waveguide Dispersion

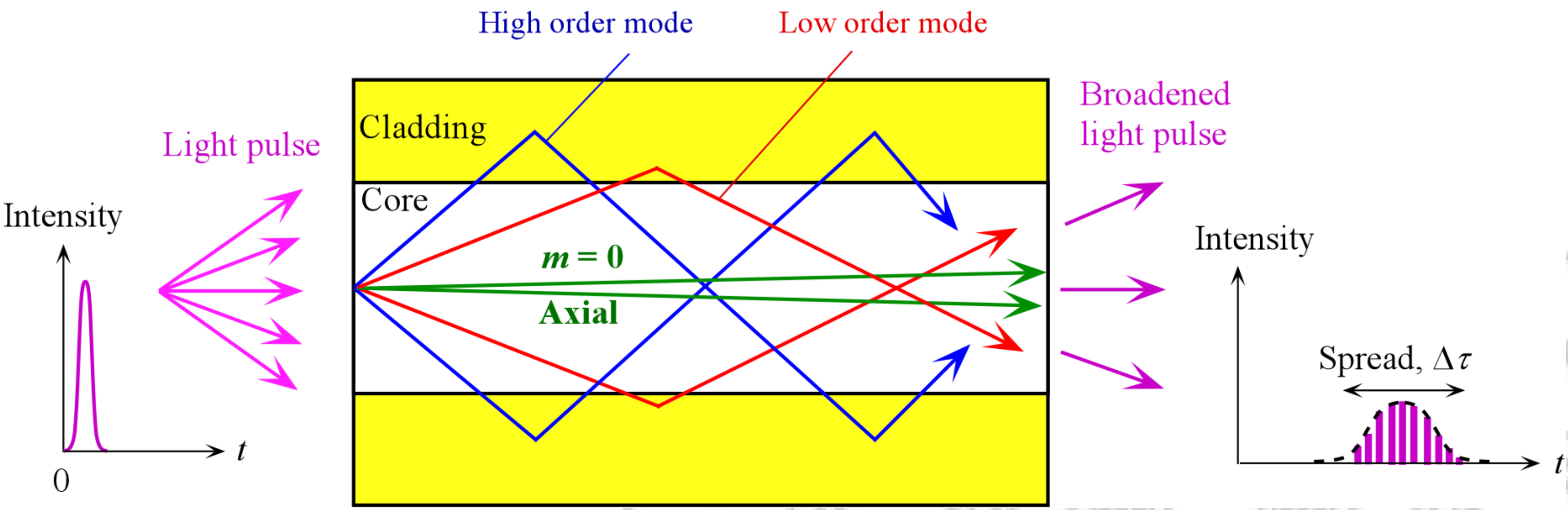
  Group velocity depends on waveguide structure
- Chromatic Dispersion

  Material dispersion + Waveguide Dispersion
- Polarization Dispersion
- Profile Dispersion

  Like material and waveguide dispersion. Add all 3

  Material + Waveguide + Profile
- Self phase modulation dispersion

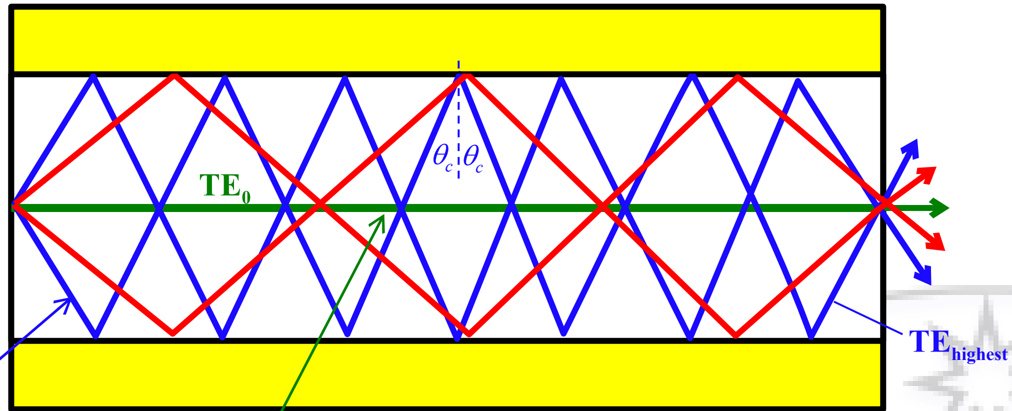
Intermode Dispersion (MMF)



$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

(Since n_1 and n_2 are only slightly different)

Intermode Dispersion (MMF)



$$v_{g\min} \approx \frac{c}{n_1}\sin\theta_c = \frac{c}{n_1}\left(\frac{n_2}{n_1}\right)$$

$$v_{g\max} \approx \frac{c}{n_1}$$

$$\Delta\tau = \frac{L}{v_{g\min}} - \frac{L}{v_{g\max}}$$

$$\frac{\Delta\tau}{L} = \frac{(n_1 - n_2)}{c}\left(\frac{n_1}{n_2}\right)$$

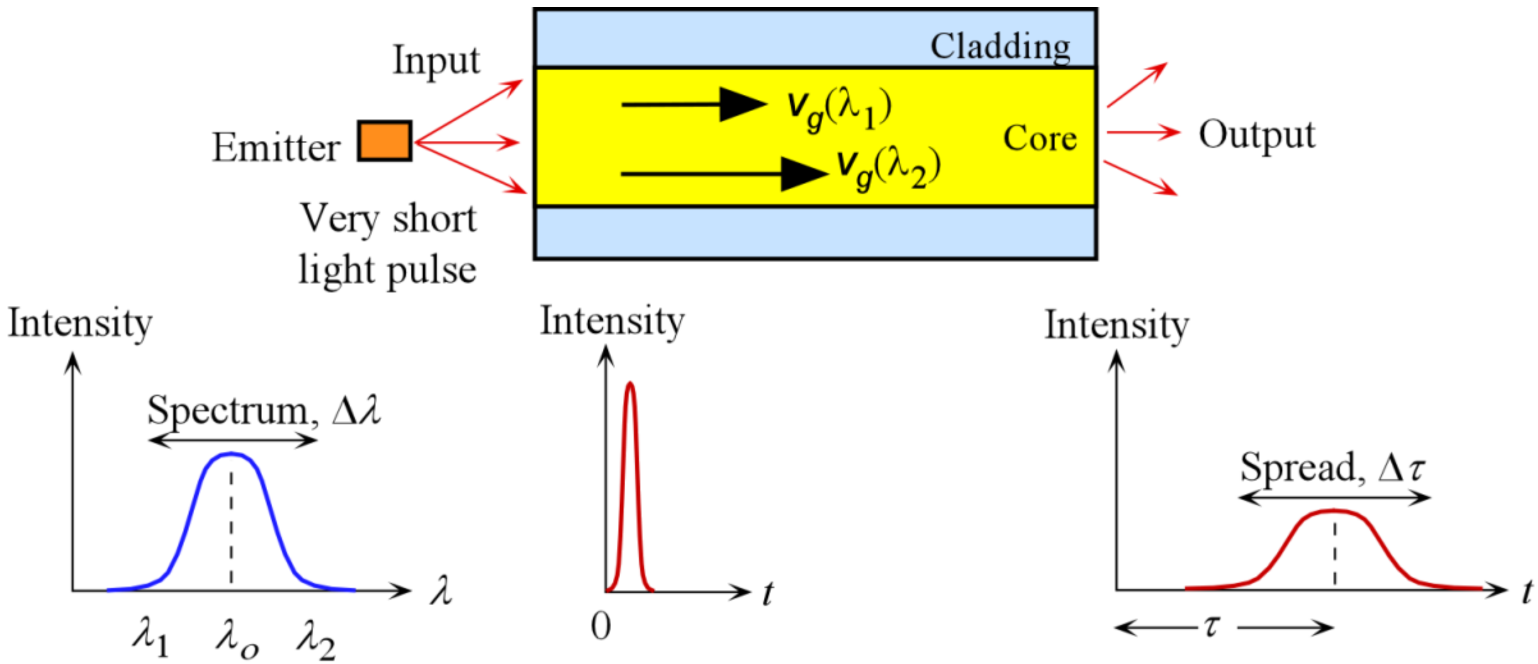
$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

$\Delta\tau/L \approx 10 - 50$ ns / km

Depends on length!

Intramode Dispersion (SMF)

Dispersion in the fundamental mode



Group Delay $\tau = L / v_g$

Group velocity v_g depends on

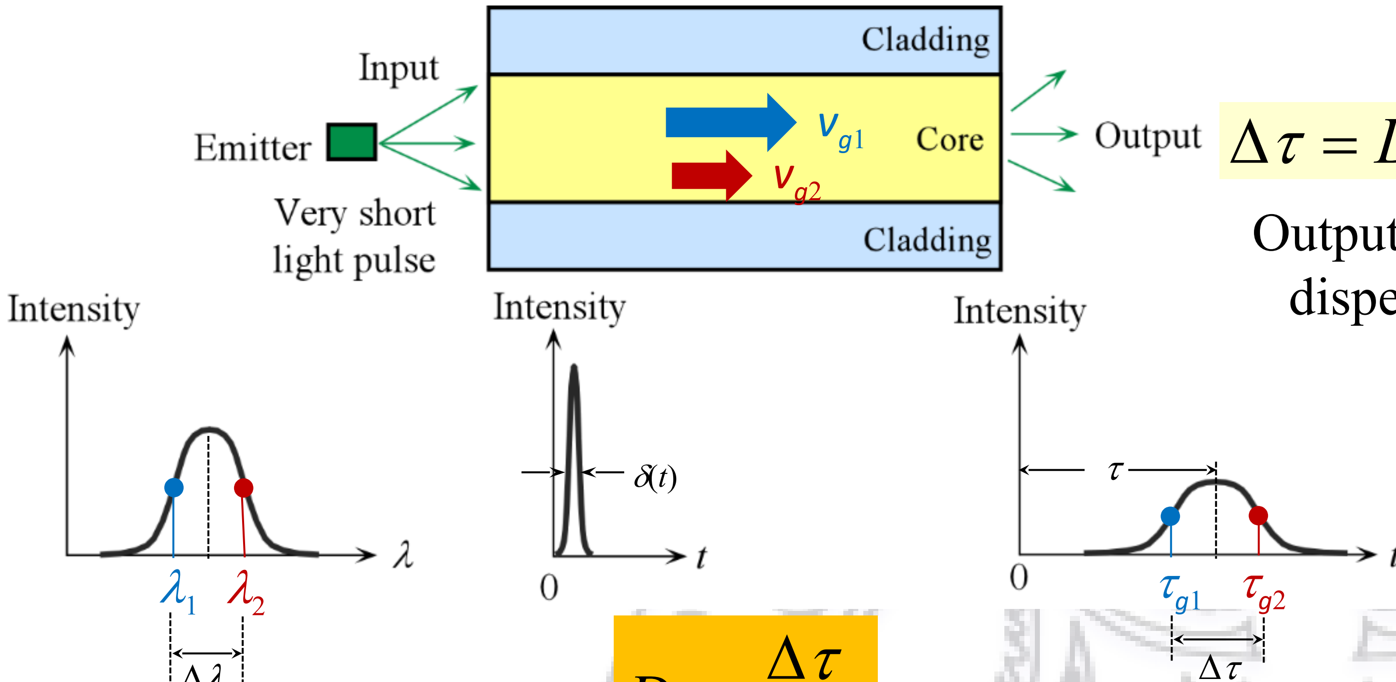
Refractive index = $n(\lambda)$ **Material Dispersion**

***V*-number = $V(\lambda)$** **Waveguide Dispersion**

$\Delta = (n_1 - n_2)/n_1 = \Delta(\lambda)$ **Profile Dispersion**

Intramode Dispersion (SMF)

Chromatic dispersion in the fundamental mode



$$\Delta\tau = DL\Delta\lambda$$

Output pulse dispersed

$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1}$$

Chromatic spread

$$D = \frac{\Delta\tau}{L\Delta\lambda}$$

OR

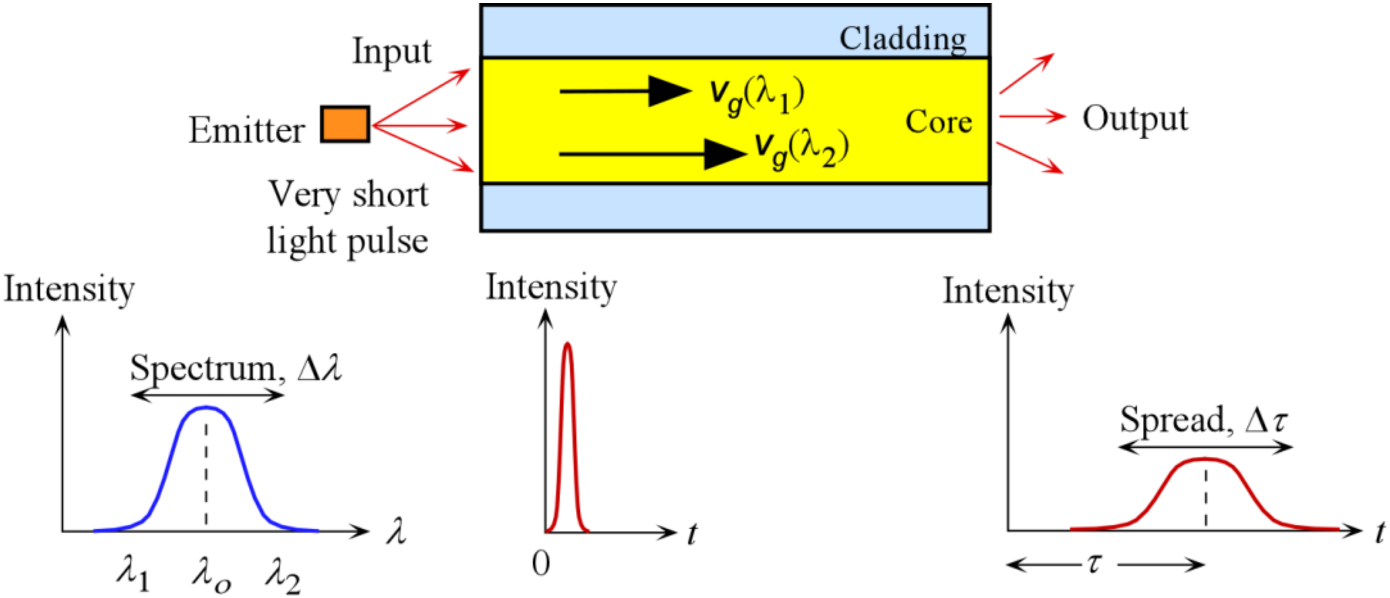
$$D = \frac{d\tau}{Ld\lambda}$$

$$\boxed{\Delta\tau = \tau_{g2} - \tau_{g1}}$$

Dispersion

Definition of Dispersion Coefficient

Material Dispersion



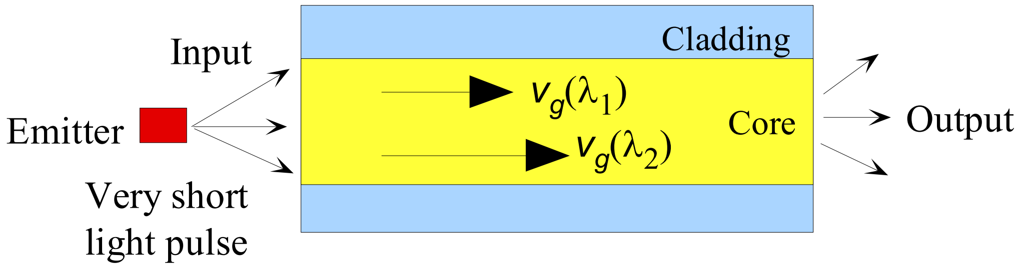
Emitter emits a spectrum $\Delta\lambda$ of wavelengths.

Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

$$\boxed{\frac{\Delta\tau}{L} = D_m \Delta\lambda}$$

D_m = Material dispersion coefficient, ps nm^{-1} km^{-1}

Material Dispersion



$$v_g = c / N_g$$

Group velocity

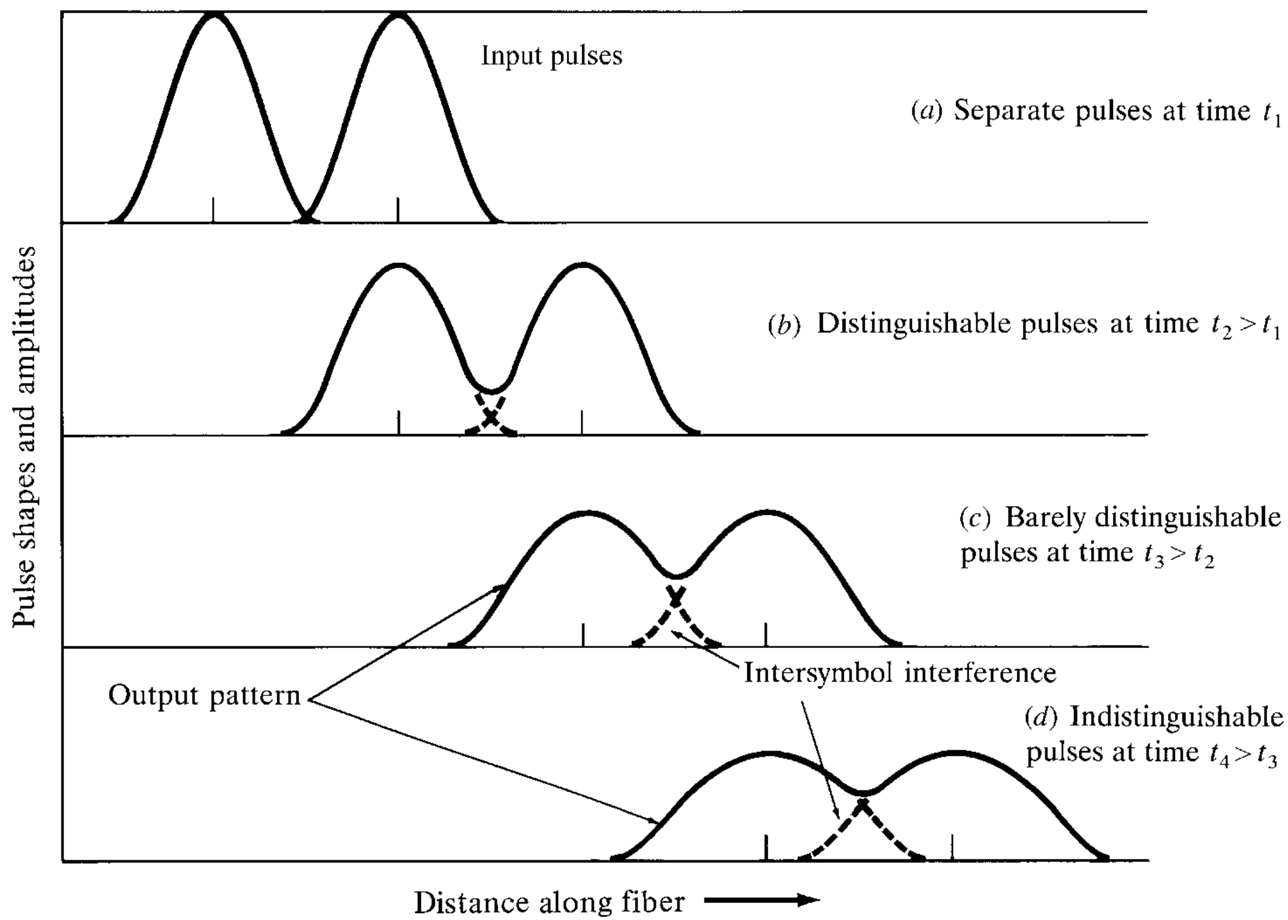
Depends on the wavelength

$$\frac{\Delta\tau}{L} = D_m \Delta\lambda$$

D_m = Material dispersion coefficient, ps nm^{-1} km^{-1}

$$D_m \approx -\frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\right)$$

Pulse broadening for communications



Wave guide dispersion

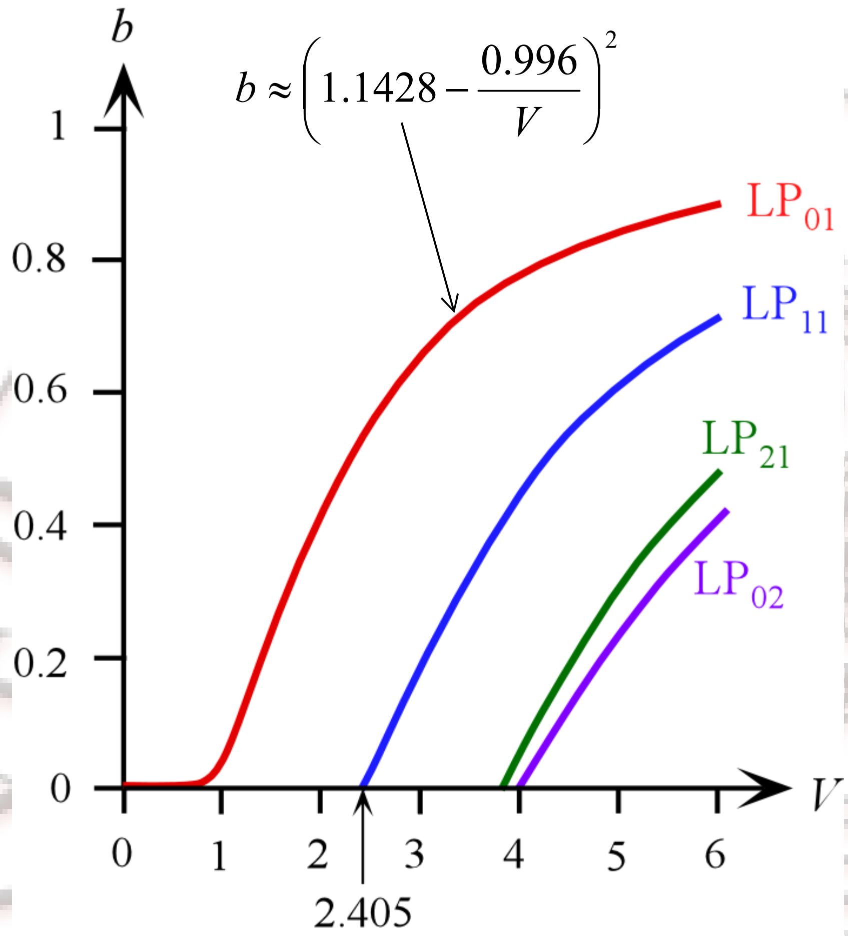
b hence β depend on *V* and hence on λ

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

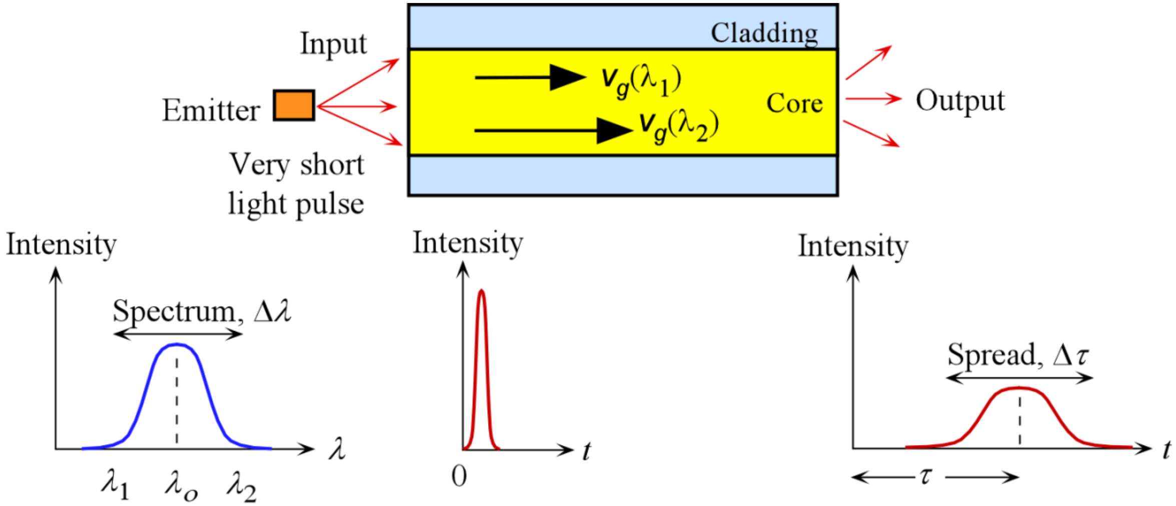
Normalized propagation constant

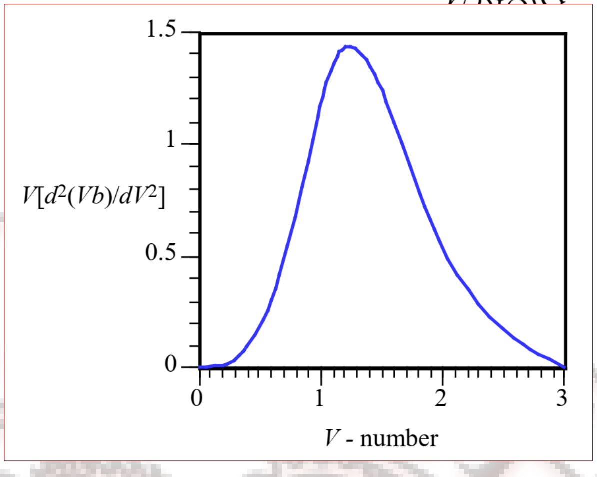
$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$$

$$k = 2\pi/\lambda$$



Waveguide Dispersion





Waveguide dispersion The group velocity $v_g(01)$ of the fundamental mode depends on the **V-number**, which itself depends on the source wavelength λ, even if n_1 and n_2 were constant. Even if n_1 and n_2 were wavelength independent (no material dispersion), we will still have waveguide dispersion by virtue of $v_g(01)$ depending on V and V depending inversely on λ. Waveguide dispersion arises as a result of the guiding properties of the waveguide which imposes a nonlinear ω vs. β_{lm} relationship.

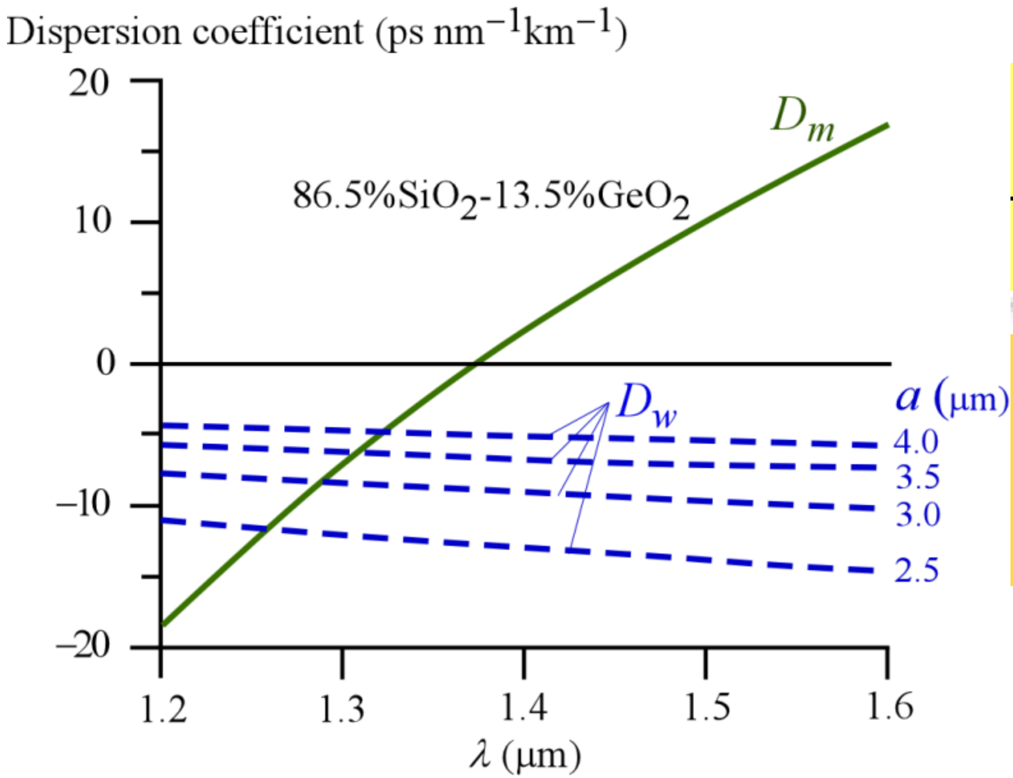
$$\frac{\Delta\tau}{L} = D_w \Delta\lambda$$

D_w = waveguide dispersion coefficient

D_w depends on the waveguide structure, ps nm^{-1} km^{-1}

$$Dw = -\frac{\Delta . n2}{c\lambda} V \frac{d2(Vb)}{dV2}$$

Waveguide Dimension and Chromatic Dispersion



$$D_w = -\frac{n_2 \Delta}{c\lambda}\left[V\frac{d^2(bV)}{dV^2}\right]$$

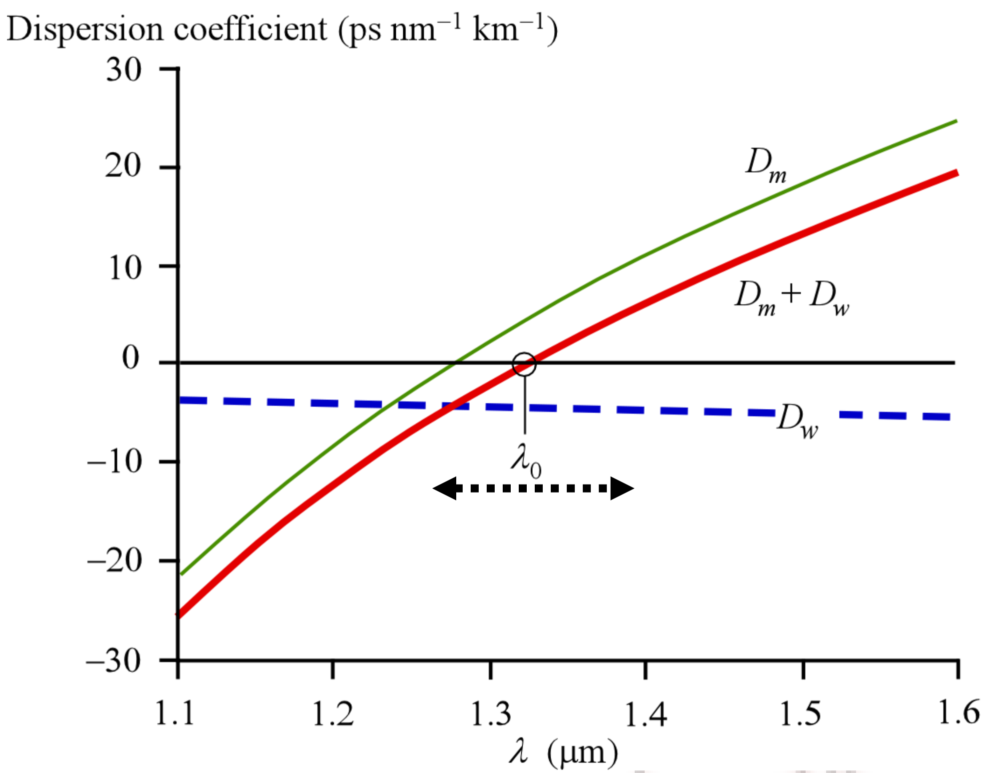
$$D_w \approx -\frac{0.025\,\lambda}{a^2 c\, n_2}$$

Can be tuned

$$D_w(\mathrm{ps\,nm^{-1}\,km^{-1}}) \approx -\frac{83.76\,\lambda(\mu\mathrm{m})}{[a(\mu\mathrm{m})]^2 n_2}$$

Waveguide dispersion depends on the guide properties

Chromatic Dispersion



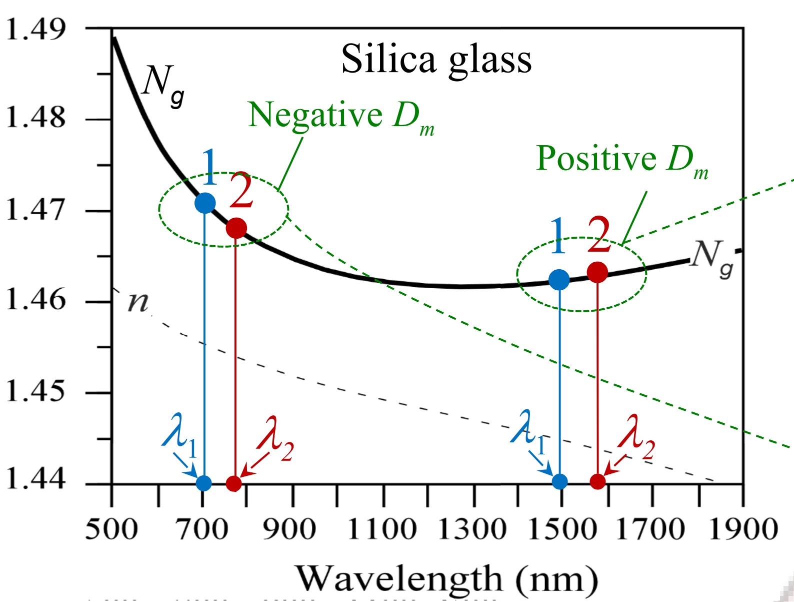
Material dispersion coefficient (D_m) for the core material (taken as SiO_2), waveguide dispersion coefficient (D_w) (a = 4.2 mm) and the total or chromatic dispersion coefficient D_{ch} (= $D_m + D_w$) as a function of free space wavelength, l

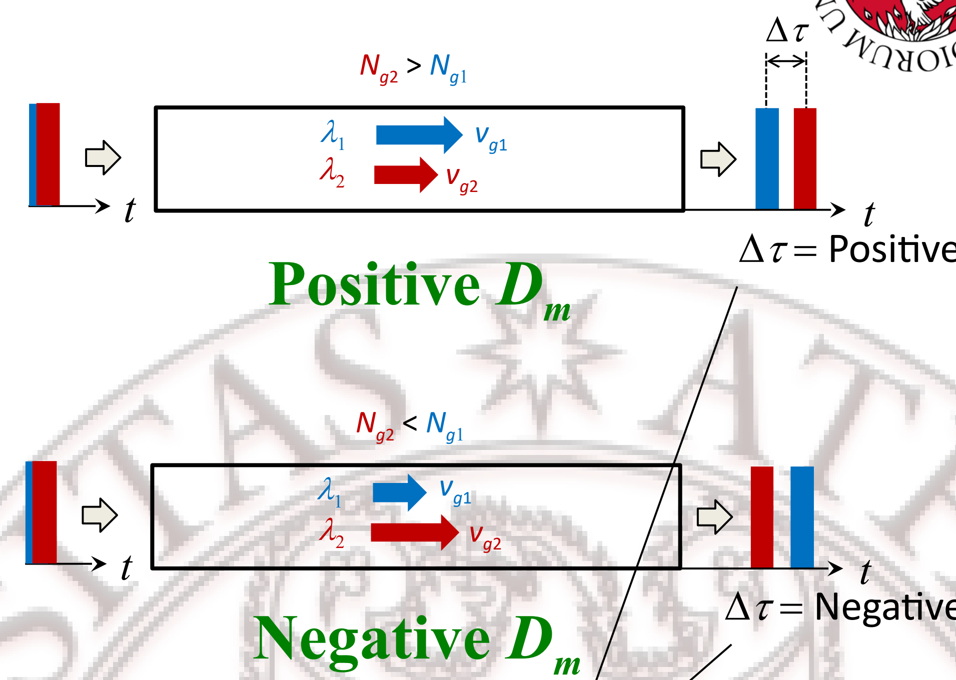
The zero crossing can be engineering playing with D_w

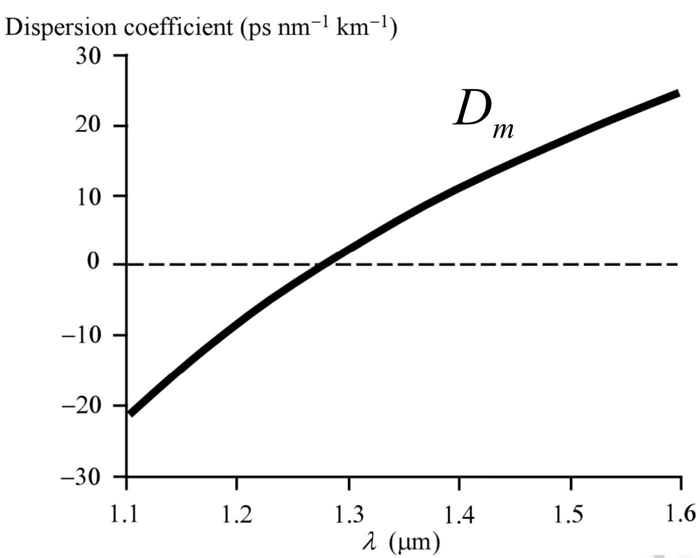
Chromatic = Material + Waveguide

$$\frac{\Delta\tau}{L} = (D_m + D_w)\Delta\lambda$$

What do Negative and Positive D_m mean?



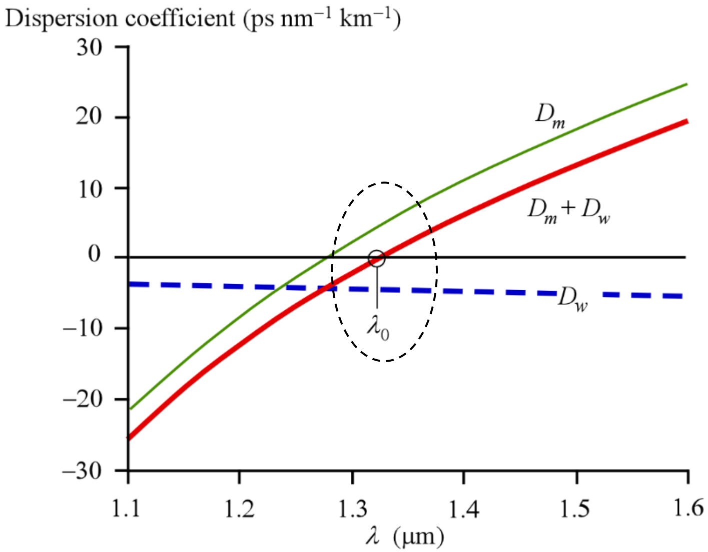




$$D_m = \frac{\Delta\tau}{L\Delta\lambda}$$

$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1}$$

Is dispersion really zero at λ_0?



The cause of $\Delta\tau$ is the wavelength spread $\Delta\lambda$ at the input

$$\Delta\tau = f(\Delta\lambda)$$

$$\Delta\tau = \tau(\lambda) - \tau(\lambda_0) = \boxed{\left[\frac{d\tau}{d\lambda}\right]_{\lambda_0}}(\Delta\lambda) + \frac{1}{2!}\left[\frac{d^2\tau}{d\lambda^2}\right]_{\lambda_0}(\Delta\lambda)^2 + \cdots$$

$$\frac{d\Delta\tau}{L d\lambda} = D_{ch}$$

$$\frac{dD_{ch}}{d\lambda} = S_0$$

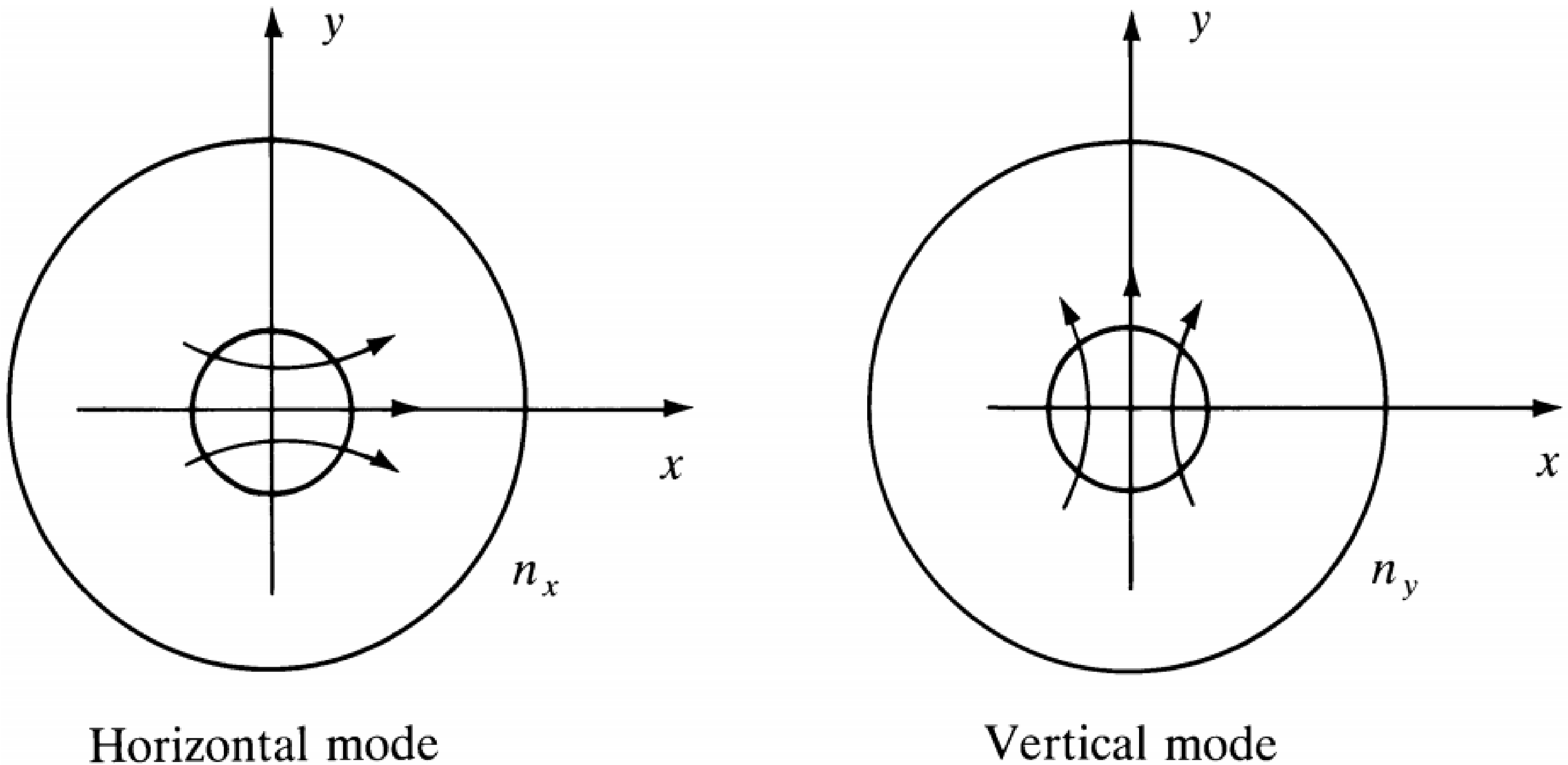
$$= 0$$

$$\Delta\tau = [L\boxed{D_{ch}(\lambda_0)}]\Delta\lambda + \frac{1}{2}\left[\frac{d}{d\lambda}\boxed{\left(\frac{d\tau}{d\lambda}\right)}\right]_{\lambda_0}(\Delta\lambda)^2$$

$$\Delta\tau = \frac{L}{2} S_0$$ ¿

Δλ for multimode laser diode

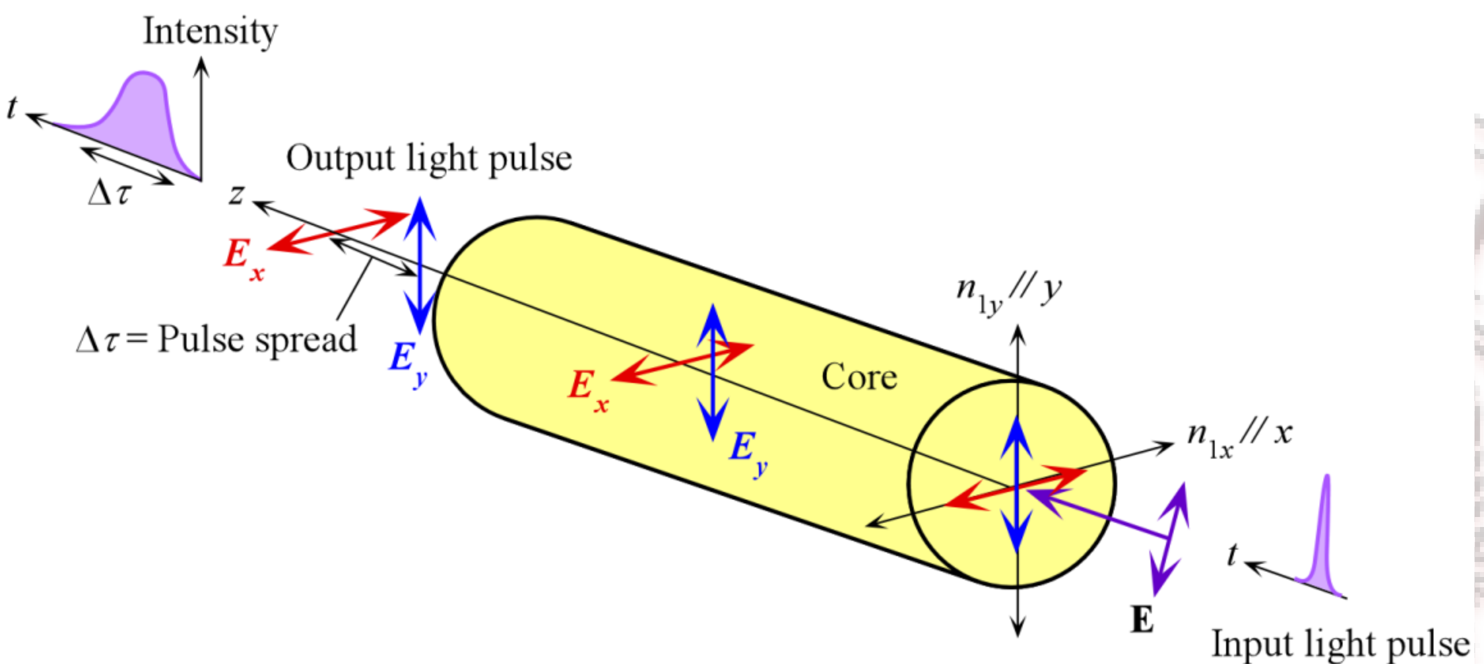
Fundamental mode polarization



Installation of optical fiber

Polarization Dispersion

n different in different directions due to induced **strains** in fiber in manufacturing, handling and cabling ➔ Birefrengence = $\delta n/n < 10^{-6}$



$$\Delta\tau = D_{\text{PMD}}\sqrt{L}$$

Polarization Mode dispersion

- The effects of fiber-birefringence on the polarization states of an optical are another source of pulse broadening. **Polarization mode dispersion** (PMD) is due to slightly different velocity for each polarization mode because of the lack of perfectly symmetric & anisotropicity of the fiber. If the group velocities of two orthogonal polarization modes are v_{gx} and v_{gy} then the differential time delay $\Delta\tau_{pol}$ between these two polarization over a distance L is

$$\Delta\tau_{pol} = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| \qquad [3\text{-}26]$$

- The rms value of the differential group delay can be approximated as:

$$\langle \Delta\tau_{pol} \rangle \approx D_{PMD} \sqrt{L}$$

D_{PMD} = Polarization dispersion coefficient

Typically D_{PMD} = 0.1 – 0.5 ps km$^{-1/2}$

Profile Dispersion

Group velocity $v_g(01)$ of the fundamental mode depends on Δ, refractive index difference.

Δ may not be constant over a range of wavelengths: $\Delta = \Delta(\lambda)$

$$\frac{\Delta\tau}{L} = D_p \Delta\lambda$$

D_p = *Profile dispersion coefficient*

$D_p < 0.1$ ps nm^{-1} km^{-1}

Can generally be ignored

NOTE

Total intramode (chromatic) dispersion coefficient D_{ch}

$$D_{ch} = D_m + D_w + D_p$$

where D_m, D_w, D_p are material, waveguide and profile dispersion coefficients respectively

Chromatic Dispersion recap

$$D_{ch} = D_m + D_w + D_p$$

$$\frac{\Delta\tau}{L} = D_{ch}\Delta\lambda$$

$S_0 =$ Chromatic dispersion slope at λ_0

Chromatic dispersion is zero at $\lambda = \lambda_0$

$$\boxed{D_{ch} = \frac{S_0\lambda}{4}\left[1 - \left(\frac{\lambda_0}{\lambda}\right)^4\right]}$$

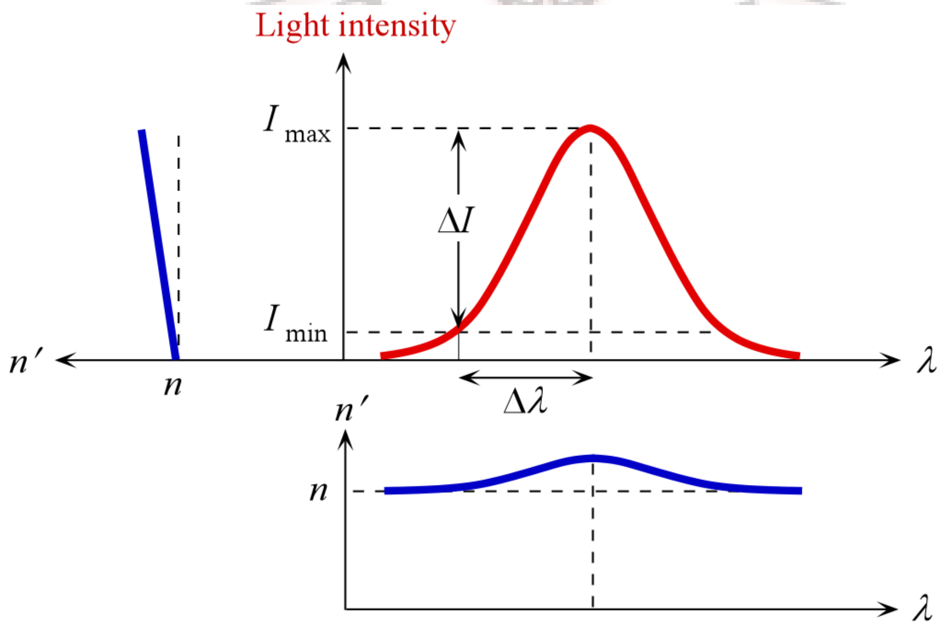
Self-Phase Modulation Dispersion : Nonlinear Effect

At sufficiently high light intensities, the refractive index of glass n' is

$n' = n + CI$

where C is a constant and I is the light intensity. **The intensity of light modulates its own phase.**

$$\boxed{\frac{\Delta\tau}{L} \approx \frac{C\Delta I}{c}}$$



What is the optical power that will giv
$\Delta\tau/L \approx$ 0.1 ps km^{-1}?

Take $C = 10^{-14}$ cm^2 W^{-1}

$\therefore\ \Delta I \approx (c/C)(\Delta\tau/L) = 3\times10^6$ W cm^{-2}

or $\Delta n \approx 3\times10^{-6}$

Given $2a \approx 10$ μm, $A \approx 7.85\times10^{-7}$ cm^2

$\therefore$ Optical power $\approx$ **2.35 W in the core**

$\therefore$ **GIVES A FIRST LIMIT TO POWER INSIDE FIBER**

In many cases, this dispersion will be less than other dispersion mechanisms

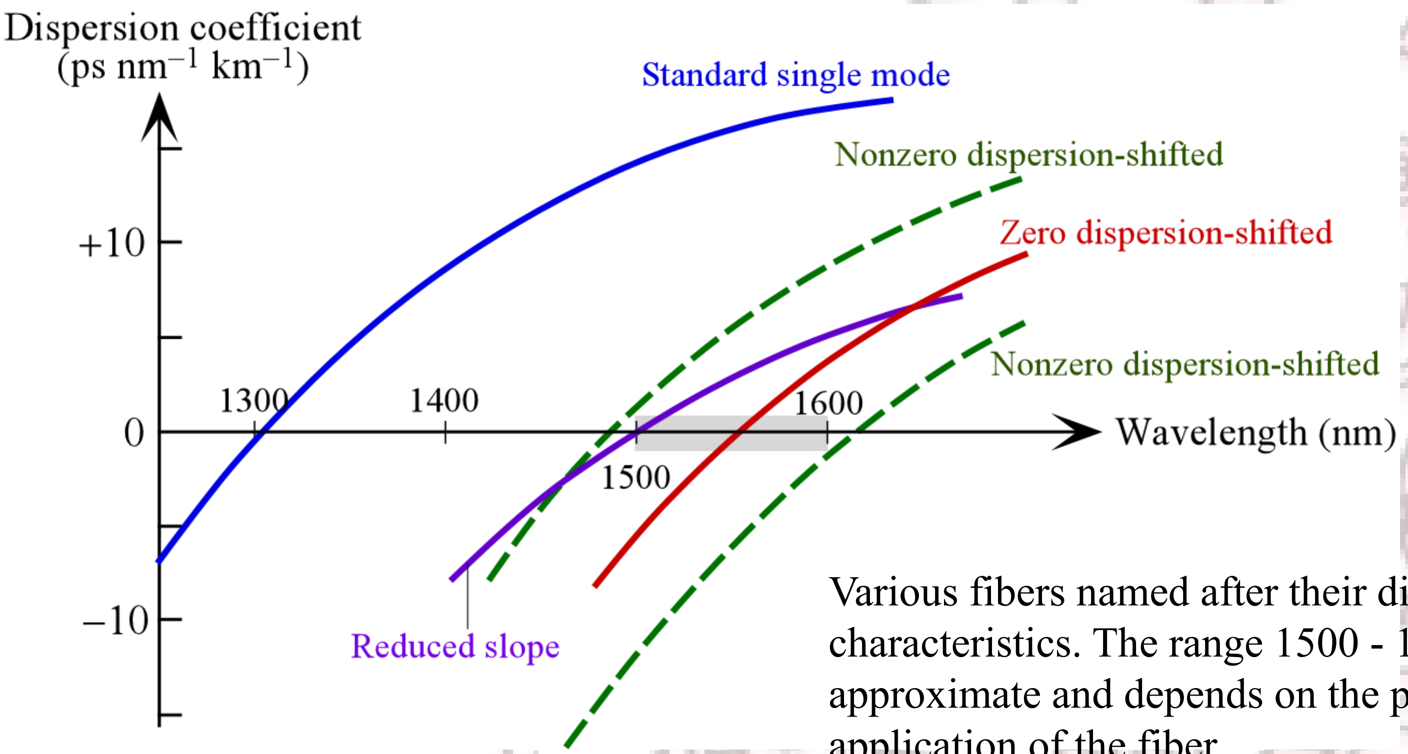
Nonzero Dispersion Shifted Fiber

For Wavelength Division Multiplexing (WDM) avoid 4 wave mixing: cross talk.

We need dispersion not zero but very small in Er-amplifer band (1525-1620 nm)

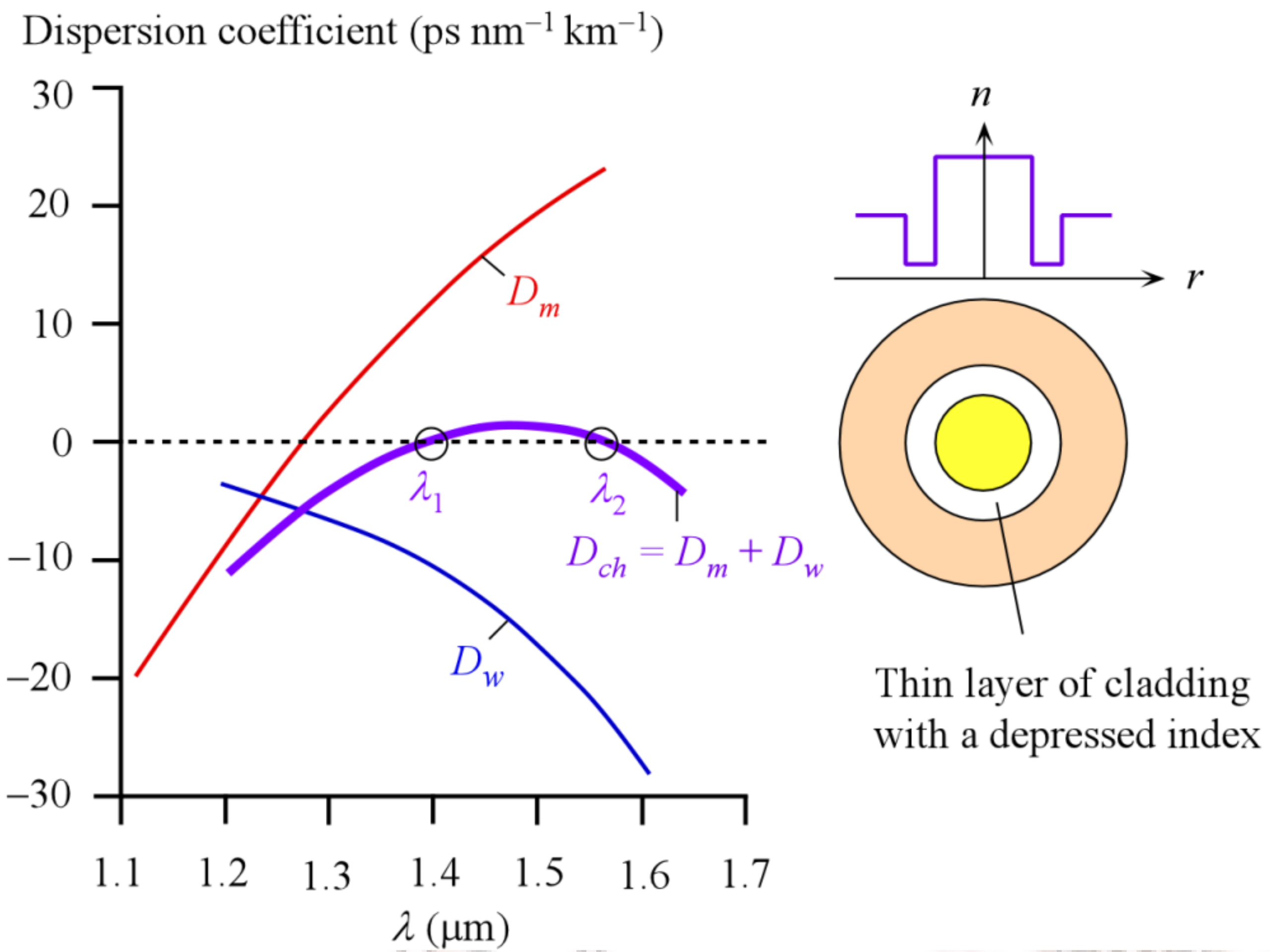
D_{ch} = 0.1 – 6 ps nm^{-1} km^{-1}.

Nonzero dispersion shifted fibers



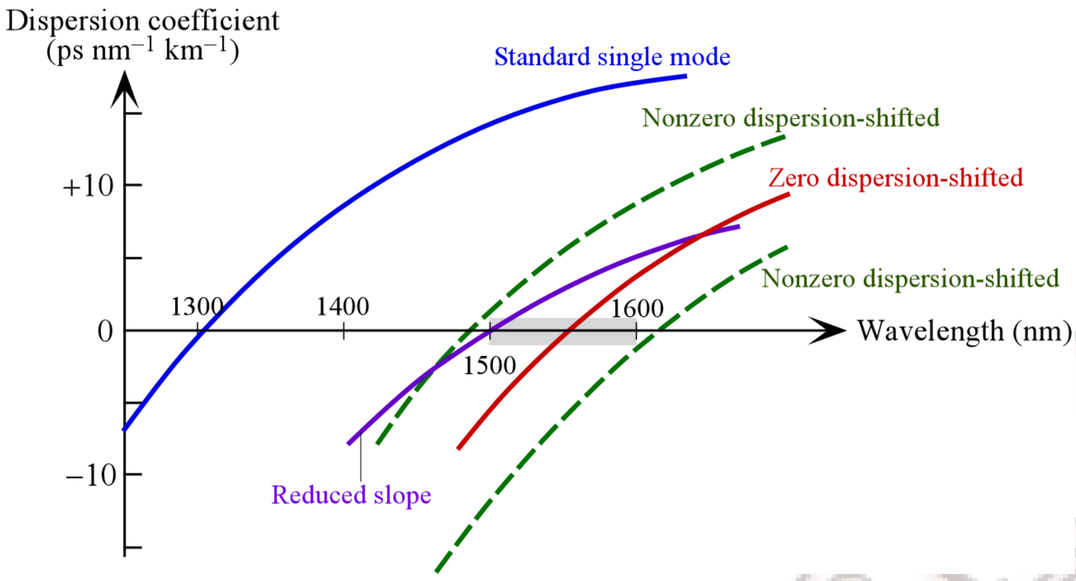
Various fibers named after their dispersion characteristics. The range 1500 - 1600 nm is only approximate and depends on the particular application of the fiber.

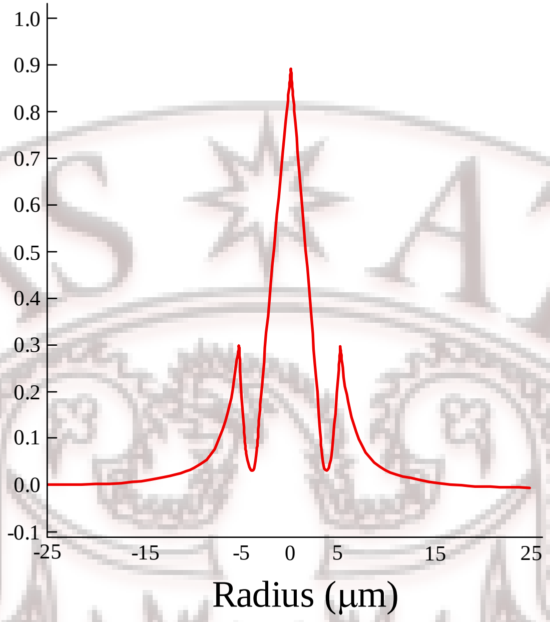
Dispersion Flattened Fiber



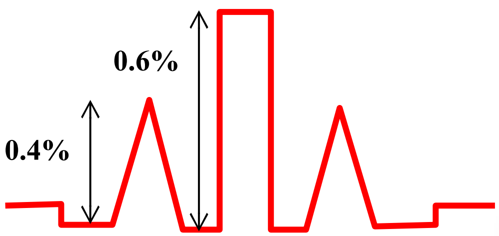
Dispersion flattened fiber example. The material dispersion coefficient (D_m) for the core material and waveguide dispersion coefficient (D_w) for the doubly clad fiber result in a flattened small chromatic dispersion between λ_1 and λ_2.

Nonzero Dispersion Shifted Fiber: More Examples





Nonzero dispersion shifted fiber (Corning)



Fiber with flattened dispersion slope (schematic)

Commercial Fibers for Optical Communications

Fiber	D_{ch} ps nm^{-1} km^{-1}	S_0 ps nm^{-2} km^{-1}	D_{PMD} ps km$^{-1/2}$	Some attributes
Standard single mode, ITU-T G.652	17 (1550 nm)	$\leq$ 0.093	< 0.5 (cabled)	D_{ch} = 0 at $\lambda_0 \approx$ 1312 nm, MFD = 8.6 - 9.5 μm at 1310 nm. $\lambda_c \leq$1260 nm.
Non-zero dispersion shifted fiber, ITU-T G.655	0.1 – 6 (1530 nm)	< 0.05 at 1550 nm	< 0.5 (cabled)	For 1500 - 1600 nm range. WDM application MFD = 8 – 11 μm.
Non-zero dispersion shifted fiber, ITU-T G.656	2 – 14	< 0.045 at 1550 nm	< 0.20 (cabled)	For 1460 - 1625 nm range. DWDM application. MFD = 7 – 11 μm (at 1550 nm). Positive D_{ch}. λ_c <1310 nm
Corning SMF28e+ (Standard SMF)	18 (1550 nm)	0.088	< 0.1	Satisfies G.652. $\lambda_0 \approx$ 1317 nm, MFD = 9.2 μm (at 1310 nm), 10.4 μm (at 1550 nm); $\lambda_c \leq$ 1260 nm.
OFS TrueWave RS Fiber	2.6 - 8.9	0.045	0.02	Satisfies G.655. Optimized for 1530 nm - 1625nm. MFD = 8.4 μm (at 1550 nm); $\lambda_c \leq$1260 nm.
OFS REACH Fiber	5.5 -8.9	0.045	0.02	Higher performance than G.655 specification. Satisfies G.656. For DWDM from 1460 to 1625 nm. $\lambda_0 \leq$ 1405 nm. MFD = 8.6 μm (at 1550 nm)

Single Mode Fibers: Selected Examples

TABLE 2.3 Selected single-mode fibers

Fiber	D_{ch} (ps nm^{-1} km^{-1})	S_0 (ps nm^{-2} km^{-1})	D_{PMD} (ps km$^{-1/2}$)	Some attributes
Standard single mode, ITU-T G.652	17 (1550 nm)	$\leq$0.093	<0.5 (cabled)	$D_{ch} = 0$ at $\lambda_0 \approx 1312$ nm. MFD = 8.6–9.5 μm at 1310 nm. $\lambda_c \leq 1260$ nm.
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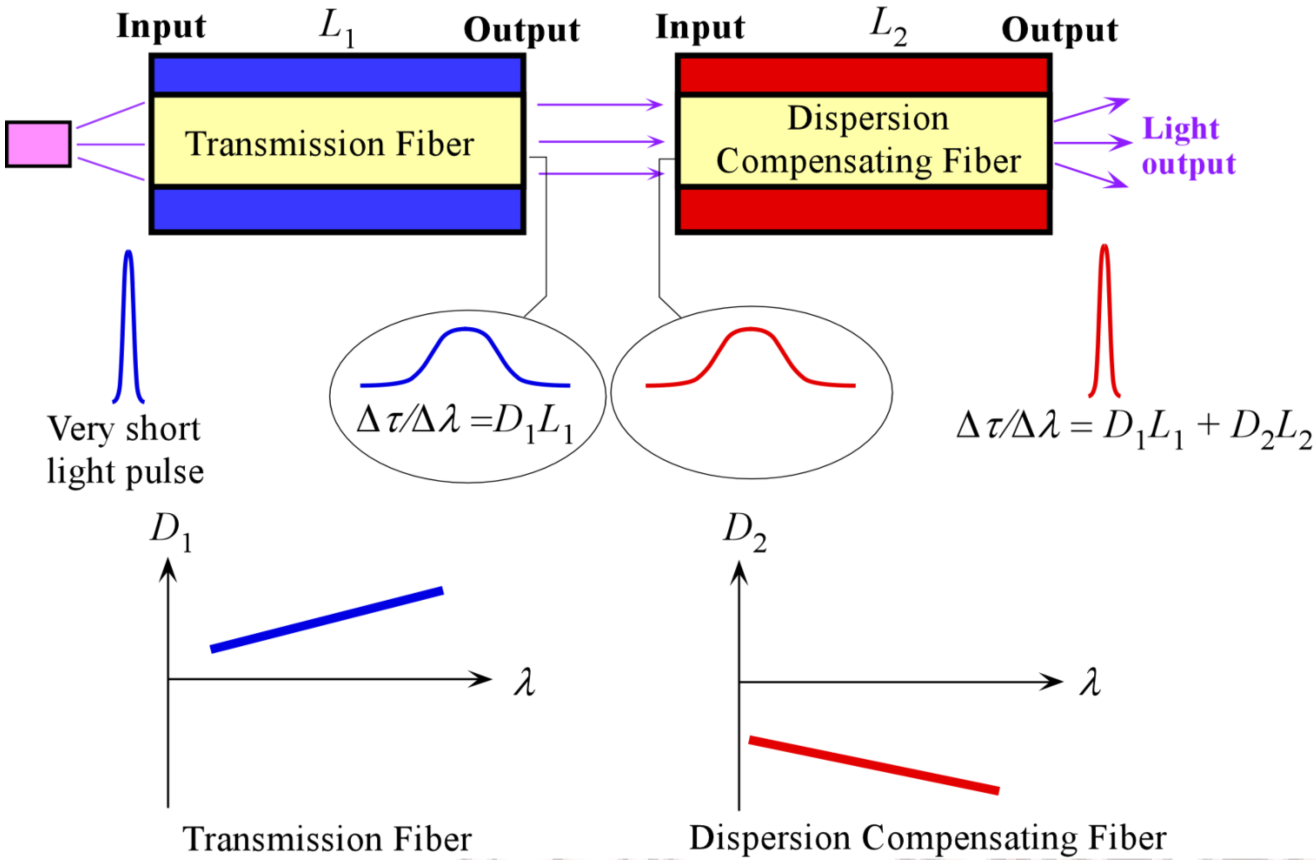
Note: ITU-T is the International Telecommunications Union with the suffix T representing the Telecommunication Standardization Sector in ITU. G.652, G.655, and G.656 are their standards for three single-mode fibers: a standard SMF, nonzero dispersion-shifted fibers for WDM (wavelength division multiplexing), and DWDM (dense WDM) applications, respectively. A few selected commercial SMF properties are also given. λ_0 is the wavelength at which $D_{ch} = 0$.

Fiber Types and Typical Specifications			
Core/Cladding	Attenuation	Bandwidth	Applications/Notes
Multimode Graded-Index			
	@850/1300 nm	@850/1300 nm	
50/125 microns	3/1 dB/km	500/500 MHz-km	Laser-rated for GbE LANs
50/125 microns	3/1 dB/km	2000/500 MHz-km	Optimized for 850 nm VCSELs
62.5/125 microns	3/1 dB/km	160/500 MHz-km	Most common LAN fiber
100/140 microns	3/1 dB/km	150/300 MHz-km	Obsolete
Singlemode			
	@1310/1550 nm		
8-9/125 microns	0.4/0.25 dB/km	HIGH! ~100 Terahertz	Telco/CATV/long high speed LANs
Multimode Step-Index			
	@850 nm	@850 nm	
200/240 microns	4-6 dB/km	50 MHz-km	Slow LANs & links
POF (plastic optical fiber)			
	@ 650 nm	@ 650 nm	
1 mm	~ 1 dB/m	~5 MHz-km	Short Links & Cars

CAUTION: You cannot mix and match fibers! Trying to connect singlemode to multimode fiber can cause 20 dB loss - that's 99% of the power. Even connections between 62.5/125 and 50/125 can cause loss of 3 dB or more - over half the power.

- From Lennie Lightwave (www.jimhayes.com/lenniew/fiber.html)

Dispersion Compensation



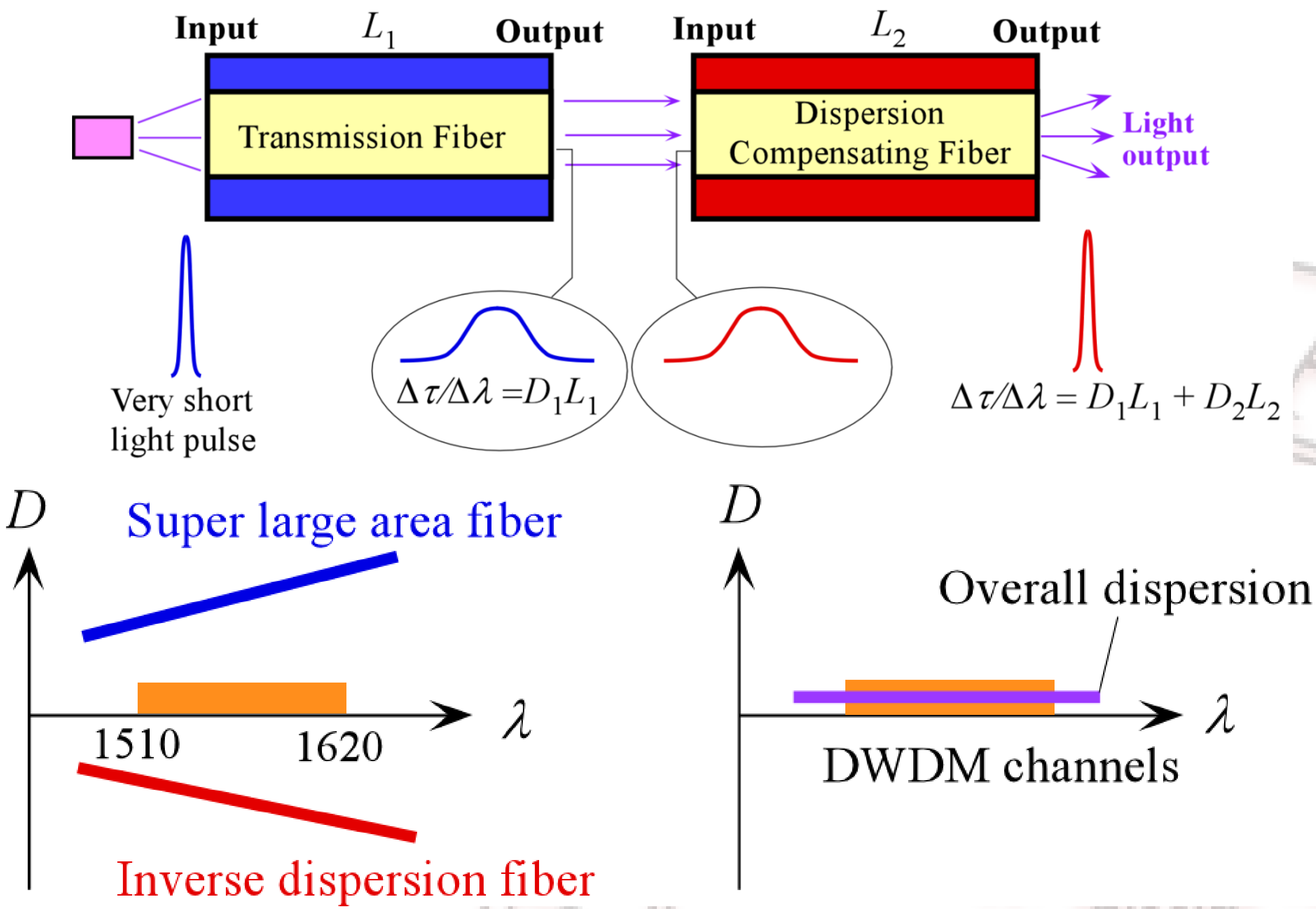
Total dispersion $= D_tL_t + D_cL_c$ **= (10 ps nm⁻¹ km⁻¹)(1000 km) +**

(–100 ps nm⁻¹ km⁻¹)(80 km)

= 2000 ps/nm for 1080 km

$D_{\text{effective}}$ **= 1.9 ps nm⁻¹ km⁻¹**

Dispersion Compensation

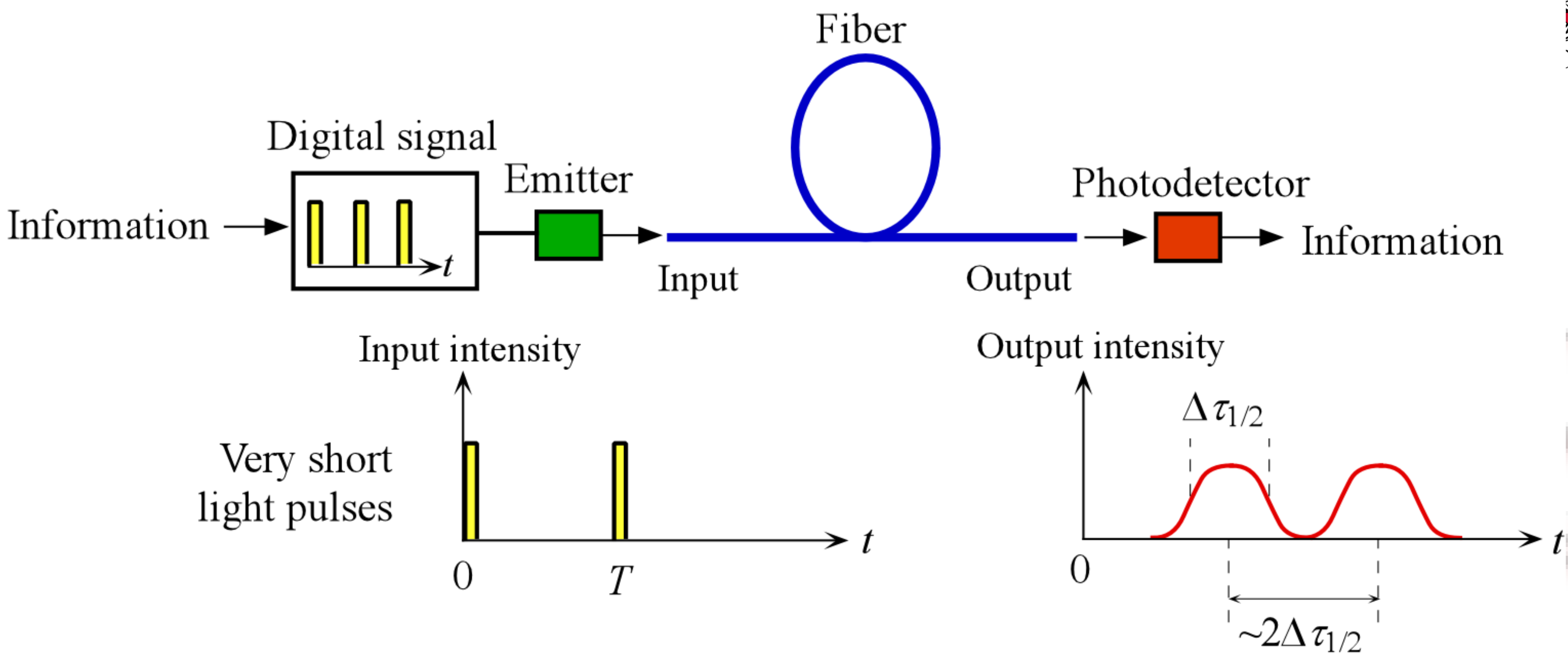


Dispersion D vs. wavelength characteristics involved in dispersion compensation. Inverse dispersion fiber enables the dispersion to be reduced and maintained flat over the communication wavelengths.

Dispersion Compensation and Management

- Compensating fiber has higher attenuation.
  Doped core. Need shorter length
- More susceptible to nonlinear effects.
  Use at the receiver end.
- Different cross sections. Splicing/coupling losses.
- Compensation depends on the temperature.
- Manufacturers provide transmission fiber spliced to inverse dispersion fiber for a well defined D vs. λ

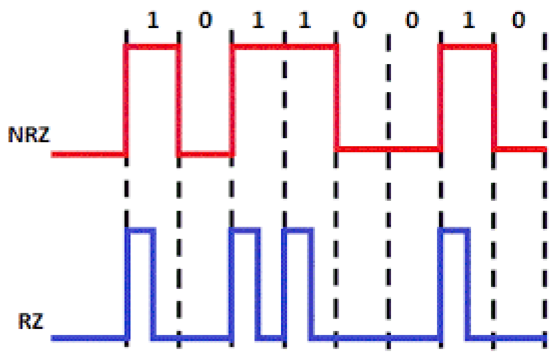
Dispersion and Maximum Bit Rate



$$B \approx \frac{0.5}{\Delta\tau_{1/2}}$$

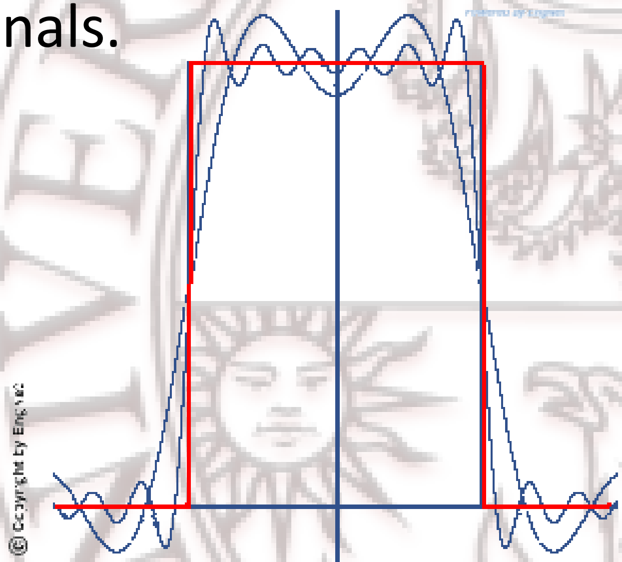
Return-to-zero (RTZ) bit rate or data rate.

Nonreturn to zero (NRZ) bit rate = 2 RTZ bitrate



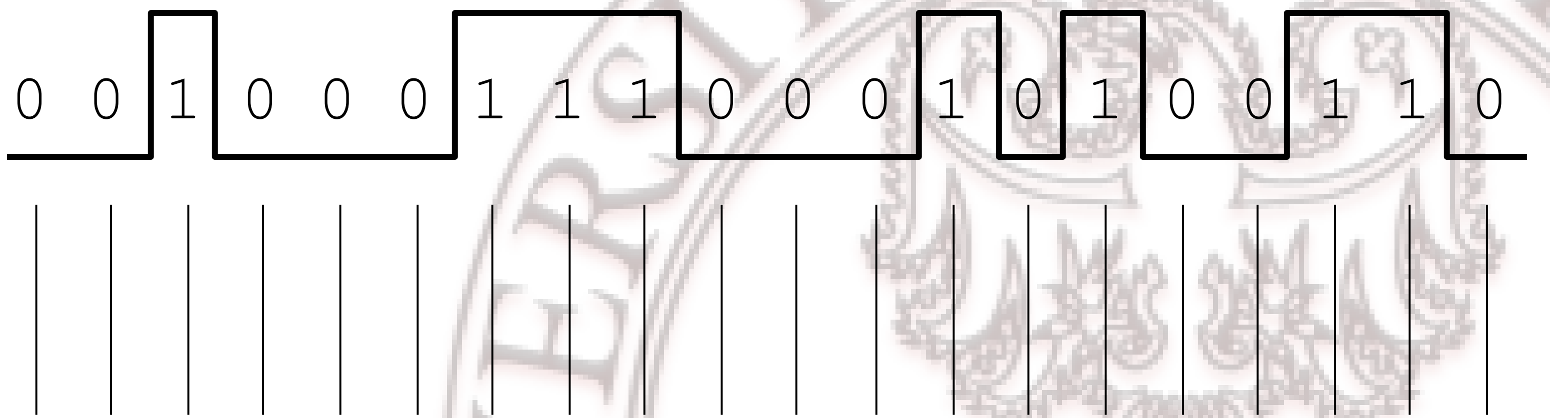
The Frequency Domain

- A (periodic) signal can be viewed as a sum of sine waves of different strengths.
  - *Corresponds to energy at a certain frequency*
- Every signal has an equivalent representation in the frequency domain.
  - What frequencies are present and what is their strength (energy)
- E.g., radio and TV signals.



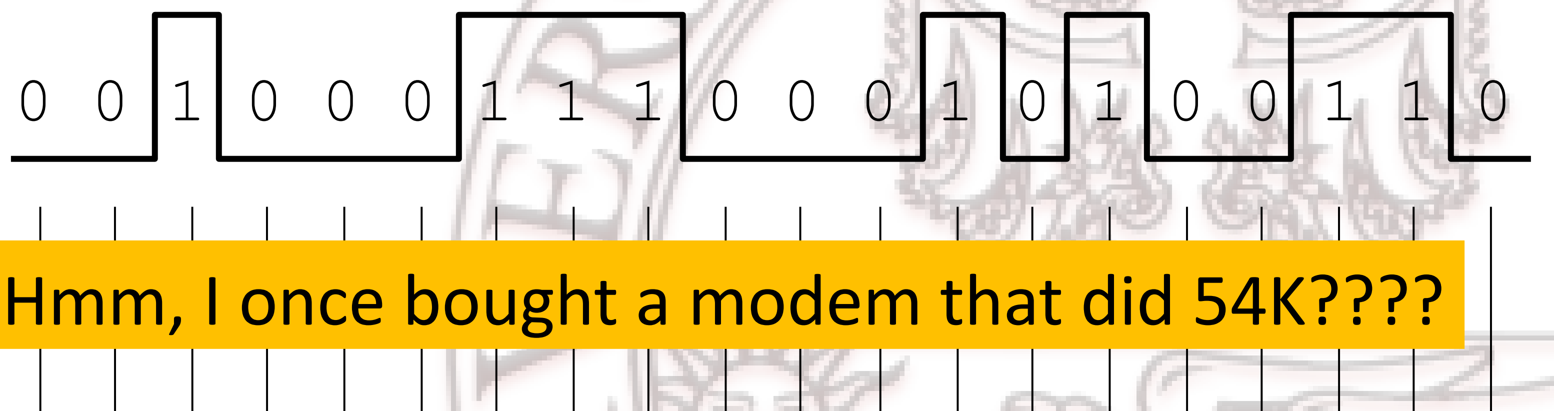
The Nyquist Limit

- A noiseless channel of width H can at most transmit a binary signal at a rate 2 x H.
  - Assumes binary amplitude encoding: 1→1.0, 0→ -1.0



The Nyquist Limit

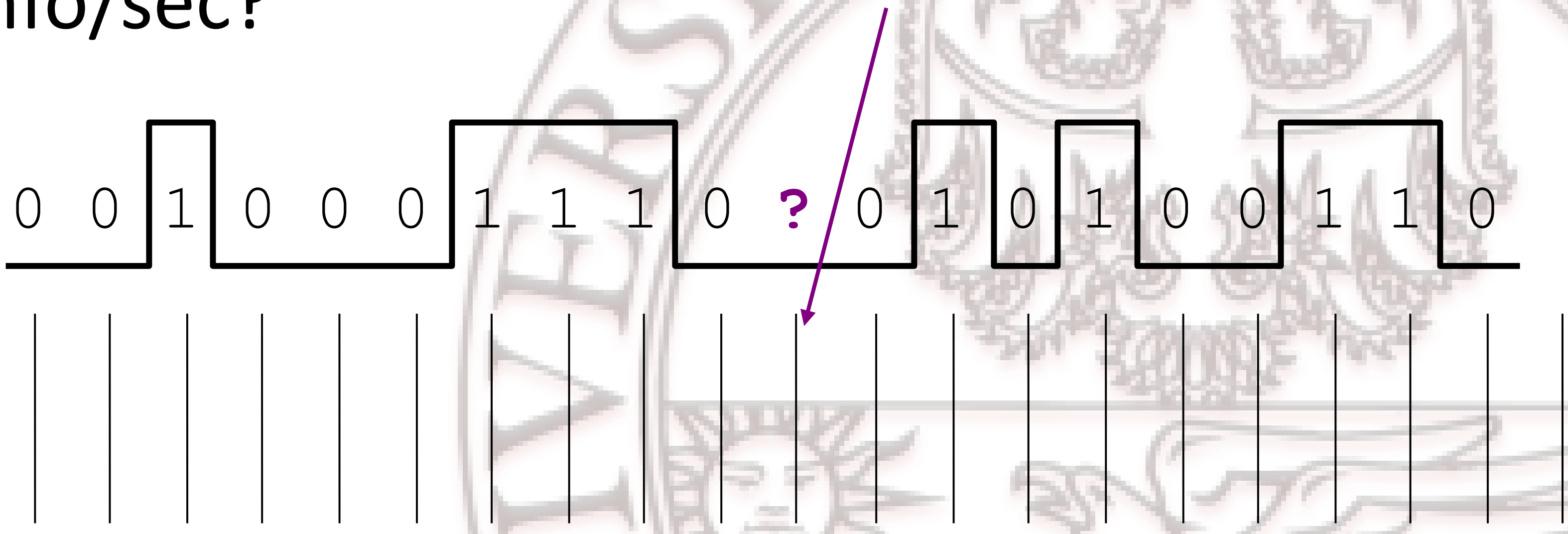
- A noiseless channel of width H can at most transmit a binary signal at a rate 2 x H.
  - Assumes binary amplitude encoding
  - E.g. a 3000 Hz channel can transmit data at a rate of at most 6000 bits/second?



Hmm, I once bought a modem that did 54K????

How to Get Past the Nyquist Limit

- Instead of 0/1, use lots of different values.
- (Remember, the channel is noiseless.)
- Can we really send an infinite amount of info/sec?



Past the Nyquist Limit

- Every transmission medium supports transmission in a certain *fixed* frequency range.
- The channel bandwidth is determined by the transmission medium and the quality of the transmitter and receivers.
- More aggressive encoding can increase the channel bandwidth ... to a point ...

Capacity of a Noisy Channel

- Can’t add infinite symbols
  - you have to be able to tell them apart.
  - This is where noise comes in.

Capacity of a Noisy Channel

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- Shannon's theorem:

$$C = B \times \log_2(1 + S/N)$$

  - C: maximum capacity (bps)
  - B: channel bandwidth (Hz)
  - S/N: signal to noise (power) ratio of the channel
    Often expressed in decibels (db) ::= 10 log(S/N)
-

Capacity of a Noisy Channel

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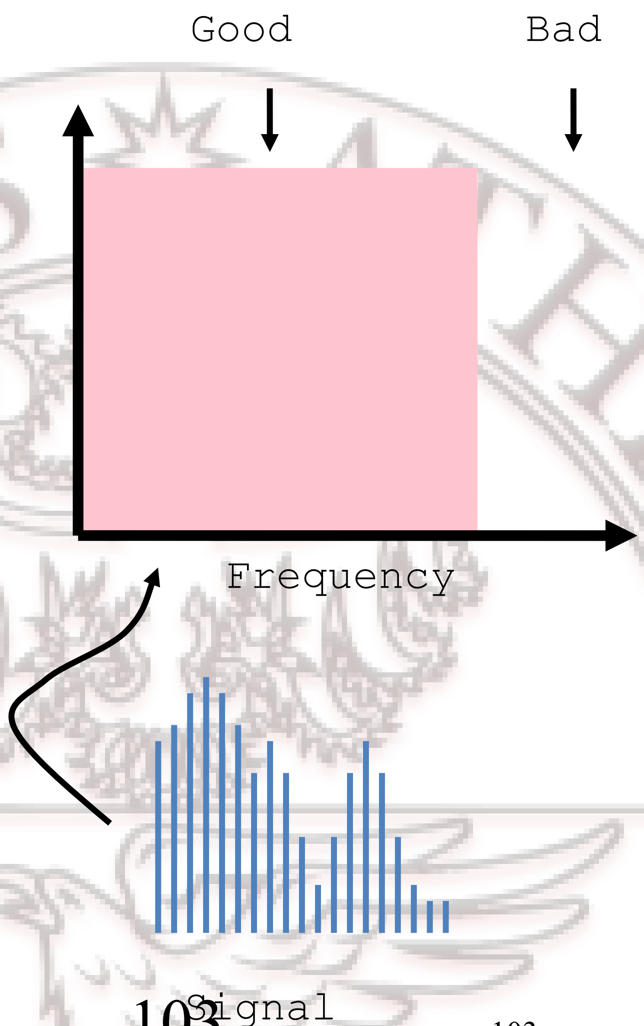
  - C: maximum capacity (bps)
  - B: channel bandwidth (Hz)
  - S/N: signal to noise ratio of the channel
    Often expressed in decibels (db) ::= 10 log(S/N)
- Example:
  - Local loop bandwidth: 3200 Hz
  - Typical S/N: 1000 (30db)
  - What is the upper limit on capacity?
    - $3200 \times \log_2(1 + 1000) = 31.895$ kbits/s

Transmission Channel Considerations

- Every medium supports transmission in a certain frequency range.
  - Outside this range, effects such as attenuation degrade the signal too much
- Transmission and receive hardware will try to maximize the useful bandwidth in this frequency band.
  - Tradeoffs between cost, distance, bit rate
- As technology improves, these parameters change, even for the same wire.



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Limits to Speed and Distance

- Noise: "random" energy is added to the signal.

- Attenuation: some of the energy in the signal leaks away.

- Dispersion: attenuation and propagation speed are frequency dependent.
  (Changes signal shape)

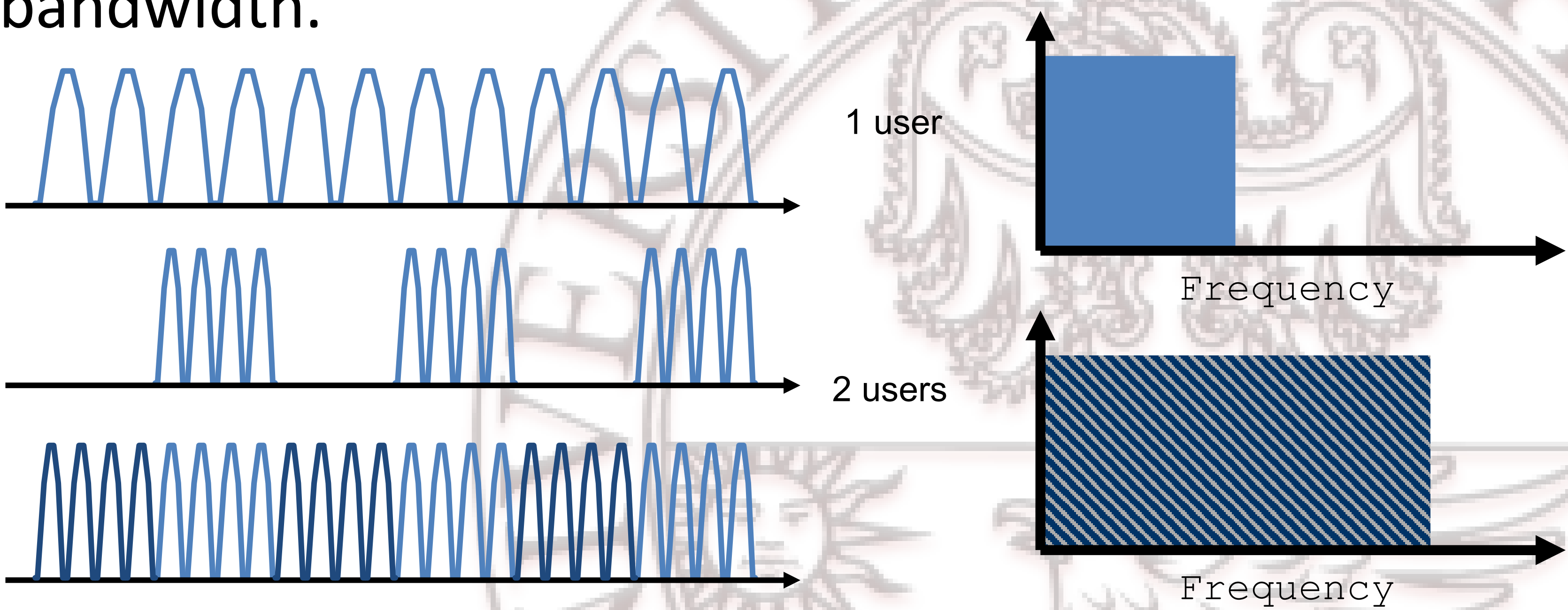
- Effects limit the data rate that a channel can sustain.
  – But affects different technologies in different ways
- Effects become worse with distance.
  – Tradeoff between data rate and distance

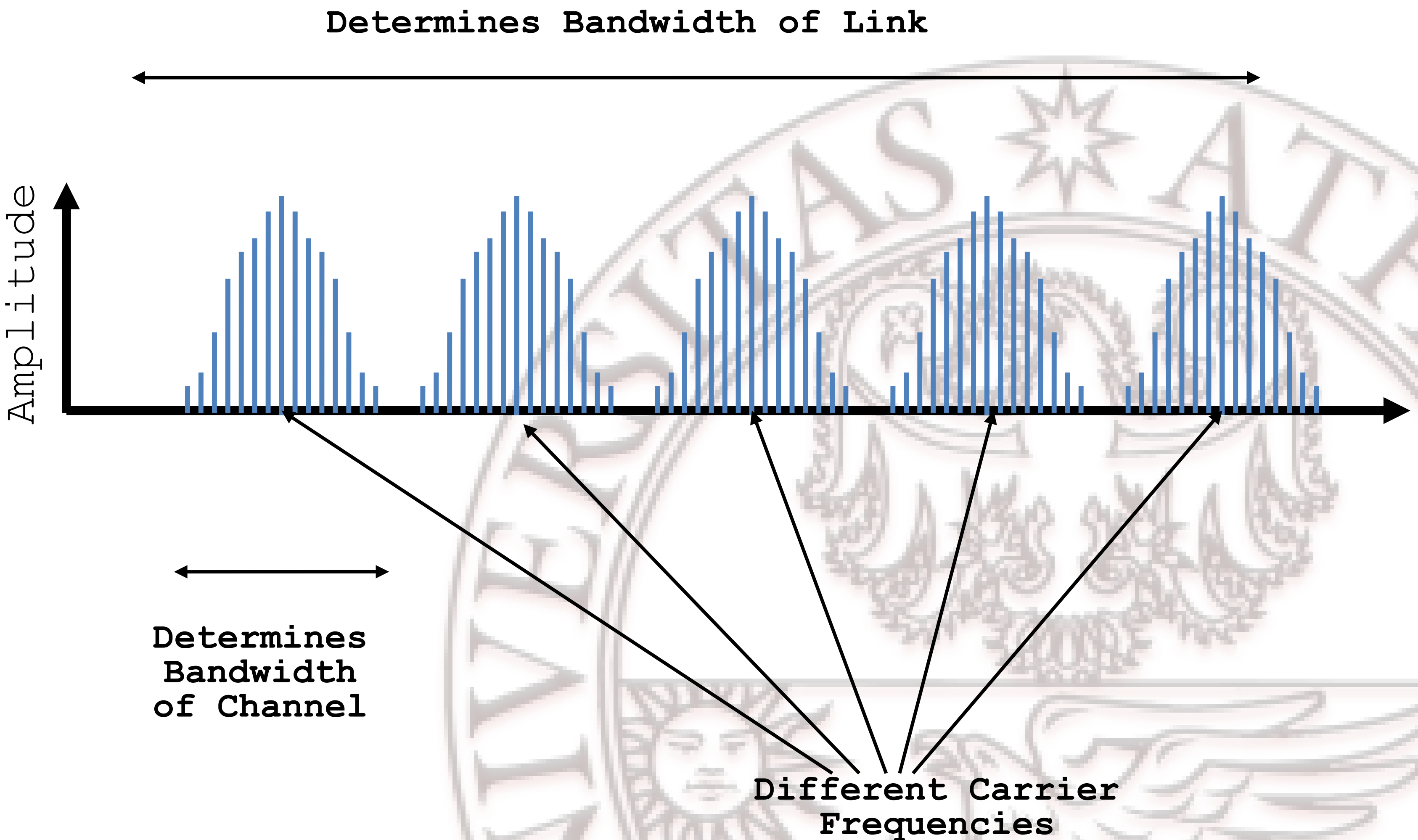
Supporting Multiple Channels

- Multiple channels can coexist if they transmit at a different frequency, or at a different time, or in a different part of the space.
  - Three dimensional space: frequency, space, time
- Space can be limited using wires or using transmit power of wireless transmitters.
- Frequency multiplexing means that different users use a different part of the spectrum.
  - Similar to radio: 95.5 versus 102.5 station
- Controlling time (for us) is a datalink protocol issue.
  - Media Access Control (MAC): who gets to send when?

Time Division Multiplexing

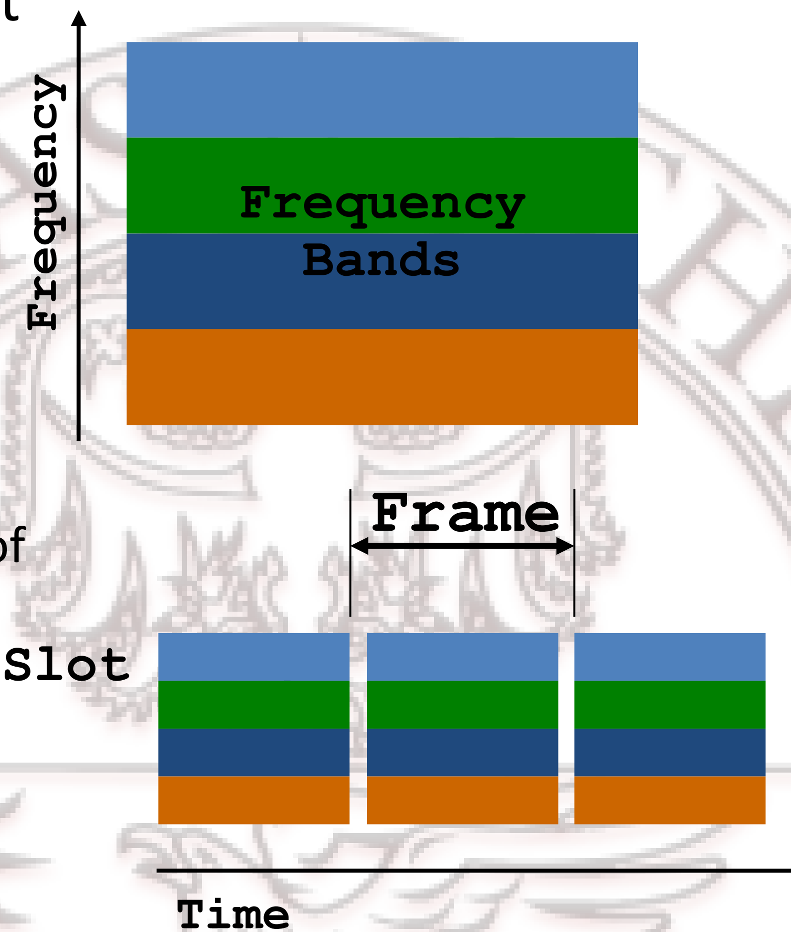
- Different users use the wire at different points in time.
- Requires spectrum proportional to aggregate bandwidth.




Determines Bandwidth of Link
Amplitude
Determines Bandwidth of Channel
Different Carrier Frequencies

Frequency versus Time-division Multiplexing

- With FDM different users use different parts of the frequency spectrum.
  - I.e. each user can send all the time at reduced rate
  - Example: roommates
- With TDM different users send at different times.
  - I.e. each user can send at full speed some of the time
  - Example: time-share condo
- The two solutions can be combined.



How to increase distance?

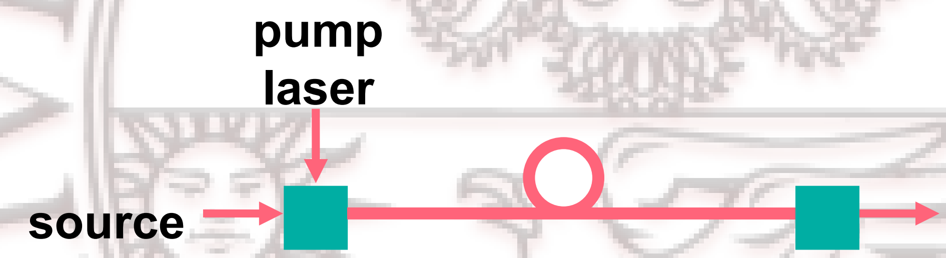
- Even with single mode, there is a distance limit.
- I.e.: How do you get it across the ocean?

How to increase distance?

- Even with single mode, there is a distance limit.
- I.e.: How do you get it across the ocean?

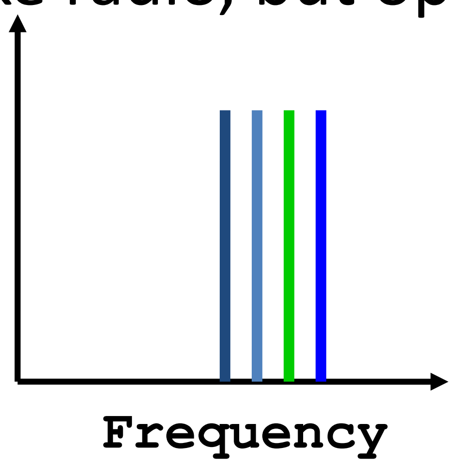
Regeneration and Amplification

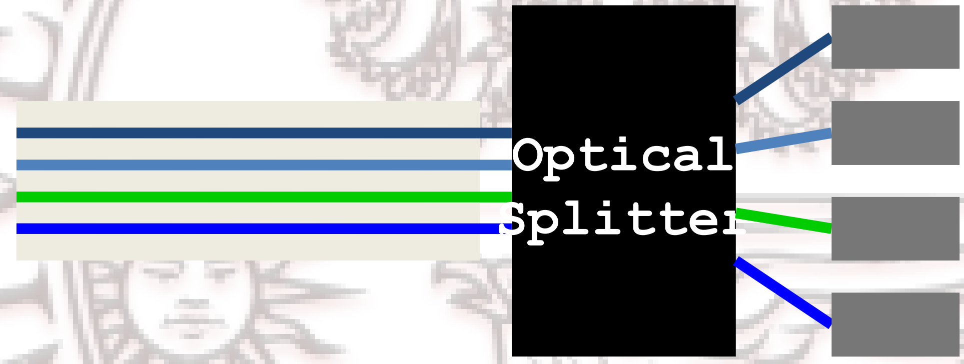
- At end of span, either regenerate electronically or amplify.
- Electronic repeaters are potentially slow, but can eliminate noise.
- Amplification over long distances made practical by erbium doped fiber amplifiers offering up to 40 dB gain, linear response over a broad spectrum. Ex: 40 Gbps at 500 km.



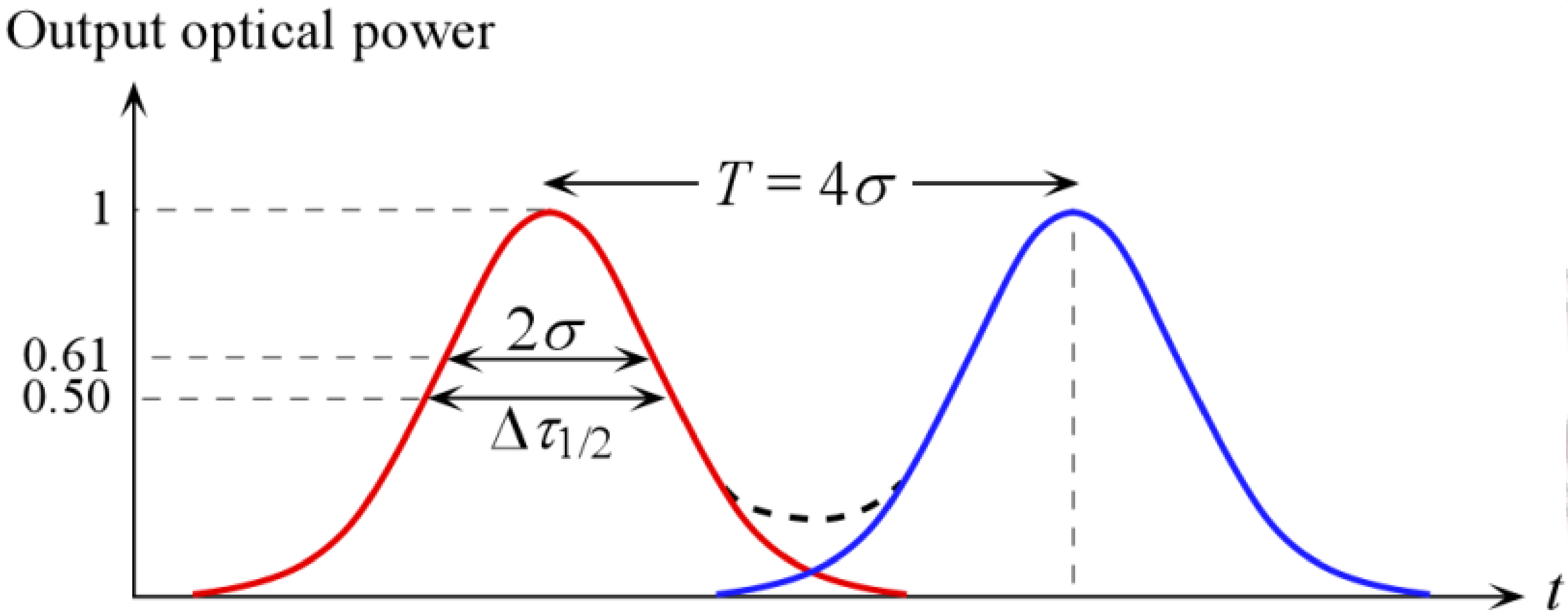
Wavelength Division Multiplexing

- Send multiple wavelengths through the same fiber.
  - Multiplex and demultiplex the optical signal on the fiber
- Each wavelength represents an optical carrier that can carry a separate signal.
  - E.g., 16 colors of 2.4 Gbit/second
- Like radio, but optical and much faster





Maximum Bit Rate *B*



A Gaussian output light pulse and some tolerable intersymbol interference between two consecutive output light pulses (*y*-axis in relative units). At time $t = \sigma$ from the pulse center, the relative magnitude is $e^{-1/2} = 0.607$ and full width root mean square (rms) spread is $\Delta\tau_{rms} = 2\sigma$. (The RTZ case)

Dispersion and Maximum Bit Rate

Maximum Bit Rate

$$B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta\tau_{1/2}}$$

Dispersion

$$\frac{\Delta\tau_{1/2}}{L} = D_{ch}\Delta\lambda_{1/2}$$

$$BL \approx \frac{0.59}{\Delta\tau_{1/2}} = \frac{0.59}{|D_{ch}|\Delta\lambda_{1/2}}$$

Bit Rate × Distance is

inversely proportional to dispersion

inversely proportional to line width of laser

(so, we need single frequency lasers!)

Dispersion and Maximum Bit Rate

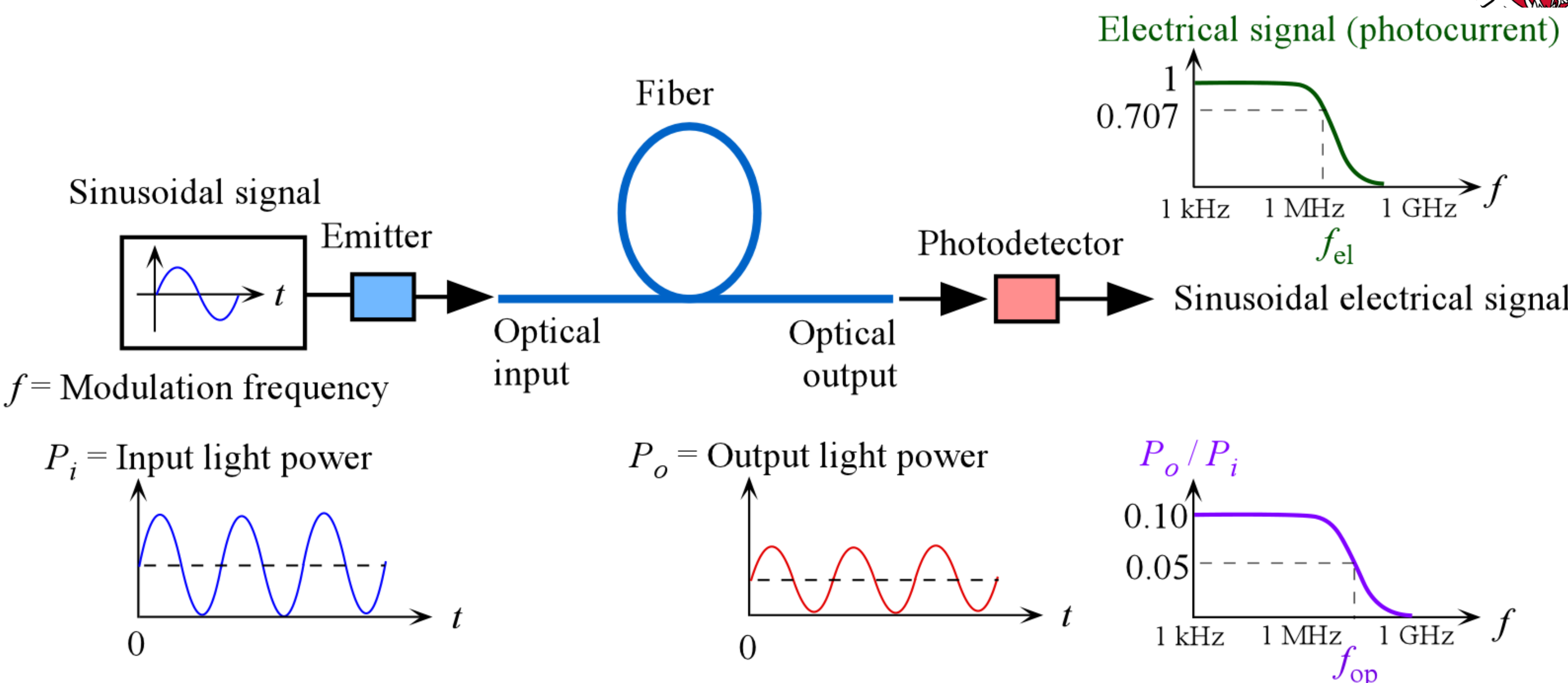
Maximum Bit Rate

$$B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta\tau_{1/2}}$$

$$\sigma^2 = \sigma^2_{\text{intermodal}} + \sigma^2_{\text{intramodal}}$$

$$(\Delta\tau_{1/2})^2 = (\Delta\tau_{1/2})^2_{\text{intermodal}} + (\Delta\tau_{1/2})^2_{\text{intramodal}}$$

Optical Bandwidth

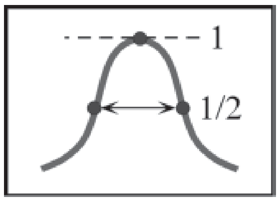
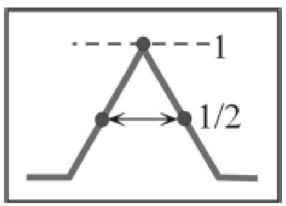
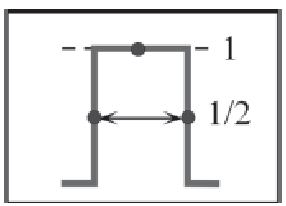


An optical fiber link for transmitting analog signals and the effect of dispersion in the fiber on the bandwidth, f_{op}.

Note that f_{el} = 0,7 f_{op} due to the difference in definition

Pulse Shape and Maximum Bit Rate

TABLE 2.4 Relationships between dispersion parameters, maximum bit rates, and bandwidths

Dispersed pulse shape	Pulse shape and FWHM width, $\Delta\tau_{1/2}$	$\Delta\tau_{1/2}$ FWHM width	B (RZ)	f_{op}
Gaussian with rms deviation σ		$\Delta\tau_{1/2} = 2.353\sigma$ $\sigma = 0.425\Delta\tau_{1/2}$	$0.25/\sigma$	$0.75B = 0.19/\sigma$
Triangular pulse with full-width ΔT		$\Delta\tau_{1/2} = 0.5\Delta T = (6^{1/2})\sigma$ $\sigma = \Delta\tau_{1/2}/2.45 = 0.408\Delta\tau_{1/2}$	$0.25/\sigma$	$0.99B = 0.247/\sigma$
Rectangular with full-width ΔT		$\sigma = 0.289\Delta T = 0.289\Delta\tau_{1/2}$ $\Delta\tau_{1/2} = \Delta T = (2)(3^{1/2})\sigma$	$<1/\Delta T$	$0.69B = 0.17/\sigma$

(*Source:* Data from J. Gowar, *Optical Communication Systems*, 2nd Edition (Prentice Hall, Pearson Education, 1993) Chapter 1.)

Note: RZ = Return-to-zero pulses.

Example: Bit rate and dispersion

Consider an optical fiber with a chromatic dispersion coefficient 8 ps km^{-1} nm^{-1} at an operating wavelength of 1.5 μm. Calculate the bit rate distance product (BL), and the optical and electrical bandwidths for a 10 km fiber if a laser diode source with a FWHP linewidth $\Delta\lambda_{1/2}$ of 2 nm is used.

Solution

For FWHP dispersion,

$$\Delta\tau_{1/2}/L = |D_{ch}|\Delta\lambda_{1/2} = (8\text{ ps nm}^{-1}\text{ km}^{-1})(2\text{ nm}) = 16\text{ ps km}^{-1}$$

Assuming a Gaussian light pulse shape, the RTZ bit rate × distance product (BL) is

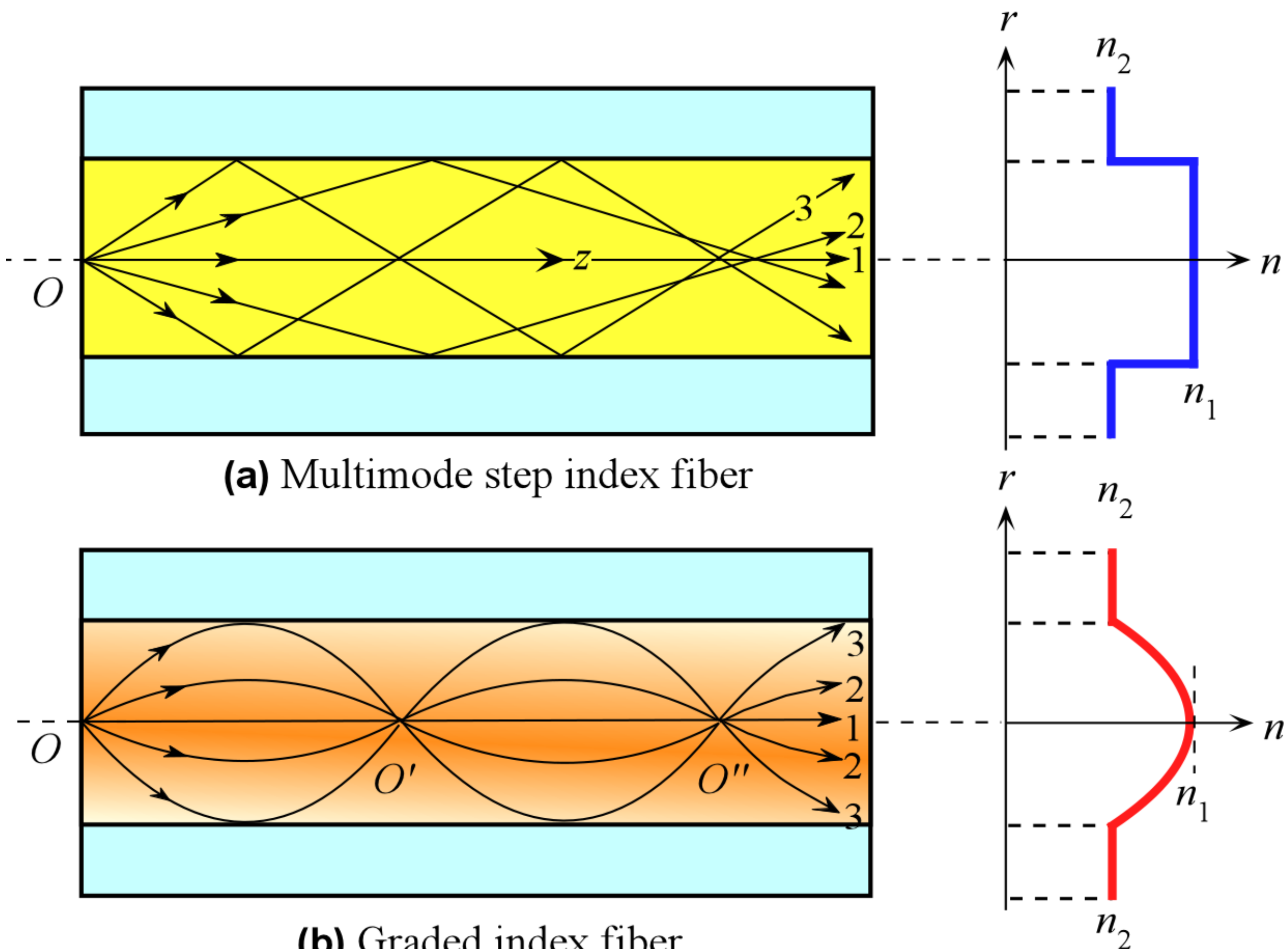
$$BL = 0.59L/\Delta\tau_{1/2} = 0.59/(16\text{ ps km}^{-1}) = 36.9\text{ Gb s}^{-1}\text{ km}$$

The optical and electrical bandwidths for a 10 km fiber are

$$f_{\text{op}} = 0.75B = 0.75(36.9\text{ Gb s}^{-1}\text{ km}) / (10\text{ km}) = 2.8\text{ GHz}$$

$$f_{\text{el}} = 0.70f_{op} = 1.9\text{ GHz}$$

Graded Index (GRIN) Fiber



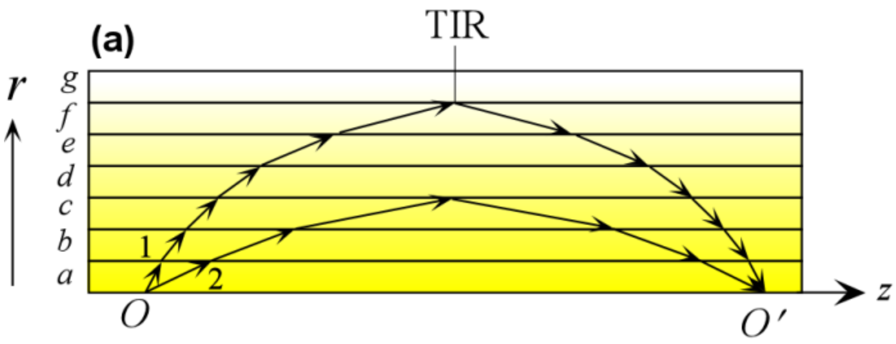
(a) Multimode step index fiber

(b) Graded index fiber

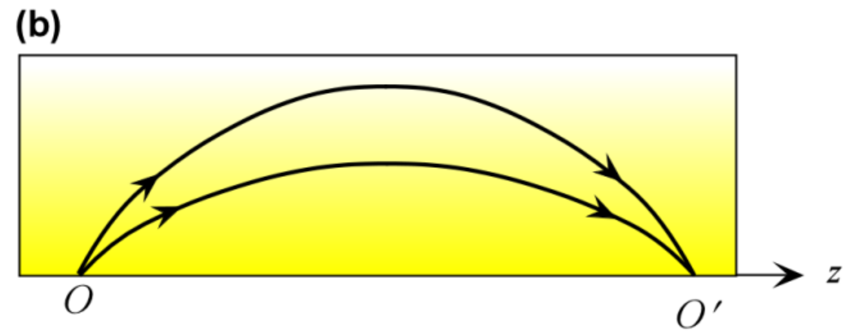
(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.

(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.

Graded Index (GRIN) Fiber



n decreases step by step from one layer to next upper layer



Continuous decrease in *n* gives a ray path changing continuously.

(a) Consider a ray in thinly stratified medium becomes refracted as it passes from one layer to the next upper layer with lower *n* (with larger velocity) and eventually its angle satisfies TIR.

(b) In a medium where *n* decreases continuously the path of the ray bends continuously.

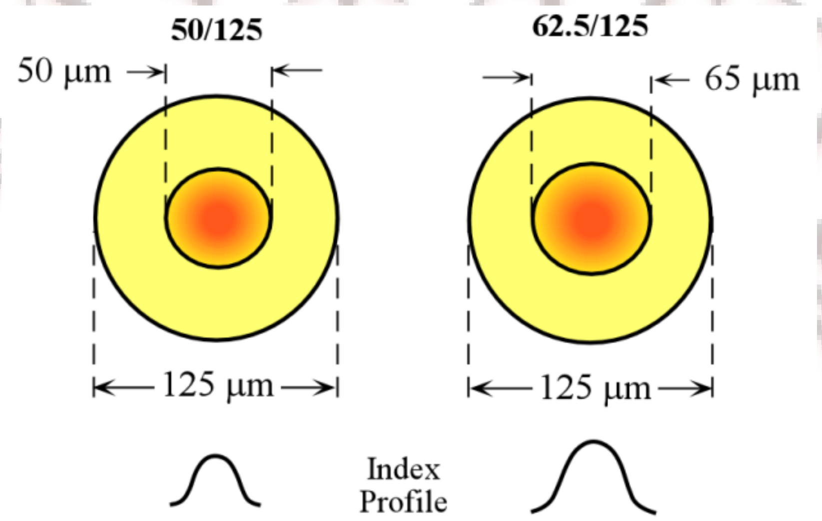
(c) A proper design allows to get the condition where the time of arrival is the same for all paths

Graded Index (GRIN) Fiber

The refractive index profile can generally be described by a power law with an index γ called the **profile index** (or **the coefficient of index grating**) so that,

$$n = n_1[1 - 2\Delta(r/a)^{\gamma}]^{1/2} \quad ; \quad r < a,$$

$$n = n_2 \quad ; r \geq a$$



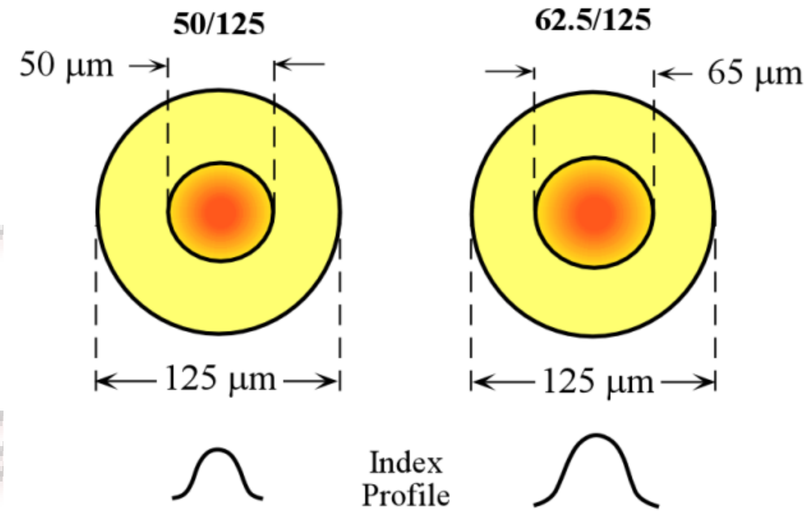
Graded Index (GRIN) Fiber

best coefficient to minimize intermodal dispersion:

$$\gamma_o \approx 2+\delta-\Delta\frac{(4+\delta)(3+\delta)}{5+2\delta}\approx 2$$

Profile dispersion parameter

$$\delta=-\left(\frac{n_1\lambda}{N_{g1}\Delta}\right)\frac{d\Delta}{d\lambda}$$

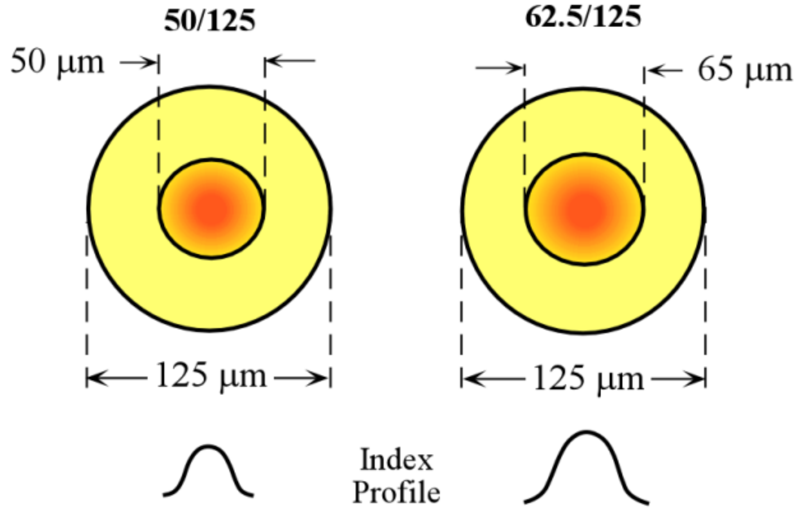


$$\boxed{\frac{\sigma_{\text{intermode}}}{L}\approx\frac{n_1}{20\sqrt{3}c}\Delta^2}$$

Minimum intermodal dispersion

Graded Index (GRIN) Fiber

$$\mathrm{NA} = \mathrm{NA}(r) = [n(r)^2 - n_2^2]^{1/2}$$



Effective numerical aperture for GRIN fibers (γ=2)

$$\boxed{\mathrm{NA}_{\mathrm{GRIN}} \approx (1/2^{1/2})(n_1^2 - n_2^2)^{1/2}}$$

Number of modes in a graded index fiber in general

$$\boxed{M \approx \left(\frac{\gamma}{\gamma+2}\right)\frac{V^2}{2}}$$

Table 2.5
Graded index multimode fibers

d = core diameter (μm), D = cladding diameter (μm). Typical properties at 850 nm. VCSEL is a vertical cavity surface emitting laser. α is attenuation along the fiber. OM1, OM3 and OM4 are fiber standards for LAN data links (ethernet). α are reported typical attenuation values. 10G and 40G networks represent data rates of 10 Gb s^{-1} and 40 Gb s^{-1} and correspond to 10 GbE (Gigabit Ethernet) and 40 GbE systems.

MMF d/D	Compliance standard	Source	Typical D_{ch} ps nm^{-1} km^{-1}	Bandwidth MHz·km	NA	α dB km^{-1}	Reach in 10G and 40G networks
50/125	OM4	VCSEL	–100	4700 (EMB) 3500 (OFLBW)	0.200	< 3	550 m (10G) 150 m (40G)
50/125	OM3	VCSEL	–100	2000 (EMB) 500 (OFLBW)	0.200	< 3	300 m (10G)
62.5/125	OM1	LED	–117	200 (OFLBW)	0.275	< 3	33 m (10G)

Example: Dispersion in a GRIN Fiber and Bit Rate

Graded index fiber. Diameter of 50 μm and a refractive index of $n_1 = 1.4750$, $\Delta = 0.010$.

The fiber is used in LANs at 850 nm with a vertical cavity surface emitting laser (VCSEL) that has very a narrow linewidth that is about 0.4 nm (FWHM). Assume that the chromatic dispersion at 850 nm is −100 ps nm^{-1} km^{-1} as shown in Table 2.5. Assume the fiber has been optimized at 850 nm, and find the minimum rms dispersion. How many modes are there? What would be the upper limit on its bandwidth? What would be the bandwidth in practice?

Solution

Given Δ and n_1, we can find n_2 from

$\Delta = 0.01 = (n_1 - n_2)/n_1 = (1.4750 - n_2)/1.4750.$

$\therefore\ n_2 = 1.4603.$

The V-number is then

$V = [(2\pi)(25\ \mu\text{m})/(0.850\ \mu\text{m})(1.4750^2 - 1.4603^2)^{1/2} = 38.39$

For the number of modes we can simply take $\gamma = 2$ and use

$M = (V^2/4) = (38.39^2/4) = 368$ modes

The lowest intermodal dispersion for a profile optimized graded index fiber for a 1 km of fiber, $L = 1$ km, is

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

$$\frac{\sigma_{\text{intermode}}}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2 = \frac{1.4750}{20\sqrt{3}(3\times10^8)}(0.010)^2$$

$$= 14.20\times10^{-15} \text{ s m}^{-1} \text{ or } 14.20 \text{ ps km}^{-1}$$

Assuming a triangular output light pulse (max bandwidth) and the relationship between σ and $\Delta\tau_{1/2}$ given in Table 2.4, the intermodal spread $\Delta\tau_{\text{intermode}}$ (FWHM) in the group delay over 1 km is

$$\Delta\tau_{\text{intermode}} = (6^{1/2})\sigma_{\text{intermode}} = (2.45)(14.20 \text{ ps}) = 34.8 \text{ ps}$$

We also need the material dispersion at the operating wavelength over 1 km, which makes up the intramodal dispersion $\Delta\tau_{\text{intramode}}$ (FWHM)

$$\Delta\tau_{\text{intramode}} = L|D_{ch}|\,\Delta\lambda_{1/2} = (1 \text{ km})(-100 \text{ ps nm}^{-1}\text{ km}^{-1})(0.40 \text{ nm}) = 40.0 \text{ ps}$$

$$\Delta\tau^2 = \Delta\tau^2_{\text{intermode}} + \Delta\tau^2_{\text{intramode}} = (34.8)^2 + (40.0)^2 \Rightarrow \Delta\tau = 53.0 \text{ ps}$$

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

$$B = \frac{0.25}{\sigma} = \frac{0.25}{0.408\Delta\tau} = \frac{0.61}{\Delta\tau} = \frac{0.61}{(53.0\times10^{-12}\,\mathrm{s})} = \mathbf{11.5\ Gb\ s^{-1}}$$

Optical bandwidth $f_{\mathrm{op}} = 0.99B =$ **11.4 GHz**

This is the upper limit since we assumed that the graded index fiber is perfectly optimized with $\sigma_{\mathrm{intermode}}$ being minimum. Small deviations around the optimum γ cause large increases in $\sigma_{\mathrm{intermode}}$, which would sharply reduce the bandwidth.

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

If this were a multimode step-index fiber with the same n_1 and n_2, then the full dispersion (total spread) would roughly be

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c} = \frac{n_1\Delta}{c} = \frac{(1.475)(0.01)}{3\times10^8}$$

$$= 4.92\times10^{-11} \text{ s m}^{-1} \text{ or } 49.2 \text{ ns km}^{-1}$$

To calculate the BL we use $\sigma_{\text{intermode}} \approx 0.29\Delta\tau$

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} = \frac{0.25}{0.29(\Delta\tau/L)} = \frac{0.25}{(0.29)(49.2\times10^{-9}\text{ s km}^{-1})} = 17.5 \text{ Mb s}^{-1}\text{ km}$$

LANs now use graded index MMFs, and the step index MMFs are used mainly in low speed instrumentation

Example: Dispersion in a graded-index fiber and bit rate

Consider a graded index fiber whose core has a diameter of 50 μm and a refractive index of $n_1 = 1.480$. The cladding has $n_2 = 1.460$. If this fiber is used at 1.30 μm with a laser diode that has very a narrow linewidth what will be the bit rate × distance product? Evaluate the *BL* product if this were a multimode step index fiber.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.48 - 1.46)/1.48 = 0.0135$.

Dispersion for 1 km of fiber is

$\sigma_{\text{intermode}}/L = n_1\Delta^2/[(20)(3^{1/2})c] = 2.6\times10^{-14}$ s m^{-1} or 0.026 ns km^{-1}.

$BL = 0.25/\sigma_{\text{intermode}}$ = **9.6 Gb s^{-1} km**

We have ignored any material dispersion and, further, we assumed the index variation to perfectly follow the optimal profile which means that in practice *BL* will be worse. (For example, a 15% variation in γ from the optimal value can result in $\sigma_{\text{intermode}}$ and hence *BL* that are more than 10 times worse.)

If this were a multimode step-index fiber with the same n_1 and n_2, then the full dispersion (total spread) would roughly be 6.67×10^{-11} s m^{-1} or 66.7 ns km^{-1} and BL = 12.9 **Mb s^{-1} km**

Note: Over long distances, the bit rate × distance product is not constant for multimode fibers and typically $B \propto L^{-\gamma}$ where γ is an index between 0.5 and 1. The reason is that, due to various fiber imperfections, there is mode mixing which reduces the extent of spreading.

Example: Combining intermodal and intramodal dispersions

Consider a graded index fiber with a core diameter of 30 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient D_m at 1300 nm is –7.5 ps nm^{-1} km^{-1}.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.474 - 1.453)/1.474 = 0.01425$. Modal dispersion for 1 km is

$$\sigma_{\text{intermode}} = Ln_1\Delta^2/[(20)(3^{1/2})c] = 2.9\times10^{-11}\ \text{s}^{-1} \text{ or } 0.029 \text{ ns.}$$

The material dispersion is

$$\Delta\tau_{1/2} = LD_m\Delta\lambda_{1/2} = (1\ \text{km})(7.5\ \text{ps nm}^{-1}\ \text{km}^{-1})(3\ \text{nm}) = 0.0225\ \text{ns}$$

Assuming a Gaussian output light pulse shape,

$$\sigma_{\text{intramode}} = 0.425\Delta\tau_{1/2} = (0.425)(0.0225\ \text{ns}) = 0.0096\ \text{ns}$$

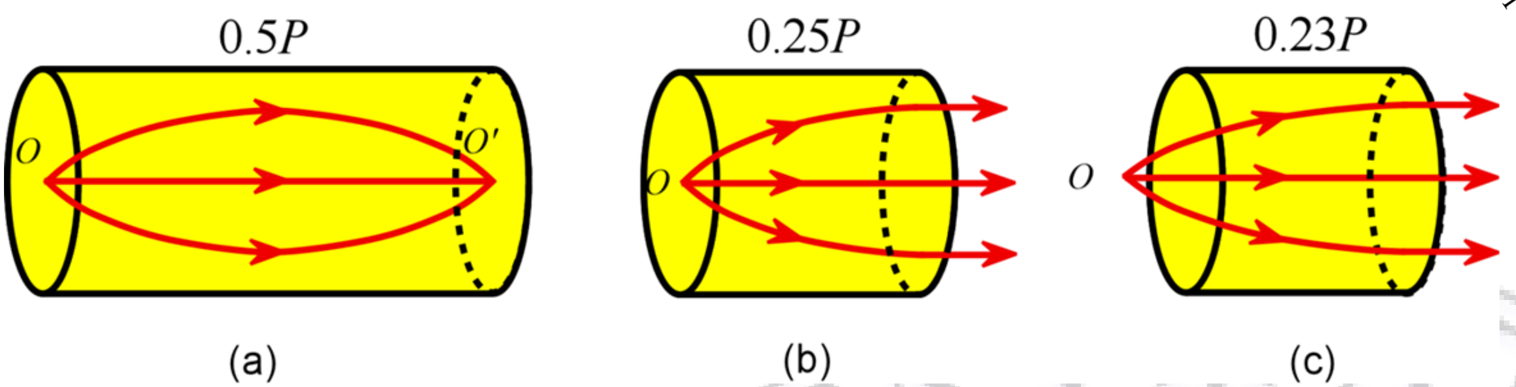
Total dispersion is

$$\sigma_{\text{rms}} = \sqrt{\sigma^2_{\text{intermode}} + \sigma^2_{\text{intramode}}} = \sqrt{0.029^2 + 0.0096^2} = 0.030\ \ \text{ns}$$

Assume L = 1 km

$B = 0.25/\Delta\tau_{\text{rms}}$ = **8.2 Gb**

GRIN Rod Lenses

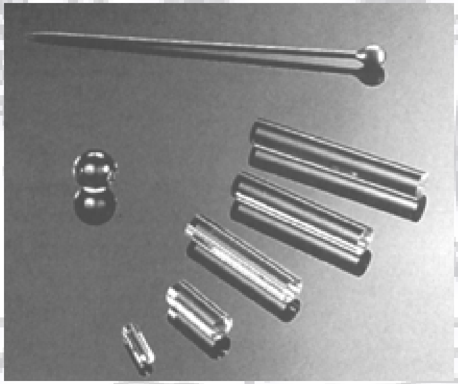


Point *O* is on the rod face center and the lens focuses the rays onto *O′* on to the center of the opposite face.

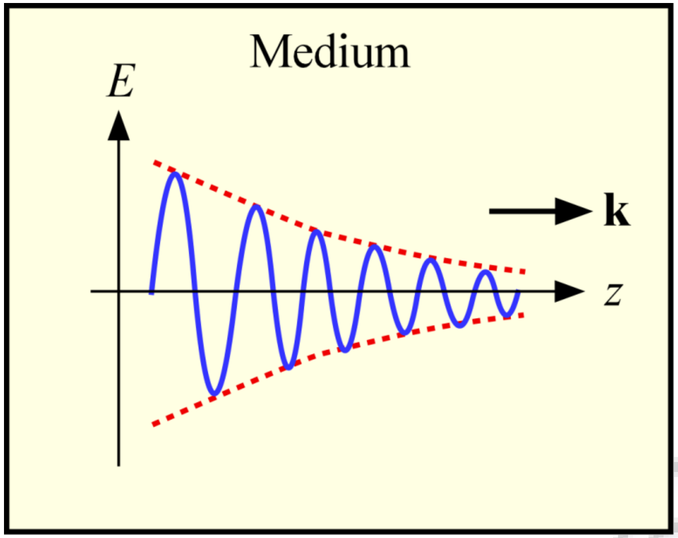
The rays from *O* on the rod face center are collimated out.

***O* is slightly away from the rod face and the rays are collimated out.**

One pitch (*P*) is a full one period variation in the ray trajectory along the rod axis.



Attenuation



The attenuation of light in a medium

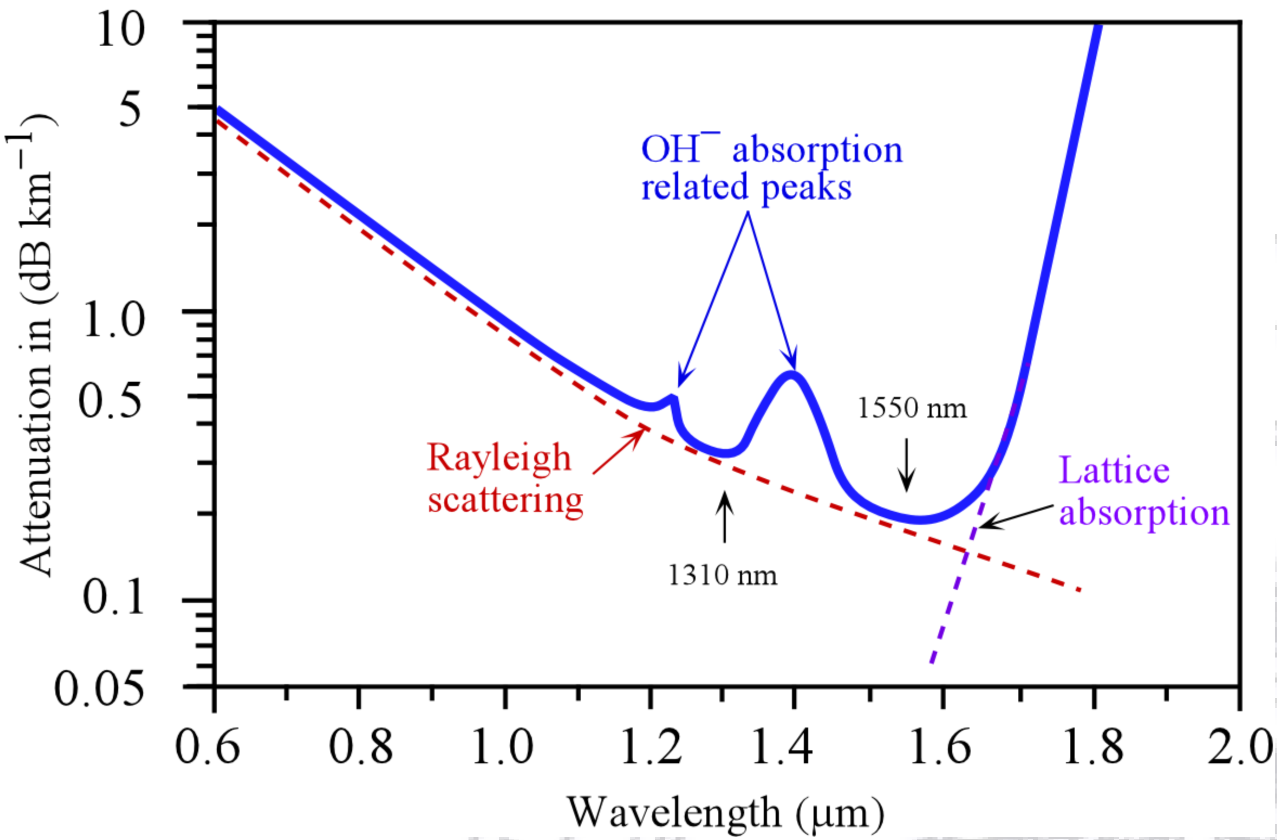
Attenuation = Absorption + Scattering

Attenuation coefficient α is defined as the *fractional decrease in the optical power per unit distance*. α is in m^{-1}.

$$P_{out} = P_{in}\exp(-\alpha L)$$

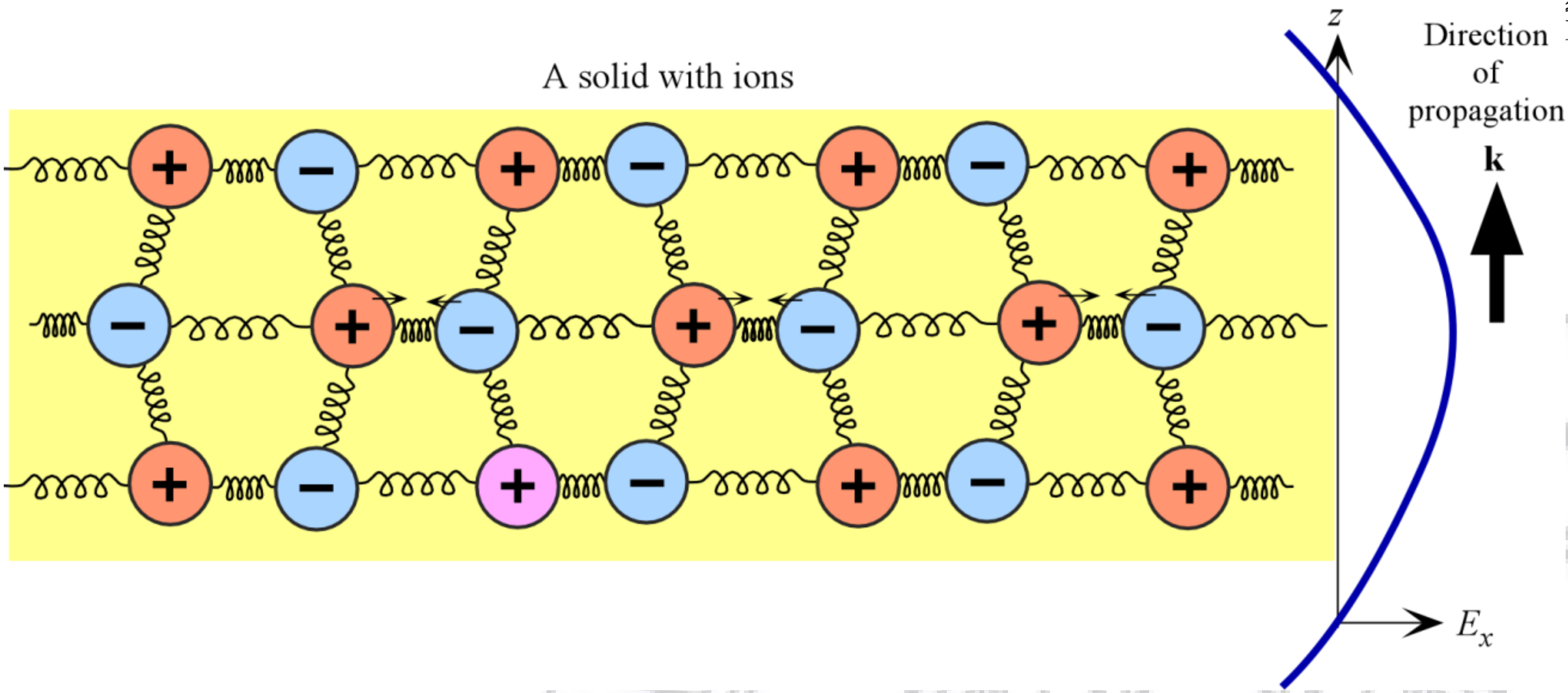
$$\alpha_{dB} = \frac{1}{L} 10 \log\left(\frac{P_{in}}{P_{out}}\right) \qquad \alpha_{dB} = \frac{10}{\ln(10)}\alpha = 4.34\alpha$$

Attenuation in Optical Fibers



Attenuation vs. wavelength for a standard silica based fiber.

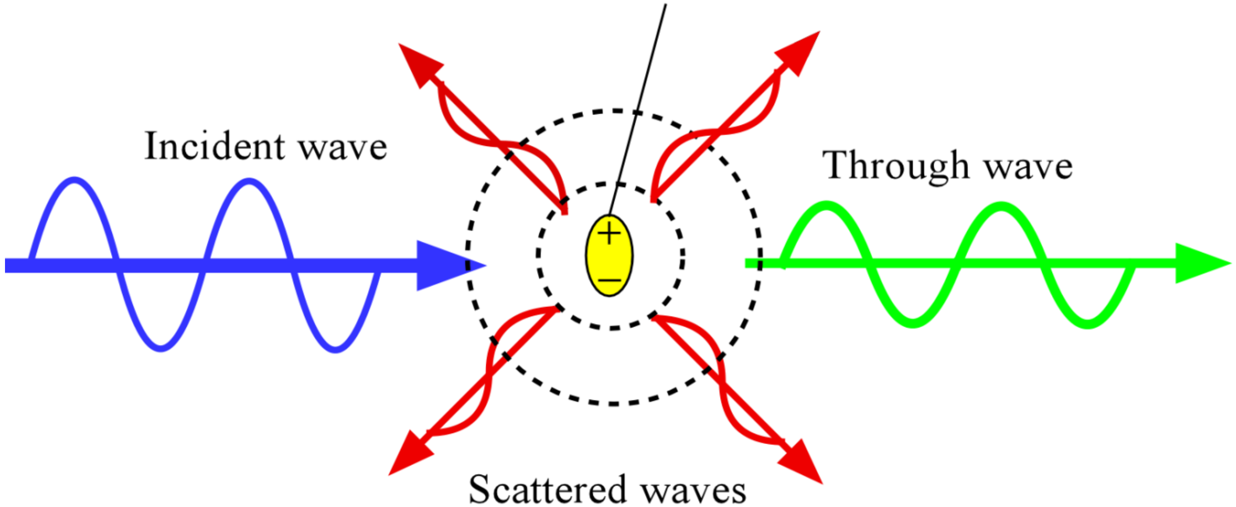
Lattice Absorption (Reststrahlen Absorption)



EM Wave oscillations are coupled to lattice vibrations (phonons), vibrations of the ions in the lattice. Energy is transferred from the EM wave to these lattice vibrations.

This corresponds to "Fundamental Infrared Absorption" in glasses

Rayleigh Scattering



Rayleigh scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}\left(n^2-1\right)^2 \beta_T k_B T_f$$

β_T = isothermal compressibility (at T_f)

T_f = *fictive temperature* (roughly the *softening temperature of glass*) where the liquid structure during the cooling of the fiber is frozen to become the glass structure

Example: Rayleigh scattering limit

What is the attenuation due to Rayleigh scattering at around the λ = 1.55 μm window given that pure silica (SiO_2) has the following properties: T_f = 1730°C (softening temperature); $\beta_T = 7\times10^{-11}$ m^2 N^{-1} (at high temperatures); n = 1.4446 at 1.5 μm.

Solution

We simply calculate the Rayleigh scattering attenuation using

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)^2\beta_T k_B T_f$$

$$\alpha_R \approx \frac{8\pi^3}{3(1.55\times10^{-6})^4}(1.4446^2-1)^2(7\times10^{-11})(1.38\times10^{-23})(1730+273)$$

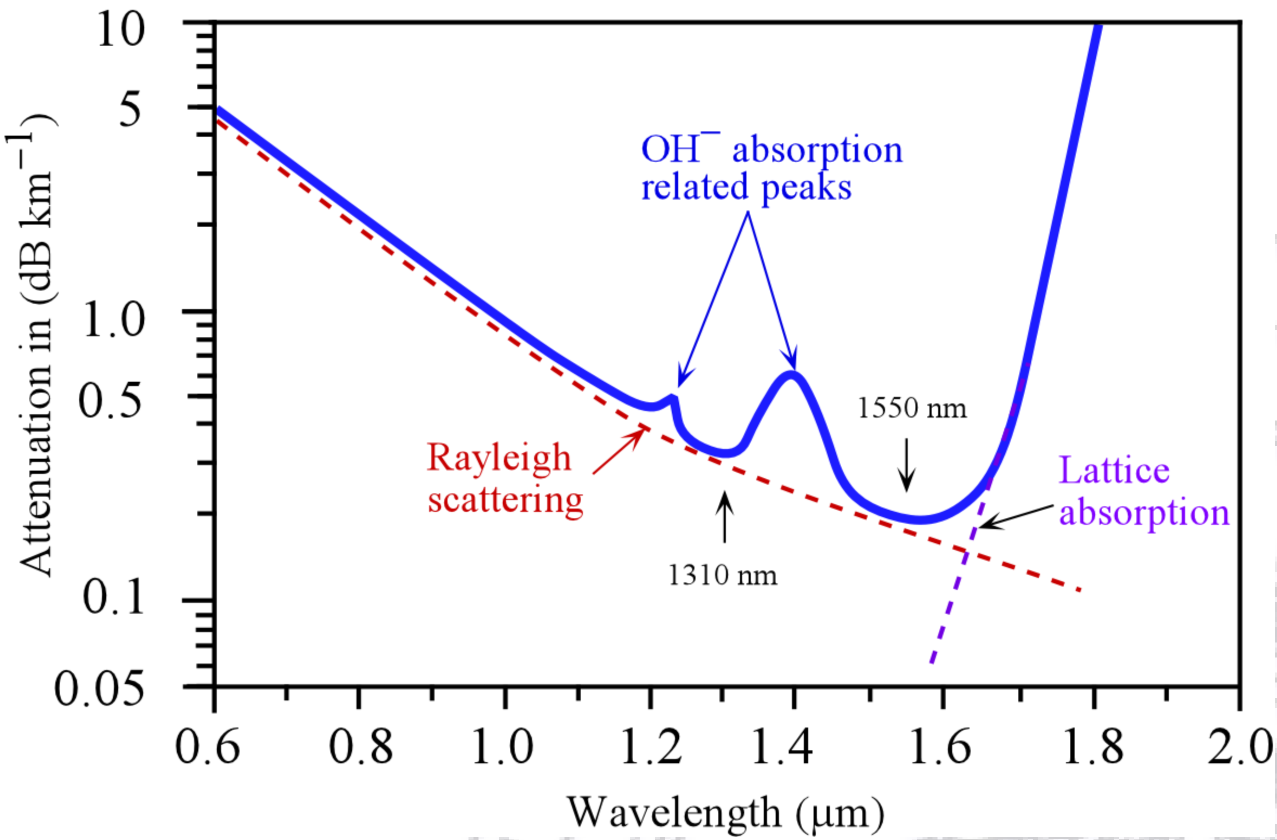
α_R = 3.276×10^{-5} m^{-1} or 3.276×10^{-2} km^{-1}

Attenuation in dB per km is

$\alpha_{dB} = 4.34\alpha_R = (4.34)(3.735\times10^{-2}$ km^{-1}) = **0.142 dB km^{-1}**

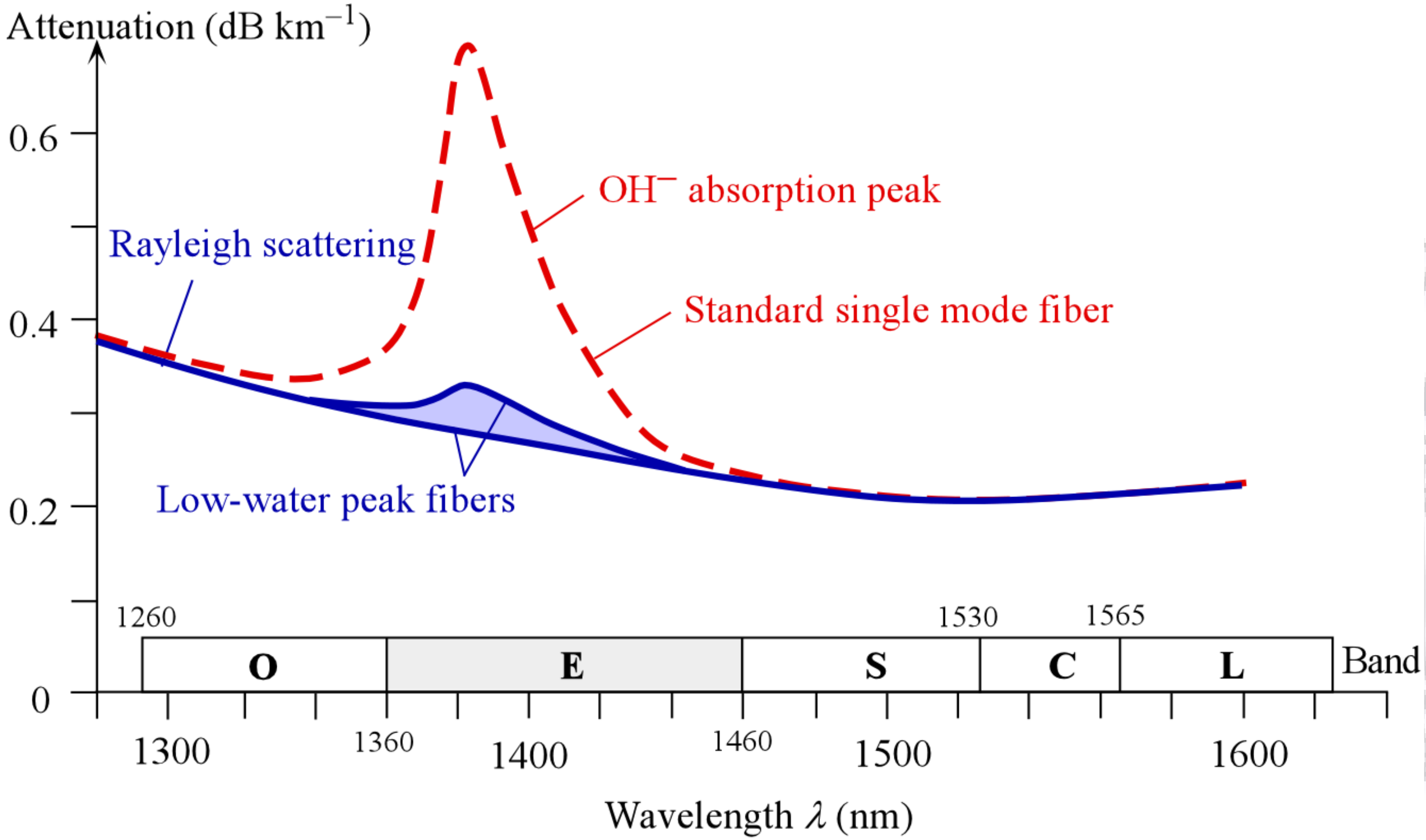
This represents the lowest possible attenuation for a silica glass core fiber at 1.55 μm.

Attenuation in Optical Fibers



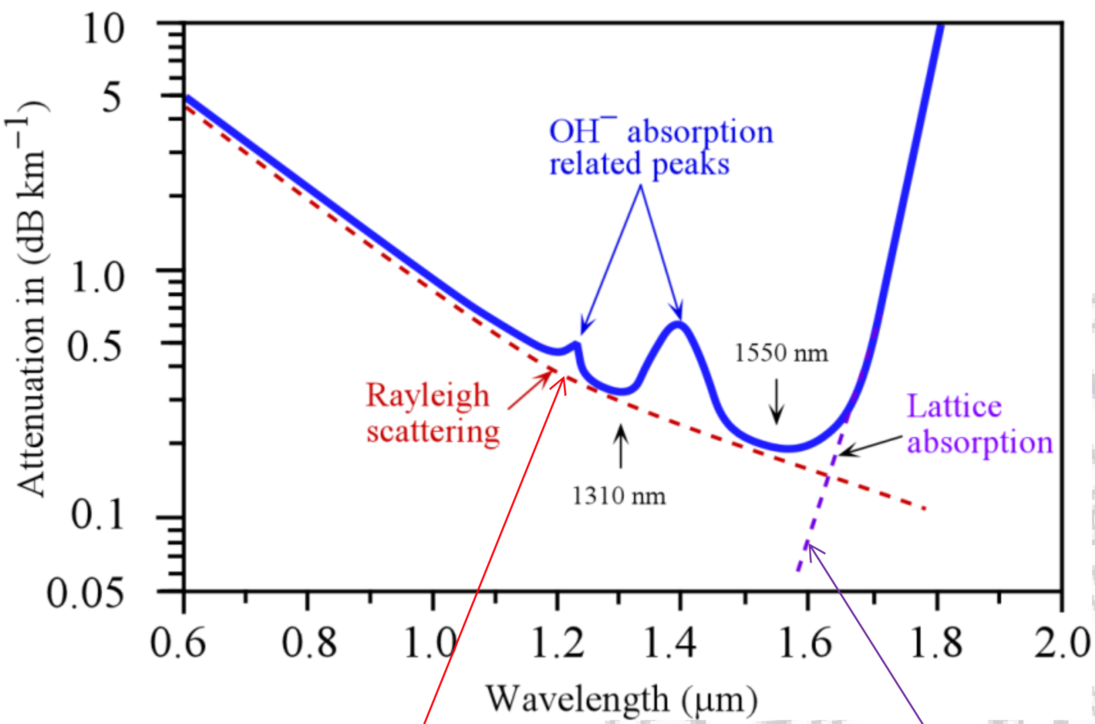
Attenuation vs. wavelength for a standard silica based fiber.

Low-water-peak fiber has no OH^- peak



E-band is available for communications with this fiber

Attenuation in Optical Fibers



$$\alpha_R = \frac{A_R}{\lambda^4}$$

$$\alpha_{\mathrm{FIR}} = A\exp(-B/\lambda)$$

Attenuation

$$\alpha_{\mathrm{FIR}} = A\exp\left(-B/\lambda\right)$$

$$\alpha_R = \frac{A_R}{\lambda^4}$$

A_R in dB km^{-1} µm^4

α_R in dB km^{-1}

λ in µm

Attenuation

Approximate attenuation coefficients for silica-based fibers for use in Equations (7) and (8). Square brackets represent concentration as a fraction. NA = Numerical Aperture. A_R = 0.59 used as reference for pure silica, and represents A_R(silica). Data mainly from K. Tsujikawa *et al*, *Electron. Letts.*, *30*, 351, 1994; *Opt. Fib. Technol. 11*, 319, 2005, H. Hughes, *Telecommunications Cables* (John Wiley and Sons, 1999, and references therein.)

Glass	A_R dB km^{-1} μm^4	Comment	Glass
Silica fiber	0.90	"Rule of thumb"	Silica fiber
SiO_2-GeO_2 core step index fiber	0.63 + 2.06×NA	NA depends on $(n_1^2 - n_2^2)^{1/2}$ and hence on the doping difference.	SiO_2-GeO_2 core step index fiber
SiO_2-GeO_2 core graded index fiber	0.63 + 1.75×NA		SiO_2-GeO_2 core graded index fiber
Silica, SiO_2	0.63	Measured on preforms. Depends on annealing. A_R(Silica) = 0.59 dB km^{-1} μm^4 for annealed.	Silica, SiO_2
65%$SiO_2$35%GeO_2	0.75	On a preform. A_R/A_R(silica) = 1.19	65%$SiO_2$35%GeO_2
$(SiO_2)_{1-x}(GeO_2)_x$	$A_{R(}$silica)×(1 + 0.62x)	x = [GeO_2] = Concentration as a fraction (10% GeO_2, x = 0.1). For preform.	$(SiO_2)_{1-x}(GeO_2)_x$

Rayleigh Scattering

Example: Consider a single mode step index fiber, which has a numerical aperture of 0.14. Predict the expected attenuation at 1.55 μm, and compare your calculation with the reported (measured) value of 0.19 - 0.20 dB km^{-1} for this fiber. Repeat the calculations at 1.31 μm, and compare your values with the reported 0.33 - 0.35 dB km^{-1} values.

Solution

First, we should check the fundamental infrared absorption at 1550 nm.

$$\alpha_{\mathrm{FIR}} = A\exp(-B/\lambda) = 7.8\times10^{11}\exp[-(48.5)/(1.55)]$$

$$= 0.020 \text{ dB km}^{-1}, \text{ very small}$$

Rayleigh Scattering

Solution

Rayleigh scattering at 1550 nm, the simplest equation with $A_R = 0.9$ dB km^{-1} μm^4, gives

$$\boldsymbol{\alpha_R = A_R/\lambda^4 = (0.90 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.55\ \mu\text{m})^4 = 0.178 \text{ dB km}^{-1}}$$

This equation is basically a rule of thumb. The total attenuation is then

$$\alpha_\mathrm{R} + \alpha_\mathrm{FIR} = 0.178 + 0.02 = \mathbf{0.198\ dB\ km^{-1}}.$$

The current fiber has NA = 0.14.

$\therefore$ $A_R = 0.63 + 2.06 \times \mathrm{NA} = 0.63 + 2.06 \times 0.14 = 0.918$ dB km^{-1} μm^4

i.e. $\alpha_R = A_R/\lambda^4 = (0.918 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.55\ \mu\text{m})^4 = 0.159$ dB km^{-1},

which gives a total attenuation of 0.159 + 0.020 or **0.179 dB km^{-1}**.

Rayleigh Scattering

Solution

We can repeat the above calculations at $\lambda = 1.31$ μm. However, **we do not need** to add α_{FIR}.

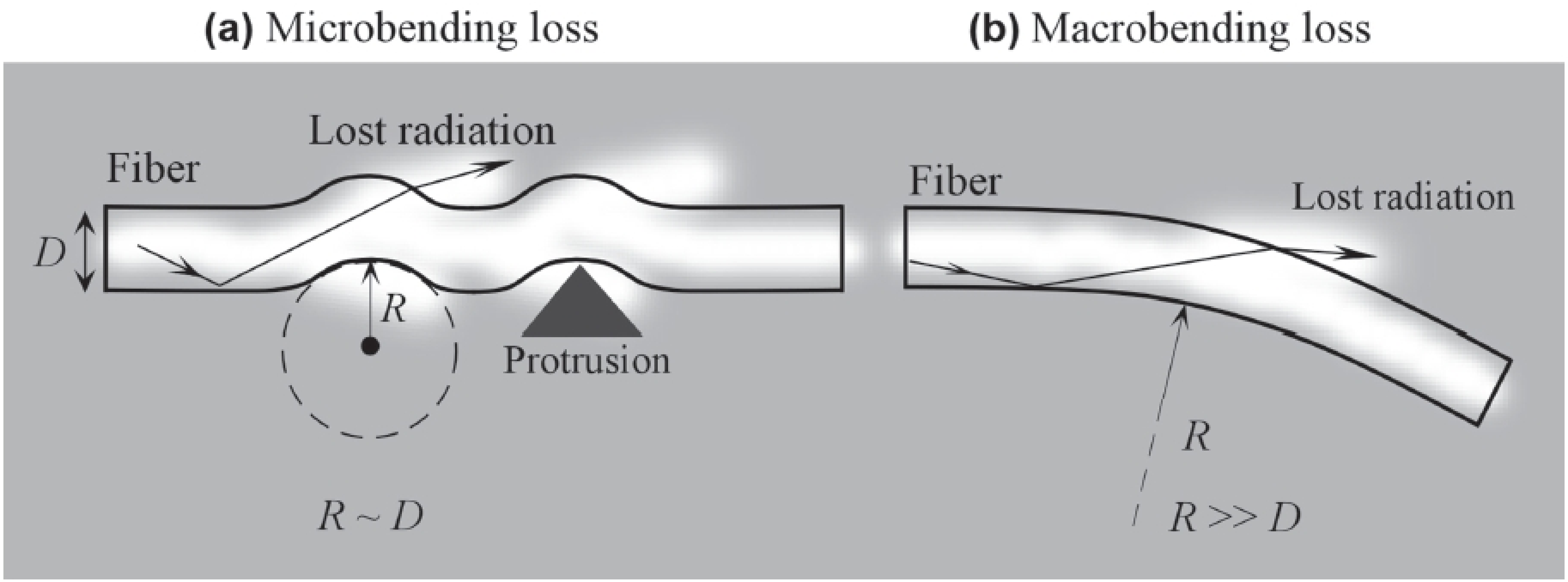
$$\alpha_R = A_R/\lambda^4 = (0.90 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.31\ \mu\text{m})^4 = \mathbf{0.306\ dB\ km^{-1}}$$

and using the NA based for A_R,

$$\alpha_R = A_R/\lambda^4 = (0.918 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.31\ \mu\text{m})^4 = \mathbf{0.312\ dB\ km^{-1}}.$$

Both close to the measured value.

Bending Loss



Definitions of (a) microbending and (b) macrobending loss and the definition of the radius of curvature, *R*. (A schematic illustration only.) The propagating mode in the fiber is shown as white painted area. Some radiation is lost in the region where the fiber is bent. *D* is the fiber diameter, including the cladding.

Bending Loss

Microbending loss

the radius of curvature *R* of the bend is sharp
bend radius is comparable to the diameter of the fiber

Typically microbending losses are significant when the radius of curvature of the bend is less than 0.1 – 1 mm.

They can arise from careless or poor cabling of the fiber or even in flaws in manufacturing that result in variations in the fiber geometry over small distances.

Bending Loss

Macrobending losses

Losses that arise when the bend is much larger than the fiber size

Typically much grater than 1 mm

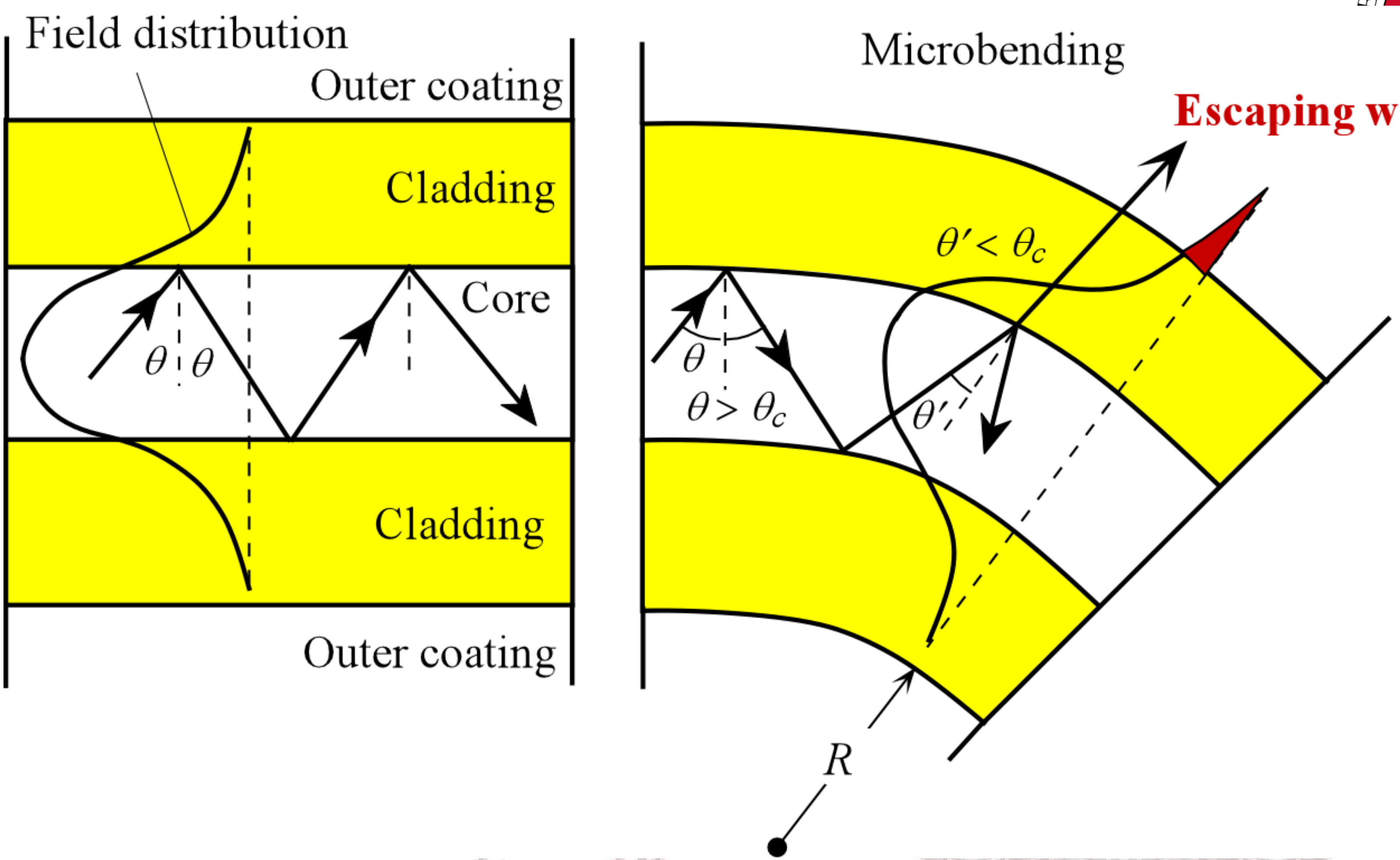
Typically occur when the fiber is bent during the installation of a fiber optic link such as turning the fiber around a corner.

There is no simple precise and sharp boundary line between microbending and macrobending loss definitions.

Both losses essentially result from changes in the waveguide geometry and properties as the fiber is subjected to external forces that bend the fiber.

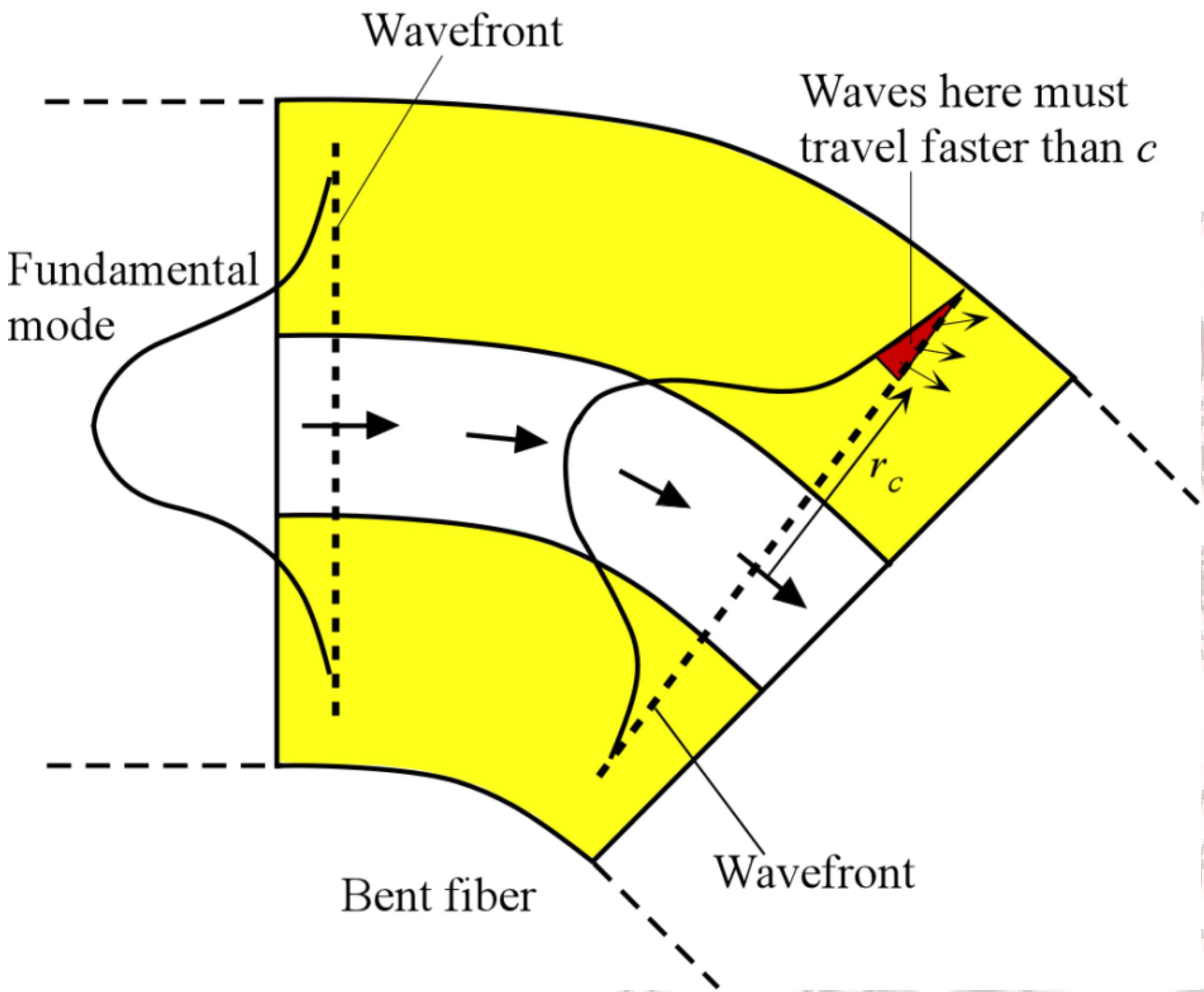
Typically, macrobending loss crosses over into microbending loss when the radius of curvature becomes less than a few millimeters.

Bending Loss



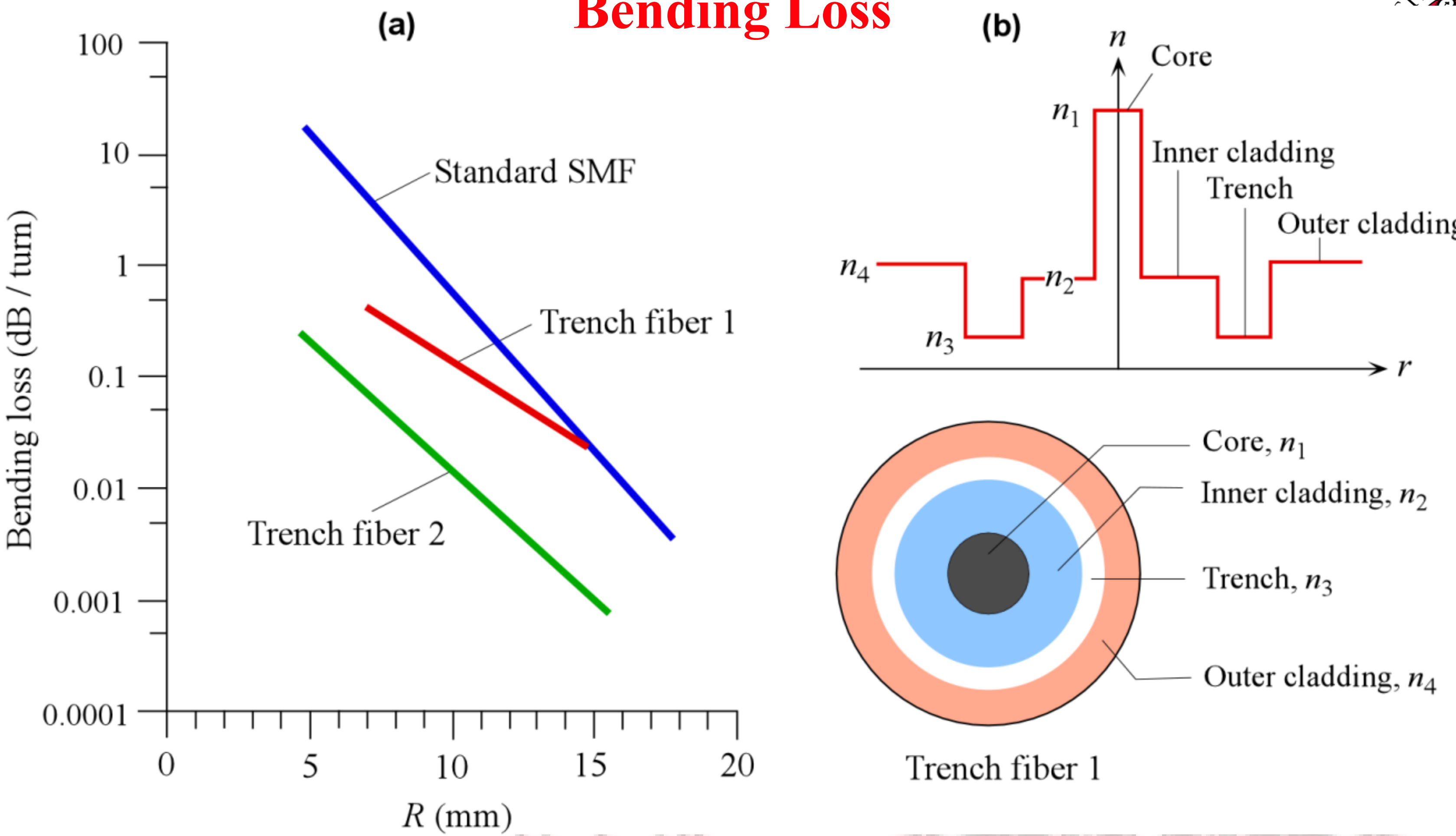
Sharp bends change the local waveguide geometry that can lead to waves escaping. The zigzagging ray suddenly finds itself with an incidence angle smaller than θ' that gives rise to either a transmitted wave, or to a greater cladding penetration; the field reaches the outside medium and some light energy is lost.

Bending Loss



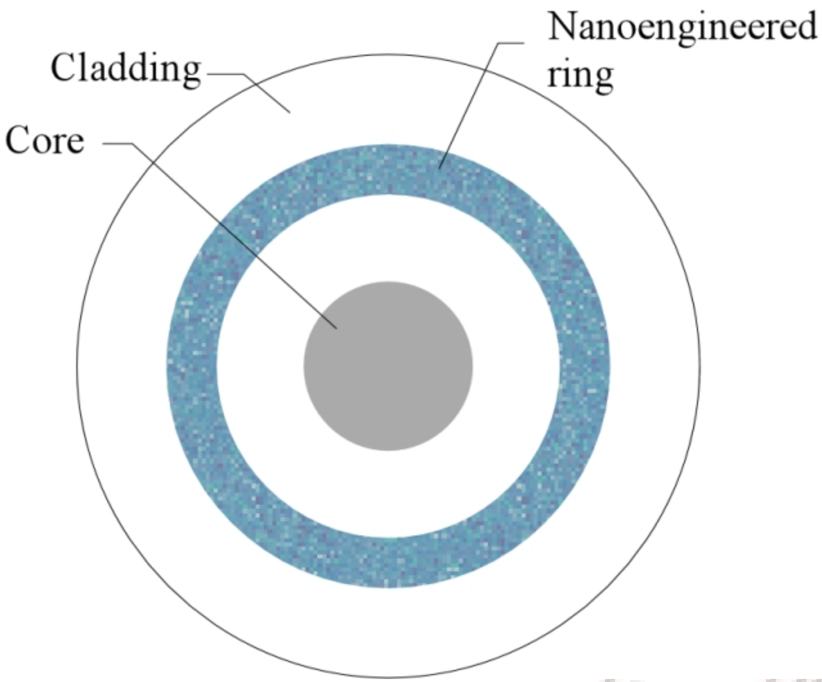
When a fiber is bent sharply, the propagating wavefront along the straight fiber cannot bend around and continue as a wavefront because a portion of it (black shaded) beyond the critical radial distance r_c must travel faster than the speed of light in vacuum. This portion is lost in the cladding- radiated away.

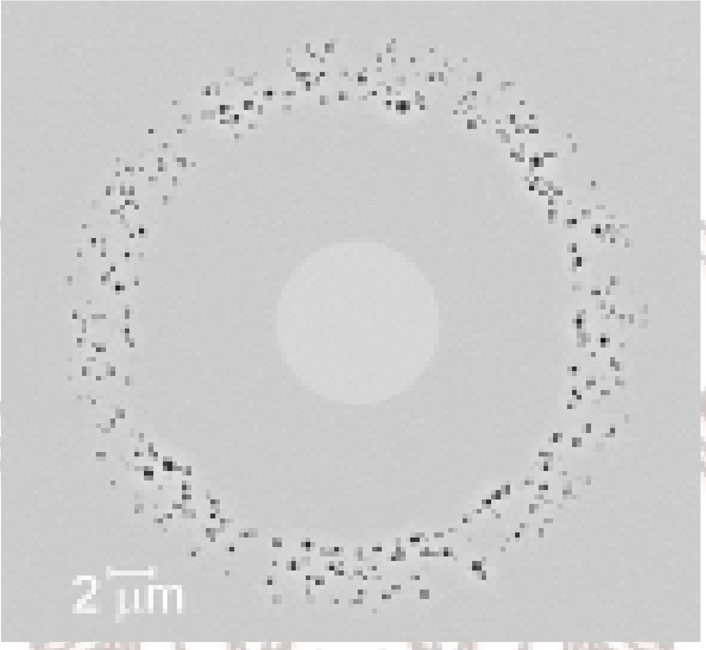
Bending Loss



(a) Bending loss in dB per turn of fiber for three types of fibers, standard single mode, and two trench fibers, around 1.55 - 1.65 µm. (b) The index profile for the trench fiber 1 in (a), and a schematic view of the fiber cross section. Experimental data have been used to generate the plots have been combined from various sources. (Standard fiber, M.-J. Li *et al*. *J. Light Wave Technol.*, 27, 376, 2009; trench fiber 1, K. Himeno *et al*, *J. Light Wave Technol.*, 23, 3494, 2005, trench fiber 2, L.-A. de Montmorillon, *et al.* "Bend-Optimized G.652D Compatible Trench-Assisted Single-Mode Fibers", *Proceedings of the 55th IWCS/Focus*, pp. 342-347, November, 2006.)

Bend Insensitive Fibers





Left, the basic structure of bend insensitive fiber with a nanoengineered ring in the cladding. Right, an SEM picture of the cross section of a nanoengineered fiber with reduced bending losses. (Courtesy of Ming-Jun Li, Corning Inc. For more information see US Patent 8,055.110, 2011)

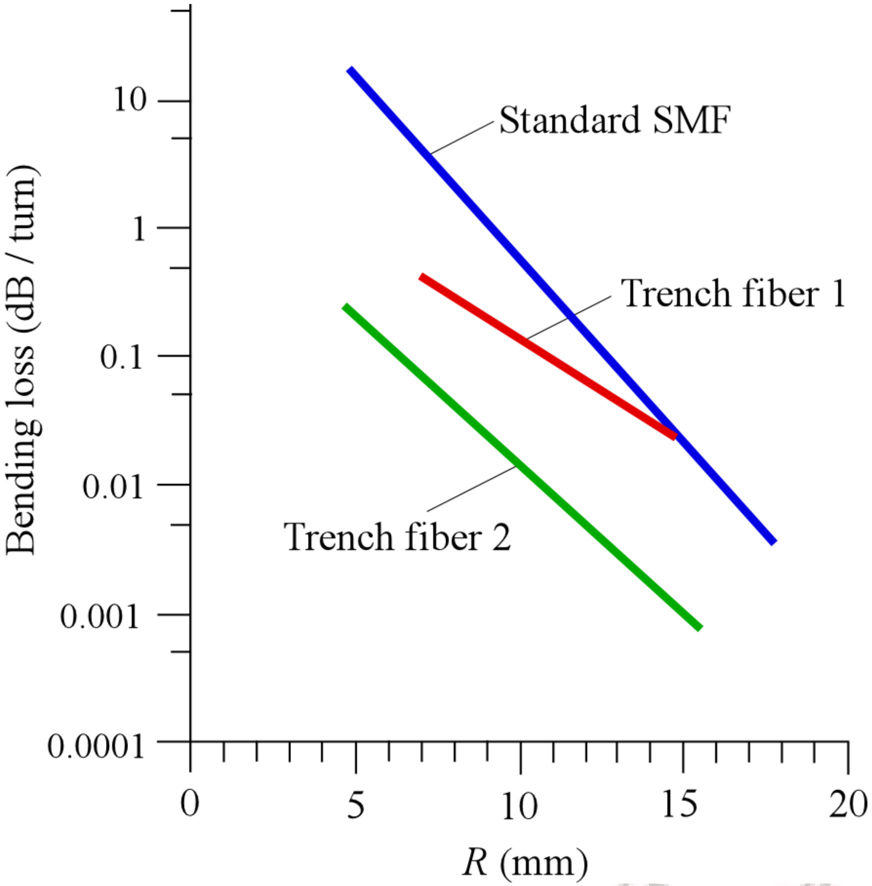
Bending Loss in Fibers
General Description

$$\alpha_B = A\exp(-R/R_c)$$

A and R_c are constants

Bending Loss

$$\alpha_B = A\exp(-R/R_c)$$

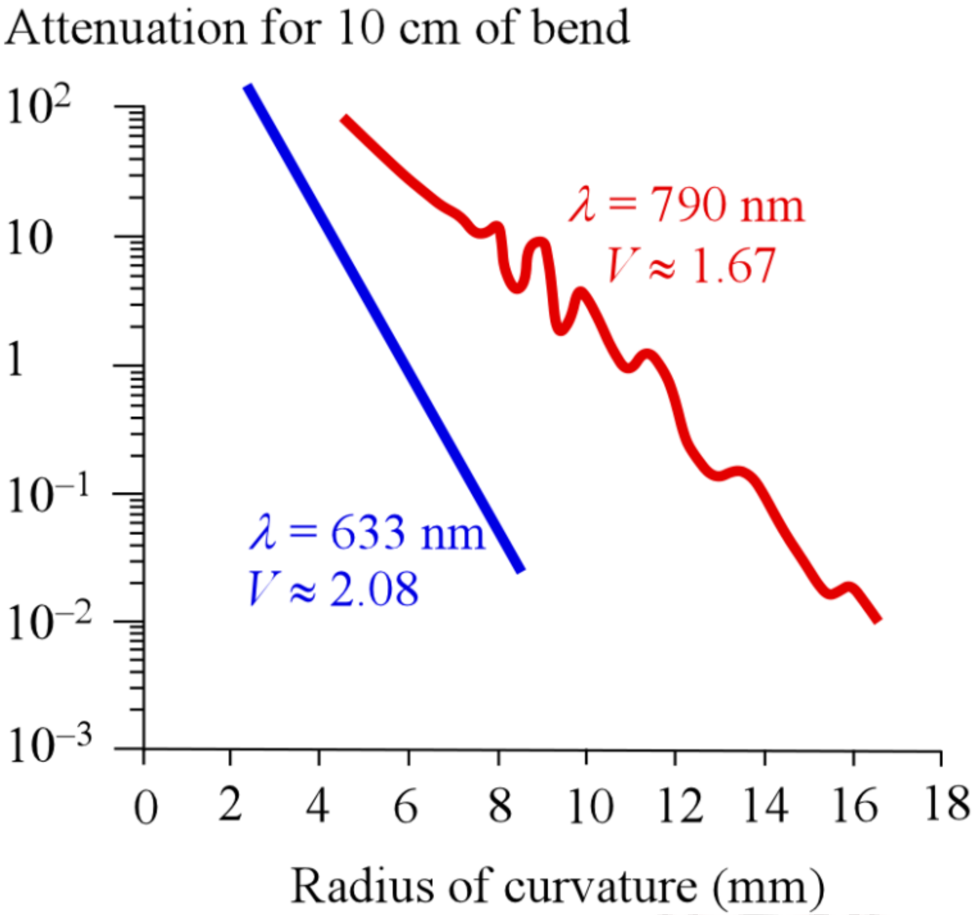


where A and R_c are constants

For single mode fibers, a quantity called a **MAC-value**, or a **MAC-number**, N_{MAC}, has been used to characterize the bending loss. N_{MAC} is defined by

$$N_{\mathrm{MAC}} = \frac{\text{Mode field diameter}}{\text{Cut-off wavelength}} = \frac{\mathrm{MFD}}{\lambda_c}$$

Microbending Loss



$$\alpha \propto \exp\left(-\frac{R}{R_c}\right) \propto \exp\left(-\frac{R}{\Delta^{-3/2}}\right)$$

Measured microbending loss for a 10 cm fiber bent by different amounts of radius of curvature R. Single mode fiber with a core diameter of 3.9 μm, cladding radius 48 μm, $\Delta = 0.00275$, NA ≈ 0.10, $V \approx 1.67$ and 2.08. Data extracted from A.J. Harris and P.F. Castle, "Bend Loss Measurements on High Aperture Single-Mode Fibers as a Function of Wavelength and Bend Radius", *IEEE J. Light Wave Technology,* Vol. LT14, 34, 1986, and repotted with a smoothed curve; see original article for the discussion of peaks in α_B vs. R at 790 nm).

Bending Loss in SMF

$$\alpha_B = A\exp(-R/R_c)$$

MAC-value, or a MAC-number, N_{MAC}

$$N_{MAC} = \frac{\text{Mode field diameter}}{\text{Cut-off wavelength}} = \frac{\text{MFD}}{\lambda_c}$$

NOTES

α_B increases with increasing MFD

α_B decreasing cut-off wavelength.

α_B increases exponentially with the MAC-number

It is not a universal function, and different types of fibers, with very different refractive index profiles, can exhibit different α_B–N_{MAC} behavior.

N_{MAC} values are typically in the range 6 – 8

Standard Fibers and Bending Loss

$$\alpha_B = A\exp(-R/R_c)$$

OM1 MMF for use at 850 and 1310

OM1

"Maximum bending (macrobending) loss of 0.5 dB when the fiber is wound 100 turns with a radius 75 mm *i.e.* a bending loss of 0.005 dB/turn for a bend radius of 75 mm."

Bend insensitive fibers have been designed to have lower bend losses. For example, some fiber manufacturers specify the allowed bend radius for a given level of attenuation at a certain wavelength (*e.g.* 1310 nm)

Bending Loss Example

Example: Experiments on a standard SMF operating around 1550 nm have shown that the bending loss is 0.124 dB/turn when the bend radius is 12.5 mm and 15.0 dB/turn when the bend radius is 5.0 mm. What is the loss at a bend radius of 10 mm?

Solution

Apply

$$\boldsymbol{\alpha_B = A\exp(-R/R_c)}$$

$$0.124 \text{ dB/turn} = A\exp[-(12.5 \text{ mm})/R_c]$$

$$15.0 \text{ dB/turn} = A\exp[-(5.0 \text{ mm})/R_c]$$

Solve for A and R_c. Dividing the first by the second and separating out R_c we find,

$$R_c = (12.5 \text{ mm} - 5.0 \text{ mm}) / \ln(15.0/0.124) = 1.56 \text{ mm}$$

and $$A = (15.0 \text{ dB/turn}) \exp[(5.0 \text{ mm})/(1.56 \text{ mm}] = 370 \text{ dB/turn}$$

At R = 10 mm

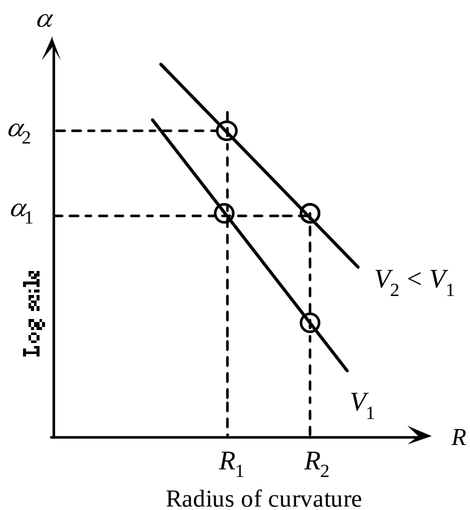
$$\boldsymbol{\alpha_B = A\exp(-R/R_c) = (370)\exp[-(10 \text{ mm})/(1.56 \text{ mm})]}$$

$$\boldsymbol{= 0.61 \text{ dB/turn.}}$$

The experimental value is also **0.61 dB/turn** to within two decimals.

Example: Microbending loss It is found that for a single mode fiber with a cut-off wavelength λ_c = 1180 nm, operating at 1300 nm, the microbending loss reaches 1 dB m^{-1} when the radius of curvature of the bend is roughly 6 mm for Δ = 0.00825, 12 mm for Δ = 0.00550 and 35 mm for Δ = 0.00275. Explain these findings.

Solution:



Microbending loss α decreases sharply with the bend radius R. (Schematic only.)

Given $\alpha = \alpha_1$, R *increases* from R_1 to R_2 when V *decreases* from V_1 to V_2.

<u>Expected</u> $R\uparrow$ with $V\downarrow$

Equivalently at one $R = R_1$, $\alpha\uparrow$ with $V\downarrow$

We can *generalize* by noting that the penetration depth into the cladding $\delta \propto 1/V$.

<u>Expected</u> $R\uparrow$ with $\delta\uparrow$

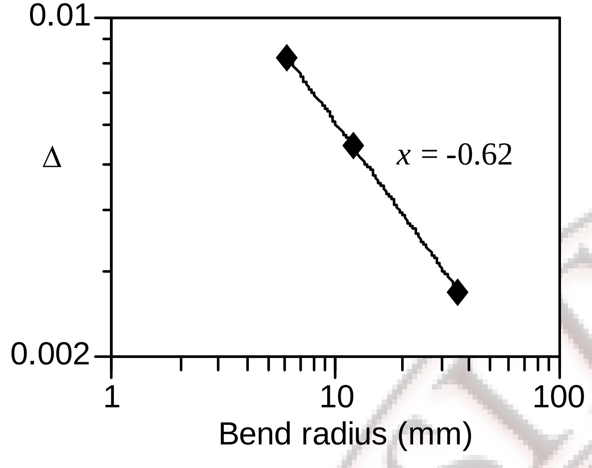
Equivalently at one $R = R_1$, $\alpha\uparrow$ with $\delta\uparrow$

Thus, microbending loss α gets worse when penetration δ into cladding increases; intuitively correct. Experiments show that for a given $\alpha = \alpha_1$, R increases with decreasing Δ. We know from basic optics $\delta\uparrow$ with $\Delta\downarrow$ i.e. δ increases with decreasing Δ.

Thus <u>expected</u> $R\uparrow$ with $\delta\uparrow$ with $\Delta\downarrow$ as observed

Example: Microbending loss It is found that for a single mode fiber with a cut-off wavelength λ_c = 1180 nm, operating at 1300 nm, the microbending loss reaches 1 dB m^{-1} when the radius of curvature of the bend is roughly 6 mm for $\Delta = 0.00825$, 12 mm for $\Delta = 0.00550$ and 35 mm for $\Delta = 0.00275$. Explain these findings.

Solution:



Log-log plot of the results of experiments on Δ vs. bend radius R for 1 dB/m microbending loss

$$\alpha \propto \exp\left(-\frac{R}{R_c}\right) \propto \exp\left(-\frac{R}{\Delta^{-3/2}}\right)$$

R_c is a constant ("a critical radius type of constant") that is proportional to $\Delta^{-3/2}$. Taking logs,

$\ln\alpha = -\Delta^{3/2}R$ + constant

We are interested in the Δ vs R behavior at a constant α. We can lump the constant into $\ln\alpha$ and obtain,

$\Delta \propto R^{-2/3}$

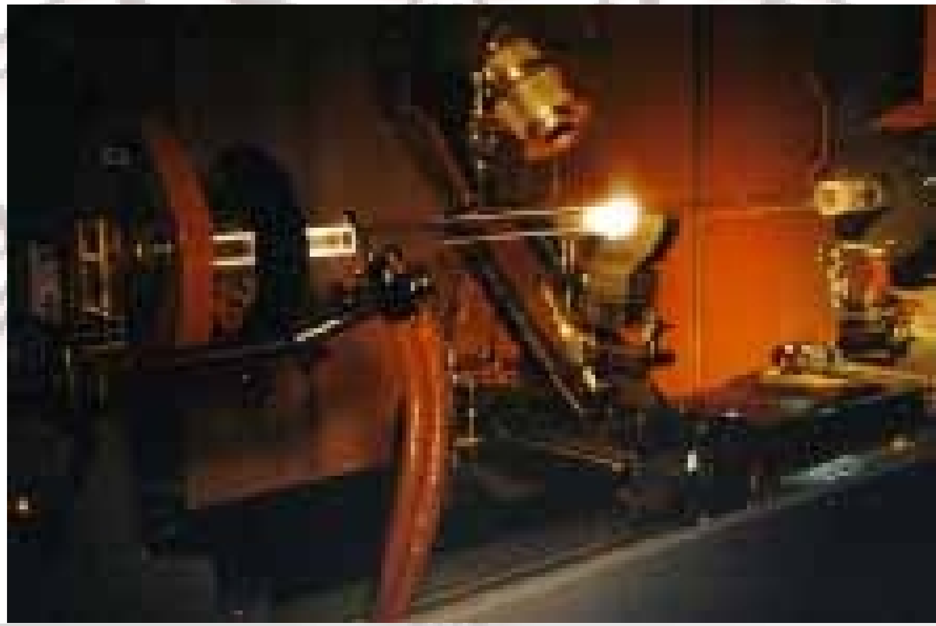
The plot in the figure gives an index close this value.

Methods of fabrication

- Modified Chemical Vapor Deposition (MCVD)
- Outside Vapor Deposition (OVD)
- Vapor Axial Deposition (VAD)

Modified Chemical Vapor Deposition (MCVD)

- A hollow, rotating glass tube is heated with a torch
- Chemicals inside the tube precipitate to form *soot*
- Rod is collapsed to crate a *preform*
- Preform is stretched in a *drawing tower* to form a single fiber up to 10 km long
  - Image from thefoa.org



Modified Chemical Vapor Deposition (MCVD)

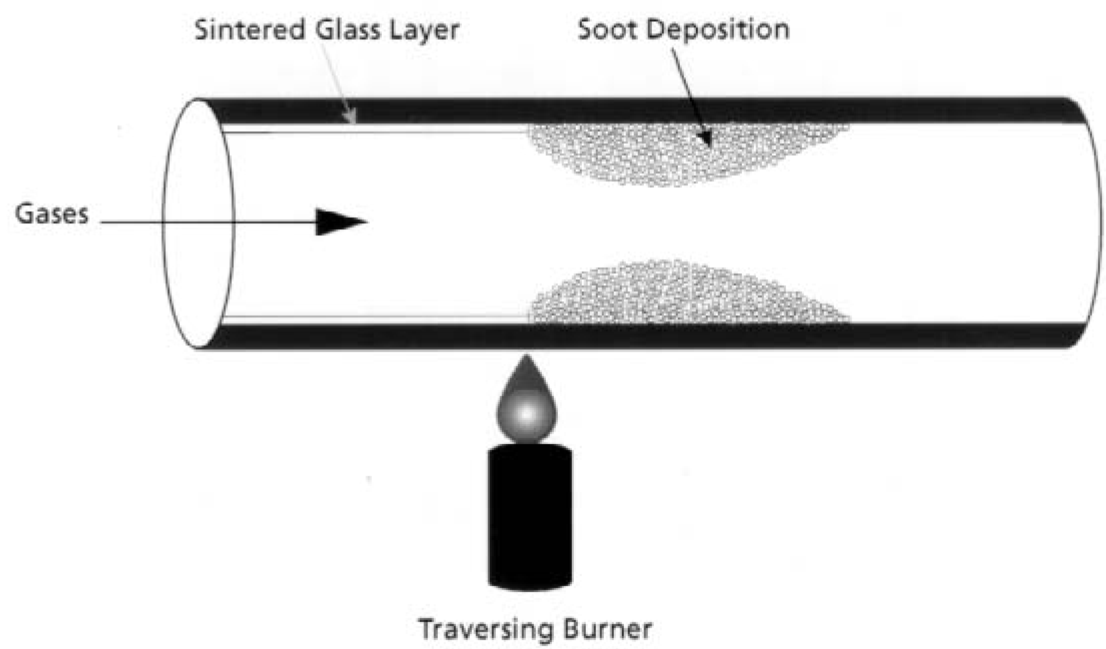


Figure 2 PREFORM FABRICATION

Outside vapor Deposition (OVD)

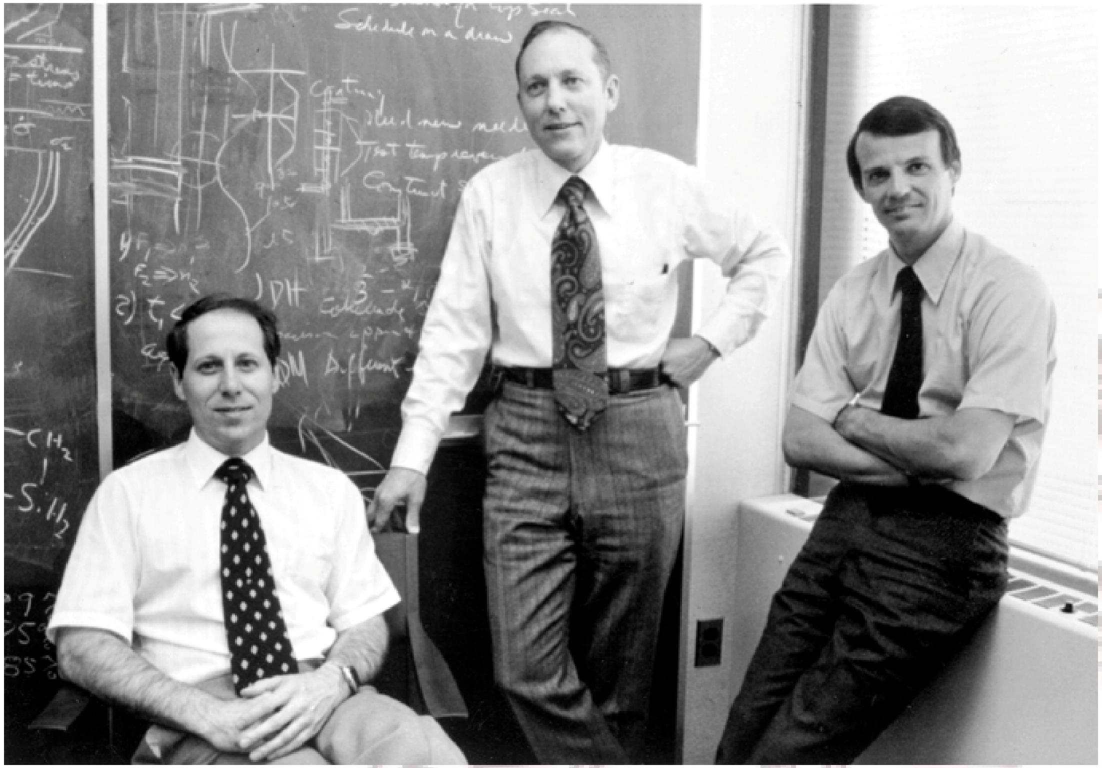
Main reactions are

Silicon tetrachloride and oxygen produces silica

$$SiCl_4(gas) + O_2(gas) \rightarrow SiO_2(solid) + 2Cl_2(gas)$$

Germanium tetrachloride and oxygen produces germania

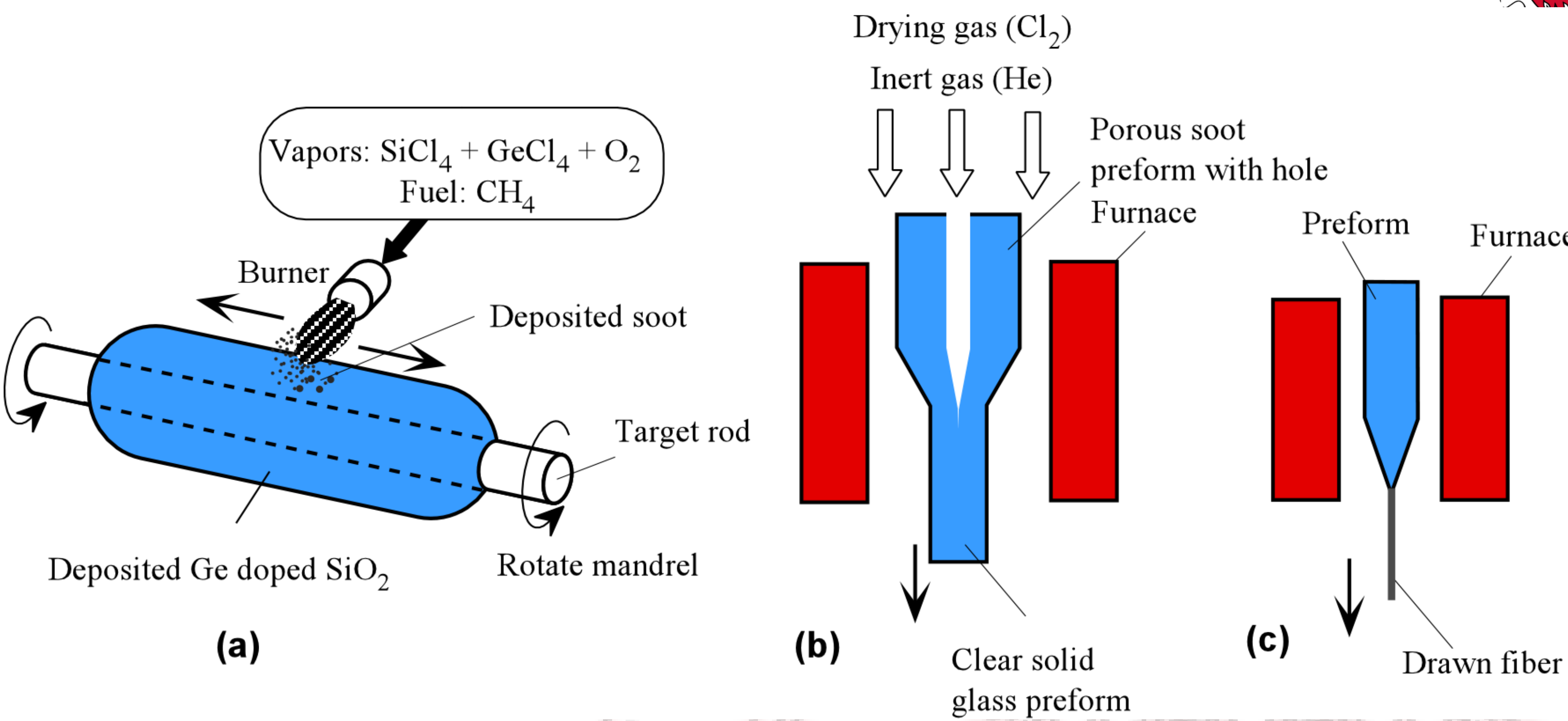
$$GeCl_4(gas) + O_2(gas) \rightarrow GeO_2(solid) + 2Cl_2(gas)$$



Donald Keck, Bob Maurer and Peter Schultz (left to right) at Corning shortly after announcing the first low loss optical fibers made in 1970. Keck, Maurer and Schultz developed the outside vapor deposition (OVD) method for the fabrication of preforms that are used in drawing fibers with low losses. Their OVD was based on Franklin Hyde’s vapor deposition process earlier at Corning in 1930s. OVD is still used today at Corning in manufacturing low loss fibers. (Courtesy of Corning)

Peter Schultz making a germania-doped multimode fiber preform using the outside vapor deposition (OVD) process circa 1972 at Corning. Soot is deposited layer by layer on a thin bait rod rotating and translating in front of the flame hydrolysis burner. The first fibers made by the OVD at that time had an attenuation of 4 dB/km, which was among the lowest, and below what Charles Kao thought was needed for optical fiber communications, 20 dB/km (Courtesy of Peter Schultz.)

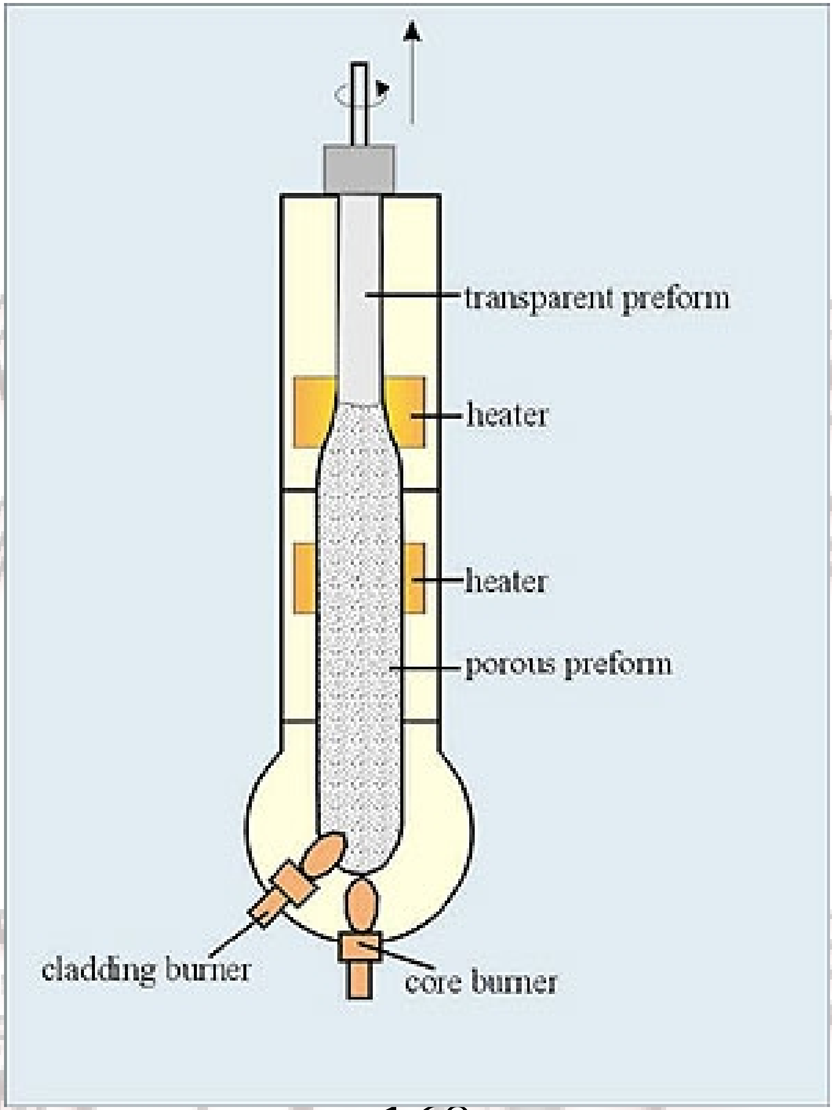
Outside Vapor Deposition (OVD)



Schematic illustration of OVD and the preform preparation for fiber drawing. (a) Reaction of gases in the burner flame produces glass soot that deposits on to the outside surface of the mandrel. (b) The mandrel is removed and the hollow porous soot preform is consolidated; the soot particles are sintered, fused, together to form a clear glass rod. (c) The consolidated glass rod is used as a preform in fiber drawing.

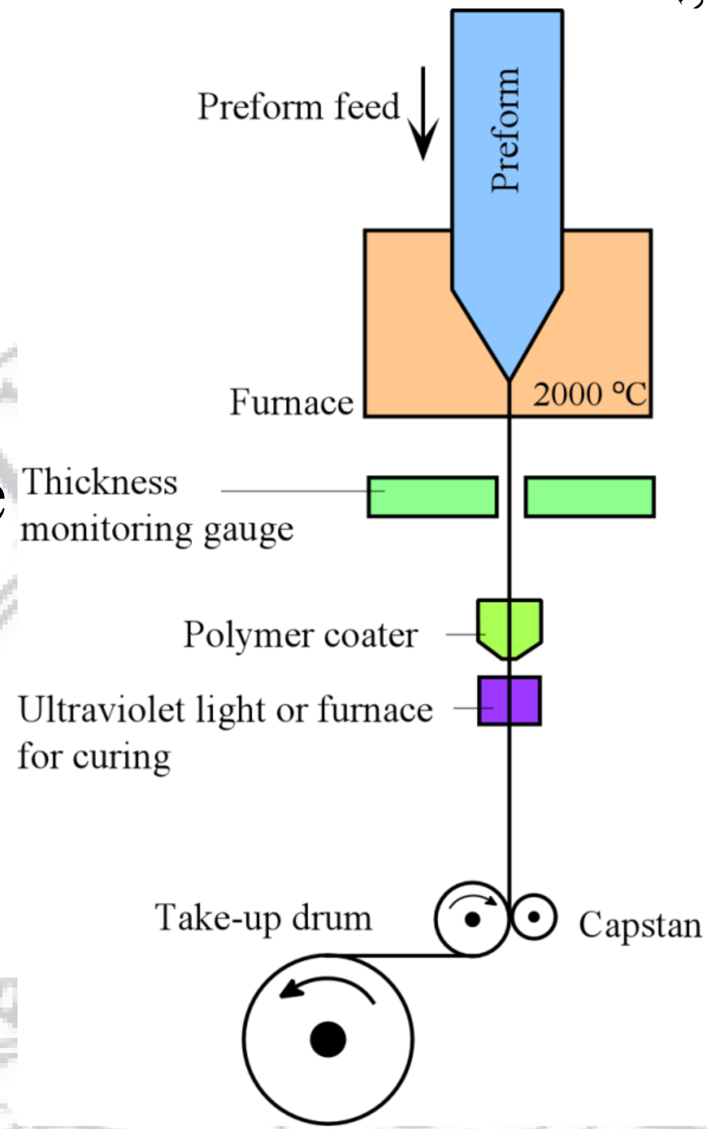
Vapor Axial Deposition (VAD)

- Preform is fabricated continuously
- When the preform is long enough, it goes directly to the drawing tower
  - Image from csrg.ch.pw.edu.pl

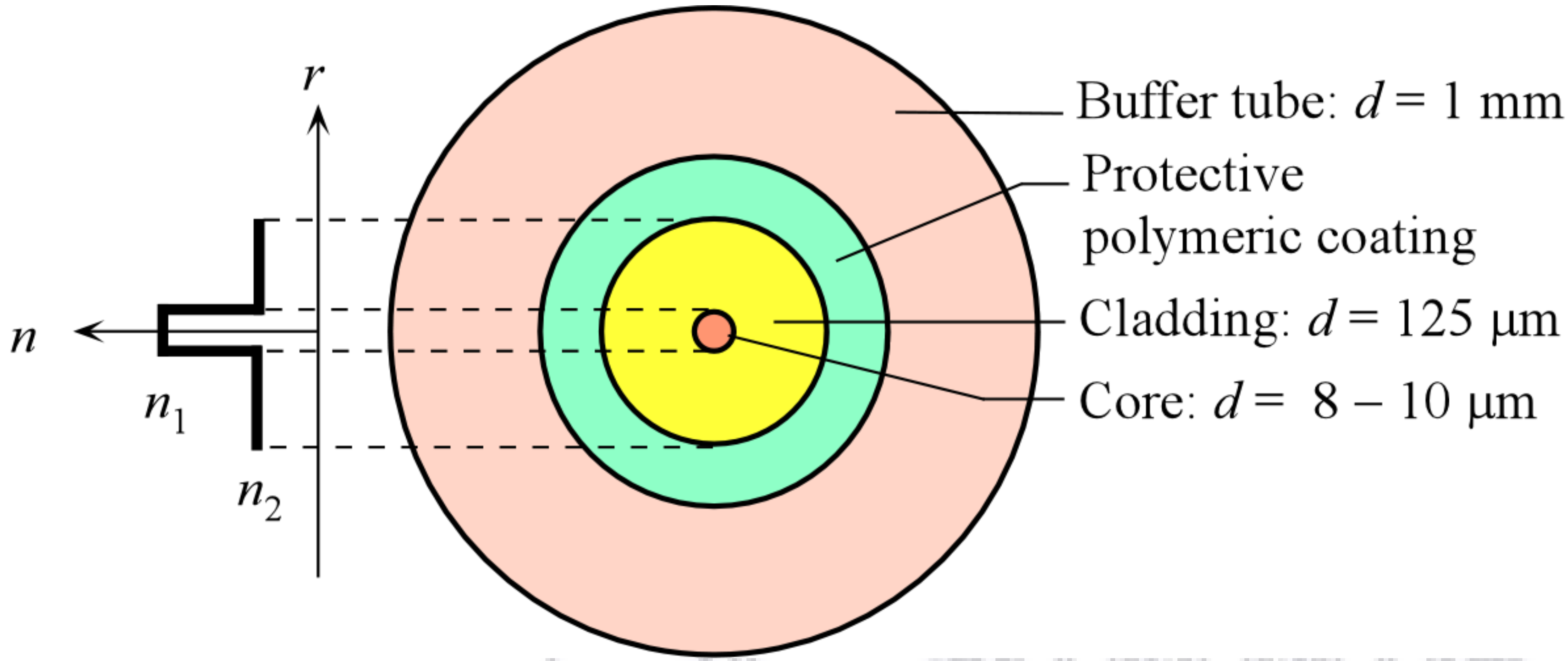


Manufacture of Optical Fibers

- The fiber is drawn from the preform and then coated with a protective coating

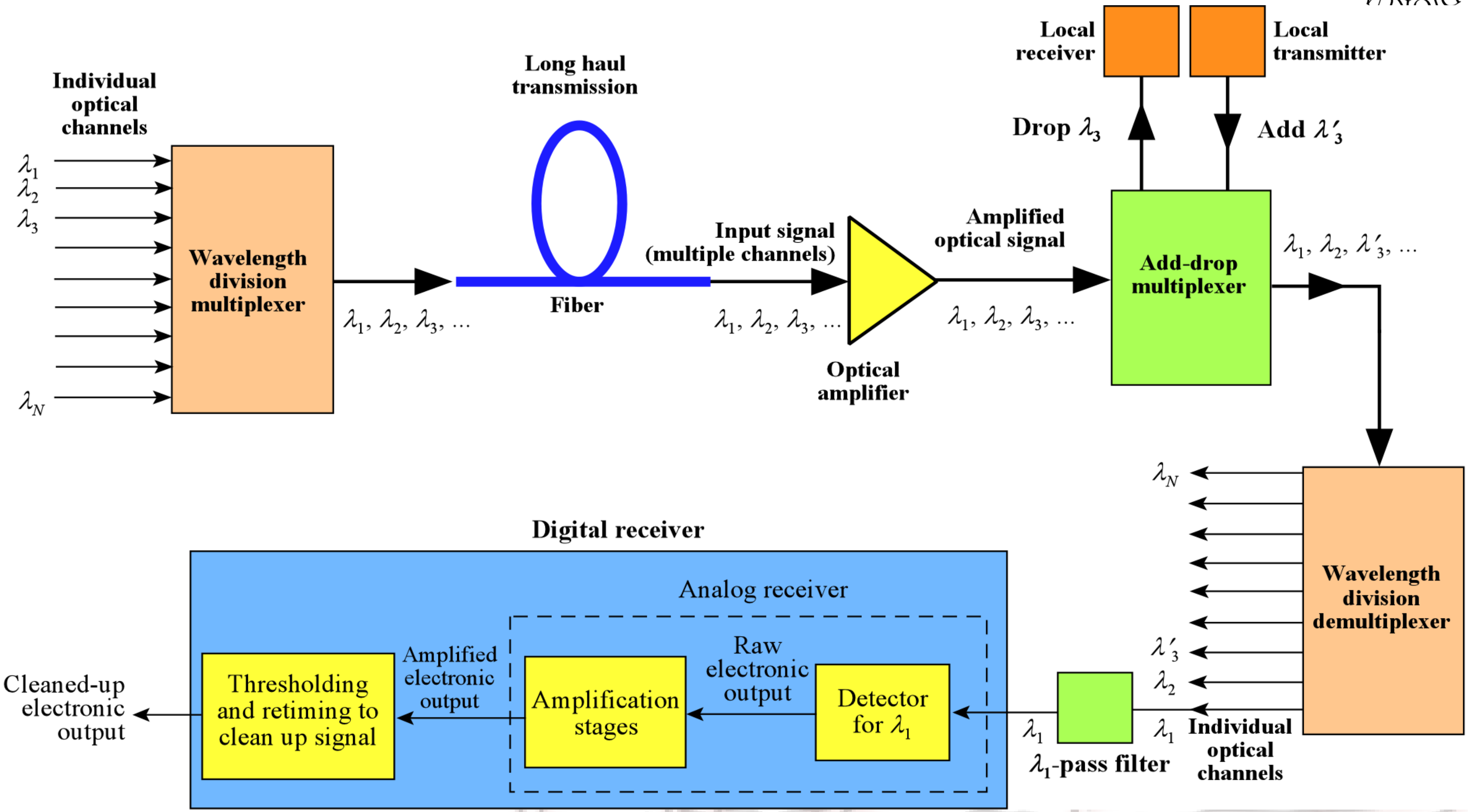


Typical Cross Section of an Optical Fiber



The cross section of a typical single-mode fiber with a tight buffer tube. (d = diameter)

WAVELENGTH DIVISION MULTIPLEXING: WDM



WAVELENGTH DIVISION MULTIPLEXING: WDM

- Generation of different wavelengths of light each with a narrow spectrum to avoid any overlap in wavelengths.
- Modulation of light without wavelength distortion; *i.e.* without *chirping* (variation in the frequency of light due to modulation).
- Efficient coupling of different wavelengths into a single transmission medium.
- Optical amplification of all the wavelengths by an amount that compensates for attenuation in the transmission medium, which depends on the wavelength.
- Dropping and adding channels when necessary during transmission.
- Demultiplexing the wavelengths into individual channels at the receiving end.
- Detecting the signal in each channel. To achieve an acceptable bandwidth, we need to dispersion manage the fiber (use dispersion compensation fibers), and to reduce cross-talk and unwanted signals, we have to use optical filters to block or pass the required wavelengths. We need various optical components to connect the devices together and implement the whole system.

WAVELENGTH DIVISION MULTIPLEXING: WDM

If $\Delta\upsilon < 200$ GHz then WDM is called **DENSE WAVELENGTH DIVISION MULTIPLEXING and denoted** as DWDM.

At present, DWDM stands typically at 100 GHz separated channels which is equivalent to a wavelength separation of 0.8 nm.

DWDM imposes stringent requirements on lasers and modulators used for generating the optical signals.

It is not possible to tolerate even slight shifts in the optical signal frequency when channels are spaced closely. As the channel spacing becomes narrower as in DWDM, one also encounters various other problems not previously present. For example, any nonlinearity in a component carrying the channels can produce intermodulation between the channels; an undesirable effect. Thus, the total optical power must be kept below the onset of nonlinearity in the fiber and optical amplifiers within the WDM system.

Passive Optical Couplers

- Passive devices operate completely in the optical domain to split and combine light streams.
- They include N × N couplers (with $N \geq 2$), power splitters, power taps, and star couplers.
- They can be fabricated either from optical fibers or by means of planar optical waveguides using material such as LiNbO3, InP, silica, silicon oxynitride, or various polymers.

- Reciprocal device

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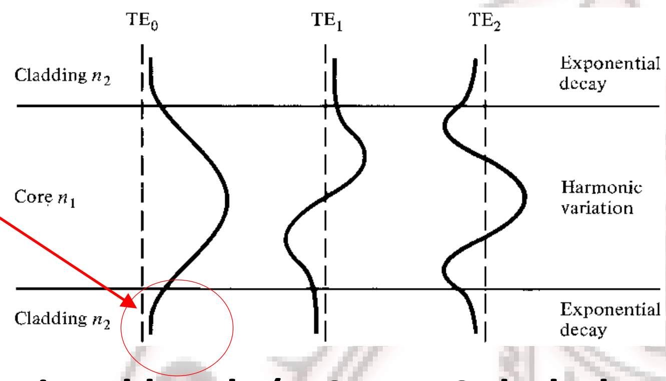
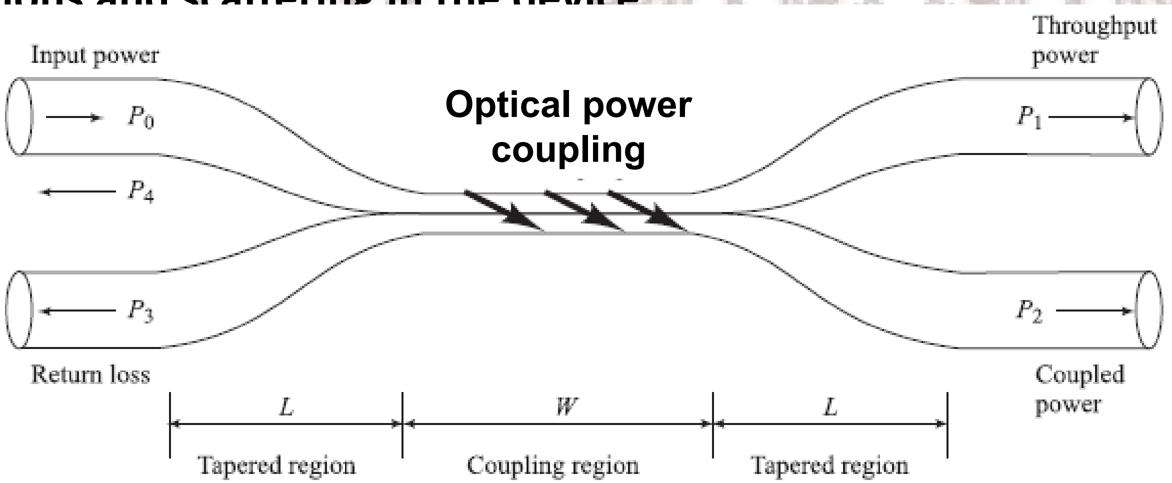
The 2 X 2 Fiber Coupler

- P_0 is the input power, P_1 is the throughout power, and P_2 is the power coupled into the second fiber.

The evanescent tail from one fiber core couples into another closely spaced fiber core



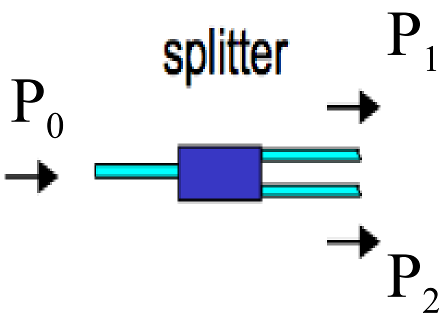
- P_3 and P_4 are extremely low signal levels (-50 to -70 dB below the input level) resulting from backward reflections and scattering in the device



Performance of an Optical

$$\text{Splitting ratio} = \left(\frac{P_2}{P_1 + P_2}\right) \times 100\%$$

- **3-dB coupler: $P_1 = P_2 = 0.5\, P_0$**
- **Tap coupler: $P_2 = 0.005\, P_0$ (- 23 dB)**



$$\text{Excess loss} = 10\log\left(\frac{P_0}{P_1 + P_2}\right)$$

$$\text{Insertion loss} = 10\log\left(\frac{P_i}{P_j}\right)$$

$$\text{Return loss} = 10\log\left(\frac{P_3}{P_0}\right)$$

Couplers examples

1x2 coupler

6x6 coupler

Example Coupler Performance

Example 10.4 A 2 × 2 biconical tapered fiber coupler has an input optical power level of $P_0 = 200\ \mu W$. The output powers at the other three ports are $P_1 = 90\ \mu W$, $P_2 = 85\ \mu W$, and $P_3 = 6.3$ nW. What are the coupling ratio, excess loss, insertion losses, and return loss for this coupler?

Solution: From Eq. 10.4, the coupling ratio is

$$\text{Coupling ratio} = \left(\frac{85}{90+85}\right) \times 100\% = 48.6\%$$

From Eq. 10.5, the excess loss is

$$\text{Excess loss} = 10 \log\left(\frac{200}{90+85}\right) = 0.58 \text{ dB}$$

Using Eq. 10.6, the insertion losses are

$$\text{Insertion loss (port 0 to port 1)} = 10 \log\left(\frac{200}{90}\right) = 3.47 \text{ dB}$$

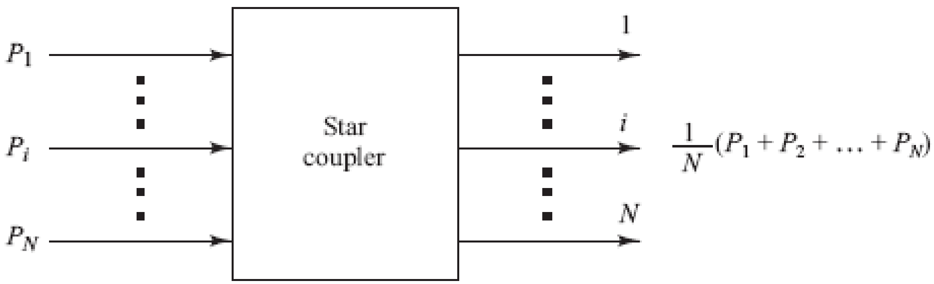
$$\text{Insertion loss (port 0 to port 2)} = 10 \log\left(\frac{200}{85}\right) = 3.72 \text{ dB}$$

The return loss is given by Eq. 10.7 as

$$\text{Return loss} = 10 \log\left(\frac{6.3 \times 10^{-3}}{200}\right) = -45 \text{ dB}$$

Star Couplers

- **In general, an N × M coupler has N inputs and M outputs**



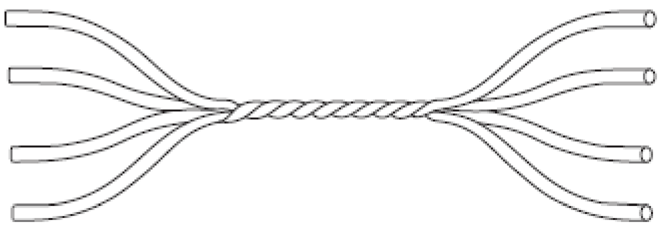


Fig. 10.13 *Generic 4 × 4 fused-fiber star coupler fabricated by twisting, heating, and pulling on four fibers to fuse them together*

$$\text{Splitting loss} = -10\log\left(\frac{1}{N}\right) = 10\log N$$

$$\text{Fiber star excess loss} = 10\log\left(\frac{P_{in}}{\sum_{i=1}^{N} P_{out,\,i}}\right)$$

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N × N Star Coupler

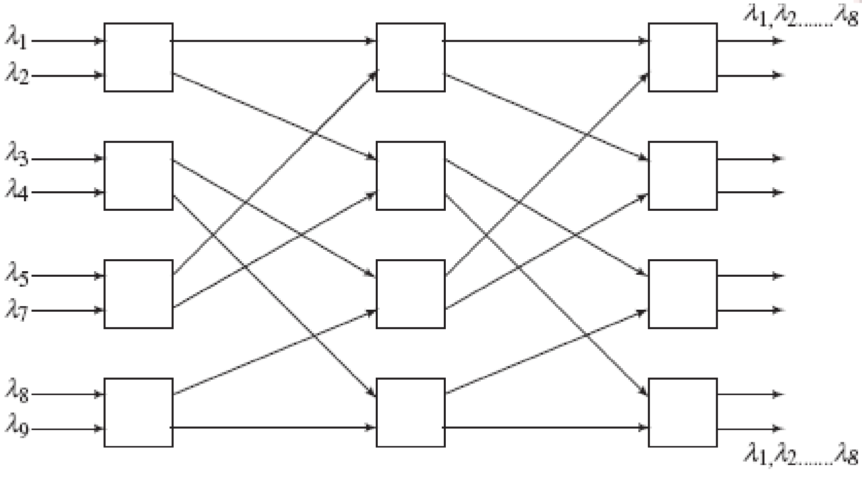
- **Can construct star couplers by cascading 3-dB couplers**
- **The number of 3-dB couplers needed to construct an N × N star is**

$$N_c = \frac{N}{2}\log_2 N = \frac{N}{2}\frac{\log N}{\log 2}$$

Example 10.8 A device engineer wants to construct a 32 × 32 coupler from a cascade of 2 × 2 3-dB single-mode fiber couplers. How many 2 × 2 elements are needed for this?

Solution: In this case there will be 16 coupler elements in the vertical direction. From Eq. (10.26), we find how many 2 × 2 elements are needed:

$$N_c = \frac{32}{2}\frac{\log 32}{\log 2} = 80$$



Example of an 8 × 8 star coupler formed by interconnecting twelve 2 × 2 couplers

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Star Coupler examples

Nonlinear Effects and DWDM

Induced Polarization, $P = a_1E + \underline{a_2E^2 + a_3E^3}$

Nonlinear effects

Nonlinear behavior for glasses $\boxed{P = a_1E + a_3E^3}$

Four photon mixing

Consider signals at 3 channels

$$E_1 = C_1\cos(2\pi\upsilon_1 t) \qquad E_2 = C_2\cos(2\pi\upsilon_2 t) \qquad E_3 = C_3\cos(2\pi\upsilon_3 t)$$

$$\text{Nonlinear } P = a_3E^3 = a_3(E_1 + E_2 + E_3)^3$$
$$= a_3(E_1{}^3) + \ldots + 2a_3(E_1E_2E_3) + \ldots + a_3(E_3{}^3)$$

Nonlinear Effects and DWDM

Nonlinear $P = a_3(E_1^3) + \ldots + 2a_3(E_1E_2E_3) + \ldots + a_3(E_3^3)$

Nonlinear $P = \ldots + 2a_3C_1C_2C_3\underline{\cos(2\pi\upsilon_1 t)\cos(2\pi\upsilon_2 t)\cos(2\pi\upsilon_3 t)} + \ldots$

Frequencies mix and can be written as sum and difference frequencies

Consider three optical signals at

υ_1

$\upsilon_2 = (\upsilon_1 + 100\text{ GHz})$

$\upsilon_3 = (\upsilon_2 + 100\text{ GHz}) = (\upsilon_1 + 200\text{ GHz})$

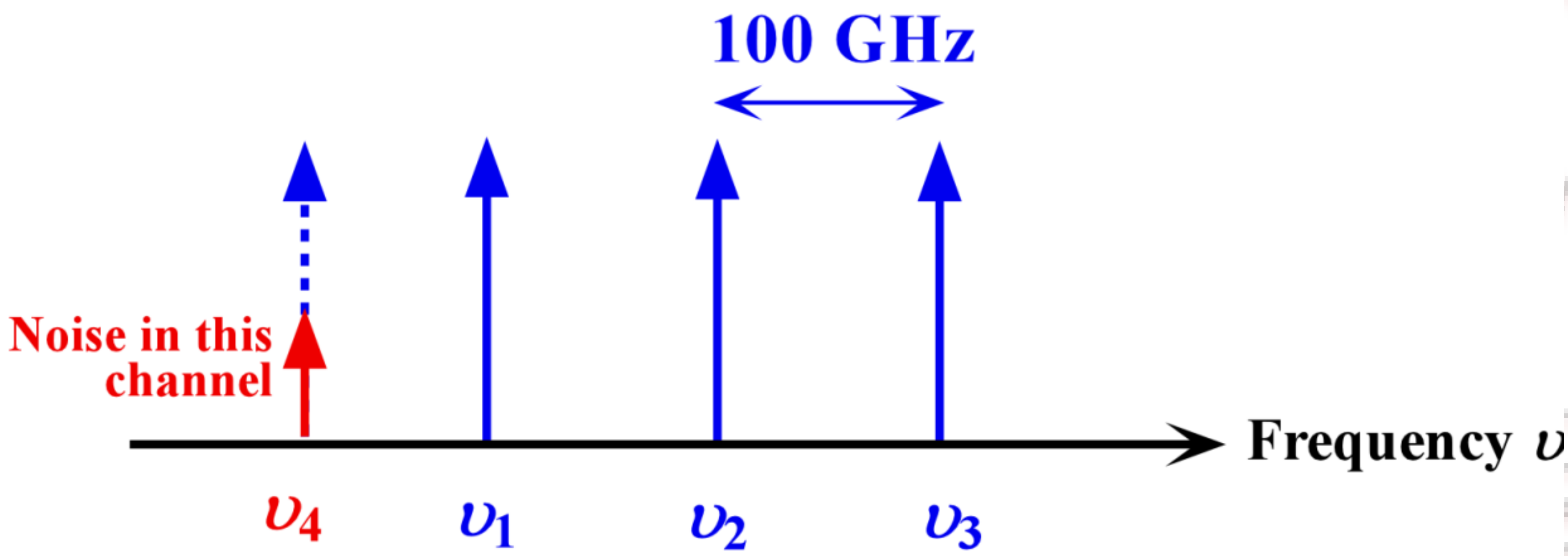
Nonlinear Effects and DWDM
Four Photon (Wave) Mixing

Three photons (υ_1, υ_2, υ_3) mix to generate a fourth photon (υ_4)

$$\upsilon_4 = \upsilon_1 + \upsilon_2 - \upsilon_3$$

$$\upsilon_4 = \upsilon_1 + (\upsilon_1 + 100 \text{ GHz}) - (\upsilon_1 + 200 \text{ GHz})$$

$\upsilon_4 = \upsilon_1 - 100$ GHz on top of previous channel



Nonlinear Effects: Frequency Chirping

$$n^2 = \varepsilon_r = \frac{P}{\varepsilon_o E} + 1 = \left(\frac{a_1}{\varepsilon_o} + 1\right) + \left(\frac{a_3}{\varepsilon_o}\right) E^2$$

$$n = n_o + n_2 I$$

Nonlinear refractive index

$$\Delta\phi = \frac{2\pi z \Delta n}{\lambda} = \frac{2\pi z n_2 I}{\lambda}$$

Self-phase modulation

$$\Delta\omega = \frac{d\Delta\phi}{dt} = \frac{2\pi z n_2}{\lambda} \frac{dI}{dt}$$

Frequency chirping

Nonlinear Effects: Frequency Chirping

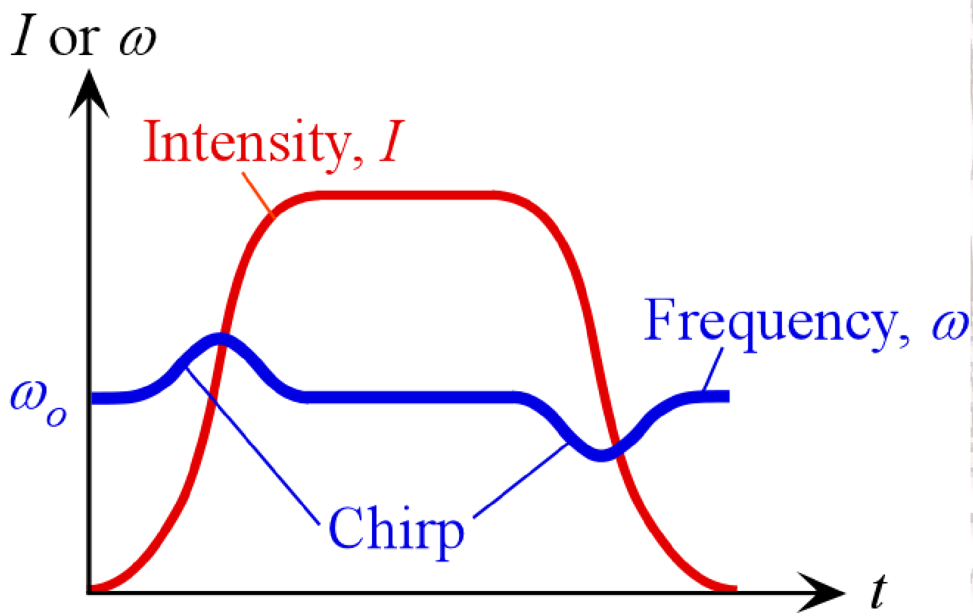
$$n = n_o + n_2 I$$

$$\Delta\phi = \frac{2\pi z \Delta n}{\lambda} = \frac{2\pi z n_2 I}{\lambda}$$

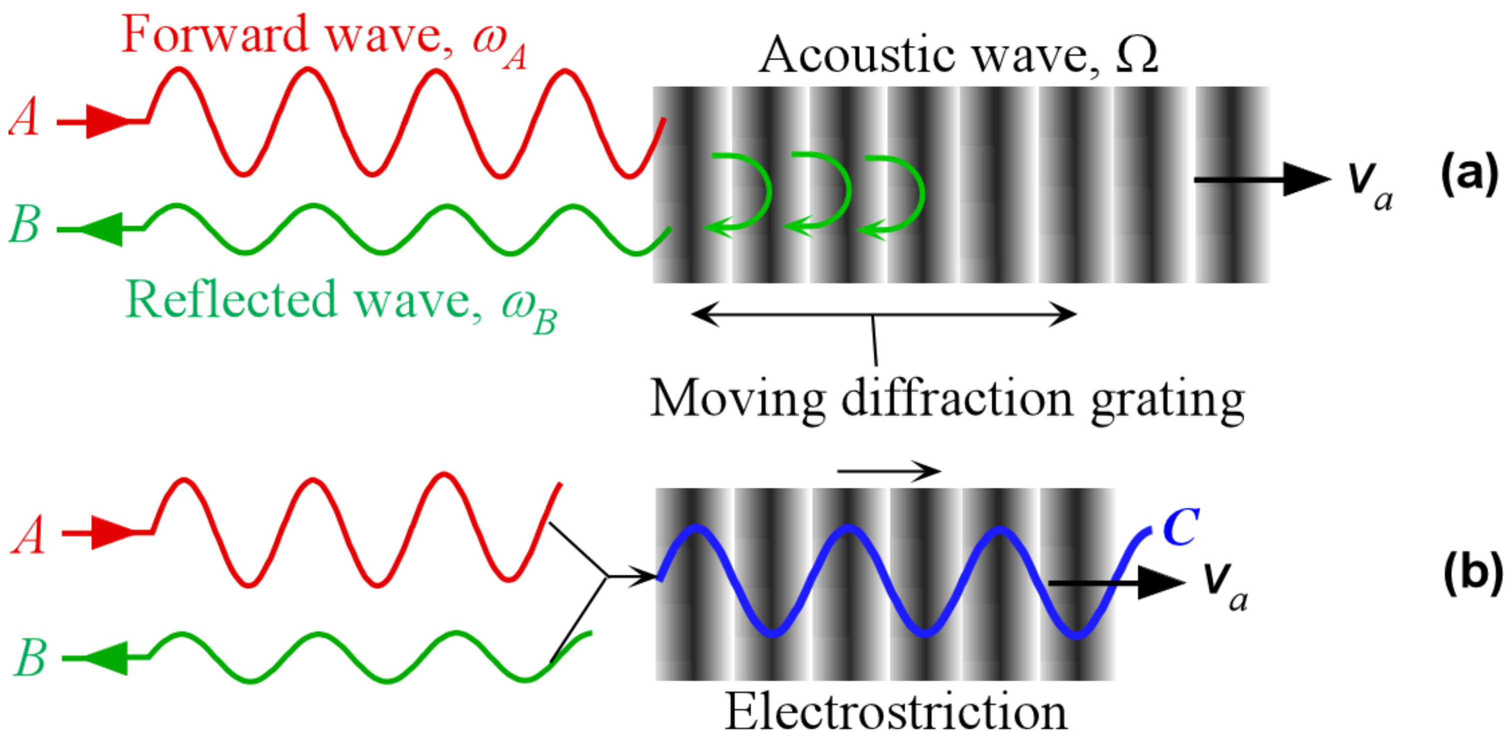
Self-phase modulation

$$\Delta\omega = \frac{d\Delta\phi}{dt} = \frac{2\pi z n_2}{\lambda}\frac{dI}{dt}$$

Frequency chirping



Stimulated Brillouin Scattering



(a) Scattering of a forward travelling EM wave *A* by an acoustic wave results in a reflected, backscattered, wave *B* that has a slightly different frequency, by Ω. (b) The forward and reflected waves, *A* and *B* respectively, interfere to give rise to a standing wave that propagates at the acoustic velocity v_a. As a result of electrostriction, an acoustic wave is generated that reinforces the original acoustic wave and stimulates further scattering.

Stimulated Brillouin Scattering (SBS)

Atomic vibrations give rise to traveling waves in the bulk – phonons. Collective vibrations of the atoms in a solid give rise to lattice waves inside the solid

Acoustic lattice waves in a solid involve periodic strain variations along the direction of propagation.

Changes in the strain result in changes in the refractive index through a phenomenon called the **photoelastic effect**. (The refractive index depends on strain.) Thus, there is a periodic variation in the refractive index which moves with an acoustic velocity v_a as depicted

The moving diffraction grating reflects back some of the forward propagating EM wave *A* to give rise to a *back scattered* wave *B* as shown. The frequency ω_B of the back scattered wave *B* is Doppler shifted from that of the forward wave ω_A by the frequency Ω of the acoustic wave *i.e.* $\omega_B = \omega_A - \Omega$.

Stimulated Brillouin Scattering (SBS)

The forward wave *A* and the back scattered wave *B* interfere and give rise to a standing wave *C*.

This standing wave *C* represents a periodic variation in the field that moves with a velocity v_a along the direction of the original acoustic wave

The field variation in *C* produces a periodic displacement of the atoms in the medium, through a phenomenon called **electrostriction**. (The application of an electric field causes a substance to change shape, *i.e.* experience strain.). Therefore, a periodic variation in strain develops, which moves at the acoustic velocity v_a.

The moving strain variation is really an acoustic wave, which reinforces the original acoustic wave and stimulates more back scattering. Thus, it is clear that, a condition can be easily reached that Brillouin scattering stimulates further scattering; **stimulated Brillouin scattering** (SBS).

Stimulated Brillouin Scattering (SBS)

The SBS effect increases as the input light power increase

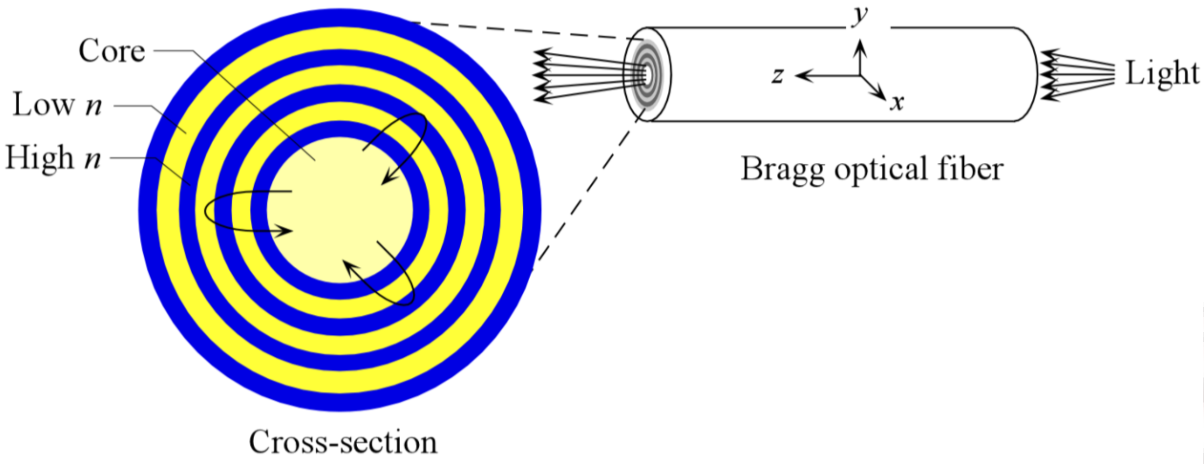
The SBS effect increases as the spectral width of the input light becomes narrower.

The onset of SBS depends not only on the fiber type and core diameter, but also on the spectral width $\Delta\lambda$ of the laser output spectrum.

SBS is enhanced as the laser spectral width $\Delta\lambda$ is narrowed or the duration of light pulse is lengthened.

Typical values: For a directly modulated laser diode emitting at 1550 nm into a single mode fiber, the onset of SBS is expected to occur at power levels greater than 20 – 30 mW. In DWDM systems with externally modulated lasers, *i.e.* narrower $\Delta\lambda$, the onset of SBS can be as low as ~10 mW. SBS is an important limiting factor in transmitting high power signals in WDM systems.

Bragg Fibers



Bragg fibers have a core region surrounded by a cladding that is made up of concentrating layers of high low refractive index dielectric media

The core can be a low refractive index solid material or simply hollow. In the latter case, we have a **hollow Bragg fiber.**

The high (n_1) and low (n_2) alternating refractive index profile constitutes a Bragg grating which acts as a dielectric mirror.

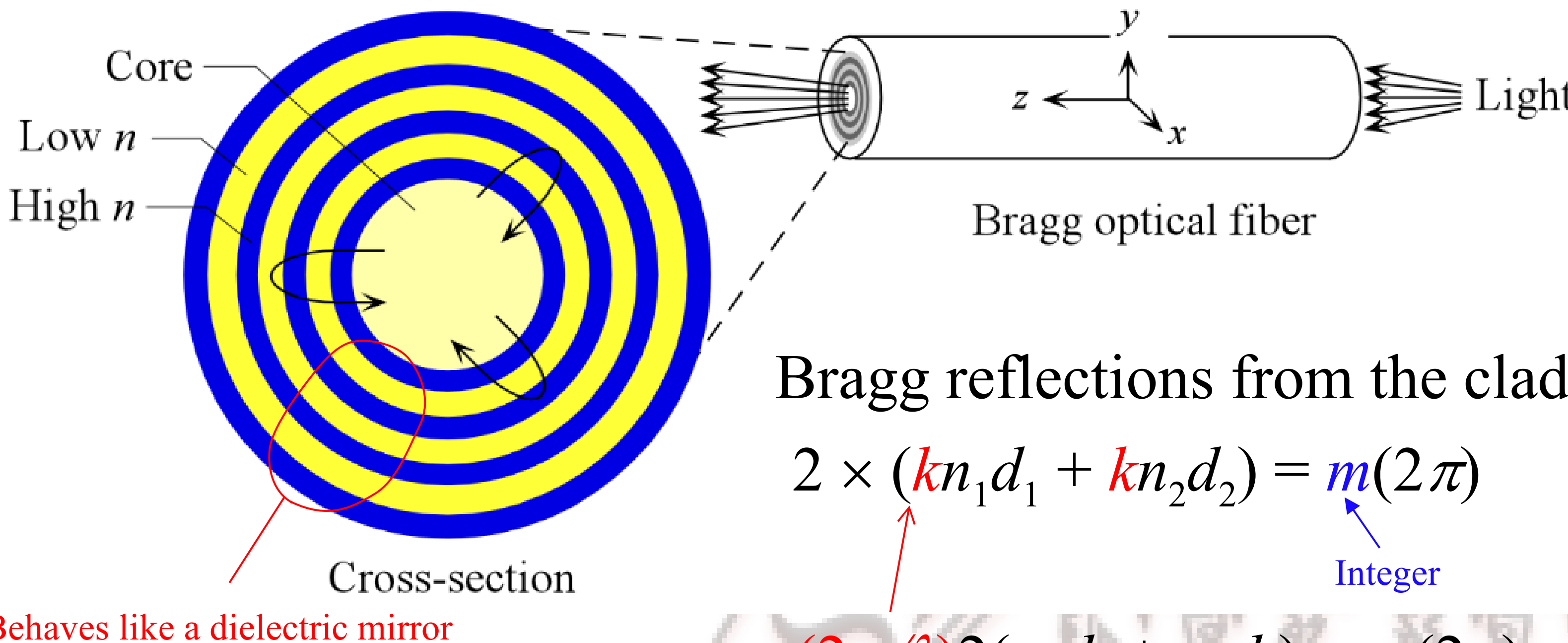
There is a band of wavelengths, forming a **stop-band**, that are not allowed to propagate into the Bragg grating.

We can also view the periodic variation in n as forming a *photonic crystal cladding in one-dimension*, along the radial direction, with a stop-band, *i.e.* a photonic bandgap.

Light is bound within the core of the guide for wavelengths within this stop-band. Light can only propagate along z. *The cladding is like a dielectric mirror*

$$d_1 = \lambda/4n_1 \text{ and } d_2 = \lambda/4n_2 \quad \therefore \quad d_1 n_1 = d_2 n_2$$

Bragg Fibers



Bragg reflections from the cladding need

$$2 \times (kn_1d_1 + kn_2d_2) = m(2\pi)$$

Integer

$$\therefore \quad (2\pi/\lambda)2(n_1d_1 + n_2d_2) = m(2\pi)$$

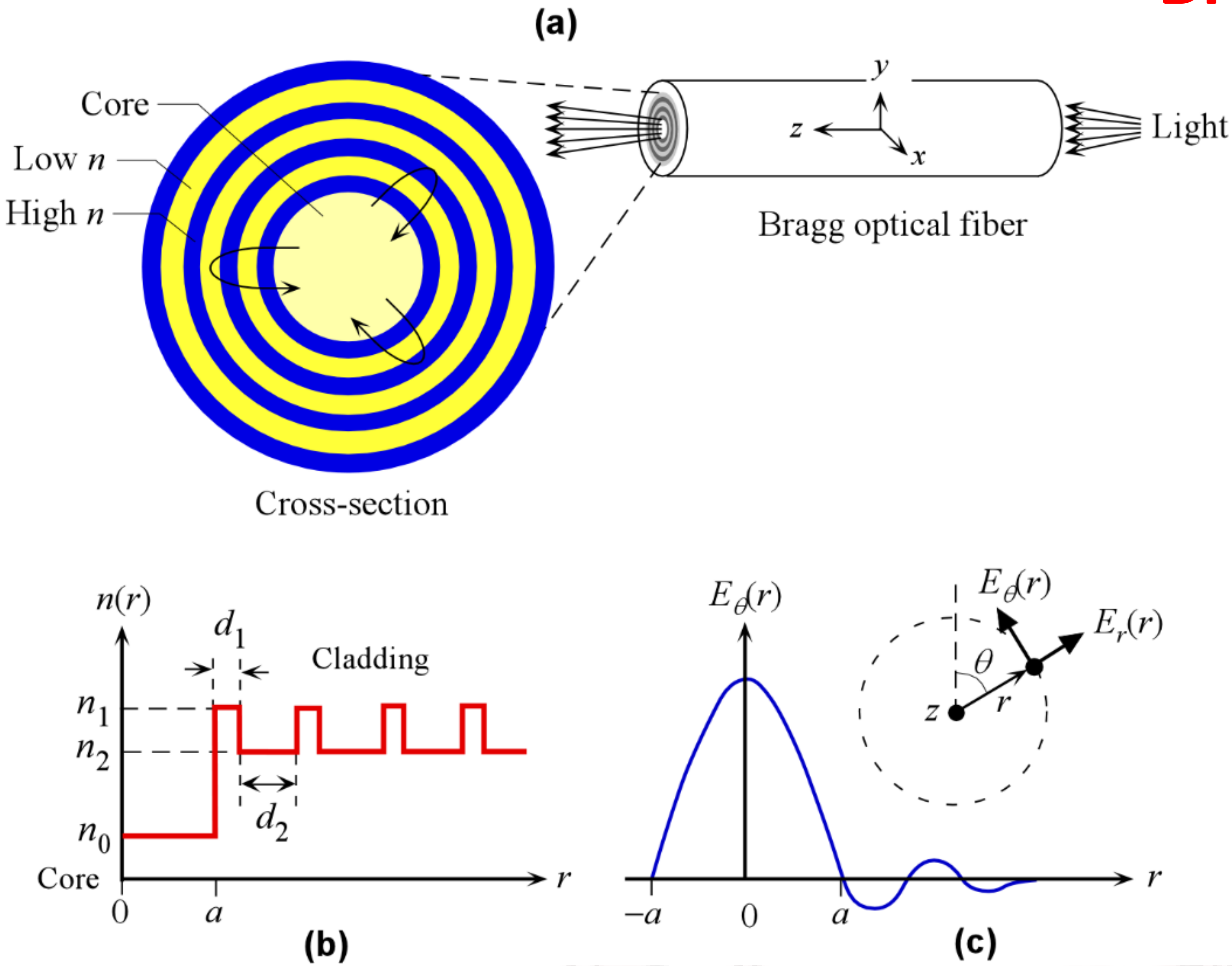
$$\therefore \quad \boxed{n_1d_1 + n_2d_2 = m(\lambda/2)}$$

This is satisfied for $d_1 = \lambda/4n_1$ and $d_2 = \lambda/4n_2$

Layers are quarter wavelength thick

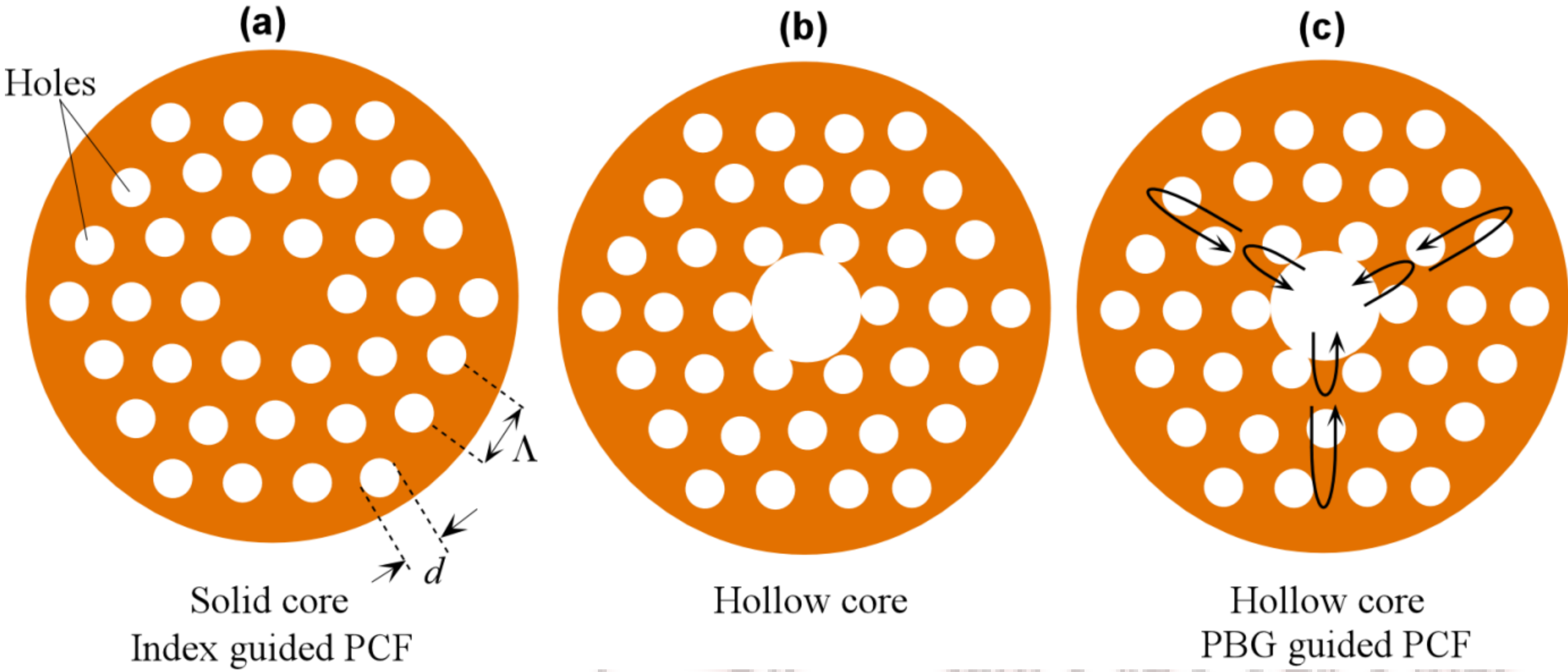
$$\mathbf{d_1n_1 = d_2n_2}$$

Bragg Fibers



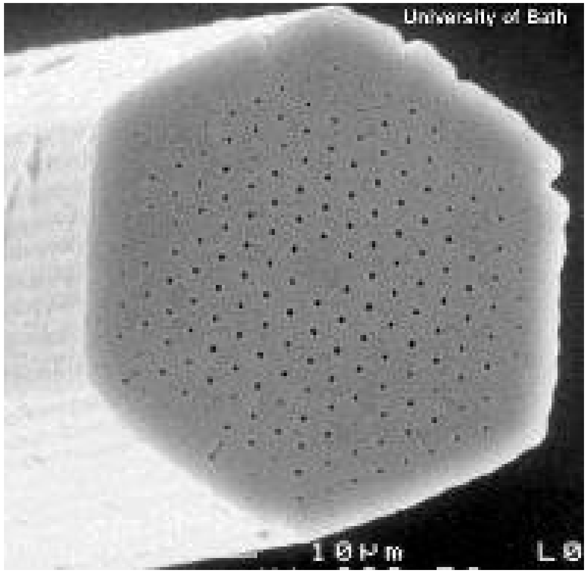
(a) A Bragg fiber and its cross section. The Bragg grating in the cladding can be viewed to reflect the waves back into the core over its stop-band. (b) The refractive index variation (exaggerated). (c) Typical field distribution for the circumferential field E_θ.

Photonic Crystal Fibers: Holey Fibers

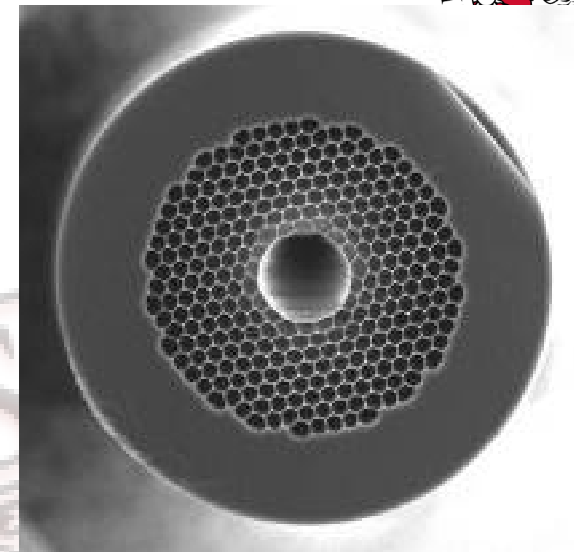


(a) A solid core PCF. Light is index guided. The cladding has a hexagonal array of holes. *d* is the hole diameter and Λ is the array pitch, spacing between the holes (b) and (c) A hollow core PCF. Light is photonic bandgap (PBG) guided.

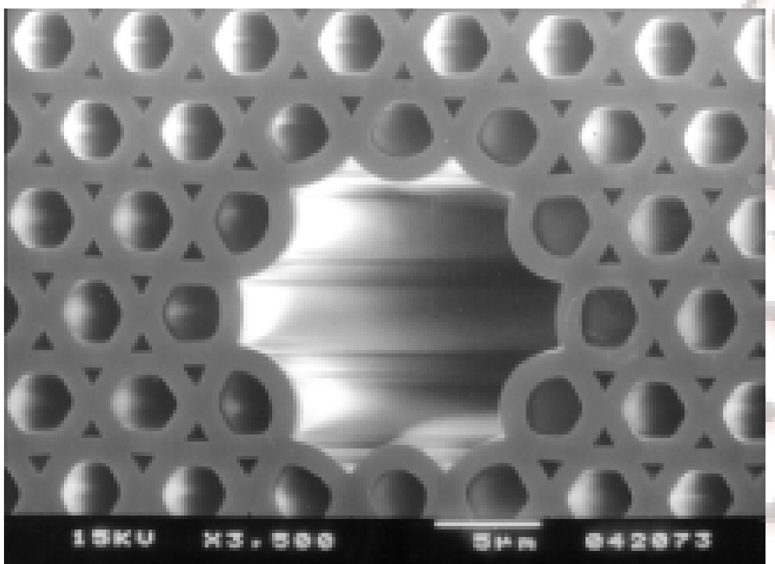
Photonic Crystal Fibers: Holey Fibers



Left: The first solid core photonic crystal fiber prepared by Philip Russell and coworkers at the University of Bath in 1996; an endlessly single mode fiber. (Courtesy of Philip Russell)

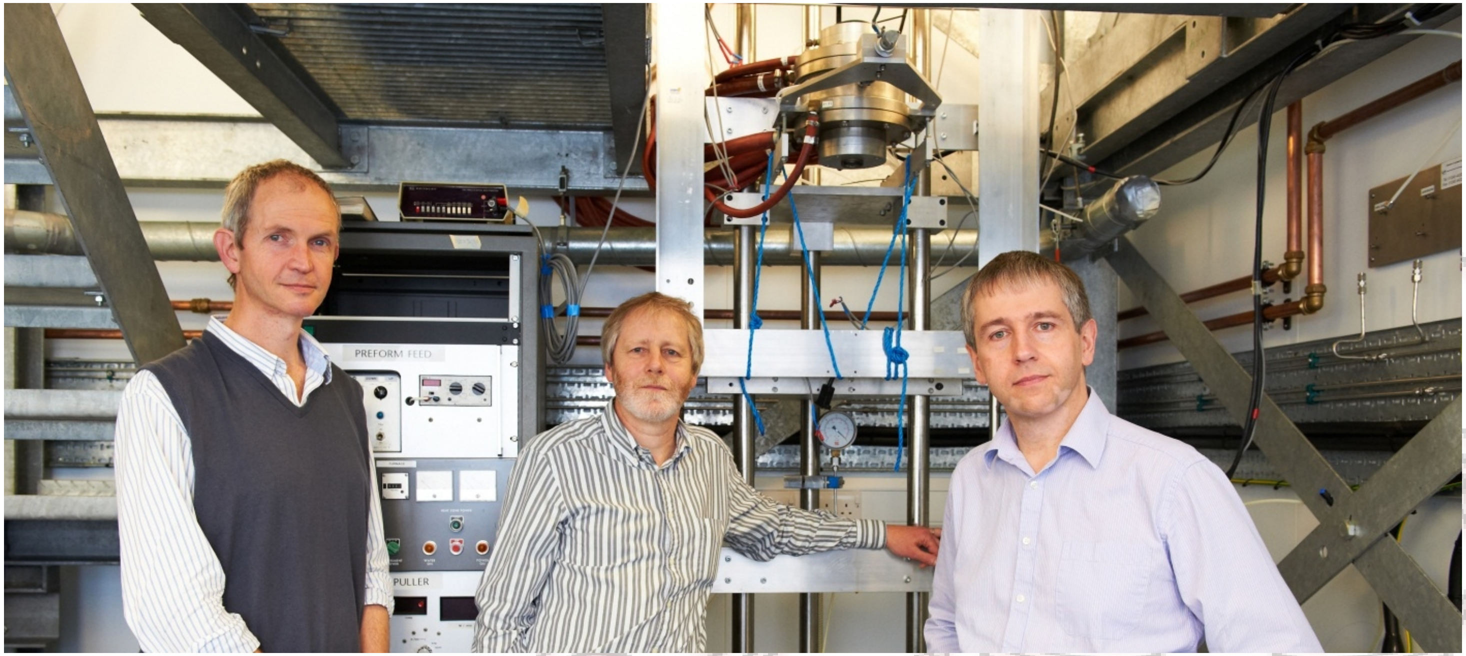


Above: A commercially available hollow core photonic crystal fiber from Blaze Photonics. (Courtesy of Philip Russell)



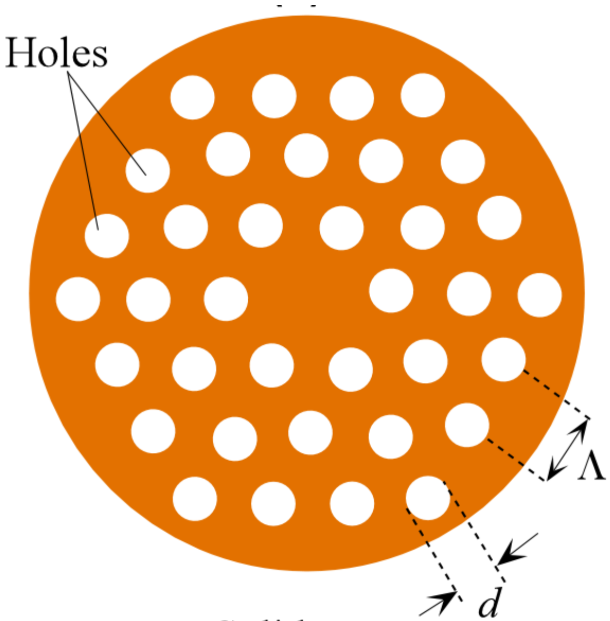
Left: One of the first hollow core photonic crystal fibers, guiding light by the photonic bandgap effect (1998) (Courtesy of Philip Russell)

Photonic Crystal Fibers: Holey Fibers



Philip Russell (center), currently at the Max Planck Institute for the Science of Light in Erlangen, Germany, led the team of two postdoctoral research fellows, Jonathan Knight (left) and Tim Birks (right) (both currently professors at the University of Bath in England) that carried out the initial pioneering research on photonic crystal fibers in the 1990s. (See reviews by P.St.J. Russell, *Science*, **299**, 358, 2003, J. C. Knight, *Nature*, **424**, 847, 2003) (Courtesy of Philip Russell)

Solid Core Photonic Crystal Fibers

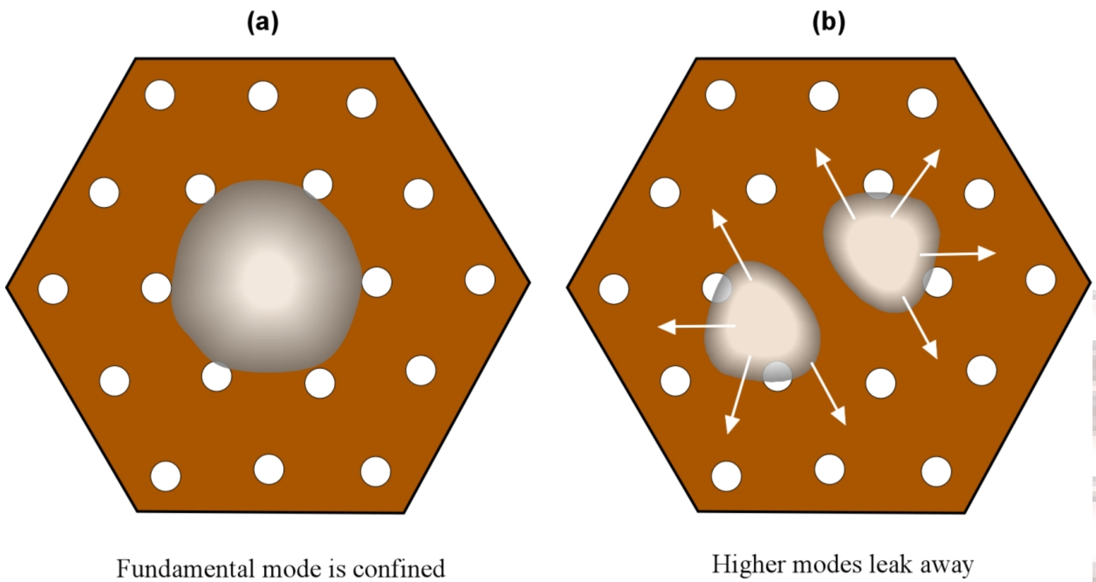


Both the core and cladding use the same material, usually silica, but the air holes in the cladding result in an effective refractive index that is lower than the solid core region.

The cladding has a lower effective refractive index than the core, and the whole structure then is like a *step index fiber*.

Total internal reflection then allows the light to be propagated just as in a step index standard silica fiber. Light is **index guided**.

Solid Core Photonic Crystal Fibers

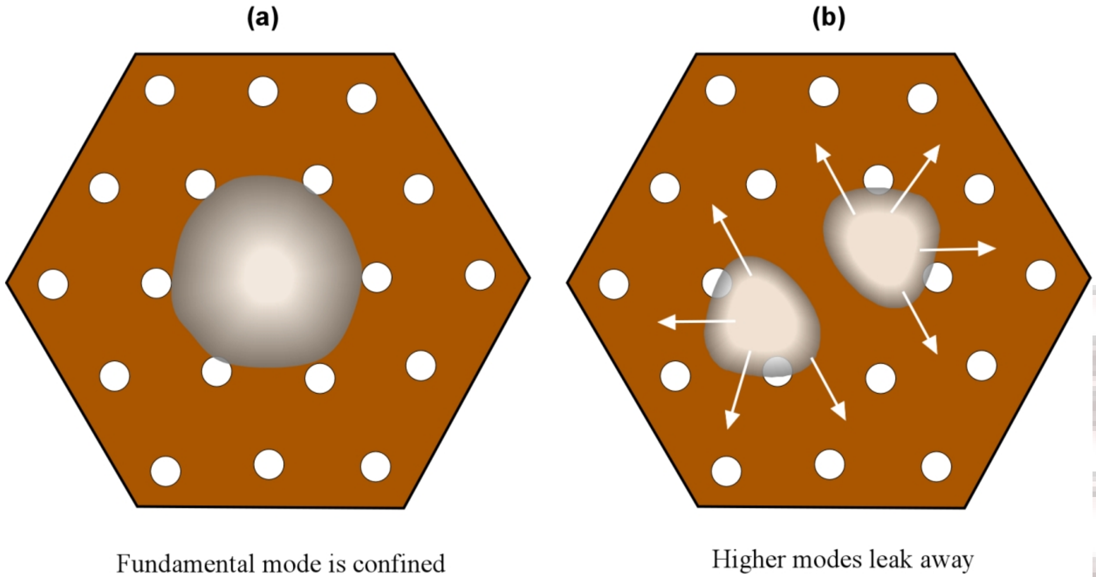


(a) The fundamental mode is confined. (b) Higher modes have more nodes and can leak away through the space.

The solid can be pure silica, rather than germania-doped silica, and hence exhibits lower scattering loss.

Single mode propagation can occur over a very large range of wavelengths, almost as if the fiber is **endlessly single mode** (ESM). The reason is that the PC in the cladding acts as a filter in the transverse direction, which allows the higher modes to escape (leak out) but not the fundamental mode.

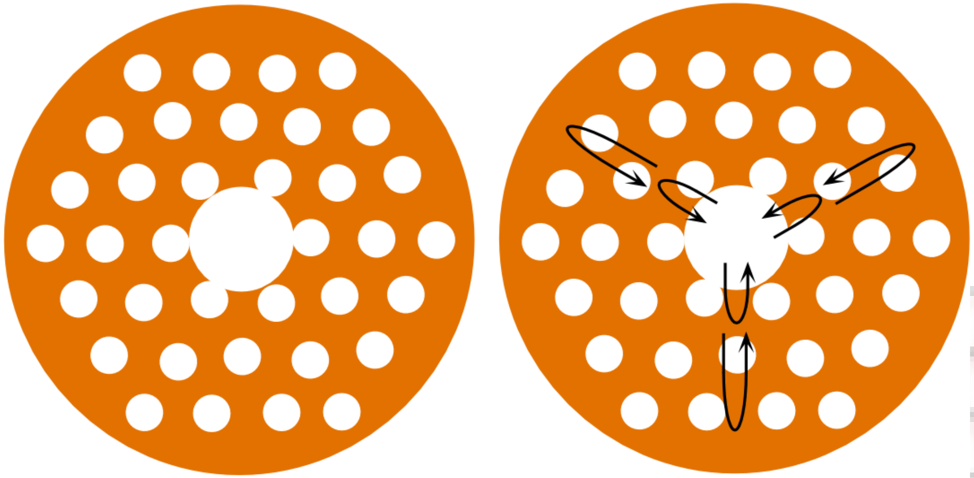
Solid Core Photonic Crystal Fibers



(a) The fundamental mode is confined. (b) Higher modes have more nodes and can leak away through the space.

ESM operation: The core of a PCF can be made quite large without losing the single mode operation. The refractive index difference can also be made large by having holes in the cladding. Consequently, PCFs can have high numerical apertures and large core areas; thus, more light can be launched into a PCF. Further, the manipulation of the shape and size of the hole, and the type of lattice (and hence the periodicity *i.e.* the lattice pitch) leads to a much greater control of chromatic dispersion.

Hollow Core Photonic Crystal Fibers: Holey Fibers



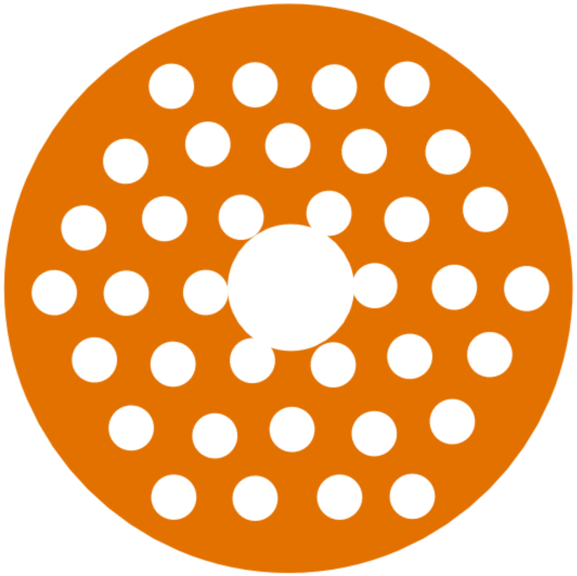
Hollow core

Hollow core
PBG guided PCF

Notice that the core has a lower refractive index (it is hollow) so that we cannot rely on total internal reflection (TIR) to explain light propagation.

Light in the transverse direction is reflected back into the core over frequencies within the stop-band, *i.e.* photonic bandgap (PBG), of the photonic crystal in the cladding. When light is launched from one end of the fiber, it cannot enter the cladding (its frequency is within the stop-band) and is therefore confined within the hollow core. Light is **photonic bandgap guided**.

Hollow Core Photonic Crystal Fibers: Holey Fibers

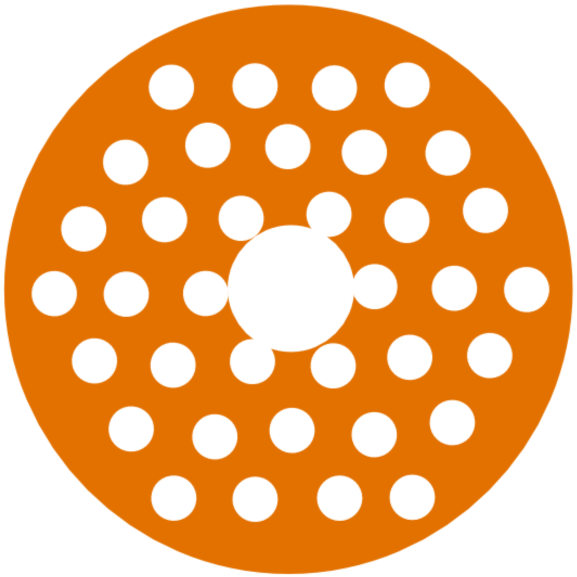


As recalled by Philip Russell:

"My idea, then, was to trap light in a hollow core by means of a 2D photonic crystal of microscopic air capillaries running along the entire length of a glass fiber. Appropriately designed, this array would support a PBG for incidence from air, preventing the escape of light from a hollow core into the photonic-crystal cladding and avoiding the need for TIR." (P. St.J. Russell, *J. Light Wave Technol, 24*, 4729, 2006)

The periodic arrangement of the air holes in the cladding creates a photonic bandgap in the transverse direction that confines the light to the hollow core.

Hollow Core Photonic Crystal Fibers: Holey Fibers



There are certain distinct advantages

Material dispersion is absent.

The attenuation in principle should be potentially very small since there is no Rayleigh scattering in the core. However, scattering from irregularities in the air-cladding interface, that is, surface roughness, seems to limit the attenuation.

High powers of light can be launched without having nonlinear effects such as stimulated Brillouin scattering limiting the propagation.

There are other distinct advantages that are related to strong nonlinear effects, which are associated with the photonic crystal cladding.

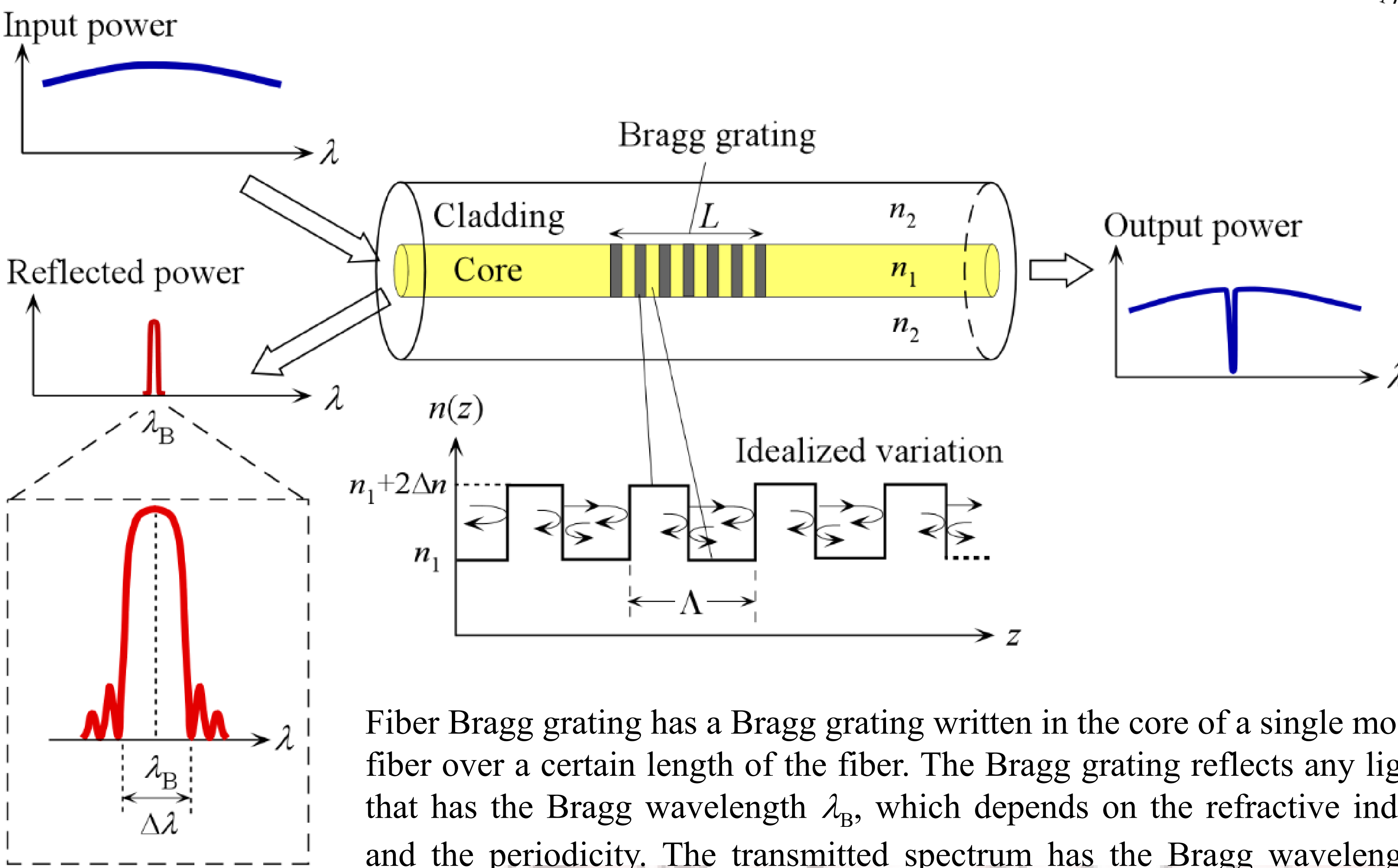
Applications of optical fibers

Fiber Bragg Gratings and FBG Sensors

Fiber Bragg Grating (FBG) based optical strain sensor for monitoring strains in civil structures *e.g.* bridges, dams, buildings, tunnels and building. The fiber is mounted on a steel carrier and maintained stretched. The carriage is welded to the steel structure. (Courtesy of Micron Optics)

Fiber Bragg Grating (FBG) based optical temperature sensors for use from -200 °C to 250 °C. FBGs are mounted in different packages: top, left, all dielectric, top right, stainless steel and bottom, copper. The sensors operate over 1510 - 1590 nm. (Courtesy of Lake Shore Cryotronics Inc.)

FBG: Fiber Bragg Grating



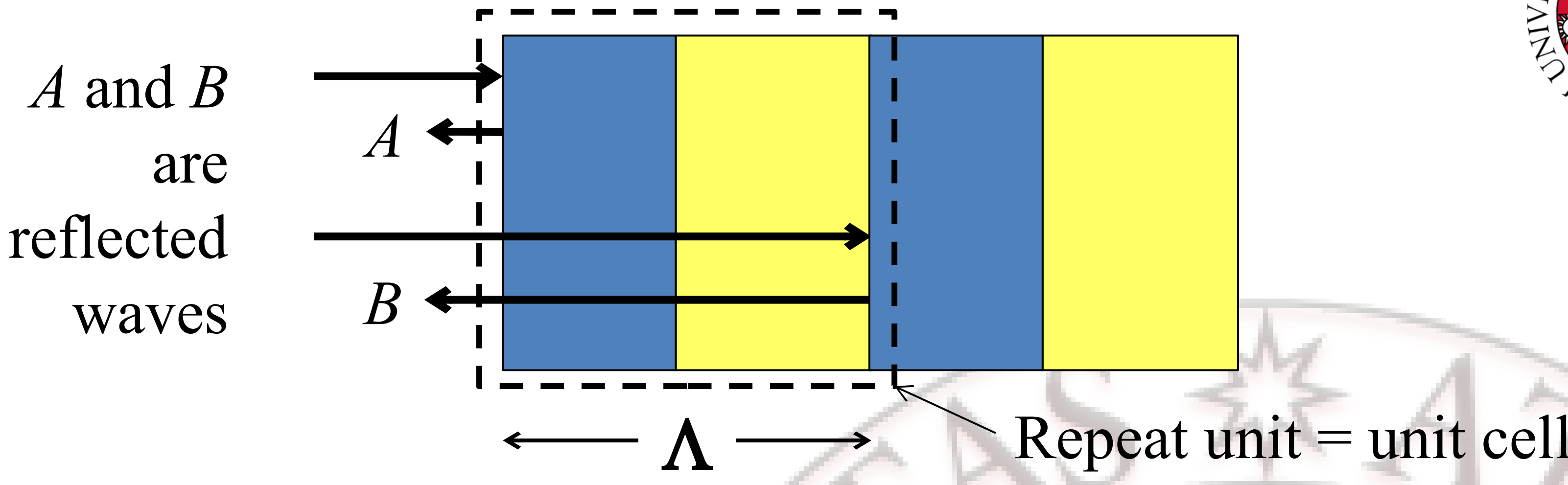
Fiber Bragg grating has a Bragg grating written in the core of a single mode fiber over a certain length of the fiber. The Bragg grating reflects any light that has the Bragg wavelength λ_B, which depends on the refractive index and the periodicity. The transmitted spectrum has the Bragg wavelength missing.

Fiber Bragg Grating: FBG

Variation in n in the grating acts like a dielectric mirror Partial Fresnel reflections from the changes in the refractive index add in phase and give rise to a back reflected (back diffracted) wave only at a certain wavelength λ_B, called the **Bragg wavelength**

$$q\lambda_B = 2\bar{n}\Lambda$$

Integer, 1,2,… normally $q = 1$ is used, the fundamental



For reflection A and B must be in phase *i.e.* phase difference should be $q2\pi$. This will be so at wavelength λ_B, **the Bragg wavelength**. The phase difference between A and B is $\Delta\Phi_{AB}$

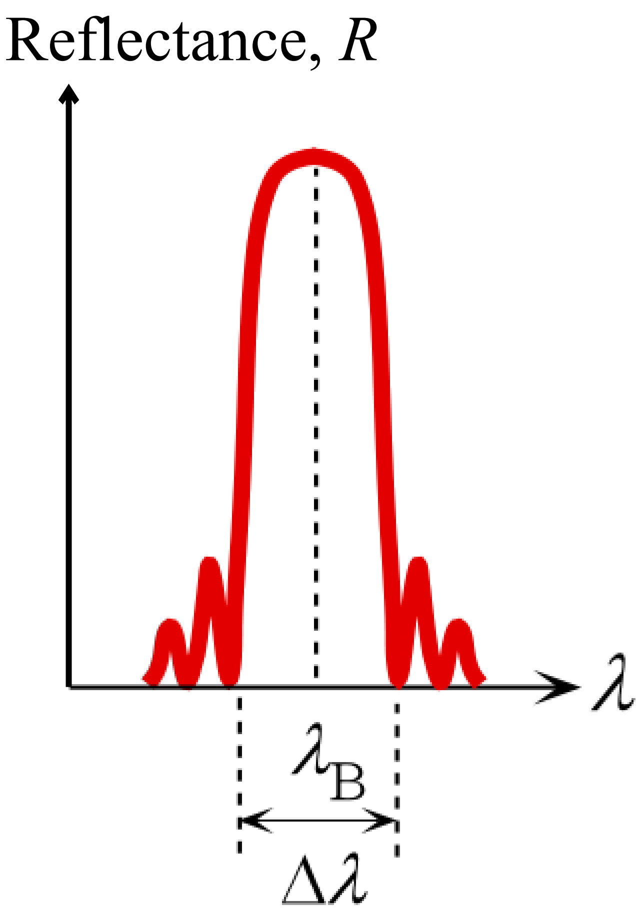
$$\Delta\Phi_{AB} = 2\bar{k}\Lambda = q(2\pi)$$

$$\bar{k} = \text{Average wavevector} = \frac{2\pi\bar{n}}{\lambda_B}$$

Average index in unit cell

$$\boxed{q\lambda_B = 2\bar{n}\Lambda}$$

Fiber Bragg Grating: FBG



$$R = \tanh^2(\kappa L)$$

$$\kappa = \pi \frac{\Delta n}{\lambda}$$

$$\Delta\lambda_{\text{weak}} = \frac{\lambda_B^2}{\bar{n} L}$$

$\kappa L < 1$ **Weak**

$$\Delta\lambda_{\text{strong}} = \frac{4\kappa\lambda_B^2}{\pi\bar{n}}$$

$kL > 1$ **Strong**

FBG Sensors

$$\delta\lambda_B = 2\Lambda\delta\overline{n} + 2\overline{n}\,\delta\Lambda$$

$$\frac{\delta\lambda_B}{\lambda_B} = (\alpha_{\mathrm{TCRI}} + \alpha_L)\delta T$$

Strain

$$\delta n = -\tfrac{1}{2} n^3 p_e \varepsilon$$

Photoelastic or elasto-optic coefficient

FBG Sensors

Consider strain which results in changes in *n* and Λ

$$\delta\lambda_B = 2\Lambda\delta\bar{n} + 2\bar{n}\,\delta\Lambda$$

$$\boxed{\delta\bar{n} = -\tfrac{1}{2}\bar{n}^3 p_e \varepsilon}$$

$$\boxed{\delta\Lambda = \varepsilon\Lambda}$$

$$\frac{\delta\lambda_B}{\lambda_B} = (1 - \tfrac{1}{2} n^2 p_e)\varepsilon$$

Fiber Bragg Grating: Manufacturing

Split Beam Interferometer Method

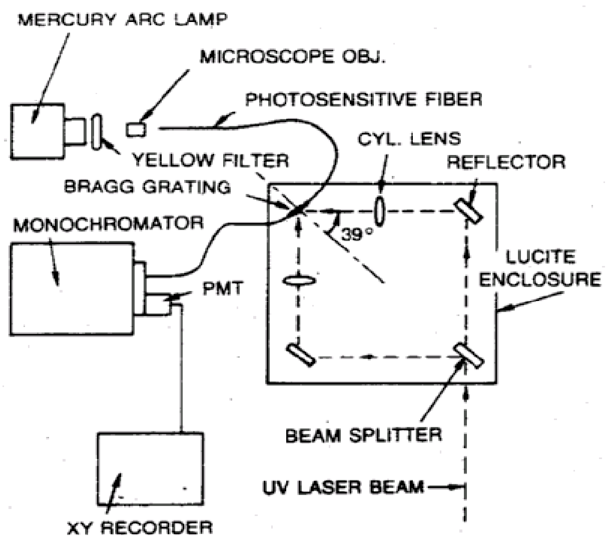
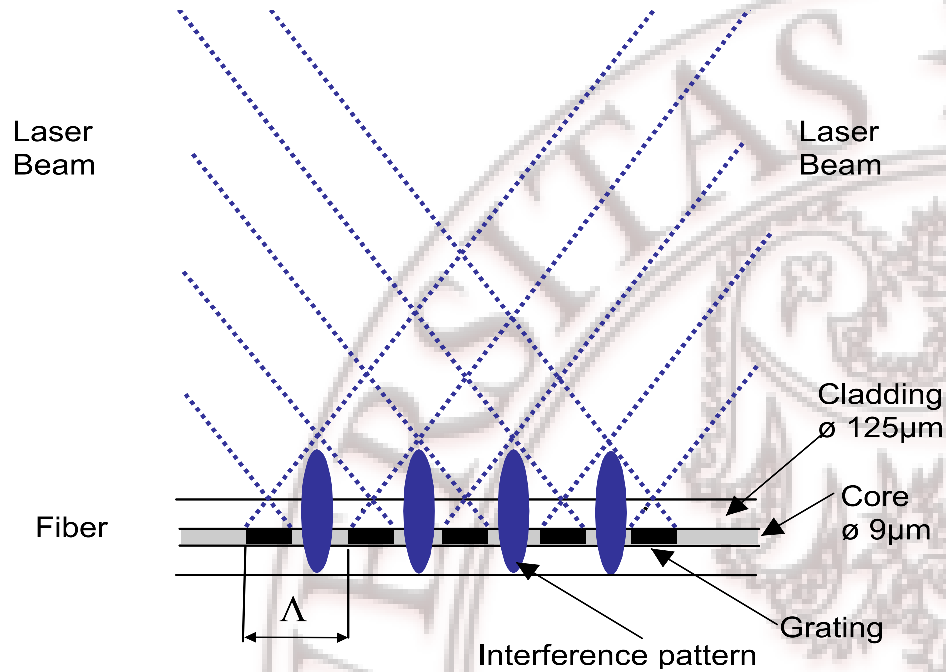


Fig. 1. Diagram of the experimental setup. A beam splitter (not shown) at the fiber input end is used with the monochromator to measure the reflection spectrum of the Bragg grating. PMT, photomultiplier tube.

Fiber Bragg Grating: Manufacturing



Fiber Bragg Grating: Manufacturing

Novel interferometer technique using a right angled prism.

Inherently more stable - because beams are perturbed similarly by any prism vibration.

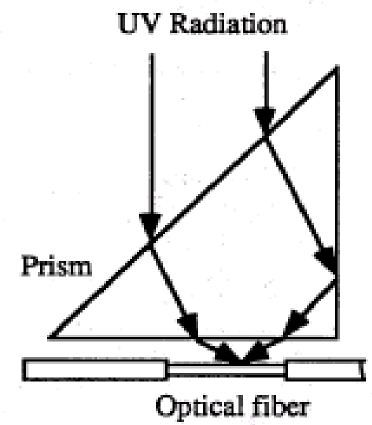


FIGURE 6.104 The novel interferometer technique.

Fiber Bragg Grating: Manufacturing

Phase Mask Technique.

- UV is diffracted into –1,0,1 orders by relief grating.
- Input mask is wavelength specific.
- Different λ_B require different phase masks.

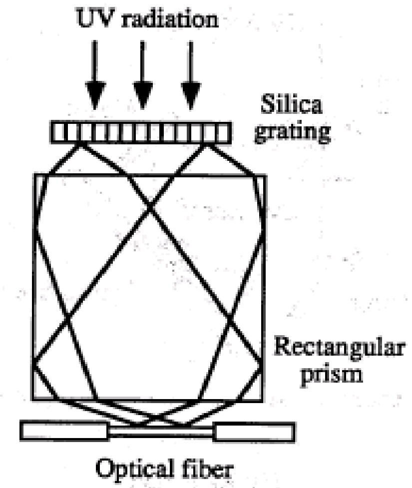


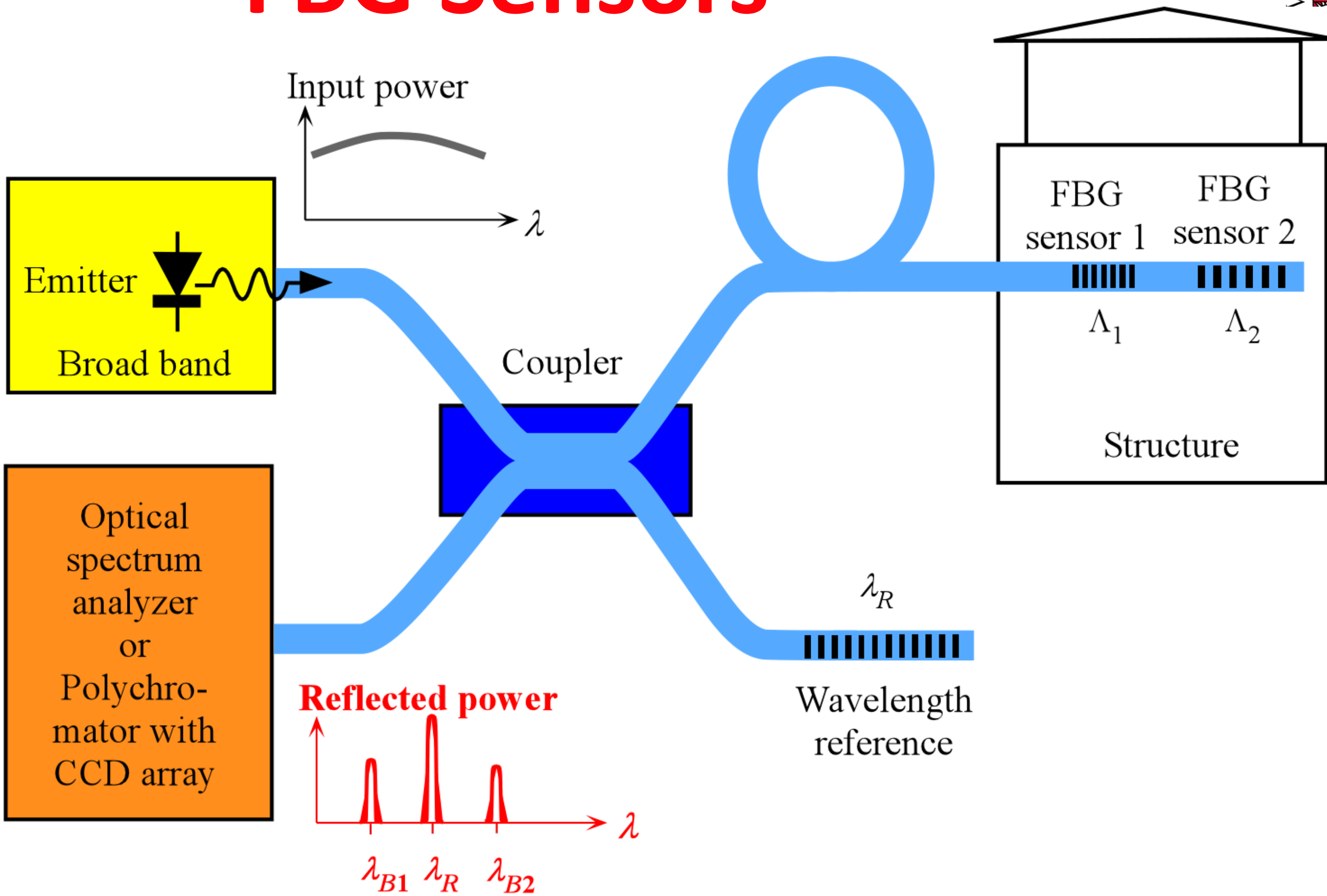
FIGURE 6.105 The phase mask technique for grating fabrication.

Fiber Bragg Grating: Manufacturers

Manufacturers:

- Advanced Optics Solutions GMBH
- Blue Road Research
- 3M Optical OEM Systems
- Alcatel Optronics
- Boeing
- Gould Fiber Optic
- MPB Communications
- OZ Optics Limited
- TeraXion Inc.
- Oxford Lasers Inc.
- Thorlabs Inc.

FBG Sensors



A highly simplified schematic diagram of a multiplexed Bragg grating based sensing system. The FBG sensors are distributed on a single fiber that is embedded in the structure in which strains are to be monitored. The coupler allows optical power to be coupled from one fiber to the other.

FBG at 1550 nm

EXAMPLE

A silica fiber based FBG is required to operate at 1550 nm. What should be the periodicity of the grating Λ? If the amplitude of index variation Δn is 10^{-4} and total length of the FBG is 10 mm, what is the reflectance at the Bragg wavelength and the spectral width of the reflected light? Assume that the effective refractive index is 1.4550

SOLUTION

We can use

$$\lambda_B = 2\bar{n}\Lambda$$

1550 nm = 2(1.450)(Λ) so that the grating periodicity Λ = **534.5 nm**

The coupling coefficient κ is by

$$\kappa = \pi\Delta n/\lambda = \pi(10^{-4})/(1550\times10^{-9}\text{ m}) = \mathbf{202.7\ m^{-1}}.$$

Thus, $\kappa L = 2.027$, so the FBG is a strong grating. The reflectance is

$$R = \tanh^2(\kappa L) = \tanh^2(2.027) = 0.933 \text{ or } \mathbf{93.3\%}$$

$$\Delta\lambda_{\text{strong}} = \frac{4\kappa\lambda_B^2}{\pi\bar{n}} = \frac{4(202.7)(1.55\times10^{-6})^2}{\pi(1.450)} = \mathbf{0.428\ nm}$$

The reflectance and spectral width values are approximate inasmuch as above equations contain a number of assumptions.

Appendix I: FBG Strain Gauge

Some researchers use

$$\frac{\delta n}{n} \propto -\frac{\delta\Lambda}{\Lambda} \quad \text{or} \quad \frac{\delta n}{n} \propto -\varepsilon \quad \therefore \quad \boxed{\frac{\delta n}{n} = -p'_e \varepsilon}$$

$$\delta\lambda_B = 2n\delta\Lambda + 2\Lambda\delta n \quad \text{and} \quad \lambda_B = 2n\Lambda$$

$$\frac{\delta\lambda_B}{\lambda_B} = \frac{2\Lambda\delta n + 2n\delta\Lambda}{2n\Lambda} = \frac{(-np'_e\varepsilon) + n(\delta\Lambda/\Lambda)}{n}$$

$$\frac{\delta\lambda_B}{\lambda_B} = (1 - p'_e)\varepsilon$$

Compare with (2.15.7)

$$\frac{\delta\lambda_B}{\lambda_B} = (1 - \tfrac{1}{2}n^2 p_e)\varepsilon$$

What is the difference?

Appendix I

$$\frac{\delta\lambda_B}{\lambda_B} = (1 - p'_e)\varepsilon$$

compare with

$$\frac{\delta\lambda_B}{\lambda_B} = (1 - \tfrac{1}{2} n^2 p_e)\varepsilon$$

$$p'_e = \tfrac{1}{2} n^2 p_e$$

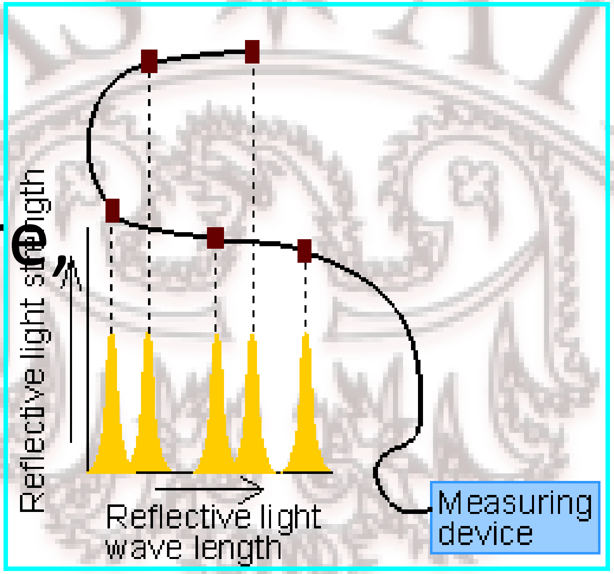
For silica glass, take $n \approx 1.44$

$$p'_e = \tfrac{1}{2}(1.44)^2 p_e = 1.04 p_e \approx p_e$$

$$\therefore \quad p'_e = \tfrac{1}{2} n^2 p_e \approx p_e$$

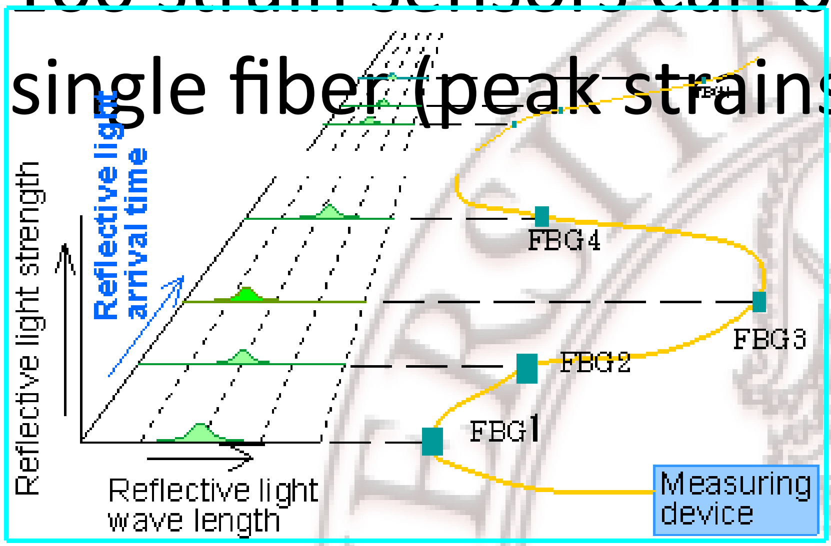
Fiber Bragg Gratings – Application, WDM

- In WDM, each FBG sensor is assigned a portion of the source spectrum.
- Enables quasi-distributed sensing of strain, temperature, chemical, etc….
- No. of FBG is a function of:
  - Source profile width
  - Grating operational bandwidth



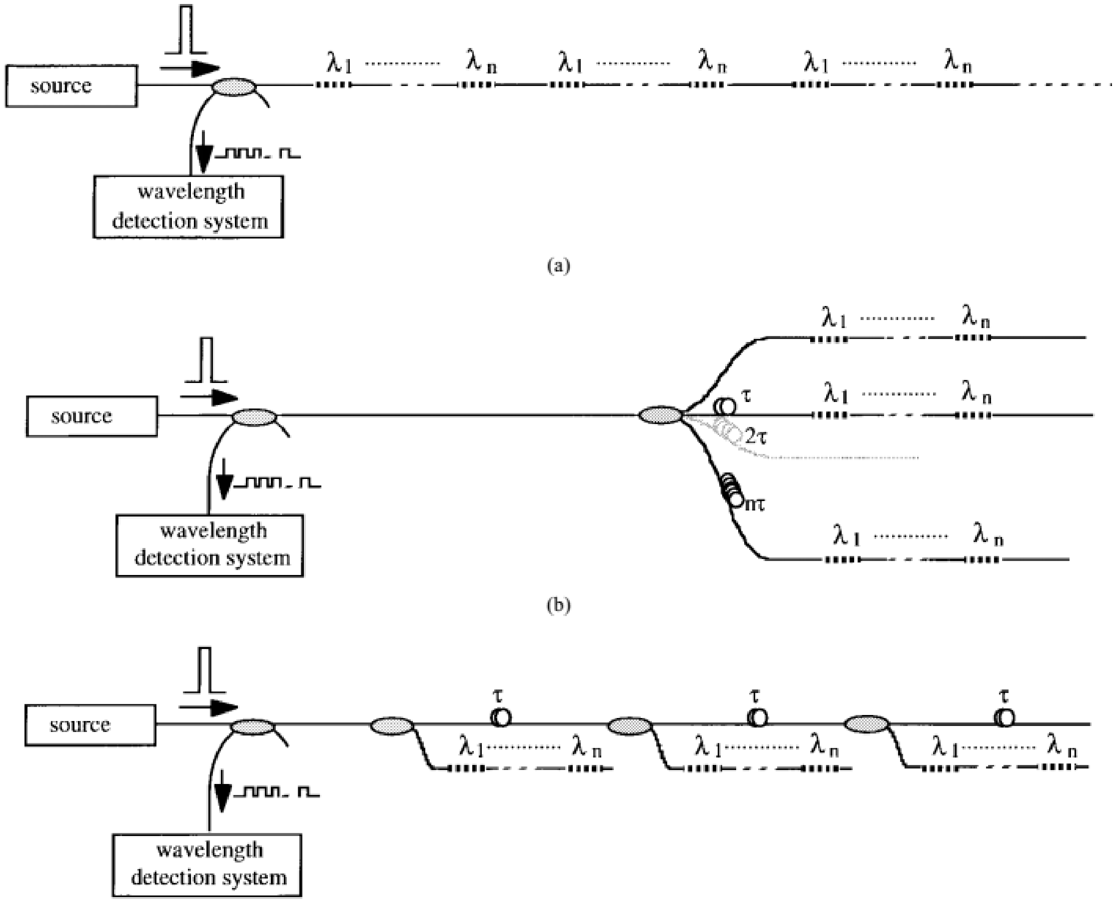
Fiber Bragg Gratings - Application

- Approx 100 strain sensors can be multiplexed along a single fiber (peak strains of $\pm 1000\mu\varepsilon$)

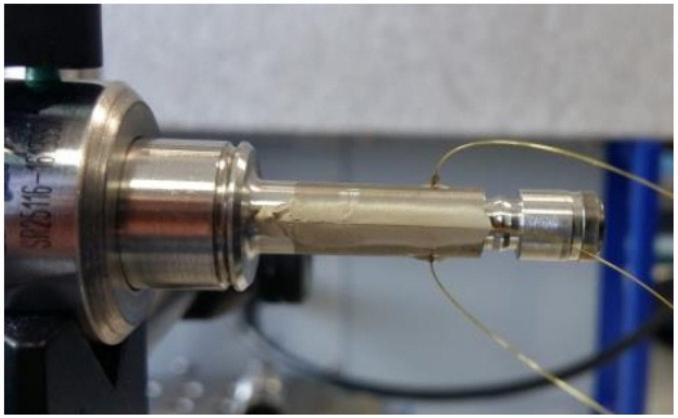


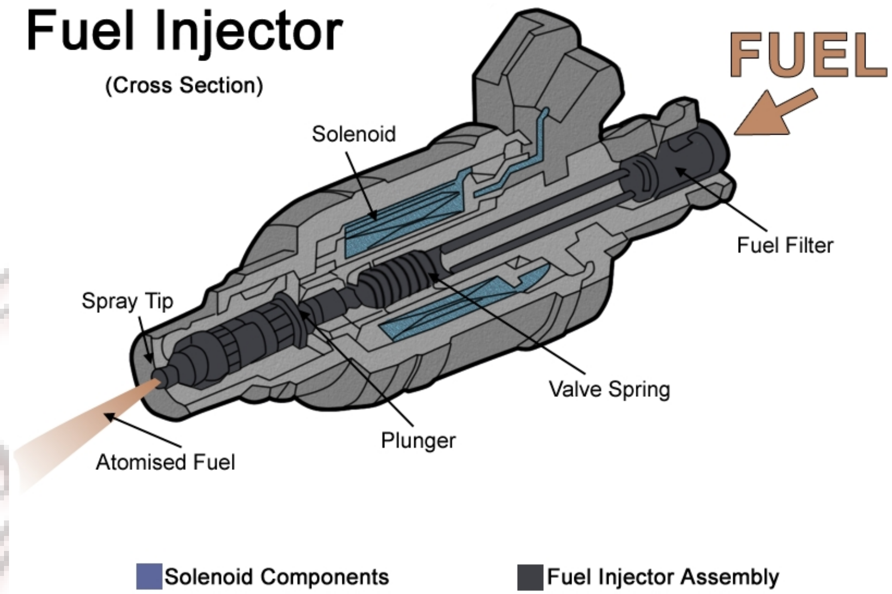
Fiber Bragg Gratings - Application

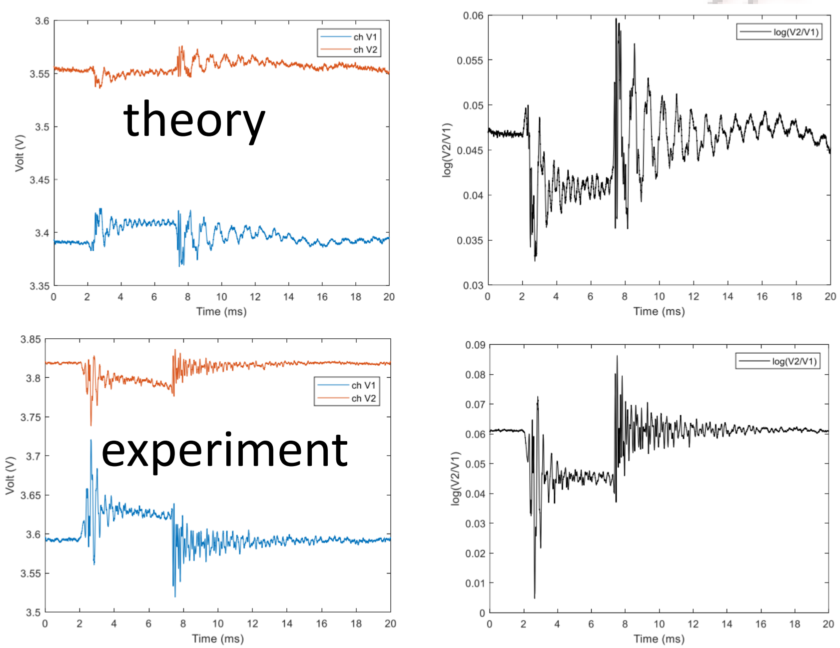
Time & Wavelength Division



Some research applications







Very tight space

Extreme temperature and pressure

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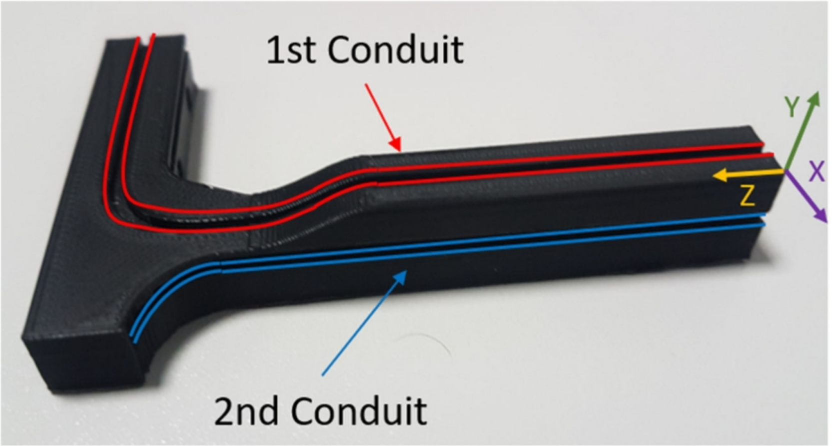
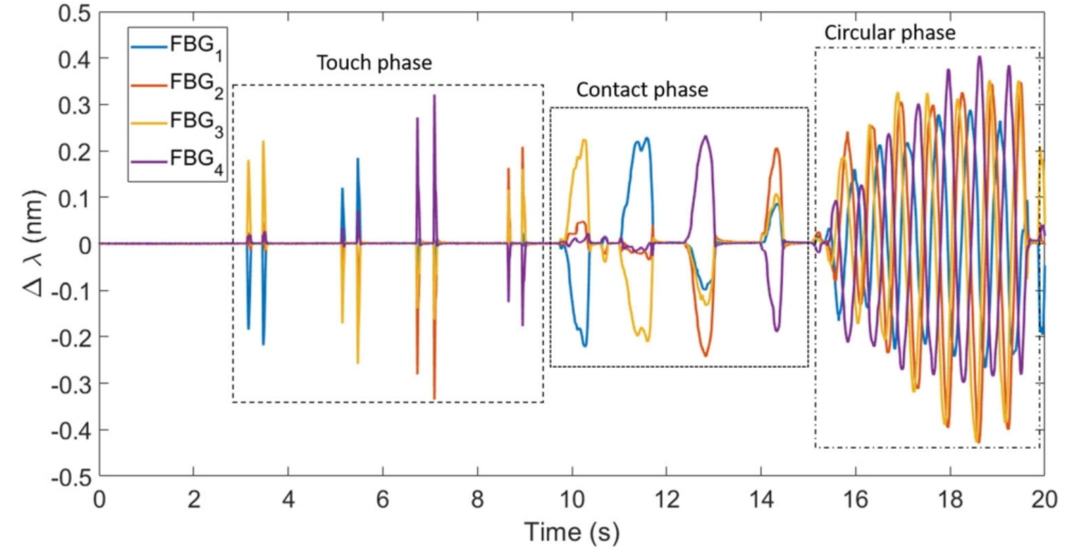
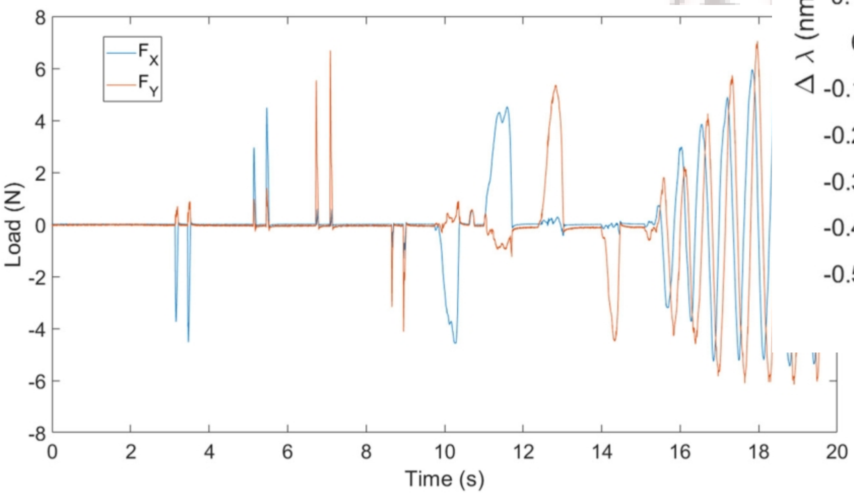
Some research applications







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Fiber Bragg Gratings - Reference

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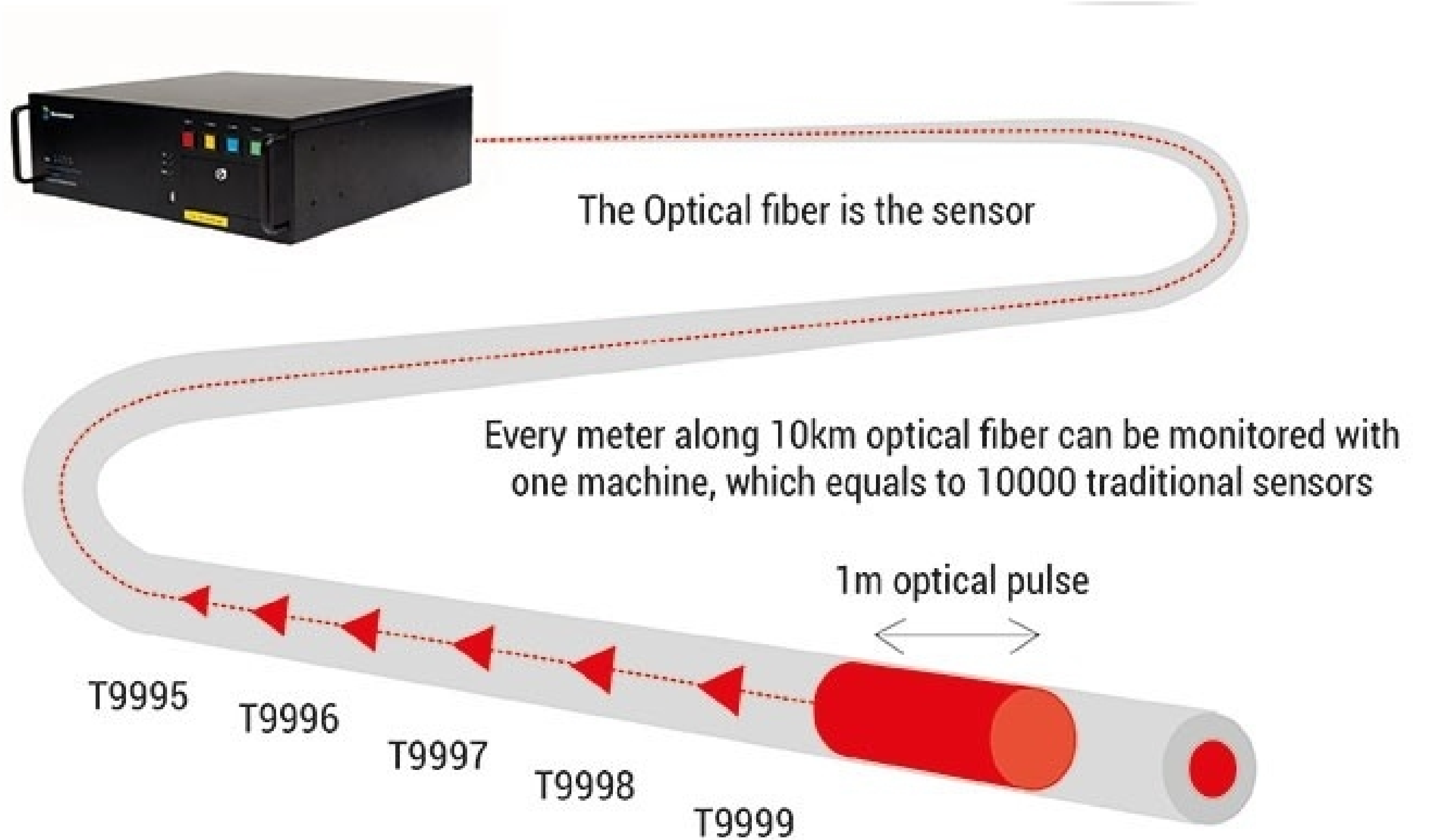
http://www.aos-fiber.com/

Why Fiber Optic Sensors

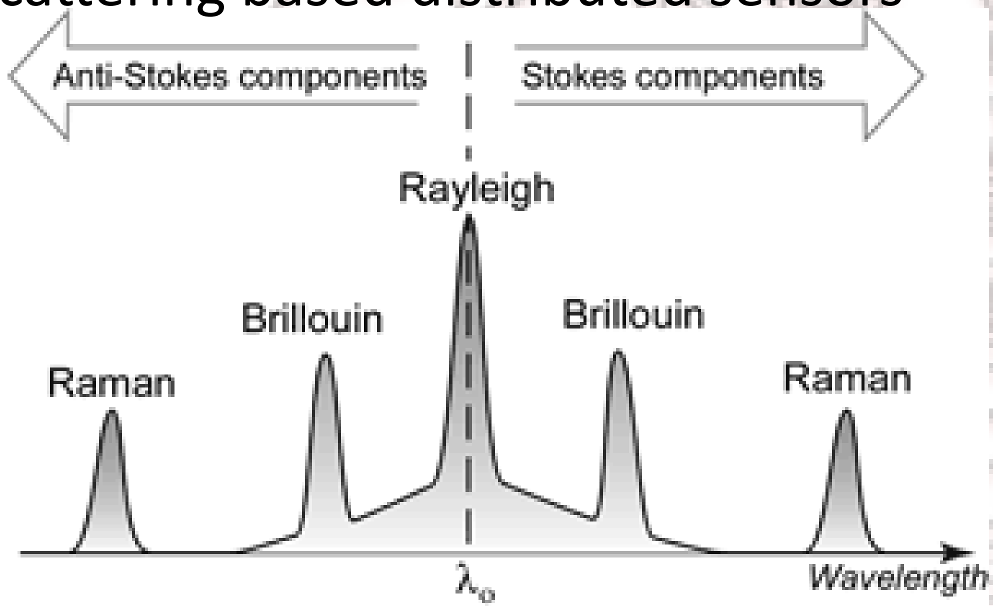
- Light properties in fiber are sensitive to environmental disturbance
- Optical fiber easily to be configured into host material or structure and not degrade the host
- Not sensitive to EMI
- Can reach high T
- Robust to harsh environment
- Do not require electrical power at the sensing point
- Point sensors can be multiplexed in time, wavelength and space domains (quasi-distributed sensing)
- ***Distributed sensing***

Why distributed sensors?



Examples of Fiber Sensors

- FBG-based sensors
- Distributed backscattering-based sensors
- Raman scattering based distributed sensors
- Brillouin scattering based distributed sensors
- Rayleigh scattering based distributed sensors



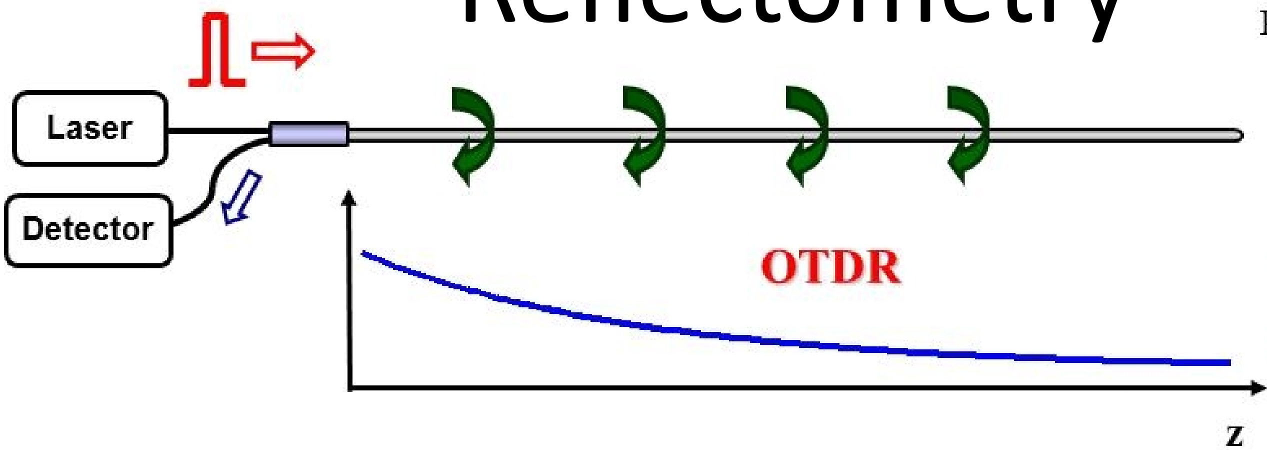
Introduction to Distributed Fiber Optic Sensors

Distributed fiber sensors allow the measurement of physical properties along fiber length (temperature, strain, etc.)

based on optical reflectometry

- ✓ ***Optical Time Domain Reflectometry (OTDR)***
- ✓ ***Optical Frequency Domain Reflectometry (OFDR)***
- ✓ ***Coherent Optical Frequency Domain Reflectometry (C-OFDR)***
- ✓ ***Optical Low Coherence Reflectometry (OLCR)***

Optical Time Domain Reflectometry



Pulse width determines the spatial resolution (e.g. 10 ns 1 m sp. res.)

Dynamic range limited by receiver sensitivity

Low SNR at long distances

- To increase SNR at the receiver:
  - Use higher peak power → limited by nonlinear effects
  - Use longer pulses → degrades the spatial resolution

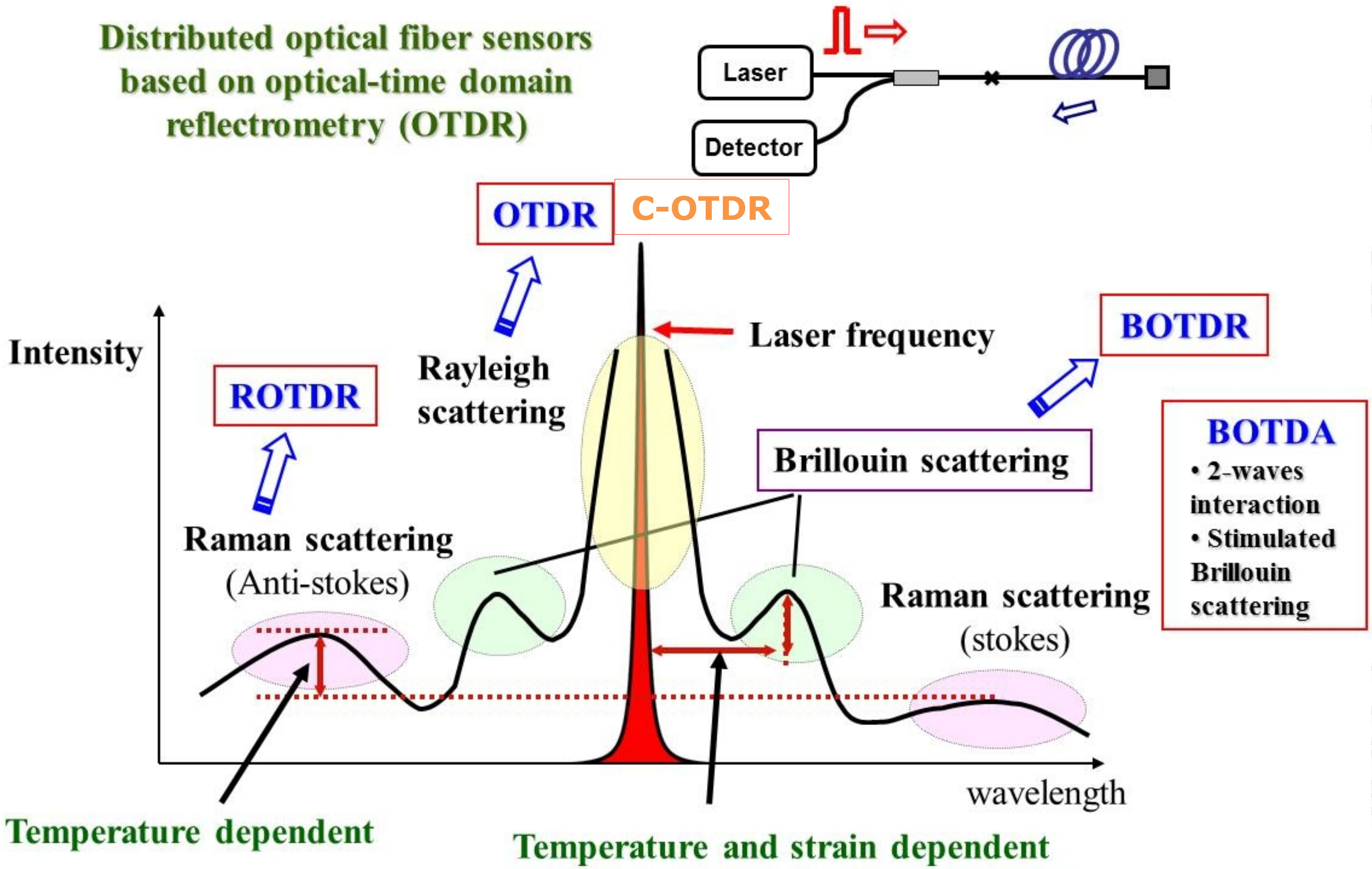
Trade off between distance range and spatial resolution

- To overcome the limitations
  - Use receiver with higher sensitivity (e.g. coherent detectors)
  - Use of optical pulse coding

Coded-OTDR techniques:

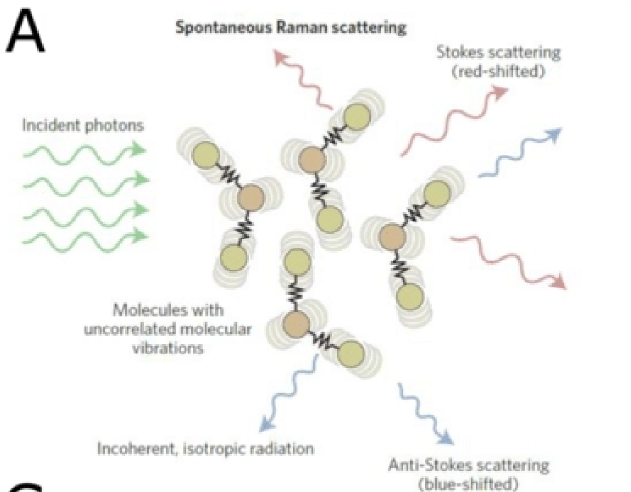
- Spreading signal in time domain
- More optical input power
- It avoids to use high peak power pulses (nonlinearities)
- It allows to improve the SNR with no impact on the spatial resolution

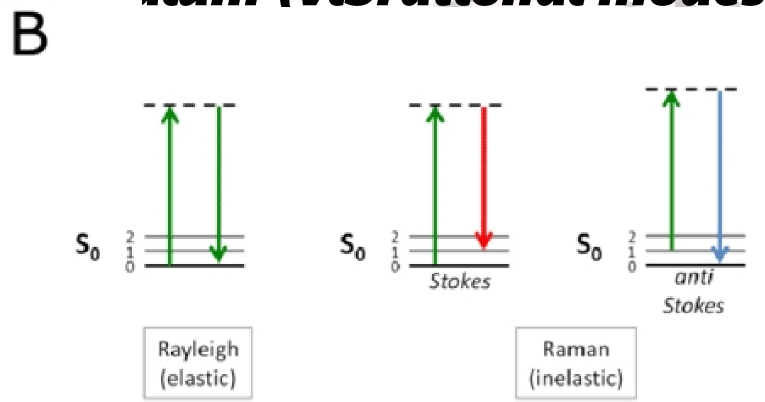
Scattering Phenomena in Distributed OFSs

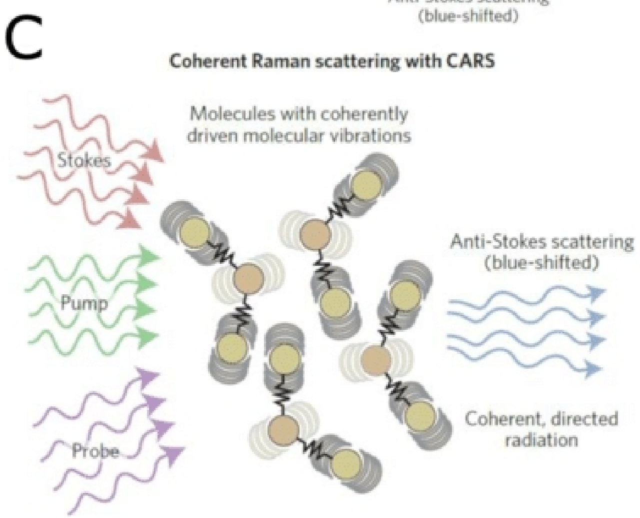


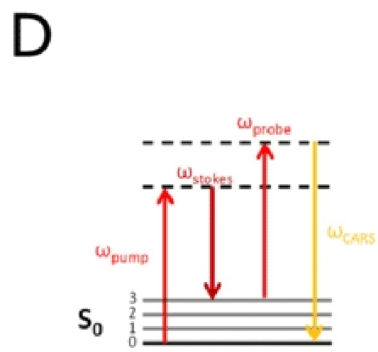
Spontaneous Raman Scattering

Raman scattering is generated by light interaction with resonant modes of the molecules in the dium (vibrational modes)





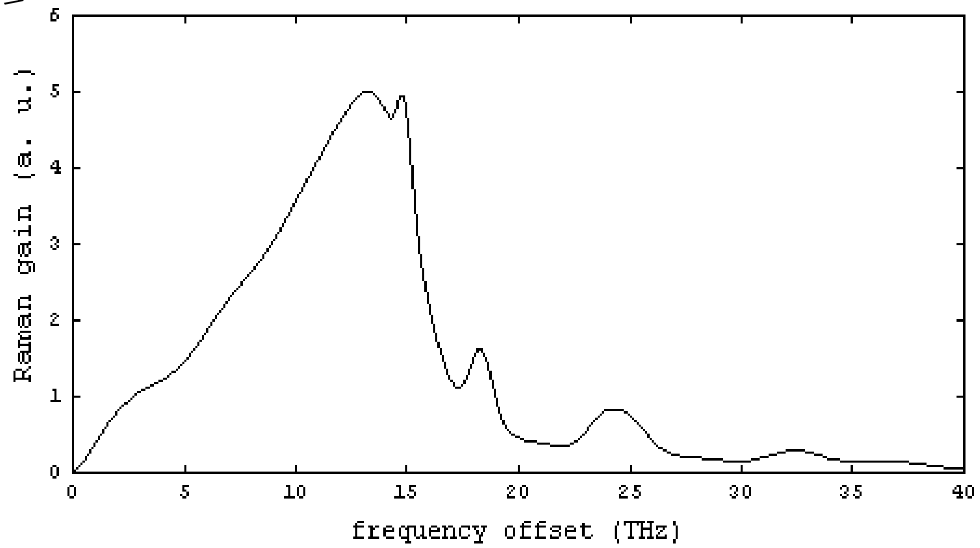




- Phonons interact with photons in inelastic scattering
- Stokes line → photon energy is given to phonon
- Anti-Stokes line → phonon gives energy to photon

Spontaneous Raman Scattering

- *The high energy of vibrational modes induces a large Raman frequency shift (~13 THz in silica fibers)*



- *For each molecular vibration two Raman components are observed:*

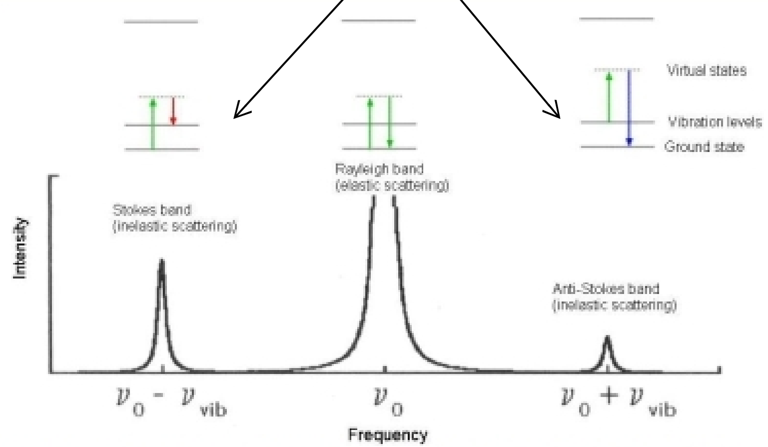
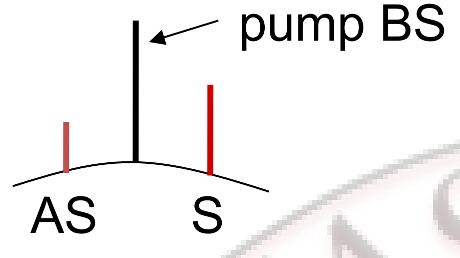


Fig. 4.1.2: Spectrum of photon transition from elemental ground level to virtual states.

- ***At different increasing temperature thermal excitation increases both Raman S and AS (asymmetry between Raman S and AS !)***

Spontaneous Raman Scattering

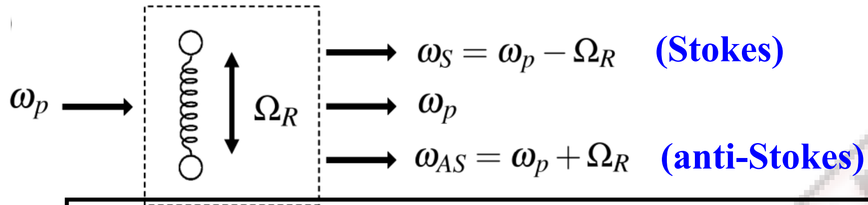


T dependence of phonon population → T dependence of SRS light

Ratio $R(T)$, P_{AS}/P_S (or P_{AS}/P_{BS}) is usually used

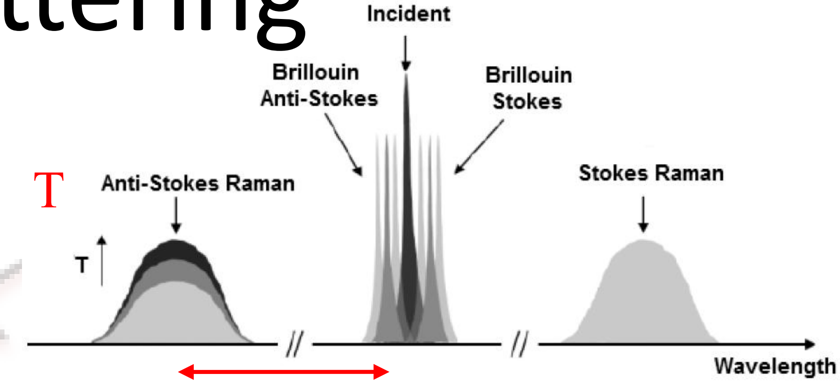
Raman temperature sensitivity: 0.8% K^{-1}

Spontaneous Raman Scattering



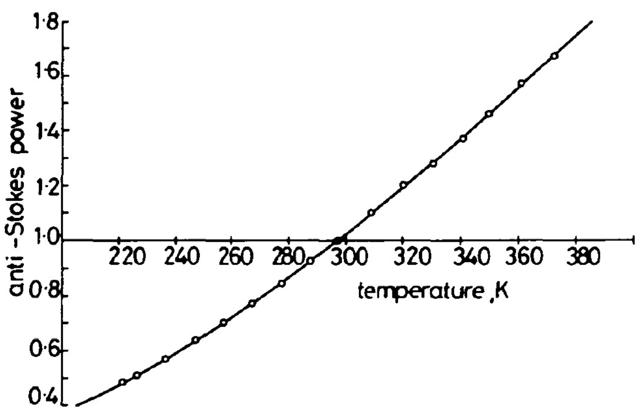
Suitable for Temperature Sensing only

T



12.5 THz
(100 nm @1550 nm)

$$R(T,z) = C_R \, exp\left(-\frac{h\Delta\nu_R}{k_B T(z)}\right)$$

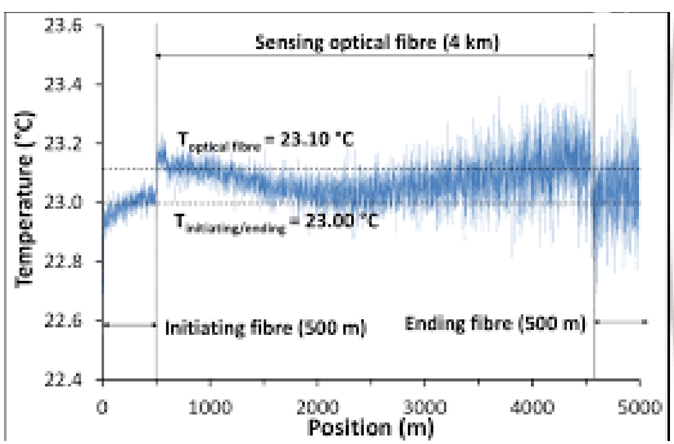


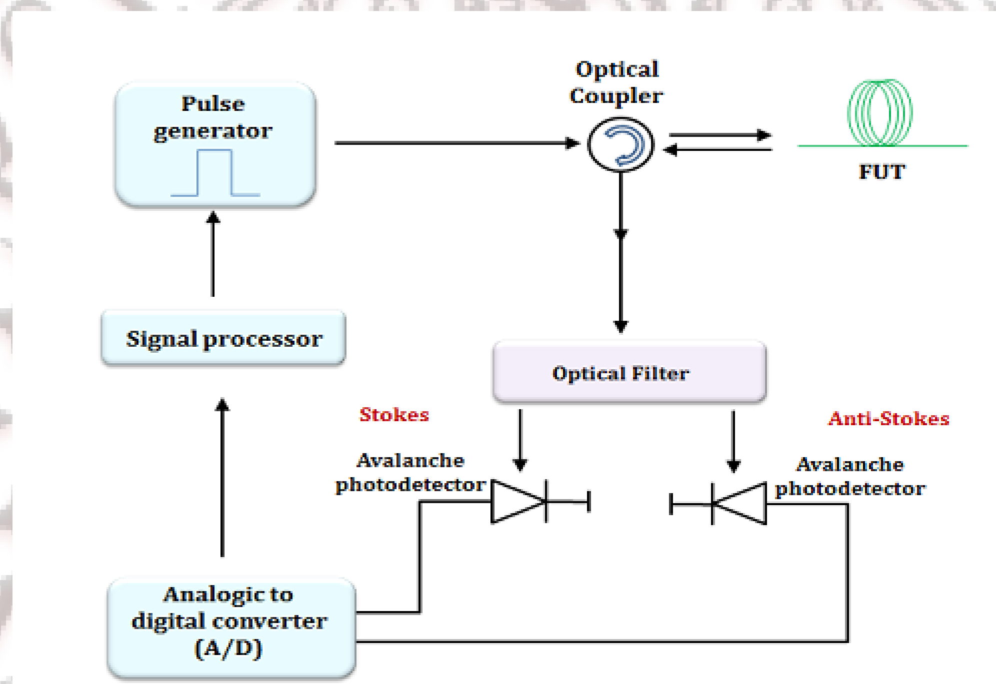
Sensitivity: 0.8 % K^{-1}

Advantages	*Disadvantages*
Easy detection	Low backscattered power
High sensitivity	High input power required

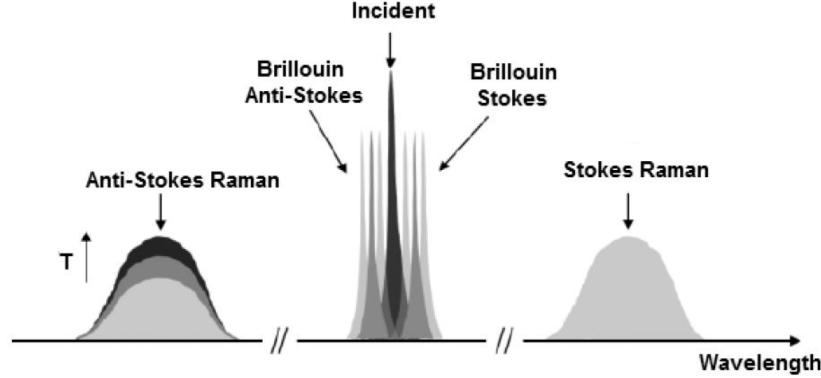
Raman DTS performances

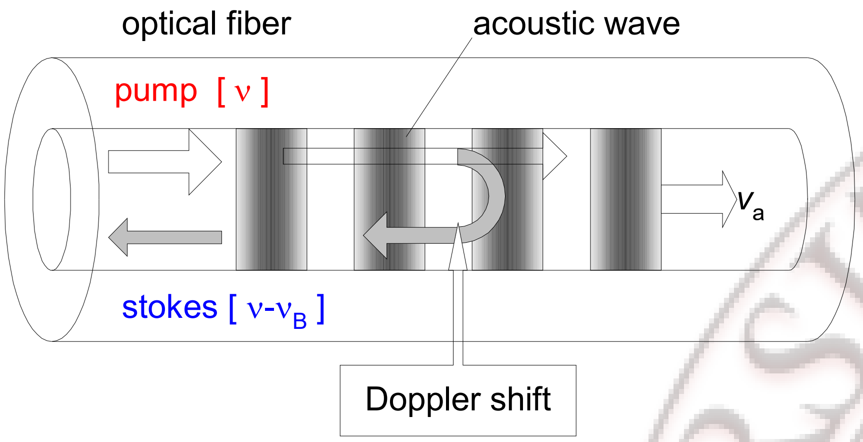
- Only measures temperature and is independent of strain.
- • The temperature resolution is 0.5°C
- • The measurement range is up to 15 km with a 1 meter spatial resolution (up to 25km with a 1.5 meter resolution) of the location of the temperature perturbation



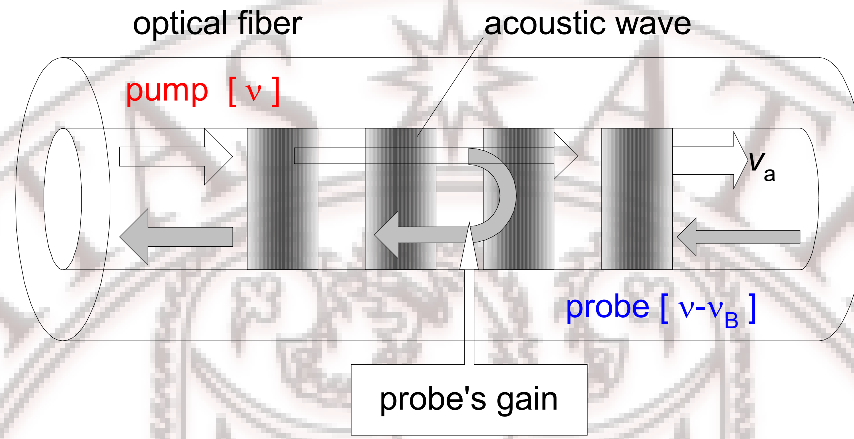


Brillouin





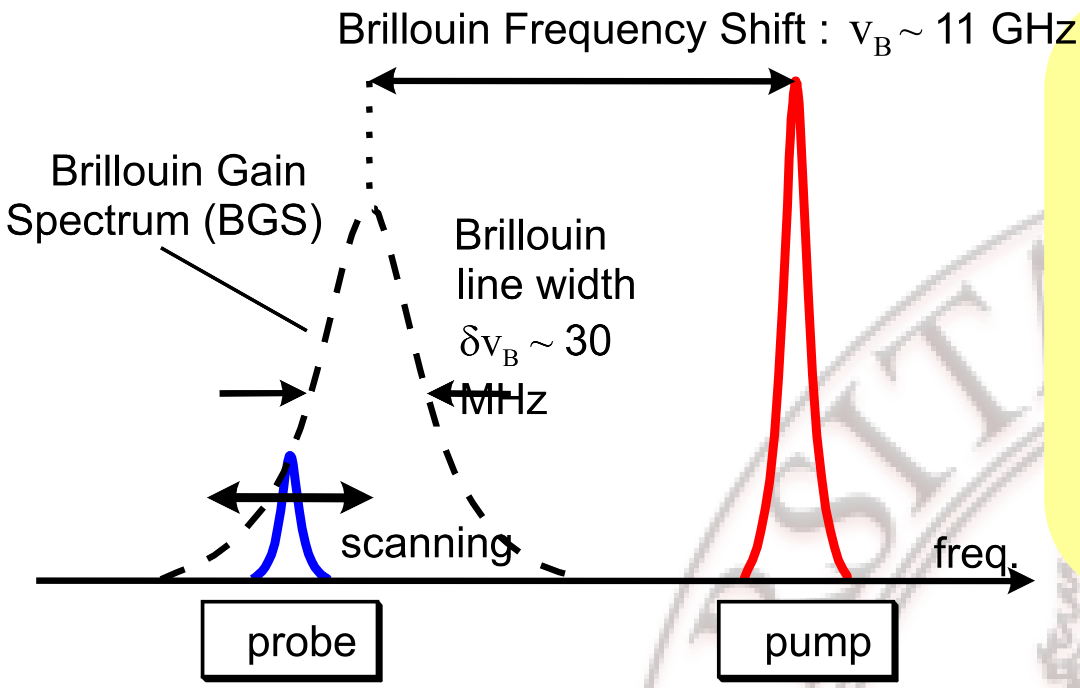
Spontaneous Brillouin Scattering



Stimulated Brillouin Scattering

- Acoustic wave works as a diffraction grating
- Stokes wave's frequency is down-shifted by Doppler shift
- Probe's gain profile is called Brillouin gain spectrum (BGS)

Sensors based on Brillouin Scattering



Brillouin frequency shift

$$\nu_B = \frac{2nV_a}{\lambda_p}$$

n: refractive index

V_a: acoustic wave velocity

λ_p: pump wavelength

- Brillouin frequency shift and intensity change linearly against tensile strain and temperature

$C_{P_BT} = 0.36\,\%\,°C^{-1}$

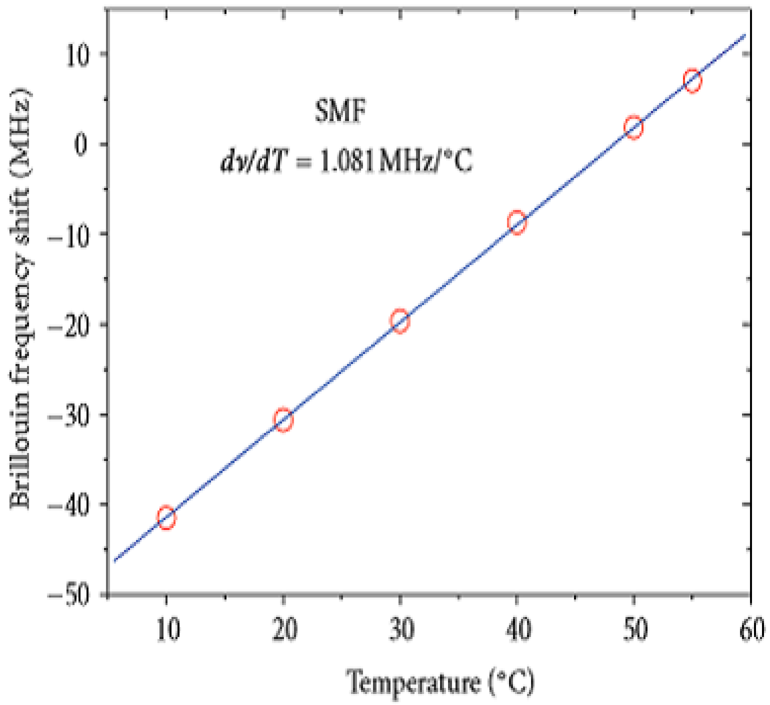
$C_{P_B\varepsilon} = -9\times10^{-4}\,\%\,\mu\varepsilon^{-1}$

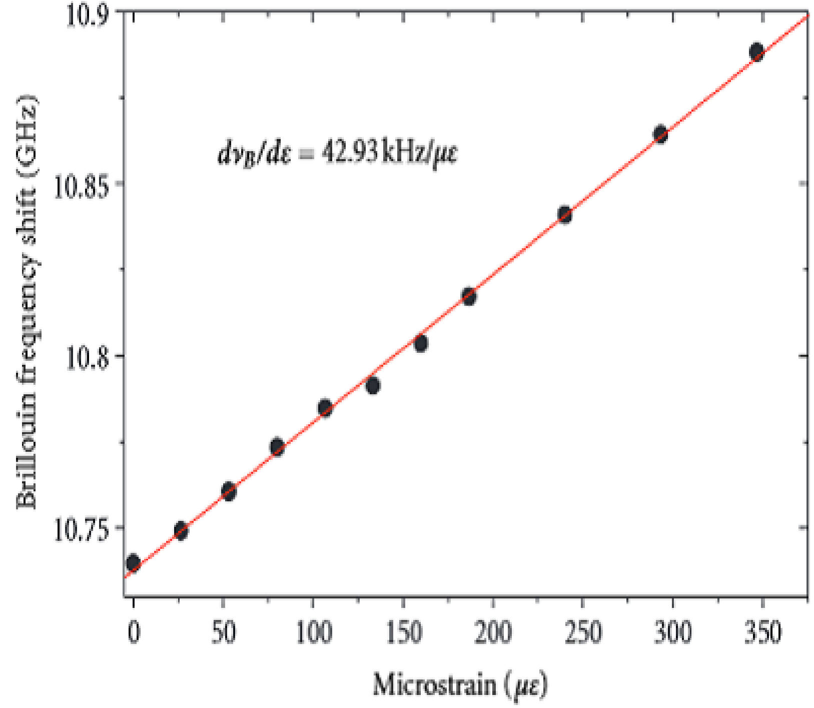
$C_{\nu_BT} = 1.07\,MHz\,°C^{-1}$

$C_{\nu_B\varepsilon} = 0.048\,MHz\,\mu\varepsilon^{-1}$

$$\begin{bmatrix}\Delta\nu_B \\ \Delta P_B\end{bmatrix} = \begin{bmatrix}C_{\nu_B\varepsilon} & C_{\nu_BT} \\ C_{P_B\varepsilon} & C_{P_BT}\end{bmatrix}\begin{bmatrix}\Delta\varepsilon \\ \Delta T\end{bmatrix}$$

Typical Brillouin frequency shift in an SMF



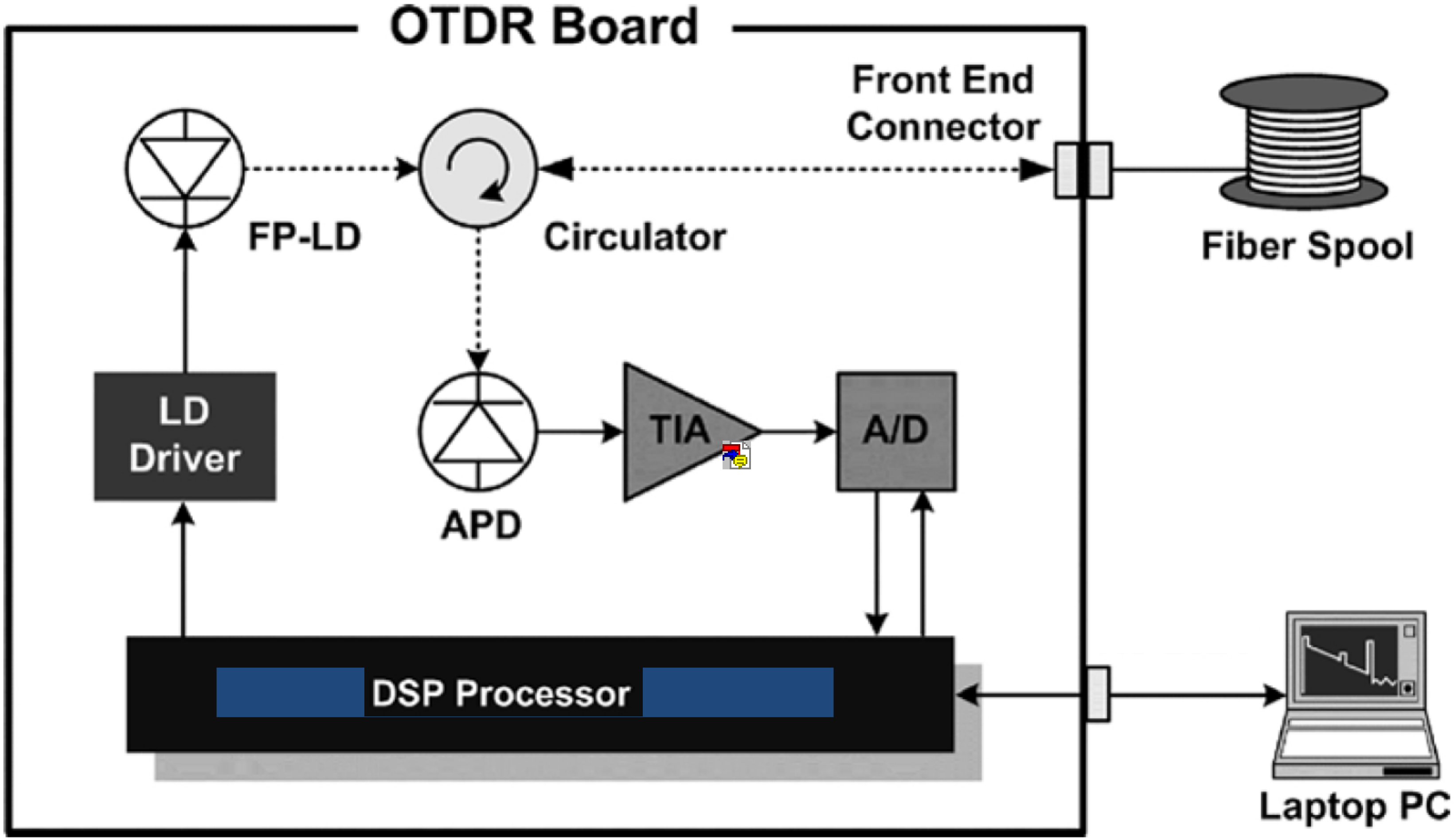


Optical time-domain reflectometry

Optical Time Domain Reflectometry

- ❑ Overview of OTDR
- ❑ Dynamic range, dead-zones and resolution
- ❑ OTDR signal analysis
- ❑ OTDR measurements
- ❑ Use in optical communication systems

Typical OTDR Scheme

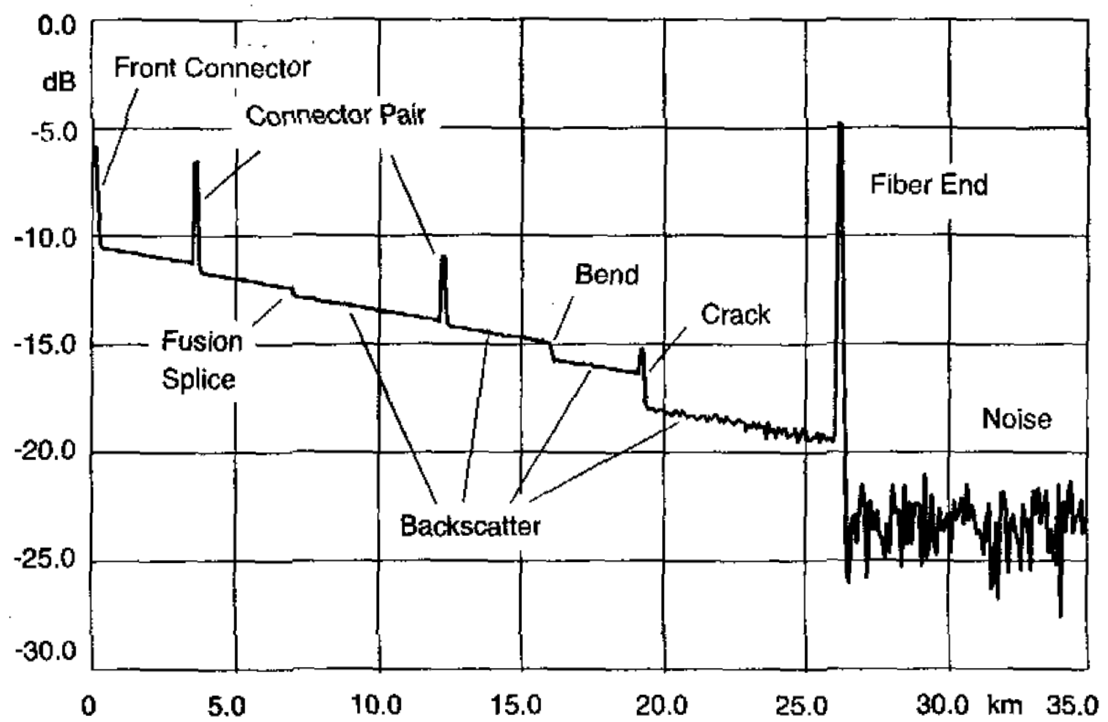


- Pulse widths between 5 ns and 1 μs (0.5 m to 100 m spatial resolutions)
- Circulator, 3 dB coupler, optical filters for detecting backscattered light
- Pin or APD photoreceiver, 1310 nm or 1550 nm, 1620 nm wavelengths

OTDR Principles

- Analog-to-digital-converter (ADC) is the interface from analog to digital signals
- The ADC sampling rate determines the spatial resolution of adjacent data samples (50 MHz sampling rate corresponds to a data spacing of ~ 2 meter)
- It is not practical to increase the resolution by increasing the sampling rate
- Interleaved processing schemes are usually used to improve the spatial resolution
- High bandwidth is required in the signal processing to avoid smoothing of the final results
- As the backscattered signal is usually weak averaging is used to improve the SNR

Example of OTDR Trace



- Time delay with respect to pulse launch is converted into distance:

$$T = \frac{2L}{v}$$

$$v = \frac{c}{n} \approx \frac{3\times10^8 m/s}{1.5} = 2\times10^8 m/s$$

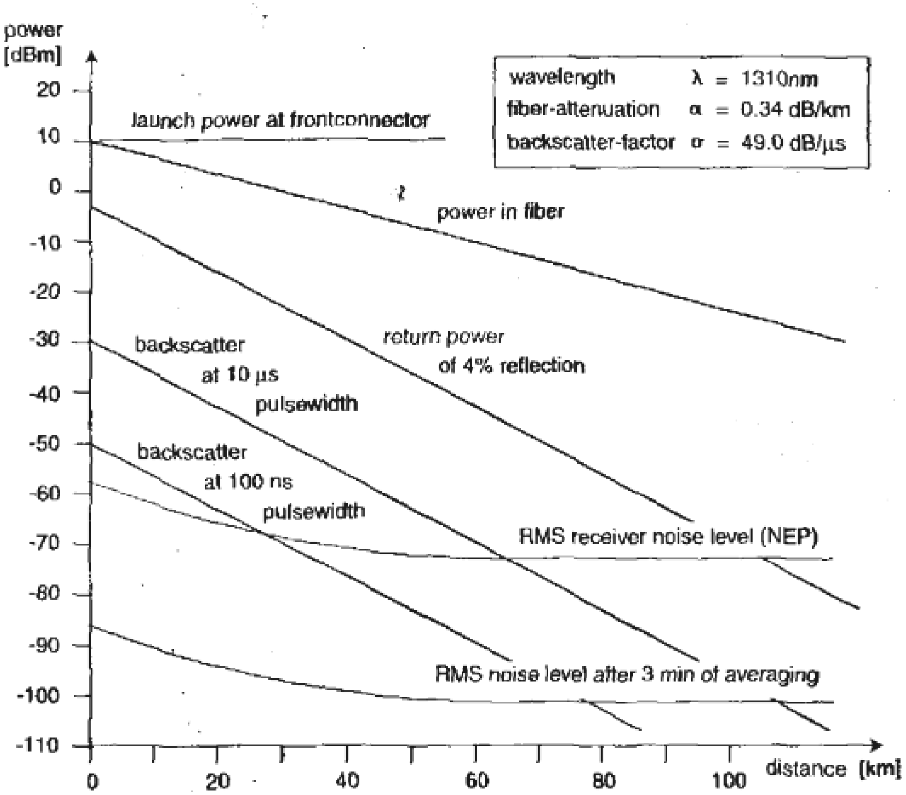
Round trip propagation delay T: $\boxed{T \sim 10\ \mu s/km}$

- One-way OTDR diagrams: $5\cdot\log_{10}(P)$
- X-axis accuracy: exact timing, group index, cabling factor $\Delta T_{acc} \sim 0.01\%$

Main Features of OTDR Traces

- Distributed Rayleigh backscattering gives rise to
- straight lines in OTDR traces
- Non-reflective events give rise to positive or negative steps
  - - fusion splices
- - bends
- Reflective events give rise to positive spikes
- - mechanical splices
- - connectors
- - cracks
- Fresnel reflection at fiber end connector is about 4% of incident light

Example of Backscattered Power Levels in OTDR



Not OTDR diagram!

- +13 dBm launch power, 3dB coupler
- Backscatter level depends on pulse width
- Slope of curves related to attenuation coefficients
- Noise is reduced by averaging

Time Averaging

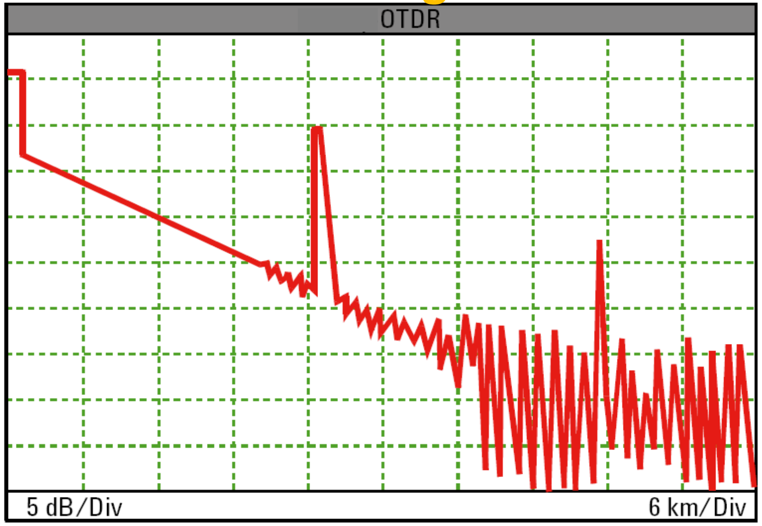
- SNR improved with time averaging
- Longer acquisition time
- Noise amplitude depends on number of acquired samples N as: $1/\sqrt{N}$

σ_{noise} Not averaged

$\frac{\sigma_{\text{noise}}}{\sqrt{N}}$ Averaged

- SNR improves with averaging samples proportionally with $\sqrt{N}$
- SNR specified typically after 3 min averaging time

no avg



3 min avg

Example of Time Averaging

*A 10 km fiber link is tested with an OTDR at 1300 nm. Compute the noise reduction that can be achieved by signal averaging compared to a single shot measurement within the first second and after 3 min, considering that **10%** of the time is needed for processing overhead.*

Round trip time for a 10 km fiber: $T = 10\frac{\mu s}{km} \times 20km = 200\mu s$

Number of averages in 1 sec: $N_{1s} = \frac{1}{200 \times 10^{-6}} \times 0.9 = 4500$

Number of averages in 3 min: $N_{3\min} = 4500 \times 180 = 810000$

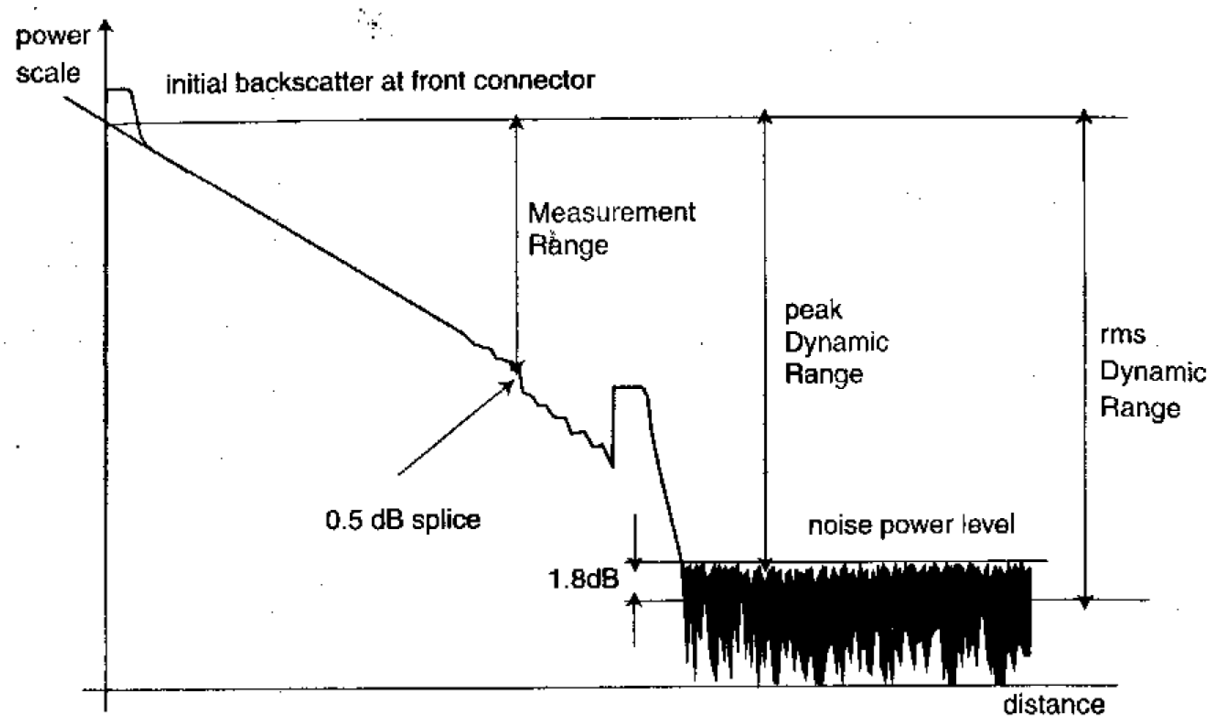
The noise reduction is proportional to the square-root of N:

$$\Delta SNR_{1s} = 10 \times \log(\sqrt{N_{1s}}) = 5 \times \log(4500) = 18.3dB$$

$$\Delta SNR_{3\min} = 10 \times \log(\sqrt{N_{3\min}}) = 5 \times \log(810000) = 29.5dB$$

Dynamic Range

- DYNAMIC RANGE: difference between initial backscatter level and noise level in 'one-way' decibel

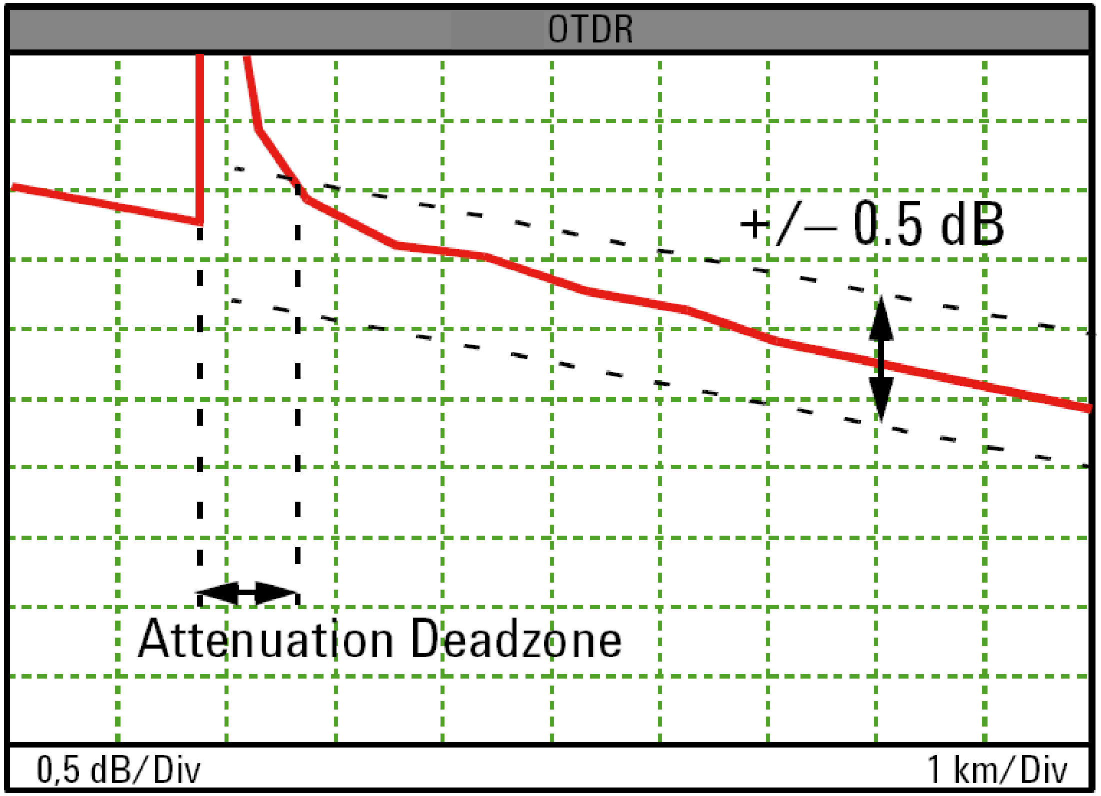


Noise level specified typically after 3 mins of measurement time

Measurement range ≠ dynamic range (measurement range deals with identification of events: 0.5 dB splice is usually chosen as the event to be identified). It is the maximum attenuation that can be inserted between the OTDR and an event for which the OTDR is still able to measure the event.

Attenuation Deadzone

- Reflective events cause saturation of the receiver, with recovery time inversely proportional to the receiver bandwidth



- ATTENUATION DEADZONE: distance from start of a reflective event and recovery within 0.5 dB of backscatter trace

Dynamic Range and Resolution

Spatial resolution: ability to detect two different events with a given distance spacing

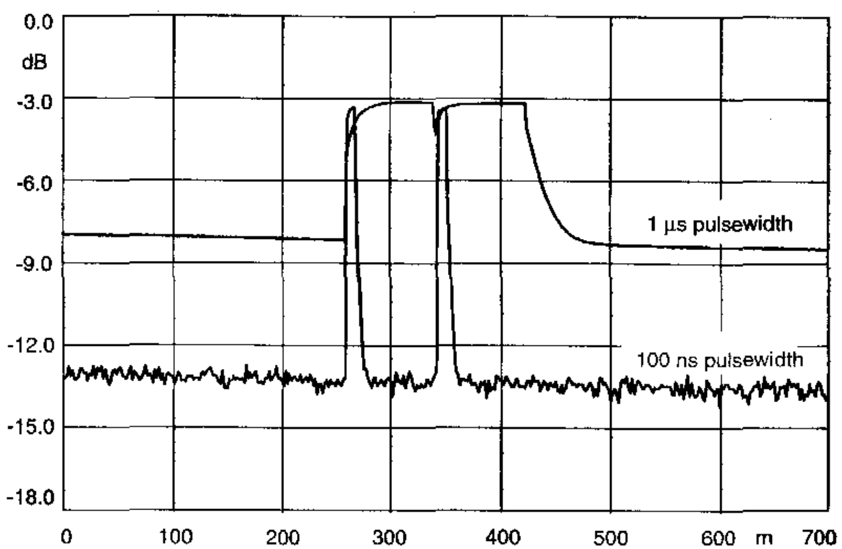
Spatial resolution typically defined as one-way distance from 10% to 90% power increase for a reflective event

RISE TIME
RESPONSE TIME

Trade-off between dynamic range and resolution: $s(t) = p(t) \otimes f(t) \otimes r(t)$

Received signal as convolution of the probing pulse p(t), backscattering impulse response of the fiber f(t) and impulse response of the receiver r(t)

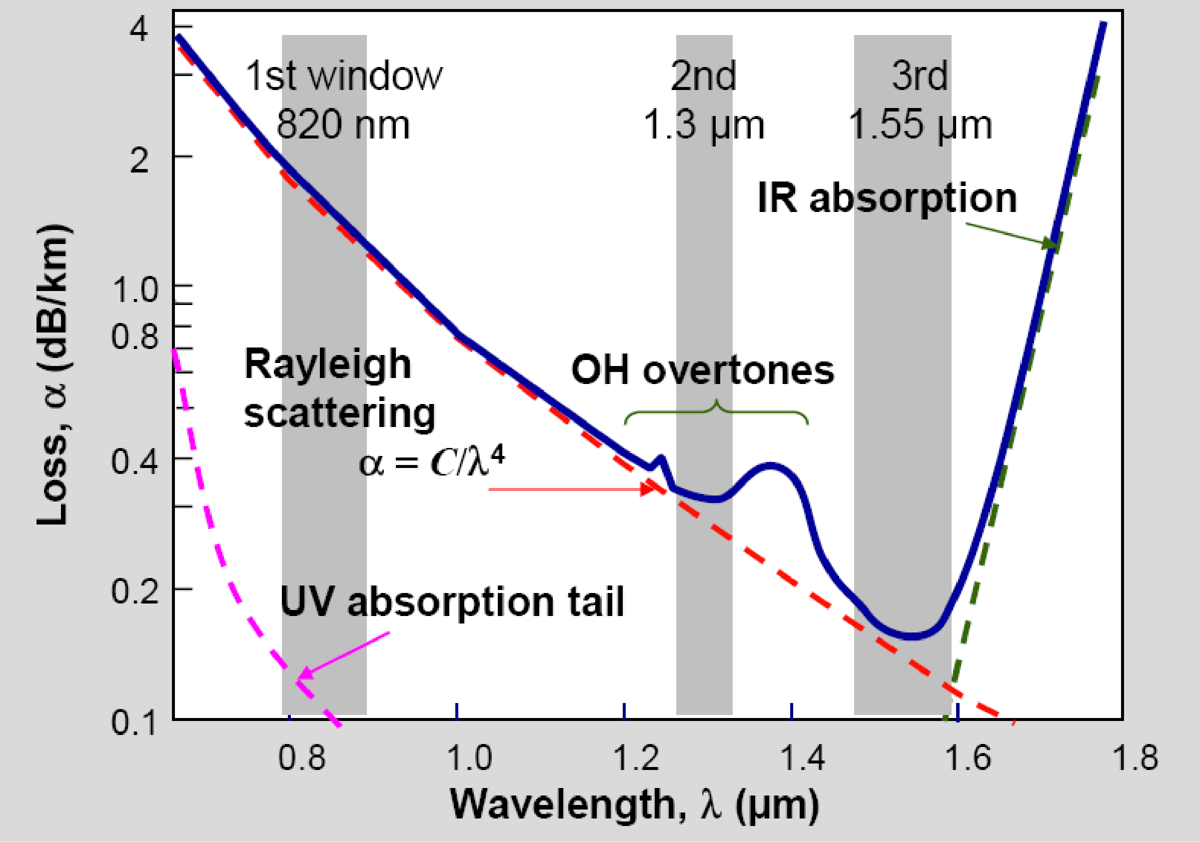
EXAMPLE



Longer pulses → higher dynamic range

→ less spatial resolution

Fiber Attenuation and Scattering - Repeat



Attenuation due to:

- Material absorption
- Bending losses
- Rayleigh scattering

$$\alpha = \alpha_R + \alpha_M + \alpha_{MB} + \alpha_{mB}$$

Evolution of optical power along fiber length *z*:

$$P(z) = P_0 \cdot e^{-\alpha_\lambda z}$$

In dB: $$P_{\mathrm{dBm}}(z) = P_{0\_\mathrm{dBm}} - \alpha_{\mathrm{dB/km}} \cdot z$$

Loss coefficient

$$\boxed{\alpha_{\mathrm{dB/km}} = \alpha \cdot 10 \log_{10} e = 4.343 \cdot \alpha}$$

Rayleigh Scattering

Rayleigh scattering:

Microscopic variations of refractive index n
with a spatial scale << than signal λ cause
scattering of lightwave signal in all directions → power loss

Rayleigh scattering absorption coefficient in Silica fibers:

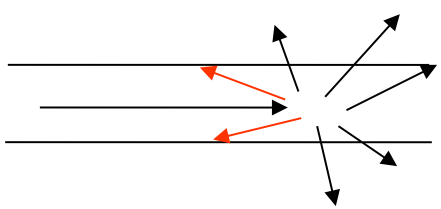
$$\alpha_s = \frac{(0.76 + 0.51\cdot\Delta n)}{(\lambda/\mu m)^4}\left[\frac{\text{dB}}{\text{km}}\right] \longrightarrow \alpha_s = \frac{c}{\lambda^4}$$

C: constant

3rd window: $a_s = \alpha_R \sim 0.1$ dB/km, absorption is dominated by Rayleigh scattering

Signal backscattering:

A fraction of scattered signal is collected by fiber NA in backward propagating direction



Rayleigh back-scattering coefficient g :

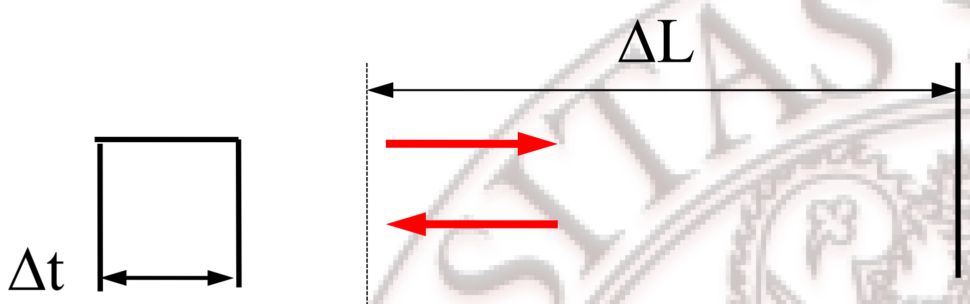
$$\gamma = S \cdot \alpha_R$$

S: capture factor (fiber geometry, NA)

$$S = \left(\frac{NA}{n_0}\right)^2 \cdot \frac{1}{m}$$

m~4.5 for SMF, S~10^{-3} *g* ~ 10^{-4} 1/km

Optical Time-Domain Reflectometry



$$\Delta L = c\Delta t\,/(2n)$$

Spatial resolution is basically determined by the pulse-width.

In glass (fiber):

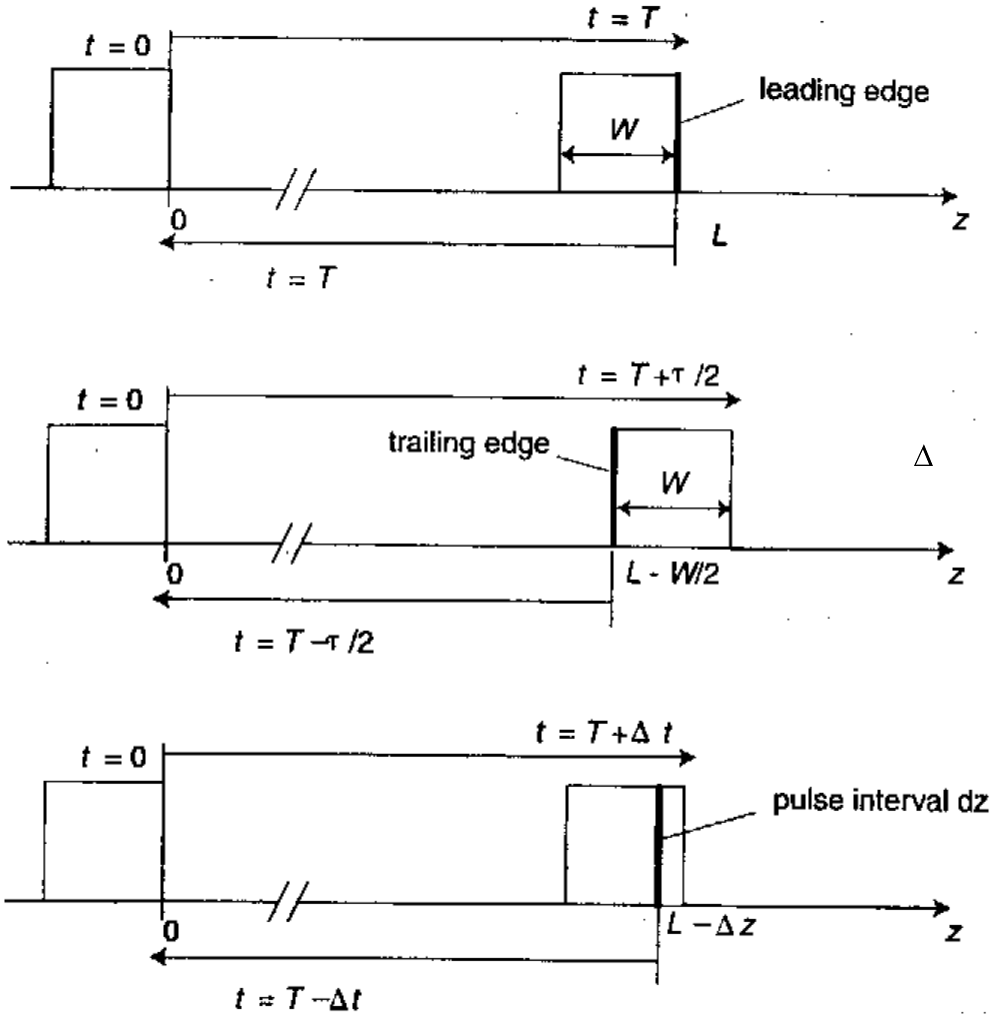
10 ns → 1 m

1 ns → 10 cm

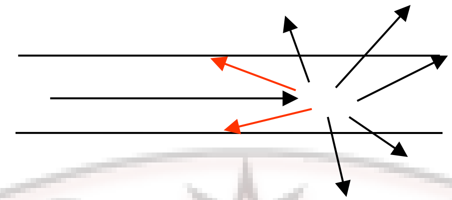
1 ps → 0.1 mm

$c = 3\times10^{8}$ m/s

Analysis of OTDR Traces



Δ



Pulse spatial width
$W = \tau \cdot v_g$

Group velocity

Pulse duration

$\gamma = S\alpha_s$

α_s: backscattering coeff. ~$1/\lambda^4$

S: capture factor

Scattered power at position z with infinitesimal length interval dz:

$$dP_s(z) = \gamma \cdot P(z) \cdot dz$$

Analysis of OTDR Traces

For short pulses the backscattered power is proportional to the pulse duration τ

Backscatter factor σ :

$$\sigma(dB) = -10\log(S\alpha_s W) = -10\log(S\alpha_s \tau v_g) \ [dB/\mu s]$$

$$W = \tau \cdot v_g$$

W: spatial width

v_g: group velocity

TYPICAL VALUES $\sigma = -10 \cdot \log(S \cdot \alpha_s \cdot W)$

λ [nm]	Fibertype	α_s [km^{-1}]	S	σ [dB/1 μs]	η [W/J]
850	MM-SI 50μ	$3.5 \cdot 10^{-1}$	$1.1 \cdot 10^{-2}$	31	385
1300	MM-GI 62.5μ	$6.5 \cdot 10^{-2}$	$1.0 \cdot 10^{-2}$	38	65
1300	MM-GI 50μ	$6.5 \cdot 10^{-2}$	$5.0 \cdot 10^{-3}$	41	32
1310	SM 9μ	$6.3 \cdot 10^{-2}$	$1.0 \cdot 10^{-3}$	49	6.3
1550	SM 9μ	$3.2 \cdot 10^{-2}$	$1.0 \cdot 10^{-3}$	52	3.2

Example: 100 km fiber is probed with an OTDR having a peak output power of 13 dBm and the pulsewidth is 10 μsec.

Calculate the backscattered power returning from the far end of the fiber having attenuation of
0.33 dB/km, a scattering coeff. α of 0.3 dB/km and capture factor S=10^{-3} at 1300 nm.

$$P_s(L) = S \cdot \alpha_s \cdot W \cdot P_0 \cdot e^{-2\alpha L}$$

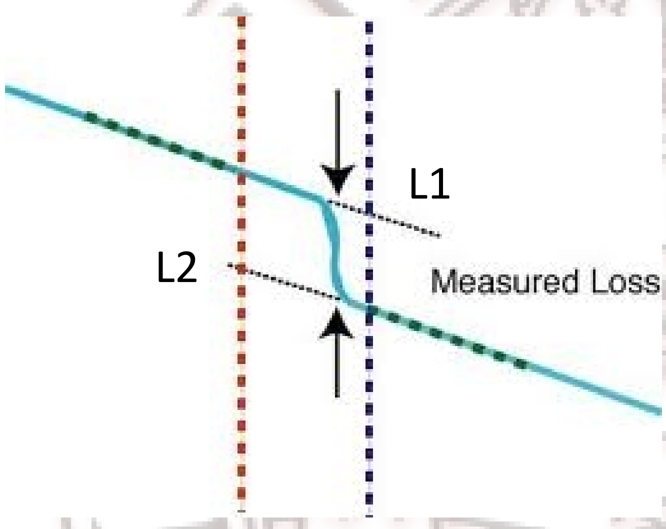
$$P_s(100km) = 0.001 \times (0.3 \times 0.23) \times 2 \times 20mW \times e^{-2\times(0.33\times0.23)\times100} =$$
$$= 35.3 \times 10^{-12} \times 20mW = 0.75\,pW = -91.5dBm$$

$$W = \tau \cdot v_g = 2km$$

Measurement of Splices and Connectors

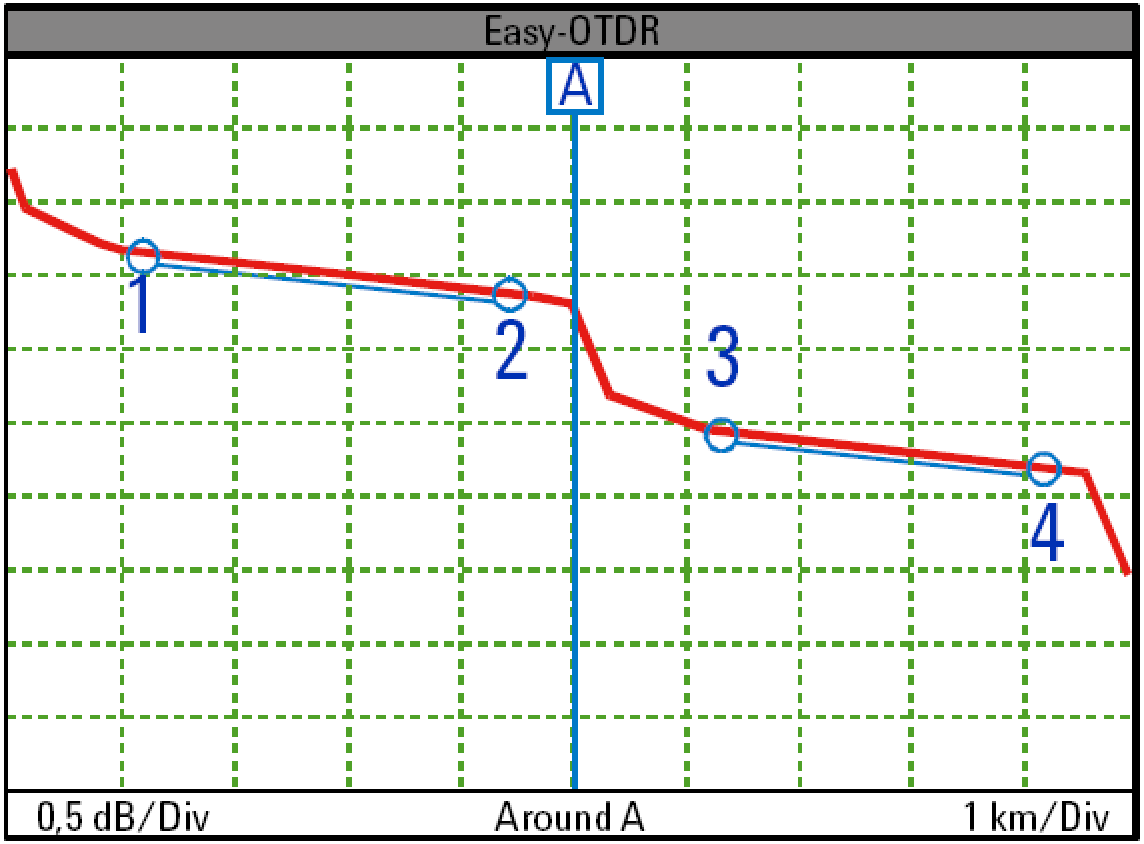
OTDR can be used to measure loss from splices, bending and connectors

Different Rayleigh backscattering coefficients before and after an event can affect the inserction loss measurement accuracy



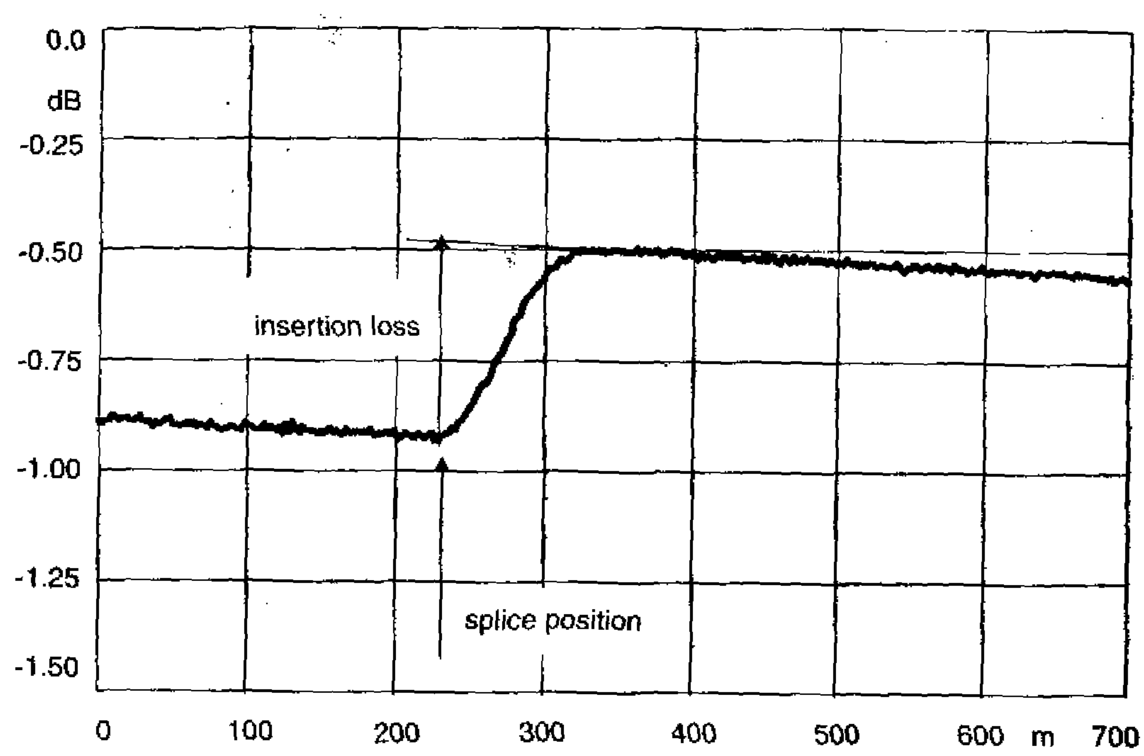
Least-square-approximation is usually used to determine the slopes and positions of the two lines L1 and L2

Splice Loss Measurement



- ❑ Least mean square approx of attenuations
- ❑ Backscatter coefficients should be the same for the two fibers

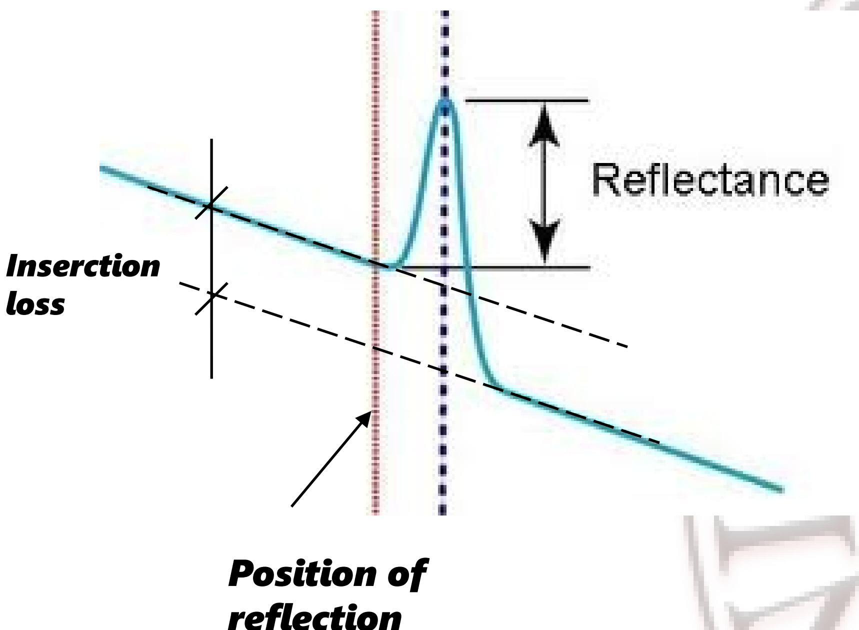
Splice Loss Measurement

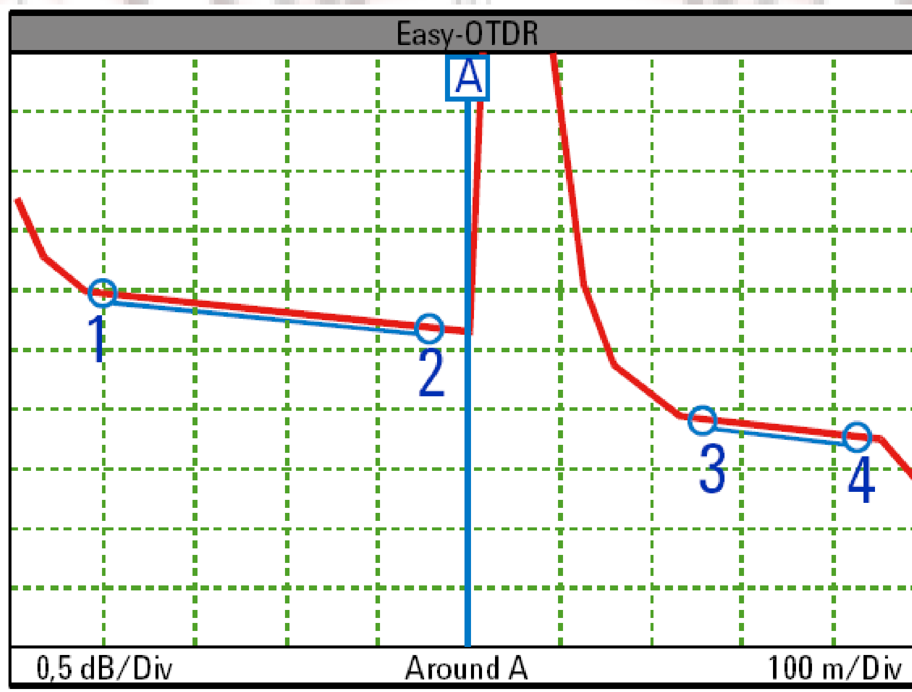


- If Back Scattering coefficients are not the same (different types of fiber), then ‘gain’ can be seen by OTDR

Insertion Loss Measurement

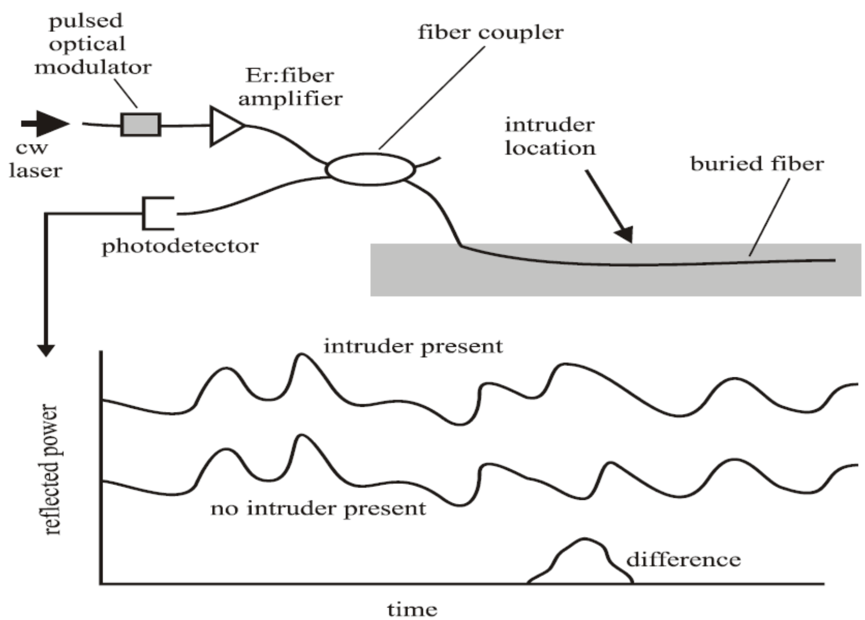
Air gap of a tiny crack, a mechanical splice, a connector are examples of reflective events.
Also misalignement of connectors, mismatch in core dimensions or NA can induce additional losses

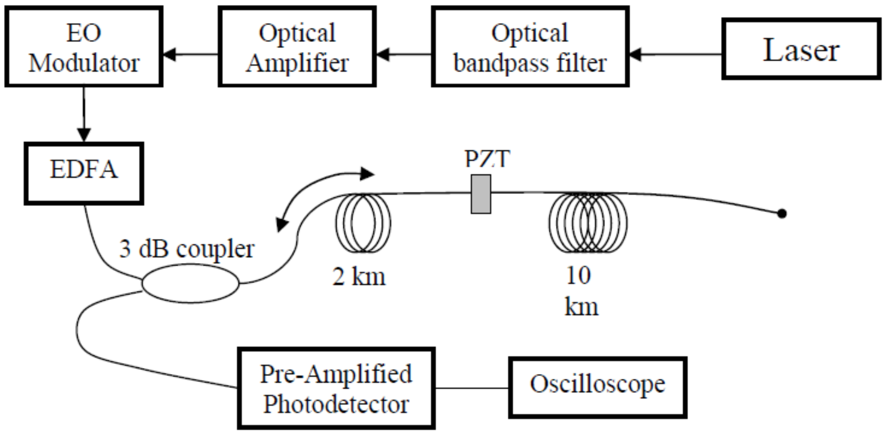


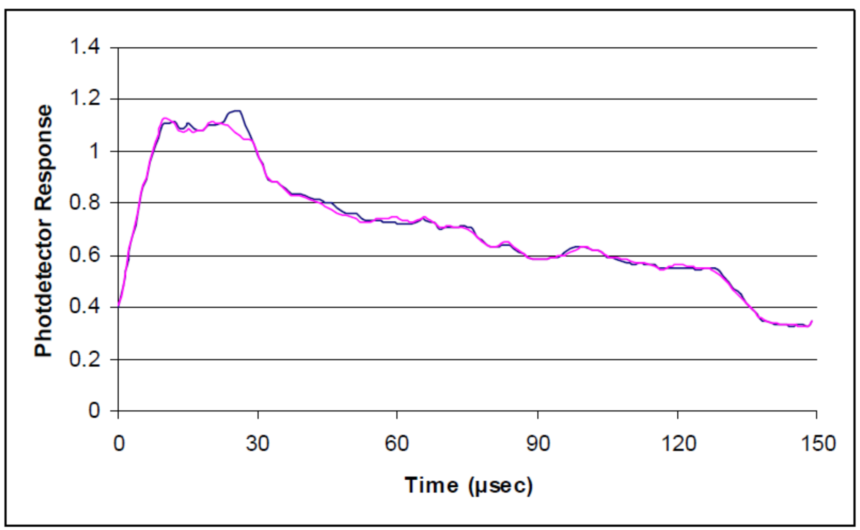


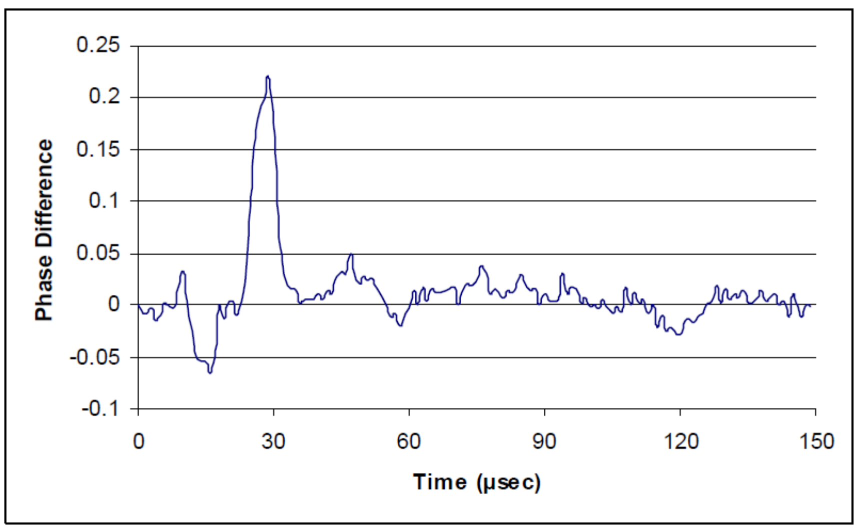
Extrapolation of reflection point for IL measurement in a reflective event

Other types of OTDR - Phase sensitive OTDR

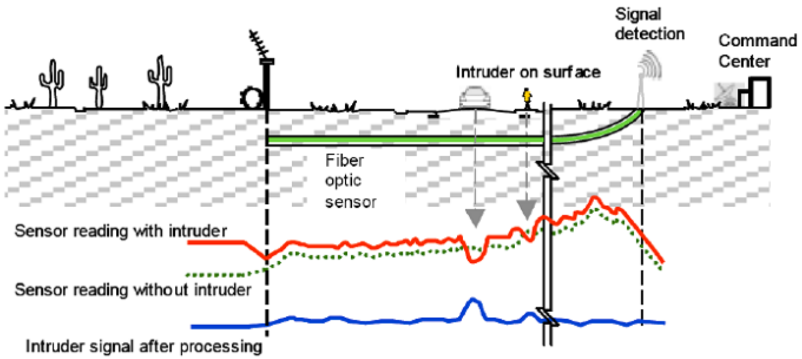


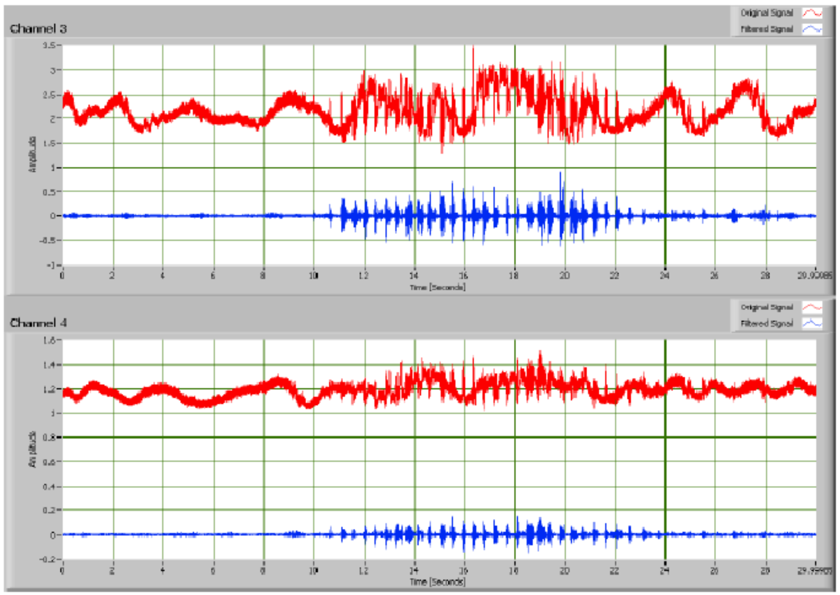




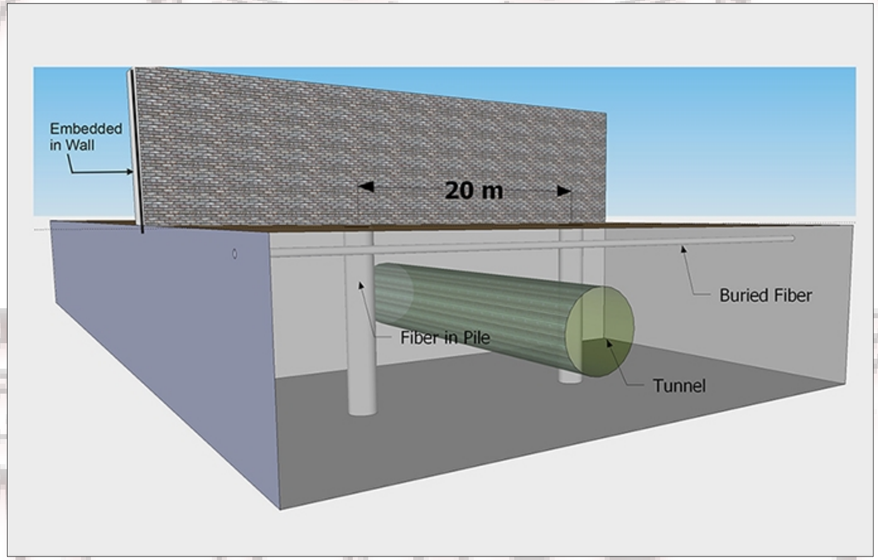


Intruder detection – border control



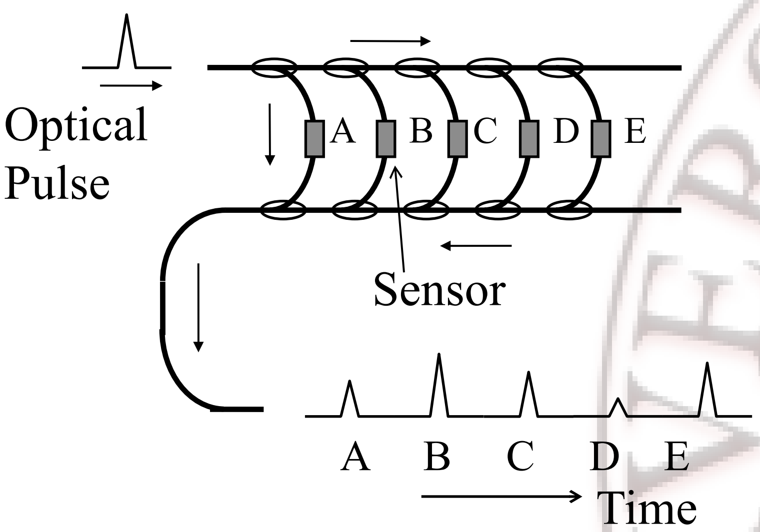


- Extracted from "Border Security Utilizing a Distributed Fiber Optic Intrusion Sensor"

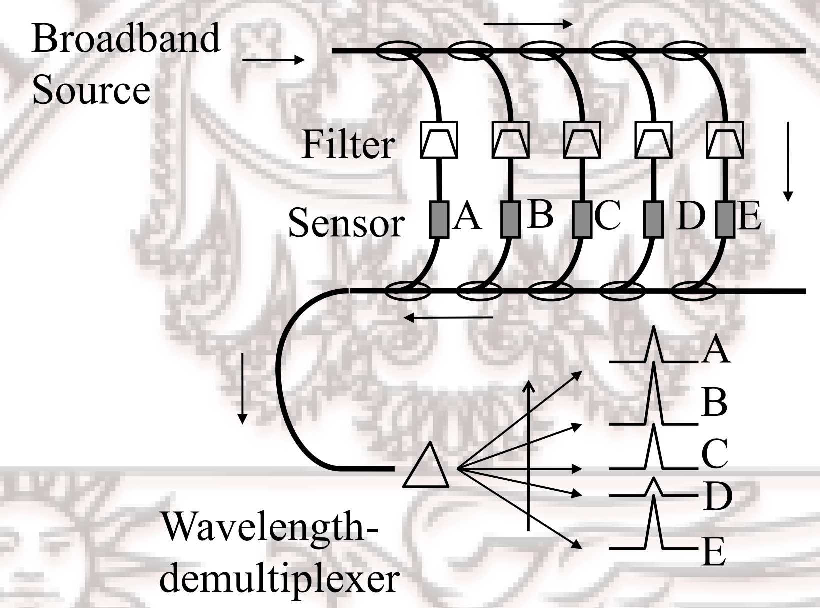


Approaches for Multiplexed Fiber Sensors

Time Division Multiplexing （TDM)



Wavelength Division Multiplexing （WDM)

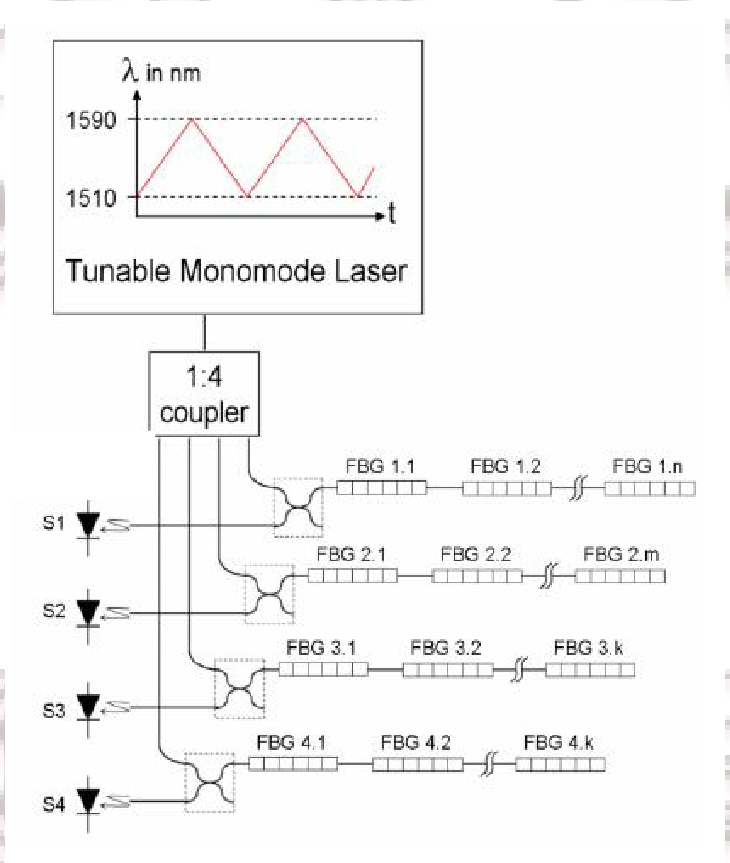


- Limited by power budget (distance and losses)

Approaches for Multiplexed Fiber Sensors

Space and Wavelength Division Multiplexing (WDM/SDM)

- Comparison with FBG sensors
- In general points of sensing are on specific critical points
- ➔ Easier implementation



Other types of optical fiber sensors

- Fabry Perot
- Interferometers

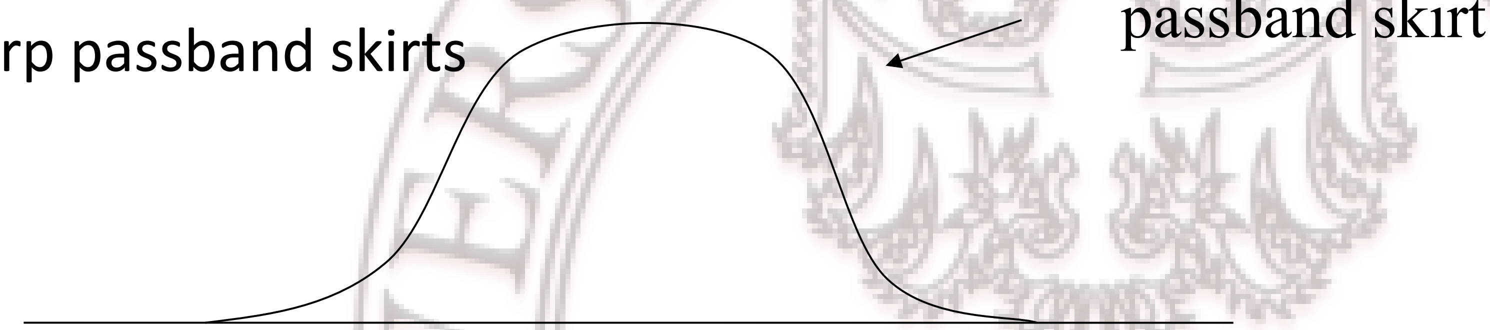
Optical Filters

- Optical filters select one specific channel
- The bandwidth of the filter must match the optical spectrum of the considered channel
- Also, the unwanted channels must be filtered properly
- Usually optical filters use wavelength dependent mecahnism, either interference or diffraction.

Optical Filters characteristic

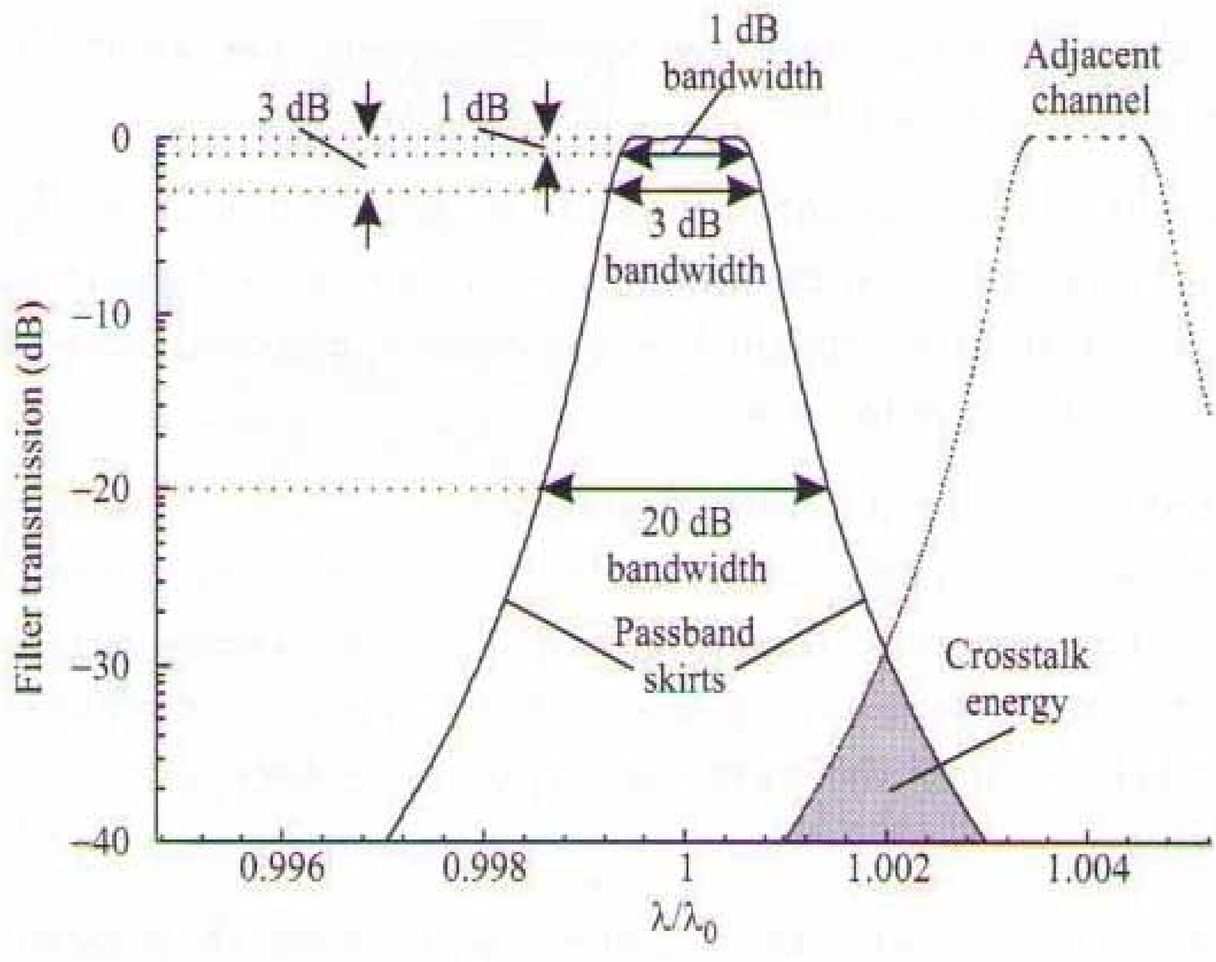
The desired characteristics of filters

1. Low insertion loss
2. Polarization-independent loss
3. Low temperature coefficient
4. Reasonable broad passbands
5. Sharp passband skirts



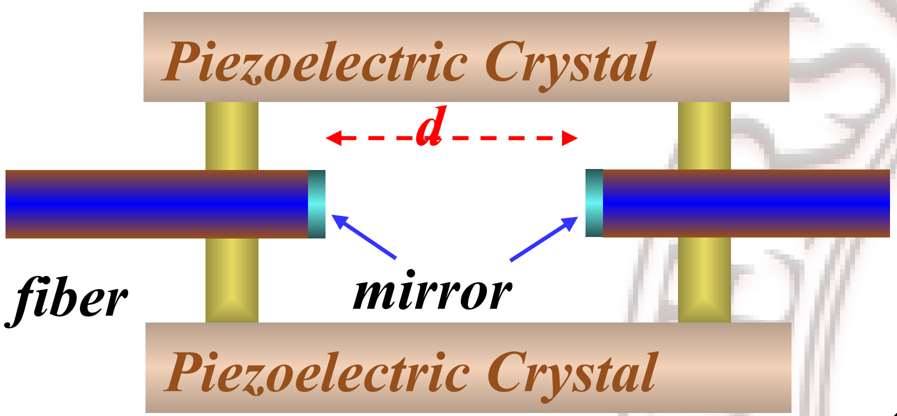
6. Low cost
   a. integrated-optic (may be polarization dependent)
   b. all-fiber devices

Optical Filters characteristic



Optical Filters – Fabry Perot Filters

Fabry Perot Filters are interferomtric filters composed of one cavity between 2 mirrors. The Length of the cavity determines the frequency and can be controlled externally.



The wavelength of resonance and the difference between peaks or Free Spectral Range.

$$\frac{2\pi\, 2\, d\, n_g}{\lambda} = 2\, m\, \pi \Rightarrow \quad \lambda = \frac{2\, L\, n_g}{m}$$

R is the reflexion coefficient of the electric field.

$$f = \frac{c}{\lambda} = \frac{c}{2\, L\, n_g} \quad 270\, \frac{\Delta f}{B} = \frac{\pi\sqrt{R}}{1-R}$$

Optical Filters – Fabry Perot Filters

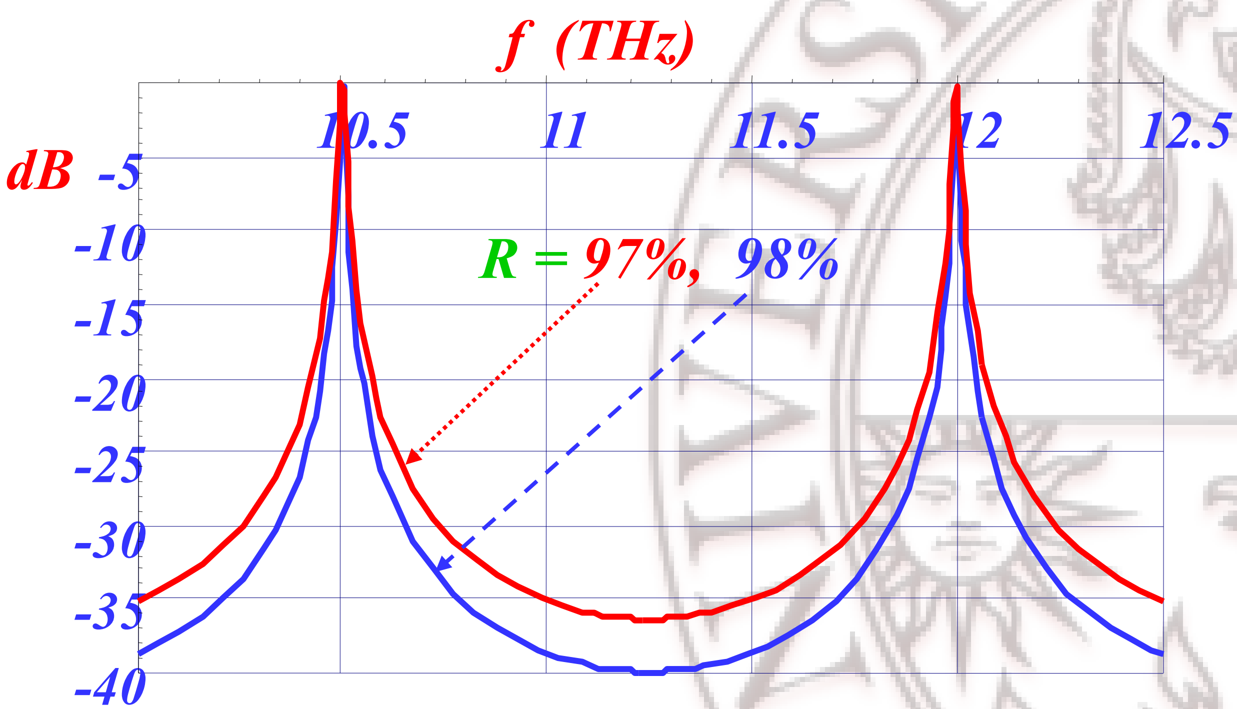
The transfer function H is function of the field reflectivity √R, the cavity length d, the group index n_g:

$$\left|H(f)\right|^2 = \frac{(1-R)^2}{1+R^2-2R\cos\left(\frac{2\,d\,n_g\,2\pi f}{c}\right)} \approx 1-\left(\frac{2\,d\,n_g\,2\pi f}{c(1-R)}\right)^2 R \;\Rightarrow\; B_{f3dB} \approx \frac{0.1\,c\,(1-R)}{d\,n_g\sqrt{R}}$$

A finess F of around 100 is obtained for 97% of reflectvity and offers a good range of tunability 100 channels of a few GHz of band, setting time quite slow around 100 μs).

Optical Filters – Fabry Perot Filters

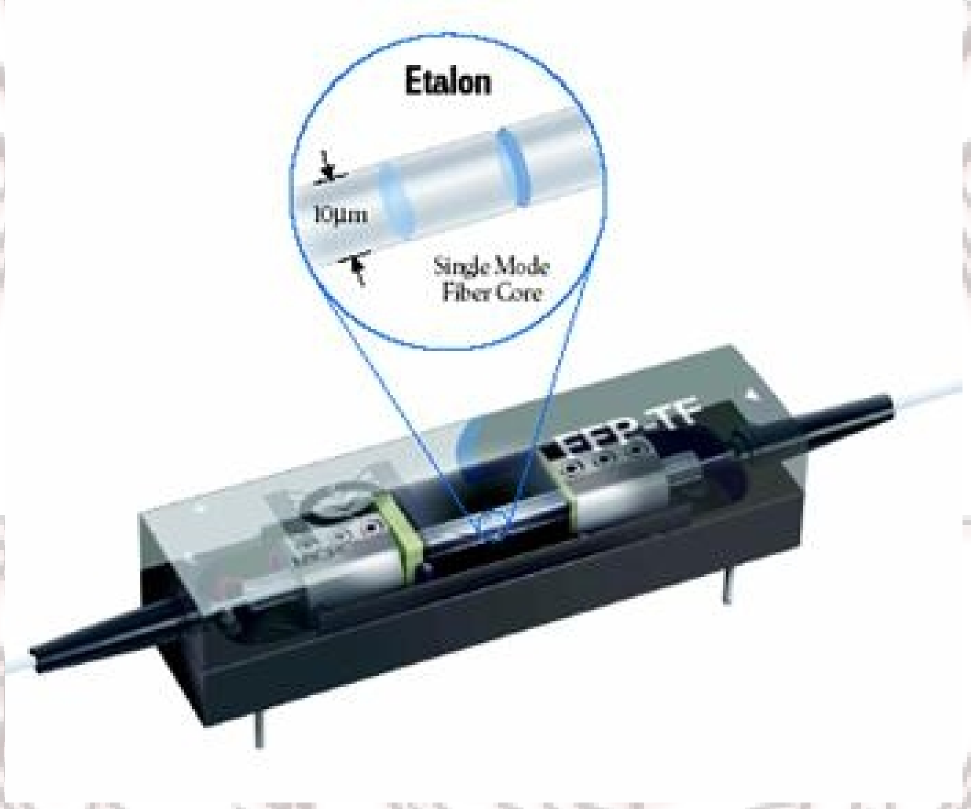
Example for different values of √R, d = 100 µm, with refractive index of 1.



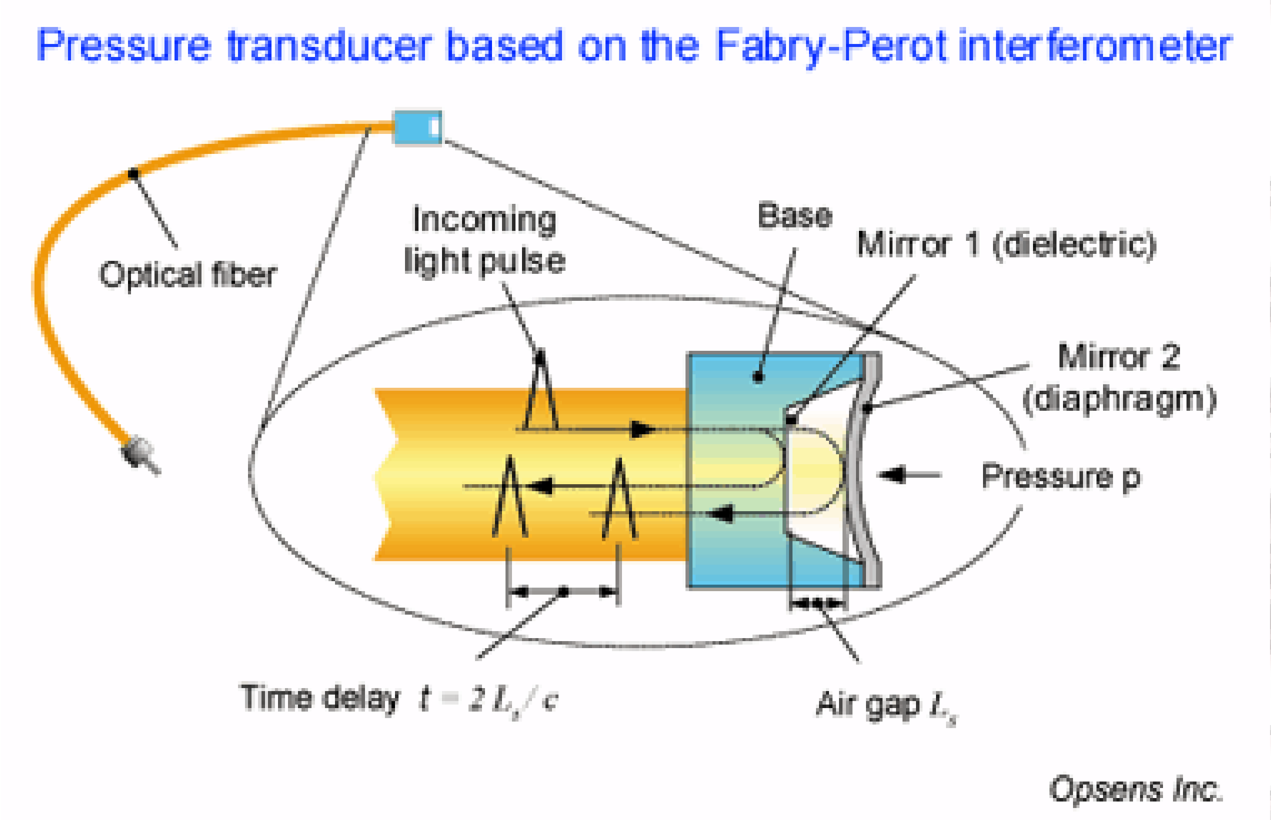
Effect of R is quite large.

Optical Filters – Fabry Perot Filters

Example of a tunable Fabry Perot based on SM optical fibres,

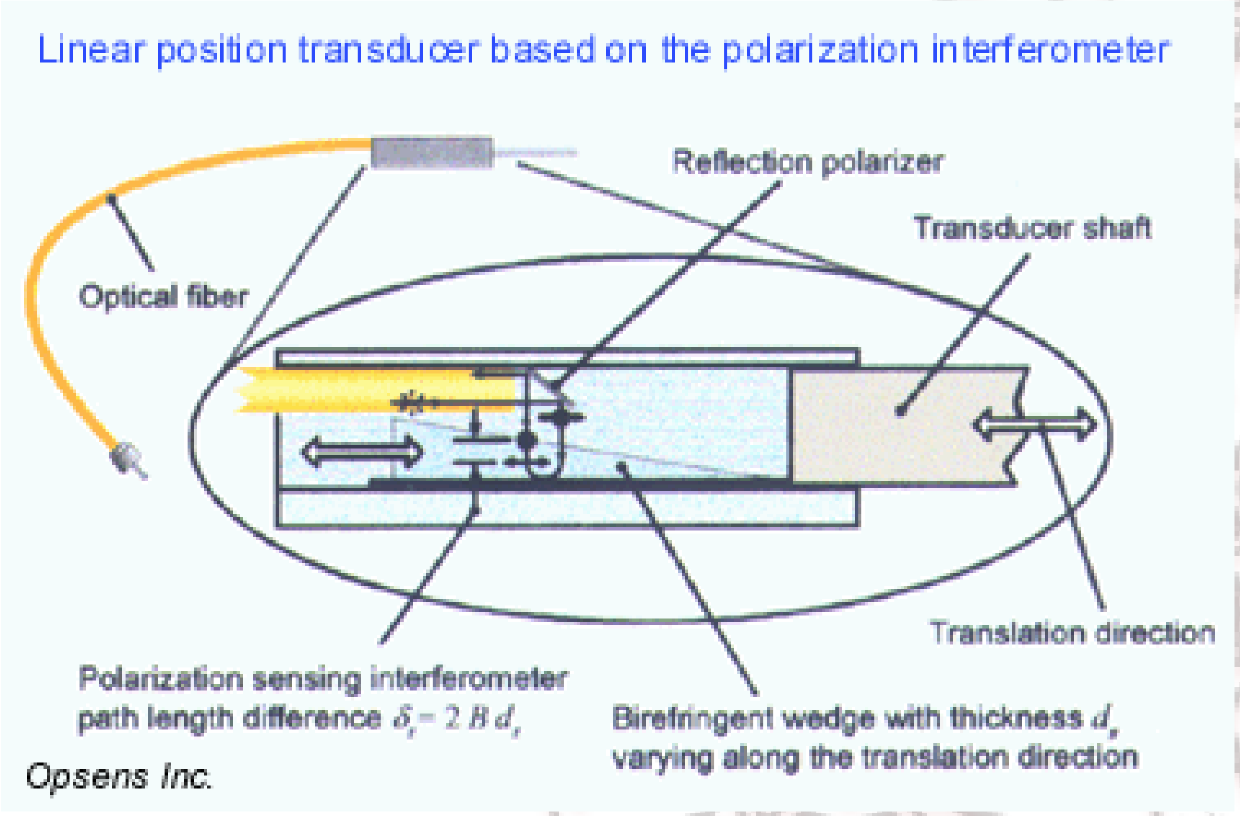


Fabry- Perot as Sensors



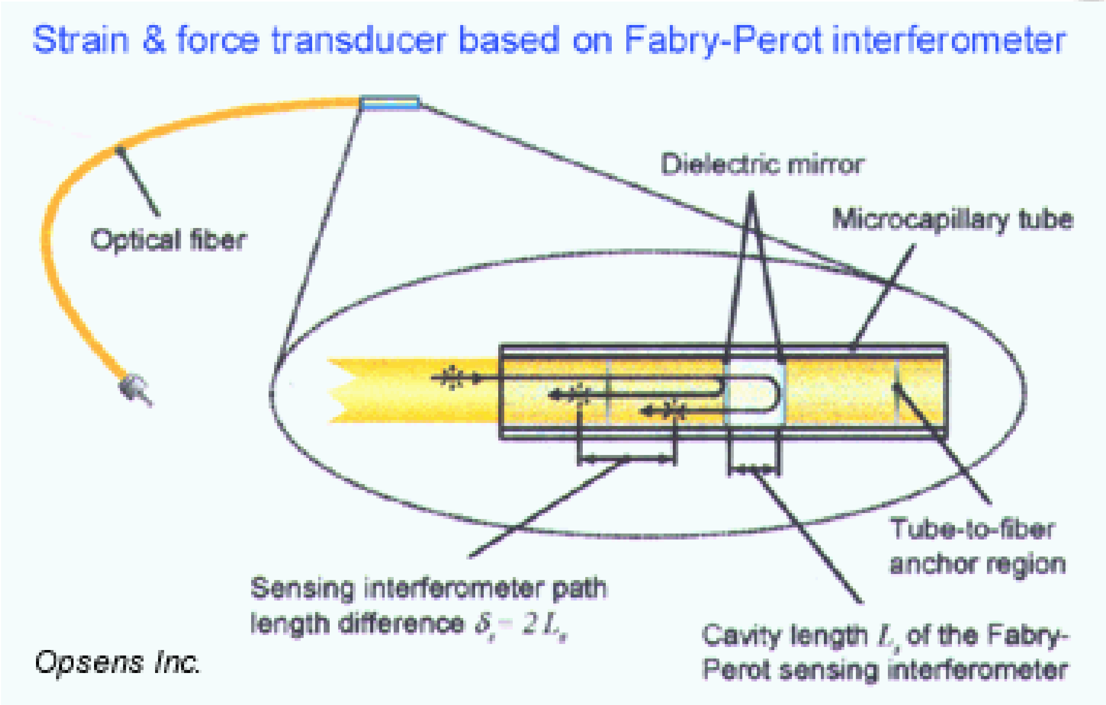
Pressure sensor

position

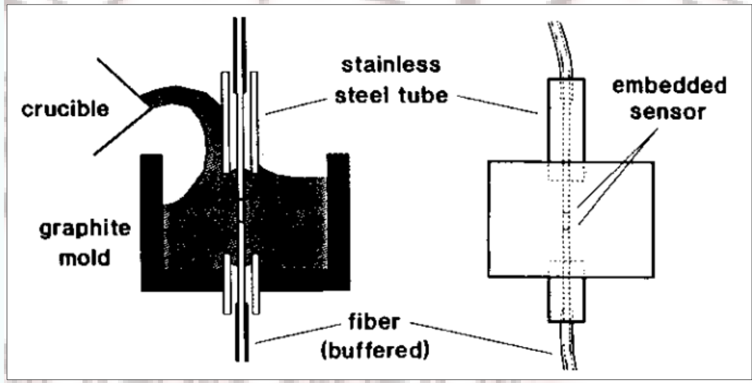


Linear position transducer based on the polarization interferometer
Reflection polarizer
Transducer shaft
Optical fiber
Translation direction
Polarization sensing interferometer
path length difference $\delta_i = 2\,B\,d_i$
Birefringent wedge with thickness d_i
varying along the translation direction
Opsens Inc.

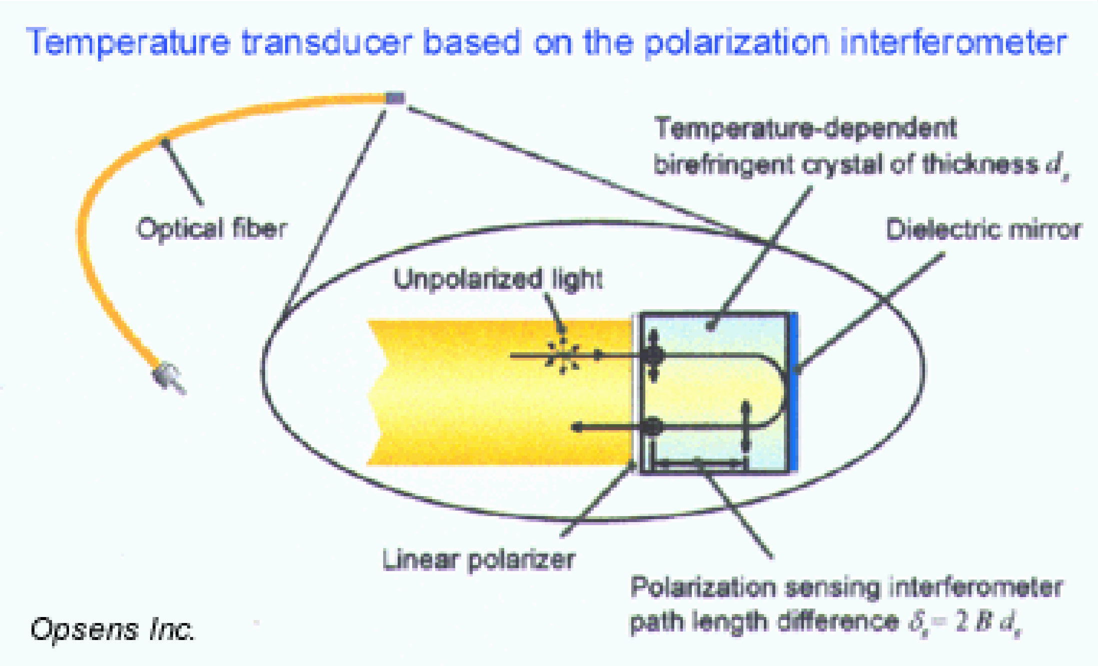
Strain and force



Axial Strain sensor



Temperature



Rotation Sensor Characteristics
SAGNAC INTERFEROMETER

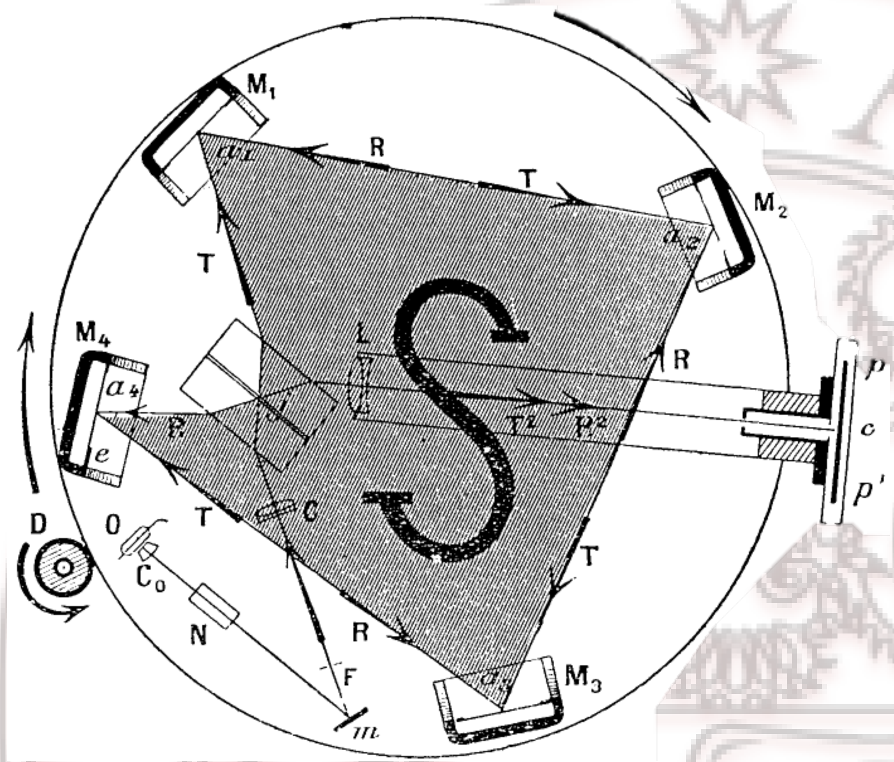
Rate Gyro

$\Omega = KV$

Ω = Rotation rate

K = Scale factor

V = Output signal



Original sagnac configuration

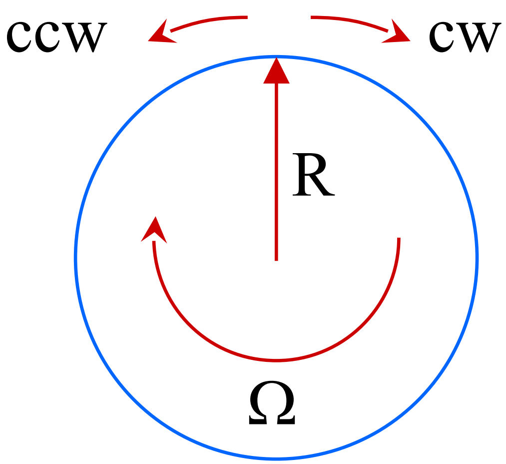
Definition of Terms

- Rate integration gyro - Integrates angular rate to get angular output
- Fixed bias - Output rotation rate with zero input rotation rate
- Bias drift - Change in output rate over time (temperature, wear, etc.)
- Scale factor - Linearity and hysteresis

Rotation Sensor Performance Factors

- Sensitivity
  - Lowest measurable rotation rate
- Spectral noise characteristics
- Dynamic range
- Turn on time

The Sagnac Effect



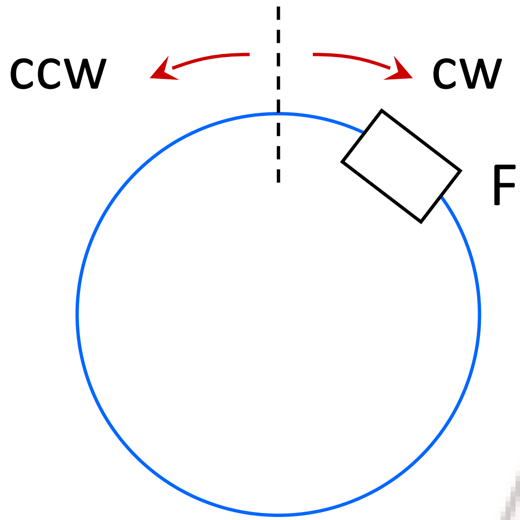
cw path: $2\pi R + \Omega RL/c$

ccw path: $2\pi R - \Omega RL/c$

Net path difference: $2\Omega RL/c$

$Z_R = 2\Omega RL/(\lambda c)$

The Sagnac Effect



$$Z_{Fcw} = (F_o+F)(Ln/c)$$

$$Z_{Fccw} = F_o Ln/c$$

$$Z_F = FLn/c$$

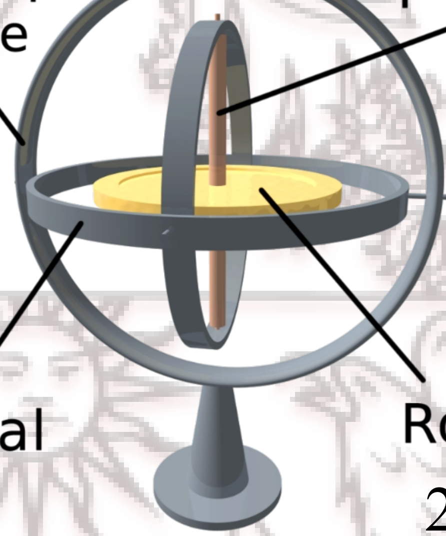
Setting $Z_R = Z\Phi$

Renders $\Phi = 2R\Omega/\lambda n$

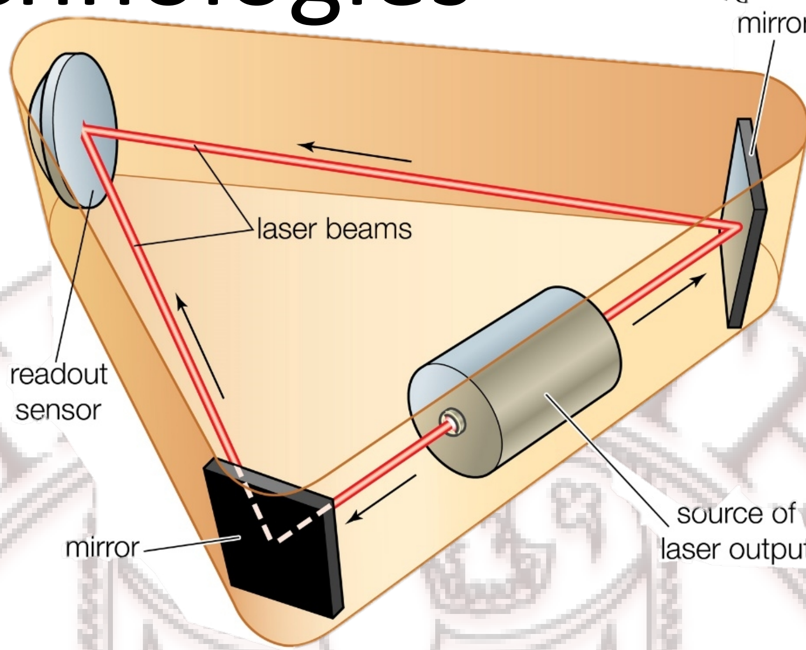
Phase shift Φ proportional to rotation speed

Gyroscope technologies

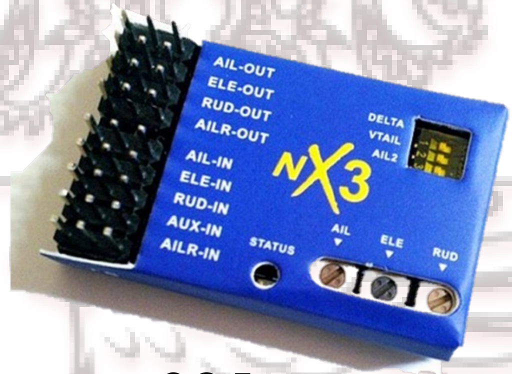
- Mechanical Gyros
  - » Advantages
- Established industrial base
  - » Disadvantages
- Bearing wear
- Start-up time
- Reliability



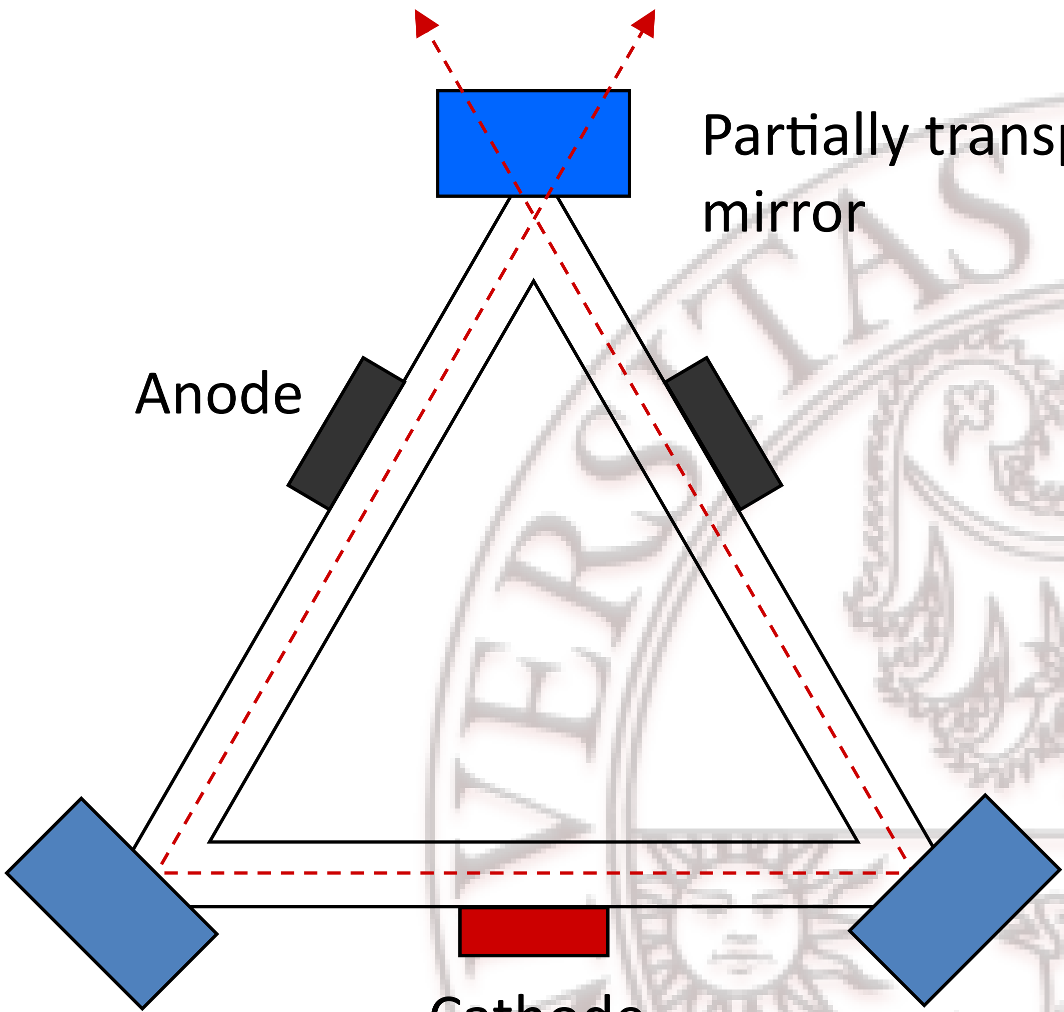
Gyroscope technologies



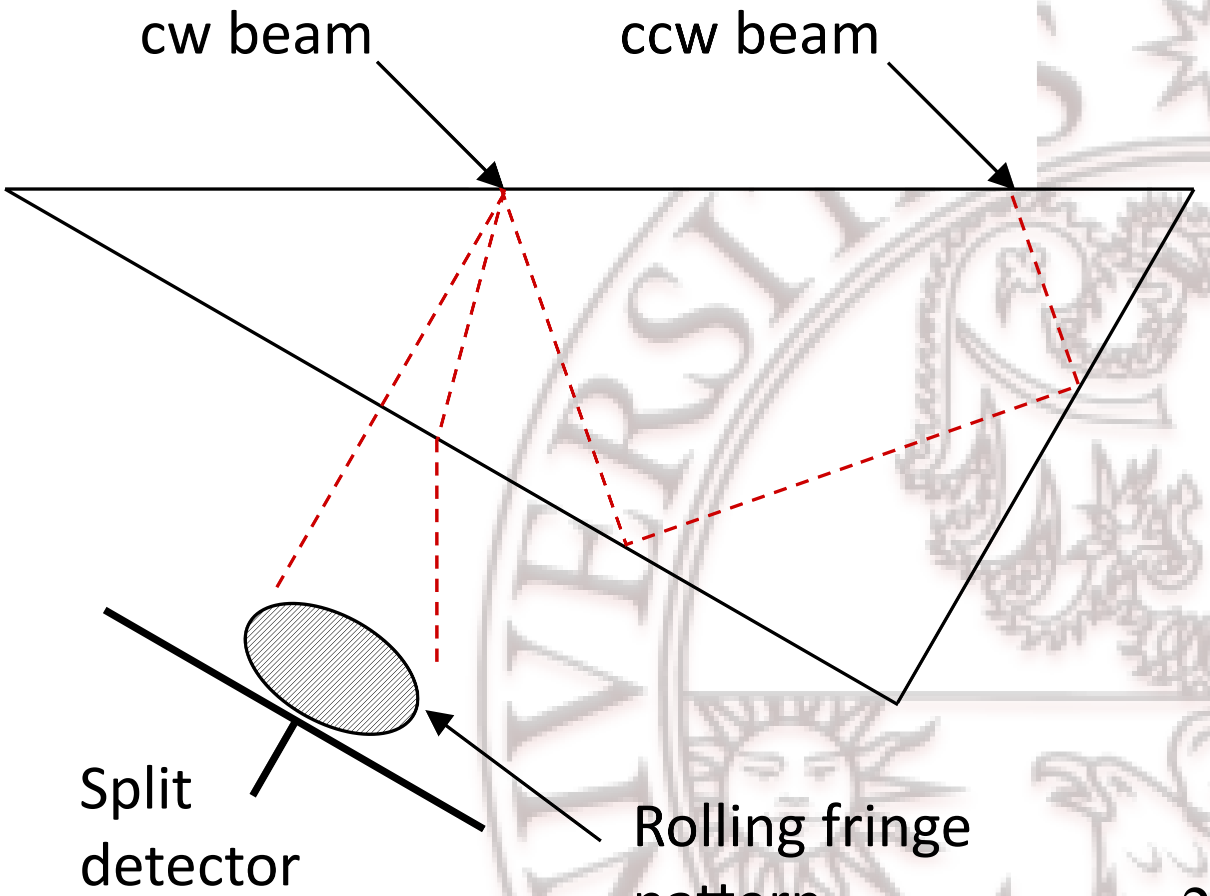
- Ring Laser Gyros
  - » Advantages
- Established industrial base
- Replaced mechanical gyros for navigation
  - » Disadvantages
- Mechanical dither
- Ultraclean vacuum tube technology



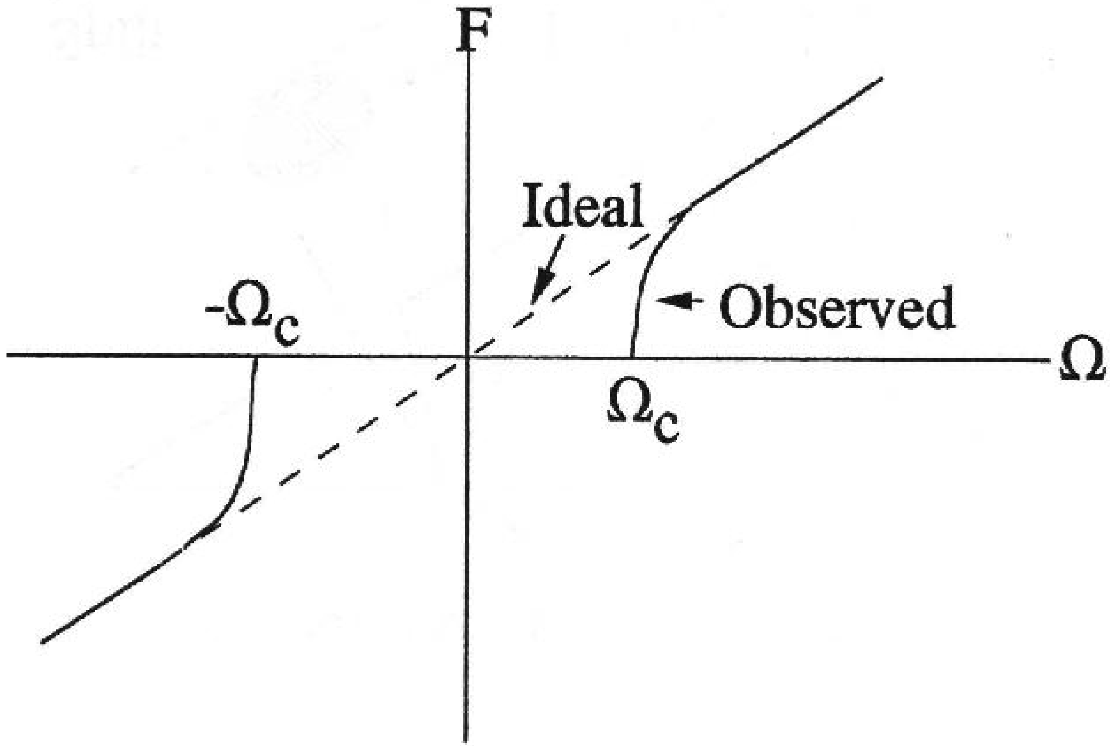
Ring Laser Gyro Assembly



Ring Laser Gyro Readout Optics



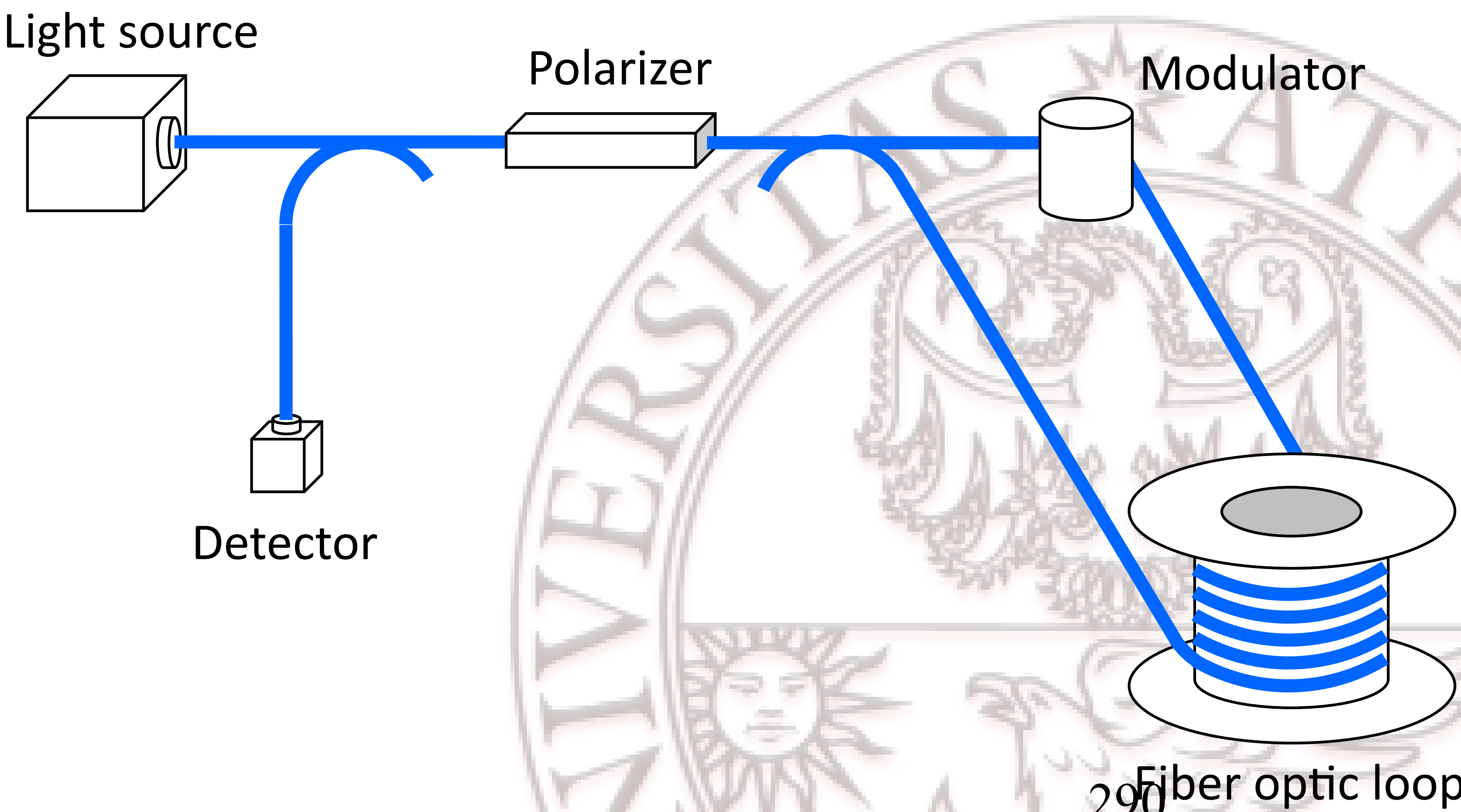
Ring Laser Lock In Zone



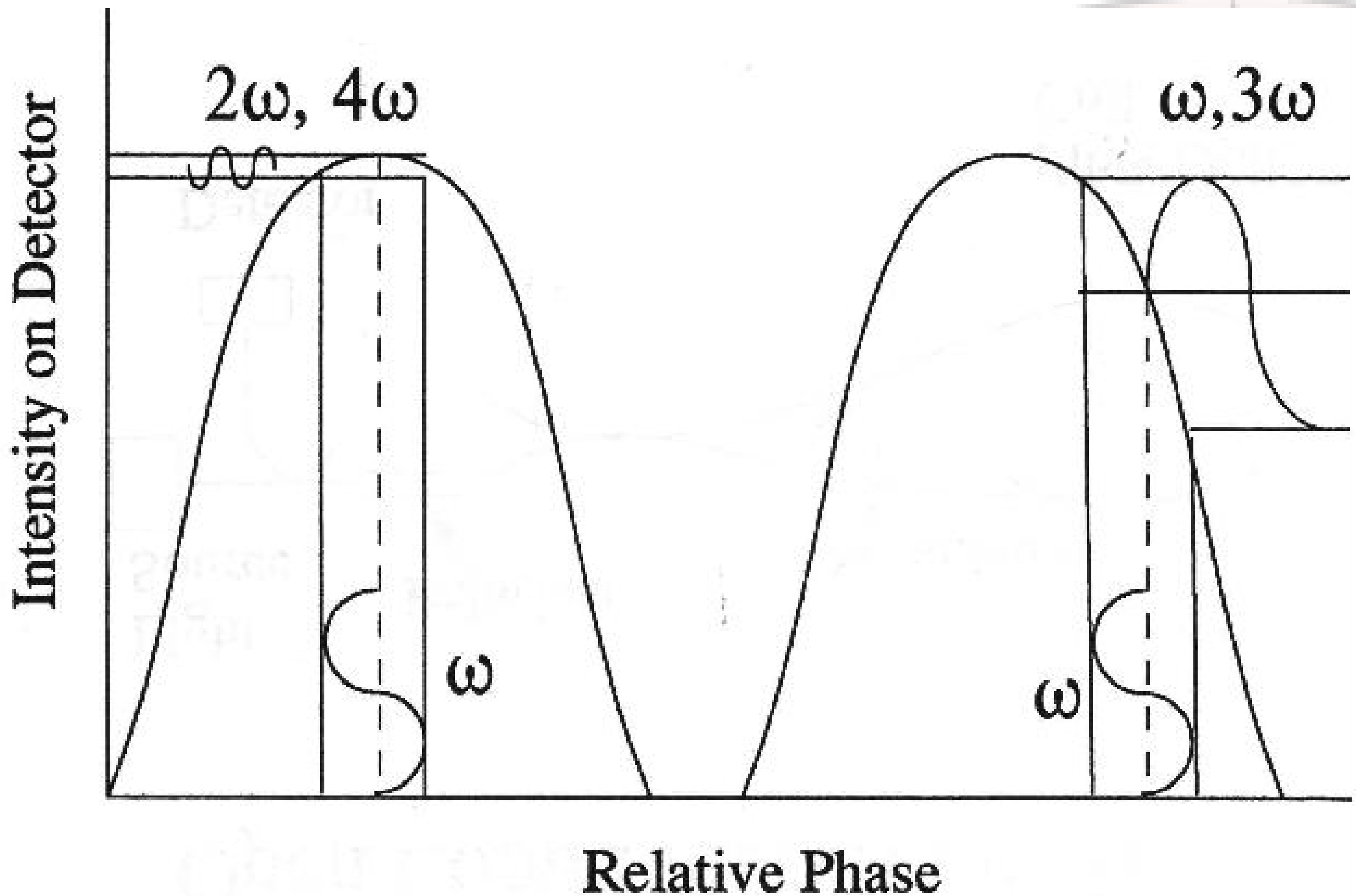
Fiber Optic Gyro Tradeoffs

- All solid state
- Packing flexibility
- Potentially very long lifetimes
- Small size
- Low cost

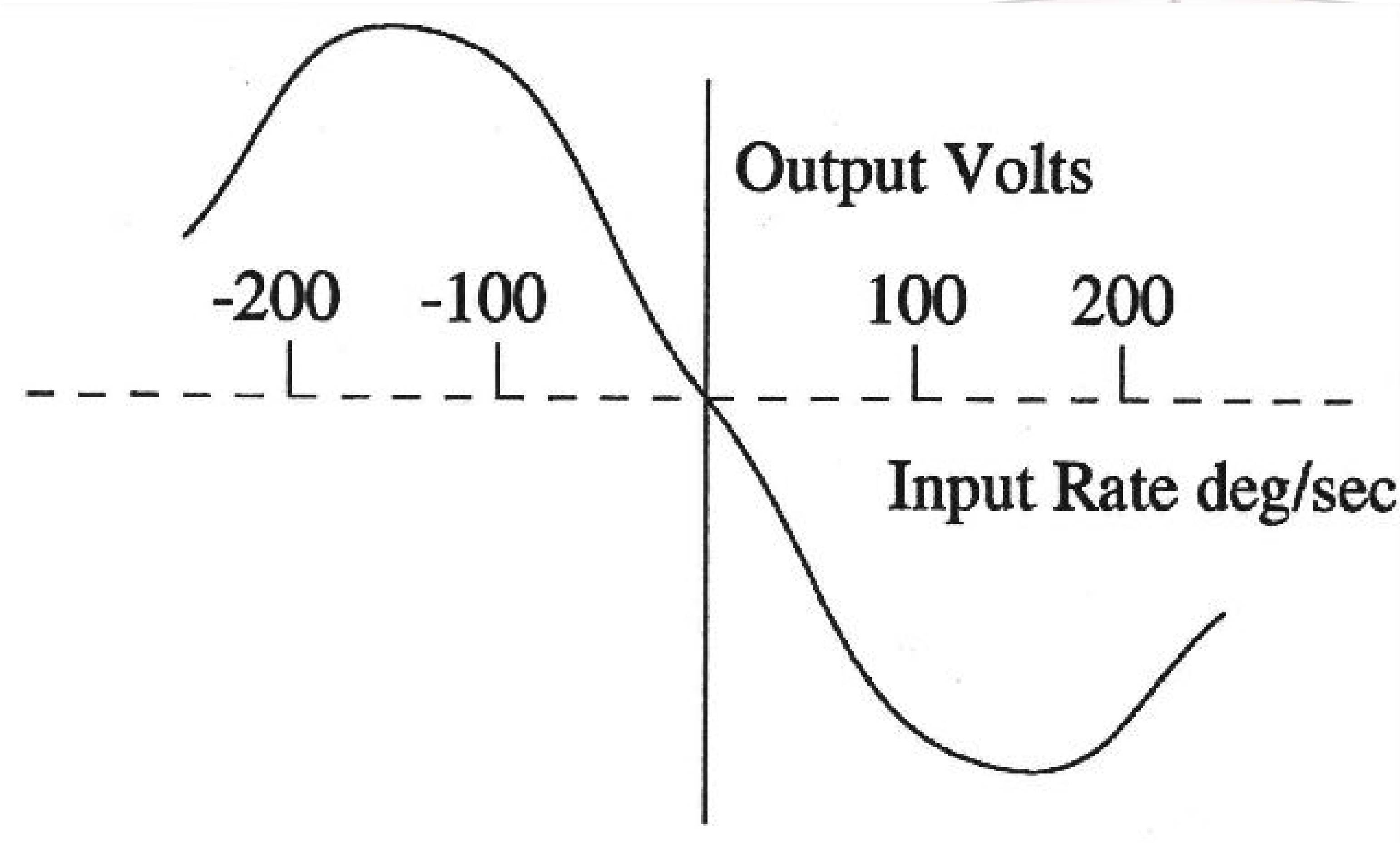
Open Loop Fiber Optic Gyro



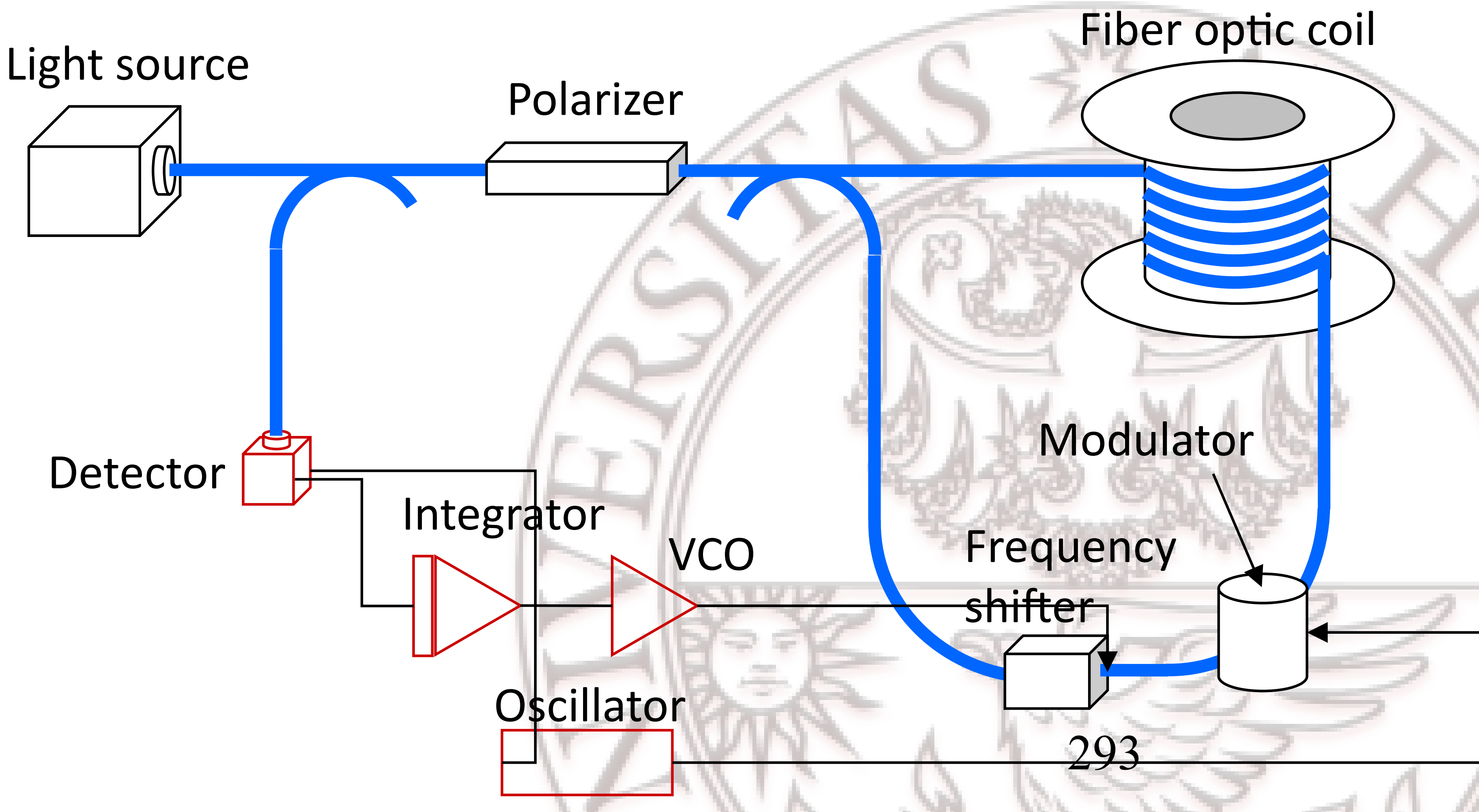
Detection Signals



Open Loop Fiber Optic Gyro Output



Closed Loop Fiber Optic Gyro



Scale Factor

Open loop fiber gyro

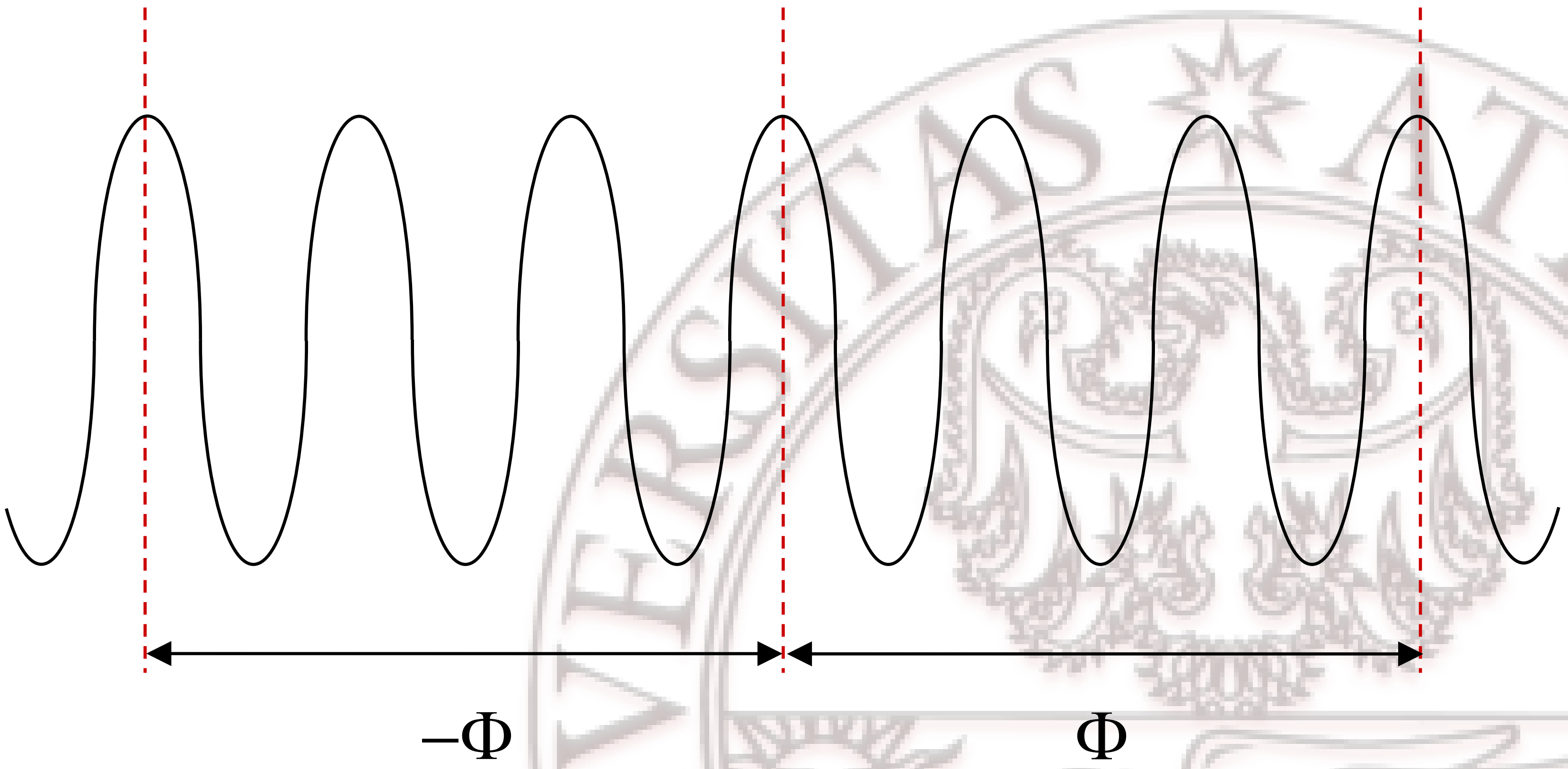
$$\Omega = Z_R[\lambda c/2RL]$$

Closed loop fiber gyro

$$\Omega = \Phi(\lambda n/2R)$$

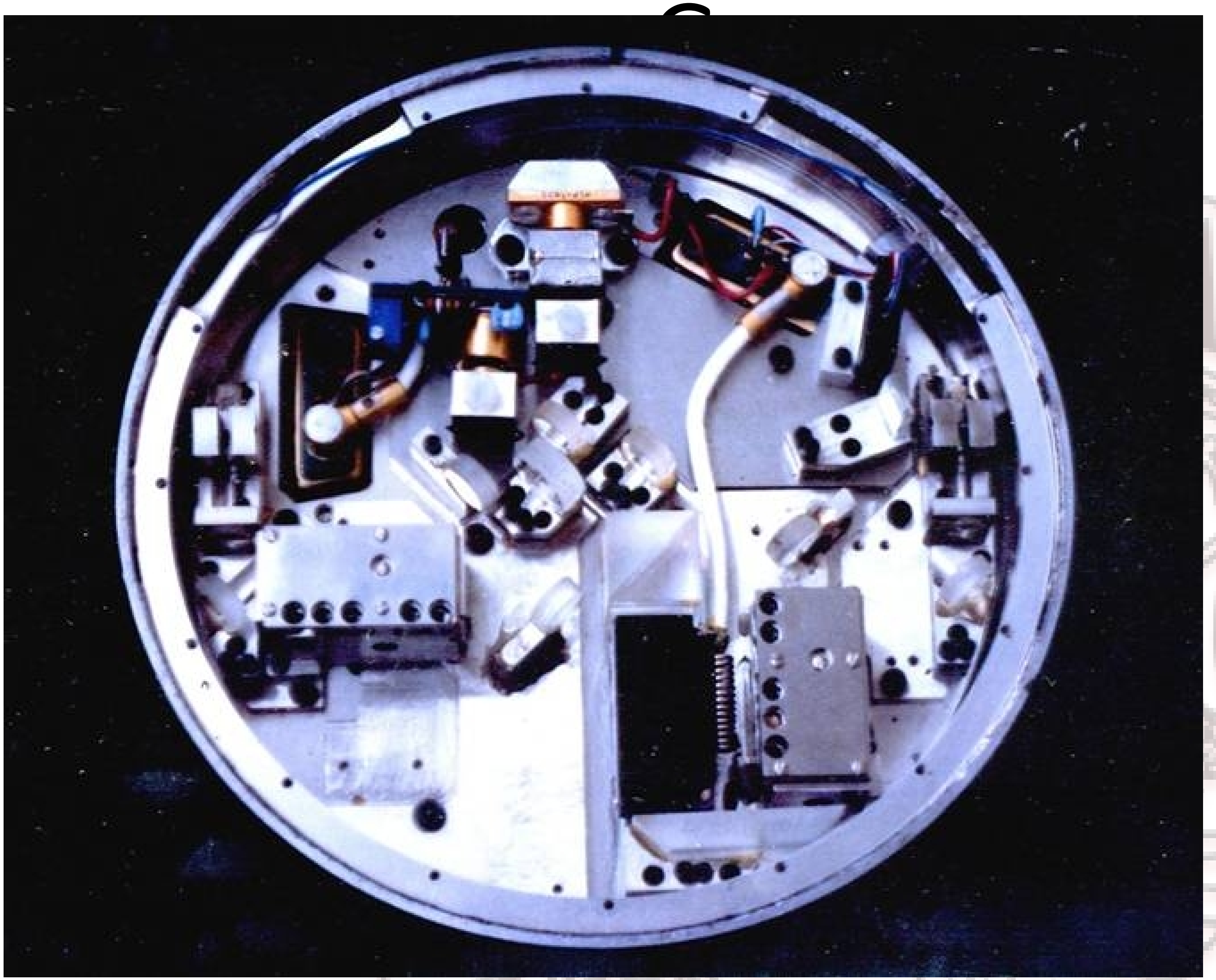
Dependence on wavelength

Correction of Scale Factor

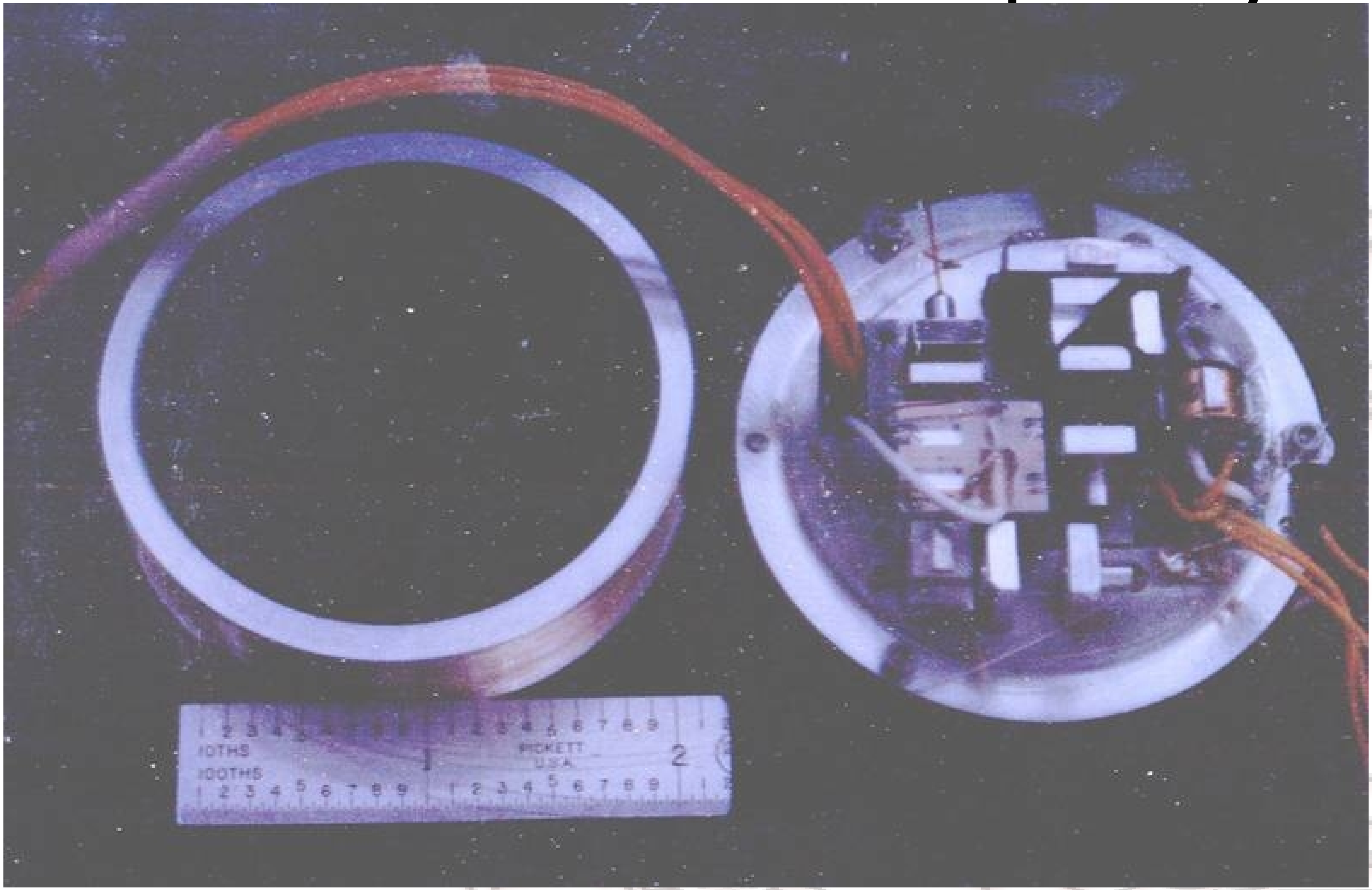


First Closed Loop Fiber Optic Gyro

First Solid State Fiber Optic S

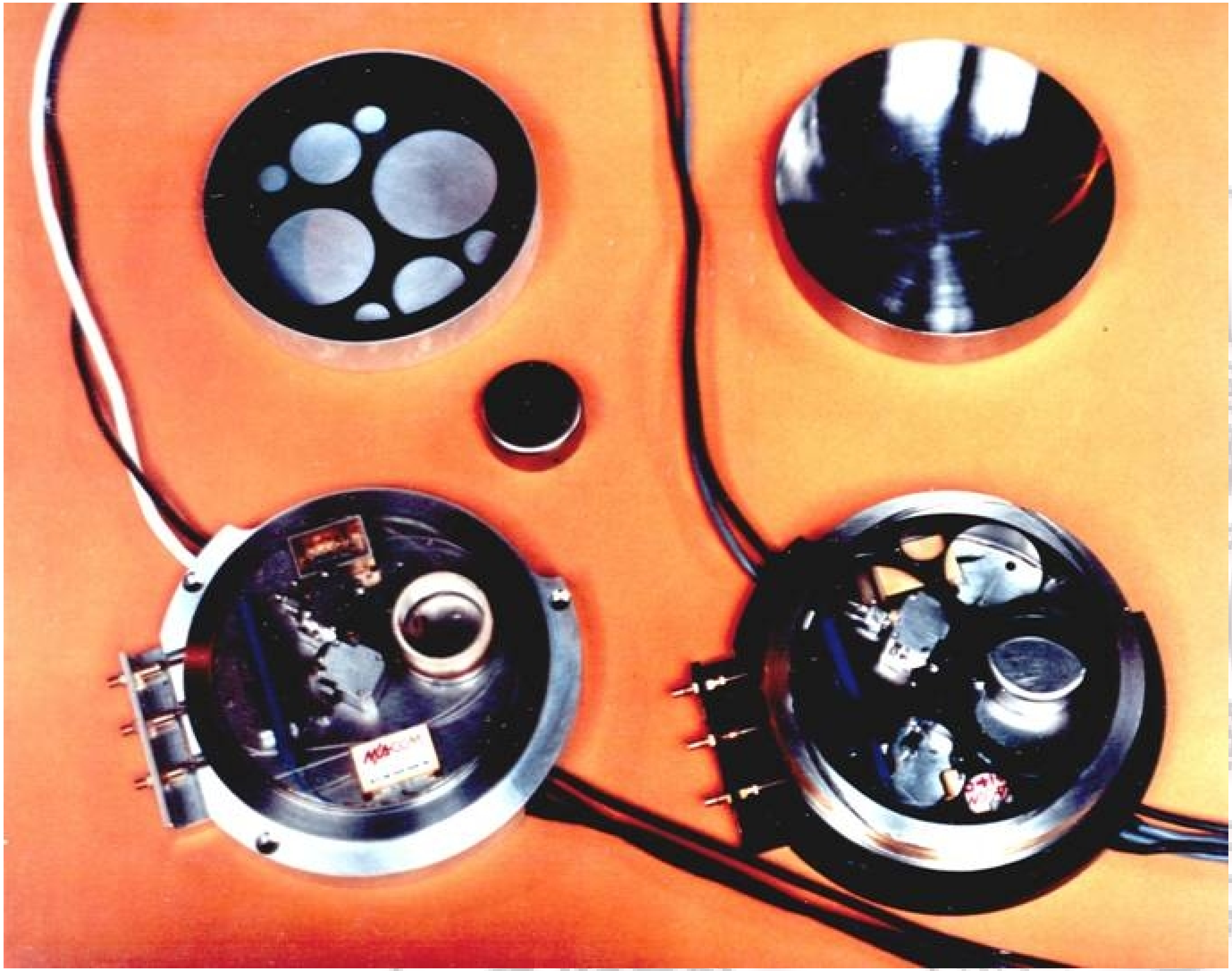


2.5” 1980 Fiber Optic Gyro

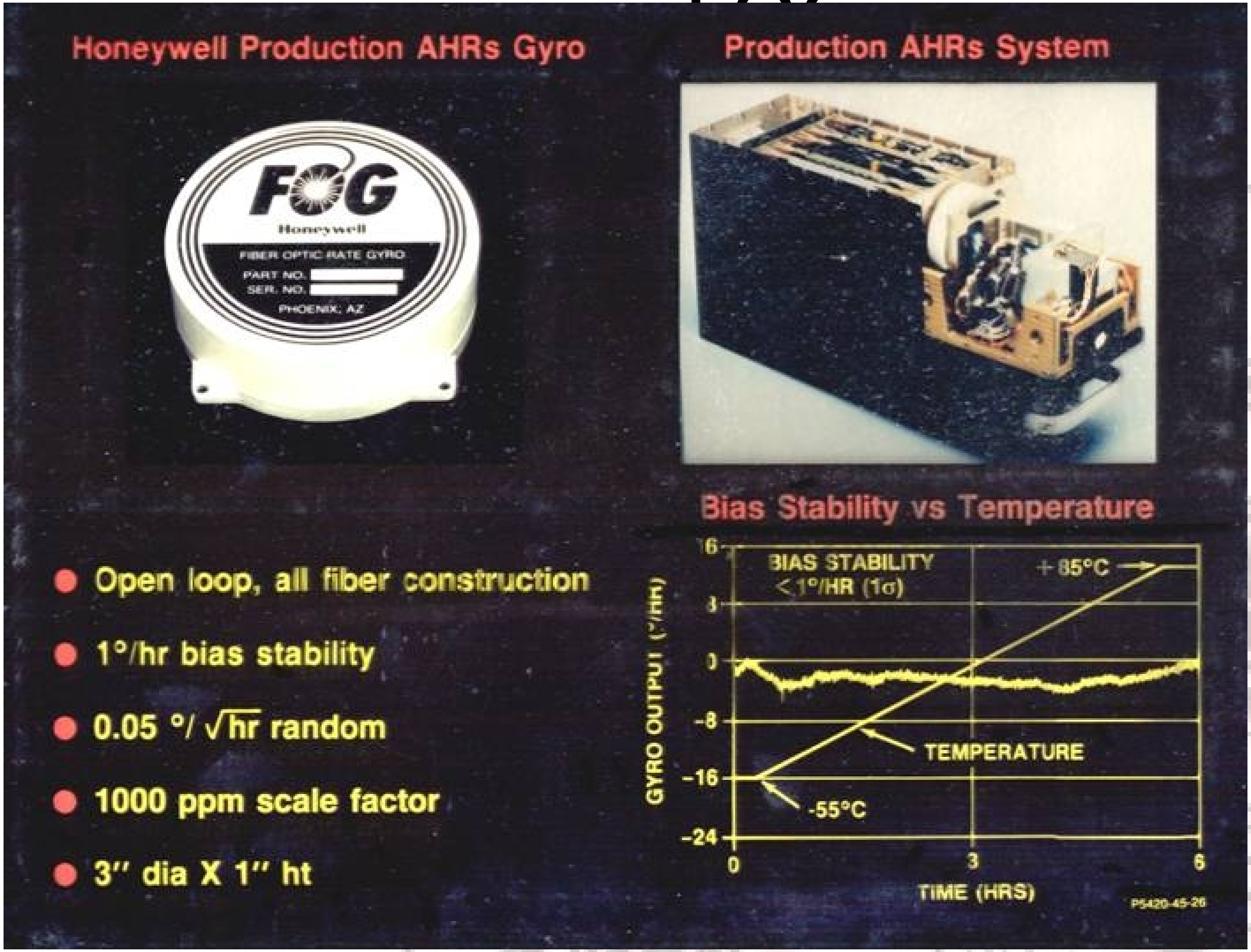


1982 Oil Drilling FOG

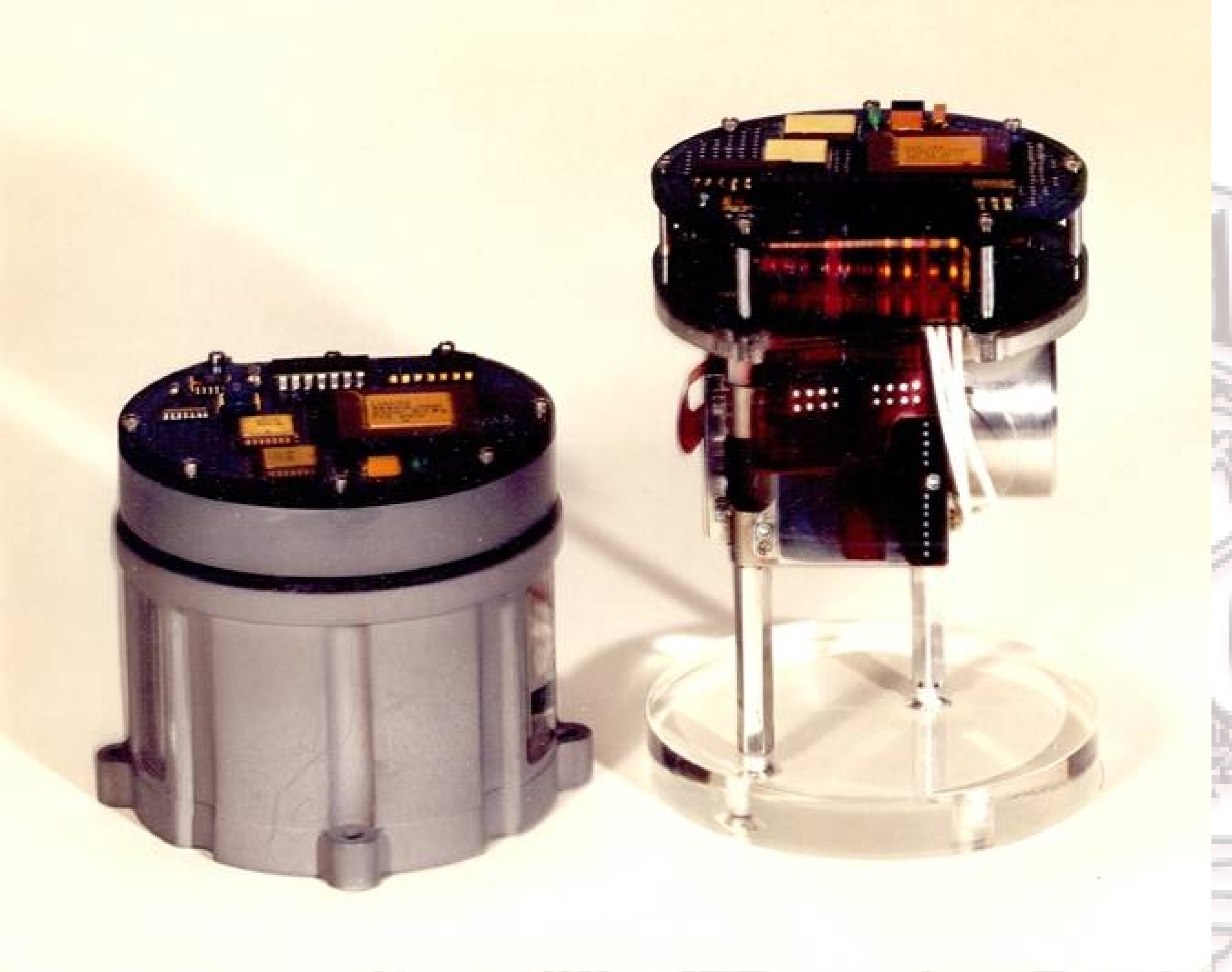
1983 Closed Loop FOGs



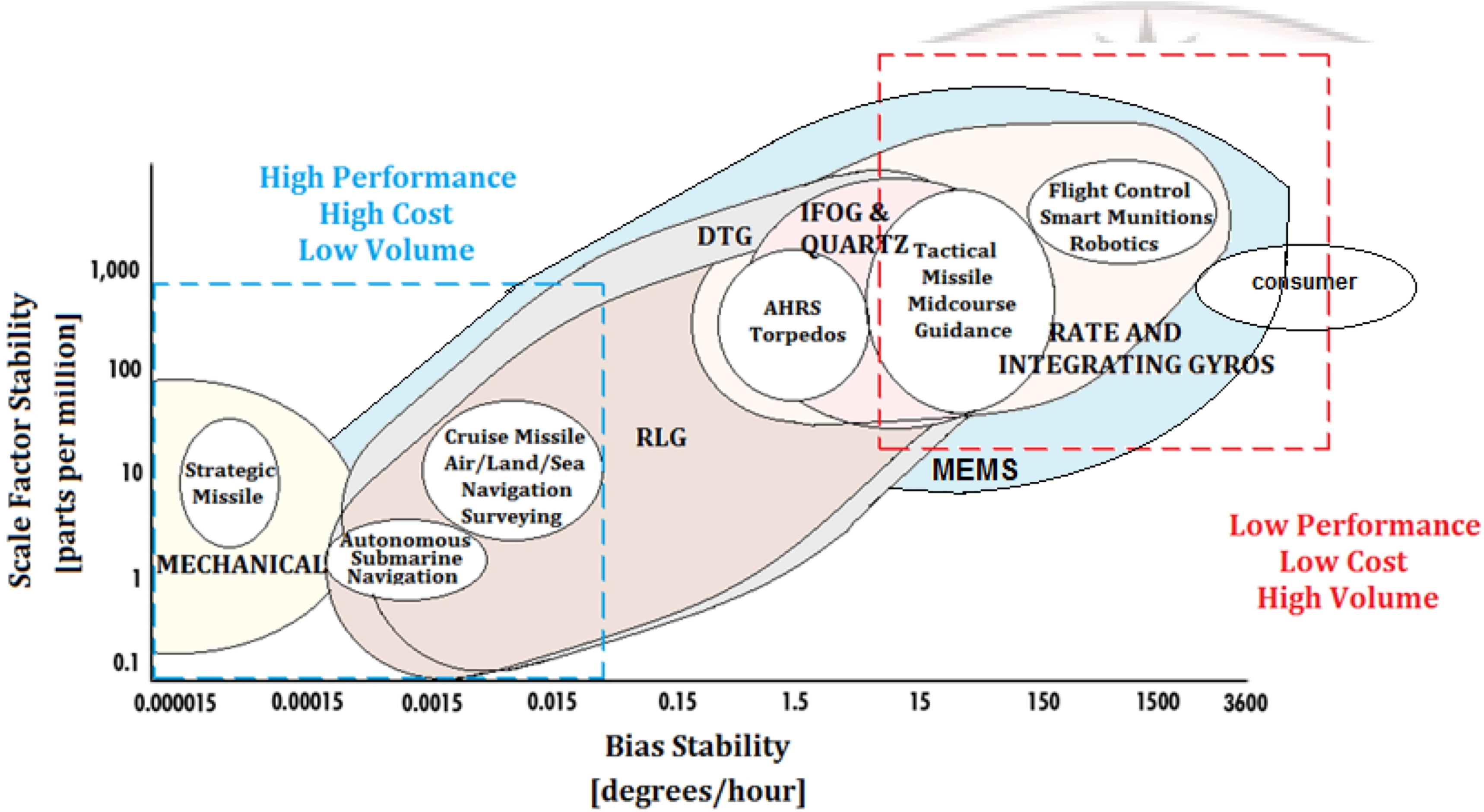
First Honeywell Production FOG



Litton (NG) FOG IMU



Different technologies market

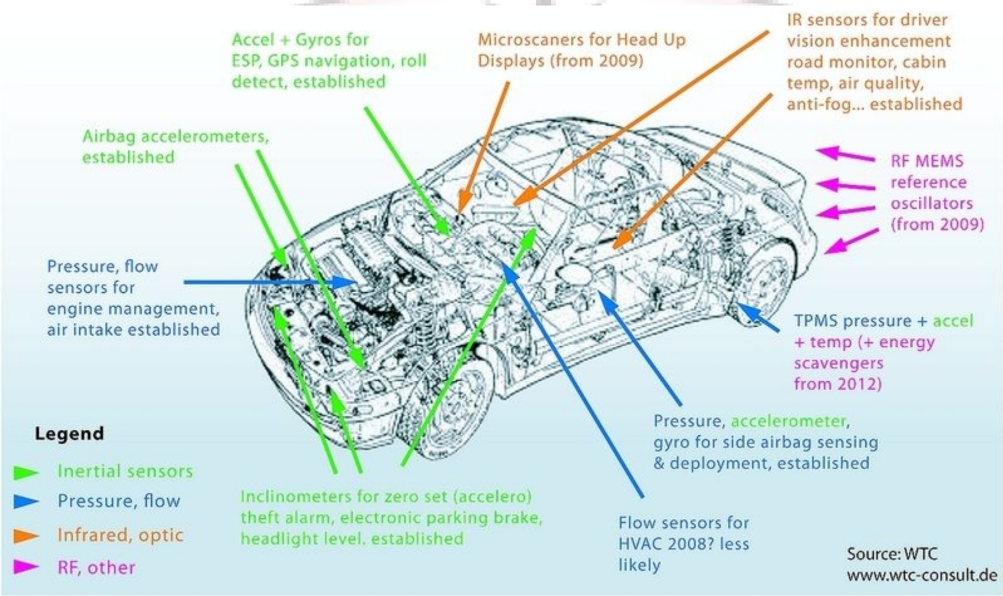


See: «Gyroscope Technology and Applications: A Review in the Industrial Perspective”

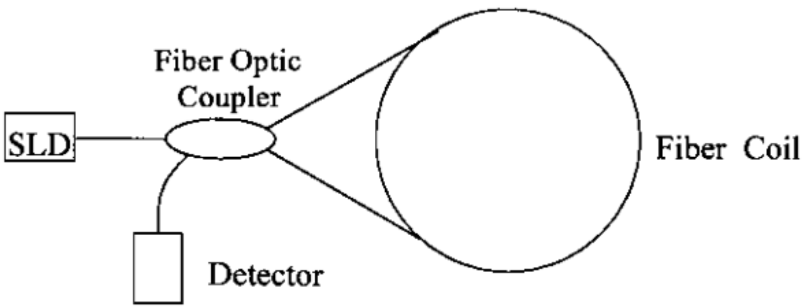
The Open Loop Fiber Optic Gyro Marketplace

- Automobiles and tru
- Pointing and trackin
- Robot navigation
- Aircraft attitude con
- Short range air navig

Sensors in modern cars



Gyroscope based on Sagnac effect



$$t_{cw} = \frac{2\pi R}{c - R\Omega}$$

$$t_{ccw} = \frac{2\pi R}{c + R\Omega}$$

- The Sagnac interferometer acts as a nonreciprocal device where the light waves propagating in one direction of a loop under rotation is not equivalent to the light waves propagating in the opposite direction.
- Sligthly transit times: