OptoElectronics Formulas

CHAPTER 1

Monochromatic plane wave electric field along x at position z at time t

$E_x(z,t) = E_0 cos(\omega t - k.z + \phi_0)$
k: propagation constant: $k = \frac{2\pi}{\lambda}$
λ: wavelength
ω: angular frequency
E_0: amplitude of the wave
ϕ_0: phase constant
$\phi = (\omega t - kz + \phi_o)$: Phase of the wave

Phase velocity: $v = \frac{\omega}{k} = v\lambda$
Spherical wave: $E = \frac{A}{r} cos(\omega t - k.r)$

Gaussian beam

Beam waist	$2w_0$ containing 86.5% of the power over radius 2w
Intensity	$I(r,z) = (\frac{2P}{\pi w}) exp(\frac{-2r^2}{w^2})$
Far field divergence	$2\theta = \frac{2w}{z} = 2\frac{\lambda}{\pi w_0}$

Length	$z_0 = \frac{\pi w_0^2}{\lambda}$
beam width	$w = w_0(1 + \frac{z}{z_0}^2)^{\frac{1}{2}}$

Refractive Index: $n = c/v = \sqrt{\epsilon_r}$
Group velocity: $v_g = \frac{d\omega}{dk} = \frac{c}{N_g}$ | $N_g = n - \lambda \frac{dn}{d\lambda}$

Poynting Vector: $\mathbf{S} = v^2 \epsilon_0 \epsilon_r \mathbf{E} \times \mathbf{B}$
Intensity: $I = S_{average} = 1/2 c\epsilon_0 n E_0^2$ for plane waves
$I = S_{average} = \frac{P_0}{4\pi r^2}$ for spherical waves

Snell's law	$\frac{sin\theta_i}{sin\theta t} = \frac{n_{refracted}}{n_{incident}}$	$sin\theta_c = \frac{n_2}{n_1}$
Lateral displacement by thin glass	$\frac{d}{L} = \frac{cos\theta_i}{\sqrt{(n/n_0)^2 - sin^2\theta_i}}$	

Fresnel Equation

$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{cos\theta_i - (n^2 - sin^2\theta_i)^{\frac{1}{2}}}{cos\theta_i + (n^2 + sin^2\theta_i)^{\frac{1}{2}}}$	$R_\perp = \lvert r_\perp \rvert^2$
$r_{//} = \frac{(n^2 - sin^2\theta_i)^{1/2} - n^2 cos\theta_i}{(n^2 - sin^2\theta_i)^{1/2} + n^2 cos\theta_i}$	$R_{//} = \lvert r_{//} \rvert^2$
$t_\perp = \frac{2cos\theta_i}{cos\theta_i + (n^2 - sin^2\theta_i)^{\frac{1}{2}}}$	$T_\perp = (\frac{n_2}{n_1})(t_\perp)^2$
$t_{//} = \frac{2ncos\theta_i}{n^2 cos\theta_i + (n^2 - sin^2\theta_i)^{\frac{1}{2}}}$	$T_{//} = (\frac{n_2}{n_1})(t_{//})^2$

Evanescent wave

Attenuation coefficient	$\alpha_2 = \frac{2\pi n_2}{\lambda}((\frac{n_1}{n_2})^2 sin^2\theta_i - 1)^{1/2}$
Penetration depth	$\delta = \frac{1}{\alpha_2}$
$E(y, z, t) \propto e^{-\alpha_2 y} expj(\omega t - k_z. z)$	$k_z = k. sin\theta_i$

Anti Reflection layer

$R_{min} = (\frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3})^2$	$d = m(\frac{\lambda}{4n_2})$

Dielectric mirror

Reflection condition	$n_1 d_1 + n_2 d_2 = m\frac{\lambda}{2}$
Reflection of N pairs	$R_N = (\frac{n_1^{2N} - (n_0/n_3)n_2^{2N}}{n_1^{2N} + (n_0/n_3)n_2^{2N}})^2$
Reflection bandwidth	$\frac{\delta\lambda}{\lambda_0} \approx (\frac{4}{\pi} arcsin(\frac{n_1 - n_2}{n_1 + n_2}))$

Complex refractive index

index defined as : $N = n - j. K = \sqrt{\epsilon_r^{'} - \epsilon_r^{''}}$ Fresnel reflection: $R = |\frac{n-j.K-1}{n-j.K+1}|^2 = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$

Fabry Perot Cavity

Transmission intensity	$I = \frac{I_0}{(1-R^2) + 4Rsin^2(k.L)}$
Cavity Length	L

Wavector in cavity	k
Mirror Reflectivity	R
Finesse	$F = \frac{\pi R^{1/2}}{1-R}$
Linewidth	$\delta \nu_m = \frac{\nu_f}{F}$
Quality factor	$Q = \frac{\nu_m}{\delta \nu_m = mF}$
Mode frequency	$\nu_m = m(\frac{c}{2nL})$

Rayleigth criterion: $sin(\Delta\theta_{min} = 1.22\frac{\lambda}{D})$

Grating condition:$d(sin\theta_m - sin\theta_i) = m\lambda$
for m=...,-2,-1,0,+1,+2,...

Chapter 2

Planar waveguide

thichness planar waveguide	2a
Waveguide Condition	$(\frac{2\pi n_{core}}{\lambda})cos\theta_m - \phi_m = m\pi$
Propagation wavector	$\beta_m = k_{core}sin\theta_m = \frac{2\pi}{\lambda}n_{core}sin\theta_m$
Lateral wavector	$\kappa_m = k_{core}cos\theta_m = \frac{2\pi}{\lambda}n_{core}sin\theta_m$
Electric field in planar waveguide	$E(y, z, t) = E_m(y)cos(\omega t - \beta_m z)$
Electric field profile	$E_a = 2Acos(m\pi/2 - \frac{y}{2a}(m\pi + \phi_m))$

Optical Fiber

V-number	$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2}$
Number of modes	$M = Int(\frac{2V}{\pi}) + 1$
Length	L
Multimodal Dispersion	$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$
Mode field	$2w_0 = 2a(\frac{V+1}{V})$
Mode electric field	$E_{LP} = E_{lm}(r, \phi)exp(j\omega - \beta_{lm} z)$
b parameter	$\frac{(\beta/k)^2 - n_2^2}{n1^2 - n_2^2}$
Approximation of b-aparameter	$b \approx 1.428 - \frac{0.996}{V}$ valid for 1.5<V<2.5
Marcuse equation for mode field	$2w = 2a(0.65 + 1.619v^{-3/2} + 2.879V^{-6})$ valid for 0.8<V<2.5
simplified equation	$2w = 2a\frac{2.6}{V} valid for 1.6 < V < 2.4$
Numerical aperture	$NA = (n_1^2 - n_2^2)^{1/2}$
V-parameter	$V = \frac{2\pi a}{\lambda} NA$
Intra mode dispersion	$D = \frac{\Delta\tau}{L\Delta\lambda}$
Material Dispersion	$\frac{\Delta\tau}{L} = D_m \Delta\lambda$
Material Dispersion coefficent	$D_m = -\frac{\lambda}{c}(\frac{d^2 n}{d\lambda^2})$
Waveguide dispersion	$\frac{\Delta\tau}{L} = D_w \Delta\lambda$
Waveguide Dispersion coefficent	$D_w = -\frac{-\Delta n_2}{c\lambda} V(\frac{d^2 Vb}{dV^2})$

Fiber Bragg Grating

FBG Bragg condition	$q\lambda_B = 2\overline{n}\Lambda$ with q=1,2,...
Reflection intensity	$R = tanh^2(\kappa L)$ where $\kappa = \pi\frac{\Delta n}{\lambda}$
Bandwidth weak grating	$\Delta\lambda_{weak} = \frac{\lambda_B^2}{\pi L}$ for $kL < 1$
Bandwidth strong grating	$\Delta\lambda_{strong} = \frac{4\kappa\lambda_B^2}{\pi L}$ for $kL > 1$
Temperature dependence	$\frac{\delta\lambda_B}{\lambda_B} = \alpha\delta T$
Strain dependence	$\frac{\delta\lambda_B}{\lambda_B} = (1 - 1/2n^2p_e))\delta\epsilon$

Chapter 3 Semiconductor physics

Metals

Density of states	$g(E) = 4\pi(2m_e^*)^{3/2}h^{-3}E^{1/2}$
Plack constannt	$h = 6.62607015.10^{-34}m^2kg/s$
Fermi Dirac Distribution	$f(E) = \frac{1}{1+exp(\frac{E-E_F}{k_BT})}$
Temperature	T
Botlzmann constant	$k_B = 1.380649 \times 10^{-23}m^2kg.s^{-2}K^{-1}$
effective mass	m_e^*
number of carriers	$n = \int_0^{E_F+\phi} g(E)f(E)dE$

$E_{F0} = (\frac{h^2}{8m_e^*})(\frac{3n}{\pi})^{2/3}$	

Semiconductors

Density of states	$g_{CB}(E) = 4\pi(2m_e^*)^{3/2}h^{-3}(E - E_C)^{1/2}$
Fermi Dirac Distribution	$f(E) = exp(-\frac{E-E_F}{k_B T})$
number of electrons	$n = N_C exp(-\frac{E_C - E_F}{k_B T})$
where	$N_C = 2(\frac{2\pi m_e^* k_B T}{h^2})^{3/2}$
number of holes	$p = N_C exp(-\frac{E_F - E_V}{k_B T})$
where	$N_V = 2(\frac{2\pi m_h^* k_B T}{h^2})^{3/2}$
Fermi level	$E_{Fi} = E_v + 1/2E_g - 1/2k_B T ln(\frac{N_C}{N_V}) = E_V + 1/2E_g - 3/4k_B T ln(\frac{m_e^*}{m_h^*})$
Average Electron Energy	$\overline{E_{CB}} = E_C + 3/2k_B T$
Fermi level difference	$\Delta E_F = eV$ where V is an external voltage
Electron Drift velocity	$V_{de} = \mu_e E_x$
Holes Drift velocity	$V_{dh} = \mu_h E_x$
Conductivity	$\sigma = en\mu_e + ep\mu_h$
Doping neutrality	$np = n_i^2$
n-type	$n = N_D \quad p = \frac{n_i^2}{N_D} \quad \sigma \approx N_D \mu_e$
p-type	$p = N_A \quad n = \frac{n_i^2}{N_A} \quad \sigma \approx N_A \mu_h$

PN junctions

Depletion width	$N_A W_p = N_D W_n$
Built-in field	$E_0 = -e\frac{N_D W_n}{\epsilon}$
Built-in voltage	$V_0 = \frac{k_B T}{e} ln(\frac{N_A N_D}{n_i^2})$
Depletion width	$W_0 = (\frac{2\epsilon(N_A+N_D)V_0}{e N_A N_D})^{1/2}$
law of junction	$\frac{p_{n0}}{p_{p0}} = exp(-\frac{eV_0}{k_B T})$ and $\frac{n_{p0}}{n_{n0}} = exp(-\frac{eV_0}{k_B T})$
Short Diode current density	$J = (\frac{eD_h}{l_n N_D} + \frac{eD_e}{l_p N_A})n_i^2(exp(\frac{eV}{k_B T}) - 1)$
Length of the neutral region n outside the depletion region	l_n
Length of the neutral region p outside the depletion region	l_p
Transit time	$\tau_t = \frac{l_n^2}{2D_h}$
Lond Diode current density	$J = (\frac{eD_h}{L_h N_D} + \frac{eD_e}{L_e N_A})n_i^2(exp(\frac{eV}{k_B T}) - 1)$
Mean distance of recombination for holes	$L_h = (D_h \tau_h)^{1/2}$
Mean distance of recombination for electrons	$L_e = (D_e \tau_e)^{1/2}$
General Diode equation	$J = J_0(exp(\frac{eV}{\eta k_B T}) - 1)$
pn junction current	$I = I_0 exp(\frac{eV}{k_B T})$
Dynamic resistance	$r_d = \frac{dV}{dI} = \frac{V_{th}}{I}$
Thermal voltage	$V_{th} = \frac{k_B T}{e} \approx 25mV$

Diffusion capacitance (short diode)	$C_{diff} = \frac{\tau_t I}{2V_{th}}$
Diffusion capacitance (long diode)	$C_{diff} = \frac{\tau_h I}{2V_{th}}$

LED emission

Emission frequency	$h\nu_0 = E_g + 1/2k_BT$
Spectral linewidth	$h\Delta\nu = mk_BT$
Spectral linewidth wavelength	$\Delta\lambda = \lambda_0^2 \frac{3l_BT}{hc}$
Energy Level in quantum wells	$E_n = E_C + \frac{h^2n^2}{8m_e^*d} \frac{\hbar k_y^2}{2m_e^*} \frac{\hbar k_z^2}{2m_e^*}$
Quantum well thickness	d

Chapter 4 - Stimulated Emission and amplification

Excited electrons can decay through:

- Non radiative with probability $1/\tau_{nr}$
- Radiative with probability $1/\tau_{rsp}$
- Radiative through stimulated emission with probability $1/\tau_{rst}$

Einstein coefficients

Consider a two-level system with energy E_1 and E_2 with population N_1 and N_2

Absorption	$R_{12} = B_{12}N_1\rho(\nu) = -\frac{dN_1}{d}t$
Spontaneous Emission	$A_{21}N_2$

Stimulated emission	$B_{21}N_2$
Total emission	$A_{21}N_2 + B_{21}N_2 = -\frac{dN_2}{dt}$

At equilibrium we have $R_{12} = R_{21}$ ankd $\frac{N_2}{N_1} = exp(-\frac{E_2-E_1}{k_BT})$

Planck's Black Body radiation law: $\rho(\nu) = \frac{8\pi h\nu^3}{c^3(exp(\frac{h\nu}{k_BT})-1)}$

Relationship at equilibrium

$\frac{A_{21}}{B_{12}} = \frac{8\pi\nu^3}{c^3}$

$\tau_{SP} = \frac{1}{A_{21}}$

Gain out of equilibrium

Gain defined as: $g = \frac{\Delta I}{I\Delta x}$ Dependence on cross section: $g(\nu) = \sigma_{em}(\nu)N_2 - \sigma_{ab}(\nu)N_1$

Totale Gain after propagation length L: $G = exp(gL)$

Laser properties

gain coefficient threshold	$g_{th} = \alpha_S + \frac{1}{2L}ln(\frac{1}{R_1R_2}) = \alpha_T$
Output power	$P_0 = A/2(1+R_1)h\nu_0 N_{ph}c/n$
Cavity modes	$m\lambda = 2nL$
External Quantum efficiency	$\eta_{EQE} = \frac{P_0/h\nu}{I/e}$
External Differential Quantum Efficiency	$\eta_{EDQE} = \frac{eP_0}{E_g(I-I_{ph})}$
Extraction efficiency	$\eta_{EE} = \frac{1}{2L}ln(\frac{1}{R_1})\frac{1}{\alpha_t}$

Laser equation	$I_0 = (\frac{hc^2\tau_{ph}(1-R)}{2en\lambda d})(J - J_{th})$

Chapter 5 - Photodetectors and Image Sensors

Absorption cut-off	$\lambda_g(\mu m) \approx \frac{1.24}{Eg(eV)}$
External Quantum efficiency	$\eta_E = \frac{I_{ph}/e}{P_0/h\nu}$
Responsivity	$\frac{I_{ph}}{P_0} = \eta_E \frac{e\lambda}{hc}$
Maximum QE	when $\frac{dR}{d\lambda} = \frac{R}{\lambda}$
Internal Quantum efficiency	$\eta_i = \frac{Number\ of\ EHP\ photogenerated}{Number\ of\ absorbed\ photons}$
PN photodiode current	$I_{ph} = \frac{e\eta_i TP_0(0)}{h\nu}(1 - exp(-\alpha w))$ for W>>L_h
width of intrinsic layer	W
Tranmission of anti reflection layer	T
Absorption	α
Pin diode transit time	$t_{drift} = \frac{W}{v_d}$
Pin diode diffusion length	$l = (2D.e.t)^{1/2}$
Avalanche photodiode amplification factor	$M = \frac{1}{1-(\frac{V_r}{V_{br}})^m} = \frac{1-k}{exp(-(1-k)\alpha_e w)-k}$
Reverse voltage	V_r
Breakdown voltage	V_{br}
Total multiplied photocureent	$I_{ph} = M.I_{ph_0}$

Ionization coefficient for holes	α_h
Ionization coefficient for electrons	α_e
Ratio k	$k = \frac{\alpha_h}{\alpha_e}$

Noise in photodiodes

equivalent current noise	$i_n = (2e(I_d + I_{ph})B)^{1/2}$
Bandwidth	B in Hz
Resistance noise	$i_{th} = (\frac{4k_B T.B}{R_L})^{1/2}$
Noise Equivalent Power (NEP)	$NEP = \frac{Input\ power\ for\ SNR=1}{\sqrt{Bandwidth}} = \frac{P_l}{B^{1/2}}$ in $W.Hz^{1/2}$
Detectivity	D=1/NEP
Specific Detectivity	$D^{*}=A^{1/2}/NEP$

Solar Cells

Photocurrent	$I_{ph} = K.\mathcal{I}_l$
Light Intensity	$\mathcal{I}_l$
Diode current	$I_0(exp(\frac{eV}{\eta k_B T}) - 1)$

Chapter 6 Light polarization and modulation

Malus's Law	$I(\theta) = I(0)cos^2\theta$

Angle of rotation with polarization	θ
Birfringent material	$\frac{1}{n_e(\theta)} = \frac{cos^2\theta}{n_o} + \frac{sin^2\theta}{n_e}$
Phase retardation between the 2 polarization	$\phi = \frac{2\pi}{\lambda}(n_e - n_o)L$

Last updated 2023-01-13 11:32:41 +0100