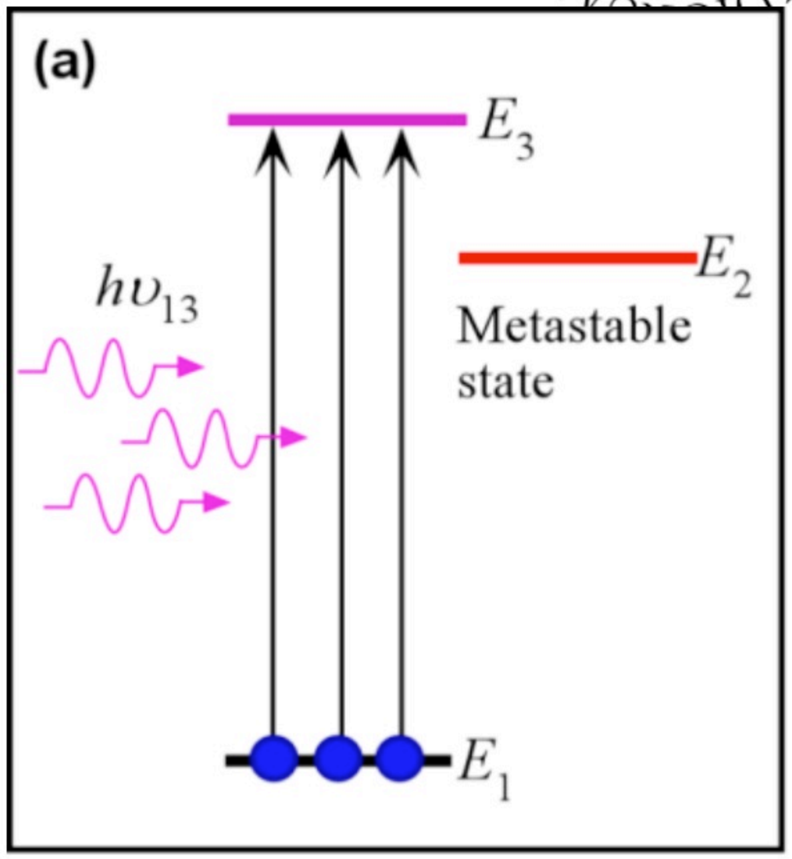
EXAMPLE: Minimum pumping power for three level laser systems

Consider the 3-level system Figure 4.2(a). Assuming that the transitions from E_3 to E_2 are fast, and the spontaneous decay time from E_2 to E_1 is τ_{sp}, show that the *minimum* pumping power P_{pmin} that must be absorbed by the laser medium per unit volume for population inversion ($N_2 > N_1$) is

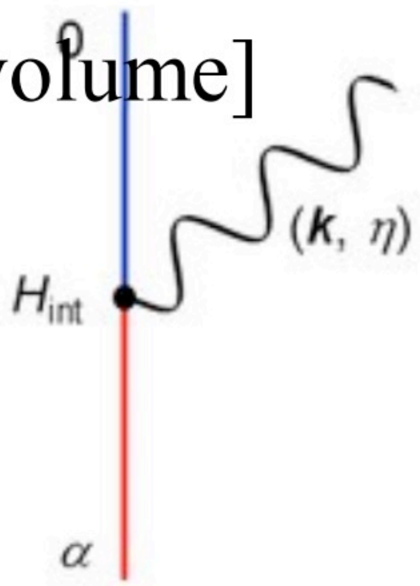
$$P_{pmin}/V = (N_0/2)h\upsilon_{13}/\tau_{sp} \quad \textit{Minimum pumping for population inversion for 3-level laser} \qquad (4.2.12)$$

where V is the volume, N_0 is the concentration of ions in the medium and hence at E_0 before pumping. Consider a ruby laser in which the concentration of Cr^{3+} ions is 10^{19} cm^{-3}, the ruby crystal rod is 10 cm long and 1 cm in diameter. The lifetime of Cr^{3+} at E_2 is 3 ms. Assume the pump takes the Cr^{3+} ions to the E_3-band in Figure 4.3 (a), which is about 2.2 eV above E_0. Estimate the minimum power that must be provided to this ruby laser to achieve population inversion.



Solution

Consider the 3-level system in Figure 4.2 (a). To achieve population inversion we need to get half the ions at E_1 to level E_2 so that $N_2 = N_1 = N_0/2$ *since* N_0 is the total concentration of Cr^{3+} ions all initially at E_1. We will need $[(N_0/2)h\upsilon_{13} \times \text{volume}]$ amount of energy to pump to the E_3-band.



EXAMPLE: Minimum pumping power for three level laser systems

Solution (continued)

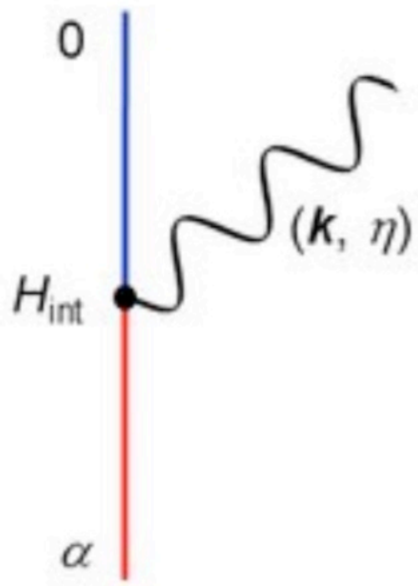
The ions decay quickly from E_3 to E_2. We must provide this pump energy before the ions decay from E_2 to E_1, that is, before τ_{sp} Thus, the *minimum* power the ruby needs to absorb is

$$\boldsymbol{P_{pmin} = V(N_0 / 2)h\upsilon_{13}/\tau_{sp}}$$

which is Eq. (4.2.12). For the ruby laser

$$\begin{aligned} P_{pmin} &= [\pi(0.5\text{ cm})^2(10\text{ cm})][(10^{19}\text{ cm}^{-3})/2](2.2\text{ eV})(1.6\times 10^{-19}\text{ J/eV})]/(0.003\text{ s}) \\ &= 4.6\text{ kW} \end{aligned}$$

The total pump energy that must be provided in less than 3 ms is **13.8 J**.



EXAMPLE: An erbium doped fiber amplifier

Consider a 3 m EDFA that has a core diameter of 5 μm, Er^{3+} doping concentration of 1×10^{19} cm^{-3}, and τ_{sp} (the spontaneous decay time from E_2 to E_1) is 10 ms. The fiber is pumped at 980 nm from a laser diode. The pump power coupled into the EDFA fiber is 25 mW. Assuming that the confinement factor Γ is 70%, what is the fiber length that will absorb the pump radiation? Find the small signal gain at 1550 nm for two cases corresponding to full population inversion and 90% inversion.

Solution

The pump photon energy $h\upsilon = hc/\lambda = (6.626\times10^{-34})(3\times10^{8})/(980\times10^{-9}) = 2.03\times10^{-19}$ J (or 1.27 eV)

Rearranging Eq. (4.3.6), we get

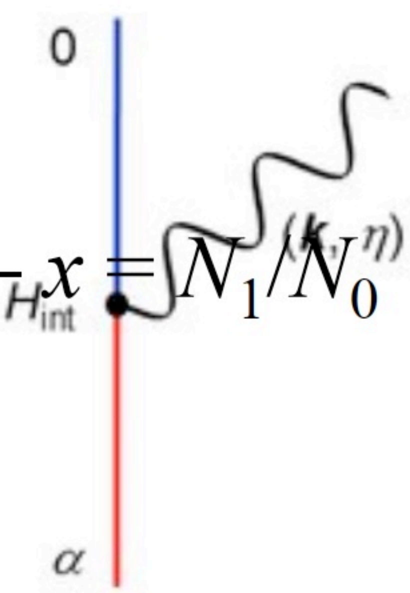
$$L_p \approx \Gamma P_p \tau_{sp} / A N h\upsilon_p$$

i.e.

$$L_p \approx (0.70)(25\times10^{-3}\text{ W})(10\times10^{-3}\text{ s}) / [\pi(2.5\times10^{-4}\text{ cm})^2(1\times10^{19}\text{ cm}^{-3})(2.03\times10^{-19}\text{ J})] = \mathbf{4.4\ m}$$

which is the maximum allowed length. The small signal gain can be rewritten as

$$g = \sigma_{em}N_2 - \sigma_{ab}N_1 = [\sigma_{em}(N_2/N_0) - \sigma_{ab}(N_1/N_0)]N_0$$



where $N_1 + N_2 = N_0$ is the total Er^{3+} concentration. Let $x = N_2/N_0$, then $1 - x = N_1/N_0$ where x represents the extent of pumping from 0 to 1, 1 being 100%.

Solution (continued)

Thus, the above equation becomes

$$g = [\sigma_{em} x - \sigma_{ab}(1 - x)]N_0$$

For 100% pumping, $x = 1$,

$$g = [(3.2 \times 10^{-21}\ \mathrm{cm}^2)(1) - 0](1 \times 10^{19}\ \mathrm{cm}^{-3}) = 3.2\ \mathrm{m}^{-1}$$

and

$$G = \exp(gL) = \exp[(3.2\ \mathrm{m}^{-1})(3\mathrm{m})] = 14{,}765 \text{ or } \mathbf{41.7\ dB}$$

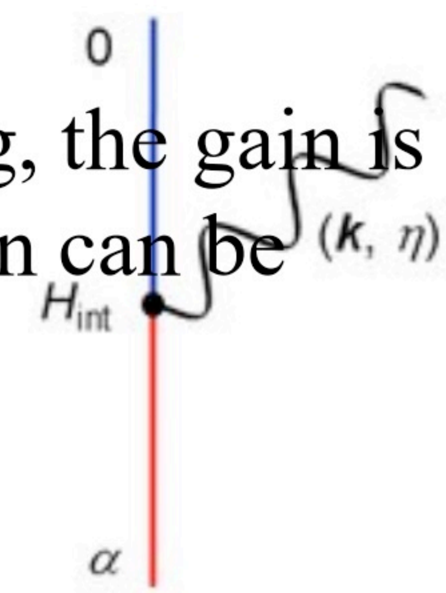
For $x = 0.9$ (90% pumping), we have

$$g = [(3.2 \times 10^{-21}\ \mathrm{cm}^2)(0.9) - (2.4 \times 10^{-21}\ \mathrm{cm}^2)(0.1)](1 \times 10^{19}\ \mathrm{cm}^{-3})$$
$$= 2.64\ \mathrm{m}^{-1}$$

and

$$G = \exp(gL) = \exp[(2.64\ \mathrm{m}^{-1})(3\mathrm{m})] = 2{,}751 \text{ or } \mathbf{34.4\ dB}$$

Even at 90% pumping the gain is significantly reduced. At 70% pumping, the gain is 19.8 dB. In actual operation, it is unlikely that 100% population inversion can be achieved; 41.7 dB is a good indicator of the upper ceiling to the gain.



EXAMPLE: Efficiency of the He-Ne laser

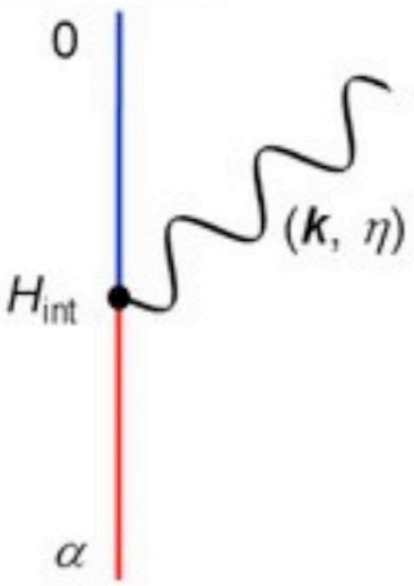
A typical low-power 5mW He-Ne laser tube operates at a dc voltage of 2000 V and carries a current of 7 mA. What is the efficiency of the laser ?

Solution

From the definition of efficiency,

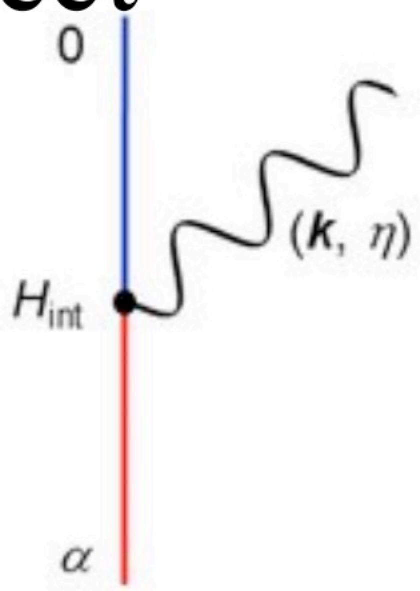
$$\text{Efficiency} = \frac{\text{Output Light Power}}{\text{Input Electrical Power}} = \frac{5 \times 10^{-3} \text{ W}}{(7 \times 10^{-3} \text{ A})(2000 \text{ V})}$$
$$= 0.036\%$$

Typically He-Ne efficiencies are less than 0.1%. What is important is the highly coherent output radiation. Note that 5 mW over a beam diameter of 1 mm is 6.4 kW m^{-2}.



EXAMPLE: He-Ne laser Doppler broadened linewidth

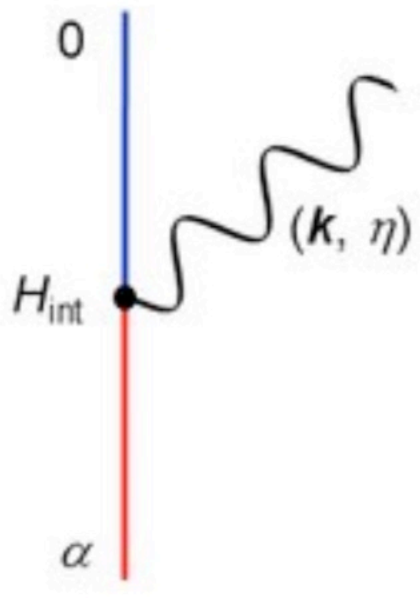
Calculate the Doppler broadened linewidths $\Delta\upsilon$ and $\Delta\lambda$ (end-to-end of spectrum) for the He-Ne laser transition for $\lambda_o = 632.8$ nm if the gas discharge temperature is about 127° C. The atomic mass of Ne is 20.2 (g mol^{-1}). The laser tube length is 40 cm. What is the linewidth in the output wavelength spectrum? What is mode number m of the central wavelength, the separation between two consecutive modes and how many modes do you expect within the linewidth $\Delta\lambda_{1/2}$ of the optical gain curve?



Solution

Due to the Doppler effect arising from the random motions of the gas atoms, the laser radiation from gas-lasers is broadened around a central frequency υ_o. The central υ_o corresponds to the source frequency. Higher frequencies detected will be due to radiations emitted from atoms moving toward the observer whereas lower frequency will be result of the emissions from atoms moving away from the observer. We will first calculate the frequency width using two approaches, one approximate and the other more accurate. Suppose that v_x is the root-mean-square (rms) velocity along the x-direction. We can intuitively expect the frequency width $\Delta\upsilon_{\mathrm{rms}}$ between rms points of the Gaussian output frequency spectrum to be

$$\Delta\upsilon_{\mathrm{rms}} = \upsilon_o\left(1+\frac{v_x}{c}\right)-\upsilon_o\left(1-\frac{v_x}{c}\right)=\frac{2\upsilon_o v_x}{c} \qquad (4.5.5)$$



EXAMPLE: Doppler broadened linewidth

Solution (continued)

We need to know the rms velocity v_x along x which is given by the kinetic molecular theory as $\frac{1}{2}M v_x^2 = \frac{1}{2}k_B T$, where M is the mass of the atom. We can therefore calculate v_x. For the He-Ne laser, it is the Ne atoms that lase, so $M = (20.2\times 10^{-3}$ kg mol^{-1}) / (6.02×10^{23} mol^{-1}) = 3.35×10^{-26} kg. Thus,

$$v_x = [(1.38\times 10^{-23}\text{ J K}^{-1})(127+273\text{K}) / (3.35\times 10^{-26}\text{ kg})]^{1/2}$$
$$= 405.9 \text{ m s}^{-1}$$

The central frequency is

$$\upsilon_o = c/\lambda_o = (3\times 10^{8}\text{ m s}^{-1}) / (632.8\times 10^{-9}\text{ m}) = 4.74\times 10^{14}\text{ s}^{-1}$$

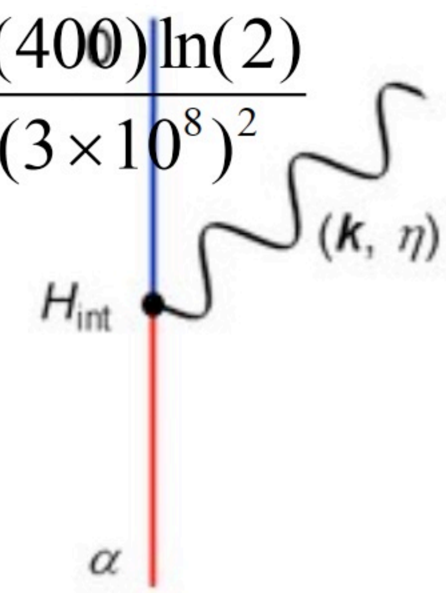
The rms frequency linewidth is approximately,

$$\Delta\upsilon_{\text{rms}} \approx (2\upsilon_o v_x)/c$$
$$= 2(4.74\times 10^{14}\text{ s}^{-1})(405.9\text{ m s}^{-1}) / (3\times 10^{8}\text{m s}^{-1})$$
$$= 1.28 \text{ GHz.}$$

The FWHM width $\Delta\upsilon_{1/2}$ of the output frequency spectrum will be given by Eq. (4.4.3)

$$\Delta\upsilon_{1/2} = 2\upsilon_o\sqrt{\frac{2k_B T\ln(2)}{Mc^2}} = 2(4.74\times 10^{14})\sqrt{\frac{2(1.38\times 10^{-23})(400)\ln(2)}{(3.35\times 10^{-26})(3\times 10^{8})^2}}$$
$$= 1.51 \text{ GHz,}$$

which is about 18% wider than the estimate from Eq. (4.5.5).



EXAMPLE: Doppler broadened linewidth

Solution (continued)

To get FWHM wavelength width $\Delta\lambda_{1/2}$, differentiate $\lambda = c/\upsilon$

$$\frac{d\lambda}{d\upsilon} = -\frac{c}{\upsilon^2} = -\frac{\lambda}{\upsilon} \qquad (4.5.6)$$

so that

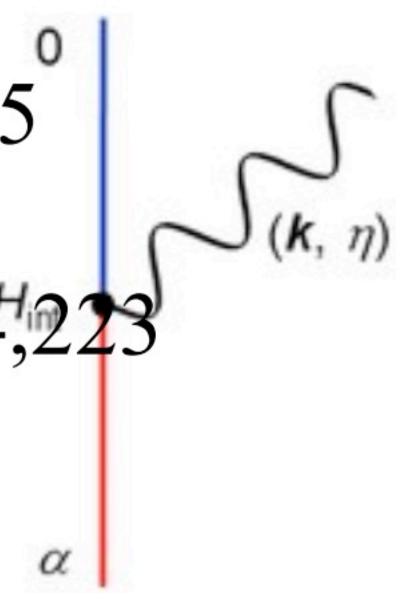
$$\Delta\lambda_{1/2} \approx \Delta\upsilon_{1/2}\,|-\lambda/\upsilon\,| = (1.51\times10^{9}\ \text{Hz})(632.8\times10^{-9}\ \text{m}) \,/\, (4.74\times10^{14}\ \text{s}^{-1})$$

or

$$\Delta\lambda_{1/2} \approx 2.02\times10^{-12}\ \text{m, or } 2.02\ \text{pm.}$$

This width is between the half-points of the spectrum. The rms linewidth would be 0.0017 nm. Each mode in the cavity satisfies $m(\lambda/2) = L$ and since L is some 4.7×10^{5} times greater than λ, the mode number m must be very large. For $\lambda = \lambda_o = 632.8$ nm, the corresponding mode number m_o is,

$$m_o = 2\text{L} \,/\, \lambda_o = (2\times0.4\ \text{m}) \,/\, (632.8\times10^{-9}\ \text{m}) = 1.264{,}222.5$$



and actual m_o has to be the closest integer value , that is, 1.264,222 or 1.264,223

EXAMPLE: Doppler broadened linewidth

Solution (continued)

The separation $\Delta\lambda_m$ between two consecutive modes (m and $m+1$) is

$$\Delta\lambda_m = \lambda_m - \lambda_{m+1} = \frac{2L}{m} - \frac{2L}{m+1} \approx \frac{2L}{m^2}$$

or $$\Delta\lambda_m \approx \frac{\lambda_o^2}{2L}$$ *Separation between modes* (4.5.7)

Substituting the values, we find $\Delta\lambda_m = (632.8\times10^{-9})^2/(2\times0.4) = 5.01\times10^{-13}$ m or 0.501 pm.

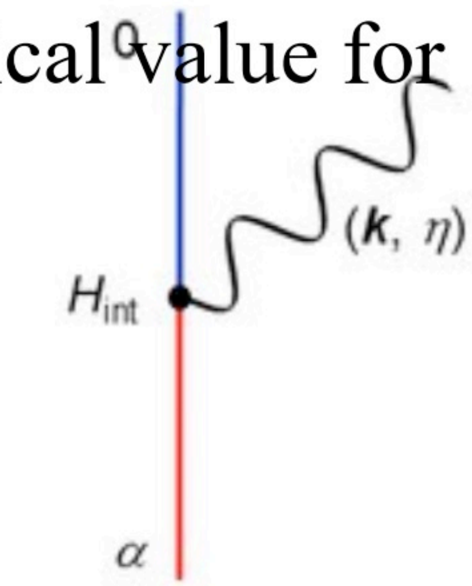
We can also find the separation of the modes by noting that Eq. (4.5.4), in which $\lambda = c/\upsilon$, is equivalent to

$$\upsilon = mc/2L$$ *Frequency of a mode* (4.5.8)

so that the separation of modes in frequency, $\Delta\upsilon_m$, is simply

$$\Delta\upsilon_m = c/2L$$ *Frequency separation of modes* (4.5.9)

Substituting $L = 0.40$ m in Eq. (4.5.9), we find $\Delta\upsilon_m = 375$ MHz. (A typical value for a He-Ne laser.)



EXAMPLE: Modes in a laser and the optical cavity length

Solution (continued)

When $L = 200$ μm,

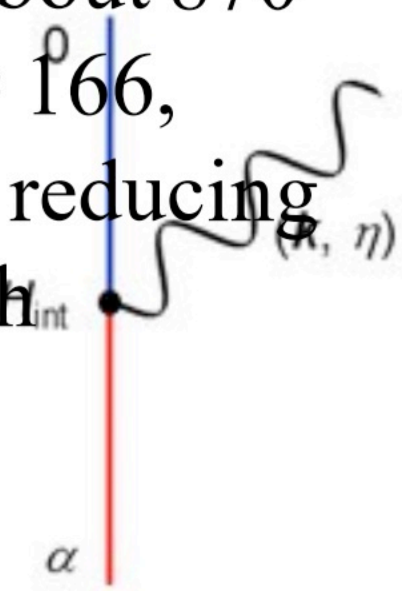
$$\Delta\lambda_m = \frac{(870\times10^{-9})^2}{2(3.6)(200\times10^{-6})} = 5.26\times10^{-10} \text{ m or } 0.526 \text{ nm}$$

If the optical gain has a bandwidth of $\Delta\lambda_{1/2}$, then there will be $\Delta\lambda_{1/2}/\Delta\lambda_m$ number of modes, or (6 nm)/(0.526 nm), that is 11 modes.

When $L = 20$ μm, the separation between the modes becomes,

$$\Delta\lambda_m = \frac{(870\times10^{-9})^2}{2(3.6)(20\times10^{-6})} = 5.26 \text{ nm}$$

Then $(\Delta\lambda_{1/2})/\Delta\lambda_m = 1.14$ and there will be one mode that corresponds to about 870 nm. In fact m must be an integer so that choosing the nearest integer, m = 166, gives $\lambda = 867.5$ nm (choosing $m = 165$ gives 872.7 nm) It is apparent that reducing the cavity length suppresses higher modes. Note that the optical bandwidth depends on the diode current.



EXAMPLE: Modes in a laser and the optical cavity length

Consider an AlGaAs based heterostructure laser diode that has an optical cavity of length 200 μm. The peak radiation is at 870 nm and the refractive index of GaAs is about 3.6. What is the mode integer m of the peak radiation and the separation between the modes of the cavity? If the optical gain vs. wavelength characteristics has a FWHM wavelength width of about 6 nm how many modes are there within this bandwidth? How many modes are there if the cavity length is 20 μm?

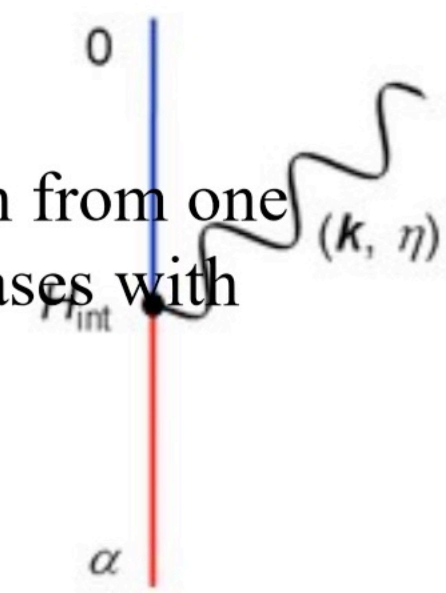
Solution

Figure 4.19 schematically illustrates the cavity modes, the optical gain characteristics, and a typical output spectrum from a laser. The wavelength λ of a cavity mode and length L are related by Eq. (4.9.1), $\boldsymbol{m(1/2)(\lambda/n) = L}$, where n is the refractive index of the semiconductor medium, so that

$$m = \frac{2nL}{\lambda} = \frac{2(3.6)(200\times10^{-6})}{(870\times10^{-9})} = 1655.1 \text{ or } 1655 \text{ (integer)}$$

The wavelength separation $\Delta\lambda_m$ between the adjacent cavity modes m and $(m+1)$ in Figure 4.19 is

$$\Delta\lambda_m = \frac{2nL}{m} - \frac{2nL}{m+1} \approx \frac{2nL}{m^2} = \frac{\lambda^2}{2nL}$$



where we assumed that the refractive index n does not change significantly with wavelength from one mode to another. Thus the separation between the modes for a given peak wavelength increases with decreasing L.

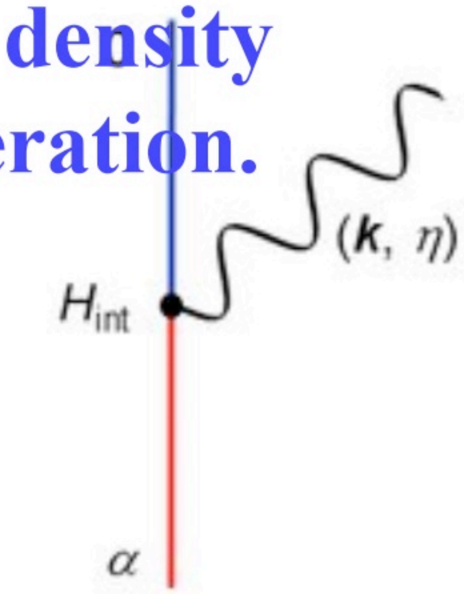
EXAMPLE: Threshold population inversion for the He-Ne laser

Solution (continued)

Note that this is the threshold population inversion for Ne atoms in configurations $2p^55s^1$ and $2p^53p^1$.

The spontaneous decay time τ_{sp} is the natural decay time of Ne atoms from E_2 ($2p^55s^1$) to E_1 ($2p^53p^1$). This time must be much longer than the spontaneous decay from E_1 to lower levels to allow a population inversion to be built-up between E_2 and E_1, which is the case in the He-Ne laser.

Equation (4.6.8) was a simplified derivation that used two energy levels; E_2 and E_1. The He-Ne case is actually more complicated because the excited Ne atom can decay from E_2 not only to E_1 but to other lower levels as well. While the He-Ne is lasing, the optical cavity ensures that the photon density in cavity promotes the E_2 to E_1 transitions to maintain the lasing operation.



EXAMPLE: Output power and photon cavity lifetime τ_{ph}

Consider the He-Ne laser in Example 4.6.1 that has a tube length of 40 cm and $R_1 = 0.95$ and $R_2 = 1$. Suppose that the tube diameter is 0.8 mm, and the output power is 2.5 mW. What are the photon cavity lifetime and the photon concentration inside the cavity? (The emission frequency υ_o is 474 THz.)

Solution

Using $L = 40$ cm, $R_1 = 0.95$, $R_2 = 1$, $\alpha_s = 0.05\ \mathrm{m}^{-1}$, gives

$$\alpha_t = \alpha_s + (1/2\ \mathrm{L})\ln(R_1R_2)^{-1}$$

$$= 0.05\ \mathrm{m}^{-1} + [2(0.4\ \mathrm{m})]^{-1}\ln[(0.95\times 1)]^{-1} = 0.114\ \mathrm{m}^{-1},$$

and hence from Eq.(4.6.12),

$$\tau_{\mathrm{ph}} = [(2)(1)(0.4)] / [(3\times 10^8)(1-e^{-2\times 0.114\times 0.4})] = 30.6\ \mathrm{ns}$$

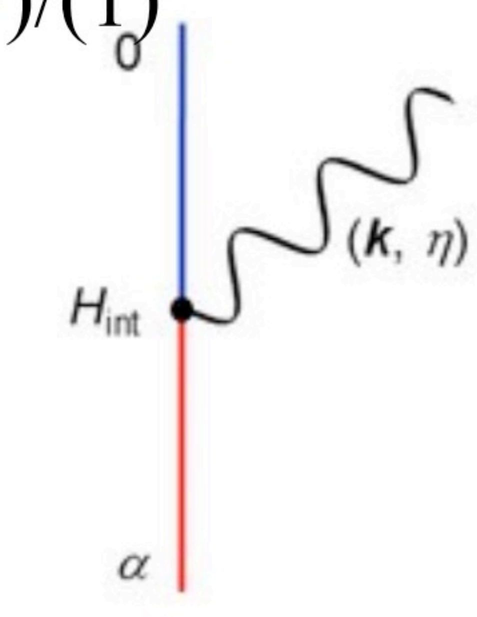
If we use Eq. (4.6.13) we would find 29.2 ns. To find the photon concentration, we use Eq. (4.6.11)

$$P_o = (0.0025\ \mathrm{W}) \approx {}^1/_2 A(1-R_1)h\upsilon_o N_{\mathrm{ph}}c/n$$

$$= ½[\pi(8\times 10^{-3}/2)^2](1-0.95)(6.62\times 10^{-34})(474\times 10^{12})N_{\mathrm{ph}}(3\times 10^8)/(1)$$

which gives

$$N_{\mathrm{ph}} \approx \mathbf{2.1\times 10^{15}\ photons\ m^{-3}}.$$



EXAMPLE: Threshold population inversion for the He-Ne laser

Consider a He-Ne gas laser operating at the wavelength 632.8 nm (equivalent to $\upsilon_o = 473.8$ THz). The tube length $L = 40$ cm and mirror reflectances are approximately 95% and 100%. The linewidth $\Delta\upsilon$ is 1.5 GHz, the loss coefficient α_s is 0.05 m^{-1}, the spontaneous decay time constant τ_{sp} is roughly 100 ns, and $n \approx 1$. What are the threshold gain coefficient and threshold population inversion?

Solution

The threshold gain coefficient from Eq. (4.6.7) is

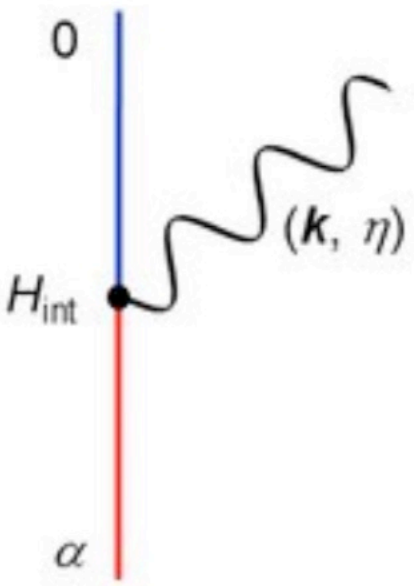
$$g_{\text{th}} = \alpha_s + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) = (0.05\ \text{m}^{-1}) + \frac{1}{2(0.4\ \text{m})}\ln\left[\frac{1}{(0.95)(1)}\right] = 0.114\ \text{m}^{-1}$$

The threshold population inversion from Eq. (4.6.9) is

$$\Delta N_{\text{th}} \approx g_{\text{th}}\frac{8\pi n^2\upsilon_o^2\tau_{\text{sp}}\Delta\upsilon}{c^2}$$

$$= (0.114\ \text{m}^{-1})\frac{8\pi(1)^2(473.8\times10^{12}\ \text{s}^{-1})^2(100\times10^{-9}\ \text{s})(1.5\times10^{9}\ \text{s}^{-1})}{(3\times10^{8}\ \text{m s}^{-1})^2}$$

$$= \ 1.1\times10^{15}\ \text{m}^{-3}.$$



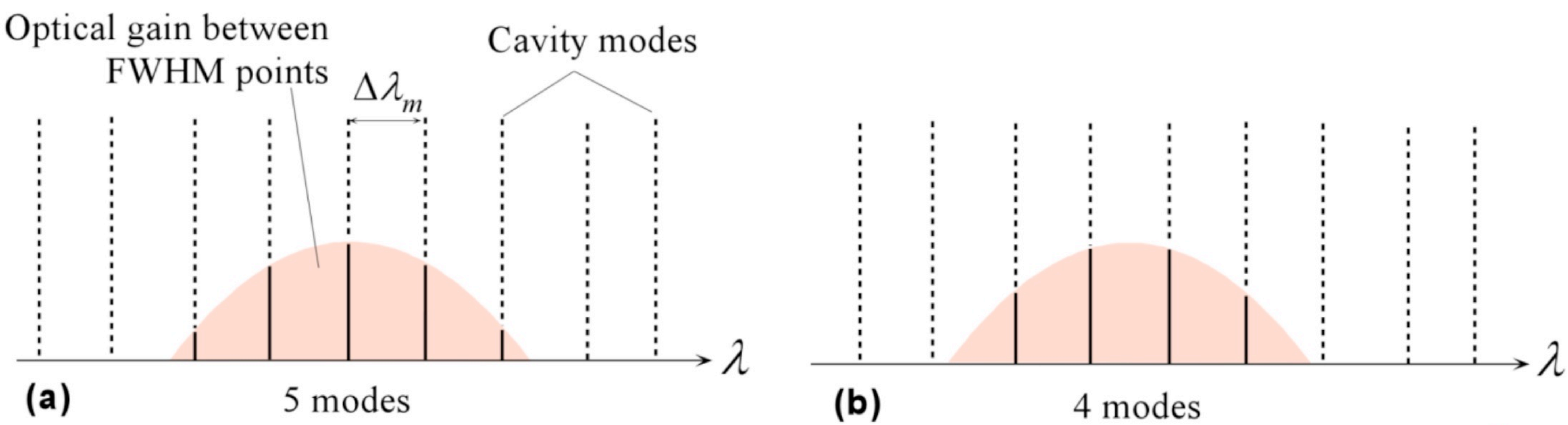
EXAMPLE: Doppler broadened linewidth

Solution (continued)

The number of modes, *i.e.* the number of m values, within the linewidth, that is, between the half-intensity points will depend on how the cavity modes and the optical gain curve coincide, for example, whether there is a cavity mode right at the peak of the optical gain curve as illustrated in Figure 4.20 . Suppose that we try to estimate the number of modes by using,

$$\text{Modes} \approx \frac{\text{Linewidth of spectrum}}{\text{Separation of two modes}} \approx \frac{\Delta\lambda_{1/2}}{\Delta\lambda_m} = \frac{2.02\ \text{pm}}{0.501\ \text{pm}} = 4.03$$

We can expect at most 4 to 5 modes within the linewidth of the output as shown in Figure 4.20. We neglected the cavity losses.



EXAMPLE: Laser diode efficiencies

Consider an InGaAs FP semiconductor laser diode that emits CW radiation at 1310 nm. The cavity length (L) is 200 μm. The internal loss coefficient $\alpha_s = 20\ \text{cm}^{-1}$, $R_1 = R_3 \approx 0.33$ (cleaved ends). Assume that internal differential quantum efficiency, IDQE, is close to 1. The threshold current is 5 mA. What is the output power P_o at $I = 20$ mA? The forward voltage is about 1.3 V. What is the EDQE and conversion efficiency?

Solution

From the definition of IDQE in Eq. (4.12.6), the number of internal coherent photons generated per second above threshold is $\eta_{\text{IDQE}}(I - I_{\text{th}})/e$. Thus,

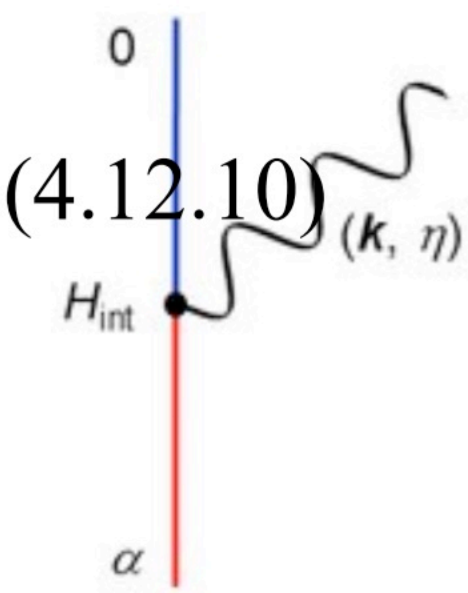
$$\textbf{Internal optical power generated} = h\upsilon \times \eta_{\text{IDQE}}(I - I_{\text{th}})/e$$

The extraction efficiency η_{EE} then couples a portion of this optical power into the output radiation. The output power P_o is then $\eta_{\text{EE}} \times h\upsilon \times \eta_{\text{IDQE}}(I - I_{\text{th}})/e$. Thus,

$$P_o = \eta_{\text{EE}}\eta_{\text{IDQE}}h\upsilon(I - I_{\text{th}})/e \qquad \textit{Output power vs current} \qquad (4.12.9)$$

The slope efficiency from Eq.(4.12.2) is

$$\eta_{\text{slope}} = \Delta P_o / \Delta I = \eta_{\text{EE}}\eta_{\text{IDQE}}(h\upsilon/e) \qquad \textit{Slope efficiency} \qquad (4.12.10)$$



EXAMPLE: Laser diode efficiencies for a sky blue LD

Solution (continued)

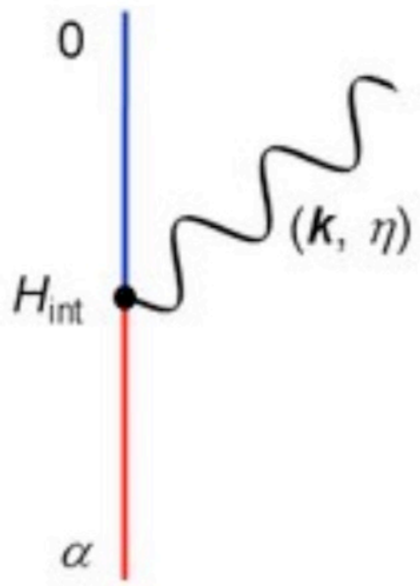
Similarly, η_{EDQE} is given by Eq. (4.12.4b) above threshold,

$$\eta_{EDQE} = (\Delta P_o / h\upsilon) / (\Delta I / e) \approx (P_o / h\upsilon) / [(I - I_{th})/e]$$

$$= [(60 \times 10^{-3})/(2.54 \times 1.6 \times 10^{-19})] / [(100 \times 10^{-3} - 30 \times 10^{-3})/1.6 \times 10^{-19})]$$

$$= \mathbf{0.34 \text{ or } 34\%}$$

The EDQE is higher than the EQE because most injected electrons above I_{th} are used in stimulated recombinations. EQE gauges the total conversion efficiency from all the injected electrons brought by the current to coherent output photons. But, a portion of the current is used in pumping the gain medium.



EXAMPLE: Laser diode efficiencies for a sky blue LD

Consider a 60 mW blue LD (Nichia SkyBlue NDS4113), emitting at a peak wavelength of 488 nm. The threshold current is 30 mA. At a forward current of 100 mA and a voltage of 5.6 V, the output power is 60 mW. Find the slope efficiency, PCE, EQE and EDQE.

Solution

From the definition in Eq. (4.12.2),

$$\eta_{\text{slope}} = P_o / (I - I_{\text{th}})$$

$$= (60\text{ mW}) / (100 - 30\text{ mA}) = \mathbf{0.86\ mW/mA^{-1}}$$

From Eq. (4.12.8), PCE is

$$\eta_{\text{PCE}} = P_o / IV$$

$$= (60\text{ mW}) / [(100\text{ mA})(5.6\text{ V})] = \mathbf{0.11} \text{ or } \mathbf{11\%}$$

We can find the EQE from Eq. (4.12.3) but we need $h\upsilon$, which is hc/λ. In eV,

$$h\upsilon\,(\text{eV}) = 1.24 / \lambda\,(\mu\text{m})$$

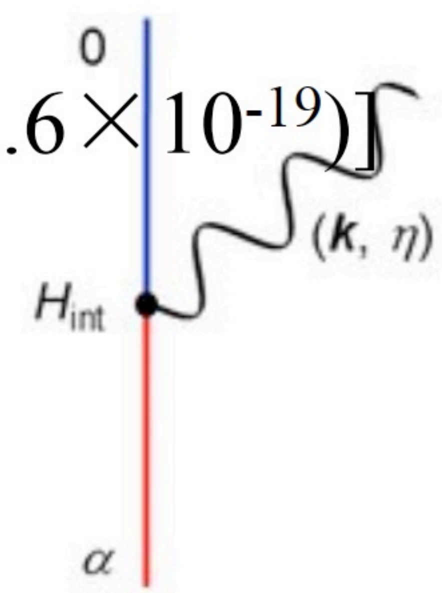
$$= 1.24 / 0.488 = 2.54\text{ eV}$$

EQE is given by Eq. (4.12.3)

$$\eta_{\text{EQE}} = (P_o / h\upsilon) / (I / e)$$

$$= [(60\times 10^{-3})/(2.54\times 1.6\times 10^{-19})]/[(100\times 10^{-3})/(1.6\times 10^{-19})]$$

$$= \mathbf{0.24} \text{ or } \mathbf{24\%}$$



EXAMPLE: Laser output wavelength variation with temperature

The refractive index n of GaAs is approximately 3.6 and it has a temperature dependence $dn/dT \approx 2.0 \times 10^{-4}\ \mathrm{K}^{-1}$. Estimate the change in the emitted wavelength at around 870 nm per degree change in the temperature for a given mode.

Solution

Consider a particular given mode with wavelength λ_m, $m\left(\dfrac{\lambda_m}{2n}\right) = L$

If we differentiate λ_m with respect to temperature,

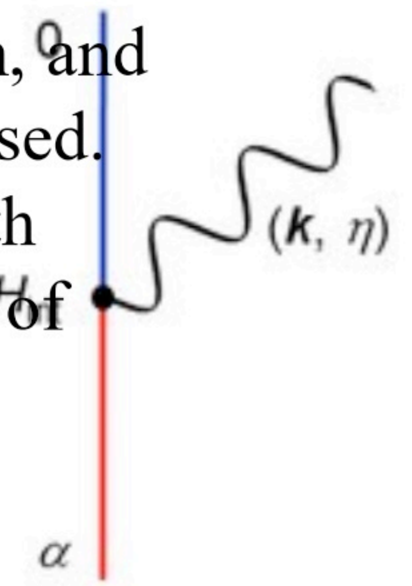
$$\frac{d\lambda_m}{dT} = \frac{d}{dT}\left[\frac{2}{m} nL\right] \approx \frac{2L}{m}\frac{dn}{dT}$$

where we neglected the change in the cavity length with temperature.

Substituting for L/m in terms of λ_m,

$$\frac{d\lambda_m}{dT} \approx \frac{\lambda_m}{n}\frac{dn}{dT} = \frac{870\ \text{nm}}{3.6}(2\times 10^{-4}\ \ \mathrm{K}^{-1}) = 0.048\ \text{nm K}^{-1}.$$

Note that we have used n for a passive cavity whereas n above should be the effective refractive index of the active cavity which will also depend on the optical gain of the medium, and hence its temperature dependence is likely to be somewhat higher than the dn/dT value we used. It is left as an exercise to show that the changes in λ_m due to the expansion of the cavity length with temperature is much less than that arising from dn/dT. The linear expansion coefficient of GaAs is $6 \times 10^{-6}\ \mathrm{K}^{-1}$.



EXAMPLE: A GaAs quantum well

Solution (continued)

where n' is the quantum number for the hole energy levels above E_v. Using $d = 10\times10^{-9}$ m, $m_h^* \approx 0.5m_e$ and $n' = 1$, we find, $\varepsilon'_1 = \mathbf{0.0075\ eV}$.

The wavelength of emission from bulk GaAs with $E_g = 1.42$ eV is

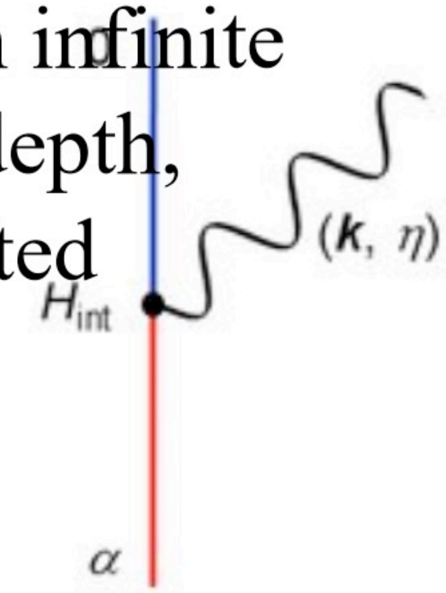
$$\lambda_g = \frac{hc}{E_g} = \frac{(6.626\times10^{-34})(3\times10^{8})}{(1.42)(1.602\times10^{-19})} = 874\times10^{-9}\ \text{m (874 nm)}$$

In the case of QWs, we must obey the selection rule that the radiative transition must have $\Delta n = n' - n = 0$. Thus, the radiative transition is from ε_1 to ε_1' so that the emitted wavelength is,

$$\lambda_{\text{QW}} = \frac{hc}{E_g + \varepsilon_1 + \varepsilon_1'} = \frac{(6.626\times10^{-34})(3\times10^{8})}{(1.42+0.0538+0.0075)(1.602\times10^{-19})}$$

$$= 838\times10^{-9}\ \text{m}\ \ (838\ \text{nm})$$

The difference is $\lambda_g - \lambda_{\text{QW}} = 36$ nm. We note that we assumed an infinite PE well. If we actually solve the problem properly by using a finite well depth, then we would find $\varepsilon_1 \approx 0.031$ eV, $\varepsilon_2 \approx 0.121$ eV, $\varepsilon'_1 \approx 0.007$ eV. The emitted photon wavelength is 848 nm and $\lambda_g - \lambda_{\text{QW}} = 26$ nm



EXAMPLE: A GaAs quantum well

Consider a very thin GaAs quantum well sandwitched between two wider bandgap semiconductor layers of AlGaAs ($Al_{0.33}Ga_{0.67}As$ in present case). The QW depths from E_c and E_v are approximately 0.28 eV and 0.16 eV respectively. Effective mass m_e* of a conduction electron in GaAs is approximately $0.07m_e$ where m_e is the electron mass in vacuum. Calculate the first two electron energy levels for a quantum well of thickness 10 nm. What is the hole energy in the QW above E_v of GaAs, if the hole effective mass $m_h^* \approx 0.50m_e$? What is the change in the emission wavelength with respect to bulk GaAs, for which $E_g = 1.42$ eV? Assume infinite QW depths for the calculations.

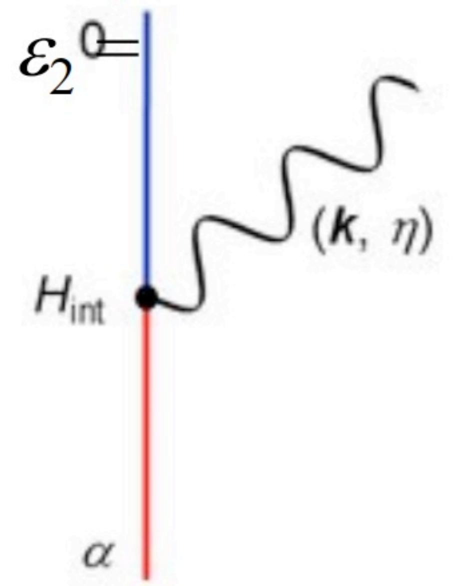
Solution

As we saw in Ch3 (Section 3.12), the electron energy levels in the QW are with respect to the CB edge E_c in GaAs. Suppose that ε_n is the electron energy with respect to E_c in GaAs, or $\varepsilon_n = E_n - E_c$ in Figure 4.40(b). Then, the energy of an electron in a one-dimensional infinite potential energy well is

$$\varepsilon_n = \frac{h^2 n^2}{8m_e^* d^2} = \frac{(6.626\times10^{-34})^2(1)^2}{8(0.07\times9.1\times10^{-31})(10\times10^{-9})^2} = 8.62\times10^{-21}\text{ J or } 0.0538\text{ eV}$$

where n is a quantum number, 1, 2,…, and we have used $d = 10\times10^{-9}$ m, $m_e^* = 0.07m_e$ and $n = 1$ to find $\varepsilon_1 = 0.054$ eV. The next level from the same calculation with $n = 2$ is $\varepsilon_2 =$ 0.215 eV.

The hole energy levels below E_v in 4.40(b) are given by $\varepsilon'_{n'} = \dfrac{h^2 n'^2}{8m_h^* d^2}$



EXAMPLE: Threshold current and optical output power from a Fabry-Perot (FP) heterostructure laser diode

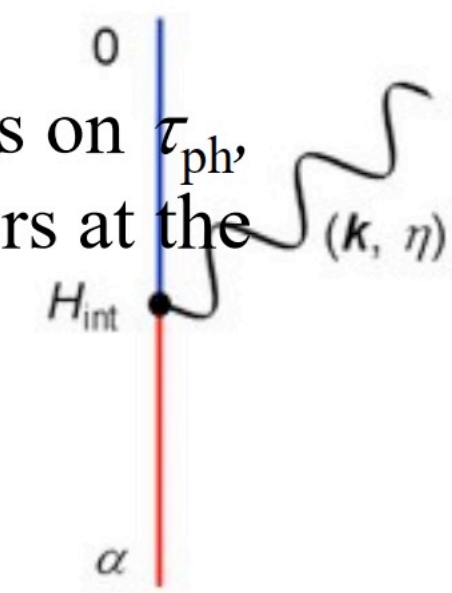
Solution (continued)

There are several important notes to this problem

• First, the threshold concentration $n_{\text{th}} \approx 2 \times 10^{18}$ cm^{-3} was obtained graphically from Figure 4.48 (b) by using the g_{th} value we need.

• Second is that, at best, the calculations represent rough values since we also need to know how the mode spreads into the cladding where there is no gain but absorption and, in addition, what fraction of the current is lost to nonradiative recombination processes. We can increase α_s to account for absorption in the cladding, which would result in a higher g_{th}, larger n_{th} and greater I_{th}. If τ_{nr} is the nonradiative lifetime, we can replace τ_r by an effective recombination time τ such that , $\tau^{-1} = \tau_r^{-1} + \tau_{nr}^{-1}$ which means that the threshold current will again be larger. We would also need to reduce the optical output power since some of the injected electrons are now used in nonradiative transitions.

• Third, is the low slope efficiency compared with commercial LDs. η_{slope} depends on τ_{ph}, the photon cavity lifetime, which can be greatly improved by using better reflectors at the cavity ends, *e.g.* ,by using thin film coating on the crystal facets to increase R.



EXAMPLE: Threshold current and optical output power from a Fabry-Perot (FP) heterostructure laser diode

Solution (continued)

That is $\boldsymbol{P_o = (0.35\ \mathrm{W\ A^{-1}})(I - I_{\mathrm{th}}) = (0.35\ \mathrm{mW\ mA^{-1}})(I - 24\ \mathrm{mA})}$

When $I = 1.5I_{th} = 36$ mA,

$$P_o = (0.35\ \mathrm{mW\ mA^{-1}})(36\ \mathrm{mA} - 24\ \mathrm{mA}) = \mathbf{4.2\ mW}$$

The slope efficiency is the slope of the P_o vs. I characteristic above I_{th},

$$\eta_{\mathrm{slope}} = \frac{\Delta P_o}{\Delta I} = \left[\frac{hc^2\tau_{\mathrm{ph}}(1-R)}{2en\lambda L}\right] = \mathbf{0.35\ mW\ mA^{-1}}$$

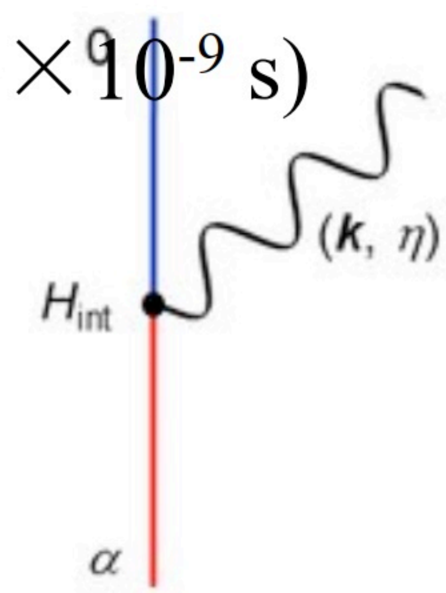
We can now repeat the problem say for $\Gamma = 0.5$, which would give $\Gamma g_{th} = \alpha_t$, so that $g_{\mathrm{th}} = 55.6\ \mathrm{cm^{-1}} / 0.5 = 111\ \mathrm{cm^{-1}}$. From Figure 4.48 (b), at this gain of 111 cm^{-1}, $n_{\mathrm{th}} \approx 2.5 \times 10^{18}\ \mathrm{cm^{-3}}$. The new radiative lifetime,

$$\tau_r = 1/Bn_{\mathrm{th}} = 1/[2.0 \times 10^{-16}\ \mathrm{m^3\ s^{-1}})(2.5 \times 10^{24}\ \mathrm{m^{-3}})] = 2.0\ \mathrm{ns}$$

The corresponding threshold current density is

$$J_{\mathrm{th}} = n_{\mathrm{th}}ed/\tau_r = (2.5 \times 10^{24}\ \mathrm{m^{-3}})(1.6 \times 10^{-19}\ \mathrm{C})(0.15 \times 10^{-6}\ \mathrm{m})/(2.0 \times 10^{-9}\ \mathrm{s})$$
$$= \mathbf{30\ A\ mm^{-2}}$$

and the corresponding threshold current I_{th} is **37.5 mA**



EXAMPLE: Threshold current and optical output power from a Fabry-Perot (FP) heterostructure laser diode

Solution (continued)

The radiative lifetime $\tau_r = 1/Bn_{\mathrm{th}} = 1/[2\times10^{-16}\ \mathrm{m^3\ s^{-1}})(\ (2\times10^{24}\ \mathrm{m^{-3}})] =$ **2.5 ns**

Since $J = I/WL$, the threshold current density from Eq. (4.13.4) is

$$J_{\mathrm{th}} = \frac{n_{\mathrm{th}}ed}{\tau_r} = \frac{(2\times10^{24}\ \mathrm{m^{-3}})(1.6\times10^{-19}\ \mathrm{C})(0.15\times10^{-6}\ \mathrm{m})}{(2.5\times10^{-9}\ \mathrm{s})}$$

$$= 1.9\times10^{7}\ \mathrm{A\ m^{-2}}\ \text{or}\ 1.9\ \mathrm{kA\ cm^{-2}}\ \text{or}\ 19\ \mathrm{A\ mm^{-2}}.$$

The threshold current itself is,

$$I_{\mathrm{th}} = (WL)J_{\mathrm{th}} = (5\times10^{-6}\ \mathrm{m})\ (250\times10^{-6}\ \mathrm{m})(1.9\times10^{7}\ \mathrm{A\ m^{-2}})$$

$= 0.024$ A or **24 mA**

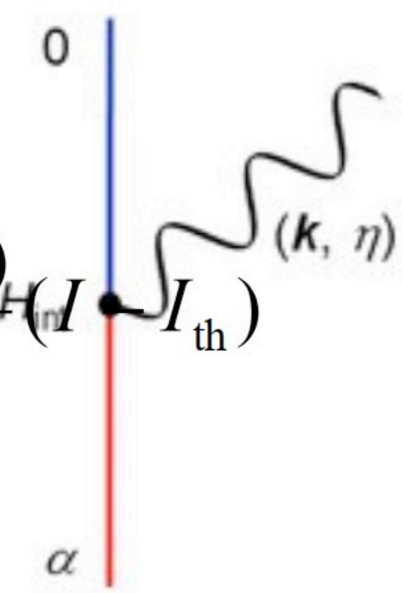
The photon cavity lifetime depends on α_t, and is given by

$$\tau_{\mathrm{ph}} = n/(c\alpha_t) = 3.6\ /\ [(3\times10^{8}\ \mathrm{m\ s^{-1}})(5.56\times10^{3}\ \mathrm{m^{-1}})]$$

$=$ **2.16 ps**

The laser diode output power is

$$P_o = \left[\frac{hc^2\tau_{\mathrm{ph}}(1-R)}{2en\lambda L}\right](I - I_{\mathrm{th}}) = \frac{(6.626\times10^{-34})(3\times10^{8})^2(2.16\times10^{-12})(1-0.32)}{2(1.6\times10^{-19})(3.6)(860\times10^{-9})(250\times10^{-6})}(I - I_{\mathrm{th}})$$



EXAMPLE: Threshold current and optical output power from a Fabry-Perot (FP) heterostructure laser diode

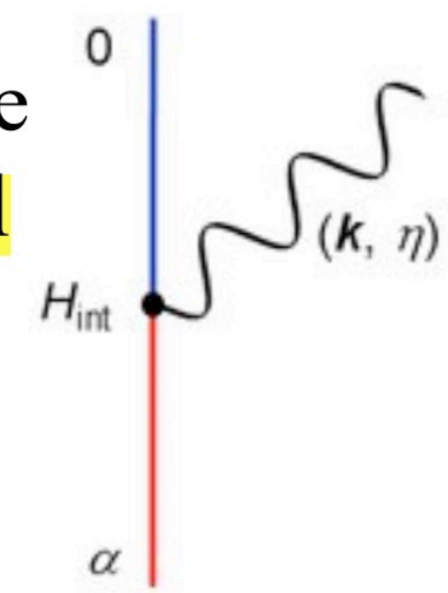
Consider GaAs DH laser diode that lases at 860 nm. It has an active layer (cavity) length L of 250 μm. The active layer thickness d is 0.15 μm and the width W is 5 μm. The refractive index is 3.6, and the attenuation coefficient α_s inside the cavity is 10^3 m^{-1}. The required threshold gain g_{th} corresponds to a threshold carrier concentration $n_{th} \approx 2\times10^{18}$ cm^{-3}. The radiative lifetime τ_r in the active region can be found (at least approximately) by using $\tau_r = 1/Bn_{th}$, where B is the direct recombination coefficient, and assuming strong injection as will be the case for laser diodes [see Eq. (3.8.7) in Chapter 3]. For GaAs, $B \approx 2\times10^{-16}$ m^3 s^{-1}. What is the threshold current density and threshold current? Find the output optical power at $I = 1.5I_{th}$, and the external slope efficiency η_{slope}. How would $\Gamma = 0.5$ affect the calculations?

Solution

The reflectances at the each end are the same (we assume no other thin film coating on the ends of the cavity) so that $R = (n-1)^2/(n+1)^2 = 0.32$. The total attenuation coefficient α_t and hence the threshold gain g_{th}, assuming $\Gamma = 1$ in Eq. (4.13.9), is

$$g_{th} = \alpha_t = (10\ \text{cm}^{-1}) + \frac{1}{(2\times250\times10^{-4}\ \text{cm})}\ln\left[\frac{1}{(0.32)(0.32)}\right] = 55.6\ \text{cm}^{-1}$$

From Figure 4.48(b), at this gain of 56 cm^{-1}, $n_{th} \approx 2\times10^{18}$ cm^{-3}. This is the threshold carrier concentration that gives the right gain under ideal optical confinement, with $\Gamma = 1$.



EXAMPLE: Laser diode efficiencies
Solution (continued)

Thus, using $I = 20$ mA in Eq. (4.12.9),

$$P_o = (0.37)(1)[(6.62\times10^{-34})(3\times10^{8})/(1310\times10^{-9})][(0.02 - 0.005)/(1.6\times10^{-19})] = \mathbf{5.2\ mW}$$

The slope efficiency from Eq. (4.12.10) is

$$\eta_{slope} = \Delta P_o / \Delta I = (5.2\text{ mW} - 0) / (20\text{ mA} - 5\text{ mA}) = \mathbf{0.35\ mW\ mA^{-1}}$$

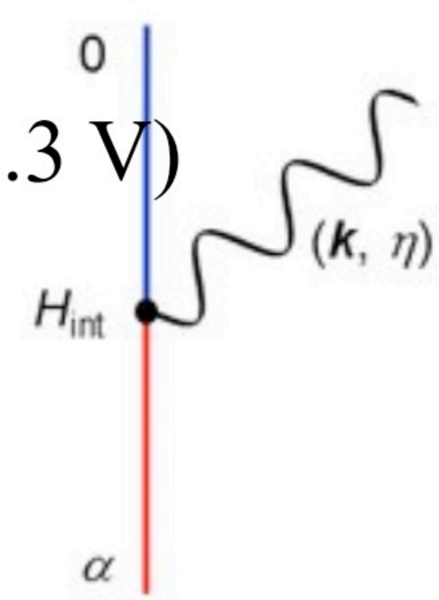
The EDQE from Eq. (4.12.11) is

$$\eta_{EDQE} = \eta_{EE}\eta_{IDQE} = \mathbf{0.37} \text{ or } \mathbf{37\%}$$

The power conversion efficiency $\eta_{PCE} = P_o / IV$

$$= 5.2\text{ mW} / (20\text{ mA} \times 1.3\text{ V})$$

$$= \mathbf{0.20} \text{ or } \mathbf{20\%}$$



EXAMPLE .3: Laser diode efficiencies
Solution (continued)

Further, from the definition of EDQE and Eq.(4.12.9) is

$$\eta_{\mathrm{EDQE}} = (\Delta P_o / h\upsilon) / (\Delta I / e) = (P_o / h\upsilon) / [(I - I_{\mathrm{th}})/e] = \eta_{\mathrm{EE}}\,\eta_{\mathrm{IDQE}}$$

External differential quantum efficiency (4.12.11)

We can now calculate the quantities needed. The total loss coefficient is

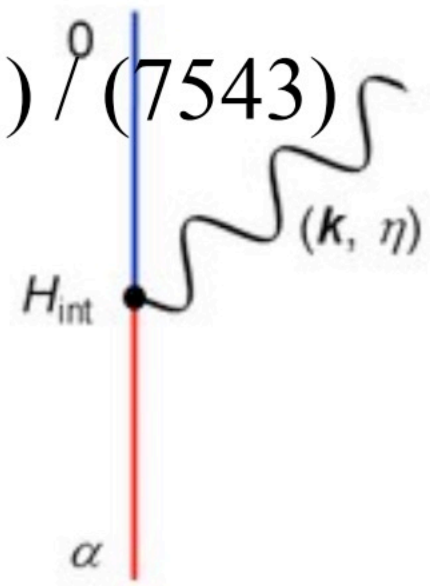
$$\alpha_t = \alpha_s + (1/2L)\ \ln\ (1 / R_1 R_2)$$

$$= 2000 + (2\times 200\times 10^{-6})^{-1} \ln(0.33\times 0.33)^{-1} = 7543\ \mathrm{m}^{-1}$$

The extraction efficiency is

$$\eta_{\mathrm{EE}} = (1/2\mathrm{L})\ln\ (1/R_1) / \alpha_t = (2\times 200\times 10^{-6})^{-1} \ln(1/0.33) / (7543)$$

$= \mathbf{0.37\ or\ 37\%}$



Example: DFB LD wavelength

Consider a DFB laser that has a corrugation period Λ of 0.22 μm and a grating length of 400 μm. Suppose that the effective refractive index of the medium is 3.5. Assuming a first order grating, calculate the Bragg wavelength, the mode wavelengths and their separation.

Solution

The Bragg wavelength is

$$\lambda_B = \frac{2\Lambda n}{q} = \frac{2(0.22\ \mu\text{m})(3.5)}{1} = 1.5400\ \mu\text{m}.$$

and the symmetric mode wavelengths about λ_B are

$$\lambda_m = \lambda_B \pm \frac{\lambda_B^2}{2nL}(m+1) = 1.5400 \pm \frac{(1.5400\ \ \mu\text{m})^2}{2(3.5)(400\ \ \mu\text{m})}(0+1)$$

so that the $m = 0$ mode wavelengths are

$$\lambda_0 = 1.53915 \text{ or } 1.54085\ \mu\text{m}.$$

The two are separated by 0.0017 μm, or 1.7 nm. Due to a design asymmetry, only one mode will appear in the output and for most practical purposes the mode wavelength can be taken as λ_B. Note: The wavelength calculation was kept to five decimal places because λ_m is very close to λ_B.