

# Plank Countdown

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Please do not actually try this one manually.

## Problem 0

What are the first four digits of  $3^{15}$ ?

Source: well known joke in the math community. Answer: 1434

## Problem 1

Let  $a$  and  $b$  be five-digit palindromes (without leading zeroes) such that  $a < b$  and there are no other five-digit palindromes strictly between  $a$  and  $b$ . What are all possible values of  $b - a$ ? (A number is a palindrome if it reads the same forwards and backwards in base 10.)

Casework on how many 'carries' there are between consecutives. 100, 110, 11.

Source: HMMT Feb 2018 C2.

## Problem 2

Quadrilateral  $ABCD$  is cyclic with  $AB = CD = 6$ . Given that  $AC = BD = 8$  and  $AD + 3 = BC$ , what is  $[ABCD]$ ?

Note

$[A_1 \dots A_n]$  denotes the area of that polygon.

Ptolemy to find the missing lengths  $AD$ ,  $BC$ , then Brahmagupta to find the area.  $33\sqrt{15}/4$ .

Source: BMT 2020 TB G2.

## Problem 3

Every face of a cube is colored one of 3 colors at random. What is the expected number of edges that lie along two faces of different colors?



Linearity of expectation. Probability that two adjacent faces are different colours is  $\frac{2}{3}$ . 8.

Source: BMT 2018 TB C1.

### Remark

BMT tiebreaker problems are also meant to be fast paced- these problems are supposed to be solved in 15 mins though most contestants finish before then. CMM TB does not work for this purpose, on the other hand....

## Problem 4

There is a unique 4-digit positive integer  $n = \underline{abcd}$  with  $\underline{ab} \cdot \underline{cd} = n/3 - 2$ .  
What is the sum of the digits of  $n$ ?

12. This is a 1434 joke invented by one of my friends. Source: 2023 AMC 12B\*/13 (a mock AMC you can find on AoPS)

## Problem 5

Find the number of ordered pairs of integers  $(a, b)$  such that  $a, b$  are divisors of 720 but  $ab$  is not.

Complementary counting. 2520.  
Source: HMMT Feb 2016 C3

## Problem 6

Compute the probability that a random permutation of the letters in BERKELEY does not have the three E's all on the same side of the Y.

Shockingly this is a BMT problem again. (2018 TB C1) We only care about the locations of the Y's and the E's.  $2 \cdot 1/4 = 1/2$ .

## Problem 7

Right triangle  $ABC$  with its right angle at  $B$  has angle bisector  $\overline{AD}$  with  $D$  on  $\overline{BC}$ , as well as altitude  $\overline{BE}$  with  $E$  on  $\overline{AC}$ . If  $\overline{DE} \perp \overline{BC}$  and  $AB = 1$ , compute  $AC$ .



Trig bash.  $(1 + \sqrt{5})/2$ .

Source: BMT 2021 TB G2

## Problem 8

Let  $\Omega$  and  $\omega$  be circles with radii 123 and 61, respectively, such that the center of  $\Omega$  lies on  $\omega$ . A chord of  $\Omega$  is cut by  $\omega$  into three segments, whose lengths are in the ratio 1 : 2 : 3 in that order. Given that this chord is not a diameter of  $\omega$ , compute the length of this chord.

Synthetic and power of a point. 42.  
Source: 2024 HMMT Feb G3

## Problem 9

In triangle  $ABC$  with  $AB = 8$  and  $AC = 10$ , the incenter  $I$  is reflected across side  $AB$  to point  $X$  and across side  $AC$  to point  $Y$ . Given that segment  $XY$  bisects  $\overline{AI}$ , compute  $BC^2$ .

Use synthetic to get  $\angle A = 60^\circ$ . 84 by some bashing.  
Source: HMMT Nov 2020 General 7

## Problem 10

Find the integer closest to

$$\frac{1}{\sqrt[4]{5^4 + 1} - \sqrt[4]{5^4 - 1}}.$$

Rationalise the denominator. 250.

Source: HMMT Feb 2014 A6

## Problem 11

Greta writes  $2, 3, \dots, 101$  on a chalkboard. Every minute she erases two chosen numbers  $x, y$  from the board, and writes  $xy/(x + y - 1)$  in their place. After 99 minutes, what number remains?



This is a contrived invariant problem. The invariant in question is

$$\prod_k \frac{1}{1 - 1/x_k}$$

where the nums on the board are  $x_1, \dots$

Source: BMT 2022 TB A3

## Problem 12

A positive integer is called extra-even if all of its digits are even. Compute the number of positive integers  $n$  less than or equal to 2022 such that both  $n$  and  $2n$  are both extra-even.

Figure out from experimenting that 0, 2, 4 are the only legal digits. 31.  
Source: BMT 2022 TB C2

## Problem

Let  $a$  and  $b$  be real numbers such that  $a + b = 16$  and  $a^3 - b^3 = ab^2 - a^2b + 2048$ . Compute  $ab^2$ .

Factor, divide, etc to obtain  $a, b$ . 192.

Source: BMT 2025 TB A2

I ran out of problem ideas

Thank you for playing!! :3