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THE INCONSTANCY  
OF THE FUNDAMENTAL PHYSICAL CONSTANTS:  
COMPUTATIONAL STATUS

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## Abstract

Ezhela V.V., Kuyanov Yu.V., Larin V.N., Siver A.S. The Inconstancy of the Fundamental Physical Constants: Computational Status: IHEP Preprint 2004-36. – Protvino, 2004. – p. 14, figs. 2, tables 3, refs.: 11.

It is argued that the CODATA recommended values of the fundamental physical constants could not be used as the reference data in searching the hypothetical space-time variations of the fundamental physical constants.

It is shown that the CODATA data permanently suffers a loss of self-consistency of the released data due to unjustified over-rounding of their estimates.

The simple estimates of the critical numbers of decimal digits that should be saved in the independently rounded correlation coefficients, the average values and uncertainties to save the self-consistency is obtained.

The set of high level quality requirements to the computerized presentation of the numerical data on the jointly measured or estimated physical values are formulated.

It is argued (once again) that the common standard for presentation of the numerical values of correlated quantities in publications and sites is urgently needed.

## Аннотация

Ежела В.В., Куйянов Ю.В., Ларин В.Н., Сивер А.С. Непостоянство фундаментальных физических постоянных: вычислительный статус: Препринт ИФВЭ 2004-36. – Протвино, 2004. – 14 с., 2 рис., 3 табл., библиогр.: 11.

Приведены свидетельства того, что рекомендуемые CODATA значения фундаментальных физических постоянных непригодны для проверки гипотезы о возможном различии значений фундаментальных постоянных в разных областях во времени и пространстве.

Показано, что публикуемые CODATA таблицы значений как на бумажных носителях, так и в электронном виде, испорчены некорректным округлением численных представлений средних значений, стандартных отклонений и коэффициентов корреляций.

Представлены простые оценки точностей корректного представления округленных средних значений, стандартных отклонений и коэффициентов корреляций. Эти оценки можно использовать для контроля корректности и согласованности значений фундаментальных физических постоянных.

Сформулированы предложения по общим требованиям к качеству представления числовых данных о совместно измеренных или оцененных физических величинах: их средних значений, стандартных отклонений и коэффициентов корреляций в публикациях, справочниках и на сайтах.

## 1. Motivation

The possible space and time variations of the fundamental physical constants (FPC) continuously attract much attention of different investigators since the time when Dirac has invented the idea. Following a recent review of J.P. Uzan [1], a general strategy for searches of the variability can be outlined as follows:

- The hypothesis of constancy of the FPC can and must be checked experimentally.
- It only make sense to consider the variations of dimensionless combinations (ratios) of the fundamental constants.
- If the FPC vary, they most probably vary jointly and slowly. This means that to notice FPC variations we should:
  - select several well separated space-time regions;
  - measure/estimate as precise as possible physics observables expressed in terms of the FPC, that refer to the same space-time region;
  - compare values of constants in the different space-time regions, but extracted from the “space-time region dependent” observables with the same current FPC evaluation and adjustment methods.

Let  $\mathbf{V}_{\mathbf{X},i}$  denotes the set of FPC related random variables to be estimated and adjusted by the method of least squares (for example) on the experimental data at space-time region  $\mathbf{X}$ . This means that after the successful adjustment we will have in the parametric  $\mathbf{V}$ -space the vector of average values  $\langle \mathbf{V}_{\mathbf{X},i} \rangle$  and the corresponding covariance matrix  $\mathbf{Cov}(\mathbf{U}_{\mathbf{X},i}, \mathbf{U}_{\mathbf{X},j})$ , characterizing the interior of the “scatter ellipsoid” centered at the end of the vector of averages

$$\sum_{ij} (\mathbf{V}_{\mathbf{X},i} - \langle \mathbf{V}_{\mathbf{X},i} \rangle) \cdot [\mathbf{Cov}(\mathbf{U}_{\mathbf{X},i}, \mathbf{U}_{\mathbf{X},j})]^{-1} \cdot (\mathbf{V}_{\mathbf{X},j} - \langle \mathbf{V}_{\mathbf{X},j} \rangle) < 1. \quad (1)$$

The same ellipsoid can be represented with the help of correlation matrix  $\mathbf{C}_{ij}(\mathbf{X}) = \mathbf{Cor}(\mathbf{U}_{\mathbf{X},i}, \mathbf{U}_{\mathbf{X},j})$  and standard deviations  $\mathbf{U}_{\mathbf{X},i} = \sqrt{\mathbf{Cov}(\mathbf{U}_{\mathbf{X},i}, \mathbf{U}_{\mathbf{X},i})}$  of  $\mathbf{V}_{\mathbf{X},i}$ .

$$\sum_{ij} \frac{\mathbf{V}_{\mathbf{X},i} - \langle \mathbf{V}_{\mathbf{X},i} \rangle}{\mathbf{U}_{\mathbf{X},i}} \cdot [\mathbf{C}_{ij}(\mathbf{X})]^{-1} \cdot \frac{\mathbf{V}_{\mathbf{X},j} - \langle \mathbf{V}_{\mathbf{X},j} \rangle}{\mathbf{U}_{\mathbf{X},j}} < 1. \quad (2)$$

To see the space-time variability we should see the well separation of the scatter ellipsoids in the  $\mathbf{V}$ -space. Let us say that vector  $\mathbf{V}$  deviates from the scatter ellipsoid obtained for  $\mathbf{X}$  space-time region by  $R_{\mathbf{X}}(\mathbf{V}, \langle \mathbf{V}_{\mathbf{X}} \rangle)$  standard deviation if

$$\sum_{ij} (\mathbf{V}_i - \langle \mathbf{V}_{\mathbf{X},i} \rangle) \cdot [\text{Cov}(\mathbf{V}_{\mathbf{X},i}, \mathbf{V}_{\mathbf{X},j})]^{-1} \cdot (\mathbf{V}_j - \langle \mathbf{V}_{\mathbf{X},j} \rangle) = R_{\mathbf{X}}^2(\mathbf{V}, \langle \mathbf{V}_{\mathbf{X}} \rangle). \quad (3)$$

Then it is easy to see that the scatter ellipsoids obtained for the  $\mathbf{X}$  and  $\mathbf{Y}$  regions will be well separated if in the whole  $\mathbf{V}$  space we will have

$$R_{\mathbf{X}}^2(\mathbf{V}, \langle \mathbf{V}_{\mathbf{X}} \rangle) + R_{\mathbf{Y}}^2(\mathbf{V}, \langle \mathbf{V}_{\mathbf{Y}} \rangle) > 2, \quad (4)$$

that means that the scatter ellipsoids do not intersect. Hence, to be able to notice the variability we should have both: accurately estimated average values and corresponding scatter ellipsoid for every space-time region where we estimate the FPC. It is the delicate problem as we will show further.

The only and the best known well elaborated procedures to evaluate and adjust fundamental physical constants are implemented at the NIST Physics Laboratory [2]. The set of FPC periodically adjusted at NIST is recommended by CODATA as the reference source of the FPC for scientific applications and technology. In any attempt to notice the space-time variability of the FPC one cannot avoid the CODATA recommended values, deemed in the physics community as the one of the best known set of FPC adjusted in the space-time region where we are. But unfortunately it is impossible. Simply because we never had the set of the recommended FPC correct enough for the testing their space-time variability. To show this let us select subsample of the dimensionless FPC from the CODATA-2002 recommended set [5], say the set:

Standard FPC name	Symbol	Value (2002)	Uncertainty
fine-structure constant	$\alpha$	7.297 352 568e-3	0.000 000 024e-3
electron-muon mass ratio	$m_e/m_\mu$	4.836 331 67e-3	0.000 000 13e-3
electron-proton mass ratio	$m_e/m_p$	5.446 170 2173e-4	0.000 000 0025e-4
electron-deuterium mass ratio	$m_e/m_d$	2.724 437 1095e-4	0.000 000 0013e-4
electron-proton magn. moment ratio	$\mu_e/\mu_p$	-658.210 6862	0.000 0066
muon-proton magn. moment ratio	$\mu_\mu/\mu_p$	-3.183 345 118	0.000 000 089
proton $g$ factor	$g_p = 2\mu_p/\mu_n$	5.585 694 701	0.000 000 056

The corresponding CODATA-2002 correlation matrix is as follows:

Cor(2002)	$\alpha$	$m_e/m_\mu$	$m_e/m_p$	$m_e/m_d$	$\mu_e/\mu_p$	$\mu_\mu/\mu_p$	$g_p = 2\mu_p/\mu_n$
$\alpha$	1.000	-0.247	0.000	0.000	-0.003	0.230	-0.002
$m_e/m_\mu$	-0.247	1.000	0.004	0.004	0.008	-0.934	0.008
$m_e/m_p$	0.000	0.004	1.000	0.894	0.000	-0.004	-0.046
$m_e/m_d$	0.000	0.004	0.894	1.000	0.000	0.012	-0.041
$\mu_e/\mu_p$	-0.003	0.008	0.000	0.000	1.000	-0.008	0.999
$\mu_\mu/\mu_p$	0.230	-0.934	-0.004	0.012	-0.008	1.000	0.350
$g_p = 2\mu_p/\mu_n$	-0.002	0.008	-0.046	0.041	0.999	0.350	1.000

This matrix is non-positive definite matrix (it has one negative eigenvalue = **-0.000293338**).

This means that we have no scatter ellipsoid, the corresponding “scatter region” is unbounded and the comparison with any other evaluations is senseless. This confusion might be due to misprints in the resource database as of 2002, but this is not the case. The same situation with non-positive definite correlation matrices is present in all releases of the FPC produced by NIST and approved/recommended by CODATA. Further examples of the wrong subsamples of the CODATA recommended FPC see in the Table 1, where we compare data from the last three releases (V.3.0, V.3.2, V.4.0). The other examples presented also in our previous papers [6, 7] on this subject.

**Table 1.** Comparison of the selected CODATA:1986, CODATA:1998, and CODATA:2002 recommended values for the triads of quantities: averages, uncertainties, correlations.

CODATA:1986		Symbol [units]	Value (uncertainty) $\times$ scale	Correlations		
Elementary charge Plank constant Electron mass $1/\alpha(0)$	$e$	[C]	$1.602\,177\,33(49) \times 10^{-19}$	$e$	$h$	$m_e$
	$h$	[J s]	$6.626\,075\,5(40) \times 10^{-34}$			
	$m_e$	[kg]	$9.109\,389\,7(54) \times 10^{-31}$			
	$\alpha(0)^{-1}$		$137.035\,989\,5(61)$			
CODATA:1998		Symbol [units]	Value (uncertainty) $\times$ scale	Correlations		
Elementary charge Plank constant Electron mass $1/\alpha(0)$	$e$	[C]	$1.602\,176\,462(63) \times 10^{-19}$	$e$	$h$	$m_e$
	$h$	[J s]	$6.626\,068\,76(52) \times 10^{-34}$			
	$m_e$	[kg]	$9.109\,381\,88(72) \times 10^{-31}$			
	$\alpha(0)^{-1}$		$137.035\,999\,76(50)$			
CODATA:2002		Symbol [units]	Value (uncertainty) $\times$ scale	Correlations		
Elementary charge Plank constant Electron mass $1/\alpha(0)$	$e$	[C]	$1.602\,176\,53(14) \times 10^{-19}$	$e$	$h$	$m_e$
	$h$	[J s]	$6.626\,0693(11) \times 10^{-34}$			
	$m_e$	[kg]	$9.109\,3826(16) \times 10^{-31}$			
	$\alpha(0)^{-1}$		$137.035\,999\,11(46)$			

The eigenvalues of these correlation sub-matrices are as follows:

$$\begin{aligned} \text{CODATA : 1986 } & \{2.99891, 1.00084, 0.000420779, -0.000172106\}; \\ \text{CODATA : 1998 } & \{2.99029, 1.01003, -0.000441572, 0.00012358\}; \\ \text{CODATA : 2002 } & \{2.99802, 1.00173, 0.000434393, -0.000183906\}. \end{aligned}$$

Definitely something is wrong with the NIST evaluation/adjustment/presentation procedures. We suspect that the origin of these permanent confusions is the unjustified independent rounding of the output interrelated quantities: vector of constant estimates, their standard deviations(uncertainties) and their correlations.

Superficial independent rounding may lead to catastrophic changes in the connection of averages, standard uncertainties and the scatter ellipsoid: the rounded average values may get out of the “etalon” scatter ellipsoid obtained after rounding the correlation matrix. The “scatter region” may turn to become hyperboloid. From the other hand any numerical calculation is performed with rounding or truncating decimal numbers.

To preserve the general properties of the FPC data structure, a special quality assurance procedures should be developed and applied. In the next section we collect the high level requirements to the set of FPC needed to guarantee the safe and correct usage of this key informational resource.

## 2. High level requirements to the set of adjusted FPC

Let us introduce a few special notations and definitions for different sets of FPC to simplify formulation and discussions of the requirements.

$\mathbf{V}^B$  or “**basic FPC**” is the set of constants that participated in the fits to the experimental data via observational equations.

$\mathbf{V}^D$  or “**derived FPC**” is the set of constants and units conversion factors that are known to be function dependent on basic constants. Symbolically  $\mathbf{V}^D = \mathbf{F}(\mathbf{V}^B)$  and they are evaluated on the basis of the  $\mathbf{V}^B$  with the proper propagation of the uncertainties with the sufficient accuracy to guarantee positive semidefiniteness of the derived covariance matrix<sup>1</sup>.

$\mathbf{V}^A$  or “**adjusted FPC**” is the  $\mathbf{V}^B \cup \mathbf{V}^D$  with cross covariances (correlations) added with sufficient accuracy to obtain combined covariance matrix as positive semidefinite matrix.

$\mathbf{V}^R$  or “**recommended FPC**” is the  $\mathbf{V}^A$  but rounded by NIST to be compactly presented in their publications and as recommended data for science and technology by CODATA.

All data sets  $\mathbf{V}^I$  defined above have the same pair of structures:

$$\mathbf{V}^I = \{\text{Average}(\mathbf{V}^I), \text{Covariance}(\mathbf{V}^I)\}$$

or

$$\mathbf{V}^I = \{\text{Average}(\mathbf{V}^I), \text{Uncertainty}(\mathbf{U}^I), \text{Correlator}(\mathbf{C}^I)\}.$$

Let us call the internal calculational accuracy of numerical presentation of all components of the  $\mathbf{V}^B$  obtained from the adjustment procedures as **etalon accuracy**.

### 2.1. Correctness and Self-consistency

If the adjustment of the constants belonging to  $\mathbf{V}^B$  is successful then we have positive definite covariance (correlation) matrix presented with an etalon accuracy, as well as the vector of average values.

We say that the  $\mathbf{V}^D$ ,  $\mathbf{V}^A$  are correct if their covariance (correlation) matrices are positive semi-definite. In other words, we have sufficient internal calculation accuracy to obtain correct results.

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<sup>1</sup>By definition the covariance (correlation) matrix for the jointly measured or estimated quantities is the positive semidefinite matrix, moreover if adjustment is performed by the least squares method the covariance (correlation) matrix if presented with the etalon accuracy should be positive definite for the successful adjustment.

We say that the  $\mathbf{V}^R$  is correct and self-consistent if one of two possibilities is true:

- 1)  $\mathbf{V}^R \equiv \mathbf{V}^A$  or
- 2) For any subset  $\mathbf{v}(\mathbf{V}^R) \subset \text{Average}(\mathbf{V}^R)$  for which corresponding covariance submatrix  $\text{Cov}(\mathbf{v}(\mathbf{V}^R))$  is positive definite we have

$$[\mathbf{v}(\mathbf{V}^R) - \mathbf{v}(\mathbf{V}^A)]_i \cdot [\text{Cov}(\mathbf{v}(\mathbf{V}^A))]_{ij}^{-1} \cdot [\mathbf{v}(\mathbf{V}^R) - \mathbf{v}(\mathbf{V}^A)]_j \leq 1$$

or

$$[\mathbf{v}(\mathbf{V}^R) - \mathbf{v}(\mathbf{V}^A)]_i \cdot [\text{Cov}(\mathbf{v}(\mathbf{V}^R))]_{ij}^{-1} \cdot [\mathbf{v}(\mathbf{V}^R) - \mathbf{v}(\mathbf{V}^A)]_j \leq 1.$$

These conditions guarantee the **self-consistency** of the  $\mathbf{V}^R$ , e.g. that the rounded and unrounded scatter ellipsoids are well intersected and unrounded and rounded subvectors belong to that intersection.

## 2.2. Reliability

We will say that the next release  $\mathbf{V}_{YY}^R$  is reliable if it is correct, selfconsistent, and if any subvector  $\mathbf{v}(\mathbf{V}_{YY}^R)$  with positive definite covariance is ended in the point inside the scatter ellipsoid for the corresponding subvector of the previous release. For example, for the 1998 and 2002 releases these conditions will read

$$[\mathbf{v}(\mathbf{V}_{02}^R) - \mathbf{v}(\mathbf{V}_{98}^A)]_i \cdot [\text{Cov}(\mathbf{v}(\mathbf{V}_{98}^A))]_{ij}^{-1} \cdot [\mathbf{v}(\mathbf{V}_{02}^R) - \mathbf{v}(\mathbf{V}_{98}^A)]_j \leq 1.$$

The reliability indicator is constructed with an assumption that the relative time variation of the fundamental constants during two successive sessions of the adjustments are negligible compared with the average relative standard deviation of the constants.

## 2.3. Availability

Next important quality indicator we propose is the availability of all data on FPC (average values, uncertainties, correlations) in computer readable forms with as maximal as possible completeness and accuracy of numerical data. The importance of the availability is hard to overestimate in the era of the Web communications and Web and GRID computations<sup>2</sup>.

It turns out that NIST and CODATA, in spite of the nicely organized affiliation web sites offer the current and archived data on the FPC in the hopeless obsolete manner, as it will be shown in the sections to follow.

## 2.4. Traceability

The traceability in the context of usage the recommended FPC is the access to all input experimental and theoretical material used in the adjustment as well as detailed descriptions of the used procedures needed to reproduce the adjustment independently in case of any suspicions on the misprints in the database, ideological or software bugs.

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<sup>2</sup>To taste the importance of the availability requirement we will recommend reader to try to check our calculations presented in the motivation section, including the correctness of data extraction from NIST publications and site.

### 3. Safety rounding off the correlated quantities

Here we derive a simple sufficient estimates on the accuracy of a safely independent rounding off the average values  $\mathbf{V}_i$ , uncertainties  $\mathbf{U}_i$ , correlations  $\mathbf{C}_{ij}$  obtained in jointly measurement or estimation procedures with sufficient etalon accuracy.

Let  $(\mathbf{V}_i, \mathbf{U}_i, \mathbf{C}_{ij}), i, j = 1, \dots, n$  be the aggregate of  $n$  jointly measured or estimated physical quantities, where numerical parts of  $\mathbf{V}_i, \mathbf{U}_i$  are the real numerical vectors,  $\mathbf{U}_i > \mathbf{0}$ ,  $\mathbf{C}_{ij}$  is the real, symmetric, and positive definite matrix with matrix elements bounded as follows:

$$C_{ii} = 1 \quad \text{for all } i = 1, \dots, n \quad \text{and} \quad |C_{i \neq j}| < 1.0.$$

Suppose that for some reason we need to store and exchange numerical data on this aggregate rounded to some accuracy  $A$  that is lower than the etalon one.

Let  $\mathbf{R}_{ij}$  be the “rounder” matrix, such that if it is added to the matrix  $\mathbf{C}_{ij}$ , the obtained matrix  $\mathbf{C}_{ij}^R = \mathbf{C}_{ij} + \mathbf{R}_{ij}$  will be real, symmetric, positive definite and all  $|C_{i \neq j}^R| < 1$  are decimal numbers with  $A$  digits to the right of the decimal point.

It is easy to see that matrix  $\mathbf{R}_{ij}$  should have the following properties:

$$R_{ii} = 0 \quad \text{for all } i = 1, \dots, n \quad \text{and} \quad |R_{i \neq j}| \leq 5.0 \times 10^{-A-1}.$$

Let further  $c_1 \leq \dots \leq c_n, \rho_1 \leq \dots \leq \rho_n$ , and  $c_1^R \leq \dots \leq c_n^R$  be the ordered sets of eigenvalues of the matrices  $\mathbf{C}_{ij}, \mathbf{R}_{ij}$ , and  $\mathbf{C}_{ij}^R$  correspondingly. Then from the Weil’s theorem for any  $l = 1, \dots, n$  we have the following inequalities [8],[9]:

$$c_l + \rho_1 \leq c_l^R \leq c_l + \rho_n.$$

>From the Gershgorin’s theorem on the distributions of the eigenvalues of the Hermitian matrices [8] it follows that

$$\rho_1 \geq -(n - 1) \cdot 5 \cdot 10^{-(A+1)} = -\frac{(n - 1)}{2} \cdot 10^{-A}$$

and hence to have the matrix  $\mathbf{C}_{ij}^R$  as positive semi definite matrix it is sufficient to demand

$$0 \leq c_1 - \frac{(n - 1)}{2} \cdot 10^{-A} \leq c_1^R.$$

>From the left inequality we have the final estimate for the threshold accuracy index for safely uniform independent rounding of the positive definite correlation matrix  $\mathbf{C}_{ij}$  with minimal eigenvalue  $c_1 = \lambda_{min}^C$

$$A \geq A_C^{th} = \left\lceil \log_{10} \left( \frac{n - 1}{2 \cdot \lambda_{min}^C} \right) \right\rceil. \quad (5)$$

**NOTE.** According to the Weil’s theorem any uniform rounding the off-diagonal matrix elements of the positive semi-definite correlation (covariance) matrix is forbidden.

Indeed, as rounder matrix is traceless Hermitian matrix, it obliged to have the negative minimal eigenvalue. Furthermore from the left inequality of the Weil’s theorem statement it follows that any rounding could lead to the matrix with negative minimal eigenvalue.

Now let us clarify to what accuracy we may round off the  $\mathbf{V}_i$  and  $\mathbf{U}_i$  in the decimal presentations. Let  $\mathbf{R}_i^V$  be the such “rounding vector” that the obtained rounded vector  $\mathbf{V}_i^R = \mathbf{V}_i - \mathbf{R}_i^V$  is still in the etalon scatter ellipsoid. Then from the condition (2) for the components of the rounding vector we will have

$$\sum_{ij} \frac{\mathbf{R}_i^V}{U_i} \cdot [C^{-1}]_{ij} \cdot \frac{\mathbf{R}_j^V}{U_j} < 1. \quad (6)$$

In the eigenbasis of the etalon correlator  $\mathbf{C}_{ij}$  the expression (6) can be transformed to

$$\sum_{ij} \sum_{mn} \frac{\mathbf{R}_i^V}{U_i} \cdot [L^{-1}]_{im} \cdot \frac{\delta_{mn}}{\lambda_m} \cdot [L]_{nj} \cdot \frac{\mathbf{R}_j^V}{U_j} < 1, \quad (7)$$

where  $L$  is a rotation matrix. As we try to find the sufficient condition for rounding vector components it is enough to demand the validity of (7) for all correlator eigenvalues replaced with minimal one. Then the inequality (7) will become

$$\sum_i \left( \frac{\mathbf{R}_i^V}{U_i} \right)^2 < \lambda_{\min}^C. \quad (8)$$

Inequality (8) means that we can round components independently only inside the maximal hypercube imbeded into scatter ellipsoid:

$$\frac{|\mathbf{R}_i^V|}{U_i} < \sqrt{\frac{\lambda_{\min}^C}{n}}. \quad (9)$$

To obtain the accuracy  $A_i^V$  for the  $i$ -th component that will be sufficient to guarantee that the end of the vector  $\mathbf{V}_i^R$  belongs to the interior of the etalon scatter ellipsoid it is sufficient to have

$$|\mathbf{R}_i^V|/[unit_i] \leq 5 \cdot 10^{-(A_i^V + 1)} [unit_i].$$

>From this bound it follows that to have the rounded vector of average values pointing to the interior of the etalon scatter ellipsoid one should save

$$A_i \geq A_i^V = \left\lceil \frac{1}{2} \log_{10} \left( \frac{n}{4 \cdot \lambda_{\min}^C \cdot (U_i/[unit_i])^2} \right) \right\rceil \quad (10)$$

digits to the right of the decimal point.

Now let us turn to the rounding of the uncertainties  $\mathbf{U}_i$ . It is the common practice to present the average values and uncertainties with the same accuracy  $A_i^V = A_i^U$ . With this rule let us rewrite inequality (9) in the form

$$\log_{10}(U_i) \geq \log_{10} \left( \frac{1}{2} \sqrt{\frac{n}{\lambda_{\min}^C}} \right) - A_i^U.$$

Taking into account the equality<sup>3</sup>

$$\lfloor \log_{10}(U_i) \rfloor + 1 = P_i^U - A_i^U,$$

where  $P_i^U$  is the precision of the  $U_i$  we will obtain

$$P_i^U \geq \left\lceil \frac{1}{2} \log_{10} \left( \frac{n}{4 \cdot \lambda_{min}^C} \right) \right\rceil. \quad (11)$$

One can see that right part of the inequality does not depend on index  $i$ , so we can introduce  $P^U$  which is the same for every  $i$ :

$$P^U = P_i^U.$$

The equation (11) give the minimal precision that should not be reduced if we adopt the rule that accuracy of the uncertainties should be equal to the accuracy of the average values.

In summary: we have obtained  $n + 1$  reference numbers  $A_C^{th}$  and  $A_i^V$  defining the levels with safety independent rounding off the decimal numerical presentation of the interrelated random quantities: average values, their uncertainties, and correlations.

Having these numbers the strategy for the safety independent rounding can be as follows:

*In self-consistent numerical presentation of interrelated random quantities ( $V_i$ ,  $U_i$ ,  $C_{ij}$ ) in decimal real numbers the average values  $V_i$  and the uncertainties  $U_i$  should have at least  $A_i^V$  digits to the right of the decimal point and the correlation coefficients  $C_{i \neq j}$  should have at least  $A_C^{th}$  digits to the right of the decimal point.*

#### 4. Do the CODATA 2002 recommended FPC meet the high level quality requirements?

In this section we present some further evidences of violations of the above high level requirements in the recent releases of the CODATA recommended values of the FPC.

##### 4.1. Correctness & Selfconsistency

In motivation section we already presented the evidences that the CODATA data on correlations are incorrect. Here we present an evidence that the average values of the recommended FPC are also questionable, because of over-rounding can easily move them out of the etalon scatter ellipsoid. To check this the whole adjustment process should be repeated with the “etalon accuracy”.

It turned out that we managed to collect enough amount of data from the NIST publications to reproduce all steps of the evaluation and adjustment of the basic set of constants [10] only for the 1998 release. We had obtained the “correct set of the basic constants” using methods

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<sup>3</sup>This equality is valid for real numbers only. For the integer number that treated as the numbers with infinite precision it is not valid.

described by NIST experts [4]<sup>4</sup> and then calculated the threshold accuracies for the elements of the correlation matrix, the averages and the uncertainties. The results are as follows:

$$\begin{aligned}\lambda_{C,\min} &\approx 7.58 \cdot 10^{-7}, \\ A_C^{th} &= 8 \text{ (versus } A_C^{CODATA} = 3), \\ P^U &= 4 \text{ (versus } P^{U,CODATA} = 2).\end{aligned}$$

One can see that the CODATA data suffers the loss of self-consistency of the released data due to unjustified over-rounding of their results.

Having the data on the FPC in the “etalon accuracy” we are able to show that the obtained estimates for the threshold rounding indices are indeed close to the real situation and should be used as regulators for the correctness of the rounding. To show that the rounding procedure can move the end of the vector-of-constants out of the etalon scatter ellipsoid we will use the sample of constants that was mentioned in [1] as the candidates to trace the large-scale space-time variability of their dimensionless combinations:

**Table 2.** Selected basic and derived constants from the IHEP adjustment based on the NIST 1998 input data.

Symbol	[units]	Average value	Uncertainty
$h$	[J s]	$6.62606875610000 \times 10^{-34}$	$5.2200000 \times 10^{-41}$
$m_e$	[kg]	$9.10938187491360 \times 10^{-31}$	$7.2057063 \times 10^{-38}$
$m_p$	[kg]	$1.67262158291420 \times 10^{-27}$	$1.3235274 \times 10^{-34}$
$m_n$	[kg]	$1.67492715608612 \times 10^{-27}$	$1.3253602 \times 10^{-34}$
$e$	[C]	$1.60217646198672 \times 10^{-19}$	$6.3181739 \times 10^{-27}$

The corresponding correlation matrix of their uncertainties in the “etalon accuracy” <sup>5</sup>

Cor	$h$	$m_e$	$m_p$	$m_n$	$e$
$h$	1.000000000	0.9957673366	0.9954294463	0.9954234131	0.9989373297
$m_e$	0.9957673366	1.000000000	0.9996433868	0.9996224521	0.9904731204
$m_p$	0.9954294463	0.9996433868	1.000000000	0.9999732991	0.9901455374
$m_n$	0.9954234131	0.9996224521	0.9999732991	1.000000000	0.9901469965
$e$	0.9989373297	0.9904731204	0.9901455374	0.9901469965	1.000000000

is the positive definite matrix with eigenvalues as follows:

$$\{4.98223, 0.0172451, 0.000495716, 0.0000263673, 6.47023 \times 10^{-10}\}.$$

Corresponding  $A_C^{th} = 10$  and it is close enough to our minimal accuracy, the rounding of the above correlator to 8 digits will make the matrix non-positive definite.

<sup>4</sup> As the correlation matrix of the uncertainties in the input experimental data is not a positive definite matrix there (supposedly by overrounding for publication), we were forced to “un-round” several matrix elements to have positive definite weight matrix in the least squares method of adjustment.

<sup>5</sup> The Plank constant is the basic one, the other selected are derived constants. In calculating the corresponding correlation matrix we use the minimal possible accuracy that give us the positive definite correlation matrix.

Now we will round average values of the constants to have accuracy below the allowed thresholds  $\mathbf{A}_i^V$ . In the Table 3 we present the values of the differences  $\langle \mathbf{V}_i \rangle - \mathbf{V}_i^r$  between calculated average values of the selected constants with the etalon accuracy and the rounded step-by-step values to show that after the predicted moment the end point of the rounded vector will be moved out of the etalon scatter ellipsoid for many standards  $\mathbf{R}(\mathbf{V}^r, \langle \mathbf{V} \rangle)$ .

**Table 3.** Evolution of the “distance” of the end point of rounded vector from the etalon scatter ellipsoid expressed in number of standard deviations squared with rounding off the vector components  $\mathbf{R}_i^V$  in steps.

Step	$h$ [J s]	$m_e$ [kg]	$m_p$ [kg]	$m_n$ [kg]	$e$ [C]	$\mathbf{R}^2(\mathbf{V}^r, \langle \mathbf{V} \rangle)$
9	<b>4.39E-42</b>	<b>2.51E-39</b>	<b>1.71E-35</b>	<b>4.39E-35</b>	<b>-1.99E-28</b>	<b>3.9E+06</b>
8	<b>3.90E-43</b>	<b>-4.91E-40</b>	<b>-2.25E-36</b>	<b>3.91E-36</b>	<b>1.40E-30</b>	<b>4.1E+04</b>
7	<b>-9.52E-45</b>	<b>8.92E-42</b>	<b>8.49E-38</b>	<b>-8.70E-38</b>	<b>1.40E-30</b>	<b>61.</b>
6	4.79 E-46	-1.08E-42	-1.51E-38	1.30E-38	4.02E-31	0.36
5	4.79E-46	-7.59E-44	4.90E-39	3.03E-39	2.29E-33	0.038
4	-2.12E-47	2.41E-44	-1.01E-40	3.16E-41	2.29E-33	0.00026
3	-1.23E-48	4.14E-45	-1.38E-42	3.16E-41	2.89E-34	2.5E-06
2	-2.32E-49	1.37E-46	-1.38E-42	1.62E-42	-1.09E-35	4.5E-09
1	-3.16E-50	3.74E-47	-3.84E-43	-3.64E-43	-9.03E-37	2.7E-09
0	-1.58E-51	-2.57E-48	1.65E-44	3.61E-44	9.69E-38	6.1E-14
$\mathbf{A}_i^V$	<b>45</b>	<b>42</b>	<b>39</b>	<b>39</b>	<b>31</b>	

We see that our indices proposed as the sufficient number of digits for the safety rounding are indeed close to the reality. They can and should be used to the quality control of the random vectors obtained by statistical estimation procedures.

Another lesson from the comparisons presented above is that the problem of the correct rounding off the FPC triad  $(\mathbf{V}_i, \mathbf{U}_i, \mathbf{C}_{ij})$  is the very important problem in the task of tracing the space-time variability of the FPC as the improper rounding will mimic the evolution of constants.

The third lesson is that the CODATA recommended values of the FPC are highly questionable as we have convinced that the correlation matrices were corrupted by the unjustified rounding.

## 4.2. Reliability

As it was mentioned in the descriptions of the high level quality requirements, it is natural to suppose that the next iteration of the adjustment will give constants more accurate and more selfconsistent than the previous adjustments.

Let us look for the time evolution of the estimates of one of the most important physical constant — the Planks constant  $h$  from the time of discovery up to the 2002 estimate. The historical perspective of the Plank constant estimates one can find in [11].

Figure 1. Plank Constant: 1969–2002. Error band show that the adjustment procedures produce estimates that still are far from been stable, though the amplitude of variation is reduced in the last two releases.

This “small-scale time variability” of the Plank constant estimates we attribute to the possible presence of the hidden (not estimated) systematic error introduced or missed by the adjustment procedures. It should be noted that systematist have to use contradictory input data which impossible to refine at the time of adjustment sessions<sup>6</sup>.

The “evidence” of the possible stabilization (see Fig. 2 is very preliminary and should be tested for the other constants simultaneously by tracing the variation of the hodograph of the “vector of basic constants” as it is outlined in the reliability requirement. Unfortunately it is not possible now because of the corrupted data on correlations in the releases. The conclusion based on the reliability indicator is that the CODATA recommended values cannot be used in searches of the possible large scale space-time variations of the FPC.

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<sup>6</sup>See discussion of this issue in the subsection: “A. Comparison of 1998 and 1986 CODATA recommended values” of the summary of the 1998 review ([4], pages 459-461).

Figure 2. Plank Constant: 1986–2002. Evidence for the possible stabilization.

### 4.3. Availability

The web access to the data on FPC offered by NIST & CODATA in the last release (V.4.0) is greatly improved. Now we have easy access to all data on average values and their uncertainties just copy the file in the ASCII format. But unfortunately in the released list the values of 7 basic constants out of 29 participating in the adjustment process did not quoted. The values of the other 28 important parameters (possible corrections to the theoretical expressions) for the whole adjustment procedure are omitted. They even did not discussed in the publications on the 1998 release.

As it was discussed in the previous sections, the ignorance of the correlations is inadmissible in the high precision physics applications. But the access to the recommended correlation coefficients remains to be the “misanthropic” one. It is hard to get data for an operative calculations with several constants simultaneously. There is no easy and safety way to get the complete data on the subsample of the triad ( $\mathbf{V}_i, \mathbf{U}_i, \mathbf{C}_{i,j}$ ) in a truly computer readable form.

To extract data on say 10 constants with the correlation matrix one have to produce about 300 flip-flops between web-pages “by hands”.

### 4.4. Traceability

Traceability means that any release of the recommended FPC set should be accompanied with full toolkit of the input data and methods to give interested user possibility to perform all steps of the adjustment process and to compare the results with the recommended values.

Unfortunately materials attached to the recommended FPC are not complete as it was stressed in the discussions of the availability indicator. Additional example is the incorrect presentation of the correlations of uncertainties in the input experimental data of the 1998 release.

The data on input correlations are presented only in the review on the paper [4] and the correlation matrix is non positive definite there [7].

It should be noted also that in the published documents related to the releases of FPC there are no discussions of the procedures used for rounding off the correlated quantities.

## 5. Summary

Summarizing the above discussions and evidences we are forced to stress that all high level quality requirements to the scientific information numerical data resource: correctness, selfconsistency, availability, reliability, and traceability are badly violated in at least the last three releases of the CODATA recommended values of the fundamental physical constants.

They could not be used as reference data to monitor the large scale space-time variability of the fundamental physical constants and moreover their usage in physics applications where the high precision calculations are needed is highly questionable.

The positive outcome from our critical treatment of the quality aspects of the central numerical scientific information resource are:

- the preliminary proposal for the safety rounding strategy in presentation the results of high precision computations of the physical observables;
- the proposal for the set of quality indicators to certify scientific information resources for the safety usage in physics applications;
- the proposal of the data structure and procedures for the complete and user friendly **Web-FPC**.

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