In memory of Prof. Jacob Bekenstein, whose insightful and elegant works have inspired my research

Bekenstein, I, and the quantum of black-hole surface area

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Professor Jacob Bekenstein was known not only for his brilliant and original physical ideas, but also for their clear presentation in his lectures and seminal research papers. I here provide a short review of Bekenstein's pioneering ideas about the quantization of black holes. I also describe my attempt, as a young and extremely naive student, to prove him wrong and how I got convinced in the correctness and utility of his deep physical intuition. Finally, my personal contribution to the ongoing attempts to understand the evenly spaced (discrete) area spectrum of quantized black holes, as originally suggested by Bekenstein in the early days of his scientific career, is described.

I. PREFACE

The remarkably deep and unique physical insights of Professor Jacob Bekenstein, that were also characterized by extremely beautiful mathematical elegancy, have inspired the imagination of many physicists, I among them, during the last decades. The sudden death of Jacob has left a huge hole in many hearts, but his original ideas still live with us. I am glad and honored for the invitation to contribute a personal essay for the special memorial volume for Jacob Bekenstein.

II. THE BEKENSTEIN QUANTIZATION OF THE BLACK-HOLE SURFACE AREA

The quantization of the black-hole horizon area was first discussed by Jacob Bekenstein, then a young and brilliant researcher in Princeton university [1], more than four decades ago [2]. Bekenstein has based his original and ground breaking quantization argument on the intriguing physical fact discussed in [3] that, while the conserved charges of a Kerr-Newman black hole (mass, electric charge, and angular momentum) may change due to the absorption of test particles, the surface area of the black hole may remain unchanged during an assimilation process of a point (structureless) particle if the later is absorbed at the black-hole horizon while being in a turning point [4] of its radial motion.

It was therefore suggested by Bekenstein that the black-hole surface area may serve as a classical adiabatic invariant. In particular, in the spirit of the Ehrenfest principle that has been formulated in the early days of quantum mechanics [5], Bekenstein has boldly conjectured that the surface area of a black hole should have a discrete (quantized) spectrum of the form [2, 6]

$$A_n = \gamma \hbar \cdot n \quad ; \quad n = 1, 2, 3, \dots , \tag{1}$$

where γ is a dimensionless constant.

Bekenstein has further suggested [2] to estimate the spacing of the black-hole area levels [that is, the value of the dimensionless physical coefficient γ in (1)] from a semi-classical gedanken experiment in which a black hole absorbs a finite-size neutral particle whose radial location and momentum are related by the quantum Heisenberg uncertainty principle [7, 8]. In particular, Bekenstein has explicitly proved that, taking cognizance of quantum effects that influence the motion of test particles in black-hole spacetimes [2, 7, 8], the minimal increase in the black-hole horizon area is given by the simple expression

$$(\Delta A)_{\min} = 8\pi\hbar . \tag{2}$$

Intriguingly, the minimal increase (2) in the black-hole surface area, as first suggested by Bekenstein in his seminal works [2, 7], is *universal* in the sense that it does not depend on the physical parameters of the black hole and the absorbed particle that have been used to derive it. This physically interesting observation naturally suggests that a lower bound of the form (2) is of fundamental importance in a quantum theory of gravity [2, 7].

III. CHALLENGING (AND THEN CONFIRMING) THE BEKENSTEIN AREA QUANTIZATION

In 1997, while I was investigating the inner structure of black holes under the insightful and enjoyable supervision of Professor Tsvi Piran at the Hebrew university of Jerusalem, I had a casual conversation with Dr. Avraham Mayo, then a student of Professor Bekenstein. The short and (for me) life changing conversation took place at the corridor of the physics department, few steps from Jacob's office. Until that moment I only knew Professor Bekenstein by his name. In particular, at that time I had no real information about Bekenstein's research interests nor about his incredible scientific achievements. Mayo had told me that his supervisor, Professor Bekenstein, is working on the quantization of black holes [9].

The intriguing concept of bringing the ideas of quantum physics into the realms of black-hole physics and gravitation was totally new for me and it had immediately caught my imagination. I remember myself running, after the short conversation with Mayo, to the library in order to search for Bekenstein's works on the quantization of black holes [2]. I have read his seminal papers with much excitement and passion. The papers were written in a remarkably clear style (which, as I later learned, characterizes the entire set of Jacob's scientific works). In particular, the brilliant and (for me) new physical ideas were presented in these seminal papers in a mathematically elegant fashion.

In the first few days after reading, for the first time, Bekenstein's papers on the quantization of the black-hole horizon area, I was incredibly enthusiastic and happy about the beauty of physics that was reflected to me from his papers. In particular, the remarkably simple and mathematically elegant quantum area spectrum (1) that emerged from Bekenstein's deep physical insights [2] seemed to me as a new and exciting window into the elusive world of quantum gravity.

However, as a young, bold, and probably extremely naive student, I took it as a scientific, and even more, as a personal challenge to prove that the physically intriguing conclusion of Prof. Bekenstein, regarding the discrete quantization of the black-hole surface area, is wrong. In particular, I have noticed that the interesting analysis of Bekenstein presented in his seminal work [2] is restricted to the regime of *neutral* particles which are absorbed by black holes. I had the feeling that an analogous gedanken experiment, which involves the absorption of *charged* (rather than neutral) particles by charged black holes, may provide a physically interesting counter-example to the intriguing lower bound (2) derived by Bekenstein on the increase in black-hole surface area.

Analyzing the absorption of a charged particle of proper mass μ and electric charge q by a charged Reissner-Nordström black hole of mass M and electric charge Q, I have found that the minimally allowed increase in the black-hole surface area is given by the simple expression [10]

$$(\Delta A)_{\min} = \frac{4\pi\mu^2}{qE_+} \; , \tag{3}$$

where $E_+ = Q/r_+^2$ is the electric field of the charged black hole at its surface $r_+ = M + (M^2 - Q^2)^{1/2}$.

Intriguingly, at first glance one may deduce from the analytically derived expression (3) that, for charged black holes, the minimal increase $(\Delta A)_{\min}$ in surface area may become arbitrarily small in the $E_+ \to \infty$ limit. For two days I was very excited from the (obviously naive) thought that I, a young researcher, have found a counter-example to Bekenstein's well known lower bound (2).

However, I have then realized that, within the framework of a quantum theory of gravity, the black-hole electric field cannot become arbitrarily large. In particular, vacuum polarization effects associated with the Schwinger pair production mechanism [11] set the upper bound [12]

$$E_+ \leq \frac{\pi\mu^2}{q\hbar} \tag{4}$$

on the strength of the black-hole electric field. Substituting (4) into (3), I have found the simple lower bound [10]

$$(\Delta A)_{\min} = 4\hbar \tag{5}$$

on the increase in surface area of charged black holes due to the absorption of charged particles.

The fact that the lower bound (5), which I have derived for the minimal increase in the horizon area of charged black holes, and the lower bound (2) which, as originally proved by Bekenstein [2], provides the minimal increase in black-hole surface area due to the absorption of neutral particles, are of the same order of magnitude has convinced me that there is something very deep and physically correct in Bekenstein's intriguing ideas about the quantization of black holes.

IV.   THE BOHR CORRESPONDENCE PRINCIPLE AND BLACK-HOLE AREA QUANTIZATION

The intriguing fact that both expressions (2) and (5) for the minimal increase in the black-hole surface area are universal (that is, independent of the physical parameters of the black hole and the absorbed particle) clearly indicates that these lower bounds may be of fundamental physical importance in black-hole physics and in the elusive quantum theory of gravity.

It should be realized, however, that the precise value of the fundamental dimensionless constant γ in the quantized Bekenstein area spectrum (1) cannot be determined by the gedanken experiments of [2, 10]. In particular, due to the inherent fuzziness of the Heisenberg uncertainty principle and due to the approximated nature of the expression (4) for the critical electric field, the estimated lower bounds (2) and (5) on the black-hole area increase can be challenged.

Furthermore, Bekenstein and Mukhanov [13, 14] have correctly argued that the thermodynamic Bekenstein-Hawking area-entropy relation [2, 15]

$$S_{\text{BH}} = \frac{A}{4\hbar} \tag{6}$$

for black holes, together with the Boltzmann-Einstein formula

$$g_n = e^S \in \mathbb{N} \tag{7}$$

for the number of microstates in statistical physics, imply that the value of the dimensionless physical constant γ in the quantum spectrum (1) should be of the simple functional form

$$\gamma = 4 \ln k \quad \text{with} \quad k = 2, 3, \dots . \tag{8}$$

Although physically convincing, the combined thermodynamic-statistical-physics argument presented in [13, 14] could not fix uniquely the value of the integer k in the functional relation (8). As a researcher taking his first steps in the exciting academic world, I felt that it is a physically worthy challenge to try to determine the value of this mysterious integer.

In particular, one of the physical questions that most intrigued me as a young student was how the observed (classical) world emerges from the fundamental (quantum) description of nature. At that time, I already knew that, using the physical concept of coherent states, one can learn about the classical description of macroscopic phenomena from the underlying microscopic quantum description. But can one learn about the fundamental quantum world from its macroscopic coarse-grained classical description?

It took me a few days to realize that, luckily enough, the answer to the above stated question is 'Yes!'. In particular, the Bohr correspondence principle that was formulated in the early days of quantum mechanics [5] allows one to relate, in the regime of large quantum numbers, the fundamental discrete properties of a quantum physical system to the corresponding macroscopic (classical) properties of the system.

In particular, I have realized that the classical resonant oscillation frequencies of black-hole spacetimes can be related, through the Bohr correspondence principle, to the underlying quantum properties of these fundamental physical objects [16, 17]. Taking cognizance of the Bekenstein quantized area spectrum (1) and the area-mass relation $A = 16\pi M^2$ for Schwarzschild black holes [2], one finds the characteristic emission frequency

$$\hbar\omega^{\text{quantum}} = M_n - M_{n-1} = \gamma \cdot \frac{\hbar}{32\pi M} \tag{9}$$

which is associated with the quantum transition $n \to n-1$ of the black-hole area state in the asymptotic semi-classical $n \gg 1$ regime.

I have therefore conjectured, in the spirit of the Bohr correspondence principle [5, 7, 16], that the characteristic classical resonant frequency [18] of these spherically symmetric black holes should have the simple functional relation [see Eqs. (8) and (9)]

$$\omega_{\text{R}}^{\text{classical}} = \frac{\ln k}{8\pi M} , \tag{10}$$

where k is the (yet unknown) integer parameter of Bekenstein and Mukhanov [13, 14].

The characteristic asymptotic resonant oscillation frequency of the Schwarzschild black hole was, at that time, known only numerically [19]. In particular, using sophisticated numerical techniques, it was computed with an accuracy of seven digits after the decimal point [19]:

$$M\omega_{\text{R}}^{\text{classical}}(n) = 0.0437123 \quad \text{for} \quad n \gg 1 . \tag{11}$$

It was Saturday morning, twenty years ago [17], that I took my calculator and, with a shaky hand, calculated the numerical value of the expression $\ln 2/8\pi$ [see Eq. (10)]. To my deep disappointment, the answer I got did not agree with (11). But I did not lose heart; I turned to calculate the value of $\ln 3/8\pi$...and, to my excitement, the calculator returned the answer 0.0437123. It now agreed, digit-by-digit, with the numerically computed [19] characteristic black-hole resonant oscillation frequency (11).

As a young student, it was amazing to feel, for the first time in my (then) short scientific career, that I can count on my physical intuition. Moreover, I was thrilled to know that I have found the last missing piece in the black-hole quantization scheme of Bekenstein and Mukhanov. In particular, substituting $k = 3$ into Eqs. (1) and (8), one obtains the discrete Bekenstein area spectrum

$$A_n = 4\hbar \ln 3 \cdot n \quad ; \quad n = 1, 2, 3, ... \tag{12}$$

of quantum Schwarzschild black holes.

It is worth emphasizing that the evenly spaced quantum black-hole area spectrum (12), as originally formulated by Bekenstein [2, 7] [see Eq. (1)], is consistent both with the thermodynamic entropy-area relation (6) of black holes, with the Boltzmann-Einstein relation (7) of statistical physics, and, as I have explicitly shown in [16], with the Bohr correspondence principle of quantum physics.

V. SUMMARY

Jacob was known for his original and highly insightful physical ideas, but not less for the beautiful and elegant ways he was using to present them in his lectures and research papers. I, as many others, benefited a lot from reading Jacob's seminal works on the physics of black holes. His ingenious insights have inspired the imagination of many physicists, I among them. Jacob's pioneering ideas about the integration of quantum effects into black-hole physics continue to inspire the physics community in her ongoing search for a unified and self-consistent quantum theory of gravity and thermodynamics.

As a final personal note, I would like to comment that, unfortunately, I have never asked Jacob what he thought about my idea to use the Bohr correspondence principle and the asymptotic resonant frequencies of black holes in order to determine the value of the dimensionless physical parameter γ in his black-hole quantum area spectrum (1). Now, that it is too late, I can only regret for not doing so.

However, as fate would have it, many years after the appearance of my paper [16] with the suggested black-hole quantization spectrum (12), I came across a nice photo of Prof. Bekenstein that was taken in his office in the occasion of him winning the 2012 Wolf Prize in physics [20]. In the background of that photo I saw Jacob standing near his famous blackboard, on which I could vividly see the fundamental factor of $4\ln 3$ [20, 21]...

[1] Bekenstein had received his Ph.D. from Princeton University at 1972 and then moved to the University of Texas at Austin.
[2] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); J. D. Bekenstein, Lett. Nuovo Cimento **11**, 467 (1974).
[3] D. Christodoulou, Phys. Rev. Lett. **25**, 1596 (1970); D. Christodoulou and R. Ruffini, Phys. Rev. D **4**, 3552 (1971).
[4] That is, for the classical absorption process to be an adiabatic one, the absorbed particle should be characterized by a zero radial momentum.
[5] M. Born, *Atomic Physics* (Blackie, London, 1969).
[6] We shall use natural units in which $G = c = 1$. In these units $\hbar$ has the dimensions of length^2.
[7] J. D. Bekenstein, Cosmology and Gravitation, M. Novello, ed. (Atlantisciences, France 2000) [arXiv:gr-qc/9808028].
[8] As emphasized by Bekenstein [7], the Heisenberg uncertainty principle implies that, within the framework of semi-classical general relativity, a quantized (finite-size) particle cannot be absorbed precisely at the black-hole horizon from an exact radial turning point of its motion.
[9] I would like to thank Dr. Avi Mayo for bringing Jacob's seminal works [2] on the quantization of black holes to my attention.
[10] S. Hod, Phys. Rev. D **59**, 024014 (1999); S. Hod, Gen. Rel. Grav. **31**, 1639 (1999).
[11] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[12] L. Parker and J. Tiomno, Astrophys. Journ. **178**, 809 (1972); T. Damour and R. Ruffini, Phys. Rev. Lett. **35**, 463 (1975).
[13] V. Mukhanov, JETP Lett. **44**, 63 (1986).
[14] J. D. Bekenstein and V. F. Mukhanov, Phys. Lett. B **360**, 7 (1995).

[15] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[16] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998).

[17] See also https://birthofidea.tecnico.ulisboa.pt/#hod .

[18] It is worth noting that black holes are characterized by an infinite set $\{\omega_n\}_{n=1}^{n=\infty}$ of damped gravitational quasinormal resonant modes. It is well known that, within the framework of classical general relativity, black holes do not emit radiation. Thus, in applying the Bohr correspondence principle to the classical oscillation frequencies of black holes, I have suggested [10, 16] to focus on the highly damped ($n \gg 1$) black-hole resonant modes, which are characterized by infinitely small lifetimes (the characteristic zero lifetimes of these highly damped modes correspond to the no emission status of classical black-hole spacetimes).

[19] H. P. Nollert, Phys. Rev. D **47**, 5253 (1993); N. Andersson, Class. Quantum Grav. **10**, L61 (1993).

[20] See http://support.huji.ac.il/news-events/news/bekenstein_wolf_prize2012/

[21] See also https://www.nytimes.com/2015/08/22/science/space/jacob-bekenstein-physicist-who-revolutionized-theory-of-black-holes-dies-at-68.html