

A Whole Cosmology View of the Hubble Constant

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The Hubble constant H_0 is the value of the cosmic expansion rate at one time (the present), and cannot be adjusted successfully without taking into account the entire expansion history and cosmology. We outline some conditions, that if not quite “no go” are “no thanks”, showing that changing the expansion history, e.g. employing dynamical dark energy, cannot reconcile disparate deductions of H_0 without upsetting some other cosmological measurement.

I. INTRODUCTION

Cosmology is the study of the universe as a whole. This is usually interpreted as *Cosmology is the study of the (universe as a whole)*, where universe as a whole means large spatial scales and large volumes. However one should also be aware that *Cosmology is the study of the universe (as a whole)*, where as a whole means both over a wide range of history and over the full panoply of observations. This has two essential implications for the Hubble constant discussion:

1. $H_0 \equiv H(z = 0)$ is merely a value at one instant of a dynamical description of cosmic expansion, and it is very little use getting it “right” without accounting properly for the whole dynamics.
2. H_0 is merely one number from the foundational functions, e.g. cosmic expansion history, cosmic growth history, cosmic gravity history, that describe cosmology, and cannot be examined in isolation.

This chapter focuses on these two aspects of the Hubble constant issue, specifically whether changing the expansion history through using a dynamical (dark energy) density can reconcile different deductions of H_0 .

We will give arguments that are as model independent as possible, and hence not list the quite numerous models that have been considered (see, for example, [1–5] and references therein for many examples). Rather we will seek to come as close to “no go” strictures as possible, though in the vast arena of trade offs between standard and created model parameters these are closer to “no thanks”. The philosophy follows that of Occam, not to multiply entities unless necessary.

One of the simplest reconciliations of the Hubble constant values is that some measurement is incorrect. Indeed, in the nearly 100 year history of the Hubble constant this has always been the answer, with much of that history having discrepancies at nominal statistical σ levels in excess of the present situation. We will say no more about this resolution beyond Figure 1, instead assuming that the discrepant deductions are all valid and seeking for a cosmological paradigm to reconcile them.

II. EXPANSION HISTORY

It is relatively easy to shift the value of H_0 by changing the cosmological model, but again H_0 is not the whole of cosmology. To repeat: cosmology is the study of the universe as a whole, or alternately, all cosmology all the time [6, 7]. We need to take into account the effects on the full expansion history $H(z)$, not just $H(z = 0)$.

Let us begin by saying that the cold dark matter plus cosmological constant, i.e. Λ CDM, cosmology is a good description of the expansion history. That is, we would like to match some expansion history model to observations that are adequately described within the (flat) cosmology

$$H^2(z) = H_2^2 \Omega_{m,2} (1+z)^3 + H_2^2 (1 - \Omega_{m,2}) . \quad (1)$$

Here $H_2 = H(z = 0)$ is the Hubble constant for this cosmology and we specify that the model has a matter density fraction $\Omega_{m,2}$. However, when making local universe measurements we find the Hubble constant is H_1 and try to reconcile the situation by allowing deviations from Λ CDM in the form of dynamical dark energy. So we take a model

$$H^2(z) = H_1^2 \Omega_{m,1} (1+z)^3 + \frac{8\pi G}{3} \rho_{de}(z) . \quad (2)$$



FIG. 1. Ladders require care in use and are most stable when anchored firmly at both ends.

We only relate these models over a finite range of redshift, where observations indicate the Λ CDM model matches the observations well. (One could add spatial curvature as well but this does not qualitatively change the results.) Thus the necessary dark energy density is

$$\frac{8\pi G}{3}\rho_{\text{de}}(z) = (H_2^2\Omega_{m,2} - H_1^2\Omega_{m,1})(1+z)^3 + H_2^2(1-\Omega_{m,2}) . \quad (3)$$

We can immediately see why it is so crucial to do cosmology as a whole, rather than only think about the Hubble constant. If we have observations determining $\Omega_m h^2$ (the cosmic microwave background (CMB) value for what should be a universal constant $\Omega_m h^2$ should be much less model dependent than the CMB value for h itself) then we are forced to conclude that our attempt to change the Hubble constant through dynamical dark energy fails: one must have $H_2 = H_1$. It is only when we ignore the rest of cosmology that we can obtain a different Hubble constant.

Suppose that we do not have $\Omega_m h^2$ determined by the CMB. In that case then Eq. (3) tells us the form of dark energy that can shift the Hubble constant – it looks like the sum of a cosmological constant and a matter-like term. In particular, if the value H_2 coming from the Λ CDM fit to the observations (e.g. the “CMB” value) is less than the value H_1 coming from the local measurements then the effective energy density will generally be negative (one has to compensate for the higher expansion rate). In any case, dark energy looking like the sum of matter and Λ has been studied for decades, for example in mucker and polytropic dark energy [8, 9]; very generically it causes growth of

structure to deviate significantly from the Λ CDM cosmology. Once again, one must use cosmology as a whole when assessing the viability of shifting the Hubble constant.

III. DISTANCE-REDSHIFT RELATION

One could attempt to sidestep the “no thanks” scenarios of the previous section by saying we do not know the expansion history that well, and are willing to allow $H(z)$ to deviate significantly from the Λ CDM form. This is tenable in 2022, but will shortly be severely constrained by radial baryon acoustic oscillation (BAO) measurements from the Dark Energy Spectroscopic Instrument [10, 11]. For this approach, rather than matching $H(z)$ over some redshift range one only matches the integral quantity of the distance-redshift relation,

$$d(z) = \int_0^z \frac{dz'}{H(z')} . \quad (4)$$

One can have $H(z)$ dip below the Λ CDM expectation at some redshifts and rise above it at others to obtain $d(z)$ near Λ CDM over some finite redshift range.

Since distances have been measured fairly accurately, and since a deviation in $H(z')$ at some redshift affects all $d(z > z')$, this is not that easy to accomplish, especially when taking into account CMB distances as well. Furthermore, changing $H(z)$ also affects growth of structure and generally the large swings needed to alter the Hubble constant throw off the growth history (including the large scale integrated Sachs-Wolfe effect in the CMB, see e.g. [12]). In fact, growth of structure is not only tightly bound to the expansion history within general relativity but nearly determined by the well measured distance to CMB last scattering.

Figure 2 shows how tightly the growth of structure is bound to the distance to CMB last scattering, not just for Λ CDM but for quite disparate models along the dynamical “mirage of Λ ” line. Conversely, at low redshift a cartoon relation is that the growth rate

$$f \approx \Omega_m^{0.55} \sim \left(\frac{\Omega_m h^2}{h^2} \right)^{0.55} ; \quad (5)$$

so for preserved $\Omega_m h^2$ one has $f \sim h^{-1.1}$, or if $f\sigma_8$ is measured then $\sigma_8 \sim h^{1.1}(f\sigma_8)$.

Thus there is a difficult balancing act in changing the Hubble constant, due to the impact of the altered expansion history on growth probes (and the CMB through the Sachs-Wolfe effect), as well as calibration of supernova distances to higher redshift (while preserving CMB and BAO distances). All these elements must be considered together. See [14–18] for some specific analyses.

IV. EARLY UNIVERSE DEVIATIONS

One might think of moving alterations of the expansion history to high redshift, above where distance-redshift constraints apply. This still affects growth however, and generally more severely because of the long lever arm. One way to get partly around this is to push modifications to before the recombination epoch. Such an approach is often called early dark energy.

In fact, early dark energy was “discovered” in CMB data in 2013 [19], along with its concomitant shift in the Hubble constant, as shown in Figure 3. However the shift was much smaller than the current discrepancy, and increasing the dark energy density – raising the early expansion rate (and secondarily decreasing the sound horizon) yet reducing the clustering matter density fraction – would have a severe effect on both the matter growth history and the CMB perturbation power spectrum. Indeed, the effect of early dark energy on the sound horizon has been familiar for decades [20–22], but the impact of the reduced perturbation potentials is more deleterious, forcing a disfavored greater amplitude of mass perturbations (σ_8) to compensate and obtain consistency with power spectrum observations. It is extremely difficult to obtain $H_0 > 70$ km/s/Mpc and viable CMB and viable distances and viable growth. See [23–25] for some specific examples regarding this. Models that push even further back, changing the primordial power spectrum beyond the power form, for example with oscillations or features, also have difficulty in substantially shifting H_0 , e.g. [26, 27] (and will be tightly constrained by tomographic redshift surveys and future CMB experiments, e.g. [28]).

For postrecombination alterations of the expansion history, one again must emphasize the importance of accounting

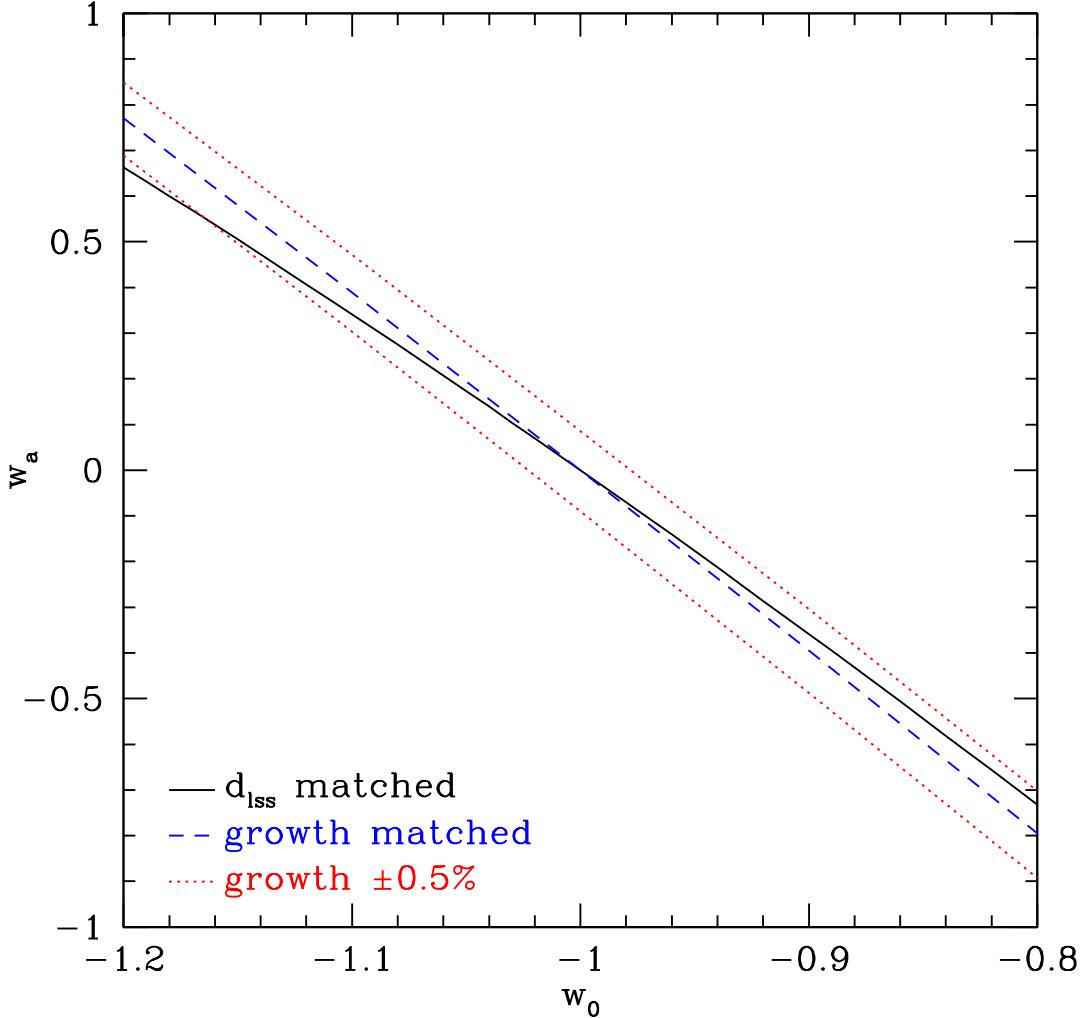


FIG. 2. Constraints on the distance to CMB last scattering d_{lss} tightly constrain structure growth as well, even for highly dynamical matching dark energy cosmologies. [From [13]]

for the impact on growth, including the Sachs-Wolfe effect. Recall that the Poisson equation imposes

$$(k/a)^2 \Phi = 4\pi \delta \rho \quad (6)$$

$$\Rightarrow \Phi \sim a^2 \delta \rho \sim a^2 \frac{\delta \rho}{\rho} \rho \quad (7)$$

$$\Rightarrow \Phi \sim (a^3 \rho) \left(a^{-1} \frac{\delta \rho}{\rho} \right) . \quad (8)$$

In the standard scenario the gravitational potential is thus time independent and no large Sachs-Wolfe effect is generated. However, if we change the energy density of clustering matter, e.g. through adding an unclustering early dark energy that influences the recombination era or through some interaction, then we affect the growth of structure $\delta \rho / \rho \sim a$ (and possibly $\rho \sim a^{-3}$) and give a time dependence to the gravitational potential Φ , resulting in a large Sachs-Wolfe effect contrary to observations. More specifically, $\delta \rho / \rho \sim a^{1-(3/5)\Omega_e}$ so only a very small fraction of early dark energy Ω_e can be tolerated, hence enabling only a small shift in the Hubble constant.

A rough rule of thumb is that

$$\frac{\delta H_0}{H_0} \sim \frac{1}{2} \frac{\delta H^2}{H^2} \sim \Omega_e , \quad (9)$$

(naively this would be $\sim \Omega_e/2$ but the shifting of other parameters to fit the CMB power spectrum pushes it closer to Ω_e [29]) so a change in H_0 to reconcile a 10% discrepancy in values would require $\Omega_e \sim 0.1$ and cause a $10^{3 \times (3/5)\Omega_e} \sim 50\%$

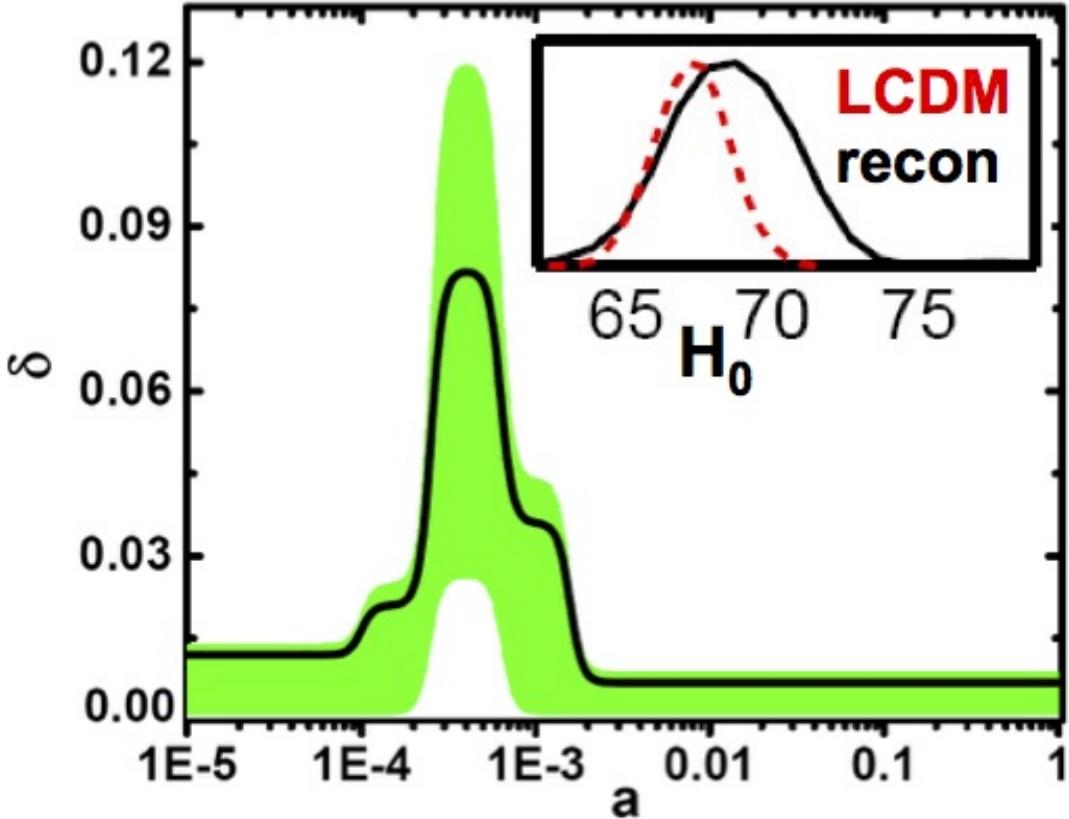


FIG. 3. Model independent reconstruction of the expansion history from CMB data in 2013. The expansion deviation $\delta \equiv \delta H^2 / H_{\Lambda\text{CDM}}^2$ shows a preference for precombination early dark energy. This in turn shifts the Hubble constant H_0 from the value derived assuming ΛCDM – but only slightly, even for 8% early dark energy. [From [19]]

change in the growth amplitude today (modulo other shifts). This gives a heuristic explanation for why it is nearly impossible to shift H_0 from, say, ~ 67 km/s/Mpc to above 70 (see, for example, [4, 30]) and still satisfy distance, growth, and CMB measurements – i.e. cosmology as a whole.

V. EXTENSIONS

Within the framework used here – matter plus (an effective) dark energy with background and perturbation equations governed by general relativity in a homogeneous and isotropic universe – reconciliation of significantly differing values of the Hubble constant are, if not quite “no go”, then substantially “no thanks”. Due to the impacts on the many other cosmological characteristics, mechanisms for shifting H_0 cause other alterations away from the observed quantities. One could add further physics to compensate, e.g. interactions, changes in initial conditions, etc., but these in turn change a variety of observables and this runs the risk of becoming epicyclic. Basically it requires multiplying entities beyond necessity – unless one’s necessity is to obtain a certain value of H_0 at any cost. Such eschewing of Occam’s Razor we regard as “no thanks”.

Furthermore, models that seek reconciliation through a mechanism taking place just before recombination or just before the present add new fine tuning issues, not simply in the model parameters but in why then questions. If one seeks a dynamical solution ideally it would arise naturally out of the cosmology history.

Gravity modified from general relativity offers one avenue of pursuit: it could compensate for the impact of the altered expansion history on the growth of structure, or change light propagation in a redshift dependent manner to better align the values of the Hubble constant derived from strong lens time delay systems. Modified gravity that achieves a whole cosmology perspective – simultaneously providing a basis for the current accelerated expansion, growth history measurements, and reconciliation of Hubble constant values – would be a theory to explore in depth. However it is difficult to see how such a theory would work: we generally want gravity to restore to general relativity at early times such as CMB recombination and on small scales such as the Cepheid distance ladder measurements so

there should be no shift between the H_0 values.

VI. CONCLUSIONS

The history of the Hubble constant across a century has always been one of disparate values. In all cases those values have shifted as measurement and modeling uncertainties have become better studied. There seems little reason to expect the current situation to be drastically different.

If a compelling, succinct physical explanation for differing values when derived from distinct probes awaited, then perhaps one might give more weight to new physics relative to systematics. However as we have reviewed here, it is remarkably difficult to shift the Hubble constant from a given value, even in the presence of quite dramatic dynamics at any point in cosmic history – *when taking into account cosmology as a whole*. While this does not rise to the level of a no go theorem, it is definite enough to weigh heavily as a no thanks likelihood.

A desideratum for a model with new physics would be one that gives an advantageous likelihood for the full panoply of cosmic observations – distances, growth, and CMB – without putting a thumb on the scale by imposing a prior on the Hubble constant, the very issue one is examining. That is postdiction not prediction. Plainly put, H_0 data or prior should not be input into the analysis whose aim is examine the value of H_0 . If the posterior for distances plus growth plus CMB observations over cosmic history is not advantaged over a standard Λ CDM model, then it does not matter what value of H_0 can be achieved with the model (the same way that if one was trying to reconcile the matter density Ω_m then one would not put a prior on that).

If an alternate model does succeed in fitting all cosmology, all the time, then one proceeds with Occam’s Razor and physical naturalness examinations. In the end, more data – with more understanding of the subtleties, selections, systematics (see for example [31, 32] for steps in this direction) – from more probes will be the path to resolution, and a firm foundation for cosmology as a study of the universe as a whole.

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