

Constraint from Lamb Shift and Anomalous Magnetic Moment on Radiatively Induced Lorentz and *CPT* Violation in Quantum Electrodynamics

W.F. Chen[†] and G. Kunstatter[‡]

*Department of Physics, University of Winnipeg, Winnipeg, Manitoba, Canada R3B 2E9
and*

Winnipeg Institute for Theoretical Physics, Winnipeg, Manitoba

Abstract

We investigate the precisely measured anomalous magnetic moment and Lamb shift as tests for the possible existence of the radiatively induced Lorentz and *CPT* violation effects in quantum electrodynamics. To this end we calculate the one-loop vacuum polarization tensor and vertex radiative correction in dimensional reduction and on-shell renormalization scheme. We explicitly show how the Lorentz and *CPT* violation sector affects the anomalous magnetic moment and Lamb shift. Remarkably, we find infrared divergences coming from Lorentz and *CPT* violating term that do not cancel in physical cross sections. This result appears to place stringent constraints on the type of Lorentz/CPT violating terms that can be added to the QED Lagrangian.

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I. INTRODUCTION

Special relativity is one of the most important cornerstones of modern physics. Its algebraic foundation, Lorentz transformation invariance and discrete *CPT* symmetry, has become a fundamental and indispensable axiom in constructing relativistic quantum field theories. However, physics is a science based on experimental observation. All of its principles must be tested and can only be confirmed to the accuracy of the experimental data. With the availability of increasingly accurate experimental data and the discovery of new phenomena, it is conceivable that even the most fundamental principles may someday have to be modified or even abandoned. It is partly in this spirit that there has recently been increasing interest in the possible breaking of Lorentz symmetry. As emphasized by Jackiw [1], the availability of higher precision instruments makes it possible to carry out a more

[†]E-mail: wchen@theory.uwinnipeg.ca

[‡]E-mail: gabor@theory.uwinnipeg.ca

precise test on the principle of special relativity, hence it is not unreasonable to make a theoretical investigation of the possible violation of Lorentz and *CPT* symmetry. In fact, if the Standard Model is considered as the low-energy limit of a more fundamental theory constructed from strings, the spontaneous breaking of Lorentz symmetry can occur [2].

A theoretical framework, the extension of the Standard Model with Lorentz and *CPT* breaking term has already been constructed [3]. Based on this model a series of predictions for possible signals of *CPT* and Lorentz violation have been suggested, including neutral-meson oscillations [4], clock-comparison experiments [5], hydrogen and anti-hydrogen spectroscopy [6] and Penning trap experiments [7] etc. The QED sector of this extended Standard Model contains a Lorentz and *CPT* violating Chern-Simons term

$$\mathcal{L}_{CS} = 1/2k_\mu \epsilon^{\mu\nu\rho\lambda} F_{\nu\rho} A_\lambda \quad (1)$$

which can lead to the birefringence of light in vacuum and was introduced earlier in QED [8]. However, this Chern-Simons term gives a negative contribution to the conserved energy and hence makes the theory unstable. Thus the coefficient k_μ should set to zero. The experimental searches for cosmological birefringence place a very stringent limit on k_μ .

Even if the Chern-Simons term Eq.(1) vanishes at classical level, it can be generated from the radiative corrections due to an explicit Lorentz and *CPT* violation term $b^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi$ in the fermion sector. A series of calculations of the generation of this term from radiative corrections was carried out, and appeared to give regularization dependent results [9–13]. Moreover, the experimental upper limit on b_μ is far less stringent than on k_μ . Of course, in an extended Standard Model, such radiative corrections from every species of quarks and leptons must cancel if the theory is anomaly-free. However, when QED is considered not embedded in a large gauge theory, it is necessary to investigate the upper limit of b_μ using the experiment data within QED itself. Significantly, explicit calculations have shown [9] that the radiatively induced Chern-Simons term in QED escapes the no-go theorem [14] which prohibits the generation of Lorentz and *CPT* violation term for any gauge invariant *CPT*-odd interaction.

It is well known that two of the most remarkable accomplishments of QED are the explanations for the anomalous magnetic moment of electron and the Lamb shift, both of which have been measured with highly precise accuracy. The introduction of a gauge invariant *CPT*-odd term, $b_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi$, in the fermionic part would inevitably affect the theoretical values of the anomalous magnetic moment and of the Lamb shift. The purpose of this paper is therefore to propose the use of the experimental data for the anomalous magnetic moment and the Lamb shift to directly impose an upper limit on b_μ and indirectly on k_μ . We think this is quite significant since it provides another experimental test on the possible existence of radiatively induced Lorentz and *CPT* violation effects.

The classical action of QED with the inclusion of *CPT*-odd term in the fermionic sector is [9,12]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i\cancel{\partial} - m - \cancel{b} \gamma_5) \psi - e \bar{\psi} \cancel{A} \psi, \quad (2)$$

where b_μ is a constant four vector with a fixed orientation in space-time. The term $\bar{\psi} b \gamma_5 \psi$ is gauge invariant, but it explicitly violates both Lorentz and *CPT* symmetries, since b_μ picks

up a preferred direction in space-time. The theory is explicitly gauge invariant under the usual gauge transformation

$$\psi(x) \longrightarrow e^{-ie\Lambda(x)}\psi(x), \quad \bar{\psi}(x) \longrightarrow e^{ie\Lambda(x)}\bar{\psi}(x), \quad A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu\Lambda(x). \quad (3)$$

The gauge symmetry determines that the vacuum polarization tensor must take the following form,

$$\begin{aligned} \Pi_{\mu\nu}(p, b) &= \epsilon_{\mu\nu\lambda\rho} p^\lambda b^\rho A(p, b) + (p^2 g_{\mu\nu} - p_\mu p_\nu) B(p, b) \\ &\quad + \left(p_\mu b_\nu + p_\nu b_\mu - \frac{p \cdot b}{p^2} p_\mu p_\nu - \frac{p^2}{p \cdot b} b_\mu b_\nu \right) C(p, b). \end{aligned} \quad (4)$$

The second term in the r.h.s of the above equation screens the electric charge. In standard QED, this term will lead to the Uehling potential correction to the Coulomb interaction at low-energy, and hence give a partial contribution to the Lamb shift when this quantum correction is applied to a Hydrogen-like atom.

For the vertex radiative correction, the introduction of the term $b^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi$ and the bosonic symmetry $p \longleftrightarrow q$, imply that the quantum on-shell vertex should have the following tensor structure:

$$\begin{aligned} \bar{u}(q)\Lambda_\mu u(p) &= \bar{u}(q) \left[F'_1 \gamma_\mu + F'_2 \frac{p_\mu + q_\mu}{m} + F'_3 \frac{b_\mu}{m} + F'_4 \gamma_5 \frac{b_\mu}{m} \right. \\ &\quad \left. + F'_5 \gamma_\mu \gamma_5 + F'_6 \sigma_{\mu\nu} \frac{b^\nu}{m} + F'_7 \sigma_{\mu\nu} \gamma_5 \frac{b^\nu}{m} \right] u(p) \\ &= \bar{u}(q) \left[F_1 \gamma_\mu + F_2 i \sigma_{\mu\nu} l^\nu + F_3 \frac{b_\mu}{m} + F_4 \gamma_5 \frac{b_\mu}{m} \right. \\ &\quad \left. + F_5 \gamma_\mu \gamma_5 + F_6 \sigma_{\mu\nu} \frac{b^\nu}{m} + F_7 i \epsilon_{\mu\nu\lambda\rho} \sigma^{\lambda\rho} \frac{b^\nu}{m} \gamma_5 \right] u(p), \end{aligned} \quad (5)$$

where F'_i and F_i , $i = 1, \dots, 7$ are the scalar functions of m , b^2 , l^2 , $l \cdot b$ and $(p + q) \cdot b$ with $l_\mu \equiv q_\mu - p_\mu$, $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$. Due to the existence of the constant vector b_μ , the form factors F_i do not depend only on the the Lorentz invariant l^2 . Note that in Eq. (5) we have made use of the following Gordon identities,

$$\bar{u}(q)\gamma_\mu u(p) = \frac{1}{2m}\bar{u}(q)[(p + q)_\mu + i\sigma_{\mu\nu}l^\nu]u(p); \quad (6)$$

$$\bar{u}(q)\gamma_\mu\gamma_5 u(p) = \frac{1}{2m}\bar{u}(q)[l_\mu\gamma_5 + i\sigma_{\mu\nu}(p + q)^\nu\gamma_5]u(p), \quad (7)$$

and the self-dual relation $\sigma_{\mu\nu}\gamma^5 = i/2\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho}$. The F_2 term of Eq.(5) will lead to the famous Schwinger anomalous magnetic moment term. Furthermore, as shown in Fig. 1, the combination of radiative corrections of vertex and the correction to the electromagnetic coupling from the vacuum polarization tensor will yield the Lamb shift.

The paper is organized as follows: In Sect. II we calculate the one-loop vacuum polarization tensor to second order in b_μ . Some of the calculation techniques are illustrated and the screening effects on the electric charge of vacuum polarization tensor are discussed. Sect. III

is devoted to the somewhat lengthy calculation of the one-loop quantum correction to the on-shell vertex to second order in b_μ . As done in standard QED, the radiative correction is extracted using the mass-shell renormalization scheme. In preparation for discussing the anomalous magnetic moment and the Lamb shift, we give the expansion of the on-shell vertex radiative correction at small photon momentum. Sects. IV and V contain detailed discussions on the effects of *CPT*-odd term on the anomalous magnetic moment and Lamb shift. The explicit b -dependence of the anomalous magnetic moment and the Lamb shift is presented. As in conventional QED, the results contain infrared (IR) divergences, so in Sect. VI we consider the soft photon emission of bremsstrahlung and demonstrate that, contrary to what happens in conventional QED, the IR divergences contained in the b -dependent part do not appear to cancel. Finally, we summarize the result, discuss the other possible effects induced by the *CPT*-odd term and emphasize the constraints from the anomalous magnetic moment and the Lamb shift on the existence of Lorentz and *CPT* violation in the electromagnetic interaction due to the non-cancellation of IR divergence.

II. VACUUM POLARIZATION TENSOR

The one-loop vacuum polarization tensor is read as

$$\Pi_{\mu\nu}(p, b) = e^2 \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \text{Tr} [\gamma_\mu S_b(k) \gamma_\nu S_b(k + p)], \quad (8)$$

where we have chosen dimensional regularization, $\epsilon_{\text{UV}} \equiv 2 - n/2$; $S_b(k)$ is the b_μ -exact fermionic propagator utilized by Jackiw and Kostelecký in calculating the radiatively induced Lorentz and *CPT* violating Chern-Simons term [9],

$$S_b(k) = \frac{i}{k - m - \not{p}\gamma_5}. \quad (9)$$

However, this propagator will give rise to complications when performing Wick rotation in order to evaluate the Feynman integral [12]. Thus motivated by the fact that the magnitude of b_μ should be small (compared to m) as well as the fact the vacuum polarization tensor should be quadratic in b to leading order, we make use of the following identity for the operators (or matrices) A and B ,

$$\frac{1}{A + B} = \frac{1}{A} - \frac{1}{A}B \frac{1}{A + B} = \frac{1}{A} - \frac{1}{A}B \frac{1}{A} + \frac{1}{A}B \frac{1}{A}B \frac{1}{A + B} = \dots \quad (10)$$

The vacuum polarization tensor up to second order b can be written as

$$\begin{aligned} \Pi_{\mu\nu}(p, b) &= e^2 \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \text{Tr} \left[\gamma_\mu \left(\frac{1}{k - m} + \frac{1}{k - m} \not{b} \gamma_5 \frac{1}{k - m} \right. \right. \\ &\quad \left. \left. + \frac{1}{k - m} \not{b} \gamma_5 \frac{1}{k - m} \not{b} \gamma_5 \frac{1}{k - m} \right) \gamma_\nu \left(\frac{1}{k + \not{p} - m} + \frac{1}{k + \not{p} - m} \not{b} \gamma_5 \frac{1}{k + \not{p} - m} \right. \right. \\ &\quad \left. \left. + \frac{1}{k + \not{p} - m} \not{b} \gamma_5 \frac{1}{k + \not{p} - m} \not{b} \gamma_5 \frac{1}{k + \not{p} - m} \right) \right] \\ &\equiv \Pi_{\mu\nu}^{(0)}(p) + \Pi_{\mu\nu}^{(1)}(p, b) + \Pi_{\mu\nu}^{(2)}(p, b), \end{aligned} \quad (11)$$

Where the Feynman diagrams corresponding to $\Pi_{\mu\nu}^{(i)}$, $i = 0, 1, 2$ is illustrated in Fig. 2. $\Pi_{\mu\nu}^{(0)}(p)$ is just the vacuum polarization tensor in the conventional QED,

$$\begin{aligned}\Pi_{\mu\nu}^{(0)}(p) &= e^2 \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \text{Tr} \left[\gamma_\mu \frac{1}{k-m} \gamma_\nu \frac{1}{k+p-m} \right] \\ &= 4e^2 \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \frac{2k_\mu k_\nu + k_\mu p_\nu + k_\nu p_\mu - g_{\mu\nu} k \cdot (k+p) + m^2 g_{\mu\nu}}{(k^2 - m^2)[(k+p)^2 - m^2]} \\ &= \left(p^2 g_{\mu\nu} - p_\mu p_\nu \right) \frac{ie^2}{2\pi^2} \left(\frac{4\pi\mu^2}{m^2} \right)^{\epsilon_{\text{UV}}} \Gamma(\epsilon_{\text{UV}}) \int_0^1 dx \frac{x(1-x)}{[1 - p^2/m^2 x(1-x)]^\epsilon} \quad (12)\end{aligned}$$

In the four-dimensional limit, we have in the case of $p^2 < 4m^2$,

$$\begin{aligned}\Pi_{\mu\nu}^{(0)}(p) &= \left(p^2 g_{\mu\nu} - p_\mu p_\nu \right) \frac{ie^2}{2\pi^2} \left[\frac{1}{6} \left(\frac{1}{\epsilon_{\text{UV}}} - \gamma + \ln 4\pi \right) + \frac{1}{6} \ln \frac{\mu^2}{m^2} + \frac{5}{18} + \frac{2}{3} \frac{m^2}{p^2} \right. \\ &\quad \left. - \left(\frac{8}{3} \frac{m^4}{p^4} + \frac{2}{3} \frac{m^2}{p^2} - \frac{1}{3} \right) \frac{p/m}{\sqrt{4-p^2/m^2}} \arctan \frac{p/m}{\sqrt{4-p^2/m^2}} \right]. \quad (13)\end{aligned}$$

This is the well-known textbook result [15].

$\Pi_{\mu\nu}^{(1)}(p, b)$ contains the linear terms in b_μ ,

$$\begin{aligned}\Pi_{\mu\nu}^{(1)}(p, b) &= e^2 \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \left[\text{Tr} \left(\gamma_\mu \frac{1}{k-m} \gamma_\nu \frac{1}{k+p-m} \not{b} \gamma_5 \frac{1}{k+p-m} \right) \right. \\ &\quad \left. + \text{Tr} \left(\gamma_\mu \frac{1}{k-m} \not{b} \gamma_5 \frac{1}{k-m} \gamma_\nu \frac{1}{k+p-m} \right) \right]. \quad (14)\end{aligned}$$

This term has been widely investigated using various regularization methods. Despite the existence of a regularization ambiguity, depending on the concrete physical renormalization condition, it indeed leads to the radiatively induced Chern-Simons term, which violate both Lorentz and *CPT* symmetries. In dimensional regularization the result is [10,12]:

$$\Pi_{\mu\nu}^{(1)}(p, b) = \frac{e^2}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} p^\beta b^\alpha \left[\frac{2m}{p} \frac{1}{\sqrt{1-p^2/4m^2}} \arctan \frac{p}{2m} - \frac{1}{4} \right]. \quad (15)$$

The radiatively induced Chern-Simons term can be defined at low-energy (or equivalently large- m) limit.

In the following we concentrate on the quadratic term in b ,

$$\begin{aligned}\Pi_{\mu\nu}^{(2)}(p, b) &= e^2 \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \left[\text{Tr} \left(\gamma_\mu \frac{1}{k-m} \gamma_\nu \frac{1}{k+p-m} \not{b} \gamma_5 \frac{1}{k+p-m} \not{b} \gamma_5 \frac{1}{k+p-m} \right) \right. \\ &\quad \left. + \text{Tr} \left(\gamma_\mu \frac{1}{k-m} \not{b} \gamma_5 \frac{1}{k-m} \gamma_\nu \frac{1}{k+p-m} \not{b} \gamma_5 \frac{1}{k+p-m} \right) \right. \\ &\quad \left. + \text{Tr} \left(\gamma_\mu \frac{1}{k-m} \not{b} \gamma_5 \frac{1}{k-m} \not{b} \gamma_5 \frac{1}{k-m} \gamma_\nu \frac{1}{k+p-m} \right) \right]. \quad (16)\end{aligned}$$

The trace calculation in Eq. (16) is a heavy task, so we first make use of the following identities

$$\begin{aligned}\not{b}\gamma_5 \frac{1}{\not{k}-m} \not{b}\gamma_5 &= \not{b} \left(\frac{1}{\not{k}-m} - \frac{2m}{k^2-m^2} \right) \not{b} = \not{b} \frac{1}{\not{k}-m} \not{b} - \frac{2mb^2}{k^2-m^2}; \\ \not{b}\gamma_5 \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \not{b}\gamma_5 &= \not{b} \left(\frac{1}{\not{k}-m} - \frac{2m}{k^2-m^2} \right) \gamma_\nu \left[\frac{1}{\not{k}+\not{p}-m} - \frac{2m}{(k+p)^2-m^2} \right] \not{b},\end{aligned}\quad (17)$$

and the differential operations

$$\begin{aligned}\frac{\partial}{\partial p_\mu} \frac{1}{\not{k}+\not{p}-m} &= -\frac{1}{\not{k}+\not{p}-m} \gamma_\mu \frac{1}{\not{k}+\not{p}-m}; \\ \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\nu} \frac{1}{\not{k}+\not{p}-m} &= \frac{1}{\not{k}+\not{p}-m} \gamma_\mu \frac{1}{\not{k}+\not{p}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \\ &\quad + \frac{1}{\not{k}+\not{p}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \gamma_\mu \frac{1}{\not{k}+\not{p}-m}.\end{aligned}\quad (18)$$

Consequently, $\Pi_{\mu\nu}^{(2)}(p, b)$ can be written in the following form,

$$\begin{aligned}\Pi_{\mu\nu}^{(2)}(p, b) &= e^2 \mu^{2\epsilon_{\text{UV}}} \left[\int \frac{d^n k}{(2\pi)^n} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \not{b} \frac{1}{\not{k}+\not{p}-m} \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \right. \\ &\quad - \int \frac{d^n k}{(2\pi)^n} \frac{2mb^2}{(k+p)^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad + \int \frac{d^n k}{(2\pi)^n} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad - \int \frac{d^n k}{(2\pi)^n} \frac{2m}{k^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad - \int \frac{d^n k}{(2\pi)^n} \frac{2m}{(k+p)^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \frac{1}{\not{k}-m} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad + \int \frac{d^n k}{(2\pi)^n} \frac{4m^2}{(k^2-m^2)[(k+p)^2-m^2]} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad + \int \frac{d^n k}{(2\pi)^n} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-\not{p}-m} \not{b} \frac{1}{\not{k}-\not{p}-m} \not{b} \frac{1}{\not{k}-\not{p}-m} \not{b} \gamma_\nu \frac{1}{\not{k}-m} \right) \\ &\quad \left. - \int \frac{d^n k}{(2\pi)^n} \frac{2mb^2}{k^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}-m} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \right] \\ &= e^2 \mu^{2\epsilon_{\text{UV}}} \left[b^\alpha b^\beta \frac{\partial}{\partial p^\alpha} \frac{\partial}{\partial p^\beta} \int \frac{d^n k}{(2\pi)^n} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \right) \right. \\ &\quad - \int \frac{d^n k}{(2\pi)^n} \frac{2mb^2}{(k+p)^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad - \int \frac{d^n k}{(2\pi)^n} \frac{2m}{k^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \\ &\quad - b^\alpha b^\beta \frac{\partial}{\partial p^\alpha} \frac{\partial}{\partial p^\beta} \int \frac{d^n k}{(2\pi)^n} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}+\not{p}-m} \gamma_\nu \frac{1}{\not{k}-m} \right) \\ &\quad \left. + b^\alpha \frac{\partial}{\partial p^\alpha} \int \frac{d^n k}{(2\pi)^n} \frac{2m}{k^2-m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \right) \right]\end{aligned}$$

$$\begin{aligned}
& -b^\alpha \frac{\partial}{\partial p^\alpha} \int \frac{d^n k}{(2\pi)^n} \frac{2m}{(k+p)^2 - m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \\
& + \int \frac{d^n k}{(2\pi)^n} \frac{4m^2}{(k^2 - m^2)[(k+p)^2 - m^2]} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \Big] \\
= & e^2 b^\alpha \frac{\partial}{\partial p^\alpha} \int \frac{d^4 k}{(2\pi)^4} \left[\frac{2m}{k^2 - m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \right) \right. \\
& - \frac{2m}{(k+p)^2 - m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \Big] \\
& + e^2 \int \frac{d^4 k}{(2\pi)^4} \left[\frac{4m^2}{(k^2 - m^2)[(k+p)^2 - m^2]} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \not{b} \gamma_\nu \not{b} \frac{1}{\not{k}+\not{p}-m} \right) \right. \\
& - \frac{2mb^2}{k^2 - m^2} \text{Tr} \left(\gamma_\mu \frac{1}{(\not{k}-m)^2} \gamma_\nu \frac{1}{\not{k}+\not{p}-m} \right) \\
& \left. \left. - \frac{2mb^2}{(k+p)^2 - m^2} \text{Tr} \left(\gamma_\mu \frac{1}{\not{k}-m} \gamma_\nu \frac{1}{(\not{k}+\not{p}-m)^2} \right) \right] . \tag{19}
\end{aligned}$$

Taking the γ -matrix trace and performing Feynman parameterization, we get

$$\begin{aligned}
\Pi_{\mu\nu}^{(2)}(p, b) = & e^2 b^\alpha \frac{\partial}{\partial p^\alpha} \int \frac{d^4 k}{(2\pi)^4} \left[\frac{8m^2}{(k^2 - m^2)^2[(k+p)^2 - m^2]} (2k_\mu b_\nu + p_\mu b_\nu + b_\mu p_\nu - g_{\mu\nu} p \cdot b) \right. \\
& - \frac{8m^2}{(k^2 - m^2)[(k+p)^2 - m^2]^2} (2k_\mu b_\nu + p_\mu b_\nu - b_\mu p_\nu + g_{\mu\nu} p \cdot b) \Big] \\
& + e^2 \int \frac{d^4 k}{(2\pi)^4} \left(\frac{16m^2}{(k^2 - m^2)^2[(k+p)^2 - m^2]^2} \{2b_\nu [k_\mu b \cdot (k+p) - b_\mu k \cdot (k+p)] \right. \\
& + (k+p)_\mu k \cdot b] - b^2 [2k_\mu k_\nu + k_\mu p_\nu + k_\nu p_\mu - g_{\mu\nu} k \cdot (k+p)] + m^2 (2b_\mu b_\nu - g_{\mu\nu} b^2) \Big\} \\
& - \frac{8m^2 b^2}{(k^2 - m^2)[(k+p)^2 - m^2]^3} \{2 [2k_\mu k_\nu + k_\mu p_\nu + k_\nu p_\mu - g_{\mu\nu} k \cdot (k+p)] \\
& + [(k+p)^2 + m^2] g_{\mu\nu}\} \\
& - \frac{8m^2 b^2}{(k^2 - m^2)^3[(k+p)^2 - m^2]} \{2 [2k_\mu k_\nu + k_\mu p_\nu + k_\nu p_\mu - g_{\mu\nu} k \cdot (k+p)] \\
& + (k^2 + m^2) g_{\mu\nu}\} \Big) \\
= & (p^2 g_{\mu\nu} - p_\mu p_\nu) \int_0^1 dx \left[\frac{x(1-x)}{[m^2 - p^2 x(1-x)]^2} \left(-i \frac{e^2 m^2 b^2}{\pi^2} \right) \right. \\
& + \frac{x^2(1-x)}{[m^2 - p^2 x(1-x)]^2} \frac{2im^2}{\pi^2} \frac{(p \cdot b)^2}{p^2} \Big] + \left(p_\mu b_\nu + p_\nu b_\mu - \frac{p^2}{p \cdot b} b_\mu b_\nu - \frac{p \cdot b}{p^2} p_\mu p_\nu \right) \\
& \times \int_0^1 dx \frac{x^2(1-x)}{[m^2 - p^2 x(1-x)]^2} \left(-\frac{2im^2 e^2 p \cdot b}{\pi^2} \right) \\
= & \left\{ (p^2 g_{\mu\nu} - p_\mu p_\nu) \left[\frac{b^2}{p^2} - \frac{(p \cdot b)^2}{p^4} \right] - \left(p_\mu b_\nu + p_\nu b_\mu - \frac{p^2}{p \cdot b} b_\mu b_\nu - \frac{p \cdot b}{p^2} p_\mu p_\nu \right) \frac{p \cdot b}{p^2} \right\} \\
& \times \frac{2ie^2}{\pi^2} \frac{m^2}{4m^2 - p^2} \left[1 + \frac{2p}{\sqrt{4m^2 - p^2}} \left(1 - \frac{2m^2}{p^2} \right) \arctan \frac{p}{\sqrt{4m^2 - p^2}} \right] . \tag{20}
\end{aligned}$$

Combining the result of Eq. (20) with Eq. (13), the vacuum polarization tensor in the absence of Lorentz and *CPT* violation term, we obtain the polarization tensor associated with the quantum correction of electric charge

$$\Pi_{\mu\nu}(p, b) = i \left(p^2 g_{\mu\nu} - p_\mu p_\nu \right) \omega(p, b) + \dots \quad (21)$$

with

$$\begin{aligned} \omega(p, b) = & \frac{2\alpha}{\pi} \left\{ \left[\left(\frac{1}{\epsilon_{\text{UV}}} - \gamma + \ln 4\pi \right) + \frac{1}{6} \ln \frac{\mu^2}{m^2} + \frac{5}{18} + \frac{2}{3} \frac{m^2}{p^2} - \left(\frac{8}{3} \frac{m^4}{p^4} + \frac{2}{3} \frac{m^2}{p^2} - \frac{1}{3} \right) \right. \right. \\ & \times \frac{p/m}{\sqrt{4-p^2/m^2}} \arctan \frac{p/m}{\sqrt{4-p^2/m^2}} \left. \right] + \frac{4}{4-p^2/m^2} \left[\frac{b^2}{p^2} - \frac{(p \cdot b)^2}{p^4} \right] \\ & \times \left. \left[1 + \left(1 - \frac{2m^2}{p^2} \right) \frac{2p/m}{\sqrt{4-p^2/m^2}} \arctan \frac{p/m}{\sqrt{4-p^2/m^2}} \right] \right\}. \end{aligned} \quad (22)$$

The on-shell renormalization condition

$$\omega_R(p, b)|_{p^2=0} = 0 \quad (23)$$

gives the radiative correction part of the vacuum polarization tensor,

$$\begin{aligned} \omega_R(p, b) = & \frac{2\alpha}{\pi} \left\{ \frac{5}{18} + \frac{2}{3} \frac{m^2}{p^2} - \left(\frac{8}{3} \frac{m^4}{p^4} + \frac{2}{3} \frac{m^2}{p^2} - \frac{1}{3} \right) \frac{p/m}{\sqrt{4-p^2/m^2}} \arctan \frac{p/m}{\sqrt{4-p^2/m^2}} \right. \\ & + \frac{4}{4-p^2/m^2} \left[\frac{b^2}{p^2} - \frac{(p \cdot b)^2}{p^4} \right] \left[1 - \frac{p^2}{3m^2} + \left(1 - \frac{2m^2}{p^2} \right) \frac{2p/m}{\sqrt{4-p^2/m^2}} \right. \\ & \times \left. \left. \arctan \frac{p/m}{\sqrt{4-p^2/m^2}} \right] \right\}, \end{aligned} \quad (24)$$

where the subscript “R” represents the radiative correction. At low-energy limit, $p^2 \rightarrow 0$, this reduces to

$$\omega_R(p, b)|_{p^2 \rightarrow 0} = \frac{2\alpha}{\pi} \left[\left(\frac{1}{30} + \frac{2}{15} \frac{b^2}{m^2} \right) \frac{p^2}{m^2} - \frac{2}{15} \frac{(p \cdot b)^2}{m^4} \right]. \quad (25)$$

The screening of the electric charge produced by the vacuum polarization is

$$e^2 \longrightarrow \frac{e^2}{1 + \omega_R(p, b)}. \quad (26)$$

III. ONE-LOOP ON-SHELL VERTEX

As in standard QED, the one-loop on-shell vertex is

$$\begin{aligned}
-i e \mu^\epsilon \bar{u}(q) \Lambda(p, q) u(p) &= \bar{u}(q) \int \frac{d^n k}{(2\pi)^n} \left\{ (-ie\gamma_\rho \mu^\epsilon) \frac{i}{\not{k} + \not{q} - m - \not{p}\gamma_5} (-ie\gamma_\mu \mu^\epsilon) \right. \\
&\quad \times \frac{i}{\not{k} + \not{p} - m - \not{p}\gamma_5} (-ie\gamma_\nu \mu^\epsilon) \frac{-i}{k^2} \left[g_{\nu\rho} - (1-\alpha) \frac{k_\nu k_\rho}{k^2} \right] \left. \right\} u(p) \\
&\equiv -ie \mu^\epsilon \bar{u}(q) [\Lambda^{(0)}(p, q) + \Lambda^{(1)}(p, q) + \Lambda^{(2)}(p, q)] u(p), \tag{27}
\end{aligned}$$

where we have used the identity Eq. (10) truncated to second order in b and ϵ denotes ϵ_{UV} or $-\epsilon_{\text{IR}}$. The relevant Feynman diagrams are shown in Fig. 3. Since the radiative correction is gauge independent, we first have a look at the gauge dependent part of the amplitude (27). Eq. (27) shows that the term proportional to $(1-\alpha)$ involves an integral with integrand

$$\frac{1}{k^4} \not{k} \frac{1}{\not{k} + \not{q} - m} (\dots) \frac{1}{\not{k} + \not{p} - m} \not{k}. \tag{28}$$

Writing the \not{k} on the left-side as $\not{k} + \not{q} - m - (\not{q} - m)$ and the \not{k} on the right-side as $\not{k} + \not{p} - m - (\not{p} - m)$ and considering the mass-shell condition, $\bar{u}(q)(\not{q} - m) = 0$, $(\not{p} - m)u(p) = 0$, one can see that gauge dependent part is independent of the external momenta and hence is absorbed by the counterterm of the vertex renormalization, $Z_1^{-1} - 1$. Therefore, we can calculate the radiative correction of the vertex in Feynman gauge, $\alpha = 1$ [16].

A. One-loop On-shell Vertex in Standard Quantum Electrodynamics

$\bar{u}(q)\Lambda_\mu^{(0)}(p, q)u(p)$ is the on-shell vertex correction in standard QED (Fig. 3a). Using

$$\begin{aligned}
\bar{u}(q)\gamma_\nu(\not{k} + \not{q} + m) &= \bar{u}(q) [-\not{k}\gamma_\nu + 2(k+q)_\nu]; \\
(\not{k} + \not{p} + m)\gamma_\nu u(p) &= [-\gamma_\nu \not{k} + 2(k+p)_\nu] u(p),
\end{aligned} \tag{29}$$

we get

$$\begin{aligned}
\bar{u}(q)\Lambda_\mu^{(0)}(p, q)u(p) &= -ie^2 \mu^{2\epsilon} \bar{u}(q) \int \frac{d^n k}{(2\pi)^n} \frac{[-\not{k}\gamma_\nu + 2(k+q)_\nu]\gamma_\mu [-\gamma_\nu \not{k} + 2(k+p)_\nu]}{k^2[(k+q)^2 - m^2][(k+p)^2 - m^2]} \\
&= -ie^2 \mu^{2\epsilon} \bar{u}(q) \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{(2-n)(2\not{k}k_\mu - k^2\gamma_\mu)}{k^2[(k+q)^2 - m^2][(k+p)^2 - m^2]} \right. \\
&\quad \left. + 4 \frac{\gamma_\mu k \cdot (p+q) + mk_\mu - \not{k}(p_\mu + q_\mu) + p \cdot q \gamma_\mu}{k^2[(k+q)^2 - m^2][(k+p)^2 - m^2]} \right\} u(p).
\end{aligned} \tag{30}$$

Given that

$$\begin{aligned}
&\mu^{-2\epsilon_{\text{IR}}} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2[(k+q)^2 - m^2][(k+p)^2 - m^2]}|_{p^2=q^2=m^2, \epsilon_{\text{IR}} \rightarrow 0} \\
&= -\frac{i}{32\pi^2} \int_0^1 dx \frac{1}{m^2 - l^2 x(1-x)} \left[\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right] \\
&= -\frac{i}{32\pi^2} \frac{1}{m^2} \left[\left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) \frac{4m}{l\sqrt{4-l^2/m^2}} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \right. \\
&\quad \left. + \int_0^1 dx \frac{\ln(1-l^2/m^2 x(1-x))}{1-l^2/m^2 x(1-x)} \right];
\end{aligned} \tag{31}$$

$$\begin{aligned}
& \int \frac{d^n k}{(2\pi)^n} \frac{k_\mu}{k^2[(k+q)^2 - m^2][(k+p)^2 - m^2]}|_{p^2=q^2=m^2} \\
&= \frac{i}{8\pi^2} \frac{p_\mu + q_\mu}{m^2} \frac{m}{l\sqrt{4-l^2/m^2}} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}}; \tag{32}
\end{aligned}$$

$$\begin{aligned}
& \mu^{2\epsilon_{\text{UV}}} \int \frac{d^n k}{(2\pi)^n} \frac{k_\mu k_\nu}{k^2[(k+q)^2 - m^2][(k+p)^2 - m^2]}|_{p^2=q^2=m^2, \epsilon_{\text{UV}} \rightarrow 0} \\
&= \frac{i}{64\pi^2} g_{\mu\nu} \int_0^1 dx \left[\frac{1}{\epsilon_{\text{UV}}} - \gamma + \ln \frac{4\pi\mu^2}{m^2} - \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right] \\
&\quad - \frac{i}{32\pi^2} \int_0^1 dx \frac{1}{m^2 - l^2 x(1-x)} \left[x(1-x) (p_\mu q_\nu + p_\nu q_\mu) + x^2 (p_\mu p_\nu + q_\mu q_\nu) \right] \\
&= \frac{i}{64\pi^2} g_{\mu\nu} \left(\frac{1}{\epsilon_{\text{UV}}} - \gamma + 3 + \ln \frac{4\pi\mu^2}{m^2} - \frac{2m}{l} \sqrt{4-l^2/m^2} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \right) \\
&\quad - \frac{i}{32\pi^2} \frac{p_\mu q_\nu + p_\nu q_\mu}{m^2} \left[-\frac{m^2}{l^2} + \frac{4m^3}{l^3} \frac{1}{\sqrt{4-l^2/m^2}} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \right] \\
&\quad - \frac{i}{32\pi^2} \frac{p_\mu p_\nu + q_\mu q_\nu}{m^2} \left[\frac{m^2}{l^2} + 2\frac{m}{l} \left(1 - 2\frac{m^2}{l^2} \right) \frac{1}{\sqrt{4-l^2/m^2}} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \right], \tag{33}
\end{aligned}$$

we obtain the result in dimensional regularization

$$\begin{aligned}
& \bar{u}(q) \Lambda_\mu^{(0)}(p, q) u(p) \\
&= \bar{u}(q) \left\{ \frac{e^2}{16\pi^2} \gamma_\mu \left(\frac{1}{\epsilon_{\text{UV}}} - \gamma + \ln \frac{4\pi\mu^2}{m^2} + \frac{6m}{l} \sqrt{4-l^2/m^2} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \right) \right. \\
&\quad + \frac{e^2}{2\pi^2} \frac{p_\mu + q_\mu}{m} \frac{m}{l} \left(1 - \frac{1}{\sqrt{4-l^2/m^2}} \right) \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \\
&\quad - \frac{e^2}{16\pi^2} \gamma_\mu \left(2 - \frac{l^2}{m^2} \right) \left[\left(\frac{1}{\epsilon_{\text{IR}}} - \ln \frac{4\pi\mu^2}{m^2} + \gamma \right) \frac{4m/l}{\sqrt{4-l^2/m^2}} \arctan \frac{l/m}{\sqrt{4-l^2/m^2}} \right. \\
&\quad \left. \left. + \int_0^1 dx \frac{\ln(1-l^2/m^2 x(1-x))}{1-l^2/m^2 x(1-x)} \right] \right\} u(p). \tag{34}
\end{aligned}$$

In above equations, $l \equiv q - p$, to distinguish the infrared and ultraviolet divergences, we especially put on the subscripts on the regulators to emphasize the difference, $\epsilon_{\text{IR}} = n/2 - 2$ and $\epsilon_{\text{UV}} = 2 - n/2$.

B. On-shell Vertex Correction at First Order of b

Now let us look at the contribution from the first order of b_μ (Fig. 3b),

$$\begin{aligned}
\Lambda_\mu^{(1)}(p, q) &= -ie^2 \mu^{-2\epsilon_{\text{IR}}} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2} \left[\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \not{b} \gamma_5 \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right. \\
&\quad \left. + \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \not{b} \gamma_5 \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right] \\
&= -ie^2 \mu^{-2\epsilon_{\text{IR}}} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2} \left\{ -b^\alpha \frac{\partial}{\partial p^\alpha} \left[\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \gamma_5 \right] \right. \\
&\quad + b^\alpha \frac{\partial}{\partial p^\alpha} \left[\gamma_5 \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right] \\
&\quad - \frac{2m}{k^2[(k+p)^2 - m^2]} \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \not{b} \gamma_\nu \gamma_5 \\
&\quad \left. + \frac{2m}{k^2[(k+q)^2 - m^2]} \gamma_5 \gamma_\nu \not{b} \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right\}. \tag{35}
\end{aligned}$$

After performing some algebraic operation and taking the derivative, we have

$$\begin{aligned}
\Lambda_\mu^{(1)}(p, q) &= -ie^2 \mu^{-2\epsilon_{\text{IR}}} \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{-\gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu \not{b} \gamma_\nu \gamma_5 + \gamma_5 \gamma_\nu \not{b} \gamma_\mu (\not{k} + \not{p} + m) \gamma_\nu}{k^2[(k+p)^2 - m^2][(k+p)^2 - m^2]} \right. \\
&\quad + \frac{2b \cdot (k+p) [\gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu (\not{k} + \not{p} + m) \gamma_\nu \gamma_5]}{k^2[(k+p)^2 - m^2]^2[(k+q)^2 - m^2]} \\
&\quad - \frac{2b \cdot (k+q) [\gamma_5 \gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu (\not{k} + \not{p} + m) \gamma_\nu]}{k^2[(k+p)^2 - m^2]^2[(k+q)^2 - m^2]} \\
&\quad - \frac{2m [\gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu (\not{k} + \not{p} + m) \gamma_\nu \gamma_5]}{k^2[(k+p)^2 - m^2]^2[(k+q)^2 - m^2]} \\
&\quad \left. + \frac{2m [\gamma_5 \gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu (\not{k} + \not{p} + m) \gamma_\nu]}{k^2[(k+p)^2 - m^2]^2[(k+q)^2 - m^2]} \right\}. \tag{36}
\end{aligned}$$

Putting Eq. (36) on mass-shell, i.e. evaluating $\bar{u}(q)\Lambda_\mu^{(1)}(p, q)u(p)$, and making use of the following γ matrices formula,

$$\begin{aligned}
\bar{u}(q)[\gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu(\not{k} + \not{p} + m) \gamma_\nu \gamma_5]u(p) &= [4k \cdot (p+q) - 2l^2 + 2m\not{k} + 2k^2] \gamma_\mu \gamma_5 \\
&\quad + [-4(p_\mu + q_\mu)\not{k} + 6m\gamma_\mu \not{k} + 8mq_\mu - 4\not{k}k_\mu] \gamma_5 + \epsilon_{\text{IR}} (4m\not{k}\gamma_\mu - 4\not{k}k_\mu + 2k^2\gamma_\mu) \gamma_5; \\
\bar{u}(q)[\gamma_5 \gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu(\not{k} + \not{p} + m) \gamma_\nu]u(p) &= \gamma_5 \gamma_\mu [4k \cdot (p+q) - 2l^2 + 2m\not{k} + 2k^2] \\
&\quad + \gamma_5 [-4(p_\mu + q_\mu)\not{k} + 6m\not{k}\gamma_\mu + 8mp_\mu - 4\not{k}k_\mu] + \epsilon_{\text{IR}} \gamma_5 (4m\gamma_\mu \not{k} - 4\not{k}k_\mu + 2\gamma_\mu k^2); \\
\bar{u}(q)[\gamma_\nu(\not{k} + \not{q} + m) \gamma_\mu(\not{k} + \not{p} + m) \not{b} \gamma_\nu \gamma_5]u(p) &= (8k_\mu + 8q_\mu - 4m\gamma_\mu)(k+p) \cdot b \gamma_5 \\
&\quad - [4k \cdot (p+q) + 4p \cdot q + 2m\not{k}]\gamma_\mu \not{b} \gamma_5 + (2m\gamma_\mu - 4q_\mu)\not{b} \not{k} \gamma_5 + (4q \cdot b \gamma_\mu \not{k} - 4p \cdot b \not{k} \gamma_\mu \\
&\quad + 4p_\mu \not{k} \not{b} - 4k^2 b_\mu) \gamma_5 - \epsilon_{\text{IR}} [4(k+p) \cdot b \not{k} \gamma_\mu + 2\not{k} \gamma_\mu \not{b} \not{k}] \gamma_5; \\
\bar{u}(q)[\gamma_5 \gamma_\nu \not{b}(\not{k} + \not{q} + m) \gamma_\mu(\not{k} + \not{p} + m) \gamma_\nu]u(p) &= \gamma_5 (k+q) \cdot b (8k_\mu + 8p_\mu - 4m\gamma_\mu) \\
&\quad - \gamma_5 \not{b} \gamma_\mu [4k \cdot (p+q) + 4p \cdot q - 2m\not{k}] + \gamma_5 \not{b} \not{k} (2m\gamma_\mu - 4p_\mu) + \gamma_5 (4p \cdot b \not{k} \gamma_\mu + 4q_\mu \not{b} \not{k} \\
&\quad - 4q \cdot b \gamma_\mu \not{k} - 4b_\mu k^2) \not{k} + \epsilon_{\text{IR}} \gamma_5 [4b \cdot (k+q) \gamma_\mu \not{k} - 2\not{k} \not{b} \gamma_\mu \not{k}], \tag{37}
\end{aligned}$$

we obtain

$$\begin{aligned}
\bar{u}(q)\Lambda^{(1)}(p, q)u(p) = & \bar{u}(q) \left\{ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{1}{m^2 - l^2 x(1-x)} \left[\left(1 - \frac{9}{2}x\right) (p+q) \cdot b\gamma_\mu \right. \right. \\
& + \frac{5}{2}x(p_\mu + q_\mu)\not{b} + m \left(\frac{1}{8} - x \right) [\not{b}, \gamma_\mu] - \frac{1}{4}mb_\mu \Big] \gamma_5 \\
& - \frac{e^2}{8\pi^2} \int_0^1 dx \frac{1}{m^2 - l^2 x(1-x)} \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2}x(1-x) \right) + \gamma \right] \\
& \times \left[-(p+q) \cdot b\gamma_\mu + (p_\mu + q_\mu)\not{b} + \frac{1}{2}m[\not{b}, \gamma_\mu] \right] \gamma_5 \\
& + \frac{e^2}{4\pi^2} \int_0^1 dx \frac{x}{[m^2 - l^2 x(1-x)]^2} \left[(p+q) \cdot b\gamma_\mu(m^2 - l^2) + m^2(p_\mu + q_\mu)\not{b} \right. \\
& \left. - \frac{ml^2}{2}[\not{b}, \gamma_\mu] + m(p_\mu + q_\mu)l \cdot b(3 + 6x - 5x^2) + l_\mu m(p+q) \cdot b(-1 + 3x + 5x^2) \right] \gamma_5 \\
& + \frac{e^2}{4\pi^2} \int_0^1 dx \frac{x}{[m^2 - l^2 x(1-x)]^2} \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2}x(1-x) \right) + \gamma \right] \\
& \times \left[2(l^2 - 2m^2)(p+q) \cdot b\gamma_\mu + m^2(p_\mu + q_\mu)\not{b} + \frac{1}{2}m(4m^2 - l^2)[\not{b}, \gamma_\mu] \right. \\
& \left. + \frac{1}{2}(p_\mu + q_\mu)ml \cdot b(16x - 11) + \frac{1}{2}l_\mu m(p+q) \cdot b(1 + 2x) \right] \gamma_5 \Big\} u(p). \tag{38}
\end{aligned}$$

With the Gordon identity (7), the superficial antisymmetric terms of Eq. (38) in p, q such as $(p_\mu + q_\mu)l \cdot b\gamma_5$ and $l_\mu(p+q) \cdot b$ can be converted into an explicit p, q symmetric form. For examples, there have

$$\begin{aligned}
\bar{u}(q)(p_\mu + q_\mu)l \cdot b\gamma_5 u(p) &= \bar{u}(q)(p+q)_\mu \left[2m\not{b}\gamma_5 + \frac{1}{2}\epsilon_{\nu\rho\alpha\beta}b^\nu(p+q)^\rho\sigma^{\alpha\beta} \right] u(p), \\
\bar{u}(q)(p+q) \cdot bl_\mu\gamma_5 u(p) &= \bar{u}(q)(p+q) \cdot b \left[2m\gamma_\mu\gamma_5 + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(p+q)^\nu\sigma^{\alpha\beta} \right] u(p). \tag{39}
\end{aligned}$$

C. Contribution to Second Order in b_μ

The on-shell quantum vertex at second order in b_μ is quite complicated. It gets contributions from three possible insertions of $\not{b}\gamma_5$ vertex as shown in Fig. 3c,

$$\begin{aligned}
\bar{u}(q)\Lambda_\mu^{(2)}(p, q)u(p) = & -ie^2\mu^{-2\epsilon_{\text{IR}}}\bar{u}(q) \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2} \left[\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \not{b}\gamma_5 \right. \\
& \times \frac{1}{\not{k} + \not{p} - m} \not{b}\gamma_5 \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \\
& + \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \not{b}\gamma_5 \frac{1}{\not{k} + \not{q} - m} \not{b}\gamma_5 \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \\
& \left. + \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \not{b}\gamma_5 \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \not{b}\gamma_5 \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right] u(p). \tag{40}
\end{aligned}$$

To reduce the number of complicated γ -matrix operations, we continue to use the identities (17) and (18). As a consequence, Eq. (40) can be rewritten in the following compact form,

$$\begin{aligned}
\bar{u}(q)\Lambda_\mu^{(2)}(p,q)u(p) = & -ie^2\mu^{2\epsilon_{\text{IR}}}\bar{u}(q)\int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2} \left\{ \frac{1}{2} \left(b^\alpha \frac{\partial}{\partial p^\alpha} + b^\alpha \frac{\partial}{\partial q^\alpha} \right)^2 \right. \\
& \times \left(\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right) \quad \textcircled{1} \\
& - \frac{2mb^2}{(k+p)^2 - m^2} \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \left(\frac{1}{\not{k} + \not{p} - m} \right)^2 \gamma_\nu \quad \textcircled{2} \\
& - \frac{2mb^2}{(k+q)^2 - m^2} \gamma_\nu \left(\frac{1}{\not{k} + \not{q} - m} \right)^2 \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \quad \textcircled{3} \\
& - \frac{2mb^2}{(k+p)^2 - m^2} b^\alpha \frac{\partial}{\partial q^\alpha} \left(\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \gamma_\mu \not{b} \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right) \quad \textcircled{4} \\
& - \frac{2mb^2}{(k+q)^2 - m^2} b^\alpha \frac{\partial}{\partial p^\alpha} \left(\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \not{b} \gamma_\mu \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right) \quad \textcircled{5} \\
& \left. + \frac{4m^2}{[(k+p)^2 - m^2][(k+q)^2 - m^2]} \gamma_\nu \frac{1}{\not{k} + \not{q} - m} \not{b} \gamma_\mu \not{b} \frac{1}{\not{k} + \not{p} - m} \gamma_\nu \right\} u(p). \quad \textcircled{6} \quad (41)
\end{aligned}$$

The last term is quite simple,

$$\begin{aligned}
\textcircled{6} = & \bar{u}(q) \left\{ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{x(1-x)}{[m^2 - l^2 x(1-x)]^2} \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right. \right. \\
& - 1 + \gamma] m^2 (2\not{b}b_\mu - b^2 \gamma_\mu) \\
& - \frac{e^2}{2\pi^2} \int_0^1 dx \frac{x(1-x)}{[m^2 - l^2 x(1-x)]^3} \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right. \\
& + 5 + 2\gamma] m^2 [-2mb_\mu b \cdot (p+q) + b^2 (p_\mu + q_\mu)] \\
& \left. + \frac{e^2}{\pi^2} \int_0^1 dx \frac{x(1-x)}{[m^2 - l^2 x(1-x)]^3} m^2 (-6m^2 + l^2) (2\not{b}b_\mu - b^2 \gamma_\mu) \right\} u(p). \quad (42)
\end{aligned}$$

The second and the third terms yield

$$\begin{aligned}
\textcircled{2} + \textcircled{3} = & \bar{u}(q) \left\{ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{x^2}{[m^2 - l^2 x(1-x)]^2} 3m^2 b^2 \gamma_\mu \right. \\
& + \frac{e^2}{8\pi^2} \int_0^1 dx \frac{x^2 m b^2 (p_\mu + q_\mu)}{[m^2 - l^2 x(1-x)]^2} \left[\frac{1}{\epsilon_{\text{IR}}} - \ln \frac{m^2}{4\pi\mu^2} - \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) - 3 + \gamma \right] \\
& - \frac{e^2}{4\pi^2} \int_0^1 dx \frac{x^2 (1-x) m b^2 l^2 (p_\mu + q_\mu)}{[m^2 - l^2 x(1-x)]^3} \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) - 3 + 2\gamma \right] \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{x^2 m b^2}{[m^2 - l^2 x(1-x)]^3} \left[(p_\mu + q_\mu) (-2m^2 + l^2(2-x)) (1-x) \right. \\
& \left. \left. + m \gamma_\mu (-16m^2 + 4m^2 x + 4l^2 - l^2 x + 2l^2 x^2) \right] \right\} u(p). \quad (43)
\end{aligned}$$

As for the fourth and fifth terms,

$$\textcircled{4} + \textcircled{5} = -ie^2 \mu^{-2\epsilon_{\text{IR}}} \bar{u}(q) \left\{ - \int \frac{d^n k}{(2\pi)^n} \frac{2m \gamma_\nu \not{b} \gamma_\mu \not{b} (\not{k} + \not{p} + m) \gamma_\nu}{k^2 [(k+p)^2 - m^2]^2 [(k+q)^2 - m^2]}$$

$$\begin{aligned}
& -\frac{2m\gamma_\nu(\not{k} + \not{q} + m)\not{b}\gamma_\mu\not{b}\gamma_\nu}{k^2[(k+p)^2-m^2][(k+q)^2-m^2]^2} \\
& +\frac{4mb\cdot(k+q)\gamma_\nu(\not{k} + \not{q} + m)\gamma_\mu\not{b}(\not{k} + \not{p} + m)\gamma_\nu}{k^2[(k+p)^2-m^2]^2[(k+q)^2-m^2]^2} \\
& +\frac{4mb\cdot(k+p)\gamma_\nu(\not{k} + \not{q} + m)\not{b}\gamma_\mu(\not{k} + \not{p} + m)\gamma_\nu}{k^2[(k+p)^2-m^2]^2[(k+q)^2-m^2]^2} \Big\} u(p), \tag{44}
\end{aligned}$$

a straightforward calculation leads to

$$\begin{aligned}
\textcircled{4} + \textcircled{5} = & \bar{u}(q) \left\{ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{x}{[m^2 - l^2 x(1-x)]^2} m [4b_\mu b \cdot (p+q) - b^2(p_\mu + q_\mu)] \right. \\
& \times \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + 2 - x + \gamma \right] \\
& + \frac{e^2}{4\pi^2} \int_0^1 dx \frac{mx}{[m^2 - l^2 x(1-x)]^2} [m (4b_\mu \not{b} - b^2 \gamma_\mu) (2x-3) - 4(1-x)b \cdot (p+q)b_\mu] \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{x(1-x)}{[m^2 - l^2 x(1-x)]^2} \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right. \\
& \left. - 1 + \gamma \right] [3mb \cdot (p+q)b_\mu - m^2 \not{b}b_\mu] \\
& - \frac{e^2}{2\pi^2} \int_0^1 dx \frac{x(1-x)}{[m^2 - l^2 x(1-x)]^3} \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi^2} \right. \\
& \left. + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) - 3 + 2\gamma \right] m (2m^2 - l^2) b \cdot (p+q)b_\mu \\
& - \frac{2e^2}{\pi^2} \int_0^1 dx \frac{x(1-x)}{[m^2 - l^2 x(1-x)]^3} [m(2m^2 - l^2)(p+q) \cdot bb_\mu \\
& \left. - m^2 [(p \cdot b)^2 + (p \cdot b)^2] + m^2(p \cdot bp_\mu + q \cdot bq_\mu) \not{b} - m^2(p \cdot bq_\mu + p \cdot bp_\mu) \not{b} \right] \Big\} u(p). \tag{45}
\end{aligned}$$

The first term is the most complicated. When the derivative is taken, it becomes

$$\begin{aligned}
\textcircled{1} = & -ie^2 \mu^{-2\epsilon_{\text{IR}}} \bar{u}(q) \int \frac{d^n k}{(2\pi)^n} \left\{ \frac{\gamma_\nu \not{b} \gamma_\mu \not{b} \gamma_\nu}{k^2[(k+p)^2-m^2][(k+q)^2-m^2]} \right. \\
& - \frac{1}{k^2} \left[\frac{2b \cdot (k+p)}{[(k+p)^2-m^2]^2[(k+q)^2-m^2]} + \frac{2b \cdot (k+q)}{[(k+p)^2-m^2][(k+q)^2-m^2]^2} \right] \\
& \times [\gamma_\nu(\not{k} + \not{q} + m)\gamma_\mu \not{b} \gamma_\nu + \gamma_\nu \not{b} \gamma_\mu(\not{k} + \not{p} + m)\gamma_\nu] \\
& - \frac{1}{k^2} \left[\frac{b^2}{[(k+p)^2-m^2]^2[(k+q)^2-m^2]} + \frac{b^2}{[(k+p)^2-m^2][(k+q)^2-m^2]^2} \right] \\
& \times [\gamma_\nu(\not{k} + \not{q} + m)\gamma_\mu(\not{k} + \not{p} + m)\gamma_\nu] \\
& + \frac{4}{k^2} \left[\frac{[b \cdot (k+p)]^2}{[(k+p)^2-m^2]^3[(k+q)^2-m^2]} + \frac{[b \cdot (k+q)]^2}{[(k+p)^2-m^2][(k+q)^2-m^2]^3} \right. \\
& \left. + \frac{[b \cdot (k+p)][b \cdot (k+q)]}{[(k+p)^2-m^2]^2[(k+q)^2-m^2]^2} \right] [\gamma_\nu(\not{k} + \not{q} + m)\gamma_\mu(\not{k} + \not{p} + m)\gamma_\nu] \Big\} u(p). \tag{46}
\end{aligned}$$

After Feynman parameterization and the momentum integration, we get

$$\begin{aligned}
\textcircled{1} = & \bar{u}(q) \left(\frac{e^2}{8\pi^2} b^2 \gamma_\mu \int_0^1 dx \frac{x^2 - 2x - 1}{m^2 - l^2 x(1-x)} - \frac{i}{4\pi^2} b_\mu \not{b} \int_0^1 dx \frac{x^2 + 2x - 1}{m^2 - l^2 x(1-x)} \right. \\
& + \frac{e^2}{4\pi^2} \int_0^1 dx \frac{mb_\mu(p+q)\cdot b}{[m^2 - l^2 x(1-x)]^2} \left\{ x^2 - 3x + 2x \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} \right. \right. \\
& \left. \left. + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + \gamma \right] \right\} \\
& + \frac{e^2}{4\pi^2} \gamma_\mu \int_0^1 dx \frac{1}{[m^2 - l^2 x(1-x)]^2} \left\{ -\frac{1}{2} [b \cdot (p+q)]^2 (x^2 + 6x) \right. \\
& - \frac{1}{2} (l \cdot b)^2 (x - 4x^2 - 2x^3 + 4x^4) - m^2 b^2 (1 + x^2) + \frac{1}{2} l^2 b^2 (x^2 + x + 1) \\
& + \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \left[\frac{7}{4} x(p \cdot b + q \cdot b)^2 + \frac{1}{4} (x - 2x^2)(l \cdot b)^2 + m^2 b^2 (1 - x)^2 \right. \\
& \left. + \frac{1}{2} l^2 b^2 (-1 + x - x^2) \right] + \left[\ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right] \left[\frac{9}{4} x(p \cdot b + q \cdot b)^2 \right. \\
& \left. + m^2 b^2 (1 - x)^2 + \frac{1}{2} l^2 b^2 (-1 + x - x^2) + \frac{1}{4} (l \cdot b)^2 (2x^2 - x) \right] \right\} \\
& + \frac{e^2}{4\pi^2} \int_0^1 dx \frac{\not{b}}{[m^2 - l^2 x(1-x)]^2} \left\{ b \cdot (p+q) (p_\mu + q_\mu) \frac{1}{2} (x^2 + 4x) \right. \\
& + \frac{1}{2} l \cdot b l_\mu (-2x + 5x^2 - 4x^3 + 4x^4) + \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + \gamma \right] \\
& \times \left[-\frac{5}{2} (p+q) \cdot b (p_\mu + q_\mu) x + \frac{1}{2} (x - 2x^2) l \cdot b l_\mu \right] \right\} \\
& + \frac{e^2}{8\pi^2} \int_0^1 dx \frac{mb^2(p_\mu + q_\mu)}{[m^2 - l^2 x(1-x)]^2} (x^2 - 2x) \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{m}{[m^2 - l^2 x(1-x)]^3} \left\{ l_\mu l \cdot b (p+q) \cdot b \frac{1}{2} (3x - 7x^2 + 12x^3 - 20x^4) \right. \\
& + (p_\mu + q_\mu) \left[(p \cdot b + q \cdot b)^2 (-4x + 3x^2) + (l \cdot b)^2 \frac{1}{2} (x + 3x^2 - 6x^4) \right] \\
& + \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + 2\gamma \right] \left[\frac{1}{2} l_\mu l \cdot b (p+q) \cdot b (-x + 4x^2 - 5x^3 \right. \\
& \left. + 3x^4) \right] + \frac{1}{4} (p_\mu + q_\mu) \left[(p \cdot b + q \cdot b)^2 (2x - x^2 - x^3 + x^4) + (l \cdot b)^2 (x^2 - x^3 + x^4) \right] \right\} \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{\gamma_\mu}{[m^2 - l^2 x(1-x)]^3} \left\{ \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + \gamma \right] \right. \\
& \times \frac{1}{4} (2m^2 - l^2) x(x-1) \left[(p \cdot b + q \cdot b)^2 + (2x-1)^2 (l \cdot b)^2 \right] \\
& + (p \cdot b + q \cdot b)^2 \left[-\frac{1}{2} m^2 (19x + 7x^2) + \frac{3}{4} l^2 (5x - 6x^2) \right] \\
& \left. + (l \cdot b)^2 \left[\frac{1}{2} m^2 (9x - 7x^2 - 12x^4) + \frac{1}{4} l^2 (-9x + 13x^2 - 4x^3 + 12x^4) \right] \right\} \Big) u(p), \quad (47)
\end{aligned}$$

where we have used

$$\begin{aligned}
p_\mu(p \cdot b)^2 + q_\mu(q \cdot b)^2 &= \frac{1}{2} \left\{ (p_\mu + q_\mu) \left[(p \cdot b)^2 + (q \cdot b)^2 \right] + l_\mu l \cdot b (p + q) \cdot b \right\}; \\
p_\mu(q \cdot b)^2 + q_\mu(p \cdot b)^2 &= \frac{1}{2} \left\{ (p_\mu + q_\mu) \left[(p \cdot b)^2 + (q \cdot b)^2 \right] - l_\mu l \cdot b (p + q) \cdot b \right\}; \\
p \cdot b p_\mu + q \cdot b q_\mu &= \frac{1}{2} [(p_\mu + q_\mu)(p + q) \cdot b + l_\mu l \cdot b]; \\
p \cdot b q_\mu + q \cdot b p_\mu &= \frac{1}{2} [(p_\mu + q_\mu)(p + q) \cdot b - l_\mu l \cdot b]; \\
(p \cdot b)^2 + (q \cdot b)^2 &= \frac{1}{2} [(p \cdot b + q \cdot b)^2 + (l \cdot b)^2]; \\
p \cdot b q \cdot b &= \frac{1}{4} [(p \cdot b + q \cdot b)^2 - (l \cdot b)^2].
\end{aligned} \tag{48}$$

Eqs. (42)–(47) give the contribution to the one-loop on-shell vertex to second order in b ,

$$\begin{aligned}
&\bar{u}(q)\Lambda_\mu^{(2)}(p, q)u(p) = \\
&\bar{u}(q) \left(\frac{e^2}{8\pi^2} \int_0^1 dx \frac{1}{m^2 - l^2 x(1-x)} \left[b^2 \gamma_\mu(x^2 - 2x - 1) - 2b_\mu \not{b}(x^2 + 2x - 1) \right] \right. \\
&+ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{mb_\mu(p+q) \cdot b}{[m^2 - l^2 x(1-x)]^2} \left\{ 7x^2 - 5x + (9x - 3x^2) \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} \right. \right. \\
&+ \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + \gamma \left. \right] \left. \right\} \\
&+ \frac{e^2}{4\pi^2} \gamma_\mu \int_0^1 dx \frac{1}{[m^2 - l^2 x(1-x)]^2} \left\{ -\frac{1}{2} [b \cdot (p+q)]^2 (x^2 + 6x) \right. \\
&- \frac{1}{2} (l \cdot b)^2 (x - 4x^2 - 2x^3 + 4x^4) - m^2 b^2 (1 + x^2) + \frac{1}{2} l^2 b^2 (x^2 + x + 1) \\
&+ \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \left[\frac{7}{4} x(p \cdot b + q \cdot b)^2 + \frac{1}{4} (x - 2x^2)(l \cdot b)^2 + m^2 b^2 (1 - x)^2 \right. \\
&+ \frac{1}{2} l^2 b^2 (-1 + x - x^2) \left. \right] + \left[\ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right] \left[\frac{9}{4} x(p \cdot b + q \cdot b)^2 \right. \\
&+ m^2 b^2 (1 - x)(1 - 3x) + \frac{1}{2} l^2 b^2 (-1 + x - x^2) + \frac{1}{4} (l \cdot b)^2 (2x^2 - x) \left. \right] \left. \right\} \\
&+ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{\not{b}}{[m^2 - l^2 x(1-x)]^2} \left\{ b \cdot (p+q)(p_\mu + q_\mu) \frac{1}{2} (x^2 + 4x) \right. \\
&+ \frac{1}{2} l \cdot b l_\mu (-2x + 5x^2 - 4x^3 + 4x^4) + m^2 b_\mu (6x^2 - 10x) + \left[\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} \right. \\
&+ \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + \gamma \left. \right] \left[-\frac{5}{2} (p+q) \cdot b (p_\mu + q_\mu) x + \frac{1}{2} (x - 2x^2) l \cdot b l_\mu \right] \left. \right\} \\
&+ \frac{e^2}{4\pi^2} \int_0^1 dx \frac{mb^2(p_\mu + q_\mu)}{[m^2 - l^2 x(1-x)]^2} \left\{ -3x + \left(\frac{x^2}{2} - x \right) \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \right. \\
&+ \left[\ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right] \left(-\frac{x^2}{2} - x \right) \left. \right\} \\
&+ \frac{e^2}{2\pi^2} \int_0^1 dx \frac{ml_\mu l \cdot b}{[m^2 - l^2 x(1-x)]^3} \left[\frac{1}{2} (p+q) \cdot b (3x + x^2 - 4x^3 - 12x^4) \right]
\end{aligned}$$

$$\begin{aligned}
& + 2m\cancel{b}x(1-x)(1-2x)^2 \Big] \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{m(p_\mu + q_\mu)}{[m^2 - l^2 x(1-x)]^3} \left\{ (p \cdot b + q \cdot b)^2 (-4x + 3x^2) + \frac{(l \cdot b)^2}{2}(x + 11x^2 \right. \\
& \quad \left. - 16x^3 + 2x^4) + l^2 b^2 x^2 (1-x)(7-2x) - 9m^2 b^2 x(1-x) \right. \\
& \quad \left. + \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + 2\gamma \right] \left[\frac{1}{4}(p \cdot b + q \cdot b)^2 (2x - x^2 - x^3 + x^4) \right. \right. \\
& \quad \left. \left. + \frac{1}{4}(l \cdot b)^2 (x^2 - x^3 + x^4) + \frac{1}{2}l^2 b^2 x^2 (1-x) - m^2 b^2 x(1-x) \right] \right\} \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{\gamma_\mu}{[m^2 - l^2 x(1-x)]^3} \left\{ \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + 2\gamma \right] \right. \\
& \quad \times \frac{1}{4}(2m^2 - l^2)x(x-1) \left[(p \cdot b + q \cdot b)^2 + (2x-1)^2(l \cdot b)^2 \right] \\
& \quad + (p \cdot b + q \cdot b)^2 \left[-\frac{1}{2}m^2(15x + 11x^2) + \frac{3}{4}l^2(5x - x^2) \right] \\
& \quad + (l \cdot b)^2 \left[\frac{1}{2}m^2(17x - 31x^2 + 32x^3 - 28x^4) + \frac{1}{4}l^2(l \cdot b)^2(-9x + 13x^2 - 4x^3 + 12x^4) \right] \\
& \quad \left. + m^2 b^2 \left[4m^2(3x - 7x^2 + x^3) + l^2(-2x + 6x^2 - x^3 + 2x^4) \right] \right\} \\
& + \frac{e^2}{2\pi^2} \int_0^1 dx \frac{mb_\mu(p+q) \cdot b}{[m^2 - l^2 x(1-x)]^3} \left\{ (8m^2 + l^2) + (4m^2 - l^2) \left[\frac{2}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} \right. \right. \\
& \quad \left. + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) + 2\gamma \right] \right\} \\
& + \frac{e^2}{\pi^2} \int_0^1 dx \frac{m^2 \cancel{b}x(1-x)}{[m^2 - l^2 x(1-x)]^2} \left[(2l^2 - 12m^2)b_\mu - 2l \cdot bl_\mu \right] u(p). \tag{49}
\end{aligned}$$

D. On-shell Vertex Renormalization and Radiative Correction

With the results given in Eqs. (34), (38) and (49), and employing the Gordon identity (6), we finally get the one-loop on-shell quantum vertex of QED with the Lorentz and *CPT* violation term in the fermionic sector to the second order in b_μ ,

$$\Lambda_\mu(p, q, b) = \Lambda_\mu^{(0)} + \Lambda_\mu^{(1)} + \Lambda_\mu^{(2)}. \tag{50}$$

The UV divergence only arises in the conventional QED part. To be consistent with the physical results of conventional QED, we define the vertex renormalization and its radiative correction in the following way

$$\Lambda_\mu = \gamma_\mu \left(Z_1^{-1} - 1 \right) + Z_1^{-1} \Lambda_\mu^R, \tag{51}$$

and the renormalization condition on the radiative correction part is

$$\Lambda_\mu^R|_{\cancel{q}=\cancel{p}=m, l^2=0, b_\mu=0} = 0. \tag{52}$$

Thus the vertex renormalization constant Z_1 is the same as the conventional QED [15],

$$Z_1 = 1 - \frac{e^2}{4\pi} \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{2}{\epsilon_{\text{IR}}} + 3 \ln \frac{4\pi}{m^2} - 4 - 3\gamma \right). \quad (53)$$

The radiative correction then consists of the conventional QED part and $\Lambda_\mu^{(1)}$, $\Lambda_\mu^{(2)}$, listed in (38) and (49),

$$\begin{aligned} \Gamma_\mu^R = & \frac{\alpha}{4\pi} \gamma_\mu \int_0^1 dx \left\{ 2 \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) - 2 \ln \frac{4\pi\mu^2}{m^2} + 3 - \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right. \\ & - \frac{2m^2 - l^2}{m^2 - l^2 x(1-x)} \left[\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{4\pi\mu^2}{m^2} + \ln \left(1 - \frac{l^2}{m^2} x(1-x) \right) \right] \\ & \left. + \frac{l^2 x(1-x) - 6m^2 x}{m^2 - l^2 x(1-x)} \right\} + \frac{\alpha}{2\pi} i \sigma_{\mu\nu} m l^\nu \int_0^1 dx \frac{x}{m^2 - l^2 x(1-x)} + \Lambda_\mu^{(1)} + \Lambda_\mu^{(2)}. \end{aligned} \quad (54)$$

In the vicinity of $l^2 = 0$, we have the radiative correction to the leading order of l^2 ,

$$\begin{aligned} \Gamma_\mu^R = & \frac{\alpha}{4\pi} \left\{ \gamma_\mu \left[\frac{2}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{m^2}{4\pi\mu^2} \right) - \frac{1}{2} \right] \frac{l^2}{m^2} + \frac{i\sigma_{\mu\nu} l^\nu}{m} \left(1 + \frac{1}{6} \frac{l^2}{m^2} \right) \right\} \\ & + \frac{\alpha}{m^2} \left\{ (p+q) \cdot b \gamma_\mu \left[-\frac{3}{4} - \frac{5}{8} \frac{l^2}{m^2} + \left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) \left(-\frac{3}{2} + \frac{5}{12} \frac{l^2}{m^2} \right) \right] \right. \\ & + (p_\mu + q_\mu) \not{b} \left[\frac{7}{4} + \frac{3}{8} \frac{l^2}{m^2} - \frac{1}{3} \frac{l^2}{m^2} \left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) \right] + m[\not{b}, \gamma_\mu] \left[\frac{3}{8} - \frac{5}{16} \frac{l^2}{m^2} \right. \\ & \left. + \left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) \left(\frac{3}{4} + \frac{1}{24} \frac{l^2}{m^2} \right) \right] - \left(\frac{1}{4} + \frac{1}{24} \frac{l^2}{m^2} \right) m b_\mu \right\} \gamma_5 \\ & + \frac{\alpha}{\pi} \gamma_\mu \left\{ -\frac{41}{6} \frac{b^2}{m^2} - \frac{77}{6} \frac{(p \cdot b + q \cdot b)^2}{m^4} + \frac{l^2}{m^2} \left[\frac{127}{120} \frac{b^2}{m^2} - \frac{8}{3} \frac{(p \cdot b + q \cdot b)^2}{m^4} \right] + \frac{5}{6} \frac{(l \cdot b)^2}{m^4} \right. \\ & + \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \left[\frac{13}{24} \frac{(p \cdot b + q \cdot b)^2}{m^4} + \frac{1}{3} \frac{b^2}{m^2} + \frac{l^2}{m^2} \left(\frac{31}{120} \frac{(p \cdot b + q \cdot b)^2}{m^4} - \frac{19}{60} \frac{b^2}{m^2} \right) \right. \\ & \left. - \frac{13}{120} \frac{(l \cdot b)^2}{m^4} \right] + \ln \frac{m^2}{4\pi\mu^2} \left[\frac{23}{24} \frac{(p \cdot b + q \cdot b)^2}{m^4} + \frac{l^2}{m^2} \left(\frac{43}{120} \frac{(p \cdot b + q \cdot b)^2}{m^4} - \frac{9}{20} \frac{b^2}{m^2} \right) \right. \\ & \left. + \frac{1}{120} \frac{(l \cdot b)^2}{m^4} \right] \left. \right\} + \frac{\alpha}{\pi} \not{b} \left\{ -\frac{34}{3} \frac{b_\mu}{m} + \frac{(p+q) \cdot b (p_\mu + q_\mu)}{m^3} \left[\frac{7}{6} - \frac{5}{4} \left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) \right] \right. \\ & + \frac{l^2}{m^2} \left[-\frac{403}{60} \frac{b_\mu}{m} + \frac{(p+q) \cdot b (p_\mu + q_\mu)}{m^3} \left(\frac{71}{120} - \frac{5}{12} \left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) \right) - \frac{29}{30} \frac{l \cdot b l_\mu}{m^3} \right] \left. \right\} \\ & + \frac{\alpha}{\pi} \frac{(p+q) \cdot b b_\mu}{m^3} \left[\frac{5}{2} + \frac{37}{6} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{29}{6} \ln \frac{m^2}{4\pi\mu^2} \right] + \frac{\alpha}{\pi} \frac{l_\mu l \cdot b}{m^3} \left(-\frac{47}{30} \frac{(p+q) \cdot b}{m^3} \right) \\ & + \frac{\alpha}{\pi} \frac{p_\mu + q_\mu}{m} \left\{ -\frac{9}{2} \frac{b^2}{m^2} - 2 \frac{(p \cdot b + q \cdot b)^2}{m^4} + \frac{l^2}{m^2} \left[-\frac{33}{40} \frac{b^2}{m^2} - \frac{323}{280} \frac{(p \cdot b + q \cdot b)^2}{m^4} \right] \right. \\ & + \frac{17}{30} \frac{(l \cdot b)^2}{m^4} + \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \left[-\frac{1}{3} \frac{b^2}{m^2} + \frac{37}{120} \frac{(p \cdot b + q \cdot b)^2}{m^4} + \frac{l^2}{m^2} \left(\frac{9}{56} \frac{(p \cdot b + q \cdot b)^2}{m^4} \right. \right. \\ & \left. \left. - \frac{11}{60} \frac{b^2}{m^2} \right) + \frac{17}{120} \frac{(l \cdot b)^2}{m^4} \right] + \ln \frac{m^2}{4\pi\mu^2} \left[-\frac{4}{3} \frac{b^2}{m^2} + \frac{37}{120} \frac{(p \cdot b + q \cdot b)^2}{m^4} \right. \\ & \left. \left. + \frac{l^2}{m^2} \left(-\frac{1}{4} \frac{b^2}{m^2} + \frac{9}{56} \frac{(p \cdot b + q \cdot b)^2}{m^4} \right) + \frac{17}{120} \frac{(l \cdot b)^2}{m^4} \right] \right\}. \end{aligned} \quad (55)$$

IV. ANOMALOUS MAGNETIC MOMENT

Applying the Gordon identities (6) and (7) in the radiative corrections given in Eq. (55), we see that in addition to the one arising in the conventional QED, there is a contribution to the anomalous magnetic moment from the terms $mb^2(p_\mu + q_\mu)$, $m(p_\mu + q_\mu)(l \cdot b)^2$ and $m(p_\mu + q_\mu)l^2b^2$ in $\Lambda_\mu^{(2)}$. The part of the interaction Hamiltonian coming from the anomalous magnetic moment of a charged spinning particle with a slowing varying external magnetic field is thus

$$\begin{aligned} \Delta H = e & \left\{ \frac{1}{4m} \frac{\alpha}{2\pi} + \frac{b^2}{2m^3} \frac{\alpha}{\pi} \left[\frac{9}{2} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{4}{3} \ln \frac{m^2}{4\pi\mu^2} \right] \right\} \int d^3x \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_c^{\mu\nu} \\ & - e \frac{\alpha}{\pi} \frac{b^2}{2m^5} \left[\frac{33}{40} + \frac{11}{60} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{1}{4} \ln \frac{m^2}{4\pi\mu^2} \right] \int \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) \partial^2 F_c^{\mu\nu} \\ & + e \frac{\alpha}{\pi} \frac{1}{2m^5} \left[\frac{17}{30} + \frac{17}{120} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{m^2}{4\pi\mu^2} \right) \right] \int \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) (b \cdot \partial)^2 F_c^{\mu\nu}(x), \end{aligned} \quad (56)$$

where $F_{c\mu\nu} = \partial_\mu A_{c\nu} - \partial_\nu A_{c\mu}$ and $A_{c\mu}$ is the classical electromagnetic potential. Choosing F to be a constant magnetic field, $B_i = -B^i = \epsilon_{ijk}F^{jk}/2$, and using $\sigma_{ij} = \epsilon_{ijk}\sigma^k$, we get the magnetic dipole energy contributed by the anomalous magnetic moment

$$-\mathbf{B} \cdot \boldsymbol{\mu} = -\mathbf{B} \cdot \left\{ \frac{e}{2m} \frac{\alpha}{\pi} \left[\frac{1}{2} + \frac{b^2}{m^2} \left(\frac{9}{2} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{4}{3} \ln \frac{m^2}{4\pi\mu^2} \right) \right] 2 \int d^3x \bar{\psi}(x) \frac{\sigma}{2} \psi(x) \right\}. \quad (57)$$

Thus the modification to the gyromagnetic ration by the quantum correction is

$$a = \frac{1}{2}(g - 2) = \frac{\alpha}{\pi} \left[\frac{1}{2} + \frac{b^2}{m^2} \left(\frac{9}{2} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{4}{3} \ln \frac{m^2}{4\pi\mu^2} \right) \right]. \quad (58)$$

Comparing with the general result in conventional QED, one can see that there arises additional contributions stemming from the Lorentz and *CPT* violation. This result makes us embarrassed about the physical validity of introducing a Lorentz and *CPT* violating term in the fermionic sector of QED and hence the radiatively induced Lorentz and *CPT* violation. The anomalous magnetic moment is a measurable physical quantity and it will yield an effective interaction Hamiltonian of QED. There is no mechanism to cancel the IR divergence in the anomalous magnetic moment terms. In conventional QED, the anomalous magnetic moment is completely free from IR divergence and hence gives a physically reasonable result. Thus this seems to strongly suggest that the way of constructing a QED model with Lorentz and *CPT* violation by directly adding an explicit Lorentz and *CPT* term breaking term should be abandoned. In the following sections, we shall calculate the Lamb shift, the prediction on which is another important achievement of QED, to see whether the IR divergence it contains can be canceled like in the conventional case, i.e., whether it can be canceled by the IR divergence contributed from bremsstrahlung.

V. LAMB SHIFT

A. Radiative Correction to Classical Coulomb Potential

Another well known achievement of QED is the precise correspondence between the theoretical prediction and experimental measurement of Lamb shift. Theoretically, the Lamb shift arises from the modification to the classical Coulomb interaction of the radiative correction. The interaction of the electron with the classical Coulomb potential produced from a nuclear of charge $-Ze$ is

$$V(r) = e\bar{\psi}(x)\gamma_0\psi(x)A_{\text{cl}}^0(x) = -Ze\frac{\psi^\dagger(x)\psi(x)}{4\pi r}, \quad r = |\mathbf{x}|, \quad (59)$$

and in momentum space it is written as

$$V = Ze\frac{\bar{u}\gamma_0 u}{|\mathbf{l}|^2} = Ze\frac{u^\dagger u}{|\mathbf{l}|^2}. \quad (60)$$

Thus to calculate the Lamb shift we need to consider all possible radiative corrections to the tree level vertex $\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu$. As shown in Fig. 1, this includes not only the vertex radiative correction from the 1PI part, but also the self-energy in the fermionic external lines and the polarization tensor in the external photon line. However, since in the on-shell renormalization schemes, the quantum correction of the fermionic self-energy takes the following form

$$\Sigma(p) = \Sigma(m) + (\not{p} - m)B(m) + (\not{p} - m)^2C(p, m). \quad (61)$$

The first term and the second one contribute to the mass renormalization and wave function renormalization of the electron, and are canceled by the corresponding counterterms. The radiative correction $\Sigma_R(p)$ of the fermionic self-energy is proportional to $(\not{p} - m)^2$, thus the contribution to the quantum vertex from the diagrams with the self-energy insertion in the fermionic external lines vanishes since the amplitude is read as

$$\bar{u}(q)\left[\Sigma_R(q)\frac{1}{\not{q} - m}\gamma_\mu + \gamma_\mu\frac{1}{\not{p} - m}\Sigma_R(p)\right]u(p). \quad (62)$$

Therefore, we only consider the contribution to the Lamb shift from on-shell vertex correction and the polarization tensor insertion in the external photon field, the radiative correction to the classical interaction vertex, according to Fig. 1, is thus read as

$$e\bar{u}(q)\gamma_\mu u(p) \longrightarrow e\bar{u}(q)\left[\gamma_\mu + \gamma^\nu D_{\nu\mu}^{(1)}(l) + \Lambda_\mu\right]u(p). \quad (63)$$

The second term of (63) comes from the insertion of vacuum polarization tensor, in the static case and Feynman gauge, $l^2 = -\mathbf{l}^2$, the observed electric charge due to the screening of vacuum polarization is

$$\frac{e}{1 + \omega_R(\mathbf{l}, b)} \simeq e \left[1 + \left(\frac{1}{15}\frac{\alpha}{\pi} + \frac{4}{15}\frac{\alpha}{\pi}\frac{b^2}{m^2} \right) \frac{\mathbf{l}^2}{m^2} + \frac{4}{15}\frac{\alpha}{\pi}\frac{(\mathbf{l}\cdot\mathbf{b})^2}{m^4} \right]. \quad (64)$$

In configuration space of an infinitely heavy nucleus of charge $-Ze$ located at the origin, the Coulomb potential is $A_{\text{cl}\mu} = -g_{0\mu}Ze/(4\pi r)$. The modification of electric charge screening to the classical Coulomb potential is thus

$$\begin{aligned}\Delta V_{\text{eff}}^{(1)}(r) &= - \left[\left(\frac{1}{15} \frac{\alpha}{\pi} + \frac{4}{15} \frac{\alpha}{\pi} \frac{b^2}{m^2} \right) \frac{\nabla^2}{m^2} + \frac{4}{15} \frac{\alpha}{\pi} \frac{(\nabla \cdot \mathbf{b})^2}{m^4} \right] A_{c\mu} \\ &= - \left\{ \frac{\alpha}{15\pi} \frac{Ze^2}{m^2} \delta^{(3)}(\mathbf{r}) + \frac{4\alpha}{15\pi} \frac{Ze^2 \mathbf{b}^2}{m^4} \delta^{(3)}(\mathbf{r}) + \frac{4\alpha}{15\pi} \frac{1}{m^4} \frac{Ze^2}{4\pi r} \left[\frac{\mathbf{b}^2}{r^2} - 3 \frac{(\mathbf{b} \cdot \mathbf{r})^2}{r^4} \right] \right\} \delta_{\mu 0}. \quad (65)\end{aligned}$$

The effects from the on-shell vertex radiative correction are quite complicated. Eq. (55) shows that only some of the terms proportional to γ_μ and $p_\mu + q_\mu$ contribute to Lamb shift. The corresponding contribution to the second term of (63) is

$$\begin{aligned}& \frac{\alpha}{\pi} \bar{u}(q) \gamma_\mu \left\{ \left[\frac{2}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \ln \frac{m^2}{4\pi\mu^2} + \gamma \right) - \frac{1}{2} \right] \frac{l^2}{4m^2} - \frac{581}{60} \frac{b^2}{m^2} - \frac{71}{120} \frac{l^2 b^2}{m^4} \right. \\ &+ \frac{59}{30} \frac{(l \cdot b)^2}{m^4} + \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \left[-\frac{1}{3} \frac{b^2}{m^2} - \frac{41}{60} \frac{l^2 b^2}{m^4} + \frac{7}{40} \frac{(l \cdot b)^2}{m^4} \right] \\ &+ \ln \frac{m^2}{4\pi\mu^2} \left[-\frac{8}{3} \frac{b^2}{m^2} - \frac{19}{20} \frac{l^2 b^2}{m^4} + \frac{7}{24} \frac{(l \cdot b)^2}{m^4} \right] \left. \right\} u(p) \\ &+ \frac{\alpha}{\pi} \bar{u}(q) \frac{i\sigma_{\mu\nu} l^\nu}{m} \left\{ \frac{1}{4} \left(1 + \frac{1}{6} \frac{l^2}{m^2} \right) + \frac{9}{2} \frac{b^2}{m^2} + \frac{33}{40} \frac{l^2 b^2}{m^4} - \frac{17}{30} \frac{(l \cdot b)^2}{m^4} + \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \right. \\ &\times \left[\frac{1}{3} \frac{b^2}{m^2} + \frac{11}{60} \frac{l^2 b^2}{m^4} - \frac{17}{120} \frac{(l \cdot b)^2}{m^4} \right] + \ln \frac{m^2}{4\pi\mu^2} \left[\frac{4}{3} \frac{b^2}{m^2} + \frac{1}{4} \frac{l^2 b^2}{m^4} - \frac{17}{120} \frac{(l \cdot b)^2}{m^4} \right] \left. \right\} u(p). \quad (66)\end{aligned}$$

In the static case, $l^2 = -\mathbf{l}^2$, with the replacement $\mathbf{l} = -i\nabla$, Eq. (66) leads to the following correction to the Coulomb potential,

$$\begin{aligned}\Delta V_{\text{eff}}^{(2)} &= \Delta V_{\text{eff}}^{(2)\prime} + \Delta V_{\text{eff}}^{(2)\prime\prime}, \\ \Delta V_{\text{eff}}^{(2)\prime} &= \frac{Z\alpha^2}{\pi} \frac{b^2}{m^2} \left[\frac{581}{60} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{8}{3} \ln \frac{m^2}{4\pi\mu^2} \right] \frac{1}{r} + 4Z\alpha^2 \left\{ \frac{1}{6m^2} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right. \right. \\ &+ \ln \frac{m^2}{4\pi\mu^2} \left. \right) - \frac{1}{8m^2} - \frac{b^2}{m^4} \left[\frac{71}{120} + \frac{41}{60} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{19}{20} \ln \frac{m^2}{4\pi\mu^2} \right] \left. \right\} \delta^{(3)}(\mathbf{x}) \\ &+ \frac{Z\alpha^2}{\pi} \frac{1}{m^4} \left[\frac{59}{30} + \frac{7}{40} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{7}{24} \ln \frac{m^2}{4\pi\mu^2} \right] \frac{3(\mathbf{x} \cdot \mathbf{b})^2 - r^2 \mathbf{b}^2}{r^5}; \\ \Delta V_{\text{eff}}^{(2)\prime\prime} &= \frac{ie\alpha}{\pi m} \left\{ \frac{1}{4} \left(1 + \frac{1}{6} \frac{\nabla^2}{m^2} \right) + \frac{b^2}{m^2} \left(\frac{9}{2} + \frac{33}{40} \frac{\nabla^2}{m^2} \right) + \frac{17}{30} \frac{(\mathbf{b} \cdot \nabla)^2}{m^4} \right. \\ &+ \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \left[\frac{b^2}{m^2} \left(\frac{1}{3} + \frac{11}{60} \frac{\nabla^2}{m^2} \right) + \frac{17}{120} \frac{(\mathbf{b} \cdot \nabla)^2}{m^4} \right] \\ &+ \ln \frac{m^2}{4\pi\mu^2} \left[\frac{b^2}{m^2} \left(\frac{4}{3} + \frac{1}{4} \frac{\nabla^2}{m^2} \right) + \frac{17}{120} \frac{(\mathbf{b} \cdot \nabla)^2}{m^4} \right] \left. \right\} [\gamma \cdot \mathbf{E}(r)], \quad (67)\end{aligned}$$

where $\mathbf{E} = -\nabla A_0 = -Ze/(4\pi)\mathbf{x}/r^3$ is the static Coulomb electric field. The term with electric field implies that the anomalous magnetic moment induces an electric dipole moment for a moving electron. Writing the electron spinor wave function in two-component form, $\psi = (\varphi, \chi)^T$, $\psi^\dagger = (\varphi^\dagger, \chi^\dagger)$, and using the Pauli's non-relativistic approximation [17], the large two-component φ and the small one χ being related by

$$\chi(r) = -i \frac{\sigma \cdot \nabla}{2m} \varphi(r), \quad (68)$$

the anomalous magnetic part can be re-written as the form with the spin-orbit angular momentum interaction,

$$\begin{aligned} \Delta V_{\text{eff}}^{(2)\prime\prime} &= \frac{Z\alpha^2}{2\pi m^2} \left\{ \frac{1}{4} + \frac{b^2}{m^2} \left[\frac{9}{2} + \frac{1}{3} \left(1 + \frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{4}{3} \ln \frac{m^2}{4\pi\mu^2} \right] \right\} \left[4\pi\delta^{(3)}(\mathbf{x}) + 4\frac{\mathbf{S} \cdot \mathbf{L}}{r^3} \right] \\ &\quad + \frac{2Z\alpha^2}{m^4} \left\{ \frac{1}{24} + \frac{b^2}{m^2} \left[\frac{33}{40} + \frac{11}{60} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{1}{4} \ln \frac{m^2}{4\pi\mu^2} \right] \right\} \\ &\quad \times \left\{ \nabla^2 \delta^{(3)}(\mathbf{x}) - 2i\mathbf{S} \cdot [\nabla \delta^{(3)}(\mathbf{x}) \times \nabla] \right\} \\ &\quad + \frac{Z\alpha^2}{2\pi m^6} \left\{ 4\pi (\mathbf{b} \cdot \nabla)^2 \delta^{(3)}(\mathbf{x}) + 12 \left[\frac{5(\mathbf{b} \cdot \mathbf{x})^2}{r^7} - \frac{b^2}{r^5} \right] \mathbf{L} \cdot \mathbf{S} \right. \\ &\quad \left. - 24 \frac{i(\mathbf{x} \cdot \mathbf{b}) \mathbf{b} \cdot (\mathbf{S} \times \nabla)}{r^5} \right\} \left[\frac{17}{30} + \frac{17}{120} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{m^2}{4\pi\mu^2} \right) \right]. \end{aligned} \quad (69)$$

where $\mathbf{S} = \sigma/2$, $\mathbf{L} = i\mathbf{x} \times \nabla$. Note that the spin-orbit interaction arises from the anomalous magnetic moment. In deriving (69) we have used

$$\begin{aligned} \psi^\dagger \frac{\gamma \cdot \mathbf{x}}{r^3} \psi &= \varphi^\dagger \frac{\sigma \cdot \mathbf{x}}{r^3} \chi - \chi^\dagger \frac{\sigma \cdot \mathbf{x}}{r^3} \varphi = \frac{1}{2im} \varphi^\dagger \left[\frac{\sigma \cdot \mathbf{x}}{r^3}, \sigma \cdot \nabla \right] \varphi, \\ [\sigma \cdot \mathbf{A}, \sigma \cdot \mathbf{B}] &= \mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A} + i\sigma (\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A}). \end{aligned} \quad (70)$$

B. Energy Level Shift of Hydrogen-Like Atom due to Radiative Correction

In the following we consider the energy-level shift in the hydrogen-like atom due to the radiative correction to the classical Coulomb potential. Let us first look at the displacement of the energy-level due to the screening of charge implied by the vacuum polarization. First, the δ -potential of Eq. (65) will lead to a displacement of the s -state energy level,

$$\begin{aligned} \delta'_1 E_{n,l} &= - \left(\frac{Z\alpha}{15\pi} \frac{e^2}{m^2} + \frac{4Z\alpha}{15\pi} \frac{e^2 \mathbf{b}^2}{m^4} \right) \int d^3 r \psi_{n,l}^*(\mathbf{r}) \delta^{(3)}(\mathbf{r}) \psi_{n,l}(\mathbf{r}) \\ &= - \left(\frac{Z\alpha}{15\pi} \frac{e^2}{m^2} + \frac{4Z\alpha}{15\pi} \frac{e^2 \mathbf{b}^2}{m^4} \right) \delta_{l,0} |\psi_{n,0}|^2 = - \left(1 + \frac{4\mathbf{b}^2}{m^2} \right) \frac{4}{15\pi} \frac{Z^4 \alpha^5}{n^3} m \delta_{l,0}, \end{aligned} \quad (71)$$

it is $1 + 4\mathbf{b}^2/m^2$ times of the corresponding displacement in the conventional QED.

The more interesting effect comes from the quadrupole-like part proportional to $\mathbf{b}^2/r^3 - 3(\mathbf{b} \cdot \mathbf{r})^2/r^5$. In the lowest order approximation, the contribution of this potential to the energy-level of hydrogen-like atom is

$$\delta''_1 E_{n,l} = - \frac{4Z\alpha}{15\pi} \frac{1}{m^4} \frac{e^2}{4\pi} \int d^3 r \psi_{n,l}^*(\mathbf{r}) \left[\frac{\mathbf{b}^2}{r^3} - 3 \frac{(\mathbf{b} \cdot \mathbf{r})^2}{r^5} \right] \psi_{n,l}(\mathbf{r}). \quad (72)$$

For convenience, taking \mathbf{b} in the direction of z -axis, we then have

$$\begin{aligned}
\delta_1'' E_{n,l} &= -\frac{4}{15\pi} \frac{\mathbf{b}^2 Z \alpha^2}{m^4} \langle nl | \frac{1}{r^3} | nl \rangle + \frac{4}{5\pi} \frac{\mathbf{b}^2 Z \alpha^2}{m^4} \langle nl | \frac{1}{r^3} | nl \rangle \langle lM | \cos^2 \theta | lM \rangle \\
&= \frac{\mathbf{b}^2 Z^4 \alpha^5}{m\pi} \frac{8}{n^3(2l+1)[(2l+1)^2-1]} \left\{ -\frac{4}{15} + \frac{4}{5} \left[\frac{(l+M)(l-M)}{(2l+1)(2l-1)} \right. \right. \\
&\quad \left. \left. + \frac{(l+M+1)(l-M+1)}{(2l+1)(2l+3)} \right] \right\}, \tag{73}
\end{aligned}$$

where we have used the orthogonality and recurrence relations of spherical function

$$\begin{aligned}
\langle LM | l'M' \rangle &= \int_0^\pi d\theta \int_0^{2\pi} d\varphi Y_{lM}^*(\theta, \varphi) Y_{l'M'}(\theta, \varphi) \sin \theta = \delta_{ll'} \delta_{MM'}; \\
\cos \theta Y_{lM} &= \sqrt{\frac{(l+M)(l-M)}{(2l+1)(2l-1)}} Y_{l-1,M} + \sqrt{\frac{(l+M+1)(l-M+1)}{(2l+1)(2l+3)}} Y_{l+1,M}; \\
\langle nl | \frac{1}{r^3} | nl \rangle &= \begin{cases} 8(mZ\alpha)^3 / \{(2l+1)n^3[(2l+1)^2-1]\}, & l > 0 \\ \infty & l = 0 \end{cases}. \tag{74}
\end{aligned}$$

The energy level shift due to the screening of electric charge is thus

$$\begin{aligned}
\delta_1 E_{nl} &= \delta_1' E_{n,l} + \delta_1'' E_{n,l} \\
&= -\frac{4}{15} \frac{Z^4 \alpha^5}{\pi n^3} m \delta_{l0} - \frac{16}{15} \frac{\mathbf{b}^2 Z^4 \alpha^5}{m} \left\{ \delta_{l0} + \frac{2}{(2l+1)[(2l+1)^2-1]} \right. \\
&\quad \times \left. \left[1 - 3 \left(\frac{(l+M)(l-M)}{(2l+1)(2l-1)} + \frac{(l+M+1)(l-M+1)}{(2l+1)(2l+3)} \right) \right] \right\}, \tag{75}
\end{aligned}$$

and

$$\delta_1 E_{n0} = -\frac{4}{15} \frac{Z^4 \alpha^5}{\pi n^3} m \left(1 + \frac{4b^2}{m^2} \right). \tag{76}$$

The modification of energy level from the vertex correction can be calculated in a similar way. Since $b^2 \ll m^2$, so we only consider the correction to the order b^2/m^4 . Using Eq. (74) and the following formula

$$\langle nl | \frac{1}{r} | nl \rangle = \frac{me^2}{n^2}, \quad \langle jlM | \mathbf{S} \cdot \mathbf{L} | jlM \rangle = \frac{1}{2} \left[j(j+1) - l(l+1) - \frac{3}{4} \right], \quad j = l \pm \frac{1}{2}, \tag{77}$$

we get the shift of energy-level in hydrogen-like atom,

$$\begin{aligned}
\delta_2 E_{nlj} &= \int d^3 \mathbf{x} \psi_{nlj}^\dagger(\mathbf{x}) \Delta V_{\text{eff}}^{(2)} \psi_{nlj}^\dagger(\mathbf{x}) \\
&= \frac{4Z\alpha^3 b^2}{n^2 m} \left[\frac{581}{60} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{8}{3} \ln \frac{m^2}{4\pi\mu^2} \right] + \frac{4Z^4 \alpha^5}{\pi n^3} m \delta_{l0} \left\{ \frac{1}{6} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right. \right. \\
&\quad \left. \left. + \ln \frac{m^2}{4\pi\mu^2} \right) - \frac{1}{8} - \frac{b^2}{m^2} \left[\frac{71}{120} + \frac{41}{60} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{19}{20} \ln \frac{m^2}{4\pi\mu^2} \right] \right\} \\
&+ \frac{8Z^4 \alpha^5 b^2}{\pi n^3 m} \left[\frac{59}{30} + \frac{7}{40} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{7}{24} \ln \frac{m^2}{4\pi\mu^2} \right] \frac{1}{(2l+1)[(2l+1)^2-1]}
\end{aligned}$$

$$\begin{aligned} & \times \left[\frac{3(l+M)(l-M)}{(2l-1)(2l+1)} + \frac{3(l+M+1)(l-M+1)}{(2l+1)(2l+3)} - 1 \right] + \frac{2Z^4\alpha^5}{\pi n^3} m \left\{ \frac{1}{4} \right. \\ & \left. + \frac{b^2}{m^2} \left[\frac{9}{2} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{4}{3} \ln \frac{m^2}{4\pi\mu^2} \right] \right\} \left[\delta_{l0} + \frac{4j(j+1) - 4l(l+1) - 3}{(2l+1)[(2l+1)^2 - 1]} \right], \end{aligned} \quad (78)$$

and

$$\begin{aligned} \delta_2 E_{n0j} = & \frac{4Z\alpha^3 b^2}{n^2 m} \left[\frac{581}{60} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) + \frac{8}{3} \ln \frac{m^2}{4\pi\mu^2} \right] \\ & + \frac{2Z^4\alpha^5}{\pi n^3} m \left\{ \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{m^2}{4\pi\mu^2} \right) \right. \\ & \left. + \frac{b^2}{m^2} \left[\frac{199}{60} - \frac{31}{30} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) - \frac{17}{30} \ln \frac{m^2}{4\pi\mu^2} \right] \right\}. \end{aligned} \quad (79)$$

The total correction on the energy-level of hydrogen-like atom is given by (74) and (78).

C. Lamb Shift

From (74) and (78), one can easily calculate the energy level splitting between the states $2S_{1/2}$ and $2P_{1/2}$ of the hydrogen atom. It is well known that in Dirac's relativistic electron theory, these two states have the same energy level. The splitting due to the radiative correction of QED leads to the Lamb shift. We have

$$\begin{aligned} E_{2S_{1/2}} - E_{2P_{1/2}} = \delta E_{2S_{1/2}} - \delta E_{2P_{1/2}} = & \frac{m\alpha^5}{4\pi} \left\{ -\frac{1}{20} + \frac{1}{3} \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma + \ln \frac{m^2}{4\pi\mu^2} \right) \right. \\ & + \frac{b^2}{m} \left[\left(\frac{257}{60} - \frac{17(6M^2 - 4)}{300} \right) - \left(\frac{83}{90} + \frac{7(6M^2 - 4)}{50} \right) \left(\frac{1}{\epsilon_{\text{IR}}} + \gamma \right) \right. \\ & \left. \left. - \left(\frac{11}{90} + \frac{7(6M^2 - 4)}{30} \right) \ln \frac{m^2}{4\pi\mu^2} \right] \right\}, \quad M = \pm 1, 0. \end{aligned} \quad (80)$$

This shows that in addition to the usual Lamb shift in QED, there arises a contribution from the *CPT*-odd fermionic term in the action. It is remarkable that in the b -dependent part, the Lamb shift has a dependence on the magnetic quantum number M , which means that the Lamb shift itself also has a hyperfine structure. The cancellation or not of the IR singularity will be discussed in the following section.

VI. NON-CANCELLATION OF IR DIVERGENCE IN LAMB SHIFT

The Lamb shift given in Eq. (80) cannot be compared to the experimental measurement, since it contains IR divergent terms. Like in the conventional QED, we now consider the contribution to the form factors of the vertex correction from the bremsstrahlung processes and hope that the IR divergence can be canceled. Unfortunately, we find that there exist several serious problems which make it impossible for the accomplishment of IR divergence

cancellation. This appears to put further in doubt the legitimacy of introducing the term $\bar{\psi}\not{b}\gamma_5\psi$.

Since the cancellation of IR divergences is expected to occur at the level of physical cross sections, we consider the scattering of an electron by the static Coulomb potential produced from a nucleus with charge $-Ze$,

$$A_0(\mathbf{x}) = A_\mu(\mathbf{x})\delta_{\mu 0} = -\frac{Ze}{|\mathbf{x}|}\delta_{\mu 0} = -\delta_{\mu 0}Ze \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \frac{1}{\mathbf{k}^2}. \quad (81)$$

The corresponding tree-level (Born) differential cross section is

$$\begin{aligned} d\sigma_0 &= \frac{E_q}{|\mathbf{q}|} 2\pi\delta(E_q - E_p) \frac{d^3 q}{(2\pi)^3} \frac{Z^2 e^4}{|\mathbf{l}|^4} \frac{m^2}{E_p E_q} |M^{(0)}|^2, \\ |M^{(0)}|^2 &= \frac{1}{2} \sum |\bar{u}(q)\gamma_0 u(p)|^2. \end{aligned} \quad (82)$$

Here and in the following the sum is over both the final and initial spin polarization states of the electron, the factor $1/2$ coming from averaging over the spin polarization of the initial electron. The contribution to the cross section from the one-loop quantum corrected vertex is

$$d\sigma_1 = \frac{E_q}{|\mathbf{q}|} 2\pi\delta(E_q - E_p) \frac{d^3 q}{(2\pi)^3} \frac{Z^2 e^4}{|\mathbf{l}|^4} \frac{m^2}{E_p E_q} |M^{(1)}|^2, \quad (83)$$

where

$$\begin{aligned} |M^{(1)}|^2 &= \frac{1}{2} \sum |\bar{u}(q)\delta_{\mu 0} [\gamma_\mu + \Lambda_\mu^{(1)}] u(p)|^2; \\ \bar{u}(q)\Lambda_\mu^{(1)}(p, q, b)u(p) &= \gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}l^\nu}{m} F_2 + \gamma_\mu \gamma_5 F_3 + \frac{(p_\mu + q_\mu)\not{b}\gamma_5}{m} F_4 \\ &\quad + \frac{\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho}b^\nu}{m} F_5 + \frac{b_\mu}{m} \gamma_5 F_6 + \frac{\not{b}b_\mu}{m^2} F_7 + \frac{\not{b}(p_\mu + q_\mu)(p + q) \cdot b}{m^3} F_8 \\ &\quad + \frac{l \cdot b(p + q) \cdot b l_\mu \not{b}}{m^6} F_9 + \frac{(p + q) \cdot b b_\mu}{m^3} F_{10} + \frac{l_\mu l \cdot b(p + q) \cdot b}{m^5} F_{11}, \end{aligned} \quad (84)$$

$F_i \equiv F_i[l^2, b^2, (p + q) \cdot b]$ ($i = 1, \dots, 11$) near $l^2 = 0$ can be read out from Eq. (55). Eqs. (82), (83) and (84) shows that due to the Lorentz and *CPT* violation term, $\bar{\psi}\not{b}\gamma_5\psi$, the radiative correction makes the tensor structure of quantum vertex become much more complicated than that in conventional QED, in which

$$|M^{(1)}|^2 = |M^{(0)}|^2 [1 + F_1(l^2)]^2 \simeq |M^{(0)}|^2 [1 + 2F_1(l^2)] \quad (85)$$

Since this is not valid any longer, there does not exist the following relation between the classical and quantum differential cross section,

$$d\sigma_1 = [1 + 2F_1(l^2)]d\sigma_0. \quad (86)$$

This is actually the first obstacle for the IR divergence cancellation.

In the following we consider the bremsstrahlung process: photon emission from an electron when it is scattered by the static Coulomb potential (81). As in conventional QED, the scattering matrix is

$$S = \frac{iZe^3}{V^{3/2}} 2\pi\delta(E_q + E_k - E_p) \frac{m}{\sqrt{E_q E_p}} \frac{1}{|\mathbf{k}|^2} \epsilon_\nu^{(\lambda)}(k) M_{1\gamma}^\nu, \quad (87)$$

where E_q and E_p are the energy of the electron before and after being scattered by the nucleus; E_k is the energy of emitted photon and $\epsilon_\mu^{(\lambda)}(k)$, $\lambda = 1, 2$ are the photon polarization vectors, which satisfy the orthogonality and the completeness relations

$$g^{\mu\nu} \epsilon_\mu^{(\lambda)}(k) \epsilon_\nu^{(\sigma)}(k) = \delta^{\lambda\sigma}, \quad \sum_{\lambda=1,2} \epsilon_\mu^{(\lambda)}(k) \epsilon_\nu^{(\lambda)}(k) = - \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right); \quad (88)$$

V is the normalization volume for the initial electron; $M_{(1\gamma)\nu}$ is the matrix element,

$$M_{(1\gamma)\nu} = \bar{u}(q) \left(\gamma_\nu \frac{1}{\not{k} + \not{q} - m - \not{p}\gamma_5} \gamma_0 + \gamma_0 \frac{1}{\not{p} - \not{k} - m - \not{p}\gamma_5} \gamma_\nu \right) u(p), \quad (89)$$

the subscript 1γ denoting that we only consider the process of single photon emission. The corresponding differential cross section is given by the amplitude, $|S|^2$, per incoming electron flux and time and summing over the final states of the electrons and photons,

$$d\sigma_{(1\gamma)} = \frac{Z^2 e^6}{|\mathbf{v}|} \frac{m}{E_p} \frac{d^3 q}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} \delta(E_q + E_k - E_p) \frac{m}{E_q} \frac{1}{2E_k} \sum_{\lambda=1,2} |\epsilon_\nu^{(\lambda)}(k) M_{(1\gamma)}^\nu|^2, \quad (90)$$

In the above, \mathbf{v} is the velocity of the incoming electron, $|\mathbf{v}| = |\mathbf{p}|/m$.

We now evaluate $|\epsilon_\nu^{(\lambda)}(k) M^\nu|$ in the limit $k_\mu \rightarrow 0$. Employing Eq. (10), we get the expansion to the second order of b ,

$$\begin{aligned} M_{(1\gamma)\nu} = & \bar{u}(q) \left[\gamma_\nu \left(\frac{1}{\not{k} + \not{q} - m} + \frac{1}{\not{k} + \not{q} - m} \not{p}\gamma_5 \frac{1}{\not{k} + \not{q} - m} \right. \right. \\ & + \frac{1}{\not{k} + \not{q} - m} \not{p}\gamma_5 \frac{1}{\not{k} + \not{q} - m} \not{p}\gamma_5 \frac{1}{\not{k} + \not{q} - m} \Big) \gamma_0 \\ & + \gamma_0 \left(\frac{1}{\not{p} - \not{k} - m} + \frac{1}{\not{p} - \not{k} - m} \not{p}\gamma_5 \frac{1}{\not{p} - \not{k} - m} \right. \\ & \left. \left. + \frac{1}{\not{p} - \not{k} - m} \not{p}\gamma_5 \frac{1}{\not{p} - \not{k} - m} \not{p}\gamma_5 \frac{1}{\not{p} - \not{k} - m} \right) \gamma_0 + \dots \right] u(p). \end{aligned} \quad (91)$$

The b^0 term in the soft photon limit (i.e. $k_\mu \rightarrow 0$) is identical to that obtained in conventional QED,

$$M_\nu(b^0) \xrightarrow{k \rightarrow 0} \bar{u}(q) \gamma_0 \left(\frac{q_\nu}{q \cdot k} - \frac{p_\nu}{p \cdot k} \right) u(p). \quad (92)$$

As for the b^1 term, it is

$$\begin{aligned}
M_\nu(b^1) &= \bar{u}(q) \left[\frac{\gamma_\nu(\not{k} + \not{q} + m)\not{b}(\not{k} + \not{q} - m)\gamma_0\gamma_5}{4(q \cdot k)^2} + \frac{\gamma_5\gamma_0(\not{p} - \not{k} - m)\not{b}(\not{p} - \not{k} + m)\gamma_\nu}{4(p \cdot k)^2} \right] u(p) \\
&\stackrel{k \rightarrow 0}{\simeq} \bar{u}(q) \left[\frac{2q_\nu(k + q) \cdot b - 2q_\nu m \not{b} - q \cdot b \not{k} \gamma_\nu - k \cdot q \gamma_\nu \not{b} + m \not{k} \gamma_\nu \not{b}}{2(q \cdot k)^2} \gamma_0 \gamma_5 \right. \\
&\quad \left. + \gamma_5 \gamma_0 \frac{2p_\nu(p - k) \cdot b - 2mp_\nu \not{b} + p \cdot b \gamma_\nu \not{k} + k \cdot p \not{b} \gamma_\nu - 2m \not{b} \gamma_\nu \not{k}}{2(p \cdot k)^2} \right] u(p). \tag{93}
\end{aligned}$$

The b^2 term is quite complicated. Making use of Eq. (17), we obtain

$$\begin{aligned}
M_\nu(b^2) &= \bar{u}(q) \left[\gamma_\nu \frac{1}{\not{k} + \not{q} - m} \not{b} \gamma_5 \frac{1}{\not{k} + \not{q} - m} \not{b} \gamma_5 \frac{1}{\not{k} + \not{q} - m} \gamma_0 \right. \\
&\quad \left. + \gamma_0 \frac{1}{\not{p} - \not{k} - m} \not{b} \gamma_5 \frac{1}{\not{p} - \not{k} - m} \not{b} \gamma_5 \frac{1}{\not{p} - \not{k} - m} \gamma_\nu \right] u(p) \\
&= \bar{u}(q) \left[\frac{\gamma_\nu (\not{k} + \not{q} + m) \not{b} (\not{k} + \not{q} + m) \not{b} (\not{k} + \not{q} + m) \gamma_0}{8(k \cdot q)^3} \right. \\
&\quad \left. - \frac{\gamma_0 (\not{p} - \not{k} + m) \not{b} (\not{p} - \not{k} + m) \not{b} (\not{p} - \not{k} + m) \gamma_\nu}{8(k \cdot p)^3} \right. \\
&\quad \left. - \frac{mb^2 \gamma_\nu (\not{k} + \not{q} + m)^2 \gamma_0}{4(k \cdot q)^3} + \frac{mb^2 \gamma_0 (\not{p} - \not{k} + m)^2 \gamma_\nu}{4(k \cdot p)^3} \right] u(p) \\
&\stackrel{k \rightarrow 0}{\simeq} \bar{u}(q) \left\{ \frac{[4(k \cdot b + q \cdot b)^2 - 2k \cdot qb^2] (2q_\nu - \not{k} \gamma_\nu) - 4(k + q) \cdot bk \cdot q \gamma_\nu \not{b}}{8(k \cdot q)^3} \gamma_0 \right. \\
&\quad \left. - \gamma_0 \frac{[4(p \cdot b - k \cdot b)^2 + 2k \cdot pb^2] (2p_\nu + \gamma_\nu \not{k}) + 4(p - k) \cdot bk \cdot p \not{b} \gamma_\nu}{8(k \cdot p)^3} \right. \\
&\quad \left. - \frac{mb^2 (\gamma_\nu k \cdot q + 2mq_\nu - m \not{k} \gamma_\nu) \gamma_0}{2(k \cdot q)^3} \right. \\
&\quad \left. + \frac{\gamma_0 mb^2 (-k \cdot p \gamma_\nu + 2mp_\nu + m \gamma_\nu \not{k})}{2(k \cdot p)^3} \right\} u(p). \tag{94}
\end{aligned}$$

In deriving Eqs. (93) and (94), we only keep the IR singular terms as $k_\mu \rightarrow 0$ and throw away the terms containing k_ν and k^2 since $\epsilon^{(\lambda)} \cdot k = 0$ and $k^2 = 0$ on-shell.

Eqs. (91) — (93) give the amplitude to the second order of b ,

$$\begin{aligned}
\epsilon_\nu^{(\lambda)} M_{(1\gamma)}^\nu &= \epsilon^{(\lambda)}(k) \cdot [M(b^0) + M(b^1) + M(b^2)] \\
&= \bar{u}(q) \left[\gamma_0 M + \gamma_0 \gamma_5 M_5 + \gamma_\mu \gamma_0 \gamma_5 M_{5\mu}^{(1)} + \gamma_5 \gamma_0 \gamma_\mu M_{5\mu}^{(2)} + \gamma_\mu \gamma_\nu \gamma_0 \gamma_5 M_{5\mu\nu}^{(1)} \right. \\
&\quad \left. + \gamma_5 \gamma_0 \gamma_\mu \gamma_\nu M_{5\mu\nu}^{(2)} + \gamma_\mu \gamma_\nu \gamma_\rho \gamma_0 \gamma_5 M_{5\mu\nu\rho}^{(1)} + \gamma_5 \gamma_0 \gamma_\mu \gamma_\nu \gamma_\rho M_{5\mu\nu\rho}^{(2)} + \gamma_\mu \gamma_0 M_\mu^{(1)} \right. \\
&\quad \left. + \gamma_0 \gamma_\mu M_\mu^{(2)} + \gamma_\mu \gamma_\nu \gamma_0 M_{\mu\nu}^{(1)} + \gamma_0 \gamma_\nu \gamma_\mu M_{\mu\nu}^{(2)} \right] u(p), \tag{95}
\end{aligned}$$

where the various M 's are listed as following,

$$M = \frac{q \cdot \epsilon^\lambda}{q \cdot k} - \frac{p \cdot \epsilon^{(\lambda)}}{p \cdot k} + \frac{q \cdot \epsilon^{(\lambda)} [2(k \cdot b + q \cdot b)^2 - k \cdot qb^2 - 2m^2 b^2]}{2(k \cdot q)^3}$$

$$\begin{aligned}
& -\frac{p \cdot \epsilon^{(\lambda)} [2(p \cdot b - k \cdot b)^2 + k \cdot pb^2 - 2m^2 b^2]}{2(k \cdot p)^3}, \\
M_5 &= \frac{q \cdot \epsilon^{(\lambda)}(k + q) \cdot b}{(q \cdot k)^2} - \frac{p \cdot \epsilon^{(\lambda)}(p - k) \cdot b}{(p \cdot k)^2}; \\
M_{5\mu}^{(1)} &= -\frac{q \cdot \epsilon^{(\lambda)} m b_\mu}{(q \cdot k)^2}; \quad M_{5\mu}^{(2)} = -\frac{p \cdot \epsilon^{(\lambda)} m b_\mu}{(p \cdot k)^2}; \\
M_{5\mu\nu}^{(1)} &= -\frac{q \cdot b k_\mu \epsilon_\nu^{(\lambda)} + k \cdot q \epsilon_\mu^{(\lambda)} b_\nu}{2(q \cdot k)^2}; \quad M_{5\mu\nu}^{(2)} = \frac{p \cdot b \epsilon_\mu^{(\lambda)} k_\nu + k \cdot p b_\mu \epsilon_\nu^{(\lambda)}}{2(q \cdot k)^2}; \\
M_{5\mu\nu\rho}^{(1)} &= \frac{m k_\mu \epsilon_\nu^{(\lambda)} b_\rho}{2(q \cdot k)^2}; \quad M_{5\mu\nu\rho}^{(2)} = \frac{m b_\mu \epsilon_\nu^{(\lambda)} k_\rho}{2(p \cdot k)^2}; \\
M_\mu^{(1)} &= -\frac{m b^2 q \cdot k \epsilon^{(\lambda)}}{2(q \cdot k)^3}; \quad M_\mu^{(2)} = -\frac{m b^2 p \cdot k \epsilon^{(\lambda)}}{2(p \cdot k)^3}; \\
M_{\mu\nu}^{(1)} &= \frac{[(k \cdot q + 2m^2)b^2 - 2(k \cdot b + q \cdot b)^2]k_\mu \epsilon_\nu^{(\lambda)} - 2(k + q) \cdot b k \cdot q \epsilon_\mu^{(\lambda)} b_\nu}{4(k \cdot q)^3}; \\
M_{\mu\nu}^{(2)} &= \frac{[(-k \cdot p + 2m^2)b^2 - 2(p \cdot b - k \cdot b)^2]\epsilon_\mu^{(\lambda)} k_\nu - 2(p - k) \cdot b k \cdot p b_\mu \epsilon_\nu^{(\lambda)}}{4(k \cdot p)^3}. \tag{96}
\end{aligned}$$

Using the following well-known formula valid for a general operator Γ ,

$$|\bar{u}(q)\Gamma u(p)|^2 = \bar{u}(q)\Gamma u(p)\bar{u}(p)\gamma_0\Gamma^\dagger\gamma_0 u(q) = \text{Tr}\left[\frac{\not{q} + m}{2m}\Gamma\frac{\not{p} + m}{2m}\gamma_0\Gamma^\dagger\gamma_0\right], \tag{97}$$

we can evaluate $\sum_\lambda |\epsilon^{(\lambda)} \cdot M|^2$. For instance,

$$\begin{aligned}
|\bar{u}(q)\gamma_0 u(p)M|^2 &= \frac{1}{m^2} (2E_p E_q - p \cdot q + m^2) |M|^2; \\
|\bar{u}(q)\gamma_0\gamma_5 u(p)M|^2 &= \frac{1}{m^2} (2E_p E_q - p \cdot q - m^2) |M_5|^2; \\
|\bar{u}(q)\gamma_\mu\gamma_\nu\gamma_0 u(p)M^{\mu\nu}|^2 &= \frac{1}{4m^2} \text{Tr} [(\not{q} + m)\gamma_\mu\gamma_\nu\gamma_0(\not{p} + m)\gamma_0\gamma_\rho\gamma_\lambda] M^{\mu\nu} M^{*\lambda\rho} \\
&= \frac{1}{m^2} \{2E_p [q_\mu(g_{\nu 0}g_{\rho\lambda} - g_{\nu\rho}g_{0\lambda} + g_{\nu\lambda}g_{0\rho}) - (\mu \longleftrightarrow \nu)] \\
&\quad - 2E_p [q_\rho(g_{\mu\nu}g_{0\lambda} - g_{\mu 0}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu 0}) - (\lambda \longleftrightarrow \rho)] \\
&\quad - [q_\mu(p_\nu g_{\rho\lambda} - g_{\nu\rho}p_\lambda - g_{\nu\lambda}p_\rho) - (\mu \longleftrightarrow \nu)] \\
&\quad - [q_\rho(g_{\mu\nu}p_\lambda - g_{\nu\lambda}p_\mu - g_{\mu\lambda}p_\nu) + (\lambda \longleftrightarrow \rho)] \\
&\quad + (2E_p E_q - p \cdot q + m^2)(g_{\mu\nu}g_{\rho\lambda} - g_{\mu\rho}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\rho})\} M^{\mu\nu} M^{*\lambda\rho}; \tag{98}
\end{aligned}$$

There are 132 terms in the expansion of the $|\epsilon \cdot M|^2$, and some of them may vanish. In principle, with this expansion, using the fact that $|b| \ll m$ and taking the non-relativistic limit, $|\mathbf{p}|, |\mathbf{q}| \ll m$,

$$p \cdot k = E_q E_k - \mathbf{p} \cdot \mathbf{k} = m\sqrt{1 + \mathbf{p}^2/m^2} E_k - \mathbf{p} \cdot \mathbf{k} \simeq m E_k - \mathbf{p} \cdot \mathbf{k}, \tag{99}$$

we can calculate the contribution from photons with long wave length and observe whether the IR divergence associated with the soft photon emission cancels the divergent term in

the vertex correction [18]. However, Eqs. (95) – (99) imply that due to the various terms relevant to b_μ in (96), the following relation in the soft photon approximation cannot be valid as in the conventional QED,

$$\sum_{\lambda=1,2} |\epsilon_\nu^{(\lambda)}(k) M_{(1)\gamma}^\nu|^2 = e^2 |M^{(0)}|^2 f(k \cdot p, k \cdot q, l^2). \quad (100)$$

As shown below, this relation plays a key role in the cancellation of IR divergences in conventional QED. Thus the non-existence of (100) further enforces the difficulty in implementing the IR divergence cancellation.

To explicitly see whether or not the IR divergences cancel, we define the physical “measurable” cross section as in conventional QED [15],

$$\sigma = \int_0^{\Delta E} d\mathcal{E} \frac{d(\sigma_1 + \sigma_{1\gamma})}{d\mathcal{E}}, \quad (101)$$

where ΔE is the energy resolution of the detection device and $\mathcal{E} = E_q - E_p$ is the energy of the emitted real photon. In standard QED, Eq. (100) leads to

$$\int_0^{\Delta E} d\mathcal{E} \frac{d\sigma_{1\gamma}}{d\mathcal{E}} \stackrel{k_\mu \rightarrow 0}{\simeq} \int_0^{\Delta E} d\mathcal{E} \frac{d\sigma_0}{d\mathcal{E}} \int_0^{|k|=E_k \leq \Delta E} \frac{d^3 \mathbf{k}}{E_k (2\pi)^3} e^2 |M_0|^2, \quad (102)$$

where the integration over $d^3 \mathbf{k}$ can be performed with dimension regularization [15], i.e. taking $3 \rightarrow n - 1 = 3 + \epsilon_{IR}$. Consequently, the differential cross section of an electron interacting with an external Coulomb potential in conventional QED has the following simple form:

$$d\sigma = d\sigma_0 \left[1 + 2F_1(l^2) + \int_0^{\Delta E} \frac{d^{3+\epsilon_{IR}} \mathbf{k}}{(2\pi)^3 |\mathbf{k}|} e^2 |M_0|^2 \right]. \quad (103)$$

with no remaining IR divergences.

Unfortunately, the relations (85), (86) and (100) do not exist in QED with Lorentz and *CPT* violation term, due to the complicated tensor structure of the matrix elements listed in Eqs. (84) and (95). This makes it impossible for the physical cross section to reduce to the form of (103). In fact, even if the relation (103) could be established, this would not guarantee the cancellation of the IR divergence. The first direct reason is that the form factors F_i with $i \geq 2$ also contain IR divergences induced by the $\bar{\psi}\psi\gamma_5\psi$ term; The second one is more catastrophic: the bremsstrahlung process in the soft photon limit contains some novel IR divergences. For example, we find the following term in evaluating $|\epsilon^{(\lambda)} \cdot M_{(1)\gamma}|^2$,

$$\bar{u}(q)\gamma_0 u(p) M \left[\bar{u}(q)\gamma_\mu \gamma_\nu \gamma_0 \gamma_5 u(p) M_5^{(1)\mu\nu} \right]^\dagger = -\frac{i}{m^2} (\epsilon_{\mu\nu\lambda\rho} p^\lambda q^\rho + 2E_p \epsilon_{0\mu\nu\rho} q^\rho) MM^{*\mu\nu}. \quad (104)$$

It is clear that there can be no such IR divergent term in the vertex correction. This fact can be verified by simply comparing the various tensor structures in Eqs. (84) and (95).

The above discussion has shown that the introduction of the Lorentz and *CPT* violating term, $\bar{\psi}\psi\gamma_5\psi$ in the fermionic sector of QED gives rise to IR divergences in the on-shell vertex radiative correction that cannot be cured by considering the soft emission of bremsstrahlung. As a consequence, the theoretical value of the Lamb shift inevitably becomes unphysical. This suggests that it may not be appropriate to investigate Lorentz and *CPT* violation effects in the electromagnetic interaction by simply modifying the fermionic sector of QED.

VII. SUMMARY AND DISCUSSION

We have calculated the one-loop polarization tensor and the on-shell vertex radiative correction to second order in b for QED with an additional CPT -odd term $\psi\gamma_5\psi$ in the fermionic sector. This term is responsible for the generation of Lorentz- CPT violation through radiative corrections. Furthermore, we showed explicitly the resulting b_μ -dependence of the electron anomalous magnetic moment and the Lamb shift, demonstrating that the Lorentz and CPT violation term in the fermionic sector gives rise to remarkable effects on these two important physical predictions of QED. However, both expressions for the anomalous magnetic moment and the Lamb shift were shown to contain IR divergences linked to the Lorentz and CPT violation. The IR divergent terms in the anomalous magnetic moment lead to non-physical effective interaction and the IR divergence in the Lamb shift cannot be canceled in physical cross-sections by the contribution from the bremsstrahlung. This seems to imply that the Lorentz and CPT violation term must vanish. Of course, our result does not negate the possible existence of Lorentz and CPT violation phenomena in the electromagnetic interaction in general. It only means that it may not be appropriate to explore theoretically the Lorentz and CPT violating effects by putting explicit violation terms in the fermionic sector. Thus alternative models may be required. Our main aim here is to reveal the possible effects of a CPT -odd term on the anomalous magnetic moment and Lamb shift data, with emphasis on their field theoretic origins and to provide constraints on theoretical models that may be used to explore Lorentz and CPT violation in electromagnetic phenomena.

Finally, we note that the b -dependent part of the vertex radiative correction leads to many new types of non-minimal coupling between the electron and photon at the second order of b such as $(b \cdot \partial)^2 \bar{\psi} A \psi$, $\bar{\psi} A (b \cdot \partial)^2 \psi$, $(b \cdot \partial) \bar{\psi} A (b \cdot \partial) \psi$ and $\bar{\psi} \psi b \cdot A$ etc. These non-minimal interactions can yield even more remarkable effects than the anomalous magnetic moment and the Lamb shift at low-energy, but they are difficult to calculate explicitly. For example, to first order in b there arise couplings of the form $(b \cdot \partial) \bar{\psi} A \gamma_5 \psi$, $\bar{\psi} A \gamma_5 (b \cdot \partial) \psi$, $\partial_\mu \bar{\psi} \psi \gamma_5 \psi A^\mu$ and $\bar{\psi} \psi \gamma_5 \partial_\mu \psi A^\mu$ etc. These are non-minimal couplings between vector field and axial vector currents and hence are not explicitly invariant under electric charge conjugation. They will lead to electrostatic interactions in which particles with the same charge attract whereas opposite charges repel.

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REFERENCES

- [1] R. Jackiw, *Chern-Simons Violation of Lorentz and PCT Symmetries in Electrodynamics*, hep-th/9811322.
- [2] V.A. Kostelecký and S. Samuel, Phys. Rev. Lett. **63** (1989) 224; Phys. Rev. **D39** (1989) 683; *ibid* **D40** (1989) 1886; V.A. Kostelecký and R. Potting, Nucl. Phys. **B359** (1991) 545; Phys. Lett. **B381** (1996) 389.
- [3] D. Coladay and V.A. Kostelecký, Phys. Rev. **D55** (1997) 6760; *ibid* **D58** (1998) 116002.
- [4] V. Alan Kostelecký, Phys. Rev. Lett. **80** (1998) 1818; Phys. Rev. **D61** (2000) 016002.
- [5] V. Alan Kostelecký, Phys. Rev. **D60** (1999) 110610.
- [6] R. Bluhm, V. Alan Kostelecký and N. Russell, Phys. Rev. Lett. **82** (1999) 2254.
- [7] R. Bluhm, V. Alan Kostelecký and N. Russell, Phys. Rev. Lett. **79** (1997) 1432; Phys. Rev. **D57** (1998) 3932.
- [8] S. Carroll, G. Field and R. Jackiw, Phys. Rev. **D41** (1990) 1231.
- [9] R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. **82** (1999) 3572.
- [10] J.M. Chung and P. Oh, Phys. Rev. **D60** (1999) 067702.
- [11] W.F. Chen, Phys. Rev. **D60** (1999) 085007.
- [12] M. Pérez-Victoria, Phys. Rev. Lett. **83** (1999) 2518; J.M. Chung, Phys. Lett. **B461** (1999) 138.
- [13] L.H. Chan, *Induced Lorentz-Violating Chern-Simons Term in QED and Anomalous Contributions to Effective Action Expansions*, hep-ph/9907349.
- [14] S. Coleman and S. Glashow, Phys. Rev. **D59** (1999) 116008.
- [15] For example, see S. Pokorski, *Gauge Field Theories* (Cambridge University Press, 1987).
- [16] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
- [17] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
- [18] W. Greiner and J. Reinhardt, *Quantum Electrodynamics* (Springer-Verlag, 1992).

FIGURES

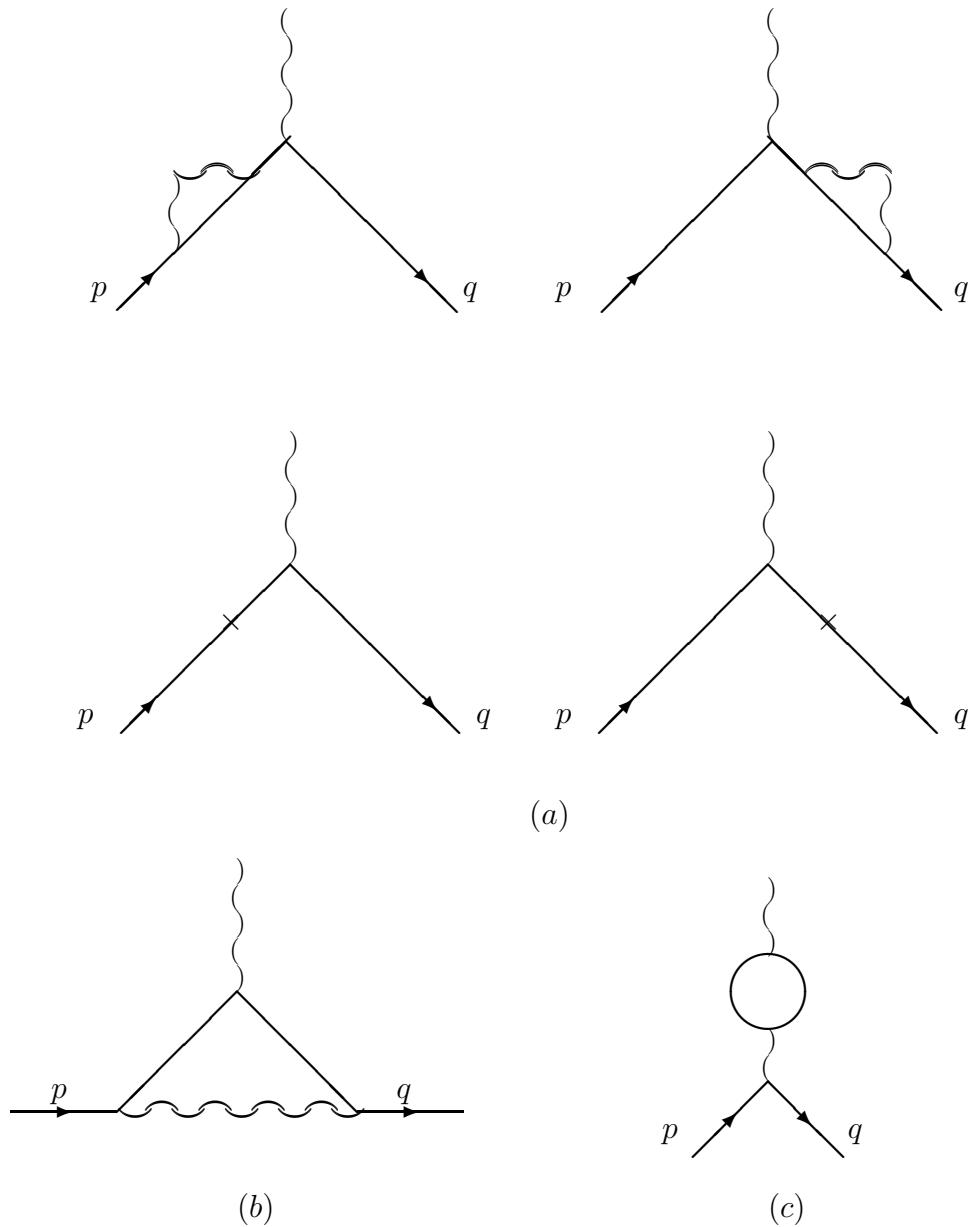


FIG. 1. One-loop Feynman diagrams contributing to Lamb shift: (a) the electron self-energy contribution, which actually vanishes in mass-shell renormalization scheme, \times representing the counterterm for electron mass renormalization; (b) the vertex radiative correction; (c) the contribution from vacuum polarization tensor.

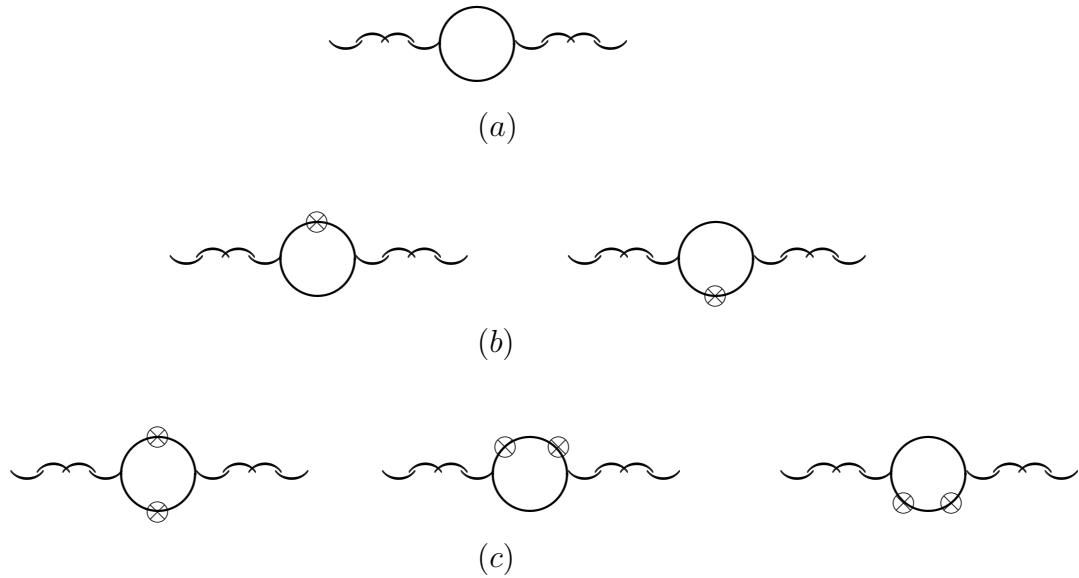
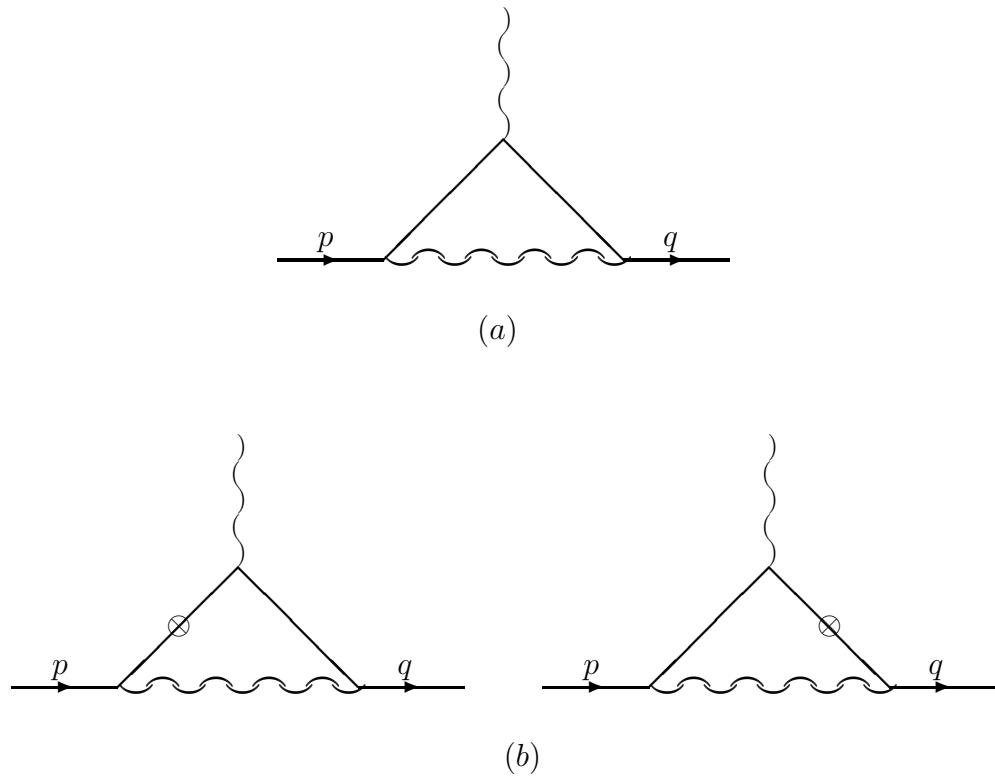


FIG. 2. Vacuum polarization up to the second order of b_μ contributed by fermionic loops with various insertions of CPT -odd vertex $\psi\gamma_5$ in the internal fermionic lines, \otimes denoting the vertex $\psi\gamma_5$.



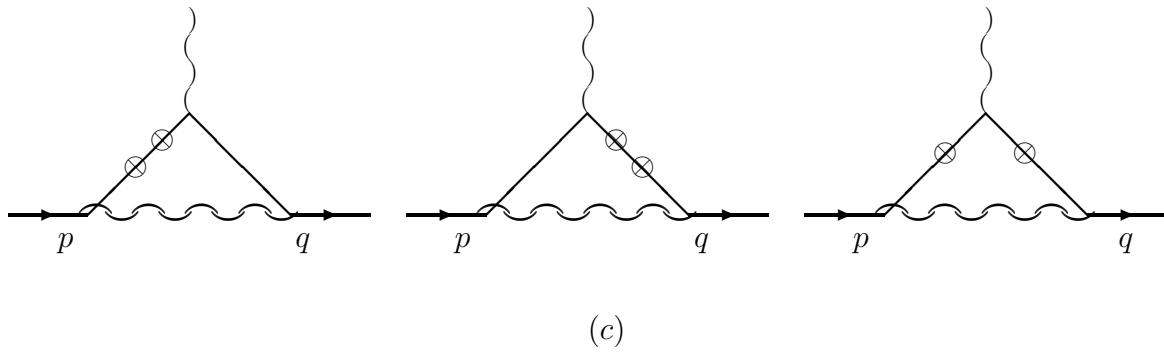


FIG. 3. One-loop vertex correction up to the second order of b_μ with various insertions of CPT -odd vertex $\not{p}\gamma_5$ in the internal fermionic lines, \otimes denoting the vertex $\not{p}\gamma_5$.