Enhancement of Radiatively Induced Magnetic Moment Form-Factors of Muon: an Effective Lagrangian Approach

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Abstract

Using an effective lagrangian approach, we identify a class of models in which the loop-induced magnetic moment form-factors of muon are enhanced by possibly large factors $(\Lambda_F^2/\Lambda^2)(m_\tau/m_\mu)\ln(m_\tau^2/\Lambda^2)$ or $(\Lambda_F^2/\Lambda^2)\ln(m_\mu^2/\Lambda^2)$, where Λ is the scale of new physics and Λ_F is the Fermi scale. These follow from left- and right-chirality mixing dimension-8 operators which for relatively small Λ, as required to explain the new $(g_\mu-2)$ measurement, dominate over dimension-6 operators. Thus significant enhancement of new physics contributions to $(g_\mu - 2)$ and, in the presence of intergenerational couplings, also to the $\mu \to e\gamma$ decay rate is possible. We discuss the compatibility of the $(g_\mu - 2)$ and $\mu \to e\gamma$ experimental data in this case and comment on the enhancement of the electron anomalous magnetic moment. An explicit model is presented to illustrate the general results.

1 Introduction

The recently announced measurement [1] of the muon anomalous magnetic moment:

$$a_\mu^{exp} = \frac{g_\mu - 2}{2} = 116592020(160) \times 10^{-11}, \tag{1}$$

which differs from the standard-model (SM) prediction [2] by 2.6σ:

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 426 \pm 165 \times 10^{-11}, \tag{2}$$

indicates that a relatively large positive new contribution to a_μ is needed, hinting thus at new physics above the electroweak scale. If the new contribution to $(g_\mu - 2)$ is induced at loop level, which is the case in models of neutrino masses [3], supersymmetric models [4], models with extra dimensions [5], models with enlarged gauge [6], Higgs [7] or fermion [8] sectors, models with leptoquarks [9] or models of compositness [10], then the largeness of Δa_μ implies usually a quite strong uper bound of order $\mathcal{O}(100)$ GeV on the new physics mass scale. While in some models there exist a mechanism to enhance the new contribution to a_μ (*e.g.* , in supersymmetric models it is enhanced by large value of $\tan\beta$), in others the masses and couplings of new particles should be tuned to satisfy the experimental value of Δa_μ.

At the same time, the new physics which gives rise to Δa_μ should likely affect also other leptonic observables. The most sensitive of them are the lepton flavour violating (LFV) processes, such as the decay $\mu \to e\gamma$, which occurs in the presence of flavour non-diagonal couplings. The connection between Δa_μ and $\mu \to e\gamma$ is particularly natural because both of them are induced by the same type of magnetic operators. In addition, if the same new physics gives also rise to non-zero neutrino masses and mixings, the LFV processes must be large [3] due to almost maximal mixings in the neutrino sector [11].

To classify the models above, an effective lagrangian description of new physics is a useful tool [12]. For Δa_μ the effective lagrangian analyses was done in Ref. [13] in which

all dimension-6 operators inducing Δa_μ were considered. Their general result agrees qualitatively with the results in each specific model: the new physics scale Λ should be relatively low, just above the Fermi scale Λ_F. On the one hand, this implies that the effective lagrangian description is not suitable for precision calculations because the higher order operators (dimension-8 and higher) may not be suppressed compared to the dimension-6 operators and in some cases may even dominate. On the other hand, because of the conceptual simplicity, the effective lagrangian language is still useful to understand the generic behaviour of certain class of models under consideration. Once the general properties are understood, the precision calculations may be performed in each model separately. This is the philosophy we adopt in this paper.

The purpose of this paper is to identify a class of models in which left- and right-chirality mixing dimension-8 effective operators inducing Δa_μ at one loop dominate over the dimension-6 operators. These operators were not considered in Ref. [13]. The factors $(\Lambda_F^2/\Lambda^2)(m_\tau/m_\mu)\ln(m_\tau^2/\Lambda^2)$ and $(\Lambda_F^2/\Lambda^2)\ln(m_\mu^2/\Lambda^2)$ occuring in the magnetic moment form-factors in these models may be large enough to significantly enhance the new physics contributions to Δa_μ and to $\mu \to e\gamma$. We analyze these two processes here, and comment also on the m_τ/m_e enhancement of the electron anomalous magnetic moment and its connection to Δa_μ. To illustrate the general result we propose a simple model in which the enhancement occurs, and perform exact calculations in that model.

2 Effective lagrangian approach

Assuming the new physics to appear at the scale Λ, the relevant terms in the effective lagrangian contributing directly to magnetic moments are

$$\mathcal{L}^{\sigma L} = \frac{\alpha_{ij}^{\sigma L}}{(4\pi)^2\Lambda^2} e\, \overline{e_{iL}}\sigma_{\mu\nu} i\, \slashed{D} e_{jL} F^{\mu\nu} + \text{h.c.}\,, \tag{3}$$

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and

$$\mathcal{L}^{\sigma R} = \frac{\alpha_{ij}^{\sigma R}}{(4\pi)^2 \Lambda^2} e \; \overline{e_{iR}} \sigma_{\mu\nu} i \not{D} e_{jR} F^{\mu\nu} + \text{h.c.} \,. \tag{4}$$

Here e_{iL} and e_{iR} are chiral charged-lepton fields, $\not{D} = \not{\partial} + ie \not{A}$, the Lorentz indices are μ, ν, and the indices i, j denote generations. Since these terms cannot be obtained from renormalizable vertices at tree level, we expect them to be generated at one loop. That is the reason we already included a factor $(4\pi)^2$ in the denominator.

We have written $\mathcal{L}^{\sigma L}$ and $\mathcal{L}^{\sigma R}$ in a particular form involving only left or right chiral fields. The two operators could be combined by using the equations of motion for the leptons. In this case we obtain

$$\mathcal{L}^{\sigma} = \frac{1}{(4\pi)^2 \Lambda^2} e \; \overline{e_L} \sigma_{\mu\nu} F^{\mu\nu} \left(\alpha^{\sigma L} m_e + m_e \alpha^{\sigma R} \right) e_R + \text{h.c.} \,, \tag{5}$$

where where m_e is the charged lepton mass matrix and the generation indices are suppressed. In chiral theories, like the ones we want to consider, magnetic moments appear always in the form eq. (5) and are proportional to the fermion masses. In more general theories with chirality explicitly broken independently of the fermion masses, operators like eq. (5) but with an arbitrary matrix M could arise.

The dimension-6 four fermion operators inducing eq. (5) at one loop level are considered in Ref. [13]. Here we consider the effective lagrangians of the type

$$\mathcal{L}^{LR} = \frac{\alpha_{ik,lj}^{LR}}{\Lambda^4} (\tilde{\Phi}^+ \Phi)(\overline{e_{iL}} e_{kL}^c)(\overline{e_{lR}^c} e_{jR}) + \text{h.c.} \,, \tag{6}$$

where Φ is the SM Higgs doublet, $\tilde{\Phi} = \tau_2 \Phi^*$ and $e_{L,R}^c = (e_{L,R})^c$ are the charge conjugated fields. The couplings $\alpha_{ik;lj}^{LR}$ are symmetric with respect to the exchanges $i \leftrightarrow k$ and $l \leftrightarrow j$. However, the pairs of indices (ik) and (lj) are totally independent. Therefore there is no need to write down the corresponding RL operator which is of the form of the hermitian conjugate of eq. (6); all such independent terms are already included in eq. (6). We assume $\alpha_{ik;lj}^{LR}$ to be
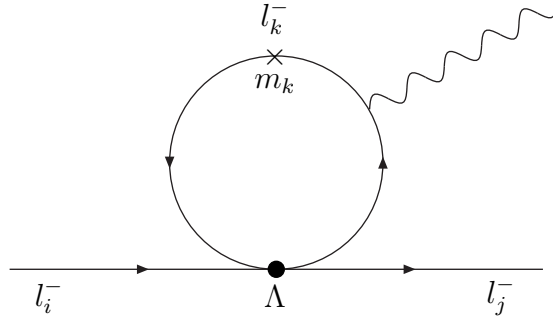
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Figure 1: *The enhanced one loop contribution to magnetic form-factors. The chirality flip in the internal lepton line is denoted by $\times$.*

real. We have chosen the form eq. (6) because it is simple for loop calculations. One could perform a Fierz transformation of eq. (6) to get rid of the charge conjugate fields. However, the simplicity will be lost in this case and the tensor operators will occur. In addition, the operators of type eq. (6) arise naturally in the class of models we shall consider.

After the Higgs boson will acquire the vacuum expectation value (vev) v, the corresponding four-fermion operator will occur which gives a contribution to magnetic moments via the one loop diagram depicted in Fig. 1. On the one hand, this is suppressed by v^2/Λ^2 compared to the similar dimension-6 four-fermion operator contributions. On the other hand, however, because of the left and right mixing chiral structure of eq. (6), the helicity flip must occur now in the internal fermion line as explicitly noted in Fig. 1. This will imply the possible enhancement[1] factors m_k/m_i compared to the usual case when the chirality flip occurs in the external line. In addition, since the fermion masses m_k are very small compared to the scale Λ, one expects large logarithms $\ln(m_k^2/\Lambda^2)$ to occur in the magnetic moment form-factors. Again, these large logarithms never occur when the chirality flip occurs in the external line. Numerically, for example for $\Lambda \sim 1$ TeV, $k = \tau$, $i = \mu$, the factor $|(m_\tau/m_\mu)\ln(m_\tau^2/\Lambda^2)| \approx 210$

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[1]In SUSY models the enhancement of Δa_μ has a similar origin [4]: for large $\tan\beta$ the slepton LR mass terms m_{LR} become large. However, no large logarithms occur in SUSY models since the superpartner masses are of the same order of magnitude.

overcomes the suppression factor $v^2/\Lambda^2 \approx 1/16$, and significant enhancement of loop induced magnetic moment form-factors will occur.

To show this with an explicit calculation, we express the relevant matrix element in the most general way as

$$
\begin{aligned}
\mathcal{M} &= e\bar{u}(p_j) \left[(f_{E0} + \gamma_5 f_{M0}) \gamma_\nu \left(g^{\lambda\nu} - \frac{q^\lambda q^\nu}{q^2} \right) \right. \\
&\quad \left. + (f_{M1} + \gamma_5 f_{E1}) \, i \, \sigma^{\lambda\nu} \frac{q_\nu}{m_i} \right] u(p_i) \epsilon_\lambda(q) \,,
\end{aligned}
\tag{7}
$$

where p_i, p_j are the lepton momenta and q is the momentum of the photon. We calculate the form factors f_{E0}, f_{E1}, f_{M0} and f_{M1} induced only by eq. (6) via the diagram in Fig. 1 (similar diagrams without the enhancement are considered in Ref. [14]). For i denoting the initial and j the final state particle and k the particle running in the loop the answer reads

$$
f_{M0} = f_{E0} = 0 \,,
\tag{8}
$$

$$
f_{M1} = \sum_k \frac{2 \left(\alpha_{jk,ki}^{LR} + \alpha_{ik,kj}^{LR} \right)}{(4\pi)^2} \frac{m_i^2}{\Lambda^2} \frac{v^2}{\Lambda^2} \frac{m_k}{m_i} F \left(\frac{m_k^2}{\Lambda^2}; \frac{-q^2}{\Lambda^2} \right) \,,
\tag{9}
$$

$$
f_{E1} = \sum_k \frac{2 \left(\alpha_{jk,ki}^{LR} - \alpha_{ik,kj}^{LR} \right)}{(4\pi)^2} \frac{m_i^2}{\Lambda^2} \frac{v^2}{\Lambda^2} \frac{m_k}{m_i} F \left(\frac{m_k^2}{\Lambda^2}; \frac{-q^2}{\Lambda^2} \right) \,,
\tag{10}
$$

where

$$
F(x; y) = 2 - \ln x + \sqrt{1 + \frac{4x}{y}} \ln \left(\frac{\sqrt{4x+y} + \sqrt{y}}{\sqrt{4x+y} - \sqrt{y}} \right) \,,
\tag{11}
$$

and we have taken $q^2 \leq 0$. In the limit of on-shell photon, which is the case we are interested in, the function F simplifies to

$$
\lim_{y \to 0} F(x; y) = 4 - \ln x \,.
\tag{12}
$$

It is clear from eq. (9),(10) that an enhancement of the form-factors by $m_k/m_i \ln(m_k^2/\Lambda^2)$ will occur. This is the case for both anomalous magnetic moments with $i = j$ as well as for the transition magnetic moments with $i \neq j$.

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Let us now turn to studies of specific observables. The new physics contribution from the effective lagrangians eq. (5) and eq. (6) to the muon anomalous magnetic moment is given by

$$\Delta a_\mu = \frac{2m_\mu^2}{(4\pi)^2\Lambda^2} \left(\alpha_{\mu\mu}^{\sigma L} + \alpha_{\mu\mu}^{\sigma R} + \sum_k 4\alpha_{\mu k,k\mu}^{LR} \frac{v^2}{\Lambda^2}\frac{m_k}{m_\mu} \left[4 - \ln\frac{m_k^2}{\Lambda^2} \right] \right) . \tag{13}$$

As we expect $\alpha^{\sigma L} \sim \alpha^{\sigma R} \sim \alpha^{LR}$ the loop induced contribution from eq. (6) clearly dominates.

Similarly, the $l_i \to l_j \gamma$ rate divided by the $l_i \to l_j \nu_i \bar{\nu}_j$ rate is given by

$$R(l_i \to l_j\gamma) = \frac{96\pi^3\alpha}{G_F^2 m_{l_i}^4} \left(|f_{M1}|^2 + |f_{E1}|^2 \right) , \tag{14}$$

where $\alpha = 1/137$ and G_F is the Fermi constant. For the decay $\mu \to e\gamma$ one has $i = \mu$, $j = e$ and the corresponding transition form-factors are

$$f_{M1} = \frac{m_\mu^2}{(4\pi)^2\Lambda^2} \left(\alpha_{e\mu}^{\sigma L} + \alpha_{e\mu}^{\sigma R} + \sum_k 2 \left(\alpha_{ek,k\mu}^{LR} + \alpha_{\mu k,ke}^{LR} \right) \frac{v^2}{\Lambda^2}\frac{m_k}{m_\mu} \left[4 - \ln\frac{m_k^2}{\Lambda^2} \right] \right) , \tag{15}$$

$$f_{E1} = \frac{m_\mu^2}{(4\pi)^2\Lambda^2} \left(\alpha_{e\mu}^{\sigma L} - \alpha_{e\mu}^{\sigma R} + \sum_k 2 \left(\alpha_{ek,k\mu}^{LR} - \alpha_{\mu k,ke}^{LR} \right) \frac{v^2}{\Lambda^2}\frac{m_k}{m_\mu} \left[4 - \ln\frac{m_k^2}{\Lambda^2} \right] \right) . \tag{16}$$

Again, a significant enhancement of the $\mu \to e\gamma$ rate is expected.

Let us now turn to discussion of our general results. Assuming that only one of the couplings $\alpha_{\mu\tau,\tau\mu}^{LR} \sim 4\pi$ or $\alpha_{\mu\mu,\mu\mu}^{LR} \sim 4\pi$ is non-zero at the time while the other vanishes, the 90% confidence-level experimental result $\Delta a_\mu \geq 215 \cdot 10^{-11}$ [2] implies upper bounds on the new physics scale Λ as shown in Table 1. Due to the enhancement the bounds are of order $\mathcal{O}(1)$ TeV rather that of order $\mathcal{O}(100)$ GeV as expected in models with no enhancement. If the off-diagonal couplings are large too, $\alpha_{ek,k\mu}^{LR} \sim \alpha_{\mu k,ke}^{LR} \sim 4\pi$, this would imply

$$R(\mu \to e\gamma) \geq 1.5 \cdot 10^{-3}, \tag{17}$$

which is orders of magnitude above the present limit $R(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$ [15]. This shows the correlation between Δa_μ and $R(\mu \to e\gamma)$ in these models. Because the experimental value

$\Delta a_\mu \geq 215 \cdot 10^{-11}$		$R(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$	$R(\mu \to e\gamma) < 2 \cdot 10^{-14}$
$\alpha^{LR}_{\mu\tau,\tau\mu} \sim 4\pi$	$\Lambda \leq 2.82$ TeV	$\alpha^{LR}_{\mu\tau,\tau e} \leq 1.1 \cdot 10^{-3}$	$\alpha^{LR}_{\mu\tau,\tau e} \leq 4.7 \cdot 10^{-5}$
$\alpha^{LR}_{\mu\mu,\mu\mu} \sim 4\pi$	$\Lambda \leq 1.47$ TeV	$\alpha^{LR}_{\mu\mu,\mu e} \leq 1.1 \cdot 10^{-3}$	$\alpha^{LR}_{\mu\mu,\mu e} \leq 4.7 \cdot 10^{-5}$

Table 1: *Upper bounds on the scale Λ if either internal τ or μ contribution to Δa_μ dominates. The upper bounds on the LFV couplings α^{LR} are given for the same Λ.*

of Δa_μ fixes Λ to be at relatively low scale, and because the process $\mu \to e\gamma$ is much more sensitive to new physics than $(g_\mu - 2)$, the only way to suppress the $\mu \to e\gamma$ rate is to suppress the LFV couplings. The upper bounds on the couplings $\alpha^{LR}_{\mu k, ke}$ obtained for the present limit on $\mu \to e\gamma$ as well as for the expected limit $R(\mu \to e\gamma) < 2 \cdot 10^{-14}$ [16] are presented in Table 1. Here we have assumed that $\alpha^{LR}_{ek,k\mu} = \alpha^{LR}_{\mu k, ke}$. It follows that the LFV couplings should be smaller than at least 10^{-3} in order not to go into conflict with the experimental data.

Let us now consider the anomalous magnetic moment of electron in our scenario. Because the necessary chirality flip occurs in the internal fermion line, also the new contribution to the electron anomalous magnetic moment Δa_e is proportional to the tau mass m_τ rather than to the electron mass m_e. This implies the enhancement factor $(\Lambda_F^2/\Lambda^2)(m_\tau/m_e)\ln(m_\tau^2/\Lambda^2)$ compared to the usual case. Therefore, if $\alpha^{LR}_{e\tau,\tau e} \approx \alpha^{LR}_{\mu\tau,\tau\mu}$ then eq. (2) automatically implies $\Delta a_e \approx (m_e/m_\mu)\Delta a_\mu \sim 10^{-11}$ in our scenario. This is more than an order of magnitude larger than the current experimental uncertainty on a_e. Therefore, either $\alpha^{LR}_{e\tau,\tau e} < \alpha^{LR}_{\mu\tau,\tau\mu}$, or one must reconsider the SM contributions to a_e (and also the quantities derived from it, such as α_{QED}).

Finally, two comments are in order. First, the origin of the enhancement of the magnetic moment form-factors discussed here is in the left-right chiral structure of the effective lagrangian eq. (6). One can easily construct similar lagrangians by replacing two of the leptons by some very heavy exotic leptons E, for example. In this case the enhancement factor m_E/m_μ from the chirality flip in the loop is huge. However, this type of models are beyond

our considerations here. Second, if the error bars in Δa_μ will be increased, the bounds in Table 1 should be revised.

3 An explicit model

Here we present and explicit model in which the operators of type eq. (6) occur. The Higgs sector of the model consists of the usual SM doublet with the $SU(2)_L \times U(1)_Y$ quantum numbers

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2) \,, \tag{18}$$

and two additional scalar fields, a triplet ξ and a singlet χ:

$$\xi = \begin{pmatrix} \xi^+/\sqrt{2} & \xi^{++} \\ -\xi^0 & -\xi^+/\sqrt{2} \end{pmatrix} \sim (3, 1) \,, \qquad \chi = \chi^{++} \sim (1, 2) \,. \tag{19}$$

The latter two fields carry lepton number -2. The triplet couples to lepton doublets L via the Yukawa interaction

$$\mathcal{L}_\xi = f_{ij} L_i^T C^{-1} i\tau_2 \, \xi \, L_j + \text{h.c.} \,, \tag{20}$$

while the lagrangian for the χ coupling to lepton singlets is

$$\mathcal{L}_\chi = h_{ij} \overline{e_{iR}^c} e_{jR} \, \chi^{++} + \text{h.c.} \,. \tag{21}$$

Here the Yukawa coupling matrices f_{ij}, h_{ij} are symmetric in the generation indices i, j. We assume them to be real.

The most general Higgs potential containing these fields is

$$
\begin{aligned}
V \;=\; & m_0^2 \, \Phi^\dagger \Phi + m_\xi^2 \, \text{Tr}[\xi^\dagger \xi] + m_\chi^2 \, \chi^\dagger \chi + \frac{1}{2}\lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2}\lambda_2 \text{Tr}[\xi^\dagger \xi]^2 + \frac{1}{2}\lambda_3 (\chi^\dagger \chi)^2 + \\
& \lambda_4 \text{Tr}[\xi^\dagger \xi^\dagger] \text{Tr}[\xi\xi] + \lambda_5 (\Phi^\dagger \Phi) \text{Tr}[\xi^\dagger \xi] + \lambda_6 \Phi^\dagger \xi^\dagger \xi \Phi + \lambda_7 (\chi^\dagger \chi) \text{Tr}[\xi^\dagger \xi] + \lambda_8 (\chi^\dagger \chi)(\Phi^\dagger \Phi) + \\
& \lambda_9 \left(\chi \Phi^\dagger \xi^\dagger \tilde{\Phi} + h.c. \right) + \left(\frac{\mu}{\sqrt{2}} \Phi^\dagger \xi \tilde{\Phi} + h.c. \right) \,,
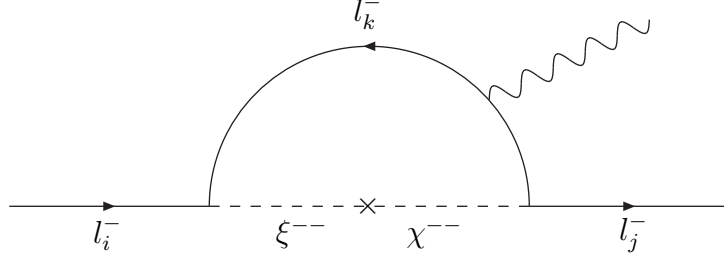\end{aligned}
\tag{22}
$$

Figure 2: *Diagrams giving rise to enhanced Δa_μ and $l_i \to l_j \gamma$ in our example model. The photon can be attached to any charged line.*

where $m_0^2 < 0$, but $m_\xi^2 > 0$. The neutral components of the fields acquire vevs as $\phi^0 \to \phi^0 + v/\sqrt{2}$, $\xi^0 \to \xi^0 + u/\sqrt{2}$. Thus the $SU(2)_L$ gauge symmetry is broken as in the SM but, because $m_\xi^2 > 0$, the lepton number is not broken spontaneously. Thus there is no Majoron in this model. This is the biggest difference between the model presented here and the original Gelmini-Roncadelli model [17]. Nevertheless, neutrinos may have Majorana masses in this model because lepton number is broken explicitly by the last dimensionful term in eq. (22). This follows from the first derivative minimization conditions

$$m_0^2 - \mu u + \frac{1}{2}\lambda_1 v^2 + \frac{1}{2}\lambda_5 u^2 = 0,$$
$$m_\xi^2 u - \frac{1}{2}\mu v^2 + \frac{1}{2}\lambda_2 u^3 + \frac{1}{2}\lambda_5 u v^2 = 0. \tag{23}$$

Therefore, $v^2 \simeq -2m_0^2/\lambda_1$ as usual, but $u \simeq \mu v^2/2m_\xi^2$, with $u \ll v$. The small vev u of the triplet, which gives masses to the neutrinos via Eq. (20), is proportional to the value of μ and inversely proportional to the square of the Higgs triplet mass, *i.e.* m_ξ^2. Thus the smallness of the neutrino mass must follow from the smallness of lepton number breaking parameter μ which can be achieved, *e.g.* , in models with extra dimensions [18]. This model has also a number of unique experimental signatures at future collider experiments [19].

For our studies here only the doubly charged Higgs bosons are important. The enhanced

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contribution to Δa_μ and $\mu \to e\gamma$ arises from the diagrams depicted in Fig. 2. In order to get the same chiral structure as in the effective lagrangian eq. (6), mixing of the left-handedly (triplet) and right-handedly (singlet) interacting particles must occur. This is explicitly shown in Fig. 2. The doubly charged Higgs boson mass matrix following from eq. (22) is in the basis (ξ^{++}, χ^{++}) given by

$$\mathcal{M}^2 = \frac{1}{2} \begin{bmatrix} (\lambda_6 + \mu/u) v^2 + 4\lambda_4 u^2 & \lambda_9 v^2 \\ \lambda_9 v^2 & 2m_\chi^2 + \lambda_8 v^2 + \lambda_7 u^2 \end{bmatrix}. \tag{24}$$

Notice that the off-diagonal entry is proportional to $\sim v^2$. Thus the mixing between the two doubly charged Higgses is roughly given by $\sim v^2/m_\chi^2$. This is nothing but the extra suppression factor v^2/Λ^2 appearing in eq. (6) after the gauge symmetry breaking. In this context the requirement of going to higher order (dimension-8) operators is explained by the requirement of having the left-right mixings of the Higgs bosons.

Explicit calculation shows that the enhanced new physics contribution to a_μ in our model is

$$\Delta a_\mu = \sum_k \frac{f_{\mu k} h_{k\mu}}{4\pi^2} \frac{m_k}{m_\mu} \sin 2\theta \sum_a (-1)^{1+a} \frac{m_\mu^2}{M_a^2} \left[\frac{7}{2} - \ln \frac{m_k^2}{M_a^2} \right], \tag{25}$$

where the second sum over $a = 1, 2$ goes over the two doubly charged Higgs boson mass eigenstates, and the angle θ is the mixing angle between them. θ can be calculated from eq. (24).

Similarly, the transition form-factors for the decay $\mu \to e\gamma$ are

$$f_{M1} = \sum_k \frac{(h_{\mu k} f_{ek} + f_{\mu k} h_{ke})}{(4\pi)^2} \frac{m_k}{m_\mu} \sin 2\theta \sum_a (-1)^{1+a} \frac{m_\mu^2}{M_a^2} \left[\frac{7}{2} - \ln \frac{m_k^2}{M_a^2} \right], \tag{26}$$

$$f_{E1} = \sum_k \frac{(h_{\mu k} f_{ek} - f_{\mu k} h_{ke})}{(4\pi)^2} \frac{m_k}{m_\mu} \sin 2\theta \sum_a (-1)^{1+a} \frac{m_\mu^2}{M_a^2} \left[\frac{7}{2} - \ln \frac{m_k^2}{M_a^2} \right]. \tag{27}$$

Notice the two differences compared to the effective lagrangian form factors. First and the less important one is that the numerical factor 7/2 appears in the square brackets rather than

factor 4. This is because we have taken into account all contributing diagrams in our model; of course, the dominant leading logarithm is the same here as in the effective lagrangian case. Secondly, there are two Higgs mass eigenstates contributing; the effective lagrangian result will be approximately achieved only if $M_2 \gg M_1 \sim \Lambda$. The exact result depends on the values of the parameters in eq. (24). For a numerical example we take $f_{\mu\tau} h_{\tau\mu} = 1$, all $\lambda = 1$, $(\mu/u)v^2 = m_\chi^2$ and $u \ll v$ in eq. (24). Then $\Delta a_\mu > 215 \cdot 10^{-11}$ implies $m_\chi < 1.4$ TeV. Therefore we emphasize that exact calculations in each particular model are important to explain quantitatively the observed Δa_μ.

Let us now discuss the decay $\mu \to e\gamma$. One of the motivations to consider this model here is that it gives an enhanced Δa_μ as well as small Majorana neutrino masses at the same time. Both of them are experimentally observed quantities. Our knowledge of the neutrino mass matrix implies that at least $f_{\mu\tau}$, and possibly also $f_{\mu e}$ entries in the Yukawa matrix f must be large. This is because of almost maximal mixing angles in the neutrino sector [11]. To satisfy the experimental constraints on $R(\mu \to e\gamma)$ one has to suppress the couplings $h_{\mu\tau} f_{e\tau}$ and $f_{\mu\tau} h_{\tau e}$. Since this is impossible for $f_{\mu\tau}$ if we want to induce the observed neutrino properties, we conclude that the LFV couplings of h_{ij} must be very much suppressed (see Table 1).

4 Conclusions

We have shown that the left-right chirality mixing dimension-8 effective operators of type eq. (6) give radiatively induced contributions to muon magnetic moment form-factors which are enhanced by $(\Lambda_F^2/\Lambda^2)(m_\tau/m_\mu) \ln(m_\tau^2/\Lambda^2)$ compared to the dimension-6 operator contributions. As the measured Δa_μ requires the scale Λ to be of order TeV, these dimension-8 operator contributions dominate over the dimension-6 ones, implying enhancement of new physics contributions to a_μ and to the decay $\mu \to e\gamma$. Using effective lagrangians we have

derived constraints on Λ from the observed Δa_μ as well as constraints on the LFV couplings from $\mu \to e\gamma$. We have illustrated this general result by an explicit model with a scalar triplet and a singlet. This model is motivated by the fact that it can generate the observed neutrino masses and the enhanced Δa_μ at the same time. Because the new physics scale Λ is low, we emphasize the importance of exact calculations in each particular model.

In this scenario also the new contribution to the anomalous magnetic moment of electron is proportional to the internal lepton mass and thus enhanced by $(\Lambda_F^2/\Lambda^2)(m_\tau/m_e)\ln(m_\tau^2/\Lambda^2)$. Numerically this exceeds the present experimental uncertainty on Δa_e and requires further suppression of the $e\tau$ couplings.

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