Guide for atomic and particle physicists to CODATA's recommended values of the fundamental physical constants

Savely G. Karshenboim

D. I. Mendeleev Institute for Metrology (VNIIM), St. Petersburg 190005, Russia
Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

Abstract. The CODATA recommended values of the fundamental constants are widely applied in particle, nuclear and atomic physics. They are a result of a complicated evaluation (adjustment) of numerous correlated data of different nature. Their application is often rather mechanical and as a result is not free of various confusions which are discussed in this note.

1 Introduction

Precision physics deals with numbers rather than with functions, but any theoretical prediction in numerical terms can appear only after one applies certain values of input parameters, the most important of which are the fundamental physical constants. The most popular are recommended values published by CODATA. Working for a while for precision physics of simple atoms, which is based on quantum electrodynamics (QED) calculations, and for fundamental constants, I have witnessed a certain number of confusions in applications of the CODATA values. This paper aims to guide to fundamental constants with a hope to avoid such confusions in future. One can consider it as a kind of 'fundamental constants for non-experts' or 'frequently necessary but not asked questions'.

Some applications of the values of certain fundamental constants to precision studies are sensitive to a choice of the values for the constant to be used. For such a case it is incorrect to apply any value of the constant blindly. The real option is to look for the origin of the result, checking what kind of measurements and calculations have been done to obtain it, what suggestions were made if any. Before any application of a particular result on the fundamental constant, one has to realize whether this application is in line with the actions done to derive the constant.

The CODATA papers [1,2] represent a very specific kind of papers, namely, reference papers. They contain very important information, which can be found on demand, but most of users are aware only about the tables of the recommended values of the fundamental constants, and even most of them did not read the papers, but access to the values through the internet (via, e.g., the NIST web site [3]) or through other compilations, such as the Review of Particle Properties [4]. In such a case they do not even have a chance to see any details of the original CODATA evaluation.

We consider this note as a supplementary paper to [1] and intentionally do not provide any references which can be found there. We also intentionally do not present any progress since the adjustment-2002 [1]. In particular, there have been a number of remarkable results improving accuracy in determination of the fine structure constant α and the Planck constant h, as well as substantial progress in understanding the muon anomalous magnetic moment.

Our purpose is not to discuss the most accurate data for a particular time period, since the data are continuously improving, but to explain how to deal with the CODATA recommendations, which may be applied to any CODATA recommendations, current and future.

Most of physicists consider CODATA as a kind of a brand for publication of the list of the best values of the constants. However, the main objective of the CODATA task group on the fundamental constants is to study the precision data, their accuracy, reliability and overall consistency. Its papers present a very detailed critical review of the experimental data which serve as input data of the adjustments.

2   The adjustment of the fundamental constants: a general view

What is the adjustment? Normally, when one performs an experiment, the final result is an average of various measurements, or a result of a simple fitting, if we cannot measure the needed values directly, but only their combinations. For instance, we can measure certain cross sections as a function of the momentum transfer and the slope of specifically normalized cross section (as a function of the momentum transfer squared, q^2) gives us a charge radius.

In the case of the fundamental constants the 'topology' of correlation links between data is cumbersome. It may be possible to measure e, h, e/h, e^2/h, e/m_e, h/m_e etc. In contrast to the mentioned scattering experiment, the accuracy of different results is high, but quite different, and the data themselves may have also substantial experimental or computational correlations in uncertainties. The adjustment is such a procedure which pretends to find the most plausible result for the output parameters.

It involves a least-square-method as a technical part; however, a crucial issue is a careful reconsideration of each inconsistency between and inside various portions of the input data. It depends on physics whether we have to treat them symmetrically or asymmetrically. A symmetric treatment may suggest, e.g., either multiplying their uncertainties by the same factor in order to reach a reasonable χ^2 value, or, in contrast, assigning to all the data equal uncertainties despite the fact they have been claimed to be very different. An example of an asymmetric treatment is the very removal of certain doubtful data as an ultimate choice.

3   The adjustment of the fundamental constants: the data

All the input data can be subdivided into a few groups as shown in Table 1 (see, e.g., [5,6] for more detail). Two 'big blocks' involve substantially correlated data of various kinds (see below). Evaluation of data of these two big blocks is the main part of the procedure of *the adjustment of the values of the fundamental constants*.

Data, which are known with a higher accuracy, can be found separately before the main adjustment of these two blocks. Those most accurate data are referred to as *auxiliary*. An example of such data is the data on the Rydberg constant R_∞ and various mass ratios like m_e/m_p (we have to mention also a few constants such as the speed of light c which numerical values are fixed in the SI by definition).

Data which are less accurate can be in principle ignored. The related constants are to be derived afterwards from the results of the adjustment. An example is a value of $h/(m_e c)$, which is in principle correlated with a value of the fine structure constant α (see below); it cannot be directly measured with high accuracy but can be extracted from adjusted data on R_∞, α etc. Such data are related to blocks, but only as their output results.

There are also certain data which are completely *uncorrelated* with the two big blocks as, e.g. the results for the Newtonian constant of gravitation G.

Constant	Value	u_r	Comment
c	$299\,792\,458\,\mathrm{m/s}$	0	exact[*]
μ_0	$4\pi \times 10^{-7}\,\mathrm{N/A^2}$	0	exact[*]
R_∞	$10\,973\,731.568\,525(73)\,\mathrm{m^{-1}}$	$[6.6 \times 10^{-12}]$	auxiliary[⋆]
m_p/m_e	$1\,836.152\,672\,61(85)$	$[4.6 \times 10^{-10}]$	auxiliary[⋆]
m_e	$5.485\,799\,094\,5(24) \times 10^{-4}\,\mathrm{u}$	$[4.4 \times 10^{-10}]$	auxiliary[⋆]
α^{-1}	$137.035\,999\,11(46)$	$[3.3 \times 10^{-9}]$	α-block[⋆]
$\lambdabar_C = \hbar/(m_e c)$	$386.159\,267\,8(26) \times 10^{-15}\,\mathrm{m}$	$[6.7 \times 10^{-9}]$	α-block[†]
$h\,N_A$	$3.990\,312\,716(27) \times 10^{-10}\,\mathrm{J\,s/\,mol^{-1}}$	$[6.7 \times 10^{-9}]$	α-block[†]
$R_K = h/e^2$	$25\,812.807\,449(86)\,\Omega$	$[3.3 \times 10^{-9}]$	α-block[⋆]
e	$1.602\,176\,53(14) \times 10^{-19}\,\mathrm{C}$	$[8.5 \times 10^{-8}]$	h-block[‡]
h	$6.626\,069\,3(11) \times 10^{-34}\,\mathrm{J\,s}$	$[1.7 \times 10^{-7}]$	h-block[⋆]
N_A	$6.022\,141\,5(10) \times 10^{23}\,\mathrm{mol^{-1}}$	$[1.7 \times 10^{-7}]$	h-block[⋆]
m_e	$0.510\,998\,918(44)\,\mathrm{Mev}/c^2$	$[8.6 \times 10^{-8}]$	h-block[†]
m_e	$9.109\,382\,6(16) \times 10^{-31}\,\mathrm{kg}$	$[1.7 \times 10^{-7}]$	h-block[†]
$K_J = 2e/h$	$483\,597.879(41) \times 10^9\,\mathrm{Hz\,V^{-1}}$	$[8.5 \times 10^{-8}]$	h-block[⋆]
G	$6.674\,2(10) \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$	$[1.5 \times 10^{-4}]$	independent

Table 1. The recommended values of some fundamental constants [1] and their subdivision into the adjustment blocks. Here, u_r is the relative standard uncertainty. Comments: [*] – fixed by the current definition of the SI units; [⋆] – measured and adjusted; [†] – derived from the adjusted data; [‡] – e is not measured directly, but its various combinations with h and N_A.

The first block is formed by the data related to the fine structure constant α. It also includes the so-called molar Planck constant hN_A and various results for the particle and atomic masses in the frequency units (i.e., in the result for the value Mc^2/h related to the mass M). The results in the frequency units are related to α because of the equation

$$
\begin{aligned}
R_\infty &= \frac{\alpha^2 m_e c}{2h} \\
&= \frac{1}{2c}\,\alpha^2\,\frac{Mc^2}{2h}\,\frac{m_e}{M} \;,
\end{aligned}
\tag{1}
$$

where M is related to the mass of the particle or atom measured in an experiment in the frequency units and we remind that the Rydberg constant and a number of important mass ratios m_e/M are known with higher accuracy.

The molar Planck constant hN_A enters this block as a conversion factor between two units in which microscopic masses can be measured with a very high accuracy, namely, the unified atomic mass units and frequency units.

The other block is formed by somewhat less accurate data related to the electron charge e, the Planck constant h and the Avogadro constant N_A. Because of the high accuracy obtained for the fine structure constant

$$
\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}
\tag{2}
$$

and the molar Avogadro constant hN_A, the final results for these three constants are strongly correlated.

4   Electrical data

An important feature of these two blocks is a substantial involvement of electric data related to standards and to some other macroscopic measurements. Two fundamental constants of quantum macroscopic effects play an important role there: the von Klitzing constant

$$
R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha} \;,
\tag{3}
$$

which describes the quantized resistance in the quantum Hall effect, and the Josephson constant

$$
K_J = \frac{2e}{h} \;.
\tag{4}
$$

The related data are often refereed in a very confusing way. For instance, in the so-called measurement of the von Klitzing constant R_K the crucial part is not the measurement proper, but a construction of a reference resistance, which should have a known value in the SI units. The only opportunity for such a resistance, or rather for an impedance, is based on a so-called calculable capacitor. Surprising for devices based on classical physics, the value of the capacitance of

certain symmetric configurations can be set with high accuracy. That is because of the special topological Thompson-Lampard theorem. Realizations of this theorem have recently provided us with classical-physics standards of the SI farad and ohm for a long period. At present a realization of the Thompson-Lampard capacitor in the only way to determine a value of R_K directly.

The watt-balance experiments do not involve any balance which deals with the power. They deal with a special kind of ampere balance which can be run in the dynamic and static regime. The static regime involves an electric current, while the dynamic one deals with an induced potential. Combining two measurements we arrive at a new quantity, power, as their product with an unknown geometric factor completely vanishing in the final equation.

A number of electric measurements deal with the gyromagnetic ratio or the Faraday constant. In practice, they do that in a very specific way. We have been numerously told from the high school time that we have to use the International System of Units, the SI, (despite certain resistance of the physical community). And that is under control of the International Committee on Weights and Measures, CIPM. However, the CIPM has sanctioned a departure from the SI system in precision electric measurements, for which so-called practical units were recommended in 1990 [7]. The latter, ohm-90 and volt-90, are based on certain fixed values of R_K and K_J [7] and all accurate electric measurements have been performed in these units.

If one declares a measurement of a certain electric quantity A (e.g., the gyromagnetic ratio of a proton in water), in practice the value actually measured in the SI units is somewhat more complicated

$$AR_K^n K_J^m \; , \tag{5}$$

where n and m are certain integer numbers $(0, \pm 1, \pm 2)$ which depend on the experiment.

This issue is so non-trivial, that measuring the same quantity, e.g., the gyromagnetic ratio of a proton, by different methods, 'in a low magnetic field' and 'in a high magnetic field', we arrive at very different results: a determination of α in former case and of h in the latter, because of difference in values of n and m. That is a kind of a metrological joke because even the units of the gyromagnetic ratio are different because of involvement of factors such as V_{90}/V. Such factors appear because in certain situations we cannot avoid applying the SI since the value of the magnetic constant μ_0 is known exactly in the SI units and we also have to deal with the practical units as long as a real measurement is concerned.

One more confusing example is a measurement of the Compton wave length of a neutron $h/(m_n c)$. The experiment consisted of two important measurements: one is related to the de Broglie wave length $\lambda_v = h/(m_n v)$ and the other to the velocity v. They were measured in a sense in quite different units. The velocity was determined in the proper SI units directly. Meanwhile the wave length λ_v was compared with the lattice spacing of a certain crystal. This crystal was indirectly compared with a so-called perfect crystal, basically used for the Avogadro project. Because of that the h/m_n result is strongly correlated with

a certain block of the data related to N_A and it is not just an isolated result related to a neutron.

Unfortunately, this customary practice with labelling the results is very confusing and for a non-expert it is hard to understand what was really measured and which data are correlated.

5   Recommended values and the 'less accurate' original results

Now, we can describe the adjustment. In the first approximation, we have to evaluate the most accurate data only (i.e., the auxiliary data), next to deal with the results from the α block and afterwards to adjust the h-block. That should give a good approximate result.

In reality, the less accurate data can still affect more accurate data, often marginally, but not always. The adjustment is very similar in a sense to a simple least-square procedure, where the statistical weight of data drops down with increase of their uncertainty. However, the less accurate data are still very important. If they agree with the main part of the data, that increases the final reliability of the evaluation, which is not just a question of the χ^2 test. We always want confirmations, even not very accurate, but independent. However, with such a large amount of data some may disagree. In such a case the less accurate data can have very important impact on the final results.

The data are strongly correlated and one may wonder what should be done by a user if certain input data are inconsistent as it actually happens from time to time. If the accuracy of the application is really sensitive to what value of the constant to take, one should avoid using the CODATA tables, and use instead the CODATA analysis of the input data. If accuracy is not important, it is better to use the same data all over the world, i.e. the data from the CODATA tables, and it should not matter whether they are well consistent or not.

6   The fine structure constant α and related data

Let us consider a situation with the fine structure constant as an example. The CODATA's result

$$\alpha^{-1} = 137.035\,999\,11(46)\,, \quad [3.3 \times 10^{-9}]\,, \tag{6}$$

is based mainly on a datum from the anomalous magnetic moment of an electron a_e. All the related contributions are shown in Fig. 1.

The fine structure constant α plays a crucial role in quantum electrodynamics (QED) and because of that a few questions may arise.

- Could we use the CODATA's value to test QED theory? The answer is negative. Comparisons of theory and experiment, which are the most sensitive to a choice of α, have already been included into deduction of the result
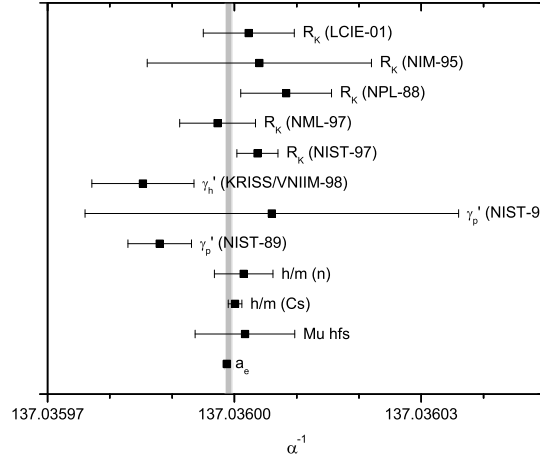
Fig. 1. The fine structure constant α. The vertical strip is related to the CODATA recommended values. The original results are explained in [1].

(6). If we like to check a particular QED effects we should apply a value of α obtained without any use of the effects under question. The CODATA adjusted value includes in principle all QED effects, for a precision test of which we need an accurate value of α.

- Which value of α can we use then? The answer depends on what kind of a test we would like to perform. If we would like to test QED 'absolutely', we should take the best non-QED value which is

$$\alpha^{-1}(\mathrm{Cs}) = 137.036\,000\,1(11)\,, \quad [7.7 \times 10^{-9}]\,, \tag{7}$$

a result, derived from the Rahman spectroscopy of the caesium atom. If we like to check consistency of QED, we can take one of QED-related values such as

$$\alpha^{-1}(a_e) = 137.035\,998\,80(52)\,, \quad [3.8 \times 10^{-9}]\,, \tag{8}$$

and use it for a calculation of other QED effects, such as the hyperfine interval in the muonium atom.

- If we calculate a value which is very sensitive to a choice of α among known values, what have we to do? The best choice is to reverse the situation, i.e., determine α and put it into Fig. 1. In this case we can see whether it agrees with various values. Sometimes the data are not in good agreement and a new value can completely change the situation.
- If we like to determine α, what is the crucial level of accuracy? Let us assume for a moment that the data are perfectly consistent. In such a case the crucial

accuracy is that of the second value in the row, which is (7). This value is vital for the reliability of the CODATA result. We remind that the dominant contribution to (6) comes from the anomalous magnetic moment and the result (8) has not been confirmed either experimentally or theoretically.

- That is not an unusual situation. The most advanced experiments and calculations are hard to repeat or confirm. Meanwhile, they have entered 'terra incognita' and despite high quality of the research teams they are most vulnerable because of lack of experience or rather a wrong 'experience' based on trusted unimportance of various phenomena which may become important. For instance, for recommendation of conservative committees of CIPM they sometimes introduce a kind of factor or reliability for accurate measurements, which may increase the uncertainty tenfold [8].

- Have we to trust all data for α? That is not exactly the case since there is no appropriate theory for the quantum Hall effect which provides us with five data points.

- However, the agreement is good, but not perfect. We note that two values with the gyromagnetic ratio of a proton are within certain disagreement with the most accurate value. From a purely scientific point of view we have a rather good general agreement (cf. with the situation on h and G below). Nevertheless, there is an application which deals with a practical unit of resistance by CIPM [7]. They conservatively estimate an uncertainty as a part in 10^7. The related value of the fine structure constant is

$$\alpha^{-1}(\mathrm{CIPM}) = 137.035\,997(14)\,, \quad [1 \times 10^{-7}]\,.$$

We should mention, however, that CIPM is overconservative because their results may have legal consequences and their examinations are for this reason not just a kind of scientific researches (see [6] for further discussion).

Actually, that is a strange story how we deal with, e.g., 3σ-off points. When they are a part of a large statistics set of similar measurements, we are satisfied by the χ^2 criterium. Meanwhile, when the data are different such as for the adjustment of the fundamental constants or QED tests with different systems, we sometimes pay special attention to such 'bad' points trying to understand what is wrong in their particular cases.

A comparison of α, extracted from a particular QED value, let us say, the muonium hyperfine interval, after a certain improvement of theory, with other α's has a number of additional reasons (in respect to a comparison with the CODATA recommended value only).

- The muonium datum has been already used for determination of α in (6). Despite the fact that is has a marginal effect, it is not appropriate to compare a certain improvement of $\alpha(\mathrm{Muhfs})$ with an average value, which includes an earlier version of $\alpha(\mathrm{Muhfs})$. The new and old values are based on the same experiment and the very appearance of the new value means that the old value is out of date.

- If we have a contradiction, we can clearly see whether the new value contradicts to one or two most accurate data but agrees with the most of the rest or so, or it disagrees with all.
- Known data experience corrections from time to time. Using a set of original data, one can introduce the proper corrections. However, there is no way to correct the CODATA value, except indeed redoing the adjustment.

The latter is a result of a complicated procedure which includes re-examination of accuracy of various data and test of their accuracy. It is not possible to update the list of recommended values very often. Because of that a substantial delay may take place. For instance, the most recent CODATA paper was published in 2005 and we can expect a new one in 2008. The deadline for the input data in [1] was the end of 2002. That means that any evaluation including data obtained since 2003 will not be available until 2008. Because of that it may be important in certain cases to consider original results reviewed in the recent CODATA paper [1] and add there new results, available since recently, if any.

7 The Planck constant h and related data

Determination of the fine structure constant has demonstrated a rather good agreement. The situation is not always so good. As an important example of a substantially worse agreement we present data related to the Planck constant h in Fig. 2.
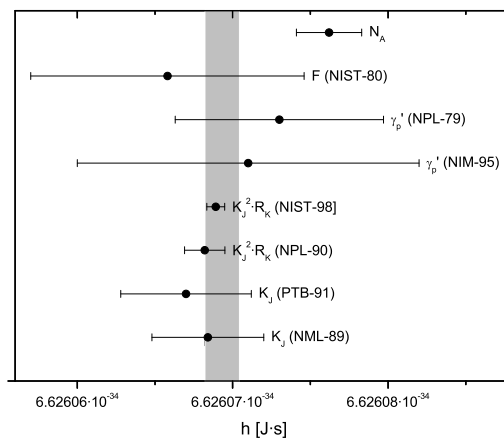

Fig. 2. The Planck constant. The vertical strip is related to the CODATA recommended values. The original results are explained in [1].

The data are not in good agreement. In particular, a result related to N_A contradicts to the most accurate data obtained from the watt-balance experiments. We will return to this result later. We need to mention that CIPM recommended a value of the Josephson constant $K_J = 2e/h$ with a conservative uncertainty of 2 parts in 10^7 while their conservative value of $R_K = h/e^2$ has uncertainty of a part in 10^7. The related value for the Planck constant is

$$h(\text{CIPM}) = 6.626\,068\,9(53) \times 10^{-34} \text{ J s}\,, \quad [8.1 \times 10^{-7}]\,.$$

8   The Newtonian constant of gravitation

The results on the Newtonian constant of gravitation G show an even much worse situation with a scatter superseding the uncertainty by many times (see Fig.3).
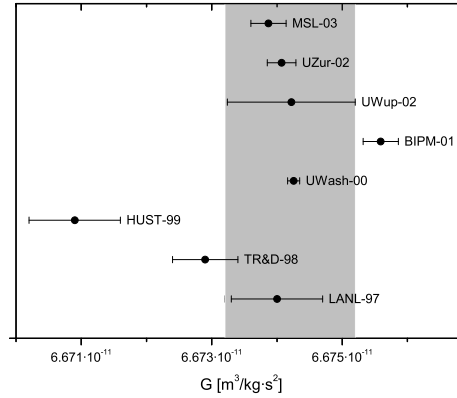

Fig. 3. The Newtonian constant of gravitation. The vertical strip is related to the CODATA recommended values. The original results are explained in [1].

Despite the gravitation constant is without any doubt one of the most fundamental constants, its accuracy does not have great importance. Fundamentality of G shows itself first of all in the application to quantum gravity where the obtained results are rather qualitative than quantitative. Another important application is due to general relativity. Precision tests of general relativity involve much higher accuracy than the one in the determination of the Newtonian constant. For actual problems, the most important constant is a product of a gravitating mass (of Sun or Earth) and G and such products have been known much more accurately than G and from completely different kind of data.

Still there is a kind of experiments of fundamental nature which are in part similar to measurements of G, namely, studies of equivalence principle in laboratory distance scale. However, such experiments are differential and essential part of uncertainty should cancel out.

As a result, we note that the determination of G is indeed an ambitious and important problem, but it is somewhat separated from both the rest of the precision data and applications of fundamental physics.

9   The fundamental constants and their numerical values

The discussion above raises a more general question on fundamental constants and their values. The numerical value of a dimensional fundamental constant involves the units and thus involves a certain kind of phenomena which are used to determine units. Such an involvement can change the physical meaning when going from the constant to its value drastically.

While the constants, such as the speed of light or the Planck constant are determined by Nature, their numerical values can be treated with a certain room for arbitrariness. We can, e.g., adopt certain numerical values by definition.

In the case of variability of the constants, the interpretation of possible changes of the constants and their numerical values is quite different (see, e.g., [9]).

Two constants, discussed above, h and G, are truly fundamental, but they are not very often needed for accurate calculations. Below we consider certain values more closely related to atomic and particle physics or, in more general terms, to microscopic physics.

10   Microscopic and macroscopic quantities

In microscopic physics nobody intends to apply any macroscopic unit such as a kilogram. However, the nature of the units is not a trivial issue. We should distinguish between their rough values and their definition. Rough values of various units have been determined historically. For most of the SI units they are macroscopic, such as for a kilogram or a second. Meanwhile, the SI kilogram is defined as a macroscopic unit, but the SI second at present is defined as a kind of atomic unit via the hyperfine interval in caesium-133 atom.

The only SI unit which has a clear historic microscopic sense is the volt. To proceed with potentials one dealt with breaking atomic or molecular bonds. A characteristic ionization potential is of a few volts and, in particular, in hydrogen it is about 13.6 V. A popular non-SI unit, the electron-volt possesses in a rough consideration a clear atomic sense. Because of this ionization issue, an energy, related to R_∞ is, indeed, 13.6 eV. However, if we look at the definition of the volt in a practical way, we find that the volt of the SI is defined via the ampere and the watt. The latter are defined via the kilogram, the metre, the second and a fixed value of the magnetic constant of vacuum μ_0. Because of presence

of the kilogram, the volt and the electron-volt have macroscopic meaning from the point of view of measurements.

Measuring microscopic values in terms of macroscopic units is always a complicated problem, which introduces serious unnecessary uncertainties. Meanwhile, the very use of the electron-volt in the atomic, nuclear and particle physics is an issue completely based on a custom and never related to real matter. It is a kind of illusion. However, for missing a difference between reality and illusion, one has to pay. The price is an unnecessary uncertainty in various data, expressed in the electron-volts and a correlation between uncertainties of various data.

The electron-volt is widely used in microscopic physics. In particular, it is customarily applied to characterize the X-ray and gamma-ray transitions by their energy and to present particle masses in units of GeV/c^2. We have to emphasize that nobody performs any precision measurement in these units in practice. The transitions are measured in relative units. To measure them absolutely one has to apply X-ray optical interferometry and either compare an X-ray and an optical wave length, or calibrate a lattice parameter in a certain crystal in terms of an optical wave length. That means that in actual precision measurements one really deals with the wave length (or related frequency) and not with the energy of the transition.

The most accurate relative measurements of hard radiation are in fact more accurate than the conversion factor between the frequency and the energy, namely, e/h (if the energy is measured in the electron-volts). The uncertainty of this coefficient is presently 8.5×10^{-8} [1]. We strongly recommend for transition frequencies measured more accurately than 1 ppm to present results in frequency units and for results in the electron-volts to present separately two uncertainties: of the measurement and of the conversion into electron-volts. It would be also helpful to specify explicitly the value of the conversion factor used.

If one even tries to measure energy in electron-volts 'by definition', the electron-volts proper are still not the best choice. CIPM recommended a practical unit, volt-90, in terms of which the Josephson constant K_J has an exactly fixed value [7]. In such a case, the result would be expressed in terms of eV_{90}, rather than in eV's. The uncertainty of the conversion factor e/h in practical units is zero.

Mev's and Gev's are also widely used for the masses of particles and for the energy excess in nuclear physics. From the point of view of accuracy, such units are not better than kilograms. The best choice is to apply direct results of relative measurements (mass ratios), when available, or to express the masses in terms of either of the two adequate microscopic units. One of the latter is the unified atomic mass unit, u, and the other corresponds to the frequency related to mc^2/h. In these two units elementary masses are known with the highest accuracy.

11 Reliability of the input data and the recommended values

The easiest part of the evaluation is their mutual evaluation. Two most important questions are related to the data.

1. Not all available data are included as input data and not all input data are exactly equal to the originally published data. The question to decide prior to the evaluation is how to treat each piece of data? Should we accept them "as they are", or assign them a corrected uncertainty, or even dismiss some of them prior to any evaluation procedure? That should be decided on base of quality of the data.

2. After initial probe mutual least-square evaluations are done, we used to see that some pieces are not in perfect agreement with the rest of the data. That cannot be avoided once we have many pieces of data. That opens another important question, to be decided at the initial stage of the evaluation. How should we treat the data when they are combined together? In other words, should we do anything with the data due to their inconsistency if any? At this stage the decision is partly based on their consistency, partly on their correlations and partly still on their initial properties.

These questions are to be decided not on base of statistics (like when in an easy case of a number of data points for the same quantity one drops the smallest and the largest results) but first of all on base of their origin, their experimental and theoretical background.

The CODATA's recommended values are the best one, but in principle that does not mean all of them are really good. They are the best because the authors perform the best possible evaluation of existing data. If data are not good enough, the result of any evaluation cannot be good. The CODATA task group are not magicians. That is why it is essential to have independent results for each important quantity. Below we consider a question of the reliability of data important in atomic and particle physics.

The conservative policy of CIPM and discrepancy in the input data (see Fig. 2) show that direct use of the CODATA result is not a single option to be considered. The CIPM treatment of the data does not contradict to the CODATA approach, because CIPM applies the CODATA analysis; however, prefers to derive a more conservative result from the CODATA's consideration.

An important illustration of reliability of the recommended values is presented in Fig. 4. While for most of them progress with time reduced the uncertainty, sometimes (e.g., for h or G) better understanding meant appearance of a discrepancy.

12 Proton properties

Among the particles listed in CODATA tables [1] two, a proton and muon, are of particular interest. The most confusing datum on a proton is its charge radius,
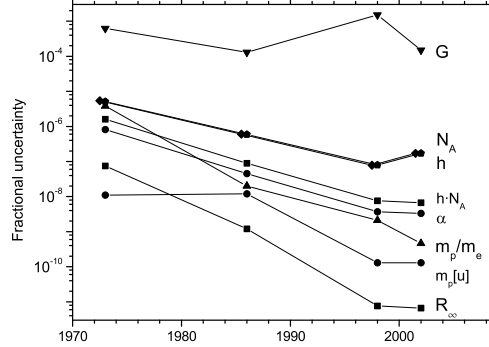
Fig. 4. Progress in determination of fundamental constants by the CODATA task group (see [1,2] and references to earlier results therein).

R_p. The CODATA paper recommends the result

$$R_p(\text{CODATA}) = 0.8750(68) \text{ fm} , \qquad (9)$$

which, in principle, is based on all available data including electron scattering and hydrogen spectroscopy. Nevertheless, we would not recommend to apply this result blindly to any sensitive issue. The dominant contribution comes from spectroscopy of hydrogen and deuterium and the related theory. The spectroscopic data included various experiments, which partly confirm each other. However, a substantial progress made in the theory (the Lamb shift) is related to surprisingly large higher-order two-loop terms [10], which are neither understood qualitatively nor independently confirmed quantitatively. I would not consider the theoretical expressions at the moment as a reliable result until their proper confirmation or understanding. Such a need for an independent confirmation is a characteristic issue for any breakthrough in either theory or experiment.

The second (in terms of accuracy) result mentioned in [1]

$$R_p(\text{Sick}) = 0.895(18) \text{ fm} , \qquad (10)$$

is the one obtained by Sick [11] from the examination of world scattering data. This piece of CODATA input data is very specific. CODATA very seldom accepts any evaluation of world data without performing a critical reconsideration. A crucial feature of the CODATA treatment of the world data is reconsideration of accuracy on experimental and theoretical results. The most important scattering results were obtained long time ago. They dealt with QED scattering corrections obtained a few decades ago. At present, the QED corrections are known better, but there is no simple way to reevaluate the existing scattering data. The Sick's examination is the most competent I have ever seen. But it is an evaluation of the data "as they were published".

The problem of correcting the experimental data because of a possibly unappropriate treatment of higher-order radiative corrections by the original authors, was not addressed in his evaluations. I would rather consider the central value of this evaluation as a valid one but would somewhat increase the uncertainty (see discussion in [12]) achieving

$$R_p = 0.895(30) \text{ fm} . \tag{11}$$

It is hard to be more precise with the uncertainty. If such a reevaluation were done in the CODATA paper, the problem should be addressed. But in was not done in [1]. A reason not to do that is twofold.

First, it is an obvious fear that the job could not be done properly because of lack of necessary information for experiments done long time ago. Next, the proton size from the scattering plays rather minor role in the adjustment. The evaluation of the auxiliary block with the Rydberg constant is sensitive to theory of a so-called state-dependent part of the Lamb shift of the n states and to theory of states with a non-zero orbital moment. Both depend on a value of the proton size marginally. That means that CODATA evaluation of the Rydberg constant needs only a very rough value of the proton size and we can accept any result for R_p for such an evaluation.

The recommended value of the proton charge radius is actually determined by the same spectroscopic study. The rest of the data can rather produce a marginal effect on the value of R_p. In particular, the second value of the radius, obtained from the scattering, is rather out of interest of the CODATA evaluation and they do not care about it. The reevaluation of the world scattering data from the CODATA side looks like an unnecessary overcomplicated problem with unclear reliability of the outcome.

One more proton property of interest is its magnetic moment, or rather electron-to-proton ratio of the magnetic moments

$$\frac{\mu_e}{\mu_p} = 658.210\,6860(66) , \quad [1 \times 10^{-8}] . \tag{12}$$

The result is completely based on an MIT experiment performed long time ago [13]. While for the most important constants such as α and h one can easily find all sources for particular results in [1], it is hard to see what result is the second in accuracy. While details of the analysis will be published elsewhere, here we conclude that the data may be obtained from a study of the muonium magnetic moment and the most accurate partial result

$$\frac{\mu_e}{\mu_p} = 658.210\,70(15) , \quad [2.3 \cdot 10^{-7}] \tag{13}$$

is much less accurate than the MIT value.

13   Muon properties

The muon data include the muon magnetic moment, mass and a_μ, the anomalous magnetic moment. The latter should not be used at all for any sensitive

issue. The CODATA can make a reasonable prediction only after the situation is settled, while for a_μ it is not. Speaking more generally, CODATA is a brand for the best constants, but not all products with this brand are equally good. Critical examination of input data can improve their reliability and reduce their scatter. I would say that is the most competent evaluation of world data on the fundamental constants. Nevertheless, there is no magics in the CODATA adjustment and the result cannot be better than the input data allow. Before trusting any particular CODATA result one has to take a look into the data analysis.

The result for a_μ has contributions from experiment and theoretical evaluations based on e^+e^- and τ data. To consider physics we should not average these partial results but reexamine and compare them.

The mass and magnetic moment have been used numerously in a quite confusing way. The experiment, most sensitive to their values, has been included into the evaluation. For instance, one can apply a value of m_e/m_μ (or μ_μ/μ_p) to the hyperfine interval in muonium, either assuming QED to determine α, or accepting a certain value of α to verify QED. However, the CODATA results

$$\frac{m_\mu}{m_e} = 206.768\,283\,8(54)\,, \quad [2.6 \times 10^{-8}]\,,$$

$$\frac{\mu_\mu}{\mu_p} = 3.183\,345\,118(89)\,, \quad [2.6 \times 10^{-8}]\,, \tag{14}$$

are dominated by a value extracted from the muonium hyperfine interval assuming a certain value of α and validity of QED. The second best set

$$\frac{m_\mu}{m_e} = 206.768\,276(24)\,, \quad [1.2 \times 10^{-7}]\,,$$

$$\frac{\mu_\mu}{\mu_p} = 3.183\,345\,24(37)\,, \quad [1.2 \times 10^{-7}]\,, \tag{15}$$

comes from separate data and may be used to either determine α or test QED.

We emphasize that all this information is contained in the CODATA papers [2,1]; however, since 'simple users' are more interested just in the tables they usually miss it.

We remind that there is a number of compilations of various kinds of data around the world and even reading carefully most of the compilations, there is no chance to find detail of input data. Sentences such as 'the uncertainty does not include systematic error' or so are often missing when a datum came from the original paper to a compilation. The CODATA paper is one of very few exceptions, however, a way of reader's treatment of the CODATA papers sometimes doesn't make use of this advantage.

14   Impact of a redefinition of the kilogram on values of the fundamental constants

To conclude the paper, we would like to discuss two issues. One is rather technical and related to a possible redefinition of the kilogram and the ampere in

terms of fixed values of h and e [14]. It is most likely that this redefinition will be adopted, but it is unclear when. The numerical values of the fundamental constants play two roles. One is that they represent in a numerical way certain experimental data. Redefining the kilogram, obviously the experimental results would not change and the information would not be added. Still, a certain pieces of the information and related uncertainty can be removed from some data. After redefinition of units, certain experiments done with a relatively low accuracy could be isolated from the fundamental constants (e.g., any direct study of the prototype of the kilogram would have no relation to basic physical quantities anymore). The other role of the numerical values is that they are reference data. As we mentioned above, it is customary to use, without any experimental or theoretical reasons, the electron-volts. The redefinition of the kilogram and the ampere would establish them as microscopic units (and the volt as well). The conversion factor e/h would be known exactly. That means that all values in electron-volts would have adequate accuracy.

15   Legacy of the adjustment of the fundamental constant

The last question to discuss here is a conceptual one. Doing precision physics, we cannot ignore the very fact that we accept a large number of physical laws. Sometimes they are proved with a certain accuracy, sometimes they are not.

For instance, there is no accepted theory which demands that the electron charge and the proton charge be of the same value. We have various direct experimental tests, but those are always limited by their accuracy. The conceptual evidence should come from a new theory, which is confirmed experimentally. We strongly expect a certain unification theory, but no evidence has been available up-to-date.

We expect that the fundamental constants are really constant, but we do not understand their origin and we (or most of us) believe that during the inflation epoch of the universe some constants such as m_e/m_p changed. So, the constancy of the constants is merely an experimental fact. What is even more important, certain physical laws are put into the very base of our system of units, the SI, and if they would occur incorrect, one may wonder whether that is detectable. The answer is positive. If we adopt a set of assumptions, either with an internal inconsistency or inconsistent with Nature, we should be able to see either an inconsistency in the interpretation of the results (e.g., a contradiction within two determinations of the same quantity) or a discrepancy between the trusted assumption and the observed reality.

To test any particular law, one has to rely on specific experiments sensitive to such a violation. The CODATA examination is mainly based on the assumption that we can follow the known physical laws. We know that any particular physical theory is an approximation. Combining the data from different fields we check the consistency of the overall picture (both: the laws and the approximations) and the result obtained is satisfactory. Up to now.

References

1. P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **77**, 1 (2005).
2. P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).
3. http://physics.nist.gov/cuu/Constants/index.html.
4. S. Eidelman *et al.*, *The Review of Particle Physics*, Phys. Lett. B**592**, 1 (2004).
5. P. J. Mohr and B. N. Taylor, Phys. Today (August 2000) BG6; (March 2001) 29.
6. S. G. Karshenboim, Physics-Uspekhi **48**, 255 (2005).
7. T. J. Quinn, Metrologia **26**, 69 (1989) (see p. 70);
   T. J. Quinn, Metrologia **38**, 89 (2001) (see p. 91);
   T. J. Quinn, Metrologia **26**, 69 (1989) (see p. 69).
8. T. J. Quinn, Metrologia **40**, 103 (2003).
9. S. G. Karshenboim, Can. J. Phys. **83**, 767 (2005).
10. K. Pachucki, Phys. Rev. A**63**, 042503 (2001); K. Pachucki and U. D. Jentschura, Phys. Rev. Lett. **91**, 113005 (2003).
11. I. Sick, Phys. Lett. B**576**, 62 (2003).
12. S. G. Karshenboim, Phys. Rep. **422**, 1 (2005).
13. P. F. Winkler, D. Kleppner, T. Myint and F. G. Walther, Phys. Rev. A**5**, 83 (1972).
14. I. M. Mills, P. J. Mohr, T. J. Quinn, B. N. Taylor and E. R. Williams, Metrologia, **42**, 71 (2005); **43**, 227 (2006).