Applications in Credit Risk_Second Part

- ## Technical indicators
  - Technical indicators are mathematical calculations or tools that assist traders or investors for making decisions by analyzing market behavior and patterns.
  - ### Overlays
    - use the same scale as prices and focus on tracking price movements and trends by directly overlaying on the price chart. Overlays are helpful to Identify price trends and key price levels.
    - #### Bollinger Bands
      - helps traders identify price movements focusing on price and volatility. **Wide Bands: the market is volatile. Narrow bands: lower volatility and a potential breakout or trend change.**
      - Middle Band (Simple Moving Average: SMA)
        - **20 days' (20 periods) average of prices** and this represents serves as the base for calculating the upper and lower bands for a volatility range.
      - Upper Band(UB) = SMA + (2 × Standard Deviation of Price)
        - When the price touches or moves above the UB, it indicates an **overbought** condition.
      - Lower Band(LB) = SMA - (2 × Standard Deviation of Price)
        - When the price touches or moves below the LB, it indicates an **oversold** condition.
    - #### Average True Range(ATR)
      - Average True Range (ATR) : A technical analysis indicator that **measures market volatility by decomposing the entire range of an asset price for that period.**
      - $$TR = \max[(H - L), |H - C_P|, |L - C_P|]$$

        $$ATR = \frac{1}{n}\sum_{i=1}^{n} TR_i$$

        - TR_i: A particular True Range
        - n: The time period employed
        - H: Current High
        - L: Current Low
        - C_P: Previous Close
  - ### Oscillators
    - shows a **momentum, strength, and potential buy/sell in a price direction**. Oscillators are helpful to Identify potential reversals or momentum shifts. Oscillators typically appear in a

separate panel.

- Stochastic Oscillator (SC): A a momentum indicator that **compares an asset's closing price to a range of its prices over a certain period.** It is primarily used to identify overbought or oversold conditions.
- Relative Strength Index (RSI): A momentum indicator that **measures the speed and change of price movements**, providing a reading from 0 to 100.
- Rate of Change (ROC): A momentum indicator that **tracks the speed at which price changes,** helping identify potential trend reversals or confirmations.
- Types
  - Stochastic Oscillator (SC)
    - %K Line(Fast Stochastic)
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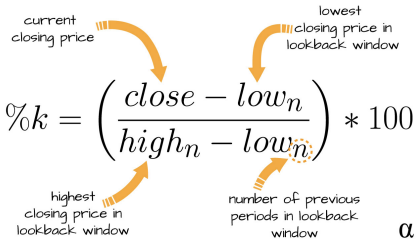

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    - %D Line(Slow Stochastic)
      - %D **represents the 3-period average of %K**
      - ▯▯▯▯ ▯▯▯▯▯▯▯▯▯▯ (▯▯, %▯) = ▯▯▯ (%▯, 3)
    - A **bearish** signal occurs when the **%K line crosses below the %D line.**
  - Relative Strength Index (RSI)
    - a momentum oscillator that measures the changes of **speed and magnitude of price.**
    - 14 days period.

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$$RS = \frac{Avg.Gain}{Avg.Loss}$$

$$RSI = 100 - \frac{100}{1 + RS}$$

      - Average Gain:
    - >70 overbought; below 30 is oversold; 50 neutral)
  - Rate of Change (ROC)
    - **measures the percent change in price from one period to a specific number of previous periods**
    -

$$Rate\ of\ Change\ (ROC) = \frac{Closing\ at\ T - Closing\ at\ (T-N)}{Closing\ at\ (T-N)} \times 100$$

- ROC goes down below 0, this signals a **bearish momentum**

- Percentage of price oscillator (PPO)
  - PPO shows a percentage value of the difference in terms of long term average.
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$$\text{PPO} = \frac{\text{12-period EMA} - \text{26-period EMA}}{\text{26-period EMA}} \times 100$$

$$\text{Signal Line} = \text{9-period EMA of PPO}$$
$$\text{PPO Histogram} = \text{PPO} - \text{Signal Line}$$
$$\textbf{where:}$$
$$\text{EMA} = \text{Exponential moving average}$$

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    - When PPO goes up above 0, this indicates a bullish momentum (short-term MA is above long-term MA).
    - When PPO goes down below 0, this indicates a bearish momentum (short-term MA is below long-term MA).
    - When PPO line crosses above its signal line, this signal indicates upward momentum.
    - When PPO line crosses below its signal line, this signal indicates downward momentum.
    - Since PPO is a value on a percentage basis, comparing assets with different price levels is more effective.

- MACD (Moving Average Convergence Divergence)
  - MACD line: the relationship between two moving averages of a security's price
  - 12 periods EMA - 26 periods EMA
  - Signal Line: 9-period EMA
  - Histogram: Difference between MACD line and signal line

Financial Statements

- **Balance sheet （Assets= liabilities + Equity）**
  - shows a company's financial position with assets, liabilities, and equity at a specific point in time.
  - Assets: Resources owned by the company.
  - Liabilities: Obligations owed to others.(debts, bonds payable and loans)
  - Equity: Owner's residuals in the company.(retained earnings, preferred stock and common stocks)
    - Equity shows how much of the assets owners truly own
- **Income Statement (Profit & Loss Statement)**

- this statement measures revenue, expenses and profit over time. shows a company's performance and profitability.
- COGS: direct costs involved in producing the goods that a company sells.
- Operating income = gross profit - operating expenses
- Net income = Operating income - taxes
- **EBIT(Earnings Before Interest and Taxes) = Net income + Interest + taxes**

  EBIT = Total Revenue – Costs of Goods Sold (COGS) – Operating Expenses* – Depreciation & Amortization

  or

  EBIT = Net Income + Interest + Taxes
- **EBITDA = Net Income + Interest + Taxes + Depreciation + Amortization**

- **Cash Flow Statement**
  - insights into operating, investing, and financing activities(**key information for accessing liquidity and solvency**)

Fundamental Analysis

- by analyzing the financial data from the financial statements to see 'fundamental' of the company.
- 1. ROI(return on investment)

$$ROI = \frac{\text{Net Profit (Net income)}}{\text{Investment Cost}} \times 100$$

  - ROI measures profitability relative to cost.
  - It helps compare efficiency across investments.
  - Used to allocate investment resources to the profitable ventures.
  - High ROI indicates favorable returns.
- 2. ROE(Return on Equity)
  - how efficiently a company uses equity

$$ROE = \frac{\text{Net Income}}{\text{Shareholders' Equity}} \times 100$$

    - ROE measures how efficiently a company uses equity.
    - It's calculated by dividing net income by shareholders' equity.
    - Higher ROE means the company is more efficient at generating profit.
    - Used to compare companies in the same industry.
- 3. PBR(Price to Book Ratio)
  - measures the valuation of a company on the trading market relative to its corresponding book value of the company.
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$$PBR = \frac{\text{Market Price per Share}}{\text{Book Value per Share}} \text{ or } \frac{\text{Market Capitalization}}{\text{Total book } value\ of\ Equity}$$

- PBR compares market value to book value.
- PBR < 1 indicates an undervalued company.
- High PBR may indicate overvaluation or currently popular company
- Important for value investors looking for undervalued stocks.
- In general, IT companies have higher PBR than traditional industries, reflecting the value of intangible assets.

- 4. BPS (Book Value per Share)
  - measures a company's equity per share of common stock.
  - $$BPS = \frac{\text{Shareholders' Equity} - \text{(Prefered Equity)}}{\text{Outstanding Shares}}$$

    - BPS shows equity available to shareholders.
    - Calculated by dividing total equity (minus preferred equity) by outstanding shares.
    - Higher BPS means better financial health.
    - Helps assess if stock is undervalued relative to its book value.
- 5. PCR (Price to Cash Flow Ratio)
  - measures the current price of the company's stock relative to operating cash generated by the company's business.
  - $$PCR = \frac{\text{Market Price per Share}}{\text{Cash Flow per Share}} \text{ or } \frac{\text{Share price} \times Number\ of\ outstanding\ share}{\text{Operating cash Flow}}$$

    - Measures price relative to cash flow per share.
    - Indicates how much investors pay per each dollar of cash flow.
    - Commonly used in capital-intensive industries.
    - Lower PCR is typically more favorable. This indicate that the company has strong cash flow relative to its price.
- 6. PSR(Price-to-Sales Ratio)
  - measures the market value of a company in relation to the total amount of sales.
  - $$PSR = \frac{\text{Market Price per Share}}{\text{Sales (Revenue) per Share}} \text{ or } \frac{\text{Share price} \times Number\ of\ outstanding\ share}{\text{Total } Sales}$$

    Compares stock price to revenue per share.
    Indicates how much investors are willing to pay for each dollar of sales.
    PSR doesn't consider or indicate a company's profitability.
    Lower PSR may indicate undervaluation.
- 7. Tobin's q
  - measures an asset's market value in relation to its total asset value
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$$Tobin's\ q = \frac{\text{Market Value of Firm}}{\text{Total } Asset\ value}$$

Measures market value relative to asset replacement cost.
Tobin's q > 1 suggests overvaluation.
Tobin's q < 1 suggests undervaluation.
Long-term investment metric.

- 8. EPS (Earnings per share)
  - measures a company's profitability divided by outstanding share of common stock.
  - $$EPS = \frac{\text{Net Income } - \text{Dividened for prefered stock}}{\text{Outstanding Shares}}$$

    - Key indicator of profitability.
    - High EPS indicates greater value because investors will pay more for stock investment with expectations of growth.
    - Commonly used to compare companies in the same industry.
- 9. PER(Price-Earnings Ratio)
  - measures a company's stock price relative to its earnings per share (EPS)
  - lower PER or Higher PER is not necessary indicates good or bad.

    $$PER = \frac{\text{Market Price per Share}}{\text{Earnings per Share}}$$

    - Higher PER can indicate investor expectations of growth.
    - Lower PER may suggest undervaluation.
    - Commonly used to compare companies in the same industry.
    - At S&P 500, average PER is around 25-30
- 10. DFL(Degree of Financial Leverage)
  - This indicates **debt impacts a company's profitability,** therefore, it can be a **potential risk of financial distress**
  - $$DFL = \frac{\%\ \text{Change in EPS}}{\%\ \text{Change in EBIT}}$$

    - Measures sensitivity of net income to changes in operating income.
    - High DFL indicates a company uses more debt to finance operations.
    - High DFL also increases risk during downturns.
    - Investors use DFL to assess a company's financial structure.

- Credit risk and risk management

- Theoretical and mathematical links Brownian Motion, Winer Process, Generalized BM, Geometric BM, Ito's lemma, BS option price model & Energy diffusion
  - Brownian Motion
    - Brownian motion refers to the random movement of particles, atoms, or molecules that are observed in fluids or gas or air.
    - **Key Properties of Brownian Motion B_t:**
      1. **Randomness**:
         - The motion is entirely random and unpredictable.
      2. **Gaussian Distribution**:
         - The increments of B_t (e.g., $B_t - B_s$ for $t > s$) are normally distributed.
      3. **Independent Increments**:
         - The increments $B_t - B_s$ and $B_u - B_v$ are independent if $[s, t] \cap [u, v] = \emptyset$.
      4. **Continuous Paths**:
         - The function B_t is continuous in t, meaning there are no jumps.
      5. **Start Point**:
         - $B_0 = 0$, indicating the motion starts at zero.
  - Winer Motion
    - The Wiener Process Wt is a specific mathematical formulation of Brownian Motion. It is widely used in probability theory and stochastic calculus to model continuous-time stochastic processes.
    - **Formal Definition of the Wiener Process:**

      The Wiener Process W_t satisfies the following:
      1. **Initial Value**:
         - $W_0 = 0$, the process starts at zero.
      2. **Independent Increments**:
         - For $0 \leq s < t$, $W_t - W_s$ is independent of W_u for $u \leq s$.
      3. **Normal Distribution**:
         - $W_t - W_s \sim \mathcal{N}(0, t - s)$, i.e., normally distributed with mean 0 and variance $t - s$.
      4. **Continuous Paths**:
         - W_t has continuous paths almost surely.
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Properties of the Wiener Process:

- **Expectation**: $\mathbb{E}[W_t] = 0$.
- **Variance**: $\text{Var}(W_t) = t$.
- **Stationarity of Increments**: Increments depend only on the time difference $t - s$.

Differential Form of Wiener Process:

In stochastic calculus, the Wiener Process is often written in differential form:

$$dW_t = \xi_t \sqrt{dt},$$

where $\xi_t \sim \mathcal{N}(0, 1)$ (standard normal random variable).

- where:
  - dW_t: Represents the infinitesimal change in the Wiener process W_t over an infinitesimal time interval dt.
  - ξ_t: A random variable drawn from a standard normal distribution, i.e., $\xi_t \sim \mathcal{N}(0, 1)$.
  - $\sqrt{dt}$: Scales the standard normal variable to reflect the variance growth proportional to the time interval dt.

- ### Applications of Wiener Process:

  1. **Stochastic Differential Equations (SDEs)**:
     - Used to describe systems influenced by random forces.
  2. **Finance**:
     - Models the random movement of asset prices (e.g., Black-Scholes model for option pricing).
  3. **Signal Processing**:
     - Models noise in systems.

- Generalized BM
  - A stochastic process with both a drift term (μ) and a Wiener process (Wt).
  - Generalized BM assumes additive randomness (Xt).
  - Generalized BM Definition

    Definition:

    The process X_t is given by:

    $$X_t = X_0 + \mu t + \sigma W_t,$$

    where:

    - X_0: Initial value of the process.
    - μ: Drift term, representing the deterministic trend over time.
    - σ: Volatility term, representing the randomness driven by the Wiener process W_t.

- **Generalized BM** Differential Form

  Differential Form:

  $$dX_t = \mu dt + \sigma dW_t,$$

  where:

  - μdt: Represents the deterministic drift component.
  - σdW_t: Represents the stochastic/random component.

- Application: Models processes with a linear drift and random volatility, such as changes in interest rates or physical systems.

- Geometric BM
  - Geometric BM assumes multiplicative randomness (St), making it suitable for modeling processes where values cannot be negative (e.g., stock prices).
  - A continuous-time stochastic process often used to model stock prices. It assumes the logarithm of the quantity follows a Brownian motion.
  - Geometric BM Definition

    Definition:

    The process S_t evolves according to:

    $$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t},$$

    where:

    - S_0: Initial value of the process.
    - μ: Drift term, representing the expected rate of return.
    - σ: Volatility term, capturing the randomness.
    - $\frac{\sigma^2}{2}$: Accounts for the variance correction (Itô's lemma).

  - Geometric BM Differential Form

    Differential Form:

    $$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

    where:

    - $\mu S_t dt$: Represents the deterministic growth proportional to the current value S_t.
    - $\sigma S_t dW_t$: Represents the stochastic growth proportional to the current value S_t.

  - Application: Widely used in finance to model stock prices and asset returns (e.g., Black-Scholes model for option pricing).

- Ito's Lemma
  - Ito's Lemma is a cornerstone of stochastic calculus. It provides a framework for computing the derivative of a time-dependent function f(t,Xt), where Xt follows a

stochastic process (often modeled as a Generalized Brownian Motion).

- Intuition for Ito's Lemma
  - It can be thought of as the **stochastic equivalent of the chain rule in calculus,** but with adjustments to account for the randomness introduced by dWt (the Wiener process increment).
  - Unlike deterministic calculus, stochastic processes involve terms proportional to *(dW_t)^2*, which are non-negligible and equal to *dt* due to the properties of *Wt*.
- Mathematical Statement

  Given a function $f(t, X_t)$, where X_t evolves according to the stochastic differential equation:

  $$dX_t = \mu dt + \sigma dW_t,$$

  Ito's Lemma states that the differential of $f(t, X_t)$ is given by:

  $$df(t, X_t) = \left(\frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2} \right) dt + \sigma \frac{\partial f}{\partial x} dW_t.$$

  Explanation of Terms:

  1. **Deterministic Terms (drift)**:
     - $\frac{\partial f}{\partial t}$: Direct change of f with time t.
     - $\mu \frac{\partial f}{\partial x}$: Change of f due to the drift term μ in X_t.
     - $\frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2}$: Correction term accounting for the quadratic variation of X_t.
  2. **Stochastic Term**:
     - $\sigma \frac{\partial f}{\partial x} dW_t$: Change in f due to the randomness in X_t, introduced by dW_t.
- Key Properties
  - Ito's Lemma captures both the deterministic (time and drift) and stochastic (random) contributions to changes in the function $f(t, X_t)$.
  - The term $\frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2}$ arises from the Taylor expansion and the property $(dW_t)^2 = dt$, unique to stochastic processes.

- Black-Scholes option price model
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  The Black-Scholes PDE for the price $V(t, S)$ of a European option is:

  $$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV.$$

  The solution for a European call option is:

  $$C = S_0 N(d_1) - Ke^{-rT} N(d_2),$$

- C: Call option price
- S_0: Current stock price
- K: Strike price of the option
- T: Time to expiration (in years)
- r: Risk-free interest rate
- $N(d)$: Cumulative distribution function of the standard normal distribution
- $d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

- Energy diffusion equation
  - $$\frac{\partial T}{\partial t} + \frac{\partial u_j T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\lambda}{\rho C_p}\frac{\partial T}{\partial x_j}\right),$$

    where:
    - T: Temperature,
    - u_j: Velocity component in the j-th direction,
    - ρ: Fluid density,
    - C_p: Specific heat at constant pressure,
    - λ: Thermal conductivity,
    - $\frac{\lambda}{\rho C_p}$: Thermal diffusivity (α).

Merton Default Risk Model

- **It leverages the Black-Scholes pricing model and seeks to establish a link between a default and its firm's capital structure.**
- **The Merton model is used to assess a company's credit risk by modeling the company's equity as a call option on its assets.**
- Fundamental Knowledge for Merton Default Risk Model
  - Financial statements
    - Balance sheet shows a company's financial position with assets, liabilities, and equity at a specific point in time.
    - Assets = Liabilities + Equity
    - Assets: Resources owned by the company.
    - Liabilities: Obligations owed to others.
    - Equity: Owner's residuals in the company.
  - Capital structure of a firm
    - A balance sheet summarizes a firm's capital structure with three components: the value of assets (A or V) on the left-hand side must equal the value of the sum of debt (D) and equity (E).
    - Assets (Firm Value): Cash flows and profits.

    - Debt: Obligation that finances assets and results in a liability
    - Equity: A capital invested by the firm's owners, and equity absorbs all losses before debt takes a loss.
    - Firm value = Value of Debt + Value of Equity
- Assume that a firm has a single outstanding zero-coupon debt with a face value (principal amount) F or debt D, payable at time T.
- There are two possible scenarios:
    - One: the value of the firm at time T, V(T) could be large enough to pay the principal amount, in which case the shareholders have a claim over the balance, i.e., V(T) − F
    - The other, the value of the firm at time T could be insufficient such that the firm is unable to settle the principal amount, in which case the equity holders receive nothing.
    - The value(payoff) of equity is E_T= Max(V_T- F, 0).
    - The payoff is a call option on firm value with exercise price as face value of debt.
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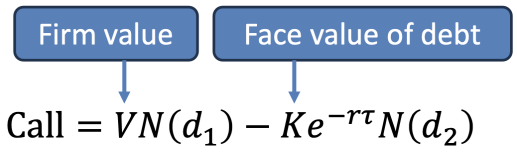

$$\text{Call} = VN(d_1) - Ke^{-r\tau}N(d_2)$$

V = value of the firm
F (K) = face value of debt maturing at T (liability)
σ = volatility of the value of the firm
r = annual interest rate
N = cumulative normal distribution
N(d2): firm's assets will exceed the debt face value
1-N(d2): firm's defaults

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$$\text{Call value} = \max(0, V(T) - F)$$
$$\begin{cases} 0 & ;0, V(T) < F \\ 0, V(T) - K & ;0, V(T) \geq F \end{cases}$$

Altman Z score method

- A numerical measurement that is used to **predict the chances of bankruptcy.**
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- **Altman Z score method**

$Z = 1.2X_1 + 1.4X_2 + 3.3X_2 + 0.6X_4 + 1.0\ X_5$
$Z = a_1x_1 + a_2x_2 + a_3x_3 + \ldots\ldots + a_nx_n$

Z = Discriminant Score (Z Score)
a_n = Discriminant Coefficients (weights)
x_n = Discriminant Variables (ratios)

X1 = Working Capital / Total Assets
X2 = Retained Earning / Total Assets
X3 = Earnings Before Interest & Tax (EBIT) / Total Assets
X4 = Market Capitalization / Total Liabilities
X5 = Sales / Total Assets

- How to evaluate the score
  - Z > 2.99: **Safe Zone (Low bankruptcy risk)**
  - 1.81 < Z < 2.99: Gray Zone (Moderate bankruptcy risk)
  - Z < 1.81: Distress Zone (High bankruptcy risk)

- Personal Credit Risk
  - FICO Score
    - Payment history
    - Amounts Owed
    - Length of Credit history
    - Credit mix
    - New Credit

- **Mortgage Loan types**
  - Principal
    - it refers to the loan amount
  - Interest
    - it is an amount (calculated as a percentage of the principal) that lenders charge you for the privilege of borrowing money that you can repay over time. ["interest" refers to the charge paid by a borrower to a lender for the privilege of borrowing money]
  - Types
    - **Conventional Loan**: Suitable for borrowers with good credit
    - **Jumbo Loan**: Finances properties exceeding FHFA limits; not backed by Fannie Mae or Freddie Mac. A mortgage used to finance properties that are too expensive for a conventional conforming loan.
    - **Government-Insured Loan**: Aimed at low-to-moderate-income first-time buyers; insured by agencies like FHA for those ineligible for conventional loans.
    - **Fixed-Rate Mortgage**: Provides stable, consistent payments throughout the loan term.

- **Adjustable-Rate Mortgage (ARM)**: Starts with a fixed rate for up to 10 years, then fluctuates with market conditions.