**This question paper consists of 19 printed pages, each of which is identified by the Code Number COMP142101.**



School of Computing

**January 2018**

**COMP142101**

Fundamental Mathematical Concepts

Answer all four questions

Time allowed: Two hours

**DO NOT REMOVE THIS PAPER FROM THE EXAM ROOM**
**Answer the question in the space provided. If you require additional space please request additional paper from the invigilators.**

| STUDENT NUMBER | | | | | | | | | |
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| SEAT NUMBER | | | | | | | | | |

| Question | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total | |

## Question 1

(a) Given the following propositions, translate the following natural language sentences into propositional logic formulas.
$a :=$ "it is raining"
$b :=$ "the weather is good"
$c :=$ "I attend the exam"

(i) If it is not raining and the weather is good then I attend the exam.

**[1 mark]** out of 1

(ii) It is raining or the weather is good

**[1 mark]** out of 1

(iii) I attend the exam if and only if it is not raining.

**[2 marks]** out of 2

(b) Given the following propositions, translate the following propositional formulas into natural language sentences.
$p :=$ "I eat cake"
$q :=$ "I drink tea"
$r :=$ "I drink coffee"

(i) $\neg(q \vee r)$

**[1 mark]**

out of 1

(ii) $(r \wedge q)$

**[1 mark]**

out of 1

(iii) $p \rightarrow ((q \vee r) \wedge \neg(q \wedge r))$

**[3 marks]**

out of 3

(c) Construct a truth table for the following propositional formula.

(i) $(p \wedge p) \rightarrow (\neg q \vee r)$

**[4 marks]**

out of 4

(ii) Is the formula in part (c)i) a contingency, a tautology or a contradiction? Justify your answer

**[1 mark]**

out of 1

(d) Translate each of the following natural language sentences into predicate logic. In each case state what domain you are using.

(i) All students study Fundamental Mathematical Concepts.

**[1 mark]**

out of 1

(ii) Some module is studied by all students.

**[1 mark]**

out of 1

(e) Translate each of the predicate logic formula into natural language.

(i) $\neg\exists x\, P(x)$ where the domain consists of all people and $P(x) := x$ is invincible.

**[1 mark]**

out of 1

(ii) $\exists x \forall y\, Q(x, y)$ where the domain consists of all people and $Q(x, y) := x$ is friends with $y$.

**[2 marks]**

out of 2

(f) For each of the following expressions state the truth value of (True or False) of the expression.

(i) $(a \vee b) \rightarrow (a \wedge b)$ where $a$ :=True and $b$ :=False.

**[1 mark]**

out of 1

(ii) $\forall x \exists y \, P(x, y)$, where the domain is the set of natural numbers and $P(x, y) := x - y = 0$

**[2 marks]**

out of 2

(g) Show that

$$\neg\big((\neg a \wedge \neg b) \vee (\neg a \wedge b)\big) \equiv a.$$

The symbol '$\equiv$' denotes logical equivalence.

**[5 marks]**

out of 5

**[Question 1 total: 27 marks]**

## Question 2

(a) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3\}$ and $B = \{2, 6\}$. Compute

(i) $A \cup B$

**[1 mark]** out of 1

(ii) $A \cap U$

**[1 mark]** out of 1

(iii) $U \setminus B$

**[1 mark]** out of 1

(iv) $A \cap B$

**[1 mark]** out of 1

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a fixed universe, and let $A = \{1, 2, 3\}$ and $B = \{2, 6\}$ be two subsets of $U$. Compute

(v) $A \cup \varnothing$

**[1 mark]**

out of 1

(vi) $\mathcal{P}(B)$

**[2 marks]**

out of 2

(vii) $\{2n \mid n \in A\}$

**[2 marks]**

out of 2

(b) Draw a Venn diagram and shade the region that corresponds to the expression

$$(\overline{A} \cap \overline{B}) \cup (A \cap B)$$

We assume that $A$ and $B$ are subsets of a universe $U$.

**[3 marks]**

out of 3

(c) Prove that the set of integers that are a multiple of 6 is a proper subset of the set of even numbers.

**[5 marks]**

out of 5

(d) Prove using induction that $|\mathcal{P}(A)| = 2^n$ for every set $A$ of size $n \in \mathbb{N}$.

**[6 marks]**

out of 6

**[Question 2 total: 23 marks]**

## Question 3

(a) Let $f\colon A \to B$ be a function. Define what it means for $f$ to be

(i) Injective

**[2 marks]**

out of 2

(ii) Bijective

**[2 marks]**

out of 2

(iii) Let $R = \{(0,0),(1,1),(2,1),(3,4),(4,3)\}$ be a relation over the set $\{0,1,2,3,4\}$. Complete the following table by writing 'Yes' or 'No' in each row to indicate whether $R$ has the property.

| Property | Yes/No |
|---|---|
| Injective | |
| Bijective | |
| Surjective | |

Each correct entry in the table will be award marks where as each incorrect entry will lose you marks. You will not get less than 0 marks. **[2 marks]**

out of 2

(b) Let $f(x) := 2x + 1$ and $g(x) := 4x - 3$ where the domain and codomain of each of the functions is the set of real numbers. Compute the following:

(i) $f(4)$

**[1 mark]**

out of 1

(ii) $(f \circ g)(3)$

**[1 mark]**

out of 1

(iii) $(f \circ g)^{-1}$

**[2 marks]**

out of 2

(c) Consider the relation $R = \{(0,0),(1,1),(1,2),(2,2),(2,1),(2,3),(3,3),(1,3)\}$ on the set $\{0,1,2,3\}$.

(i) Complete the table by writing "Yes" or "No" in each of the boxes to indicate if $R$ has the property or not.

| Property | Yes/No |
|---|---|
| Reflexive | |
| Symmetric | |
| Transitive | |
| Antisymmetric | |

Each correct entry in the table will be awarded marks and each incorrect entry will lose you marks. You will not get less than 0 marks. **[3 marks]**

out of 3

(ii) Draw the graph of the relation $R$.

**[1 mark]**

out of 1

(iii) Find the smallest relation that contains $R$ and is symmetric.

**[3 marks]**

out of 3

(iv) Prove or disprove the statement "$R$ is a partial order".

**[3 marks]**

out of 3

(d) Let $q : A \to B$ and $w : B \to C$ be two bijective functions where $A$, $B$ and $C$ are sets. Prove that $q \circ w$ is a bijective function.

**[5 marks]**

out of 5

**[Question 3 total: 25 marks]**

## Question 4

(a) Using mathematical induction, show that $6^n - 1$ is divisible by 5 for all positive whole numbers.

**[5 marks]**

**[Question 4 total: 5 marks]**

out of 5

**[Grand total: 80 marks]**