This question paper consists of 3 printed pages, each of which is identified by the Code Number COMP151101.

A non-programmable calculator may be used.

School of Computing

May/June 2018

COMP151101

Introduction to Discrete Mathematics

Answer all 5 questions

Time allowed: 2 hours

The marks available for each question are given in brackets after the question. You need to look at these marks and allocate the time you spend on each question accordingly.

Question 1

Let $\mathcal{S}$ be the set of all sequences of length 5 whose elements are letters of the English alphabet.

(a) How many sequences does $\mathcal{S}$ contain? **[3 marks]**

(b) How many sequences from $\mathcal{S}$ consist of letters that are all different? **[3 marks]**

(c) How many sequences from $\mathcal{S}$ do not start with A and end with B? **[3 marks]**

[question 1 total: 9 marks]

Question 2

Consider distributing 28 identical balls into 8 distinct boxes numbered $1, \ldots, 8$.

(a) In how many ways can this be done? **[3 marks]**

(b) What is the number of such distributions in which for every $i \in \{1, 2, 3, 4\}$, box i receives at least i balls? **[3 marks]**

(c) What is the number of such distributions in which all boxes receive an even number of balls? **[3 marks]**

[question 2 total: 9 marks]

Question 3

(a) What is the coefficient of x^4y^3 when the expression $(2x - y)^7$ is expanded? **[3 marks]**

(b) What is the coefficient of x^4y^3 when the expression $(x+y+2)^9$ is expanded? **[3 marks]**

[question 3 total: 6 marks]

Question 4

An experiment consists of two fair, different coloured dice being rolled (the dice are 6-sided and the sides show numbers $1, \ldots, 6$). Let A be the event that the two dice are both showing numbers that are greater than 3, and let B be the event that the sum of the numbers shown on the two dice is 8.

(a) What is the probability of A? **[3 marks]**

(b) What is the probability of B? **[3 marks]**

(c) What is the probability of $A \cap B$? **[3 marks]**

(d) Are the events A and B independent? (You must justify your answer!) **[3 marks]**

[question 4 total: 12 marks]

Question 5

(a) Define the following (you may use terms *graph*, *vertex*, *edge* and *path* without defining them):

- a *connected graph*;
- a *cycle*;
- a *cut-edge*.

[6 marks]

(b) Let G be a connected graph. Prove that an edge e of G is not a cut-edge if and only if it belongs to a cycle.

In your proof you may use the following lemma (that you do not need to prove).
Lemma: *Let G be a graph and u and v two vertices of G. There is a path from u to v in G if and only if there is a simple path from u to v in G.* **[6 marks]**

[question 5 total: 12 marks]

[grand total: 48 marks]

END