

This question paper consists
of 19 printed pages, each
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Code Number COMP142101.

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School of Computing

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COMP142101

Fundamental Mathematical Concepts

Answer all four questions

Time allowed: Two hours

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Answer the question in the space provided. If you require additional space please request additional paper from the invigilators.

STUDENT NUMBER									
SEAT NUMBER									

Question	Score
1	
2	
3	
4	
Total	

Question 1

- (a) Given the following propositions, translate the following natural language sentences into propositional logic formulas.

$a :=$ "it is raining"

$b :=$ "the weather is good"

$c :=$ "I attend the exam"

- (i) If it is not raining and the weather is good then I attend the exam.

[1 mark]

out of 1

- (ii) It is raining or the weather is good

[1 mark]

out of 1

- (iii) I attend the exam if and only if it is not raining.

[2 marks]

out of 2

- (b) Given the following propositions, translate the following propositional formulas into natural language sentences.

$p :=$ "I eat cake"

$q :=$ "I drink tea"

$r :=$ "I drink coffee"

(i) $\neg(q \vee r)$

[1 mark]

out of 1

(ii) $(r \wedge q)$

[1 mark]

out of 1

(iii) $p \rightarrow ((q \vee r) \wedge \neg(q \wedge r))$

[3 marks]

out of 3

(c) Construct a truth table for the following propositional formula.

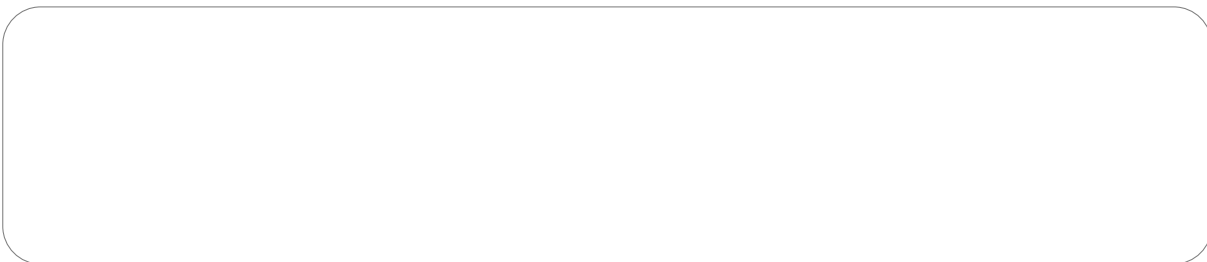
(i) $(p \wedge p) \rightarrow (\neg q \vee r)$



[4 marks]

(ii) Is the formula in part (c)i) a contingency, a tautology or a contradiction? Justify your answer

out of 4



[1 mark]

out of 1

(d) Translate each of the following natural language sentences into predicate logic. In each case state what domain you are using.

(i) All students study Fundamental Mathematical Concepts.

[1 mark]

out of 1

(ii) Some module is studied by all students.

[1 mark]

out of 1

(e) Translate each of the predicate logic formula into natural language.

(i) $\neg \exists x P(x)$ where the domain consists of all people and $P(x) := x$ is invincible.

[1 mark]

(ii) $\exists x \forall y Q(x, y)$ where the domain consists of all people and $Q(x, y) := x$ is friends with y .

out of 1

[2 marks]

out of 2

(f) For each of the following expressions state the truth value of (True or False) of the expression.

(i) $(a \vee b) \rightarrow (a \wedge b)$ where $a := \text{True}$ and $b := \text{False}$.

[1 mark]

(ii) $\forall x \exists y P(x, y)$, where the domain is the set of natural numbers and $P(x, y) := x - y = 0$

out of 1

[2 marks]

out of 2

(g) Show that

$$\neg((\neg a \wedge \neg b) \vee (\neg a \wedge b)) \equiv a.$$

The symbol ' \equiv ' denotes logical equivalence.

[5 marks]

[Question 1 total: 27 marks]

out of 5

Question 2

(a) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3\}$ and $B = \{2, 6\}$. Compute

(i) $A \cup B$

[1 mark]

out of 1

(ii) $A \cap U$

[1 mark]

out of 1

(iii) $U \setminus B$

[1 mark]

out of 1

(iv) $A \cap B$

[1 mark]

out of 1

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a fixed universe, and let $A = \{1, 2, 3\}$ and $B = \{2, 6\}$ be two subsets of U . Compute

(v) $A \cup \emptyset$

[1 mark]

out of 1

(vi) $\mathcal{P}(B)$

[2 marks]

out of 2

(vii) $\{2n \mid n \in A\}$

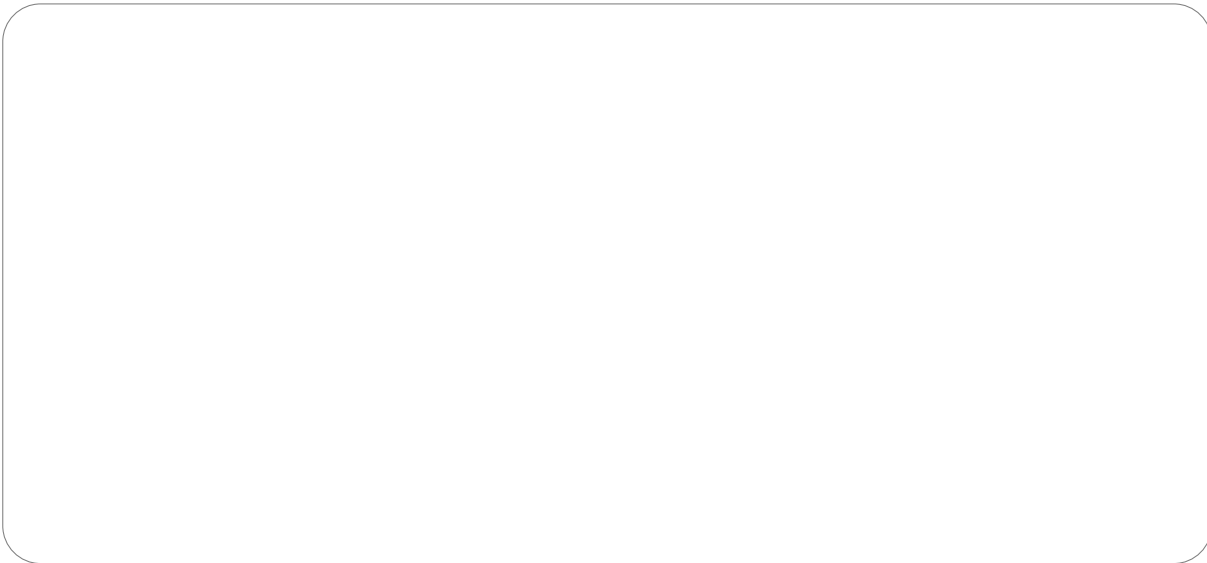
[2 marks]

out of 2

- (b) Draw a Venn diagram and shade the region that corresponds to the expression

$$(\overline{A} \cap \overline{B}) \cup (A \cap B)$$

We assume that A and B are subsets of a universe U .



[3 marks]



out of 3

- (c) Prove that the set of integers that are a multiple of 6 is a proper subset of the set of even numbers.

[5 marks]

out of 5

(d) Prove using induction that $|\mathcal{P}(A)| = 2^n$ for every set A of size $n \in \mathbb{N}$.



[6 marks]

[Question 2 total: 23 marks] out of 6

Question 3

(a) Let $f: A \rightarrow B$ be a function. Define what it means for f to be

(i) Injective

[2 marks]

out of 2

(ii) Bijective

[2 marks]

out of 2

(iii) Let $R = \{(0, 0), (1, 1), (2, 1), (3, 4), (4, 3)\}$ be a relation over the set $\{0, 1, 2, 3, 4\}$. Complete the following table by writing ‘Yes’ or ‘No’ in each row to indicate whether R has the property.

Property	Yes/No
Injective	
Bijective	
Surjective	

Each correct entry in the table will be award marks where as each incorrect entry will lose you marks. You will not get less than 0 marks.

[2 marks]

out of 2

(b) Let $f(x) := 2x + 1$ and $g(x) := 4x - 3$ where the domain and codomain of each of the functions is the set of real numbers. Compute the following:

(i) $f(4)$

[1 mark]

out of 1

(ii) $(f \circ g)(3)$

[1 mark]

out of 1

(iii) $(f \circ g)^{-1}$

[2 marks]

out of 2

(c) Consider the relation $R = \{(0, 0), (1, 1), (1, 2), (2, 2), (2, 1), (2, 3), (3, 3), (1, 3)\}$ on the set $\{0, 1, 2, 3\}$.

(i) Complete the table by writing “Yes” or “No” in each of the boxes to indicate if R has the property or not.

Property	Yes/No
Reflexive	
Symmetric	
Transitive	
Antisymmetric	

Each correct entry in the table will be awarded marks and each incorrect entry will lose you marks. You will not get less than 0 marks. **[3 marks]**

(ii) Draw the graph of the relation R .

out of 3

[1 mark]

(iii) Find the smallest relation that contains R and is symmetric.

out of 1

[3 marks]

(iv) Prove or disprove the statement “ R is a partial order”.

out of 3

[3 marks]

out of 3

- (d) Let $q : A \rightarrow B$ and $w : B \rightarrow C$ be two bijective functions where A , B and C are sets. Prove that $q \circ w$ is a bijective function.

[5 marks]

[Question 3 total: 25 marks]

out of 5

Question 4

- (a) Using mathematical induction, show that $6^n - 1$ is divisible by 5 for all positive whole numbers.

[5 marks]

[Question 4 total: 5 marks]

out of 5

[Grand total: 80 marks]