This question paper consists of 4 printed pages, each of which is identified by the Code Number COMP5823M.

This is a closed book examination. No material is permitted.

School of Computing

May 2018

COMP5823M

Animation and Simulation

Answer 6 out of 6 questions

Time allowed: 2 hours

TURN OVER FOR QUESTIONS

Question 1

(a) Controlling motions in animation is often done by first computing arc lengths on motion trajectories then deciding a control strategy. Depending on the nature of the motion, there are several strategies to compute arc lengths, such as analytical approach, forward differencing and numerical integration. Explain how they work and their comparative advantages and disadvantages. **[9 marks]**

[question 1 total: 9 marks]

Question 2

(a) Interpolation schemes are very important in making smooth and natural animations. During interpolation, smoothness is a good indicator to describe the underlying functions. Explain what Smoothness/Continuity is and how we use it to classify functions. Also, if you are using interpolation to control a point, how different degrees of smoothness could cause different visual effects? **[3 marks]**

[question 2 total: 3 marks]

Question 3

(a) Hermite interpolation, Calmull-Rom interpolation and B-Splines are three commonly used techniques for interpolating a series of points. When does one want to use Hermite interpolation instead of the other two? **[1 mark]**

(b) B-splines decouple the number of the control points from the degree of the resulting polynomials. It has a knot vector establishing the relations between the parametric value and the control points. Describe how an uniform cubic B-spline which uses four control points in [0,1] works. **[2 marks]**

(c) Assuming 5 positions are given to generate a motion trajectory for a point starting at the first position. We would like to specify the velocity only at the beginning and the end. Which one of the three methods is the best and why? **[2 marks]**

[question 3 total: 5 marks]

Question 4

CONTINUED ON FACING PAGE

(a) What are Forward Kinematics (FK) and Inverse Kinematics (IK)? How are they used in character animation? **[4 marks]**

(b) In using IK to control characters, computing the inverse of Jacobian, J^{-1}, is crucial. Give the basic version of IK using the following notations: a posture is specified by a vector of joint angles q with an end-effector position e. You also have a desired position g for the end-effector. **[2 marks]**

(c) There is no guarantee that J^{-1} exists all the time. Why? Give an example. **[2 marks]**

(d) If J^{-1} does not exist, a pseudo-inverse can be used. Give the equations to explain how pseudo-inverse works. **[2 marks]**

(e) Using the basic pseudo-inverse approach, when reaching for a point that is out of reach, the motion tends to be jittering. Explain why and how to mitigate it by modifying the pseudo-inverse. **[2 marks]**

[question 4 total: 12 marks]

Question 5

(a) Mass-spring models can be used for simulating simple interactions between multiple rigid bodies. In a 2D scenario (Fig. 1), two springs are connected at X_m and are connected to the two walls at X_l and X_r. The rest lengths of the two springs are l_1 and l_2 respectively and the distance between two walls is l_3 where $l_3 > l_1 + l_2$.

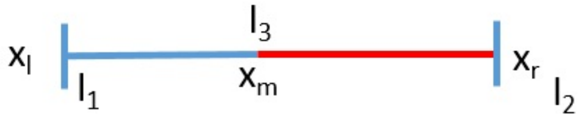

Figure 1: 2D two spring simulation.

Now we want to animate the two springs by updating the position of X_m for every time step. $X_l = 0$, $X_r = 3$, $X_M^0 = 0.5$ where the superscript indicates the time step. $l_1 = l_2 = 1$ and $l_3 = 3$. The two stiffness coefficients of the two springs are $s_1 = s_2 = 1$. Assume the time step Δt is 1 second. The mass of the springs can be ignored. The only mass is a lumped mass point (1 kilogram) attached to X_m. The gravity can be ignored. Use explicit Euler to compute step 1 and 2. For the spring modelling, a simple model with stiffness can be used. Assuming v and a are the velocity and acceleration of mass point at X_m. So $v^0 = 0$. Use a spring model $f = -s(L_s - L_r)((p_2 - p_1)/||p_2 - p_1||)$ where s is the stiffness, L_s is the current length and L_r is the rest length. p_1 and p_2 are two end points of the spring. Give detailed mathematical deductions and calculations. **[5 marks]**

(b) An alternative way to compute the animation above is to use energy-minimisation. All conditions are the same and use an energy minimisation scheme to compute step 1 and step 2. Use time step $\Delta t = 0.01$ this time. To model the spring energy, use energy function $E = s(||p_2 - p_1|| - l)^2$ where s is the stiffness, l is the rest length, p_1 and p_2 are two end points of the spring. **[5 marks]**

(c) The above mass-spring scheme can be extended to model complex deformable objects such as cloths. However, it causes problems such as super-elasticity (overly stretched). Why? What can be done to mitigate the problem? **[4 marks]**

[question 5 total: 14 marks]

Question 6

(a) Computational Fluid Dynamics (CFD) has been used for animation purposes to simulated natural phenomena such as clouds, oceans and fires. In CFD for animation, there are two mainstream approaches: Euler and Lagrangian. Explain how they differ in modelling fluids and what the advantages and disadvantages are. **[4 marks]**

(b) CFD focuses on solving Navier-Stokes equations:

$$\bigtriangledown \cdot u = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} = -(u \cdot \bigtriangledown)u - \frac{1}{\rho} \bigtriangledown p + v \bigtriangledown^2 u + f \tag{2}$$

The above equations are often used to simulate water. Why is Eqn 1 needed? **[1 mark]**

Which part of the right hand side of Eqn 2 is the convection and which part models the viscosity? **[2 marks]**

[question 6 total: 7 marks]

[grand total: 50 marks]