17 FEBBRAIO 2020

ES. 1

1) $f(z) = Log(Log\, z)$ $\begin{cases} Re(Log\, z) \\ Im(Log\, z) \end{cases} \Rightarrow \begin{cases} Log|z| < 0 \\ Arg(z) = 0 \end{cases} \Rightarrow \begin{cases} |z| < 1 \\ Arg(z) = 0 \end{cases}$

(d)

2) $f_n(x) = \log\left(\frac{x^2}{n} + 1\right)$ Poiché $\forall x$ ~~q.o.~~ $\Rightarrow f(x) \xrightarrow{n \to \infty} 0$

Per ogni $x < +\infty$, allora possiamo dire che $x \in \mathbb{R}$ $\boxed{\text{Risposta: a}}$

3) $\cos(Log\, i) \Rightarrow \frac{e^{i Log\, i} + e^{-i Log\, i}}{2} = \frac{e^{i(\cancel{log}\,|i| + i Arg\, i)} + e^{-i(\cancel{log}\,|i| + i Arg\, i)}}{2} =$

$= \frac{e^{-\pi/2} + e^{\pi/2}}{2} \Leftrightarrow \frac{e^{\pi/2} + e^{-\pi/2}}{2} \Rightarrow \cosh(\pi/2)$ $\boxed{\text{Risposta: b}}$

4) Dom. di $f(z) = Log(|z-1|)$

è connesso poiché il suo dominio ricopre $\mathbb{C}^*$, però tranne nel punto $z=1$, quindi è connesso.

$\boxed{\text{Risposta: c}}$

5) $z = 2e^{-\frac{\pi}{4}i} \Rightarrow z = 2\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right)\right) = 2\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = \sqrt{2} - i\sqrt{2}$

Es. 2

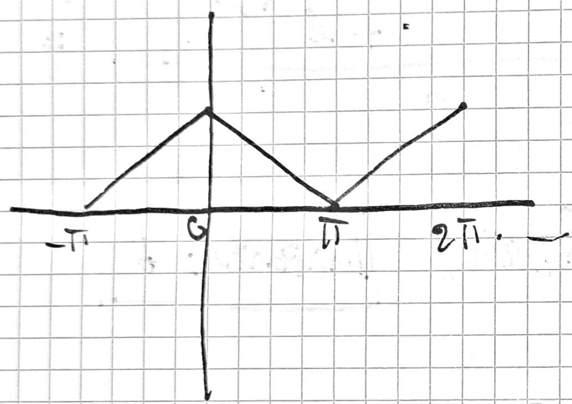
(i) Una normale Serie di Fourier è espressa come:

$$S_m(x) = \frac{a_0}{2} + \sum_{h=1}^{m} a_k \cos(hx+2\pi) + b_h \sin(hx+2h)$$

Nel caso di una serie 2π-Periodica Pari abbiamo $b_h=0$, quindi:

$$S_m^{Pari}(x) = \frac{a_0}{2} + \sum_{n=1}^{m} a_h \cos(hx+2\pi)$$

(ii) $f(x) = |x-\pi|$, $x \in \,]0, 2\pi]$



f è Pari, quindi $b_h=0$

$$f(x) = \frac{a_0}{2} + \sum_{h=1}^{m} a_h \cos(hx)$$

$a_0 = \frac{\pi}{2}$

$$a_h = \frac{2}{\pi}\int_0^\pi f(x)\cos hx\,dx = \frac{2}{\pi}\int_0^\pi (x-\pi)\cos hx =$$

$$= \frac{2}{\pi}\int_0^\pi x\cos hx - \pi\cos hx = \frac{2}{\pi}\left[\int_0^\pi x\cos hx - \pi\int_0^\pi \cos hx\right] =$$

$$= \frac{2}{\pi}\left[\cancel{\frac{x\sin kx}{k}}\Big|_0^\pi - \left(\frac{-\cos hx}{h^2}\right)\Big|_0^\pi - \frac{\pi}{k}(\cancel{\sin kx})\Big|_0^\pi\right] = \frac{2}{\pi}\left[\frac{\cos hx}{h^2}\right]_0^\pi =$$

$$= \frac{2}{\pi h^2}\Big[\underbrace{-\cos h\pi}_{-(-1)^h} + \overset{1}{\cos 0}\Big] = \frac{2}{\pi h^2}\left(-(-1)^h+1\right)$$ Quindi diventa

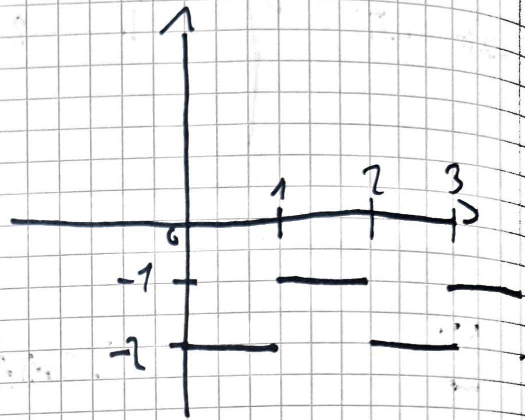
$$S(x) = \frac{\pi}{2} + \sum_{h=1}^{m} \left(-(-1)^h+1\right)\cdot\frac{2}{\pi h^2}$$

Es. 3

(i) una f è $\mathcal{L}$-trasformabile quando $\exists \int_0^{+\infty} \left| e^{-st} f(t) \right| dt < +\infty$

(ii) $$\mathcal{L}(s) = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)\, dt$$

(iii) $$f(t) = \begin{cases} -2 & 2n \le t \le 2n+1, \quad n \ge 0 \\ -1 & 2n+1 < t < 2n+2 \\ 0 & \text{altrimenti} \end{cases}$$



$$\mathcal{L}(s) = \frac{1}{1-e^{-2s}} \left(\int_0^1 e^{-st} \cdot (-2)\, dt + \int_1^2 e^{-st}\, dt \right) =$$

$$= \frac{1}{1-e^{-2s}} \left(-2\left[-e^{-st} \right]_0^1 - \left[-e^{-st} \right]_1^2 \right) = \frac{1}{1-e^{-2s}} \left(-2\left(-e^{-s} + 1 \right) - \left(-e^{-2s} + e^{-s} \right) \right) =$$

$$= \frac{2e^{-s} - 2 + e^{-2s} - e^{-s}}{1-e^{-2s}} = \cancel{3e^{-s}} \; \frac{e^{-s} + e^{-2s} - 2}{1-e^{-2s}}$$

Es. 4

(i) CRI $\Rightarrow \dfrac{\partial f}{\partial x} = -i \dfrac{\partial f}{\partial y}$ CRI $\Rightarrow \dfrac{\partial u}{\partial x} + i \dfrac{\partial v}{\partial x} = -i \left(\dfrac{\partial u}{\partial y} + i \dfrac{\partial v}{\partial y} \right)$

(ii) $$\int_\gamma \frac{e^z}{z-1}\, dz = 2\pi i \operatorname{res}(f, 1) = \boxed{2\pi i e}$$

↳ polo singolo, $\operatorname{res}(f,1) \Rightarrow \lim_{z \to z_0 = 1} \cancel{(z-1)} \cdot \dfrac{e^z}{\cancel{(z-1)}} = e$

$$\int \frac{e^z}{(z-1)^{8 \to n+1}} = 2\pi i \, res(f,1) = 2\pi i \cdot \frac{1}{7!} \cdot e = \boxed{\frac{2\pi i e}{7!}}$$

↳ polo di ordine 8, $res(f,1) = \underset{\underset{7!}{\shortparallel}}{\frac{1}{n!}} \cdot \lim_{z \to 1} e^z$

ES.5

$$(ii) \sum_{n=1}^{+\infty} (-1)^n n^{\log x} \quad , x > 0$$

Cambio la forma per renderla Armonica

$$\sum_{n=1}^{+\infty} (-1)^n \cdot \frac{1}{n^{-\log x}}$$

$-\log x > 1 \Rightarrow \log x < -1$ quindi

$x < \frac{1}{e}$

TUTACE?