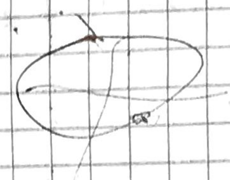
ESERCIZIO 1

I) $\mathrm{Log}(\mathrm{Log}(z)) =$

NON OLOMORFA IN $\begin{cases} \mathrm{Re}(\mathrm{Log}\, z) < 0 \\ \mathrm{Im}(\mathrm{Log}\, z) = 0 \end{cases}$ $\begin{cases} \log|z| < 0 \\ \mathrm{Arg}(z) = 0 \end{cases}$ $\begin{cases} |z| < 1 \\ \mathrm{Arg}(z) = 0 \end{cases}$

e

$\mathbb{C}^{**}$



RISPOSTA D

II) $f_n(x) = \log\left(\frac{x^2}{n} + 1\right)$

$\lim_{n \to \infty} \log\left(\frac{x^2}{n} + 1\right) = 0 \quad \forall x \in \mathbb{R}$ puntualmente

$g_n = \sup_{x \in \mathbb{R}} \left| \log\left(\frac{x^2}{n} + 1\right) \right| = \sup_{x \in [-a, a]} \left| \log\left(\frac{x^2}{n} + 1\right) \right| = \log\left(\frac{a^2}{n} + 1\right)$

$a \in \mathbb{R}^+$

$\lim_{n \to \infty} g_n = 0$

RISPOSTA A

III) $\cos(\mathrm{Log}\, i) = \cos(\log 1 + i\, \mathrm{Arg}(i)) = \cos\left(i \frac{\pi}{2}\right) = \frac{e^{-\frac{\pi}{2}} + e^{\frac{\pi}{2}}}{2}$

$= \cosh\left(\frac{\pi}{2}\right)$

RISPOSTA B

IV) $f(z) = \mathrm{Log}(|z-1|) \Rightarrow |z+1| \neq 0$

Risposta A

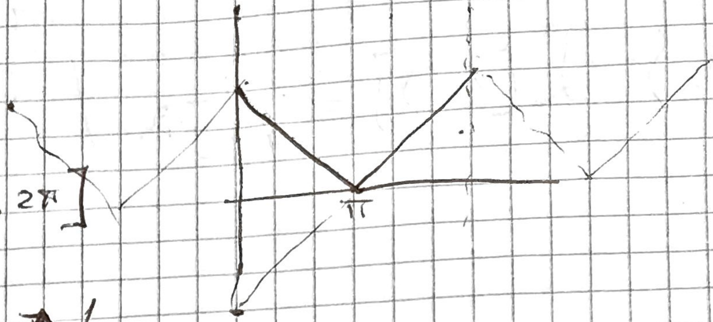
V) $2e^{-\frac{\pi}{4} i} = 2 \cdot \left(\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} - i\, 2 \cdot \frac{\sqrt{2}}{2}$

$= \sqrt{2} - i\sqrt{2}$

RISPOSTA C

Esercizio 2

$$\sum_{k\geq 0} a_k \cos kx$$



ii) $f(x) = |x - \pi| \quad x \in (0, 2\pi]$

pari $\Rightarrow b_k = 0 \quad \forall k \in \mathbb{N}$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} |x-\pi|\,dx = \cancel{\frac{1}{\pi}\int_0^{\pi}} \; \frac{2}{\pi}\int_0^{\pi} (\pi - x)\,dx = \frac{2}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left(\pi^2 - \frac{\pi^2}{2}\right) = \pi$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} |x-\pi| \cdot \cos kx \, dx = \frac{1}{\pi}\int_0^{\pi} (\pi - x)\cos kx\,dx + \frac{1}{\pi}\int_{\pi}^{2\pi} (x-\pi)\cos kx$$

$$\frac{1}{\pi}\int_0^{\pi} \pi\cos kx - x\cos kx\,dx = \frac{1}{\pi}\left[\frac{\pi}{k}\sin kx\right]_0^{\pi} = \frac{\sin k\pi}{k} = 0$$

$$\to -\frac{1}{\pi}\int_0^{\pi} x\cos kx\,dx = \frac{1}{\pi}\left[\cancel{\frac{x\sin kx}{k}}\Big|_0^{\pi} + \int_0^{\pi}\frac{\sin kx}{k}\right]$$

$$= \frac{1}{\pi}\left[\frac{-\cos kx}{k^2}\right]_0^{\pi} = \frac{1}{k^2\pi}(-\cos k\pi + \cos 0)$$

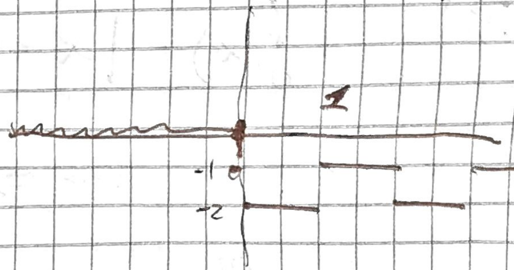
$$= \frac{1}{k^2\pi}(1 - \cos k\pi) \quad [(-1)^k]$$

$$\frac{1}{\pi}\int_{\pi}^{2\pi} x\cos kx = \frac{1}{\pi}\left[\frac{\cos kx}{k^2}\right]_{\pi}^{2\pi} = \frac{1}{\pi k^2}(\cos k\cdot 2\pi - \cos k\pi)$$

$$a_k = \frac{2}{\pi k^2}(1 - (-1)^k) \qquad f(x) = \frac{\pi}{2} + \sum_{k\geq 1}\frac{2}{\pi k^2}(1-(-1)^k)$$

Esercizio 3

ii) $\mathcal{L}[f(t)] = \frac{1}{1-e^{-Ts}} \int_0^T f(t)\, e^{st}\, dt$



$$\mathcal{L}[f(t)] = \frac{1}{1-e^{-2s}} \int_0^2 f(t)\, e^{st}\, dt =$$

$$= \frac{1}{1-e^{-2s}} \left(\int_0^1 -2 \cdot e^{st}\, dt + \int_1^2 -1 \cdot e^{st}\, dt \right) = \frac{1}{1-e^{-2s}} \left(-2 \cdot \frac{e^{st}}{s} \Big|_0^1 - \frac{e^{st}}{s} \Big|_1^2 \right)$$

$$\frac{1}{s(1-e^{-2s})} \left(-2e^{s} + 2 - e^{2s} + e^{s} \right) = \frac{-e^{2s} - e^{s} + 2}{s(1-e^{-2s})}$$

Esercizio 4

$$\int_\gamma \frac{e^z}{z-1} = 2\pi i \, \mathrm{Res}(f, 1) = 2\pi i \lim_{z\to 1} e^z = 2\pi e i$$

$$\int_\gamma \frac{e^z}{(z-1)^8} = 2\pi i \, \mathrm{Res}(f, 1) = 2\pi i \frac{1}{7!} \lim_{z\to 1} \frac{d^7}{dz^7} (e^z) = \frac{2\pi i}{7!} \lim_{z\to 1} e^z$$

$$= \frac{2\pi e}{7!} i$$

Esercizio 5

ii)

$$\sum_{n\geq 1} (-1)^n n^{\log x} = \sum_{n\geq 1} (-1)^n e^{\log x \cdot \log n} \qquad x>0$$

PUNTUALE:

converge puntualmente se ~~log x < 0 ⟹ x < 1~~

la successione $g_n(x) = n^{\log x}$ converge a 0

cioè se $\log x < 0 \Rightarrow x < 1$

ASSOLUTA

$$\sum_{n\geq 1} \left| n^{\log x} \right|$$ converge per $x < 1$

TOTALE

converge totalmente in $x \in [b, a]$ $\quad a < 1$, $b > 0$

$\Downarrow$

converge unif. in $x \in [b, a]$

$$\left|(-1)^n n^{\log x}\right| \leq n^{\log a}$$

$\sum n^{\log a}$ è conv.