

$$\begin{aligned}
 &\text{Max } 2x_1 + x_2 \\
 &-x_1 + 2x_2 \leq +4 \\
 &2x_1 + 3x_2 \leq 13 \\
 &4x_1 + x_2 \leq 16 \\
 &x_1, x_2 \geq 0
 \end{aligned}
 \quad \textcircled{P}$$

$$\begin{aligned}
 &\text{Min } 4y_1 + 13y_2 + 16y_3 \\
 &-y_1 + 2y_2 + 4y_3 \geq 2 \\
 &2y_1 + 3y_2 + y_3 \geq 1 \\
 &y_1, y_2, y_3 \geq 0
 \end{aligned}
 \quad \textcircled{D}$$

$$A = \begin{pmatrix} -1 & 2 \\ 2 & 3 \\ 4 & 1 \end{pmatrix} \quad b^T = (4 \quad 13 \quad 16)$$

$$\bar{x} = \left( \frac{7}{2} \quad 2 \right)$$

$$c = (2 \quad 1)$$

- Strong complementarity

$$\begin{aligned}
 &y^T(b - Ax) = 0 \\
 &(A^T y - c)^T x = 0
 \end{aligned}
 \rightarrow
 \begin{cases}
 y_1(x_1 - 2x_2 + 4) = 0 \\
 y_2(-2x_1 + 3x_2 + 13) = 0 \\
 y_3(-4x_1 - x_2 + 16) = 0
 \end{cases}
 \rightarrow
 \begin{cases}
 \frac{7}{2}y_1 = 0 \\
 0 = 0y_2 \\
 0 = 0y_3
 \end{cases}$$

$$\begin{aligned}
 &\begin{cases}
 (-y_1 + 2y_2 + 4y_3 - 2)x_1 = 0 \\
 (2y_1 - 3y_2 + y_3 - 1)x_2 = 0 \\
 y_1 = 0
 \end{cases}
 \rightarrow
 \begin{cases}
 2y_2 + 4y_3 = 2 \\
 -3y_2 + y_3 = 1
 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 &\downarrow \\
 &\begin{cases}
 y_2 = \frac{1}{5} \\
 y_3 = \frac{2}{5}
 \end{cases}
 \quad \bar{y} = (0 \quad \frac{1}{5} \quad \frac{2}{5})
 \end{aligned}$$

$$c^T \bar{x} = c^T \bar{y} \rightarrow g = g$$