• Simplesso

$$\text{Max} \quad 3x_1 + 5x_2$$
$$x_1 + x_2 \le 12$$
$$x_1 \le 10$$
$$x_2 \le 6$$
$$x_1 + 2x_2 \le 16$$
$$x_1, x_2 \ge 0$$

$\longrightarrow$

$$\text{Min} \quad -3x_1 - 5x_2$$
$$x_1 + x_2 + x_3 = 12$$
$$x_1 + x_4 = 10$$
$$x_2 + x_5 = 6$$
$$x_1 + 2x_2 + x_6 = 16$$
$$x_i \ge 0 \quad i = 1, \dots, 6$$

$$A_1 = \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \end{array}$$
$$A_1 = \Big(\underbrace{\begin{matrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{matrix}}_{N_1} \; \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{B_1} \Big) \qquad b^T = (12 \quad 10 \quad 6 \quad 16) \qquad c_1^T = \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array}$$
$$c_1^T = \Big(\underbrace{-3 \quad -5}_{N_1} \; \underbrace{0 \quad 0 \quad 0 \quad 0}_{B_1} \Big)$$

OSS

Slide 27 (lez. 8)
- $(x, \alpha) = (0, b) = (0 \; 0 \; 12 \; 10 \; 6 \; 16)$ punto che soddisfa tutti i vincoli
- $A = (A' \cdot I_m) \to R(A) = m \quad (I_{4\times4})$
- B è ammissibile per il problema $(B^{-1}b = b \ge 0_m)$
- $B^{-1}N = N$ e $B^{-1}b = b$

• Calcolo vettore costi ridotti

$$\gamma_1 = c^T_{N_1} - c^T_{B_1} B^{-1} N = (-3 \quad -5)$$

Criterio ottimalità non soddisfatto

• Verifico c. illimitatezza

$$\forall i : \gamma_i < 0 \Rightarrow \pi_i \le 0_m \to \text{Non soddisfatto}$$

• Cambio base

$h = 1 \to x_1$ entrante

- Cerco k con criterio rapporto minimo

$$\min \left\{ \frac{(B_1^{-1} b)_i}{(\pi_h)_i} \right\} = \min \left\{ \frac{12}{1} \quad \frac{10}{1} \quad \cancel{\frac{6}{0}} \quad \frac{16}{1} \right\}$$

$k = 2 \to x_4$ uscente

$$A_2 = \Big(\underbrace{\begin{matrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 2 \\ x_4 & x_2 \end{matrix}}_{N_2} \; \underbrace{\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ x_3 & x_1 & x_5 & x_6 \end{matrix}}_{B_2} \Big) \qquad b^T = (12 \quad 10 \quad 6 \quad 16) \qquad c_2^T = \begin{array}{cccccc} x_4 & x_2 & x_3 & x_1 & x_5 & x_6 \end{array}$$
$$c_2^T = \Big(\underbrace{0 \quad -5}_{N_2} \; \underbrace{0 \quad -3 \quad 0 \quad 0}_{B_2} \Big)$$

$$\begin{array}{l} k=2 \to \\ M = \end{array} \begin{array}{cccc} \pi_h & e_k & \pi_2 & B_1^{-1}b \\ \end{array}$$
$$M = \left(\begin{array}{c|cc|c} 1 & 0 & 1 & 12 \\ 1 & 1 & 0 & 10 \\ 0 & 0 & 1 & 6 \\ 1 & 0 & 2 & 16 \end{array} \right) = \left(\begin{array}{c|cc|c} 0 & -1 & 1 & 2 \\ 1 & 1 & 0 & 10 \\ 0 & 0 & 1 & 6 \\ 0 & -1 & 2 & 6 \end{array} \right)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad B_2^{-1}N_2 \qquad B_2^{-1}b$$

• Costi ridotti

$$\gamma_2 = c^T_{N_2} - c^T_{B_2} B_2^{-1} N_2 = (0 \quad -5) - (0 \; -3 \; 0 \; 0) \begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 2 \end{pmatrix} =$$
$$= (0 \quad -5) - (-3 \quad 0) = (+3 \quad -5)$$