- Simplesso

$$\max\ 2x_1 + x_2 + 3x_3 \qquad\longrightarrow\qquad \min\ -2x_1 - x_2 - 3x_3$$

$$\begin{array}{lcl} x_1 + x_2 + x_3 \le 2 & & x_1 + x_2 + x_3 + x_4 = 2 \\ 2x_1 + 3x_2 + 8x_3 \le 12 & \longrightarrow & 2x_1 + 3x_2 + 8x_3 + x_5 = 12 \\ x_1, x_2, x_3 \ge 0 & & x_i \ge 0 \quad i = 1, \dots, 5 \end{array}$$

$$A_0 = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 0 \\ \underbrace{2 \;\; 3 \;\; 8}_{N_0} & & & \underbrace{0 \;\; 1}_{B_0} & \end{array} \right) \end{array} \qquad b^T = (2 \quad 12) \qquad c_0^T = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ (\underbrace{-2 \;\; -1 \;\; -3}_{N_0} \; \underbrace{0 \;\; 0}_{B_0}) \end{array}$$

- $\gamma_0 = c^T_{N_0} - c^T_{B_0} B_0^{-1} N_0 = (-2 \ -1 \ -3)$
  - ottimalità e illimitatezza non soddisfatti

- Cambio di base

$h = 1 \to x_1$ entrante (Bland)

- scelgo k (CRM)

$$\min \left\{ \frac{B_0^{-1} b}{(\pi_h)_i} \right\} = \min \left\{ \frac{2}{1} \quad \frac{12}{2} \right\} \Rightarrow k = 1 \to x_4 \text{ uscente}$$

$$A_1 = (\underbrace{x_4 \; x_2 \; x_3}_{N_1} \; \underbrace{x_1 \; x_5}_{B_1}) \qquad c_1^T = \begin{array}{c} \begin{array}{ccccc} x_4 & x_2 & x_3 & x_1 & x_5 \end{array} \\ (\underbrace{0 \;\; -1 \;\; -3}_{N_1} \; \underbrace{-2 \;\; 0}_{B_1}) \end{array}$$

$$\begin{array}{l} \\ k = 1 \to \end{array} M = \begin{array}{c} \begin{array}{ccccc} \pi_h & e_k & \pi_2 & \pi_3 & B_0^{-1} b \end{array} \\ \left(\begin{array}{c|ccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 0 & 3 & 8 & 12 \end{array} \right) \end{array} = \left(\begin{array}{c|ccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & 1 & 6 & 8 \end{array} \right)$$

$$\qquad\qquad\qquad\qquad\qquad\qquad B_1^{-1} N_1 \qquad B_1^{-1} b$$

- $\gamma_1 = c^T_{N_1} - c^T_{B_1} B_1^{-1} N_1 = (0 \ -1 \ -3) - (-2 \ 0) \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & 6 \end{pmatrix} =$

$$= (0 \ -1 \ -3) - (-2 \ -2 \ -2) = (2 \ 1 \ -1)$$

- ottimalità non soddisfatta
- illimitatezza non soddisfatta

- Cambio di base

$h = 3 \to x_3$ entrante

- scelgo k

$$\min \left\{ \frac{(B_1^{-1} b)_i}{(\pi_h)_i} \right\} = \min \left\{ \frac{2}{1} \quad \frac{8}{6} \right\} \to k = 2 \to x_5 \text{ uscente}$$

$$c_2^T = \begin{array}{c} \begin{array}{ccccc} x_4 & x_2 & x_5 & x_1 & x_3 \end{array} \\ (\underbrace{0 \;\; -1 \;\; 0}_{N} \; \underbrace{-2 \;\; -3}_{B}) \end{array} \qquad \bar{x} = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ (2/3 \;\; 0 \;\; 4/3 \;\; 0 \;\; 0) \end{array}$$

$$\begin{array}{l} \\ k = 2 \to \end{array} M = \begin{array}{c} \begin{array}{ccccc} \pi_h & \pi_1 & \pi_2 & e_k & B_1^{-1} b \end{array} \\ \left(\begin{array}{c|ccc|c} 1 & 1 & 1 & 0 & 2 \\ 6 & -2 & 1 & 1 & 8 \end{array} \right) \end{array} = \begin{array}{c} \begin{array}{cc} B_2^{-1} N_2 & B_2^{-1} b \end{array} \\ \left(\begin{array}{c|ccc|c} 0 & 4/3 & 5/6 & -1/6 & 2/3 \\ 1 & -1/3 & 1/6 & 1/6 & 4/3 \end{array} \right) \end{array}$$

- $\gamma_2 = c^T_{N_2} - c^T_{B_2} B_2^{-1} N_2 = (0 \ -1 \ 0) - (-2 \ -3) \begin{pmatrix} 4/3 & 5/6 & -1/6 \\ -1/3 & 1/6 & 1/6 \end{pmatrix} = \left(\frac{5}{3}, \frac{7}{6}, \frac{1}{6} \right)$