Vision and Perception

Motion estimation and Optical flow

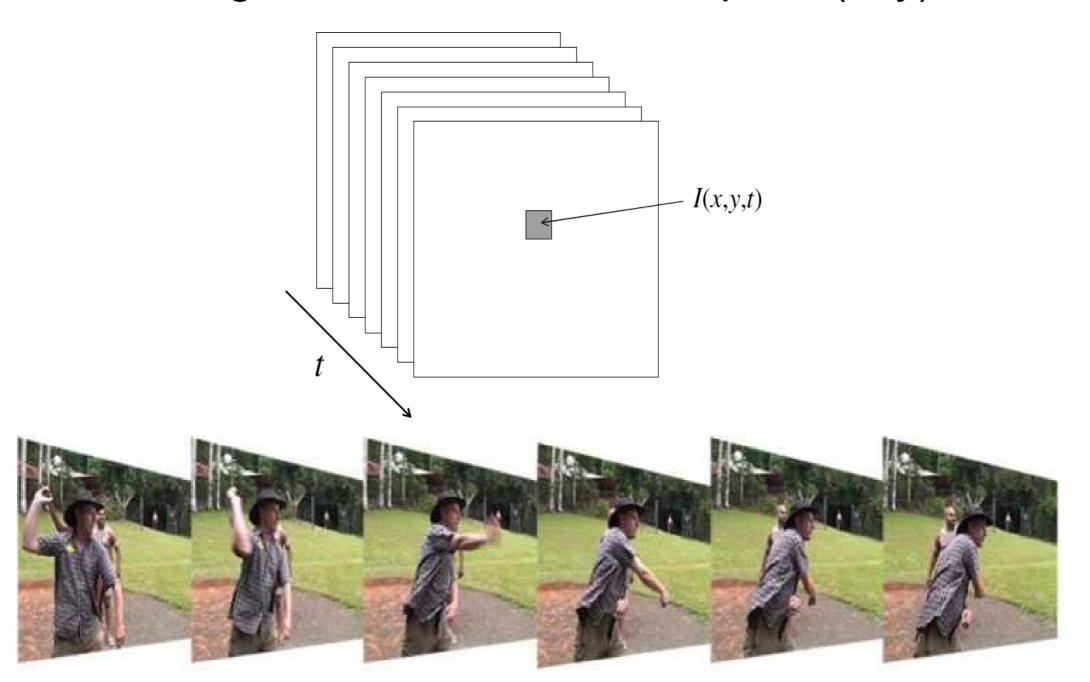


References

• Basic reading: Szeliski, Chapter 9.3-9.4

From images to videos

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



Uses of motion

The estimation of every pixel in a sequence is a problem with many applications in computer vision

- Improving video quality
 - Motion stabilization
 - Super resolution
- Segmenting objects based on motion cues
- Tracking objects
- Recognizing events and activities

Super-resolution

Example: a set of low quality images

Most of the test data of couple of exceptions. I low-temperature solde, investigated (or some of manufacturing technol monwetting of 40 ln 40 Structural coarse that cycling of 58 Bi 42 Structural cycl	Most of the test data o couple of exceptions. I how temperature solder investigated (or some omanufacturing technol nonwetting of 40 in 40% nuicrostructural coarse mal cycling of 58842%.	Most of the test data of couple of exceptions. I low-temperature solds investigated (or some manufacturing technol nonwetting of 40th405 microstructural coarse mal cycling of 58Bi425
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- Irani, M.; Peleg, S. (June 1990). "Super Resolution From Image Sequences". International Conference on Pattern Recognition
- Fast and Robust Multiframe Super Resolution, Sina Farsiu, M. Dirk Robinson, Michael Elad, and Peyman Milanfar, EEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 13, NO. 10, OCTOBER 2004

Super-resolution

Each of these images looks like this:

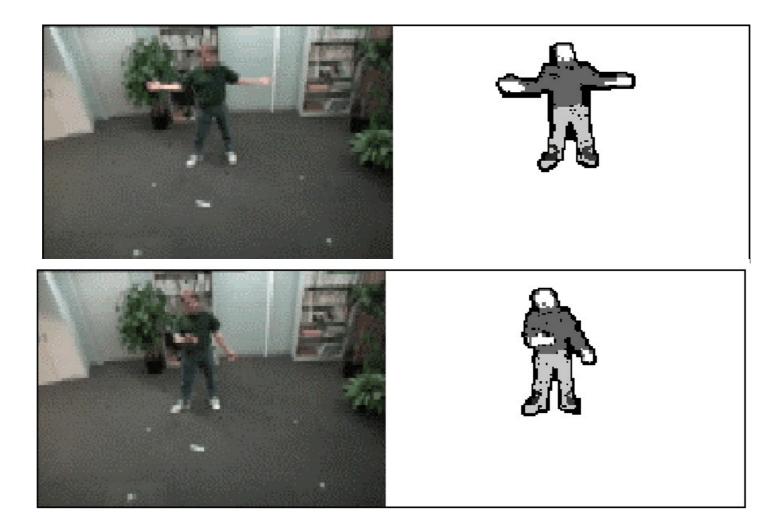
Most of the test data of couple of exceptions. T low-temperature solder investigated (or some o manufacturing technolmonwetting of 40In40Se microstructural coarse mail cycling of 58Bi42Si

The recovery result:

Most of the test data of couple of exceptions. T low-temperature solder investigated (or some o manufacturing technol nonwetting of 40In40Sr microstructural coarse mal cycling of 58Bi42Si

Segmenting objects based on motion cues

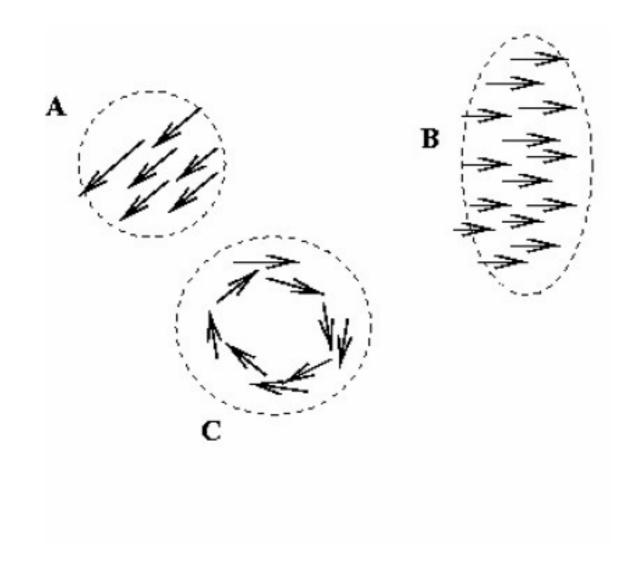
- Background subtraction
 - A static camera is observing a scene
 - Goal: separate the static background from the moving foreground

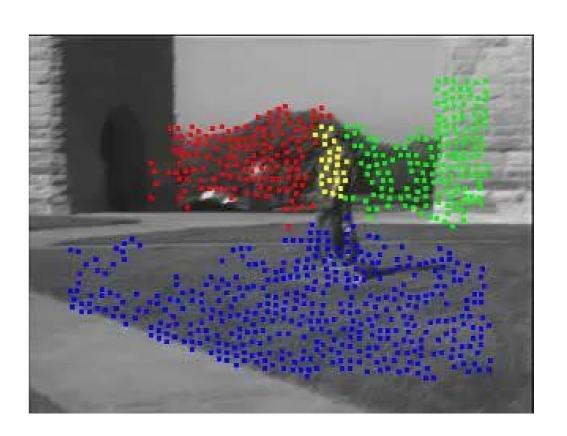


https://www.youtube.com/watch?v=YAszeOaInUM

Segmenting objects based on motion cues

- Motion segmentation
 - Segment the video into multiple coherently moving objects

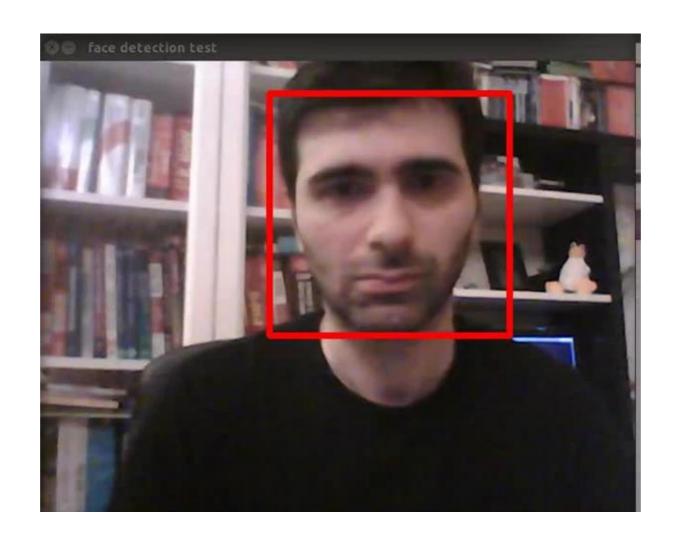




S. J. Pundlik and S. T. Birchfield, Motion Segmentation at Any Speed, Proceedings of the British Machine Vision Conference 2006

Tracking objects

Facing tracking on openCV

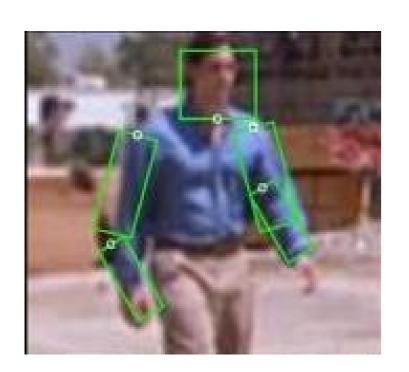


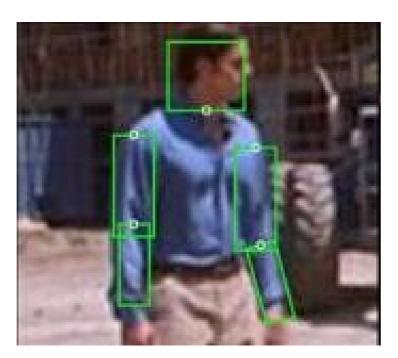
OpenCV's face tracker uses an algorithm called Camshift (based on the meanshift algorithm)

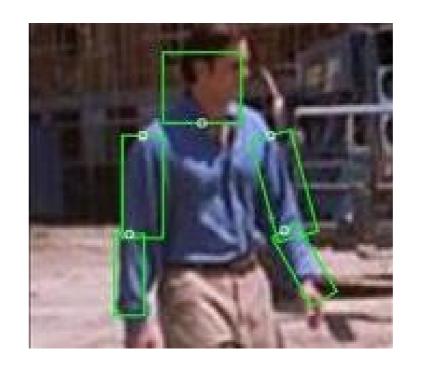
http://www.youtube.com/watch?v=HTk_UwAYzVk

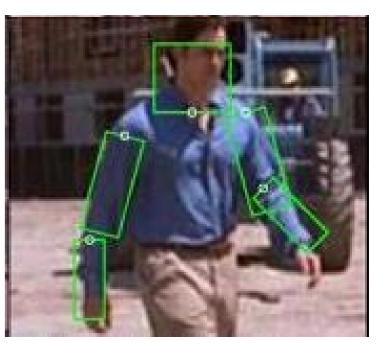
http://learnopencv.com/wp-content/uploads/2017/02/real-time-face-tracking.gif

Tracking body parts



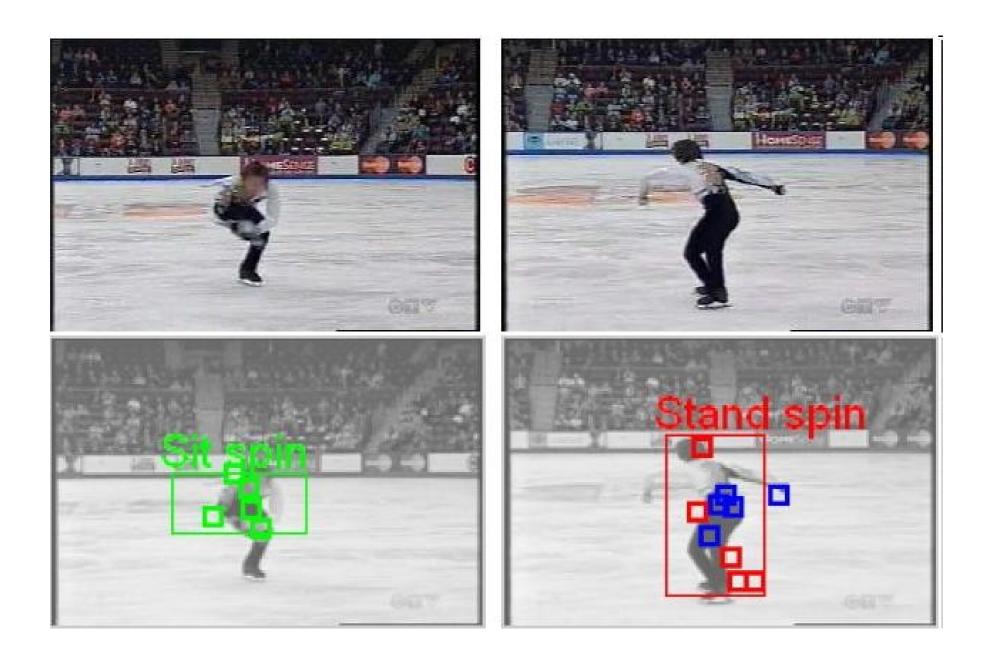






Courtesy of Benjamin Sapp

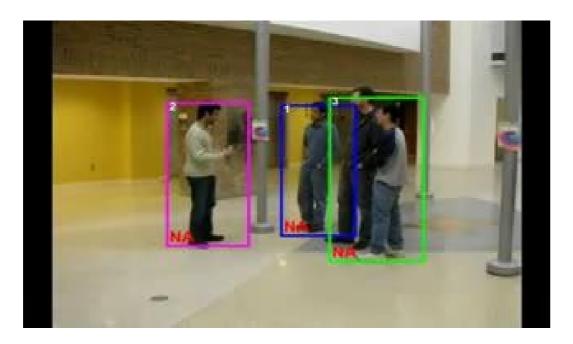
Recognizing events and activities



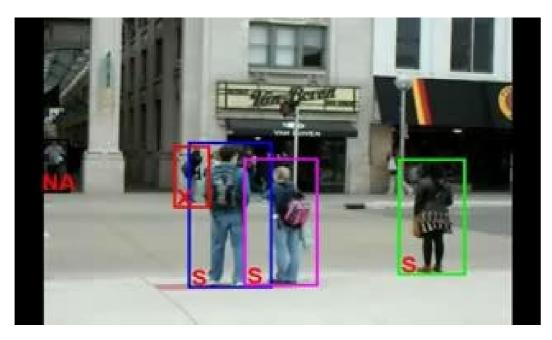
Juan Carlos Niebles, Hongcheng Wang and Li Fei-Fei, Unsupervised Learning of Human Action Categories Using Spatial-Temporal Words, (*BMVC*), Edinburgh, 2006.

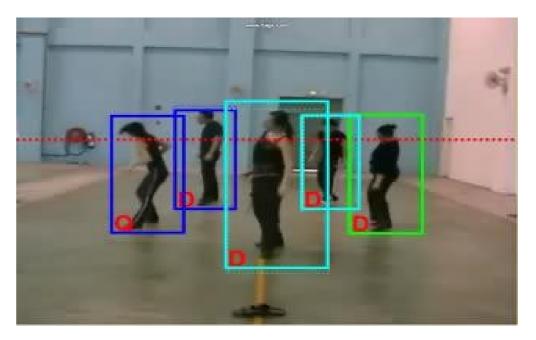
Recognizing group activities

Crossing – Talking – Queuing – Dancing – jogging









X: Crossing, S: Waiting, Q: Queuing, W: Walking, T: Talking, D: Dancing

Motion estimation techniques

Optical flow

 Recover image motion at each pixel from spatio-temporal image brightness variations

Feature-tracking

 Extract visual features (corners, textured areas) and "track" them over multiple frames

Tracking features

Tracking object regions frame to frame



Courtesy of Jean-Yves Bouguet – Vision Lab, California Institute of Technology

Steve Gu, Ying Zheng, and Carlo Tomasi. 2010. Efficient visual object tracking with online nearest neighbor classifier. In Proceedings of the 10th Asian conference on Computer vision - Volume Part I (ACCV'10). Springer-Verlag, Berlin, Heidelberg, 271–282.

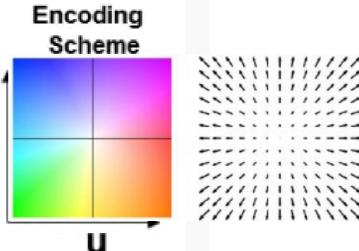
Optical flow

Optical flow is used to see how every point is moving frame to frame in a video sequence



Colour code for visualisation

HSV encoding scheme: hue, saturation, value Vector field: displacement, velocity



Optical flow

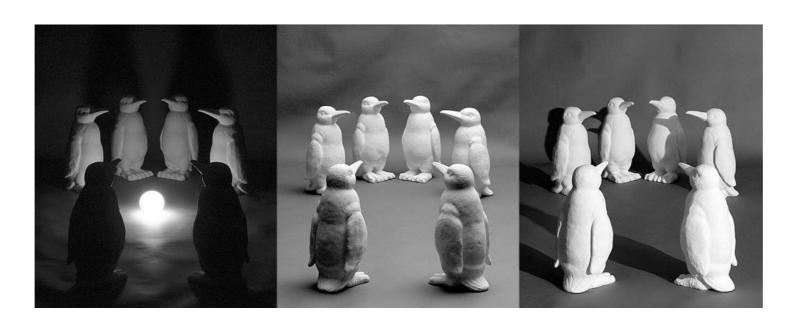
- Sparse Optical Flow: this method processes the flow vectors of only a few of the most interesting pixels from the entire image, within a frame.
- 2. **Dense Optical Flow**: the flow vectors of all pixels in the entire frame are processed which, in turn, makes this technique slower but more accurate.

Optical flow

Definition: optical flow is the *apparent* motion of brightness patterns in the image

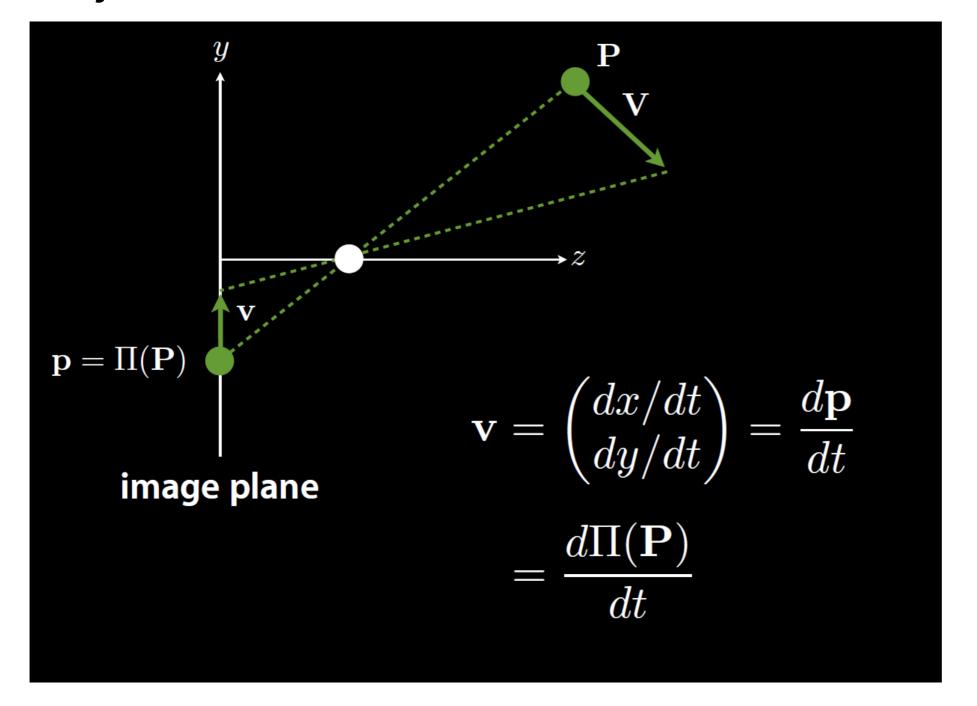
GOAL: Recover image motion at each pixel by optical flow. Pattern of motion of pixels between two consecutive frames. The motion can be caused either by the movement of a scene or by the movement of the camera.

Note: apparent motion can be caused by lighting changes without any actual motion (optical flow is different from the concept of motion field)

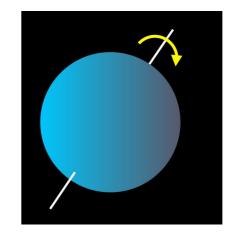


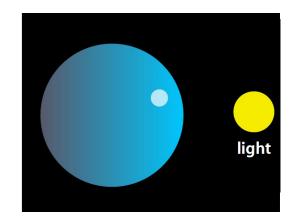
Motion field

Projection of the 3D scene velocities onto the image plane

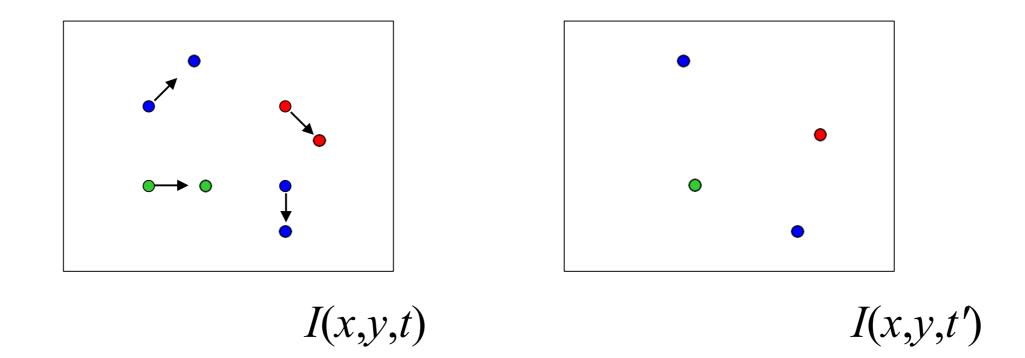


It is a geometric concept
The optical flow is a photometric concept
Ideally OF e MF are equivalent





Estimating optical flow

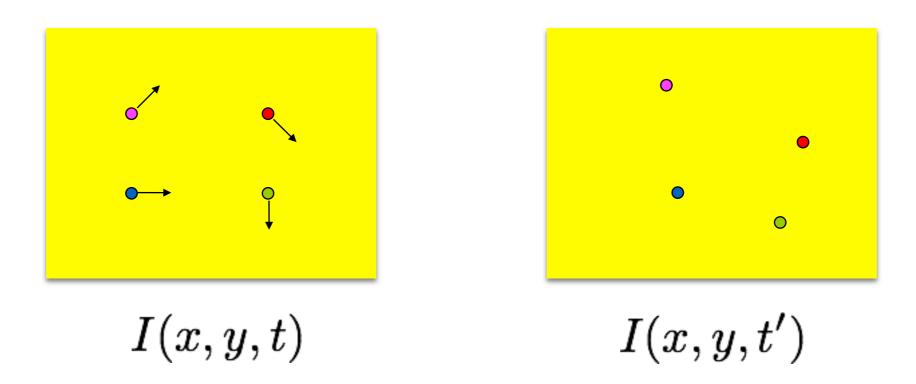


Given two subsequent frames, estimate the apparent motion field u(x,y), v(x,y) between them.

Key assumptions:

- Brightness constancy: projection of the same point looks the same in every frame
- Small motion: points do not move very far
- Spatial coherence: points move like their neighbors

The approach



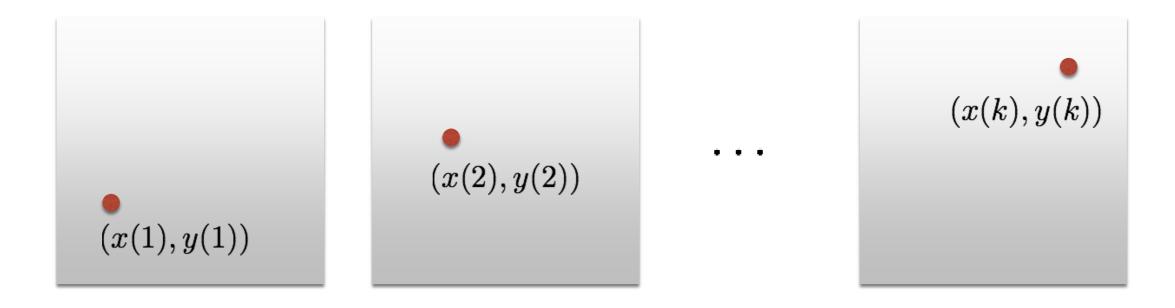
Look for nearby pixels with the same color

(small motion)

(color constancy)

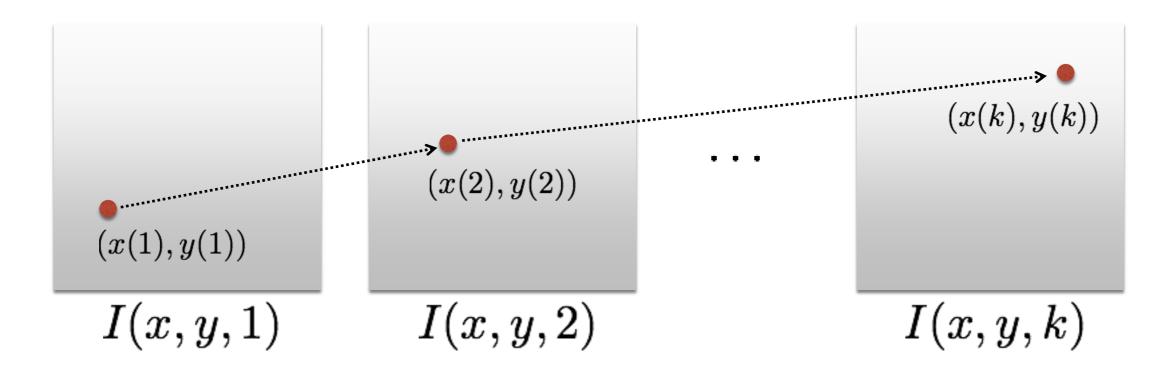
Brightness constancy

Scene point moving through image sequence



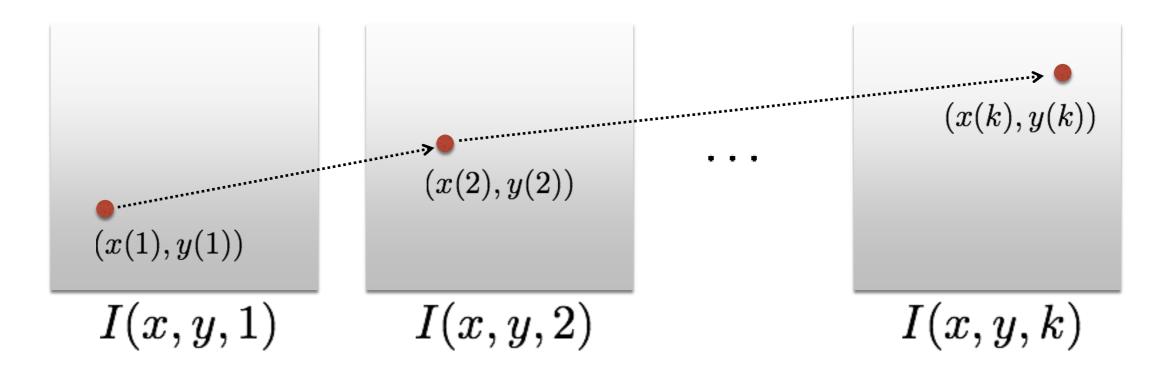
Brightness constancy

Scene point moving through image sequence



Brightness constancy

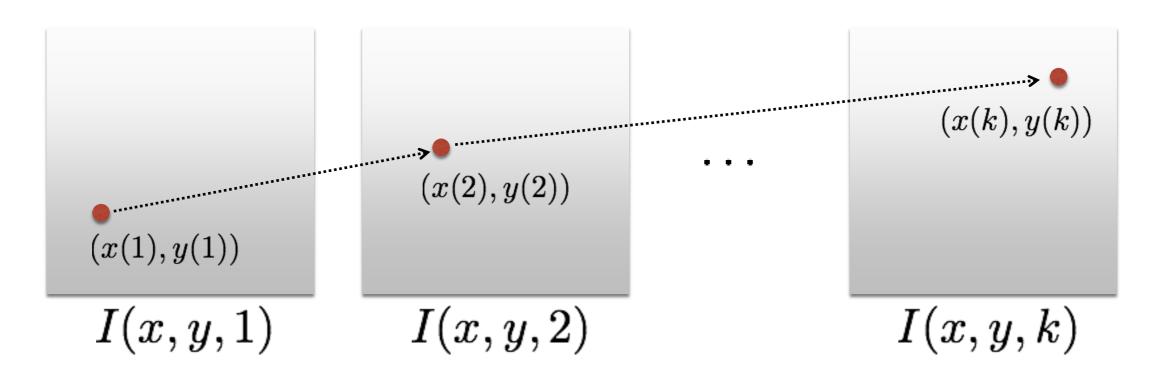
Scene point moving through image sequence



Assumption: brightness of the point will remain the same

Brightness constancy

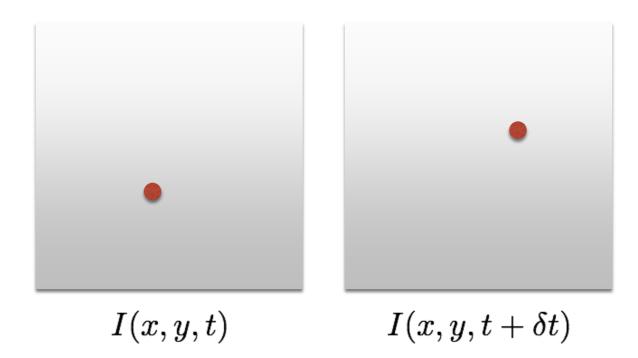
Scene point moving through image sequence



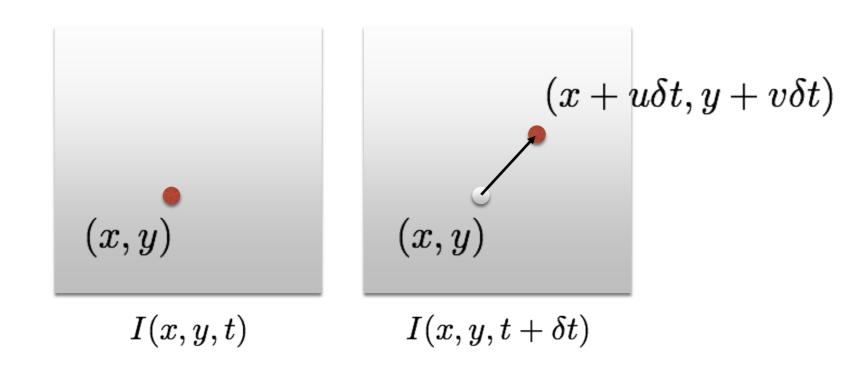
Assumption: Brightness of the point will remain the same

$$I(x(t),y(t),t)=C$$

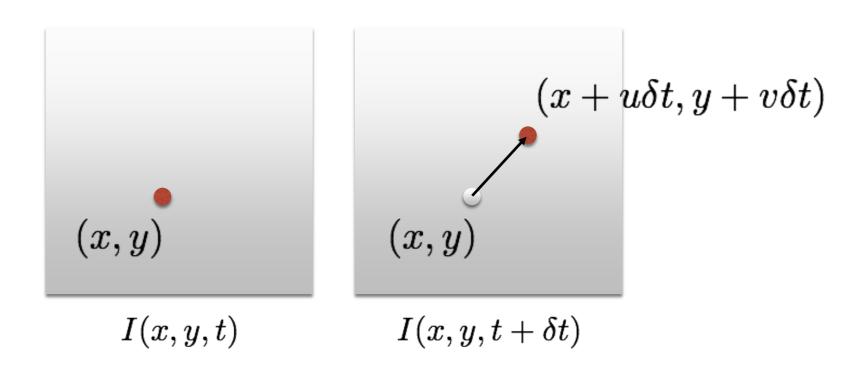
Small motion



Small motion

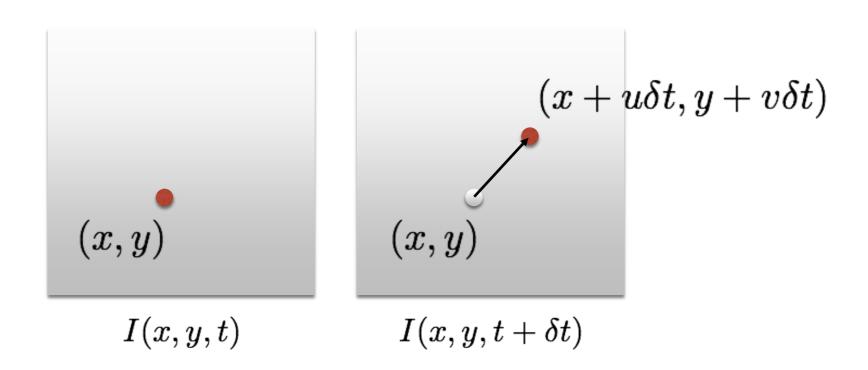


Small motion



Optical flow (velocities): (u,v) Displacement: $(\delta x,\delta y)=(u\delta t,v\delta t)$

Small motion



Optical flow (velocities): (u,v) Displacement: $(\delta x,\delta y)=(u\delta t,v\delta t)$

For a *small space-time step*

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

the brightness between two consecutive image frames is the same

Taylor series expansion

For small space-time step, brightness of a point is the same

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

$$\delta x \qquad \delta y$$

Insight:

If the time step is really small, we can *linearize* the intensity function with first order approximation of the Taylor series expansion

Expand a function as an infinite sum of its derivatives For one variable

$$f(x + \delta x) = f(x) + \frac{\partial f}{\partial x} \delta x + \frac{\partial^2 f}{\partial x^2} \frac{\delta x^2}{2!} + \dots + \frac{\partial^n f}{\partial x^n} \frac{\delta x^n}{n!}$$

If δx is small:

$$f(x + \delta x) = f(x) + \frac{\partial f}{\partial x} \delta x + O(\delta x^2)$$
 Almost Zero

Taylor series expansion

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$



$$I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = I(x, y, t)$$

assuming small motion

fixed point

Partial derivative

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0$$

cancel terms

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0$$

divide by $\,\delta t$

take limit $\delta t o 0$

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Brightness Constancy Equation

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Brightness Constancy Equation

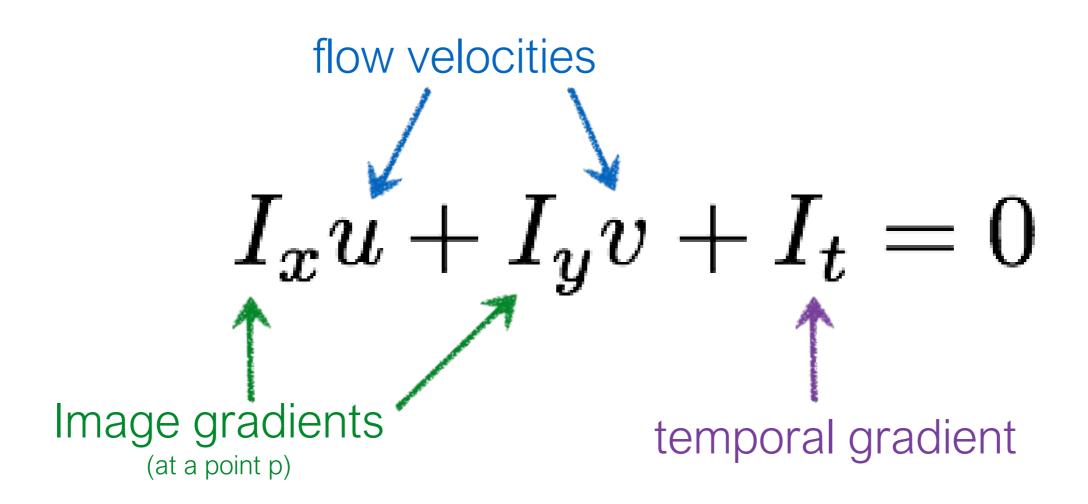
$$I_{m{x}}u+I_{m{y}}v+I_{m{t}}=0$$
 (x-flow) (y-flow)

shorthand notation

$$abla I^{ op} \boldsymbol{v} + I_t = 0$$

vector form

What do the term of the brightness constancy equation represent?



$$I_x u + I_y v + I_t = 0$$

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

Forward difference Sobel filter Derivative-of-Gaussian filter $I_t = \frac{\partial I}{\partial t}$

temporal derivative

Frame differencing

. . .

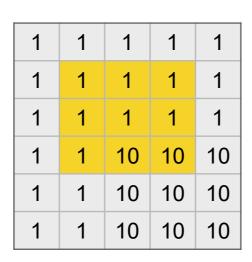
Frame differencing

Example of a forward difference

t

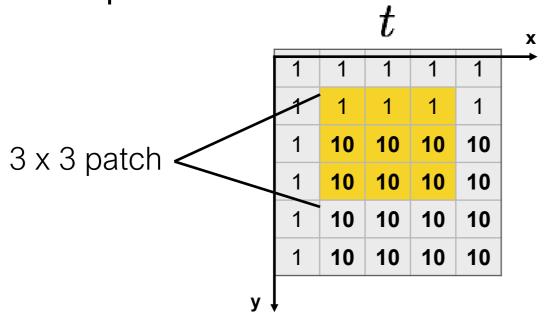
t+1

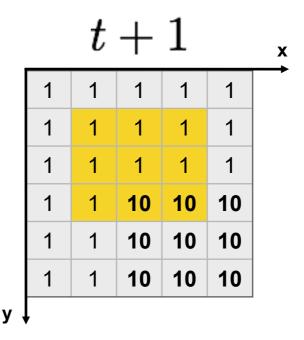
$$I_t = \frac{\partial I}{\partial t}$$

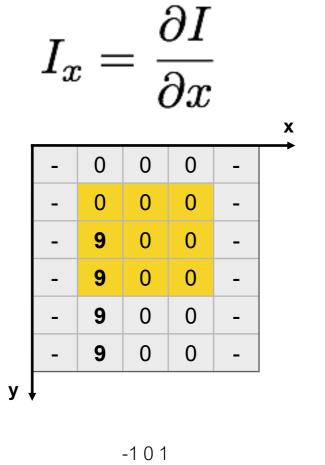


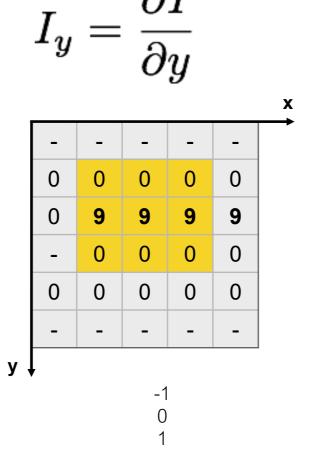
0	0	0	0	0
0	0	0	0	0
0	9	9	9	9
0	9	0	0	0
0	9	0	0	0
0	9	0	0	0

Example:









	$I_t = \frac{\partial I}{\partial t}$						
	0	0	0	0	0		
	0	0	0	0	0		
	0	9	9	9	9		
	0	9	0	0	0		
	0	9	0	0	0		
	0	9	0	0	0		
y .	,						

$$I_x u + I_y v + I_t = 0$$

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

Forward difference Sobel filter Derivative-of-Gaussian filter $u=rac{dx}{dt} \quad v=rac{dy}{dt}$ optical flow

unknown

$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

Frame differencing

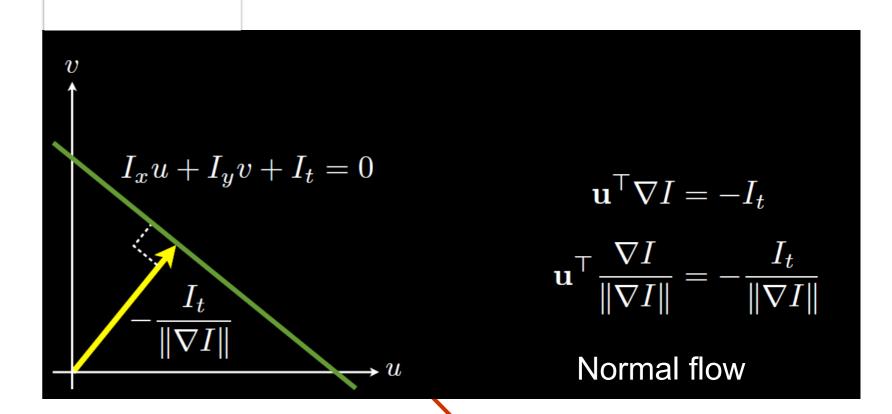
. . .

Brightness constancy equation

Solution lies on a straight line

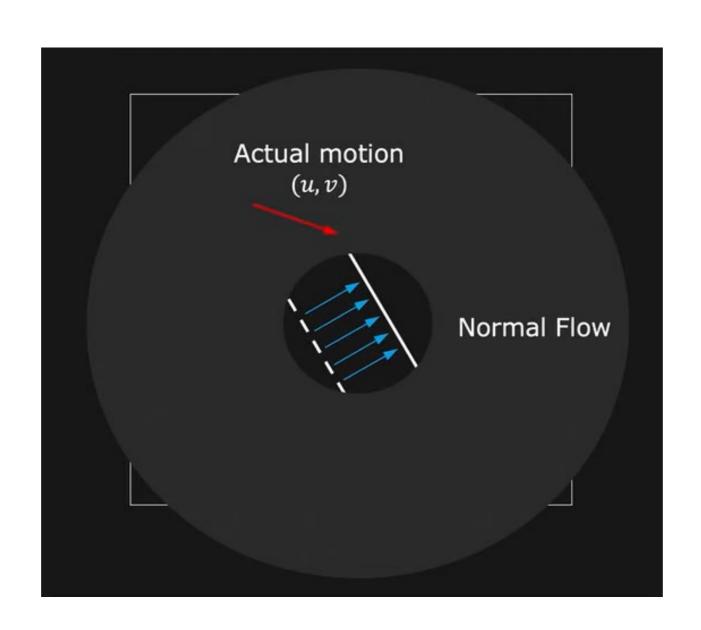
$$I_x u + I_y v + I_t = 0$$

many combinations of u and v will satisfy the equality



Optical flow can be split in two components: normal flow and parallel flow

The solution cannot be determined uniquely with a single constraint (a single pixel). The solution is underconstrained Where do we get more equations (constraints)?



Locally we can only determine normal flow

Two main methods

Lucas-Kanade Optical Flow (1981)
 method of differences
 'constant' flow (flow is constant for all pixels)
 local method (sparse)

Horn-Schunck Optical Flow (1981)

brightness constancy, small motion 'smooth' flow (flow can vary from pixel to pixel) global method (dense)

Lucas-Kanade Optical Flow

Assumptions

Assume that the surrounding patch has 'constant flow' Neighboring pixels have same displacement: i.e for each pixel assume motion field, and hence optical flow (u,v), is constant within a small neighborhood.

Using a 5 x 5 image patch, gives us 25 equations

$$I_x(\boldsymbol{p}_1)u + I_y(\boldsymbol{p}_1)v = -I_t(\boldsymbol{p}_1)$$

 $I_x(\boldsymbol{p}_2)u + I_y(\boldsymbol{p}_2)v = -I_t(\boldsymbol{p}_2)$

:
$$I_x(\mathbf{p}_{25})u + I_y(\mathbf{p}_{25})v = -I_t(\mathbf{p}_{25})$$

n^2 (25) equations, 2 unknown: find least square solution

Least square solution

Solve linear system Ax=b

- that it is equivalent to write (least squares using pseudo-inverse)

'Lucas-Kanade Optical Flow'
$$\begin{bmatrix} \sum\limits_{p\in P}I_xI_x & \sum\limits_{p\in P}I_xI_y \\ \sum\limits_{p\in P}I_yI_x & \sum\limits_{p\in P}I_yI_y \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} \sum\limits_{p\in P}I_xI_t \\ \sum\limits_{p\in P}I_yI_t \end{bmatrix}$$

where the summation is over each pixel **p** in patch **P**

$$x = (A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}b$$

When does optical flow estimation work?

$$x = (A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}b$$

 $A^{\mathsf{T}}A$ should be invertible (det !=0)

 $A^{\mathsf{T}}A$ should be well conditioned

 λ_1 and λ_2 should not be too small (both are significant enough)

 λ_{1} should not be too large respect to λ_{2} (λ_{1} =larger eigenvalue)

When does optical flow estimation work?

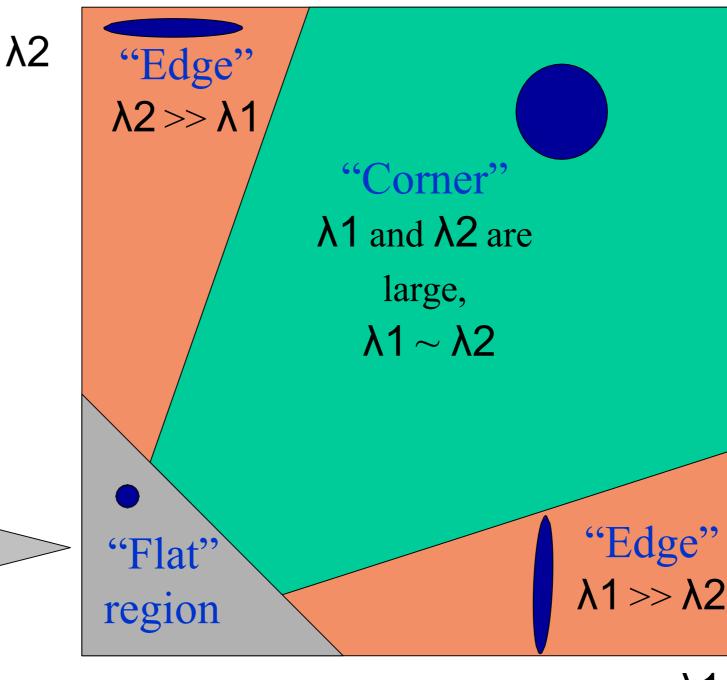
A^TA is the second moment matrix (Harris corner detector)

$$A^{T}A = \begin{bmatrix} \sum I_{x}I_{x} & \sum I_{x}I_{y} \\ \sum I_{x}I_{y} & \sum I_{y}I_{y} \end{bmatrix}$$

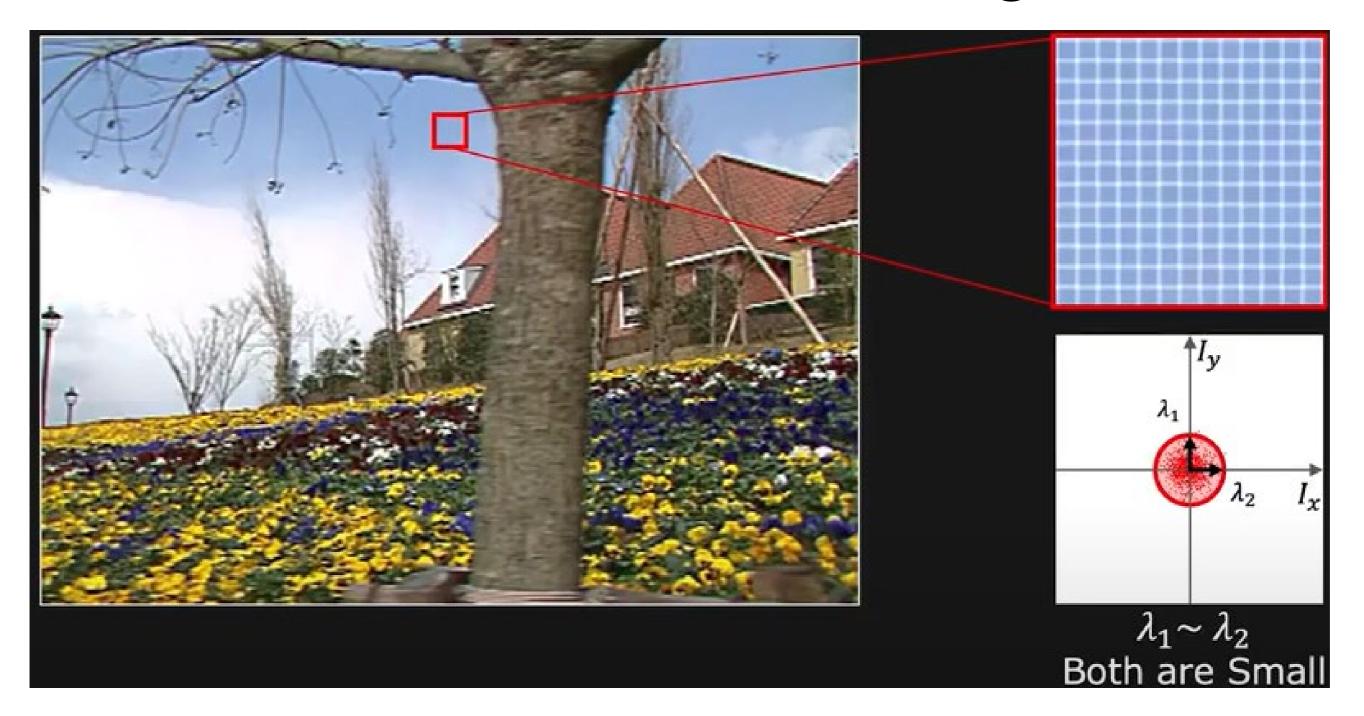
- Eigenvectors and eigenvalues of A^TA relate to edge direction and magnitude
 - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change
 - The other eigenvector is orthogonal to it

Interpreting the eigenvalues

Classification of image points using eigenvalues of the second moment matrix:

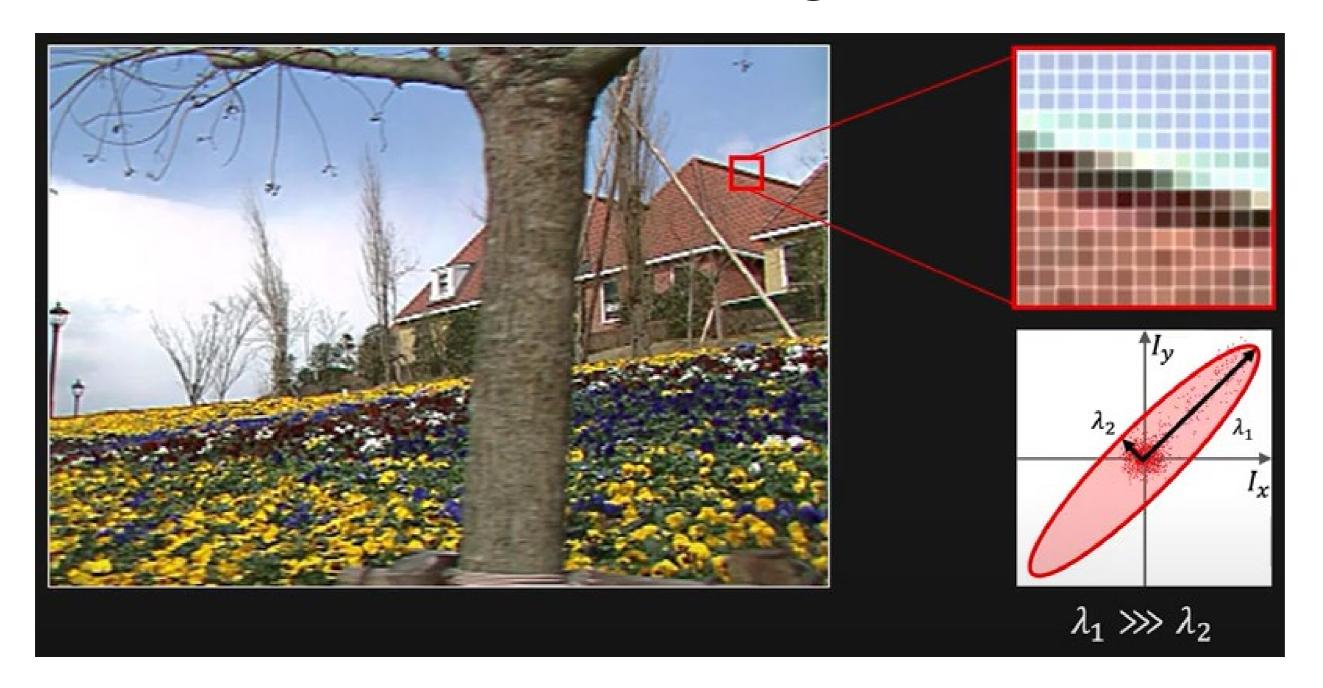


Low-texture region



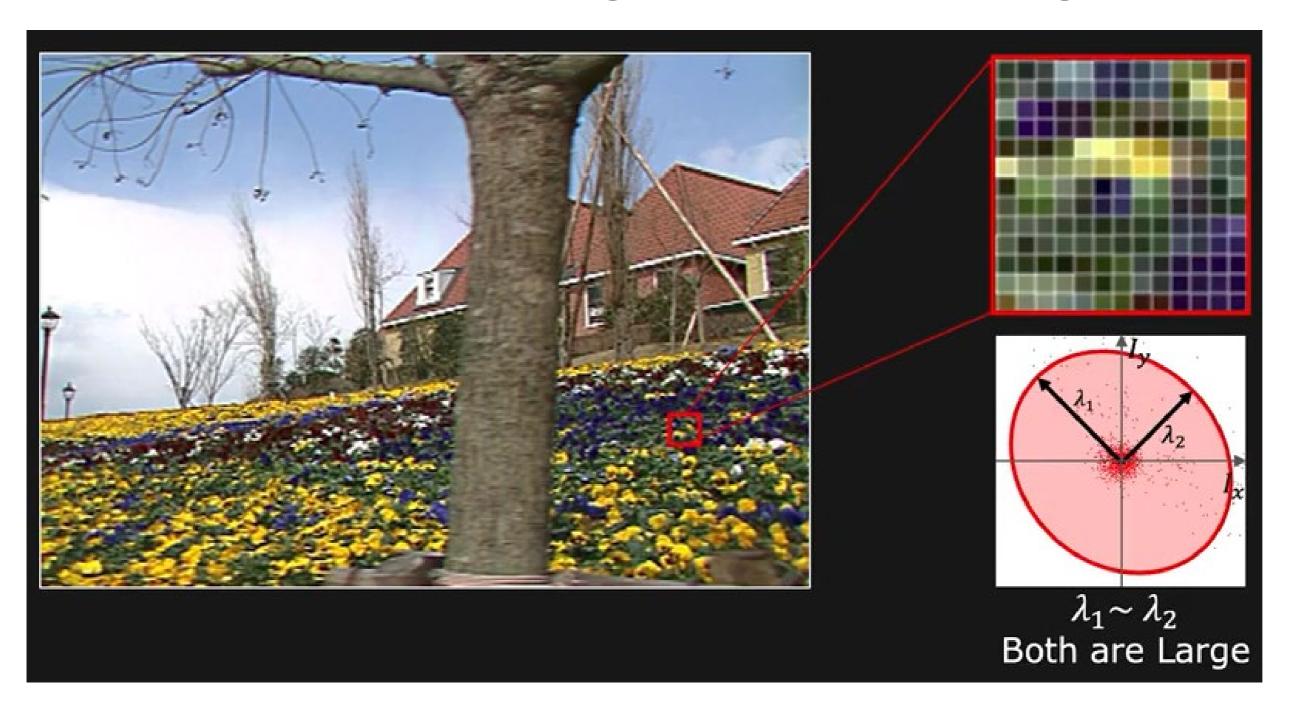
- Equations for all pixels in window are more or less the same
- Gradients have small magnitude
- Can not reliably compute the flow

Edges



- gradients very large or very small
- prominent gradient in one direction-→ badly conditioned
- can not reliably compute the flow

High-texture region



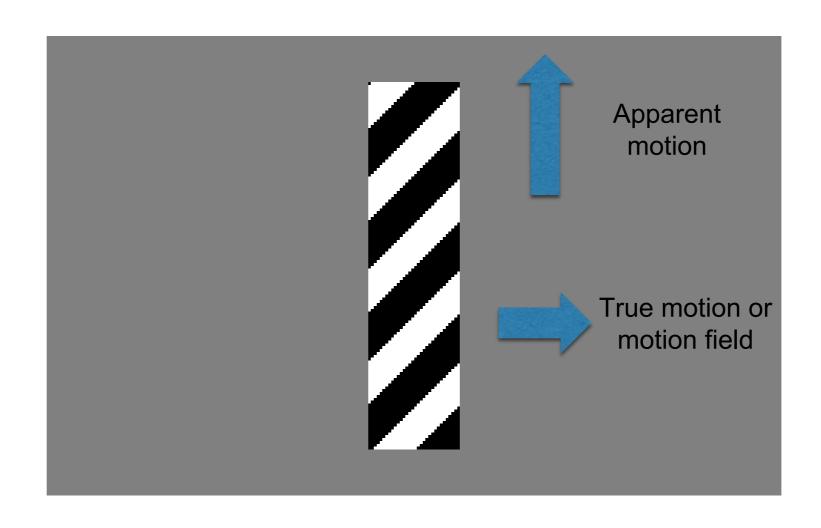
- gradients are different, large magnitudes
- well conditioned
- Can reliably compute optical flow

Implications

- Corners and high texture regions are when λ1, λ2 are big; this is also when Lucas-Kanade optical flow works best
- Corners and high texture regions are regions with two different directions of gradient (at least)
- Corners and high texture regions are good places to compute flow

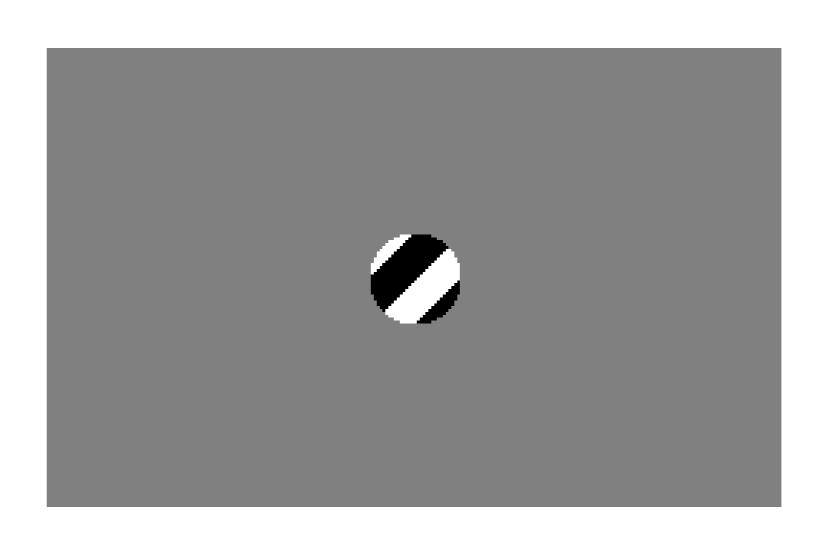
Barber's pole illusion

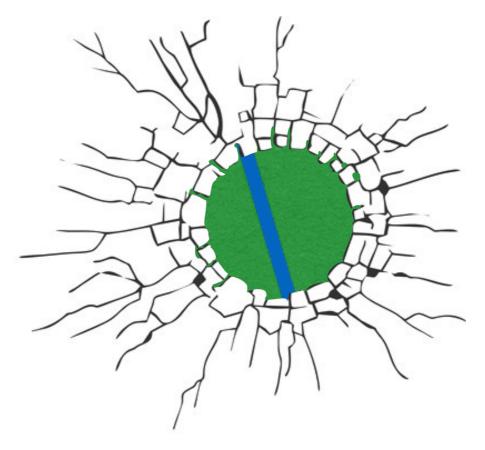




Barber's pole illusion

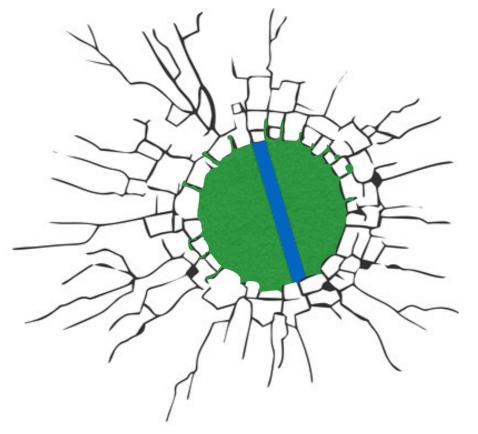






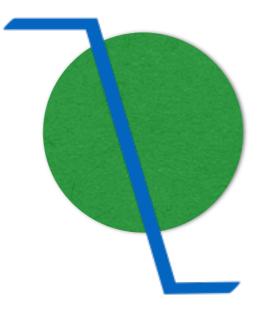
small visible image patch

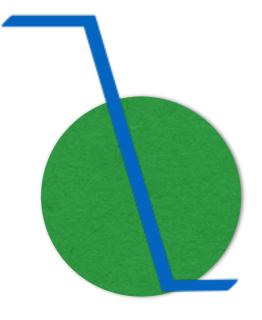
In which direction is the line moving?

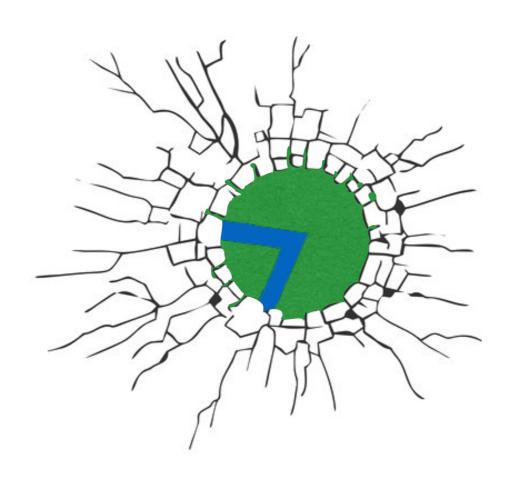


small visible image patch

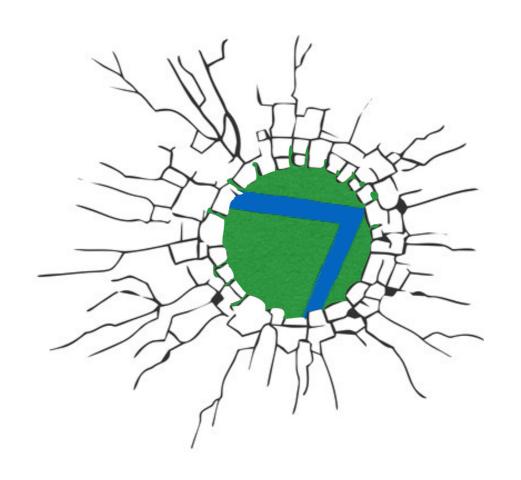
In which direction is the line moving?





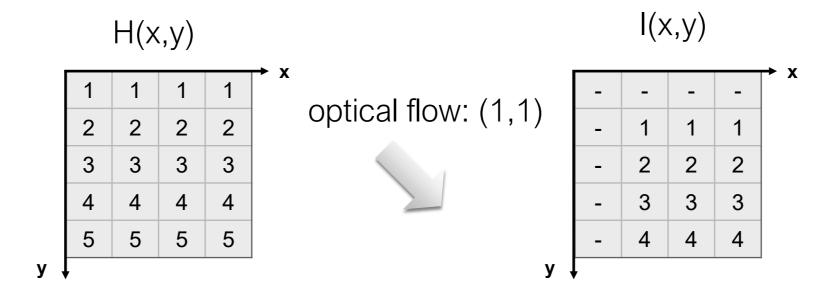


Want patches with different gradients to avoid the aperture problem



Want patches with different gradients to avoid the aperture problem

Aperture Problem: example



$$I_x u + I_y v + I_t = 0$$

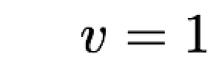
Compute gradients

$$I_x(3,3) = 0$$

$$I_y(3,3) = 1$$

$$I_t(3,3) = I(3,3) - H(3,3) = -1$$

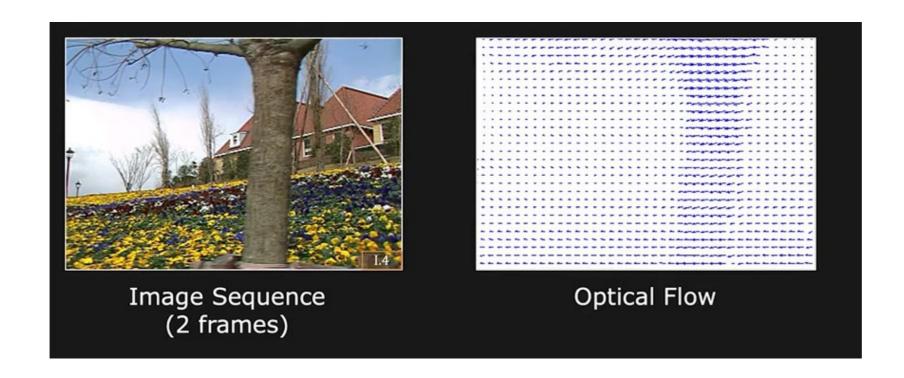
Solution:



We recover the v of the optical flow but not the u.

Lukas-Kanade method

- Decide for local neighborhood of WxW pixels and apply uniformly in frames (smoothing frame first with Gaussian filter with a small standard deviation sigma=1.5)
- At frame t, t+1 calculate the derivatives Ix, Iy, It
- For each couple of frames obtain equational system and solve in the least square sense calcuating the eigenvalues
- Plot the optical flow vectors (directions and magnitude)



Two main methods

- Lucas-Kanade Optical Flow (1981)
 method of differences
 'constant' flow (flow is constant for all pixels)
 local method (sparse)
- Horn-Schunck Optical Flow (1981)
 brightness constancy, small motion
 'smooth' flow (flow can vary from pixel to pixel)
 global method (dense)

Key idea

(of Horn-Schunck optical flow)

In order to compute optical flow:

Enforce

brightness constancy

Enforce smooth flow field

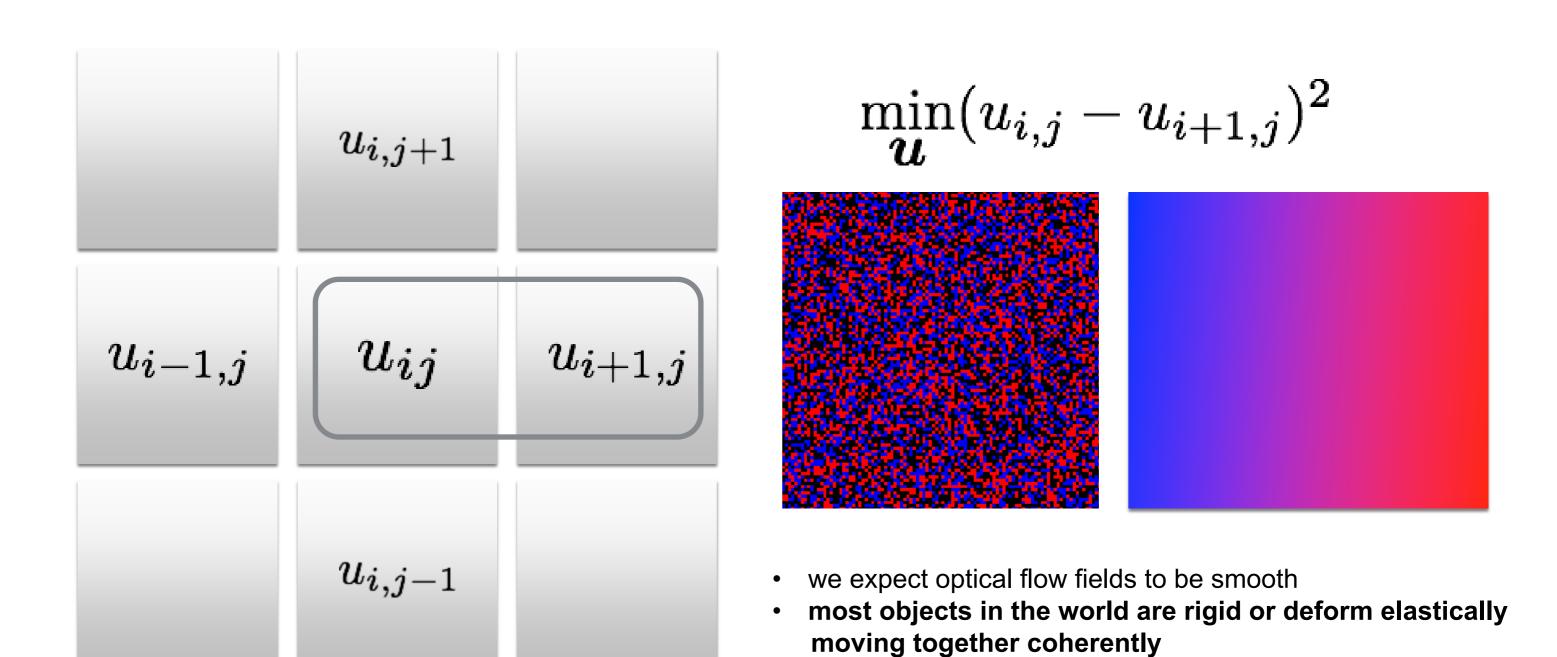
Enforce brightness constancy

$$I_x u + I_y v + I_t = 0$$

For every pixel, we need to solve the second moment matrix that is approximated by a quadratic form as you remember from HCD

$$\min_{u,v} \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

Enforce smooth flow field



u-component of flow

Horn-Schunck (HS) optical flow

$$\min_{m{u},m{v}} \sum_{i,j} \left\{ E_s(i,j) + \sum_{m{v} \in \mathcal{U}} E_d(i,j)
ight\}$$
 weight

Optimization problem

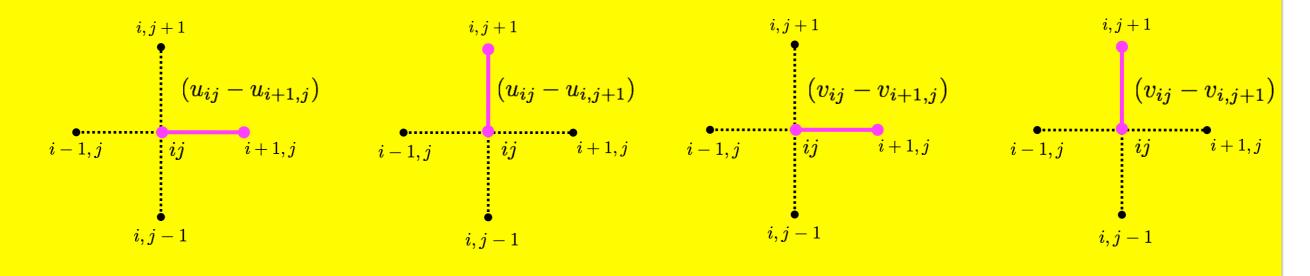
- HS algorithm assume smoothness in the flow over the whole image
- The flow is formulated as a global energy function which is then sought to be minimized
- Compute partial derivative, derive update equations (gradient descent!)

HS optical flow objective function

Brightness constancy
$$E_d(i,j) = \left[I_x u_{ij} + I_y v_{ij} + I_t
ight]^2$$

Smoothness

$$E_s(i,j) = \frac{1}{4} \left[(u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2 \right]$$



Horn-Schunck Optical Flow Algorithm

1. Precompute image gradients

- $I_y \quad I_x$
- 2. Precompute temporal gradients I_{t}
- 3. Initialize flow field

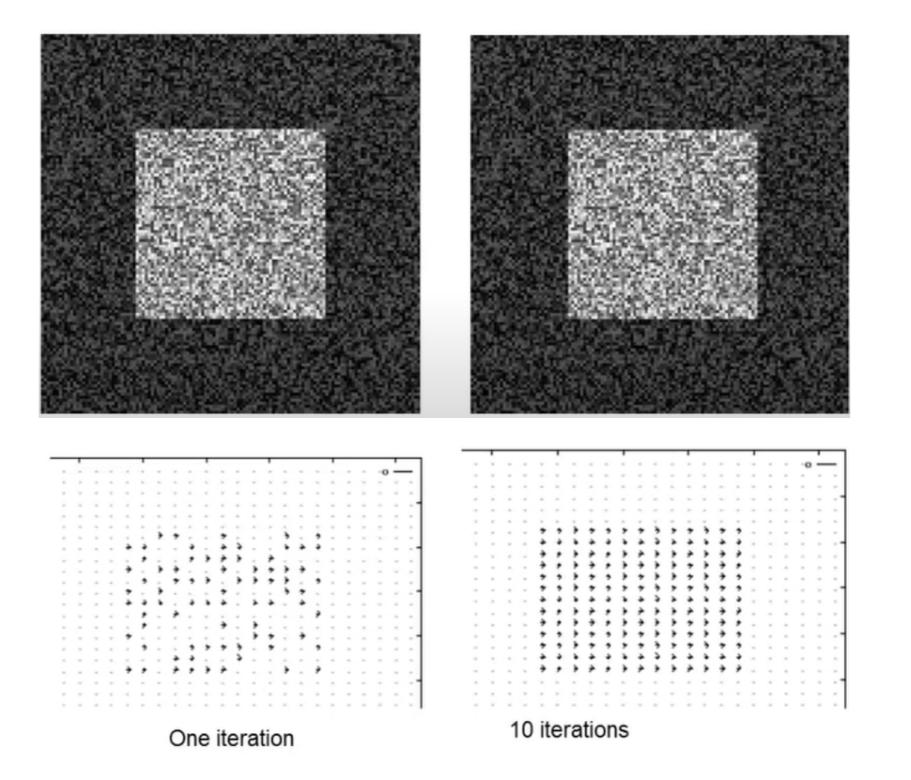
- $oldsymbol{u}=oldsymbol{0}$
- v = 0

4. While not converged

Compute flow field updates for each pixel:

$$\hat{u}_{kl} = \bar{u}_{kl} - \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} I_x \qquad \hat{v}_{kl} = \bar{v}_{kl} - \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} I_y$$

Horn – Shunck example

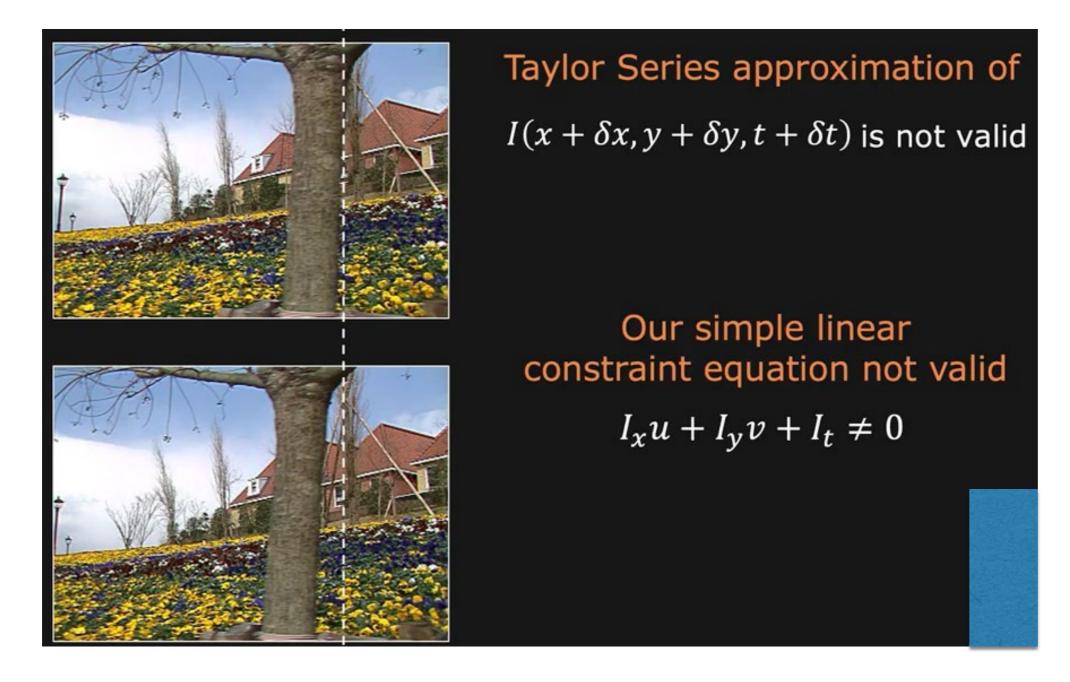


Recap

Key assumptions

- Small motion: points do not move very far
- Brightness constancy: projection of the same point looks the same in every frame
- Spatial coherence: points move like their neighbors

Revisiting the small motion assumption

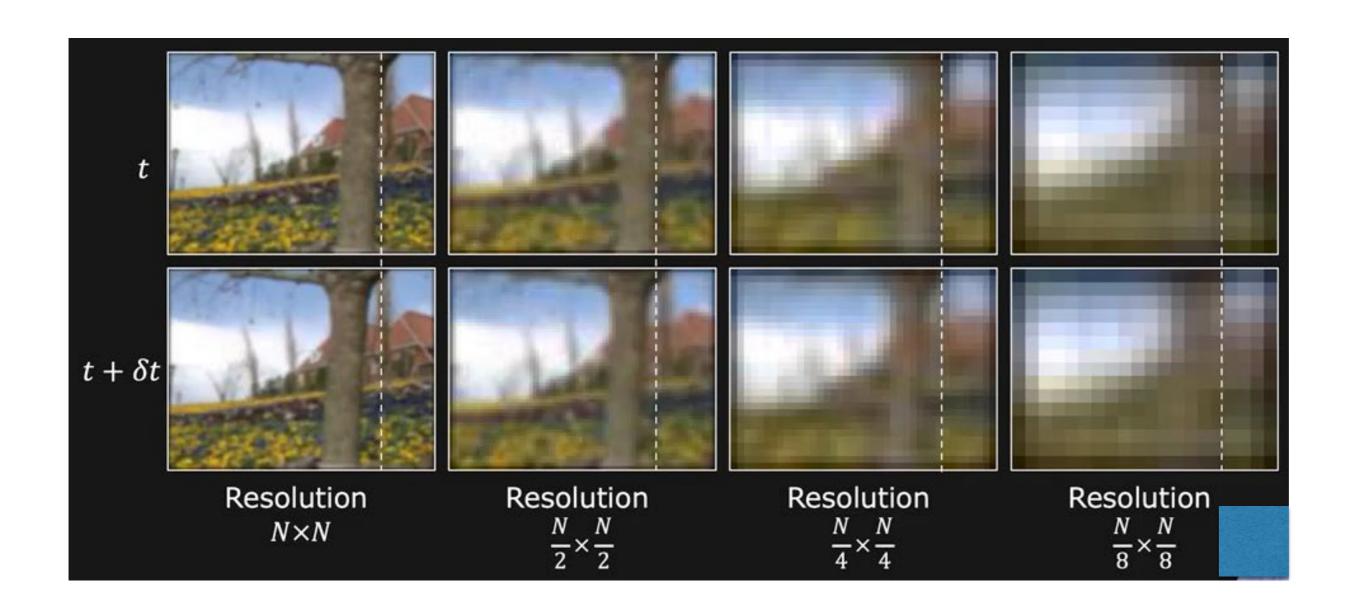


Case of large motion

Is this motion small enough?

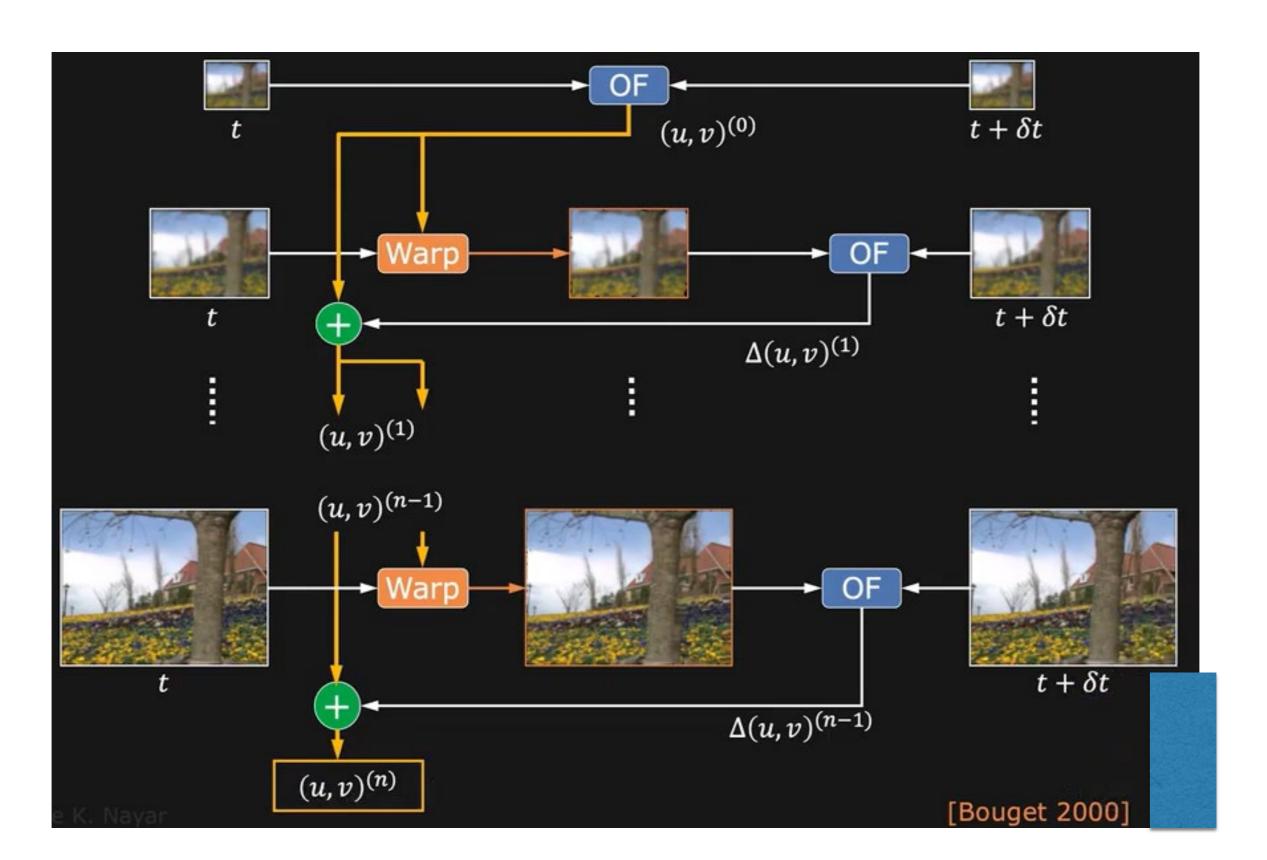
- Probably not: it's much larger than one pixel (2nd order terms dominate)
- How might we solve this problem?

Reduce the resolution

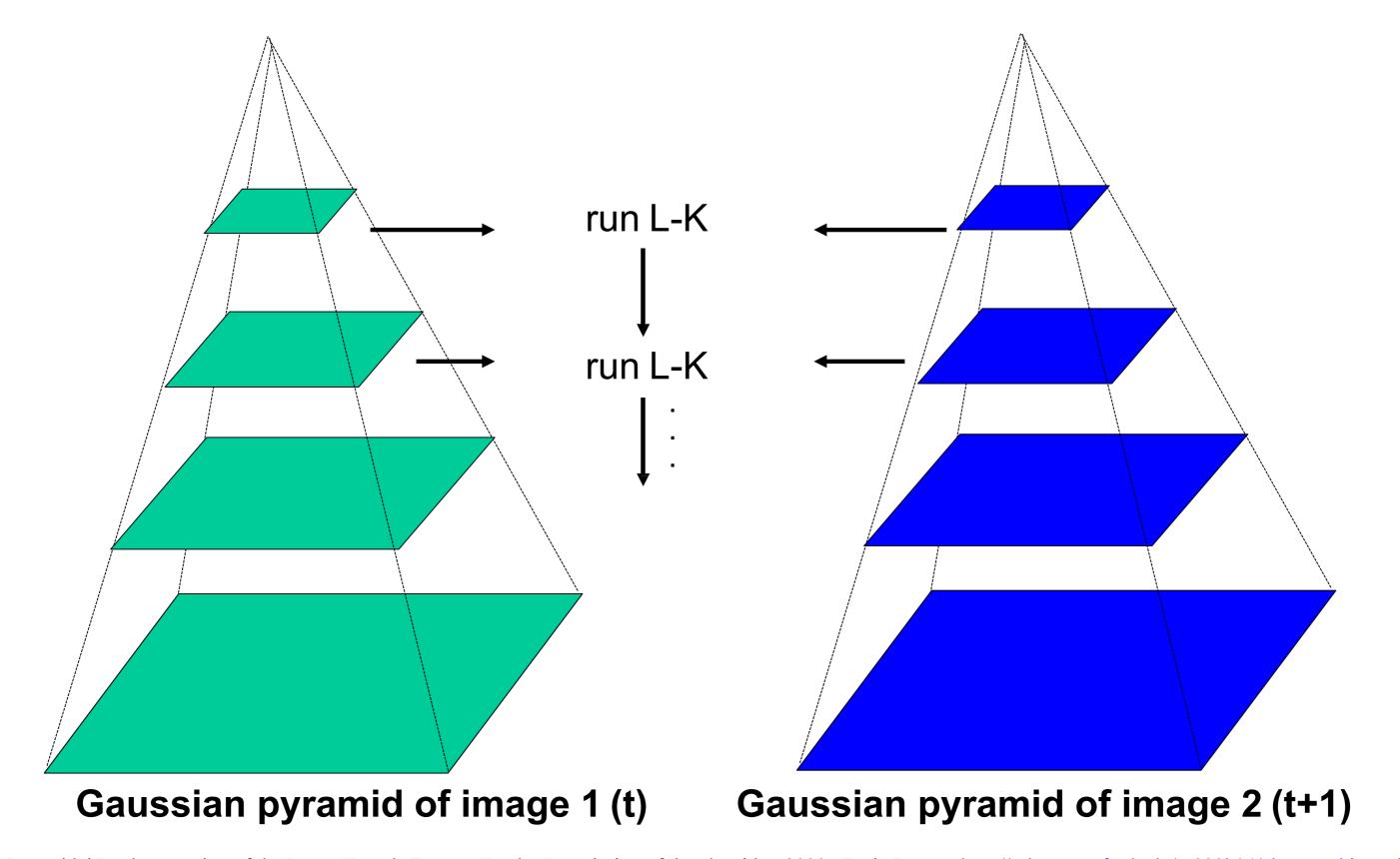


At lowest resolution, motion <=1 pixel

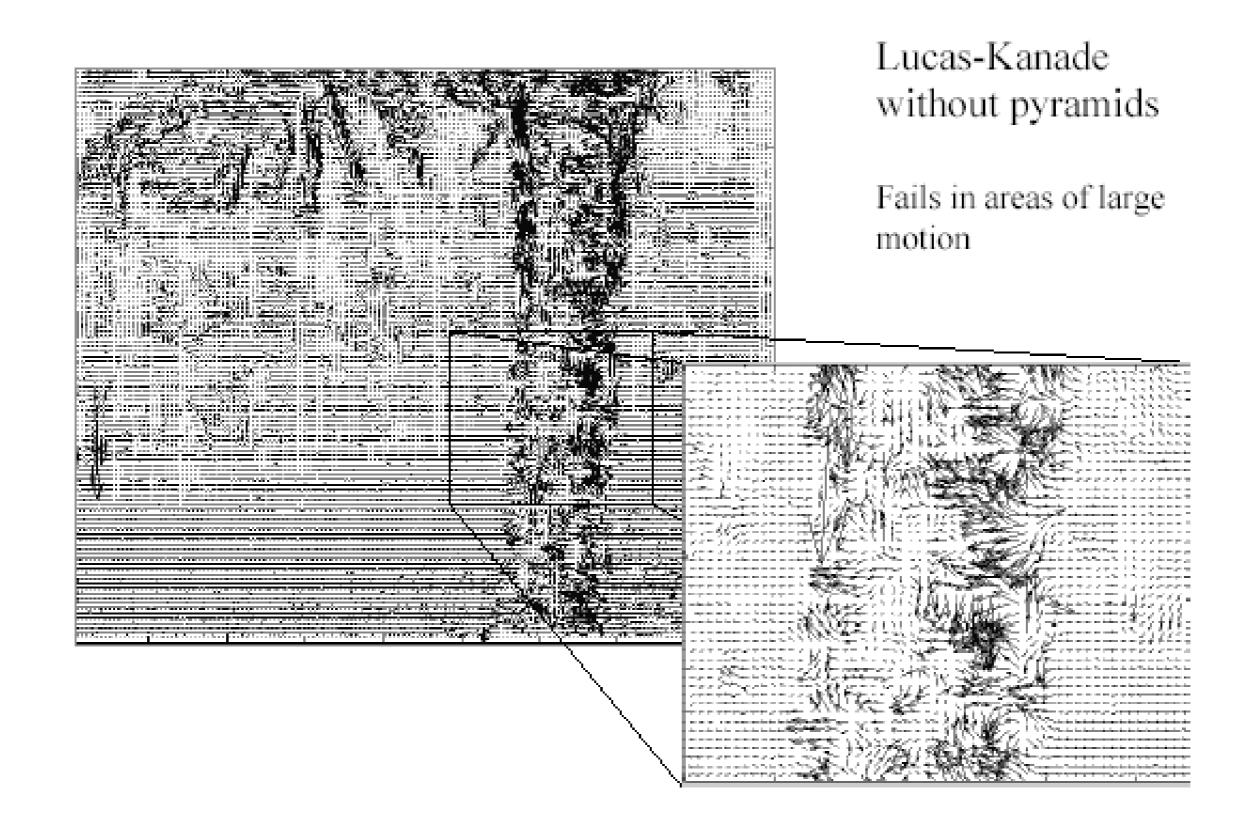
Coarse-to-fine Flow Estimation



Coarse-to-fine optical flow estimation

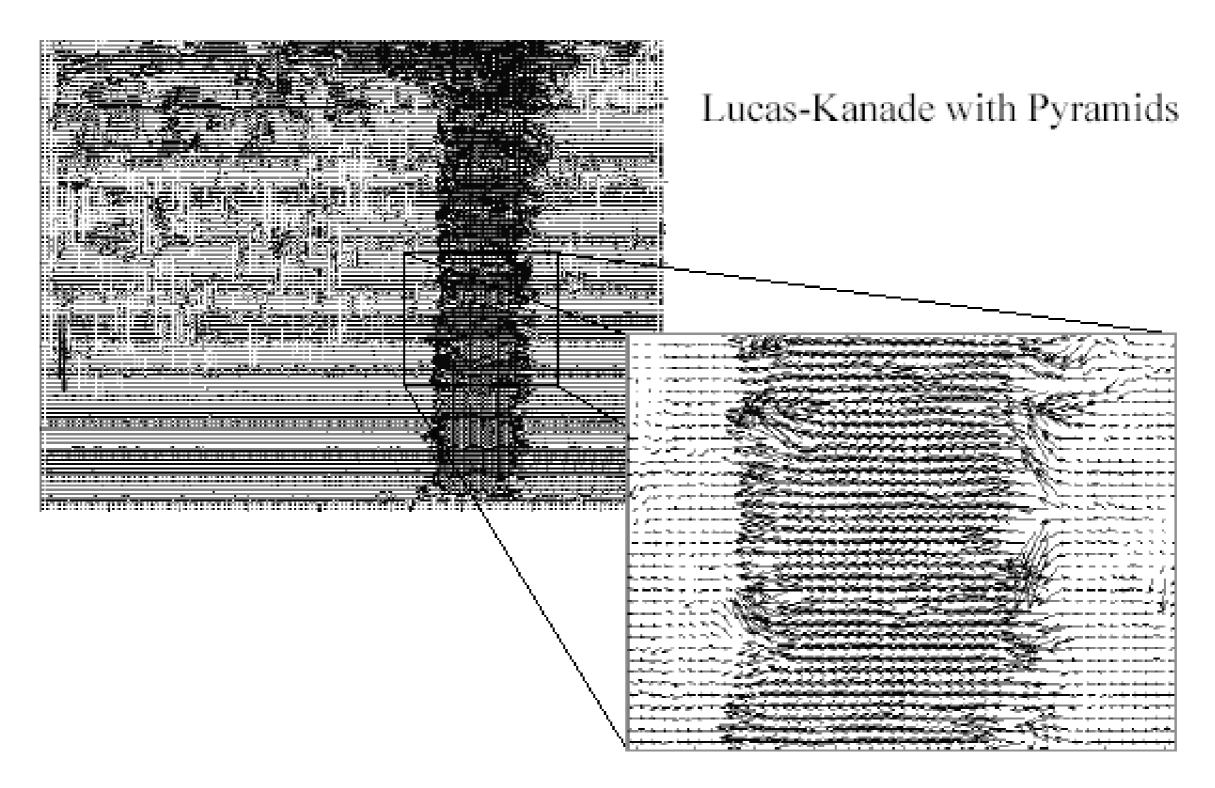


Optical Flow Results

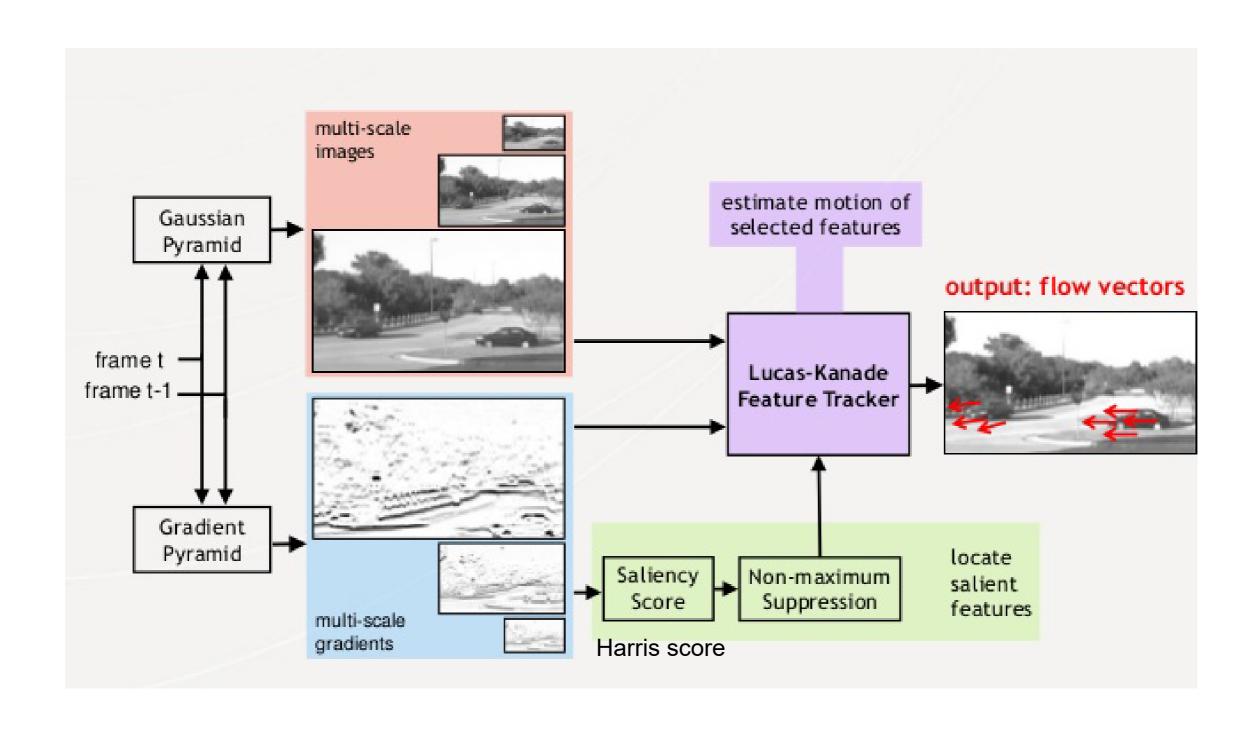


^{*} From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Optical Flow Results



Lucas- Kanade feature tracker



Recap

Key assumptions

- Small motion: points do not move very far
- Brightness constancy: projection of the same point looks the same in every frame
- Spatial coherence: points move like their neighbors

Motion segmentation

How do we represent the motion in this scene?

• Break image sequence into "layers" each of which has a coherent (affine) motion





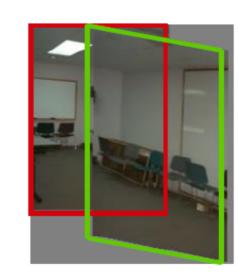
Affine motion

$$u(x, y) = a_1 + a_2 x + a_3 y$$

 $v(x, y) = a_4 + a_5 x + a_6 y$

Substituting into the brightness constancy equation:

$$I_{x} \cdot u + I_{y} \cdot v + I_{t} \approx 0$$



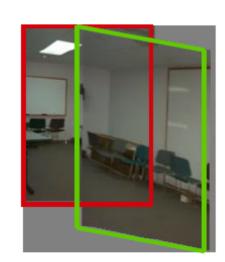
- An affine model is used to approximate the flow patterns consistent with all types
 of camera motion
- Affine parameters a1,..,a6 are calculated by minimizing the least squares error
 of the motion vectors

Affine motion

$$u(x, y) = a_1 + a_2 x + a_3 y$$

 $v(x, y) = a_4 + a_5 x + a_6 y$

Substituting into the brightness constancy equation:



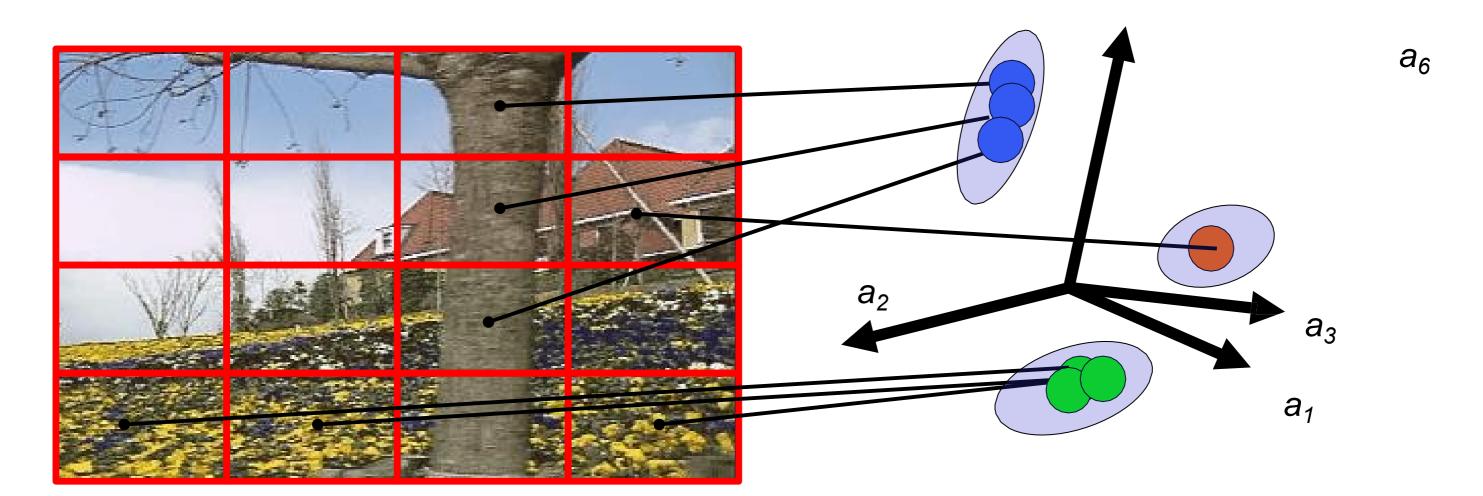
$$I_x(a_1 + a_2x + a_3y) + I_y(a_4 + a_5x + a_6y) + I_t \approx 0$$

- Each pixel provides 1 linear constraint in 6 unknowns
- If we have at least 6 pixels in a neighborhood,
 a₁... a₆ can be found by least squares minimization:

$$Err(\vec{a}) = \sum \left[I_x(a_1 + a_2x + a_3y) + I_y(a_4 + a_5x + a_6y) + I_t \right]^2$$

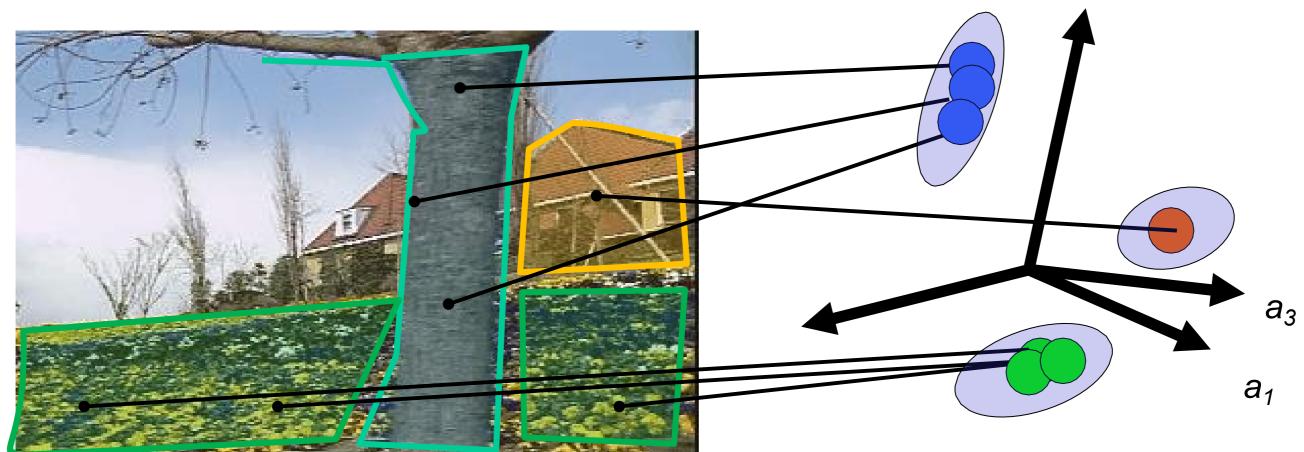
How do we estimate the layers?

- 1. Obtain a set of initial affine motion hypotheses
 - Divide the image into blocks and estimate affine motion parameters in each block by least squares
 - Eliminate hypotheses with high residual error
- 2. Map into motion parameter space
- 3. Perform k-means clustering on affine motion parameters
 - Merge clusters that are close and retain the largest clusters to obtain a smaller set of hypotheses to describe all the motions in the scene



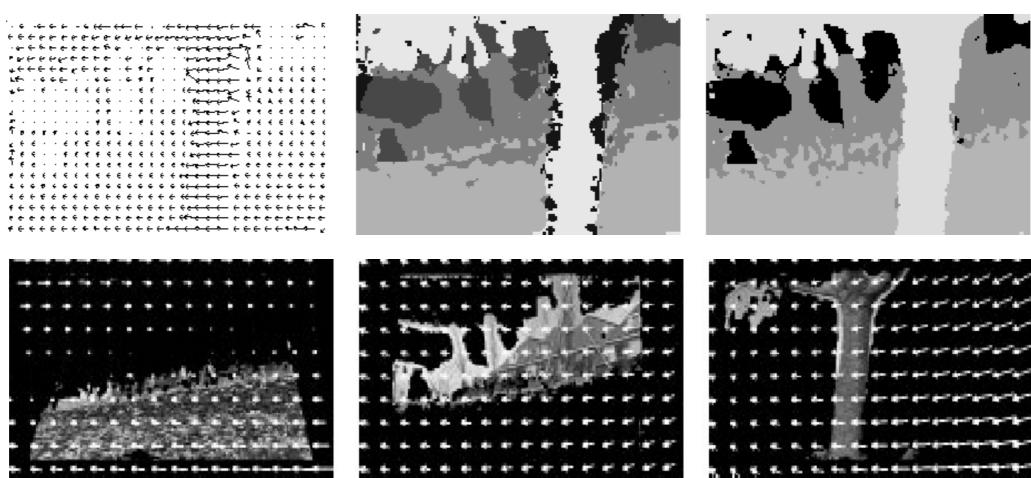
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- 4. Assign each pixel to best hypothesis --- iterate



Example result



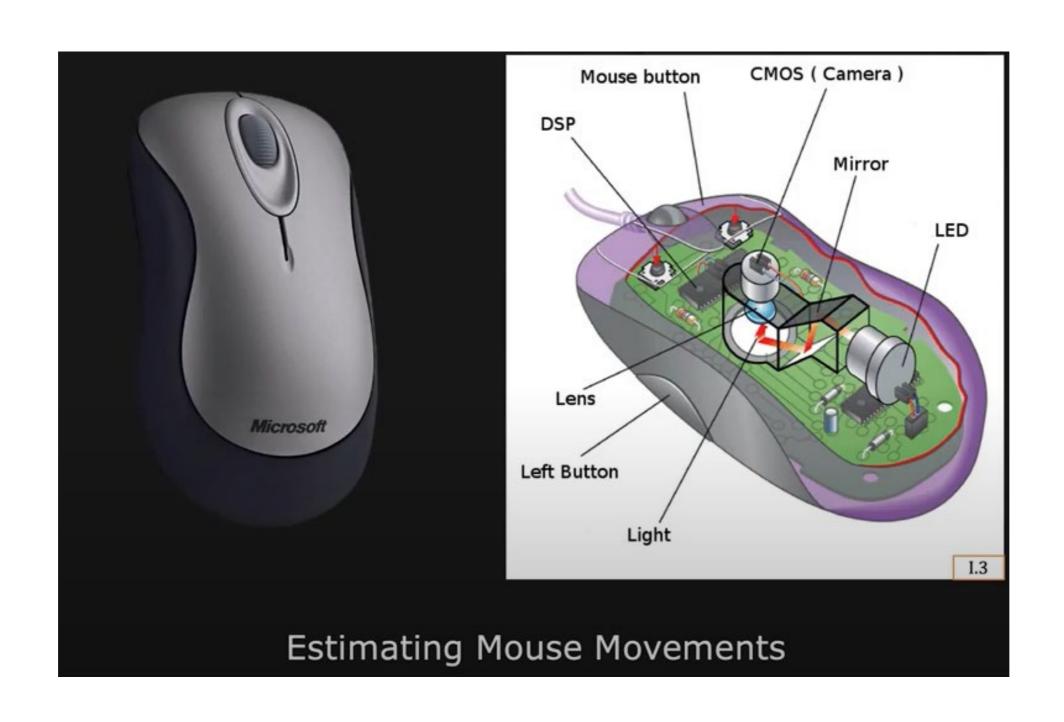


J. Wang and E. Adelson. <u>Layered Representation for Motion Analysis</u>. *CVPR 1993*.

Applications of optical flow

- Video retiming (determine intermediate frames to produce slow motion effects)
- Image stabilization (removing camera shake)
- Face tracking (i.e. eye blinking)
- Games (flow based player interaction)

Optical Mouse



Traffic monitoring

