Building Models

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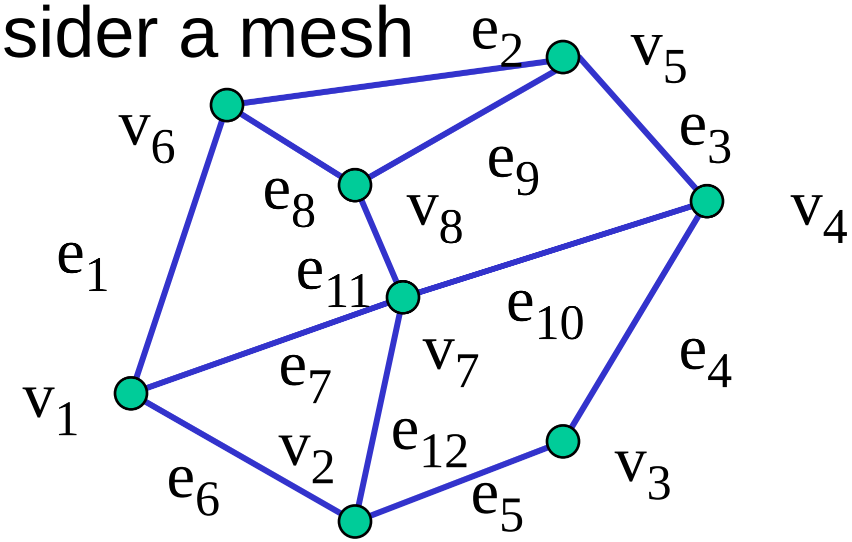
Objectives

- Introduce simple data structures for building polygonal models
    - Vertex lists
    - Edge lists

Representing a Mesh

- Consider a mesh



- There are 8 nodes and 12 edges
  - 5 interior polygons
  - 6 interior (shared) edges
- Each vertex has a location $v_i = (x_i\ y_i\ z_i)$

Simple Representation

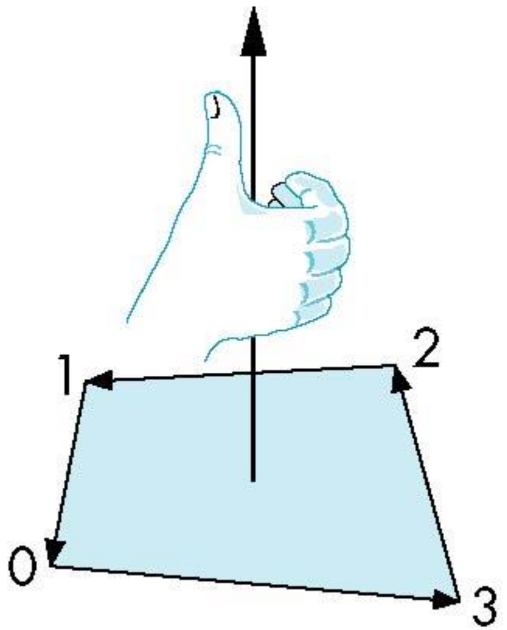
- Define each polygon by the geometric locations of its vertices
- Leads to WebGL code such as

```
vertex.push(vec3(x1, y1, z1));
vertex.push(vec3(x6, y6, z6));
vertex.push(vec3(x7, y7, z7));
```

- Inefficient and unstructured
  - Consider moving a vertex to a new location
  - Must search for all occurrences

Inward and Outward Facing Polygons

- The order $\{v_1, v_6, v_7\}$ and $\{v_6, v_7, v_1\}$ are equivalent in that the same polygon will be rendered by OpenGL but the order $\{v_1, v_7, v_6\}$ is different
- The first two describe *outwardly facing* polygons
- Use the *right-hand rule* = counter-clockwise encirclement of outward-pointing normal
- OpenGL can treat inward and outward facing polygons differently

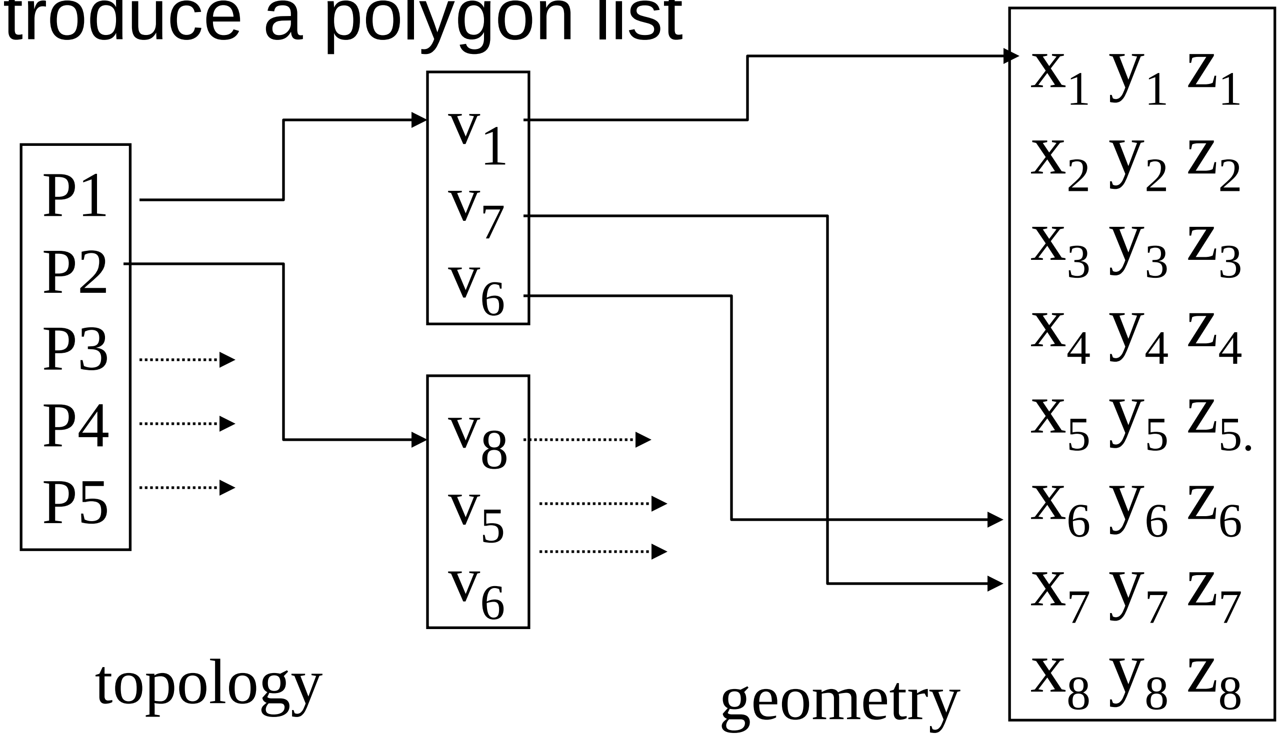


Geometry vs Topology

- Generally it is a good idea to look for data structures that separate the geometry from the topology
  - Geometry: locations of the vertices
  - Topology: organization of the vertices and edges
  - Example: a polygon is an ordered list of vertices with an edge connecting successive pairs of vertices and the last to the first
  - Topology holds even if geometry changes

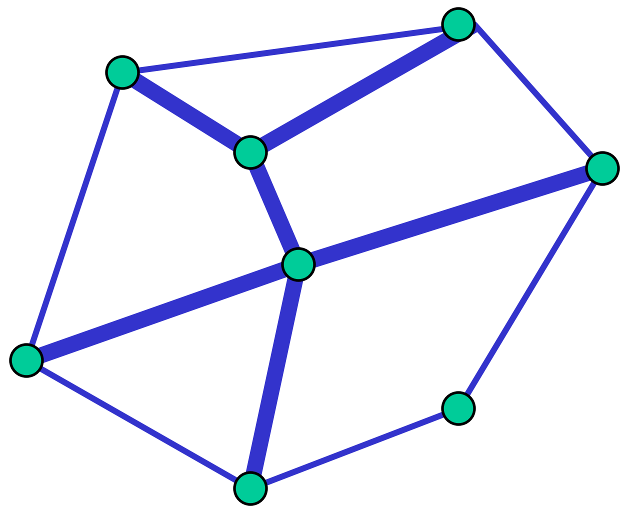
Vertex Lists

- Put the geometry in an array
- Use pointers from the vertices into this array
- Introduce a polygon list



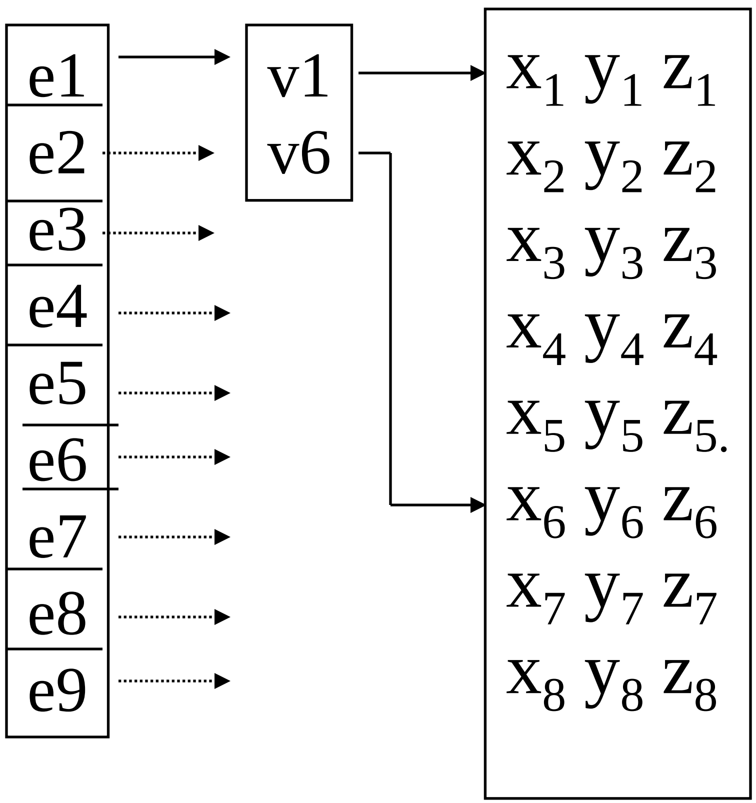
Shared Edges

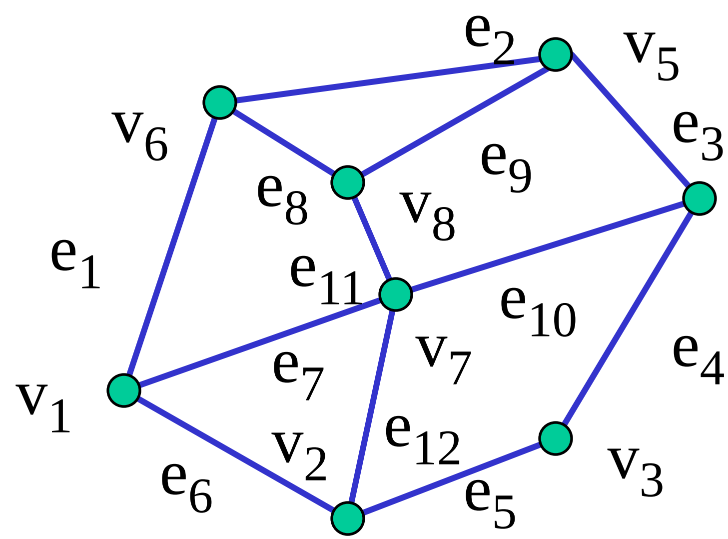
- Vertex lists will draw filled polygons correctly but if we draw the polygon by its edges, shared edges are drawn twice



- Can store mesh by *edge list*

Edge List

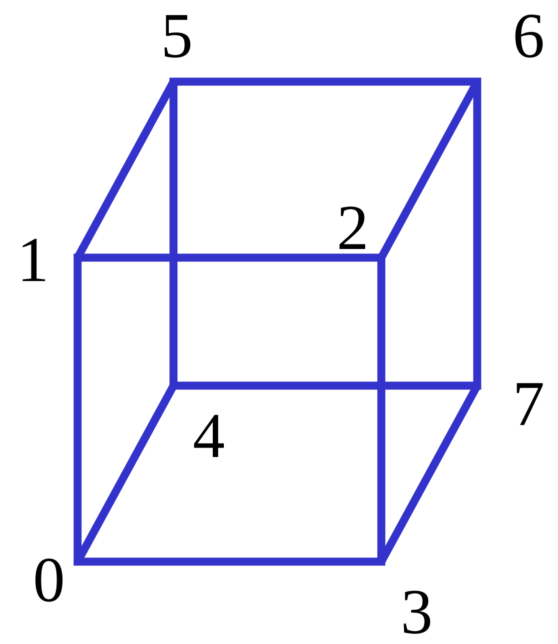




Note polygons are not represented

Draw cube from faces

```
var colorCube( )
{
    quad(0,3,2,1);
    quad(2,3,7,6);
    quad(0,4,7,3);
    quad(1,2,6,5);
    quad(4,5,6,7);
    quad(0,1,5,4);
}
```



Introduction to Computer Graphics with WebGL

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The Rotating Square

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Objectives

- Put everything together to display rotating cube
- Two methods of display
  - by arrays
  - by elements
- The first example (by arrays) is in 04/cube.html and 04/cube.js
- The second example (by elements) is in 04/cubev.html and 04/cubev.js

Modeling a Cube

Define global array for vertices

```
var vertices = [
        vec3( -0.5, -0.5,  0.5 ),
        vec3( -0.5,  0.5,  0.5 ),
        vec3(  0.5,  0.5,  0.5 ),
        vec3(  0.5, -0.5,  0.5 ),
        vec3( -0.5, -0.5, -0.5 ),
        vec3( -0.5,  0.5, -0.5 ),
        vec3(  0.5,  0.5, -0.5 ),
        vec3(  0.5, -0.5, -0.5 )
    ];
```

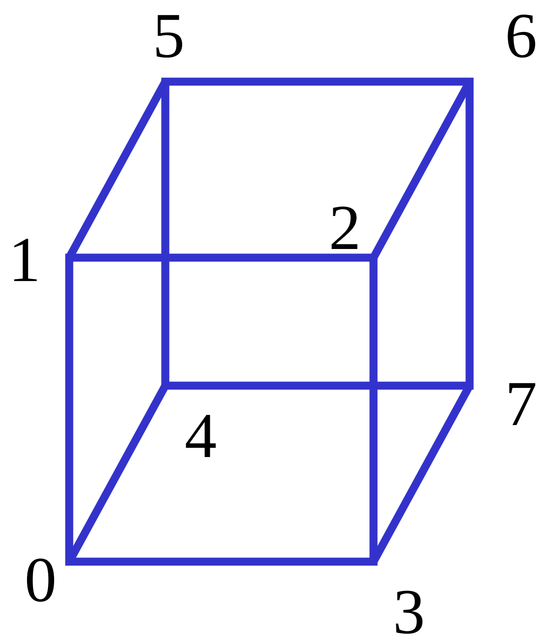
Colors

Define global array for colors

```
var vertexColors = [
        [ 0.0, 0.0, 0.0, 1.0 ],  // black
        [ 1.0, 0.0, 0.0, 1.0 ],  // red
        [ 1.0, 1.0, 0.0, 1.0 ],  // yellow
        [ 0.0, 1.0, 0.0, 1.0 ],  // green
        [ 0.0, 0.0, 1.0, 1.0 ],  // blue
        [ 1.0, 0.0, 1.0, 1.0 ],  // magenta
        [ 0.0, 1.0, 1.0, 1.0 ],  // cyan
        [ 1.0, 1.0, 1.0, 1.0 ]   // white
    ];
```

Draw cube from faces

```
function colorCube( )
{
    quad(0,3,2,1);
    quad(2,3,7,6);
    quad(0,4,7,3);
    quad(1,2,6,5);
    quad(4,5,6,7);
    quad(0,1,5,4);
}
```



Note that vertices are ordered so that we obtain correct outward facing normals

Each quad generates two triangles

Initialization

```
var canvas, gl;
var numVertices  = 36;
var points = [];
var colors = [];

window.onload = function init(){
    canvas = document.getElementById("gl-canvas");
    gl = canvas.getContext('webgl2');
    if (!gl) alert("WebGL 2.0 isn't available");
    colorCube();

   gl.viewport( 0, 0, canvas.width, canvas.height );
   gl.clearColor( 1.0, 1.0, 1.0, 1.0 );
   gl.enable(gl.DEPTH_TEST);
// rest of initialization and html file
// same as previous examples
```

The quad Function

Put position and color data for two triangles from a list of indices into the array **`vertices`**

```
var quad(a, b, c, d)
{
   var indices = [ a, b, c, a, c, d ];
   for ( var i = 0; i < indices.length; ++i ) {

      points.push( vertices[indices[i]]);
      colors.push( vertexColors[indices[i]] );

      // for solid colored faces use
      //colors.push(vertexColors[a]);

 }
}
```

Render Function

```
function render(){
    gl.clear( gl.COLOR_BUFFER_BIT |gl.DEPTH_BUFFER_BIT);
    if(flag) theta[axis] += 2.0;
    gl.uniform3fv(thetaLoc, theta);
    gl.drawArrays( gl.TRIANGLES, 0, numVertices );
    requestAnimFrame( render );
}
```

Mapping indices to faces

```
var indices = [
1,0,3,
3,2,1,
2,3,7,
7,6,2,
3,0,4,
4,7,3,
6,5,1,
1,2,6,
4,5,6,
6,7,4,
5,4,0,
0,1,5     ];
```

Rendering by Elements

- Send indices to GPU

```
var iBuffer = gl.createBuffer();
gl.bindBuffer(gl.ELEMENT_ARRAY_BUFFER, iBuffer);
gl.bufferData(gl.ELEMENT_ARRAY_BUFFER,
       new Uint8Array(indices), gl.STATIC_DRAW);
```

- Render by elements

```
gl.drawElements( gl.TRIANGLES, numVertices,
    gl.UNSIGNED_BYTE, 0 );
```

- Even more efficient if we use triangle strips or triangle fans

Adding Buttons for Rotation

```
var xAxis = 0;
var yAxis = 1;
var zAxis = 2;
var axis = 0;
var theta = [ 0, 0, 0 ];
var thetaLoc;

document.getElementById( "xButton" ).onclick =
function () {          axis = xAxis;     };
document.getElementById( "yButton" ).onclick =
function () {          axis = yAxis;     };
document.getElementById( "zButton" ).onclick =
function () {          axis = zAxis;     };
```

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Render Function

```
function render(){
    gl.clear( gl.COLOR_BUFFER_BIT |gl.DEPTH_BUFFER_BIT);
    theta[axis] += 2.0;
    gl.uniform3fv(thetaLoc, theta);
    gl.drawArrays( gl.TRIANGLES, 0, numVertices );
    requestAnimFrame( render );
}
```

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Classical Viewing

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Objectives

- Introduce the classical views
- Compare and contrast image formation by computer with how images have been formed by architects, artists, and engineers
- Learn the benefits and drawbacks of each type of view

Classical Viewing

- Viewing requires three basic elements
  - One or more objects
  - A viewer with a projection surface
  - Projectors that go from the object(s) to the projection surface
- Classical views are based on the relationship among these elements
  - The viewer picks up the object and orients it how she would like to see it
- Each object is assumed to constructed from flat *principal faces*
  - Buildings, polyhedra, manufactured objects

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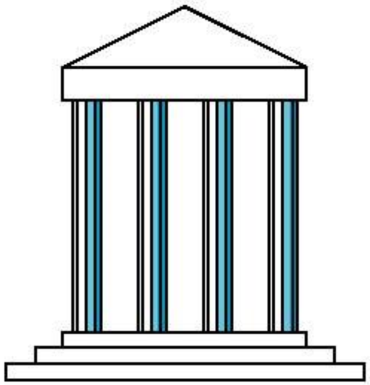
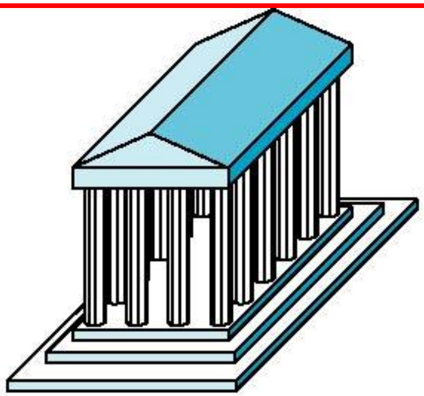
Planar Geometric Projections

- Standard projections project onto a plane
- Projectors are lines that either
  - converge at a center of projection
  - are parallel
- Such projections preserve lines
  - but not necessarily angles
- Nonplanar projections are needed for applications such as map construction

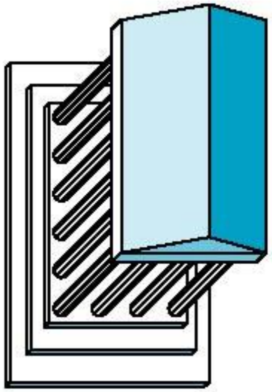
Classical Projections



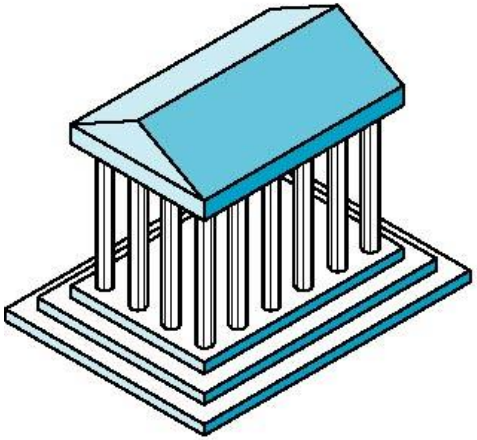
Front elevation



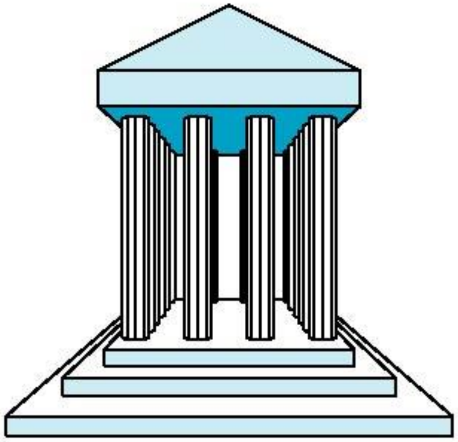
Elevation oblique



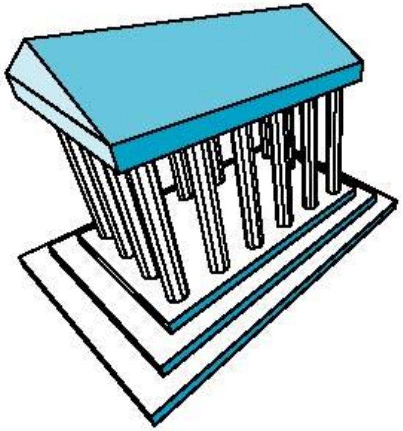
Plan oblique



Isometric



One-point perspective

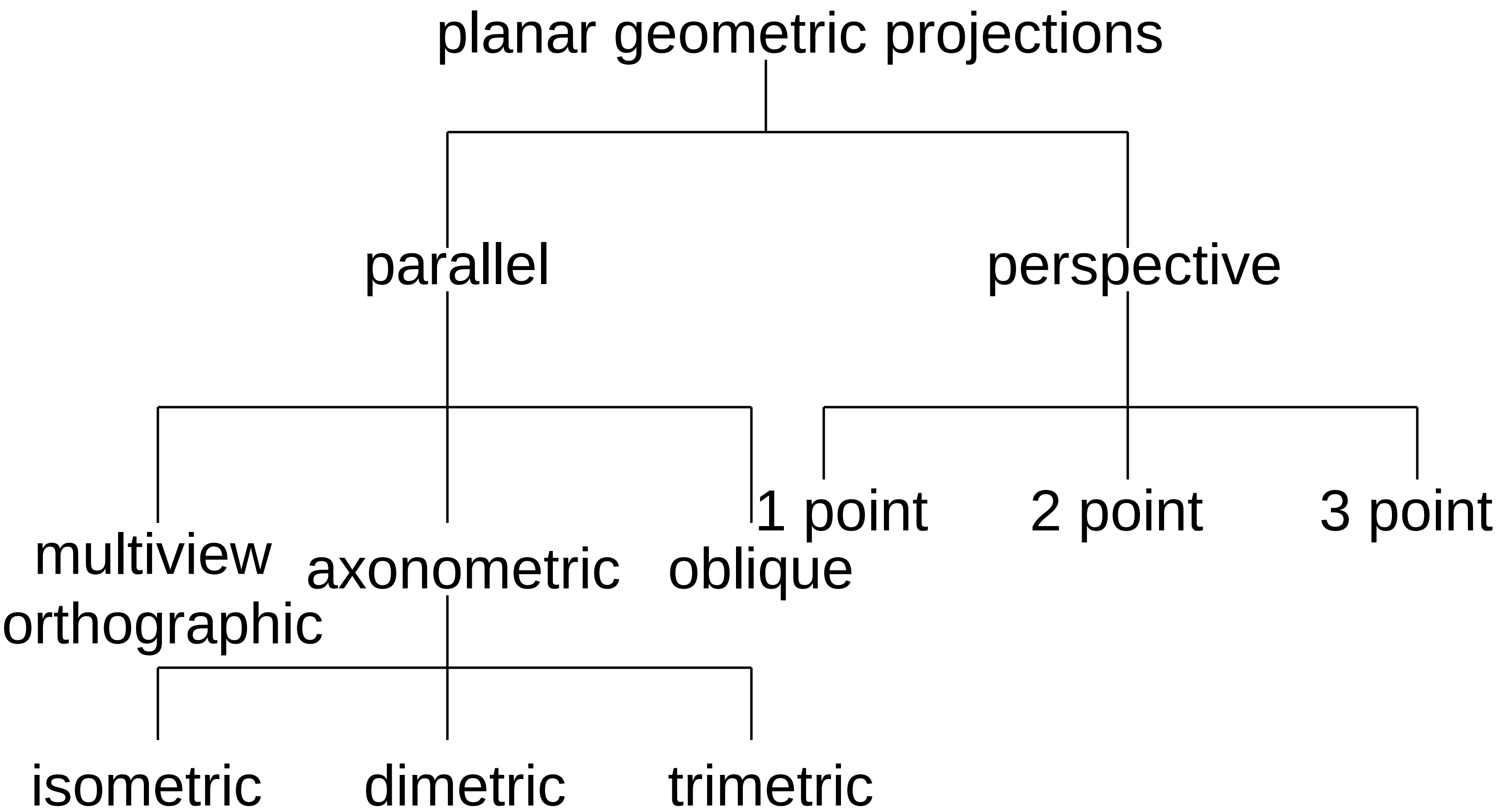


Three-point perspective

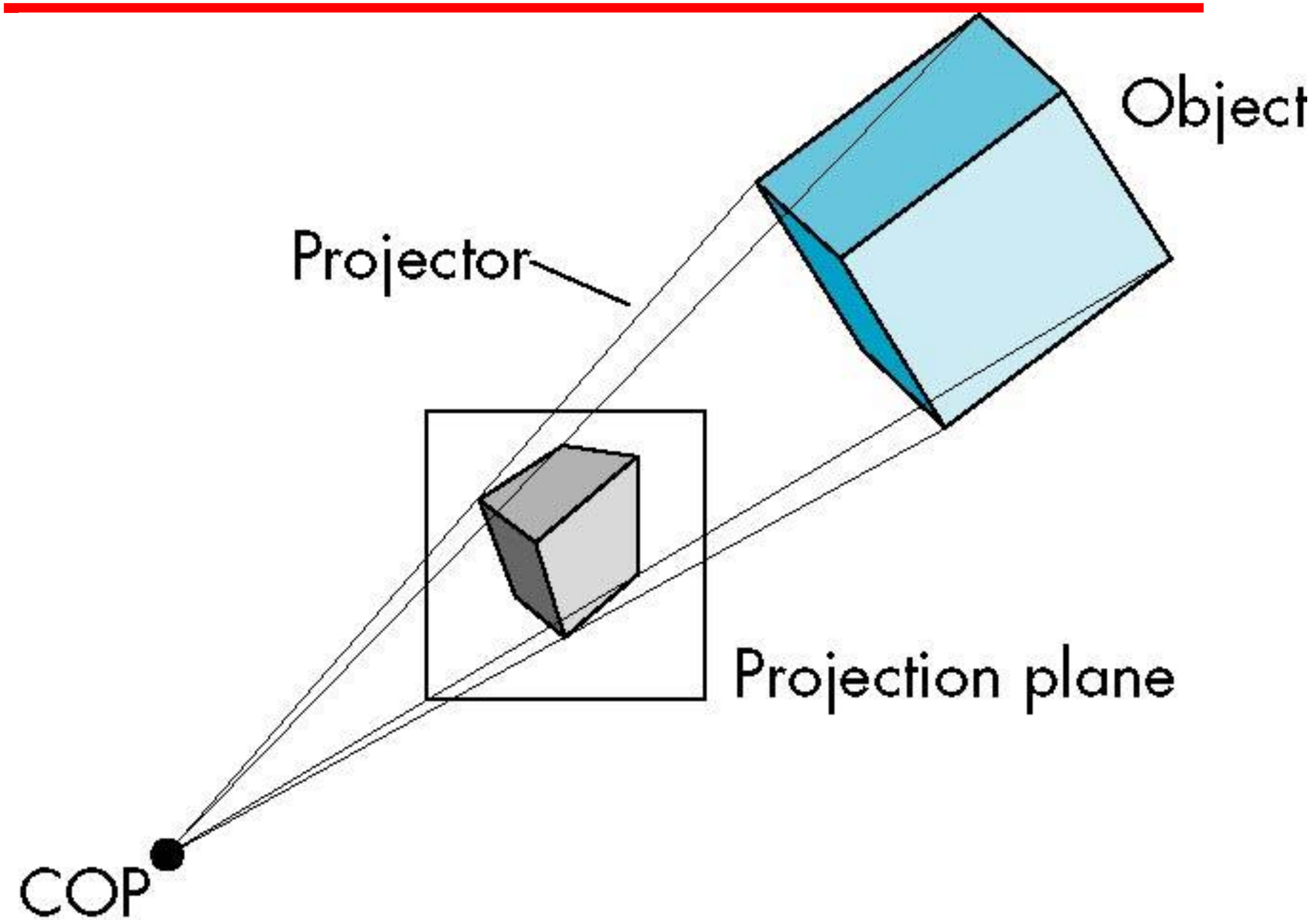
Perspective vs Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline
- Classical viewing developed different techniques for drawing each type of projection
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing

Taxonomy of Planar Geometric Projections



Perspective Projection



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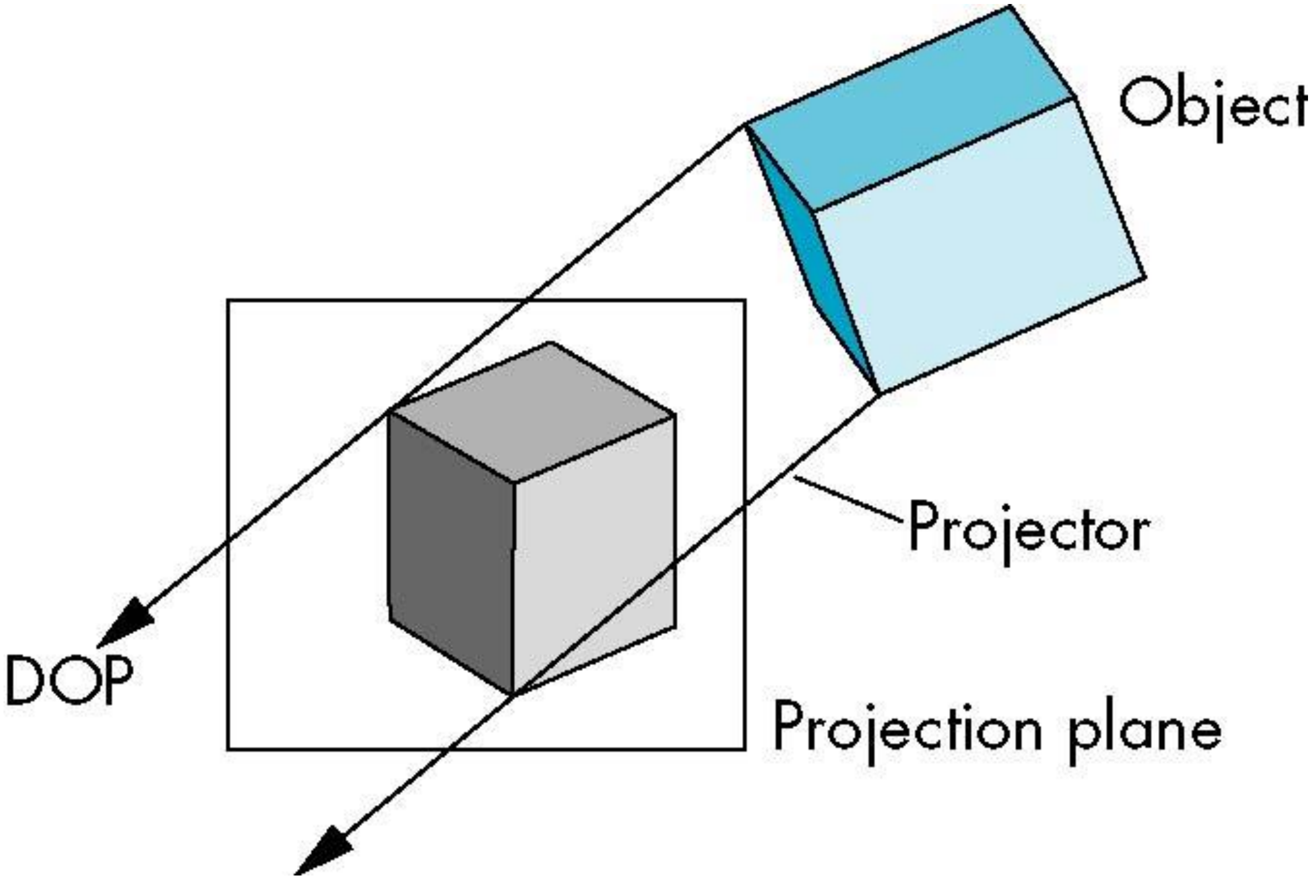
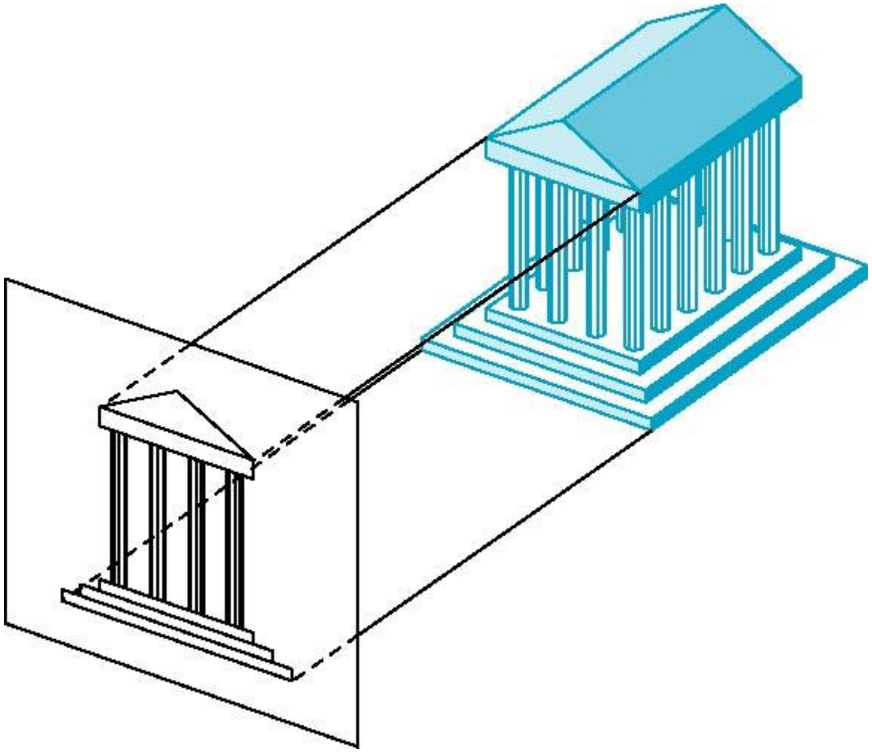
Parallel Projection



Orthographic Projection

Projectors are orthogonal to projection surface



Multiview Orthographic Projection

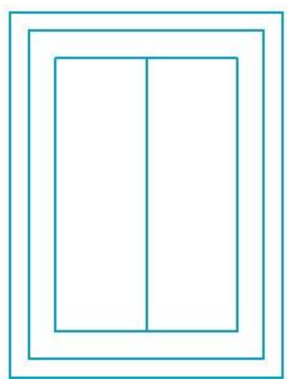
- Projection plane parallel to principal face
- Usually form front, top, side views

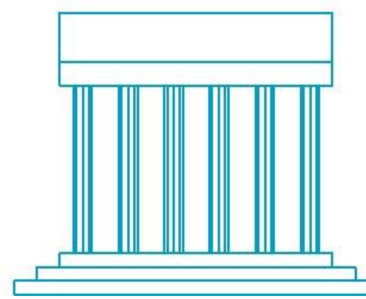
isometric (not multiview orthographic view)

front

in CAD and architecture, we often display three multiviews plus isometric

top





side

Advantages and Disadvantages

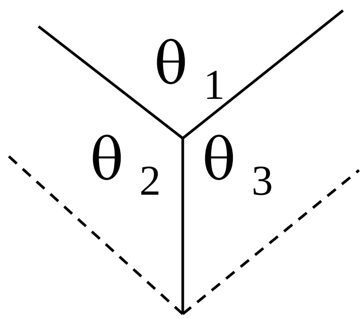
- Preserves both distances and angles
  - Shapes preserved
  - Can be used for measurements
    - Building plans
    - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
  - Often we add the isometric

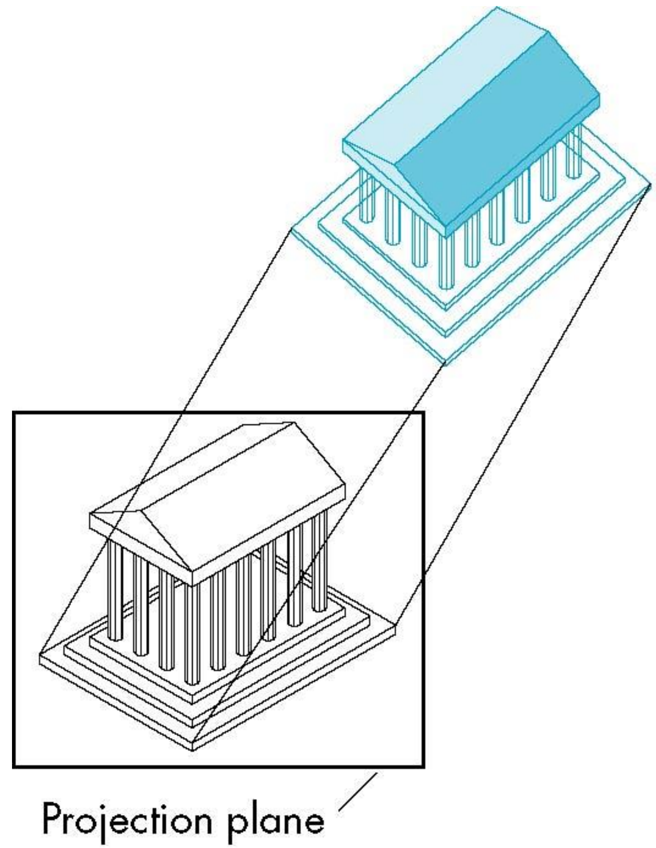
Axonometric Projections

Allow projection plane to move relative to object

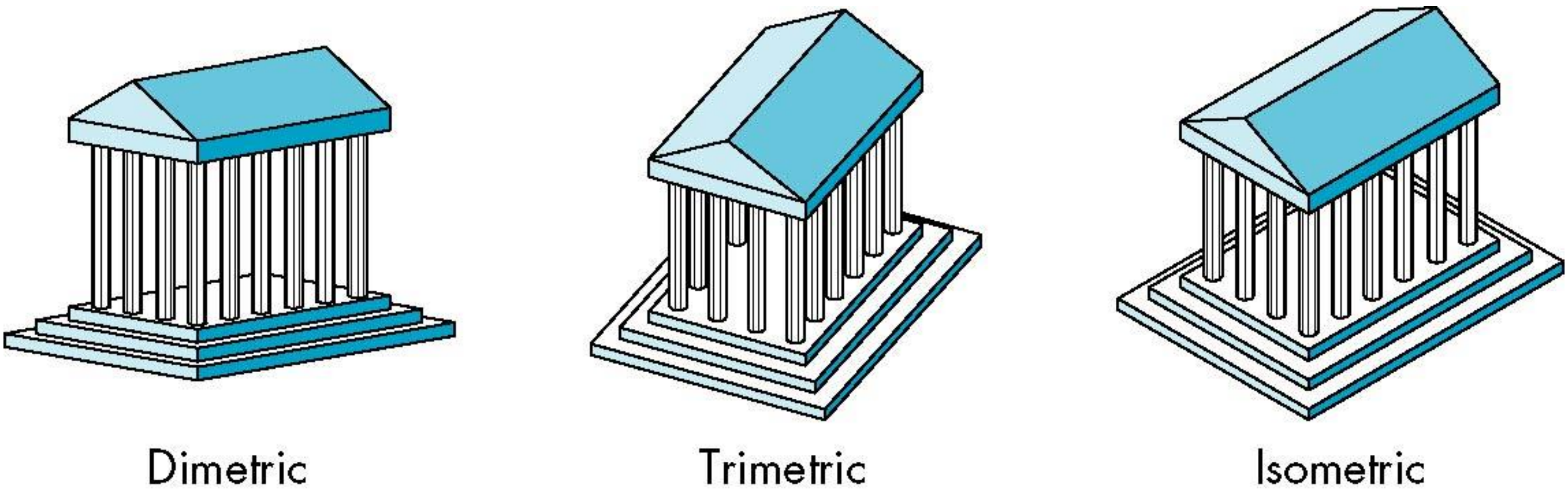
classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric





Types of Axonometric Projections

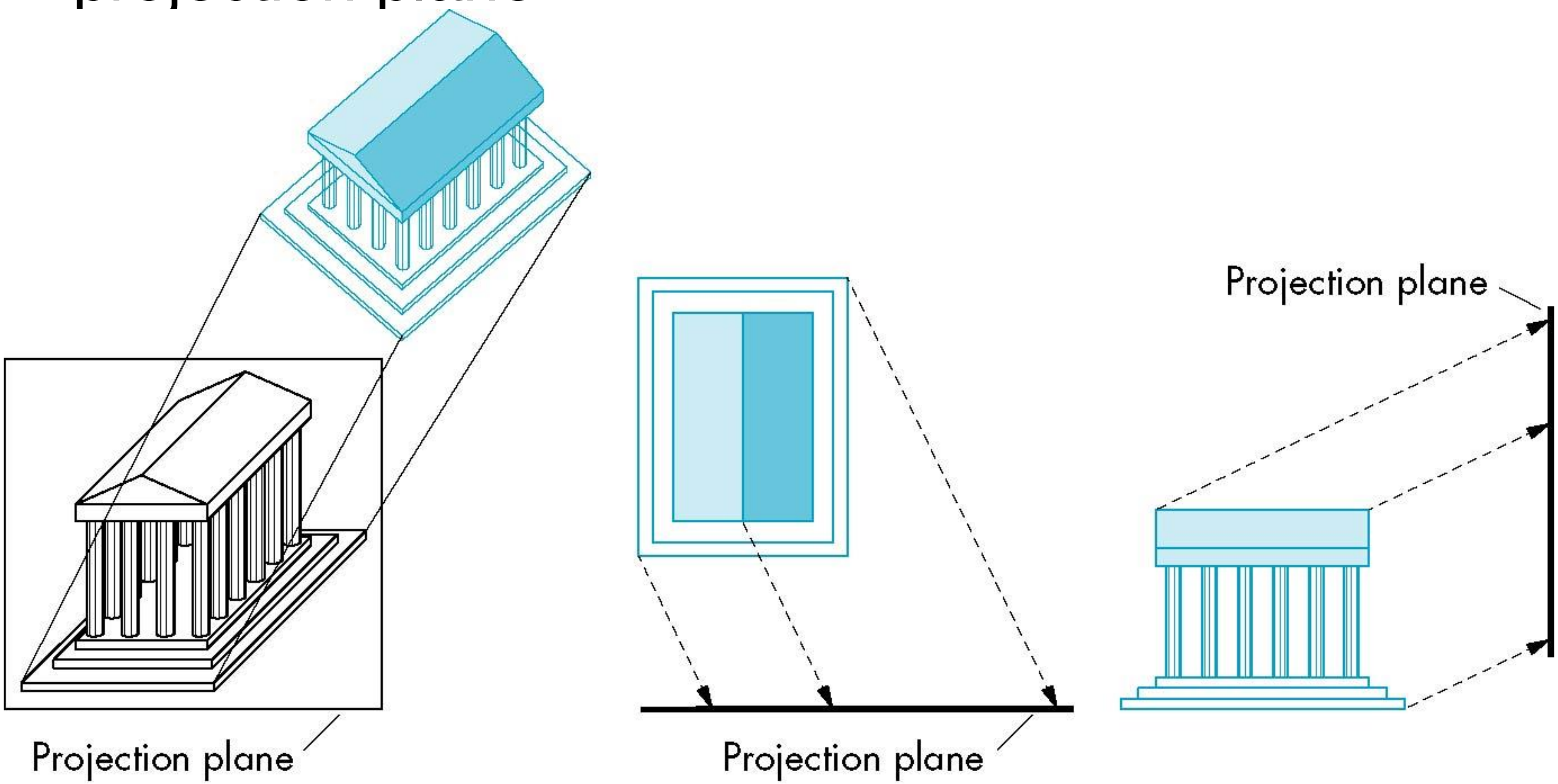


Advantages and Disadvantages

- Lines are scaled (*foreshortened*) but can find scaling factors
- Lines preserved but angles are not
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
  - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications

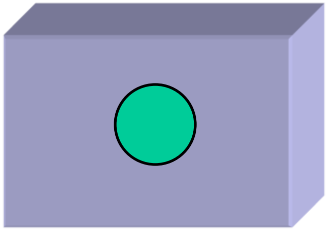
Oblique Projection

Arbitrary relationship between projectors and projection plane



Advantages and Disadvantages

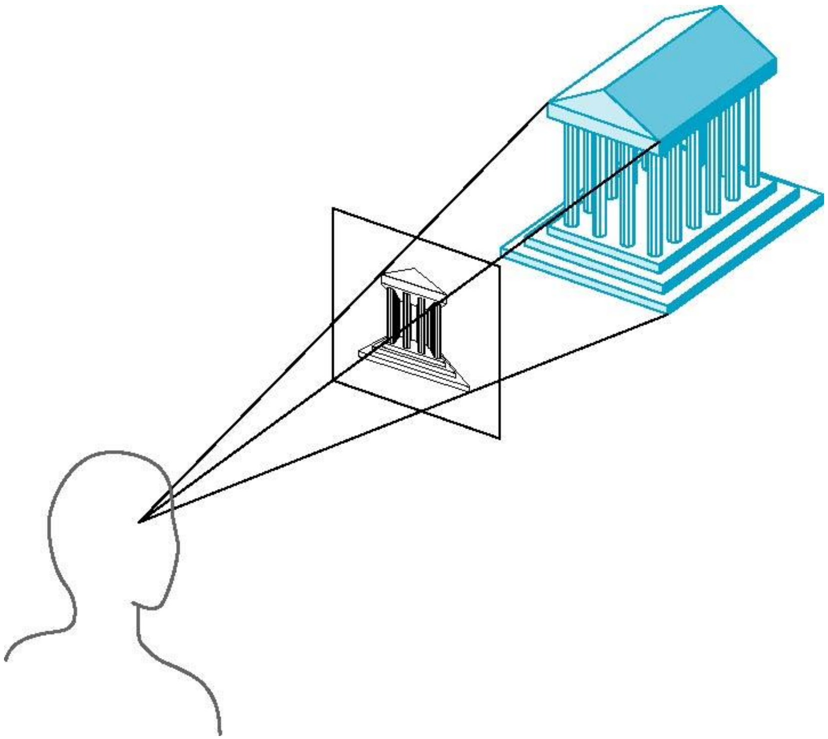
- Can pick the angles to emphasize a particular face
  - Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see “around” side



- In physical world, cannot create with simple camera; possible with bellows camera or special lens (architectural)

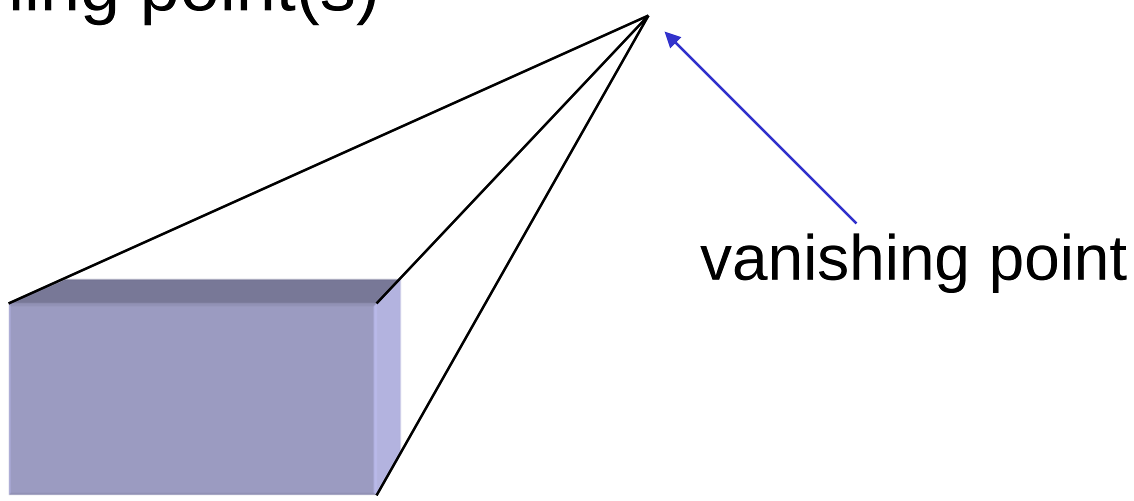
Perspective Projection

Projectors coverge at center of projection



Vanishing Points

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the *vanishing point*)
- Drawing simple perspectives by hand uses these vanishing point(s)



Three-Point Perspective

- No principal face parallel to projection plane
- Three vanishing points for cube

Two-Point Perspective

- On principal direction parallel to projection plane
- Two vanishing points for cube

One-Point Perspective

- One principal face parallel to projection plane
- One vanishing point for cube

Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (*diminution*)
  - Looks realistic
- Equal distances along a line are not projected into equal distances (*nonuniform foreshortening*)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)

Introduction to Computer Graphics with WebGL

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Computer Viewing
Positioning the Camera

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Objectives

- Introduce the mathematics of projection
- Introduce WebGL viewing functions in MVnew.js
- Look at alternate viewing APIs

From the Beginning

- In the beginning:
  - fixed function pipeline
  - Model-View and Projection Transformation
  - Predefined frames: model, object, camera, clip, ndc, window
- After deprecation
  - pipeline with programmable shaders
  - no transformations
  - clip, ndc window frames
- MVnew.js reintroduces original capabilities

Computer Viewing

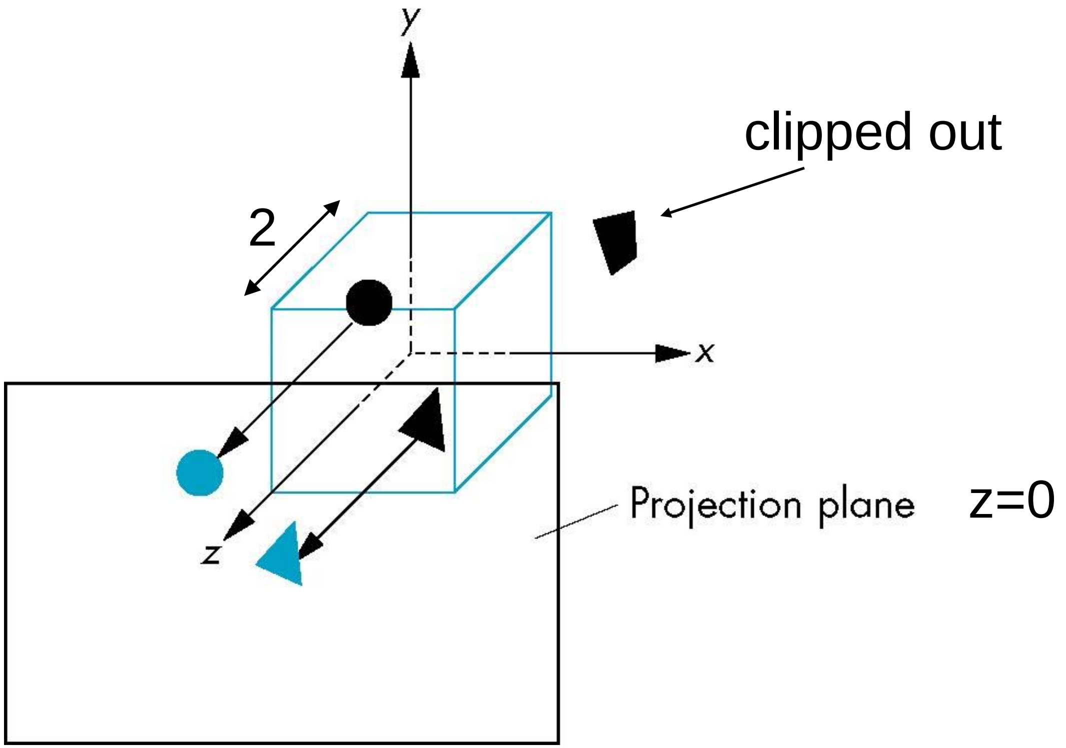
- There are three aspects of the viewing process, all of which are implemented in the pipeline,
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix
  - Clipping
    - Setting the view volume

The WebGL Camera

- In WebGL, initially the object and camera frames are the same
  - Default model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- WebGL also specifies a default view volume that is a cube with sides of length 2 centered at the origin
  - Default projection matrix is an identity

Default Projection

Default projection is orthogonal



Moving the Camera Frame

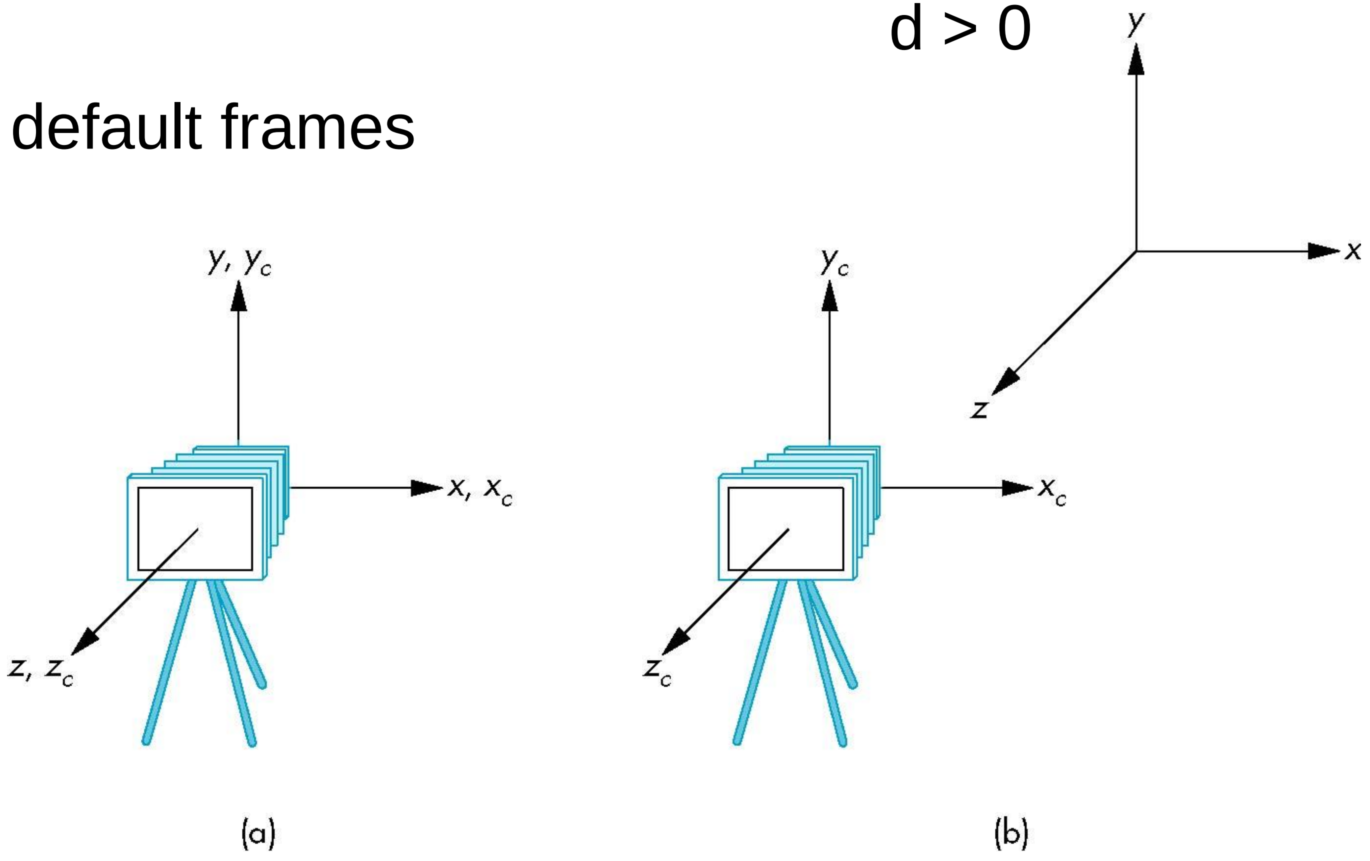
- If we want to visualize objects with both positive and negative z values we can either
  - Move the camera in the positive z direction
    - Translate the camera frame
  - Move the objects in the negative z direction
    - Translate the world frame
- Both of these views are equivalent and are determined by the model-view matrix
  - Want a translation (`translate(0.0,0.0,-d);`)

  `-d > 0`

Moving Camera back from Origin

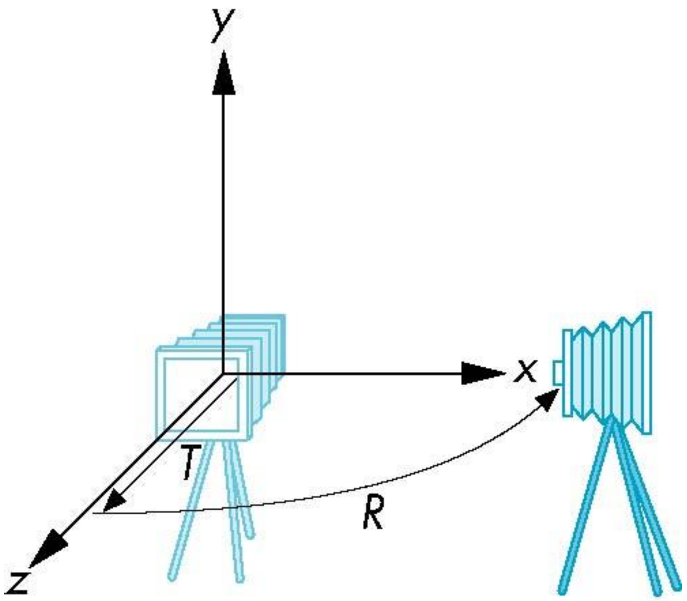
frames after translation by –d
d > 0

default frames



Moving the Camera

- We can move the camera to any desired position by a sequence of rotations and translations
- Example: side view
  - Rotate the camera
  - Move it away from origin
  - Model-view matrix C = TR



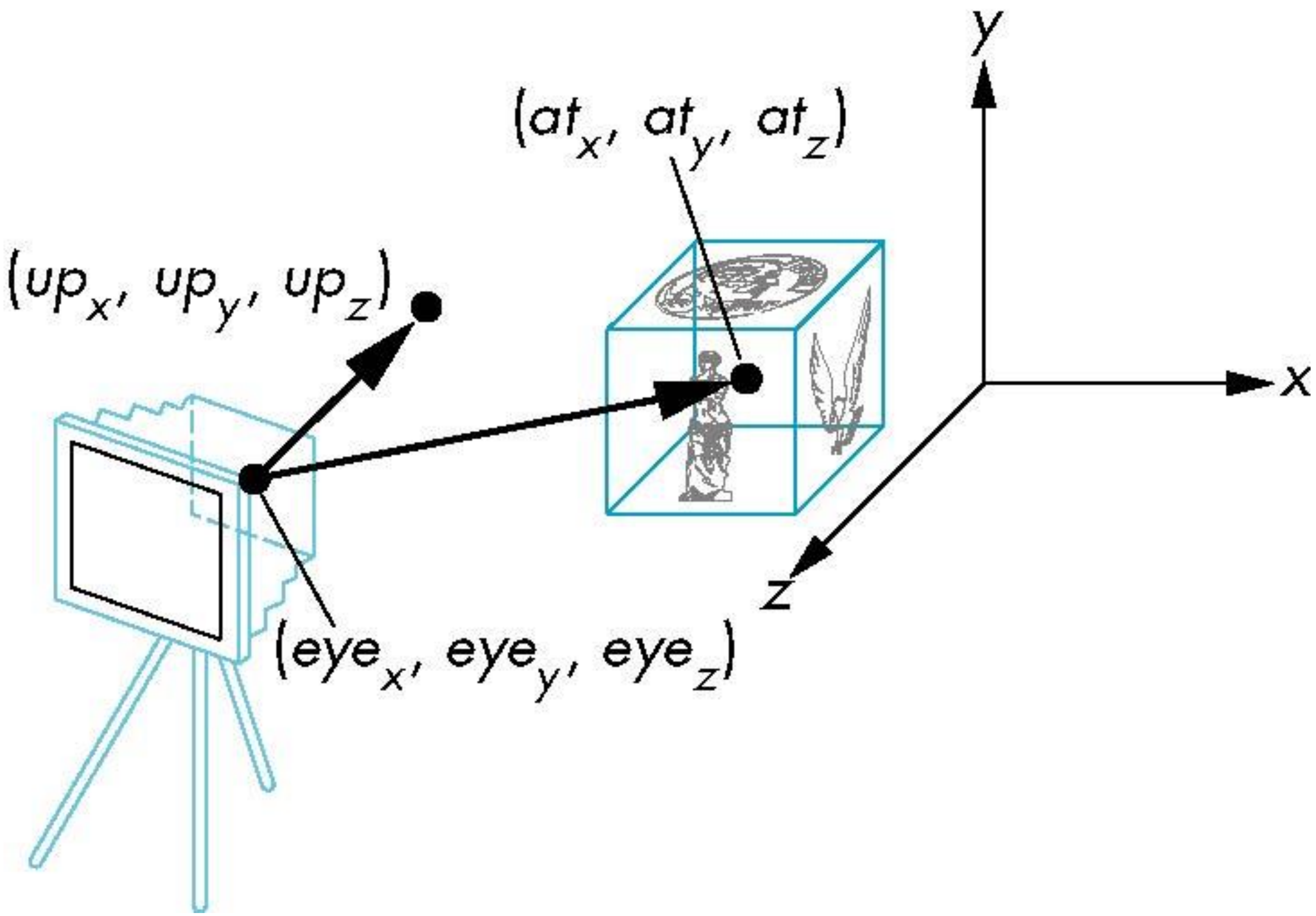
WebGL code

- Remember that last transformation specified is first to be applied

```
// Using MVnew.js

var t = translate (0.0, 0.0, -d);
var ry = rotateY(90.0);
var m = mult(t, ry);

or

var m = mult(translate (0.0, 0.0, -d),
         rotateY(90.0););
```

lookAt

```
LookAt(eye, at, up)
```



The lookAt Function

- The GLU library contained the function gluLookAt to form the required modelview matrix through a simple interface
- Note the need for setting an up direction
- Replaced by lookAt() in MV.js
  - Can concatenate with modeling transformations
- Example: isometric view of cube aligned with axes

```
var eye = vec3(1.0, 1.0, 1.0);
var at = vec3(0.0, 0.0, 0.0);
var up = vec3(0.0, 1.0, 0.0);
var mv = LookAt(eye, at, up);
```

Other Viewing APIs

- The LookAt function is only one possible API for positioning the camera
- Others include
  - View reference point, view plane normal, view up (PHIGS, GKS-3D)
  - Yaw, pitch, roll
  - Elevation, azimuth, twist
  - Direction angles

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Computer Viewing
Projection

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Objectives

- Introduce the mathematics of projection
- Add WebGL projection functions in MVnew.js

Projections and Normalization

- The default projection in the eye (camera) frame is orthogonal
- For points within the default view volume

$$x_p = x$$
$$y_p = y$$
$$z_p = 0$$

- Most graphics systems use *view normalization*
  - All other views are converted to the default view by transformations that determine the projection matrix
  - Allows use of the same pipeline for all views

Homogeneous Coordinate Representation

default orthographic projection

$x_p = x$
$y_p = y$
$z_p = 0$
$w_p = 1$

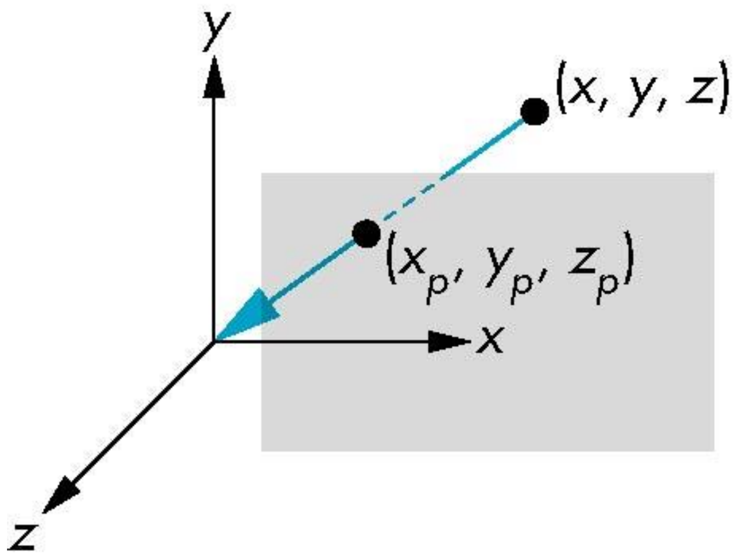
$$\mathbf{p}_p = \mathbf{Mp}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let $\mathbf{M} = \mathbf{I}$ and set the z term to zero later

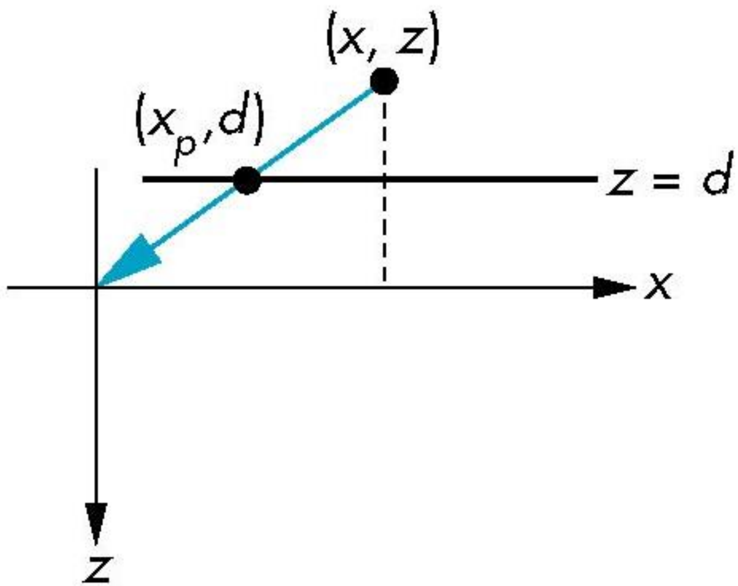
Simple Perspective

- Center of projection at the origin
- Projection plane $z = d$, $d < 0$

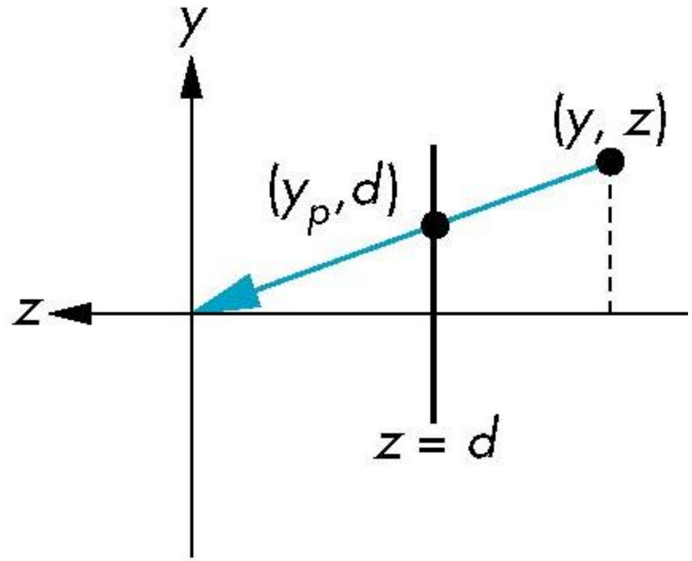


Perspective Equations

Consider top and side views



$$\frac{x}{z/d}$$



$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

Homogeneous Coordinate Form

consider $\mathbf{q} = \mathbf{Mp}$ where $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \Rightarrow \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

Perspective Division

- However $w \neq 1$, so we must divide by w to return from homogeneous coordinates
- This *perspective division* yields

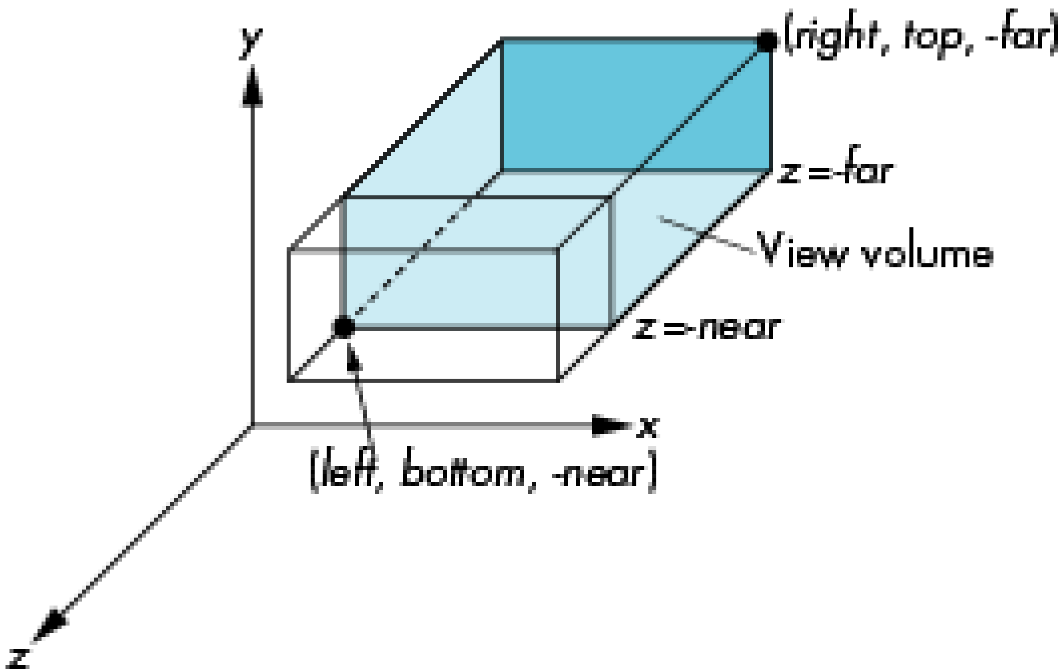
$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

the desired perspective equations

- We will consider the corresponding clipping volume with mat.h functions that are equivalent to deprecated OpenGL functions

WebGL Orthogonal Viewing

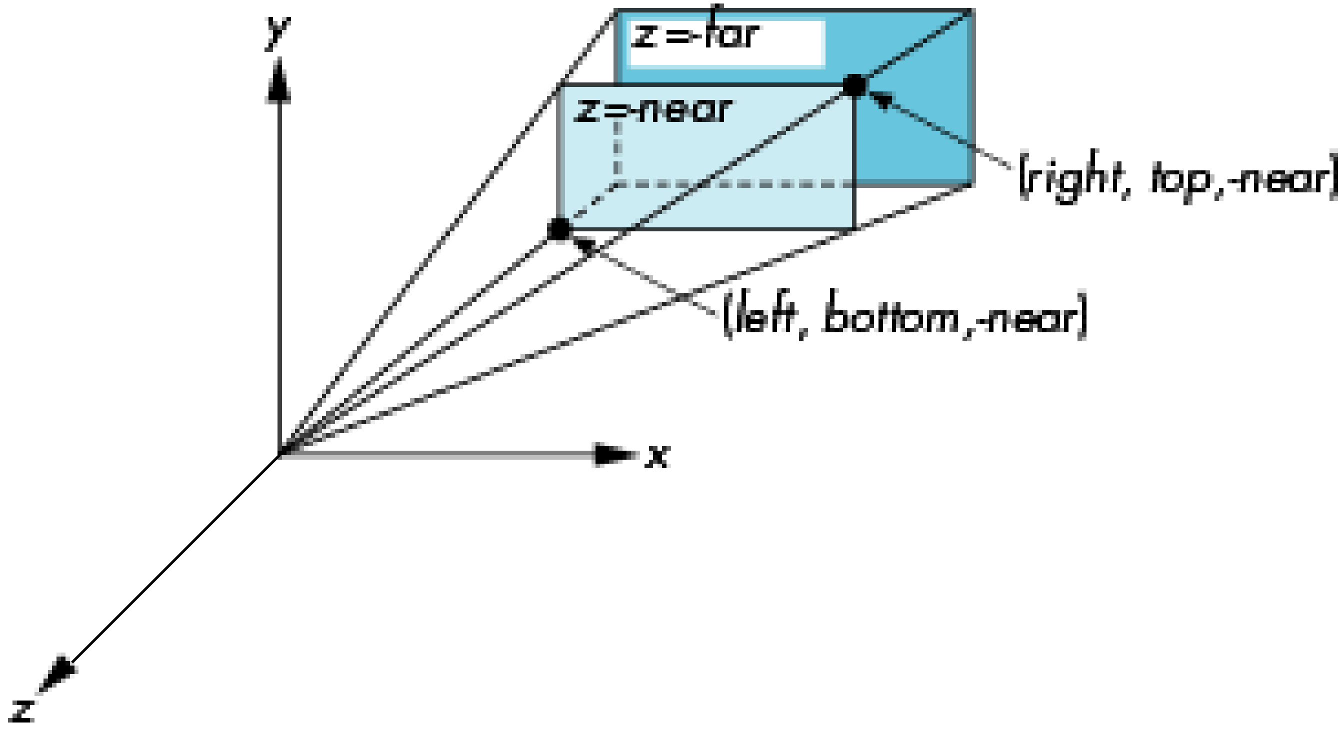
`ortho(left,right,bottom,top,near,far)`



`near` and `far` measured <u>from</u> camera

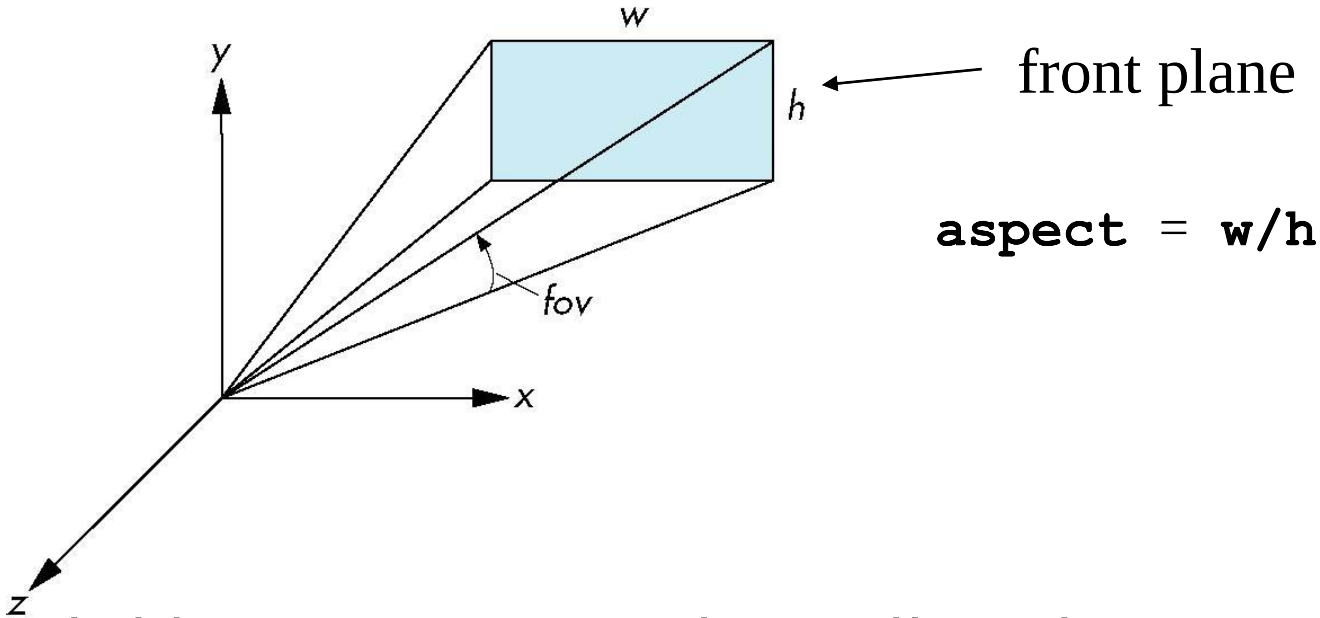
WebGL Perspective

`frustum(left,right,bottom,top,near,far)`



Using Field of View

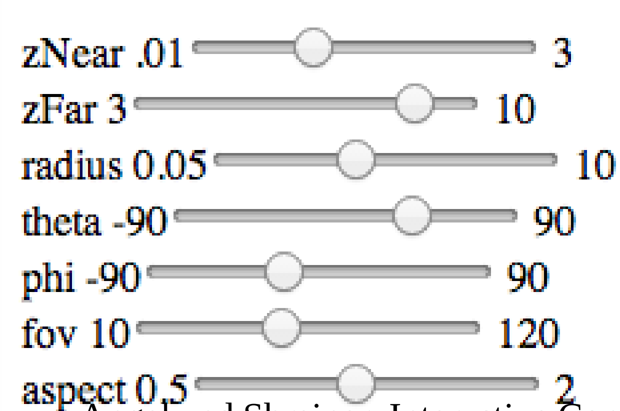
- With **`frustum`** it is often difficult to get the desired view
- **`perpective(fovy, aspect, near, far)`** often provides a better interface

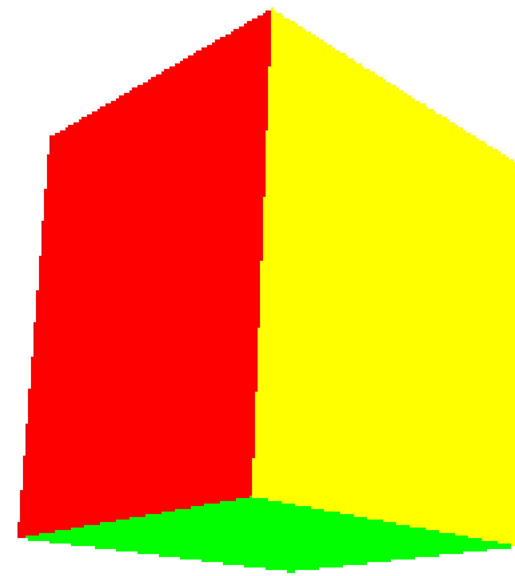


`aspect = w/h`

Computing Matrices

- Compute in JS file, send to vertex shader with gl.uniformMatrix4fv
- Dynamic: update in render() or shader





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perspective2.js

```
var render = function(){
    gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT);
     eye = vec3(radius*Math.sin(theta)*Math.cos(phi),
        radius*Math.sin(theta)*Math.sin(phi), radius*Math.cos(theta));
    modelViewMatrix = lookAt(eye, at , up);
    projectionMatrix = perspective(fovy, aspect, near, far);
    gl.uniformMatrix4fv( modelViewMatrixLoc, false,
         flatten(modelViewMatrix) );
    gl.uniformMatrix4fv( projectionMatrixLoc, false,
        flatten(projectionMatrix) );
    gl.drawArrays( gl.TRIANGLES, 0, NumVertices );
    requestAnimFrame(render);
}
```

vertex shader

```
#version 300 es
in  vec4 aPosition;
in  vec4 aColor;
out vec4 vColor;
uniform mat4 uModelViewMatrix;
uniform mat4 uProjectionMatrix;

void main(){
    gl_Position = uProjectionMatrix*uModelViewMatrix*aPosition;
    vColor = aColor;
}
```

Introduction to Computer Graphics with WebGL

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The Virtual Trackball

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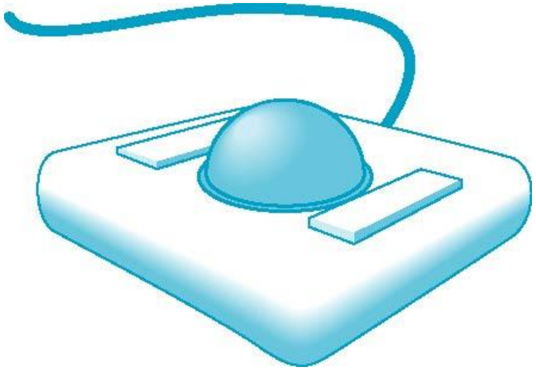
Objectives

- This is an optional lecture that
  - Introduces the use of graphical (virtual) devices that can be created using WebGL
  - Reinforce the benefit of not using direction angles and Euler angles
  - Makes use of transformations
  - Leads to reusable code that will be helpful later

Physical Trackball

- The trackball is an "upside down" mouse



- If there is little friction between the ball and the rollers, we can give the ball a push and it will keep rolling yielding continuous changes
- Two possible modes of operation
  - Continuous pushing or tracking hand motion
  - Spinning

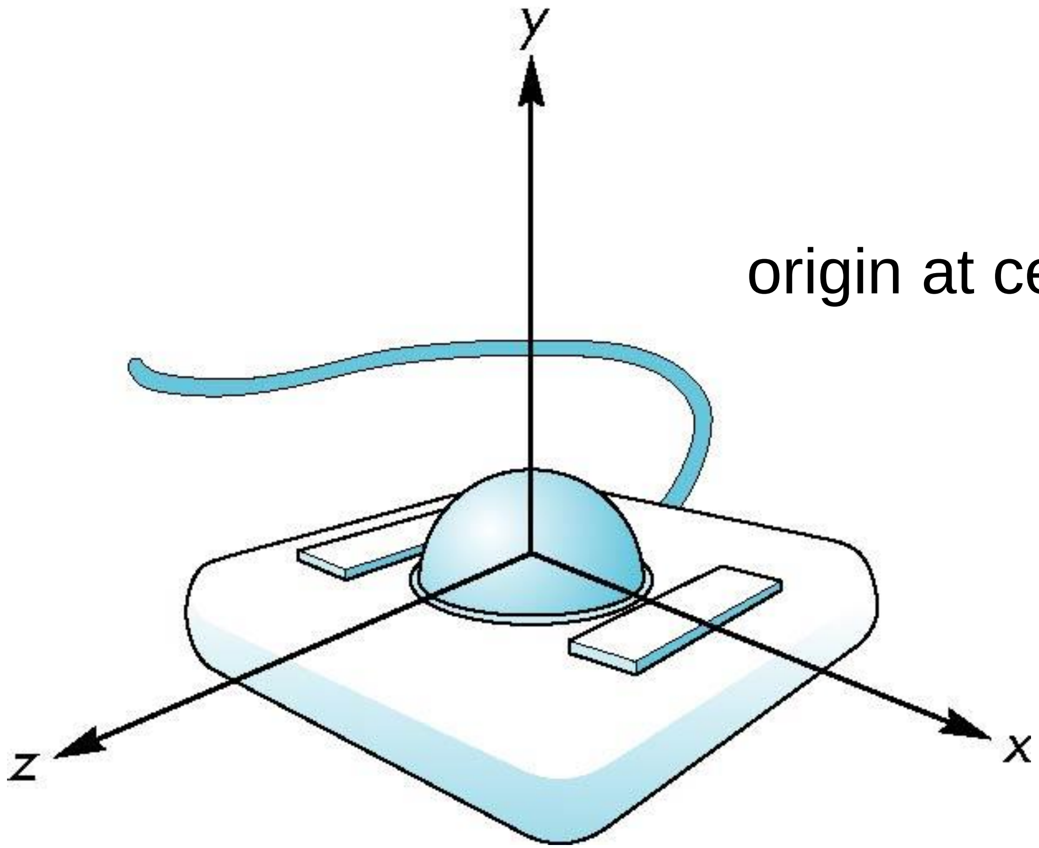
A Trackball from a Mouse

- Problem: we want to get the two behavior modes from a mouse
- We would also like the mouse to emulate a frictionless (ideal) trackball
- Solve in two steps
  - Map trackball position to mouse position
  - Use event listeners to handle the proper modes

Using Quaternions

- Quaternion arithmetic works well for representing rotations around the origin
- Can use directly avoiding rotation matrices in the virtual trackball
- Code was made available long ago (pre shader) by SGI
- Quaternion shaders are simple

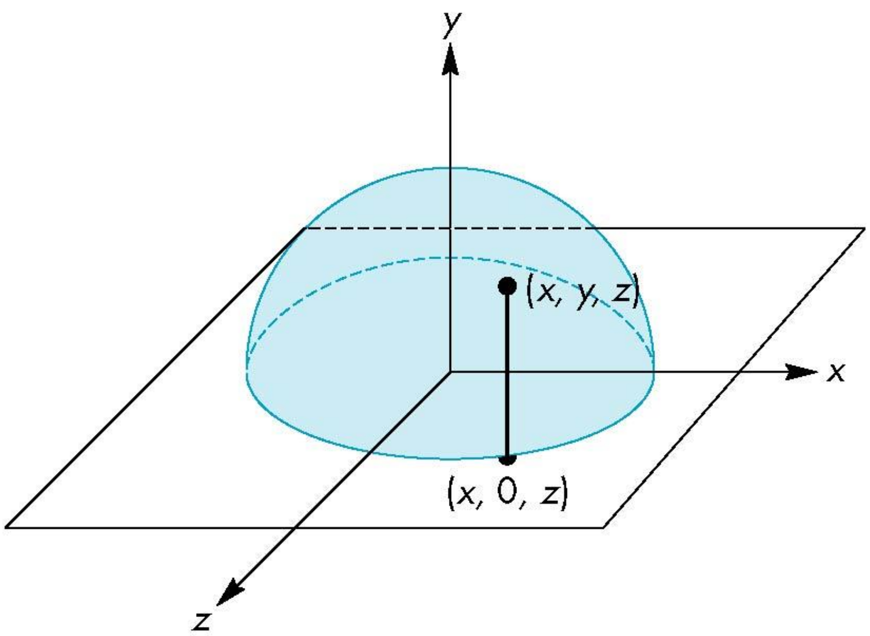
Trackball Frame



origin at center of ball

Projection of Trackball Position

- We can relate position on trackball to position on a normalized mouse pad by projecting orthogonally onto pad



Reversing Projection

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- Because both the pad and the upper hemisphere of the ball are two-dimensional surfaces, we can reverse the projection
- A point (x,z) on the mouse pad corresponds to the point (x,y,z) on the upper hemisphere where

$$y = \sqrt{r^2 - x^2 - z^2} \qquad \text{if } r \geq |x| \geq 0,\ r \geq |z| \geq 0$$

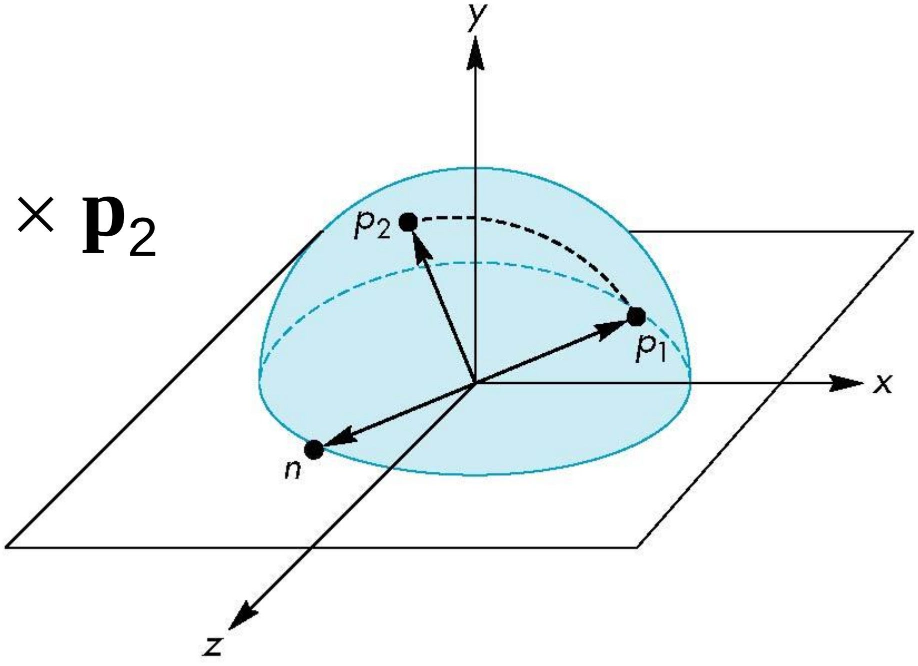
Computing Rotations

- Suppose that we have two points that were obtained from the mouse.
- We can project them up to the hemisphere to points $\mathbf{p}_1$ and $\mathbf{p}_2$
- These points determine a great circle on the sphere
- We can rotate from $\mathbf{p}_1$ to $\mathbf{p}_2$ by finding the proper axis of rotation and the angle between the points

Using the cross product

- The axis of rotation is given by the normal to the plane determined by the origin, $\mathbf{p}_1$, and $\mathbf{p}_2$

$$\mathbf{n} = \mathbf{p}_1 \times \mathbf{p}_2$$



Obtaining the angle

- The angle between $\mathbf{p}_1$ and $\mathbf{p}_2$ is given by

$$|\sin\theta| = \frac{|\mathbf{n}|}{|\mathbf{p}_1||\mathbf{p}_2|}$$

- If we move the mouse slowly or sample its position frequently, then θ will be small and we can use the approximation

$$\sin\theta \approx \theta$$

Implementing with WebGL

- Define actions in terms of three booleans
- **`trackingMouse`**: if true update trackball position
- **`redrawContinue`**: if true, idle function posts a redisplay
- **`trackballMove`**: if true, update rotation matrix

Vertex Shader I

```
in vec4 vPosition;
in vec4 vColor;
out vec4 color;
uniform vec4 rquat; // rotation quaternion

// quaternion multiplier

vec4 multq(vec4 a, vec4 b)
{
   return(vec4(a.x*b.x - dot(a.yzw, b.yzw),
       a.x*b.yzw+b.x*a.yzw+cross(b.yzw, a.yzw)));
}
```

Vertex Shader II

```
// inverse quaternion
vec4 invq(vec4 a)
{ return(vec4(a.x, -a.yzw)/dot(a,a)); }

  void main() {
  vec3 axis = rquat.yxw;
  float theta = rquat.x;
  vec4 r, p;
  p = vec4(0.0, vPosition.xyz);  // input point quaternion
  p = multq(rquat, multq(p, invq(rquat))); // rotated point quaternion
  gl_Position = vec4( p.yzw, 1.0); // back to homogeneous coordinates
  color = vColor;
}
```