23/1/2019 EX1

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Exercise 1 Given the following $\mathcal{ALC}$ TBox:

$$\begin{array}{rcl} A & \sqsubseteq & \neg F \\ B & \sqsubseteq & C \sqcap G \\ C & \sqsubseteq & F \sqcup \exists R.G \\ D & \sqsubseteq & E \sqcap F \\ E & \sqsubseteq & \exists R.A \\ F & \sqsubseteq & \forall R.B \\ G & \sqsubseteq & \forall R.\neg G \end{array}$$

(a) tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;

(b) tell whether the concept B is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where B is satisfiable;

(c) tell whether the concept D is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where D is satisfiable;

(d) given the ABox $\mathcal{A} = \{G(a), R(a, b)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ entails the assertion $G(b)$, explaining your answer.

a)

Delta^I = {a}

A^I=B^I=C^I=D^I=E^I=F^I=G^I=r^I = empty set

I is model for T, I satisfies alla axioms in Tbox

b)

Delta^I' = {a}
A^I'=E^I'=D^I'=r^I'= empty set
B^I' = {a}
C^I'={a}
G^I'={a}
F^I'={a}

(forall r.B)^I'={a}
(forall r. not G)^I' = {a}

I is a model for T and B^I' is non empty, B is satisfiable

c)

No there is a contradiction because we cannot have have the same elements in E and F because E should be non empty so there is at least one element of A that should belong of role r in the second position and A is non empty if not F is non empty but we need also to have F is non empty, so for every participation of first element in the role we need to have the second element that belong to the role to take this first element. Ex: (a,a), (b,a) where a is in B, we take a,b, instead is we have (a,b),(a,a) we don't take a because in another participation of a in the role the second element does not belong to B. We create an empty intersection because if we have an element of not F in second position in the role r we take the first element but this could not be the same element in that we obtain in forall r. B because we cannot have that element because in the second position there is an element that does not belong to B. So F and E should not have an element in common. There is no model for D, D should be empty

d)

A={G(a),r(a,b)}
A0 = {G(a),r(a,b), not G(b)}

C_GCI = (not A or not F) and (not B or (C and G)) and (not C or F or Exists r. G) and (not D or (E and F)) and (not E or Exists r. A) and (not F or Forall r. B) and (not G or Forall r. not G)

(C_GCI-rule) A1 = A0 union {((not A or not F) and (not B or (C and G)) and (not C or F or Exists r. G) and (not D or (E and F)) and (not E or Exists r. A) and (not F or Forall r. B) and (not G or Forall r. not G))(a)}
(and-rule) A2 = A1 union {(not A or not F)(a), (not B or (C and G))(a), (not C or F or Exists r. G) (a), (not D or (E and F)) (a), (not E or Exists r. A)(a), (not F or Forall r. B) (a), (not G or Forall r. not G)(a)}
(C_GCI-rule) A3 = A2 union {((not A or not F) and (not B or (C and G)) and (not C or F or Exists r. G) and (not D or (E and F)) and (not E or Exists r. A) and (not F or Forall r. B) and (not G or Forall r. not G))(b)}
(and-rule) A4 = A3 union {(not A or not F)(b), (not B or (C and G))(b), (not C or F or Exists r. G) (b), (not D or (E and F)) (b), (not E or Exists r. A)(b), (not F or Forall r. B) (b), (not G or Forall r. not G)(b)}
(or-rule) A5 = A4 union {not G(a)} - CLASH
A6 = A4 union {(Forall r. not G)(a)}
(or-rule) A7 = A6 union {not F(a)}
A8 = A6 union {Forall r. B}
(or-rule) A9 = A7 union{ F(a)} - CLASH
A10 = A7 union {not C(a)}
A11 = A7 union {(Exists r. G)(a)}
(or-rule) A12 = A10 union {(C and G)(a)}
A13 = A10 union {not B(a)}
(and-rule) A14 = A12 union {C(a), G(a)} - CLASH
(or -rule) A15 = A13 union {(E and F)(a)}
A16= A13 union {not D(a)}
(and-rule) A17 = A15 union {E(a), F(a)} - CLASH
(or-rule) A18 = A16 union {not E(a)}
A19 = A16 union {(Exists r. A)(a)}
(or-rule) A20 = A18 union {not F(b)}
A21 = A18 union {(Forall r. B)(b)}
(or-rule) A22 = A20 union {not E(b)}
A23 = A20 union {(Exists r. A)(b)}
(or-rule) A24 = A22 union {(E and F)(b)}
A25 = A22 union {not D(b)}
(and-rule) A26 = A24 union {E(b),F(b)} - CLASH
(or-rule) A27 = A25 union {F(b)} - CLASH
A28 = A25 union {not C(b)}
A29 = A25 union {(Exists r. G)(b)}
(or-rule) A30 = A28 union {(C and G)(b)}
A31 = A28 union { not B(b)} - open and complete
(and-rule) A32 = A30 union {C(b), G(b)} - CLASH

Tableau return true, so the instance checking problem return false