21/7/2017 EX1

domenica 3 gennaio 2021 09:09

Exercise 1 Given the following $\mathcal{ALC}$ TBox:

$$\begin{array}{lcl} A & \sqsubseteq & \neg F \\ B & \sqsubseteq & \neg F \\ B & \sqsubseteq & C \\ C & \sqsubseteq & D \sqcup E \\ D & \sqsubseteq & \exists R.A \\ E & \sqsubseteq & \exists R.B \end{array}$$

(a) tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;

(b) given the ABox $\mathcal{A} = \{C(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A}\rangle$ is satisfiable (consistent), and if so, show a model for $\langle \mathcal{T}, \mathcal{A}\rangle$;

(c) given the ABox $\mathcal{A}' = \{C(a), \forall R.F(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A}'\rangle$ is satisfiable (consistent), and if so, show a model for $\langle \mathcal{T}, \mathcal{A}'\rangle$;

(d) given the ABox $\mathcal{A} = \{C(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A}\rangle$ entails the assertion $\exists R.\neg F(a)$, explaining your answer.

a) This TBox is general because we have the same concept name more than one time in the left side of the primitive concept definition.

A TBOX is satisfiable if there exist a model, so if an interpretation I satisfies all axioms in T.

$\Delta^I = \{a\}$

$A^I = B^I = C^I = D^I = E^I = F^I = r^I = \emptyset$

1) $A^I \subseteq (\neg F)^I$ OK because $\emptyset \subseteq \{a\}$

2) $B^I \subseteq (\neg F)^I$ OK because $\emptyset \subseteq \{a\}$

3) $B^I \subseteq C^I$ OK because $\emptyset \subseteq \emptyset$

4) $C^I \subseteq (D \sqcup E)^I$ OK because $\emptyset \subseteq \emptyset$

5) $D^I \subseteq (\exists r A)^I$ OK because $\emptyset \subseteq \emptyset$

6) $E^I \subseteq (\exists r. B)^I$ OK because $\emptyset \subseteq \emptyset$

All axioms are true in I, so I is a model for T

b) We apply Tableau algorithm, starting with ABOX $A = \{C(a)\}$

because with tableau algorithm we prove satisfiability of KB/concept. In particular, Tableau return YES if the KB is satisfiable, otherwise it return No.

Satisfiability is proven when we apply tableau rule and we find an ABOX that it open and complete, so it doesn't contain atomic contradictions.

Instead, if all ABoxes produced by apply Tableau rules have contradictions, so they are closed then the tableau algorithm says that KB is unsatisfiable so it return No.

In this case we have a general TBox so we are going to define the C_{GCI}

$C_{GCI} \equiv (\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)$

Now, for every individual in ABox it should belong to C_{GCI}. We build C_{GCI} rule for individual a

(C_{GCI}-rule) $A_1 = A_0 \cup \{((\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B))(a)\}$

(and-rule) $A_2 = A_1 \cup \{(\neg A \sqcup \neg F)(a), (\neg B \sqcup \neg F)(a), (\neg B \sqcup C)(a), (\neg C \sqcup D \sqcup E)(a), (\neg D \sqcup \exists r.A)(a), (\neg E \sqcup \exists r.B)(a)\}$

(or-rule) $A_3 = A_2 \cup \{\neg C(a)\}$ CLASH

$A_4 = A_2 \cup \{D(a)\}$ —

$A_5 = A_2 \cup \{E(a)\}$

(or-rule) $A_6 = A_4 \cup \{\neg D(a)\}$ CLASH

$A_7 = A_4 \cup \{(\exists r.A)(a)\}$ —

(or-rule) $A_8 = A_7 \cup \{\neg E(a)\}$ —

$A_9 = A_7 \cup \{(\exists r B)(a)\}$

(or-rule) $A_{10} = A_9 \cup \{\neg B(a)\}$ —

$A_{11} = A_{10} \cup \{\neg F(a)\}$

(or-rule) $A_{12} = A_9 \cup \{\neg A(a)\}$ —

$A_{13} = A_9 \cup \{\neg F(a)\}$

($\exists$-rule) $A_{14} = A_{12} \cup \{A(x), r(a, x)\}$

(C_{GCI}-rule) $A_{15} = A_{14} \cup \{((\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B))(x)\}$

(and-rule) $A_{16} = A_{15} \cup \{(\neg A \sqcup \neg F)(x), (\neg B \sqcup \neg F)(x), (\neg B \sqcup C)(x), (\neg C \sqcup D \sqcup E)(a), (\neg D \sqcup \exists r.A)(x), (\neg E \sqcup \exists r.B)(x)\}$

(or-rule) $A_{17} = A_{16} \cup \{\neg A(x)\}$ CLASH

$A_{18} = A_{16} \cup \{\neg F(x)\}$ —

(or-rule) $A_{19} = A_{18} \cup \{\neg B(x)\}$ —

$A_{20} = A_{18} \cup \{C(x)\}$

(or-rule) $A_{21} = A_{19} \cup \{\neg D(x)\}$ —

$A_{22} = A_{19} \cup \{(\exists r.A)(x)\}$

(or-rule) $A_{23} = A_{21} \cup \{\neg E(x)\}$ —

$A_{24} = A_{21} \cup \{(\exists r.B)(x)\}$

(or-rule) $A_{25} = A_{23} \cup \{D(x)\}$ CLASH

$A_{26} = A_{23} \cup \{E(x)\}$ CLASH

(or-rule) $\mathcal{A}_{25} = \mathcal{A}_{23} \cup \{D(x)\}$ CLASH

$\mathcal{A}_{26} = \mathcal{A}_{23} \cup \{E(x)\}$ CLASH

$\mathcal{A}_{27} = \mathcal{A}_{23} \cup \{\neg C(x)\}$ — open and complete

the tableau algorithm return YES, so KB is satisfiable

c) $\mathcal{A}' = \{C(a), (\forall r.F)(a)\}$

(Gci-rule) $\mathcal{A}_1 = \mathcal{A}' \cup \{(\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)(a)\}$

(and-rule) $\mathcal{A}_2 = \mathcal{A}_1 \cup \{(\neg A \sqcup \neg F)(a), (\neg B \sqcup \neg F)(a), (\neg B \sqcup C)(a), (\neg C \sqcup D \sqcup E)(a), (\neg D \sqcup \exists r.A)(a), (\neg E \sqcup \exists r.B)(a)\}$

(or-rule) $\mathcal{A}_3 = \mathcal{A}_2 \cup \{\neg C(a)\}$ CLASH

$\mathcal{A}_4 = \mathcal{A}_2 \cup \{D(a)\}$

$\mathcal{A}_5 = \mathcal{A}_2 \cup \{E(a)\}$

(or-rule) $\mathcal{A}_6 = \mathcal{A}_4 \cup \{\neg D(a)\}$ CLASH

$\mathcal{A}_7 = \mathcal{A}_4 \cup \{(\exists r.A)(a)\}$

(or-rule) $\mathcal{A}_8 = \mathcal{A}_7 \cup \{\neg E(a)\}$

$\mathcal{A}_9 = \mathcal{A}_7 \cup \{(\exists r.B)(a)\}$

(or-rule) $\mathcal{A}_{10} = \mathcal{A}_8 \cup \{\neg B(a)\}$

$\mathcal{A}_{11} = \mathcal{A}_8 \cup \{\neg F(a)\}$

(or-rule) $\mathcal{A}_{12} = \mathcal{A}_{10} \cup \{\neg A(a)\}$

$\mathcal{A}_{13} = \mathcal{A}_{10} \cup \{\neg F(a)\}$

($\exists$-rule) $\mathcal{A}_{14} = \mathcal{A}_{12} \cup \{A(x), r(a, x)\}$

(Gci-rule) $\mathcal{A}_{15} = \mathcal{A}_{14} \cup \{(\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)(x)\}$

(and-rule) $\mathcal{A}_{16} = \mathcal{A}_{15} \cup \{(\neg A \sqcup \neg F)(x), (\neg B \sqcup \neg F)(x), (\neg B \sqcup C)(x), (\neg C \sqcup D \sqcup E)(x), (\neg D \sqcup \exists r.A)(x), (\neg E \sqcup \exists r.B)(x)\}$

(or-rule) $\mathcal{A}_{17} = \mathcal{A}_{16} \cup \{\neg A(x)\}$ CLASH

$\mathcal{A}_{18} = \mathcal{A}_{16} \cup \{\neg F(x)\}$

($\forall$-rule) $\mathcal{A}_{19} = \mathcal{A}_{17} \cup \{F(x)\}$ CLASH

($\exists$-rule) $\mathcal{A}_{20} = \mathcal{A}_{13} \cup \{A(x), r(a, x)\}$

(Gci-rule) $\mathcal{A}_{21} = \mathcal{A}_{20} \cup \{(\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)(x)\}$

(and-rule) $\mathcal{A}_{22} = \mathcal{A}_{21} \cup \{(\neg A \sqcup \neg F)(x), (\neg B \sqcup \neg F)(x), (\neg B \sqcup C)(x), (\neg C \sqcup D \sqcup E)(x), (\neg D \sqcup \exists r.A)(x), (\neg E \sqcup \exists r.B)(x)\}$

(or-rule) $\mathcal{A}_{23} = \mathcal{A}_{22} \cup \{\neg A(x)\}$ CLASH

$\mathcal{A}_{24} = \mathcal{A}_{22} \cup \{\neg F(x)\}$

($\forall$-rule) $\mathcal{A}_{25} = \mathcal{A}_{24} \cup \{F(x)\}$ CLASH

$A_{24} = A_{22} \cup \{\neg F(x)\}$

(∀-rule) $A_{25} = A_{24} \cup \{F(x)\}$ CLASH

(∃-rule) $A_{26} = A_{11} \cup \{A(x), r(a, x)\}$

(Ccc-rule) $A_{27} = A_{26} \cup \{(\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)(x)\}$

(and-rule) $A_{28} = A_{27} \cup \{(\neg A \sqcup \neg F)(x), (\neg B \sqcup \neg F)(x), (\neg B \sqcup C)(x), (\neg C \sqcup D \sqcup E)(x), (\neg D \sqcup \exists r.A)(x), (\neg E \sqcup \exists r.B)(x)\}$

(or-rule) $A_{29} = A_{28} \cup \{\neg A(x)\}$ CLASH

$A_{30} = A_{28} \cup \{\neg F(x)\}$

(∀-rule) $A_{31} = A_{30} \cup \{F(x)\}$ CLASH

(∃-rule) $A_{32} = A_{9} \cup \{A(x), r(a, x)\}$

(Ccc-rule) $A_{33} = A_{32} \cup \{(\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)(x)\}$

(and-rule) $A_{34} = A_{33} \cup \{(\neg A \sqcup \neg F)(x), (\neg B \sqcup \neg F)(x), (\neg B \sqcup C)(x), (\neg C \sqcup D \sqcup E)(x), (\neg D \sqcup \exists r.A)(x), (\neg E \sqcup \exists r.B)(x)\}$

(or-rule) $A_{35} = A_{34} \cup \{\neg A(x)\}$ CLASH

$A_{36} = A_{34} \cup \{\neg F(x)\}$

(∀-rule) $A_{37} = A_{36} \cup \{F(x)\}$ CLASH

(or-rule) $A_{38} = A_{5} \cup \{\neg E(a)\}$ CLASH

$A_{39} = A_{5} \cup \{(\exists r.B)(a)\}$

(∃-rule) $A_{40} = A_{39} \cup \{B(x), r(a, x)\}$

(Ccc-rule) $A_{41} = A_{40} \cup \{(\neg A \sqcup \neg F) \sqcap (\neg B \sqcup \neg F) \sqcap (\neg B \sqcup C) \sqcap (\neg C \sqcup D \sqcup E) \sqcap (\neg D \sqcup \exists r.A) \sqcap (\neg E \sqcup \exists r.B)(x)\}$

(and-rule) $A_{42} = A_{41} \cup \{(\neg A \sqcup \neg F)(x), (\neg B \sqcup \neg F)(x), (\neg B \sqcup C)(x), (\neg C \sqcup D \sqcup E)(x), (\neg D \sqcup \exists r.A)(x), (\neg E \sqcup \exists r.B)(x)\}$

(or-rule) $A_{43} = A_{42} \cup \{\neg B(x)\}$ CLASH

$A_{44} = A_{42} \cup \{\neg F(x)\}$

(∀-rule) $A_{45} = A_{44} \cup \{F(x)\}$ CLASH

All Aboxes are closed, so tableau algorithm return NO

KB is unsatisfiable

d) We have a problem of instance checking because we want to prove if $(\exists r.\neg P)(a)$ is entailed by KB

To ... we need to transform this problem of want to prove if $(\exists v\ \neg F)(a)$ is entailed by KB)

To check it we need to transform this problem of instance checking in an satisfiability problem because we are going to use Tableau algorithm that it is an algorithm that check satisfiability. We negate our thesis and we check the unsatisfiability, so we compute $\neg((\exists v.\ \neg F))(a)$ and we apply the Tableau. If tableau return TRUE, so the instance checking problem is false, otherwise if tableau return False the instance checking return TRUE

NNF: $(\neg(\exists v.\ \neg F))(a) \rightarrow (\forall v\ F)(a)$

$A_0 = A \cup \{(\forall v\ F)(a)\}$

but in the previous point we have already prove that tableau return False, so it return unsatisfiability and this mean that instance checking problem is true and so $(\exists v\ \neg F)(a)$ is entailed by KB.