23/07/2018 EX1

venerdì 1 gennaio 2021 09:31

Exercise 1 Given the following $\mathcal{ALC}$ TBox:

$$\begin{array}{lcl} A & \sqsubseteq & \exists R.C \\ B & \sqsubseteq & \forall R.D \\ D & \sqsubseteq & \neg C \\ E & \sqsubseteq & A \sqcup \forall R.G \\ F & \sqsubseteq & B \sqcup \exists R.C \\ G & \sqsubseteq & D \end{array}$$

(a) tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;

(b) tell whether the concept E is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where E is satisfiable;

(c) tell whether the concept $E \sqcap F$ is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where $E \sqcap F$ is satisfiable;

(d) given the ABox $\mathcal{A} = \{E(a), R(a,b)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ entails the assertion $C(b)$, explaining your answer.

a)

A^I=B^I=C^I=D^I=E^I=F^I=G^I=r^I=empty set
I is a model for TBox because it satisfies all axioms

b)

E is satisfiable if there is a model I for T and the interpretation I of E is non empty

If E is non empty A or Forall r. G should be non empty. So either A is non empty or Forall r. G is non empty.
A is non empty if there is at least one participation of element C in the role r.
forall r. G is non empty if forall r. D is non empty, so if forall r. not C is non empty

We can have a model for T because I cannot see any contraddictions

Delta^I' = {a}

A^I'= {a}
C^I'= {a}
r^I' = {(a,a)}
E^I' = {a}
B^I'=D^I'=F^I' = G^I' = empty set

I is a model for T and B^I' is non empty so B is satisfiable.

c)
If the interpretations of E and F, E^I and F^I, should be non empty and the interpretation I is a model for T then E and F is satisfiable

If E is non empty A or Forall r. G should be non empty. So either A is non empty or Forall r. G is non empty.
A is non empty if there is at least one participation of element C in the role r.
forall r. G is non empty if forall r. D is non empty, so if forall r. not C is non empty
If F is non empty B or Exist r. C should be non empty. So either B is non empty or Exists r. C is non empty.
B is non empty if Forall r. D is non empty, so forall r. not C.

We can see that there an interpretation that it is also a model fot T in which F and E are non empty because in both concept we need that there is at least one participation of element C in the role r

Delta^I'' = {a}
A^I''= {a}
C^I''= {a}
r^I'' = {(a,a)}
E^I'' = {a}
F^I'' = {a}
B^I'=D^I' = G^I' = empty set

E and F is satisfiable

d)

A0 = {E(a), r(a,b), not C(b)}

C_GCI = (not A or Exists r. C) and (not B or Forall r. D) and (not D or not C) and (not E or A or Forall r. G) and (not F or B or Exist r. C) and (not G or D)
(C_GCI-rule) A1 = A0 union {((not A or Exists r. C) and (not B or Forall r. D) and (not D or not C) and (not E or A or Forall r. G) and (not F or B or Exist r. C) and (not G or D))(a)}
(and-rule) A2 = A1 union {(not A or Exists r. C)(a), (not B or Forall r. D)(a),(not D or not C) (a), (not E or A or Forall r. G)(a), (not F or B or Exist r. C)(a), (not G or D)(a)}
(or-rule) A3 = A2 union {not E(a)} - CLASH
A4 = A2 union {(Forall r. G)(a)}
A5 = A2 union {A(a)}
(Forall-rule) A6 = A4 union {G(b)}
(or-rule) A7 = A6 union {not F(a)}
A8 = A6 union {B(a)}
A9 = A6 union {(Exist r. C)(a)}
(or-rule) A10 = A7 union {not G(a)}
A11 = A7 union {D(a)}
(or-rule) A12 = A10 union {not A(a)}
A13 = A10 union {(Exists r. C)(a)}
(or-rule) A14 = A12 union {not B(a)}
A15 = A13 union {(Forall r. D)(a)}
(or-rule) A16 = A14 union {not D(a)}
A17 = A14 union {not C(a)}
(C_GCI-rule) A18 = A17 union {((not A or Exists r. C) and (not B or Forall r. D) and (not D or not C) and (not E or A or Forall r. G) and (not F or B or Exist r. C) and (not G or D))(b)}
(and-rule) A19 = A18 union {(not A or Exists r. C)(b), (not B or Forall r. D)(b),(not D or not C) (b), (not E or A or Forall r. G)(b), (not F or B or Exist r. C)(b), (not G or D)(b)}
(or-rule) A20 = A19 union {not A(b)}
A21 = A19 union {(Exist r. C)(b)}
(or-rule) A22 = A20 union {A(b)} - CLASH
A23 = A20 union {not E(b)}
A24 = A20 union {(Forall r. G)(b)}
(or-rule) A25 = A23 union {not B(b)}

A26 = A23 union {(Forall r. D)(b)}
(or-rule) A27 = A25 union {B(b)} - CLASH
A28 = A25 union {not F(b)}
A29 = A25 union {(Exists r. C)(b)}
(or-rule) A30 = A28 union {not G(b)} - open and complete
A31 = A28 union {D(b)}

Tableau return true, instance checking problem is false