

Exercise 2 Given the following ASP program P:

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r(x,y) :- p1(x,y).
r(x,y) :- p2(y,x).
r(x,z) :- r(x,y), r(y,z).
s(x,y) :- p1(x,y).
t(x,y) :- r(x,y), not s(x,y).
t(x,y) :- t(y,x).
v(x,y) :- r(x,y), not t(x,y).
w(x,y) :- t(x,y), not v(x,y).
p1(a,b). p1(b,c). p1(c,d). p2(c,d). p2(d,e). p2(e,f).

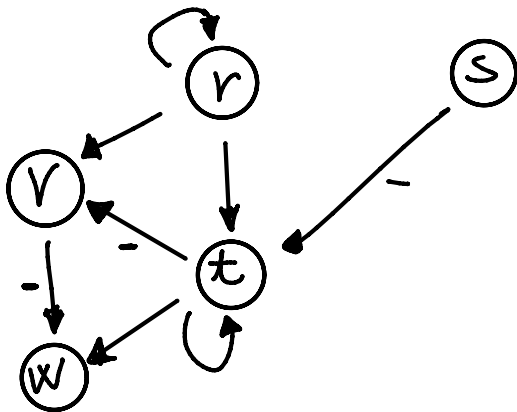
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- tell whether P is stratified;
- compute the answer sets of P.

a)

IDB = {r/2, s/2, t/2, v/2, w/2}

EDB = {p1/2, p2/2}

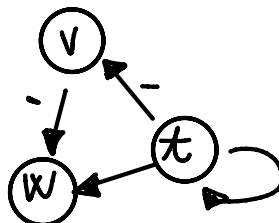


P is stratified

b)

The answer set is unique and coincide with MM(P)

S1 = {r,s}



S2 = {t}



S3 = {v}



$S3 = \{v\}$



$S4 = \{w\}$

$MM0 = \{p1(a,b), p1(b,c), p1(c,d), p2(c,d), p2(d,e), p2(e,f)\}$

$P(S1) = \{$
 $r(x,y) :- p1(x,y).$
 $r(x,y) :- p2(y,x).$
 $r(x,z) :- r(x,y), r(y,z).$
 $s(x,y) :- p1(x,y).$
 $\}$

Apply semi naive algorithm

$I = MM0$

$I' = Tp(I) = \{ r(a,b), r(b,c), r(c,d), r(d,c), r(e,d), r(f,e), s(a,b), s(b,c), s(c,d) \}$

$\Delta I = \{ \Delta r(a,b), \Delta r(b,c), \Delta r(c,d), \Delta r(d,c), \Delta r(e,d), \Delta r(f,e), \Delta s(a,b), \Delta s(b,c), \Delta s(c,d) \}$

$\Delta P = \{$
 $\Delta r(x,z) :- \Delta r(x,y), \Delta r(y,z).$
 $\}$

1-iteration

$I = I \cup \{ r(a,b), r(b,c), r(c,d), r(d,c), r(e,d), r(f,e), s(a,b), s(b,c), s(c,d) \}$

$\Delta I = \{ \Delta r(a,b), \Delta r(b,c), \Delta r(c,d), \Delta r(d,c), \Delta r(e,d), \Delta r(f,e), \Delta s(a,b), \Delta s(b,c), \Delta s(c,d) \}$

$\Delta I = \{ \Delta r(a,c), \Delta r(b,d), \Delta r(c,c), \Delta r(e,c), \Delta r(f,d) \}$

2-iteration

$I = I \cup \{ r(a,c), r(b,d), r(c,c), r(e,c), r(f,d) \}$

$\Delta I = \{ \Delta r(a,c), \Delta r(b,d), \Delta r(c,c), \Delta r(e,c), \Delta r(f,d) \}$

$\Delta I = \{ \Delta r(a,d), \Delta r(f,c) \}$

3-iteration

$I = I \cup \{ r(a,d), r(f,c) \}$

$\Delta I = \{ \Delta r(a,d), \Delta r(f,c) \}$

$\Delta I = \{ \Delta r(f,d) \}$

4-iteration

$I = I \cup \{ r(f,d) \}$

$\Delta I = \{\Delta r(f,d)\}$

$\Delta' I = \{\}$

$MM1 = MM0 \text{ union } \{ r(a,b), r(b,c), r(c,d), r(d,c), r(e,d), r(f,e), r(a,c), r(b,d),$
 $r(c,c), r(e,c), r(f,d), r(a,d), r(f,c),$
 $s(a,b), s(b,c), s(c,d) \}$

$P(S2) = \{$
 $t(x,y) :- r(x,y), \text{ not } s(x,y).$
 $t(x,y) :- t(y,x).$
 $\}$

$MM2 = MM1 \text{ union } \{ t(d,c), t(e,d), t(f,e), t(a,c), t(b,d), t(c,c), t(e,c), t(f,d), t(a,d), t(f,c), t(c,d),$
 $t(d,e), t(e,f), t(c,a), t(d,b), t(c,e), t(d,f), t(d,a), t(c,f) \}$

$P(S3) = \{$
 $v(x,y) :- r(x,y), \text{ not } t(x,y).$
 $\}$

$MM3 = MM2 \text{ union } \{ v(a,b), v(b,c), v(c,d) \}$

$P(S4) = \{$
 $w(x,y) :- t(x,y), \text{ not } v(x,y).$
 $\}$

$MM4 = MM3 \text{ union } \{ w(d,c), w(e,d), w(f,e), w(a,c), w(b,d), w(c,c), w(e,c), w(f,d), w(a,d), w(f,c),$
 $w(d,e), w(e,f), w(c,a), w(d,b), w(c,e), w(d,f), w(d,a), w(c,f) \}$

$MM(P) = \{ p1(a,b), p1(b,c), p1(c,d), p2(c,d), p2(d,e), p2(e,f),$
 $r(a,b), r(b,c), r(c,d), r(d,c), r(e,d), r(f,e), r(a,c), r(b,d), r(c,c), r(e,c), r(f,d), r(a,d), r(f,c),$
 $s(a,b), s(b,c), s(c,d),$
 $t(d,c), t(e,d), t(f,e), t(a,c), t(b,d), t(c,c), t(e,c), t(f,d), t(a,d), t(f,c), t(c,d), t(d,e), t(e,f),$
 $t(c,a), t(d,b), t(c,e), t(d,f), t(d,a), t(c,f),$
 $w(d,c), w(e,d), w(f,e), w(a,c), w(b,d), w(c,c), w(e,c), w(f,d), w(a,d), w(f,c), w(d,e),$
 $w(e,f), w(c,a), w(d,b), w(c,e), w(d,f), w(d,a), w(c,f) \}$