Knowledge Representation and Semantic Technologies

Exercises on Datalog and ASP (part 2)

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Exercise 3

Given the following ASP program P:

```
r(X,Y) :- s(X,Y).
r(X,Y) :- r(X,Z), s(Z,Y).
t(X,Y) :- r(X,Y), not s(X,Y).
s(a,b).  s(b,c).
```

1) tell whether P is stratified;
2) compute the answer sets of P.

Exercise 3 - Solution

We start by observing that there are no negated atoms involving IDB predicates (the only negated atom is relative to the EDB predicate s).

Therefore, the labeled dependency graph of P does not contain any negated edge, and hence no cycle containing a negated edge.

Consequently, program P is stratified.

Exercise 3 - Solution

Since P is stratified, it has only one answer set, and we can compute such an answer set through semi-naive evaluation of each single stratum of the program (P has actually only one stratum).

Notice that, in the computation, we will have to evaluate the negated atom not s(x,y) over the initial interpretation I=EDB(P), i.e. the set of facts in P, which establishes the interpretation of the predicate s (in other words, every fact that is not in EDB(P) must be considered false).

Exercise 3 - Solution

The semi-naive algorithm starts with
$I' = T_P(I) = \{s(a,b), s(b,c), r(a,b), r(b,c)\}$

The program ΔP is the following:
$\Delta' r(X,Y) :- \Delta r(X,Z), s(Z,Y).$
$\Delta' t(X,Y) :- \Delta r(X,Y), \text{not } s(X,Y).$

Step 1: $\Delta' I = \{ r(a,c) \}$

Step 2: $\Delta' I = \{ t(a,c) \}$

Step 3: $\Delta' I = \{ \}$

Exercise 3 - Solution

The answer set of P is thus the following:

{ s(a,b), s(b,c), r(a,b), r(b,c), r(a,c), t(a,c) }

Exercise 4

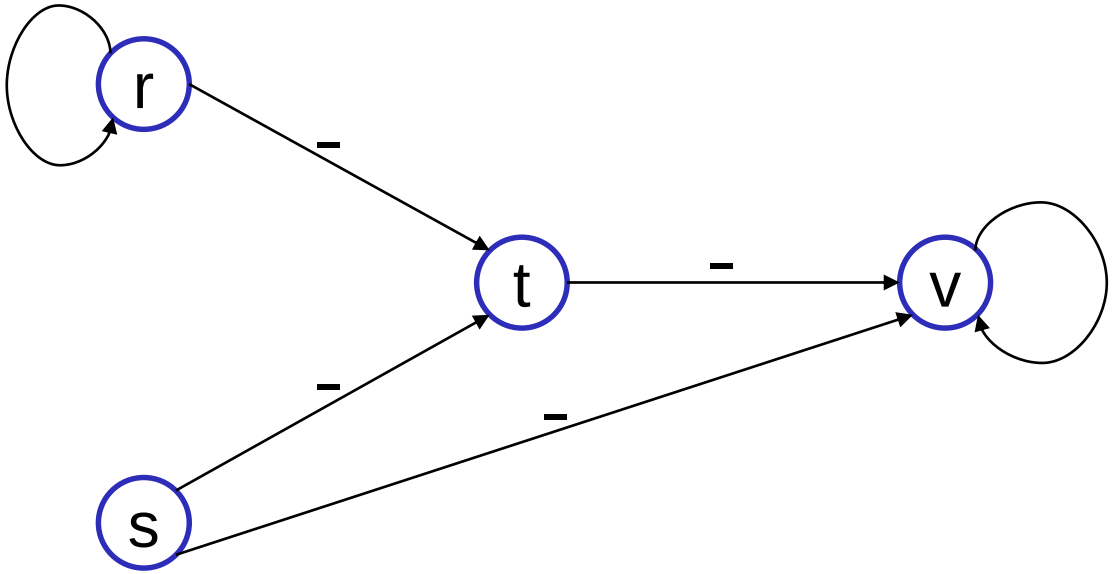
Given the following ASP program P:

```
r(X,Y) :- p(X,Y).
r(X,Y) :- p(X,Z), r(Z,Y).
s(X,Y) :- q(X,Y).
t(X,Y) :- r(X,Y), not s(X,Y).
t(X,Y) :- s(X,Y), not r(X,Y).
v(X,Y) :- t(X,Y), not s(Y,X).
v(X,Y) :- v(X,Z), v(Z,Y), not t(Z,X).
p(a,b).  p(b,c).  q(a,b).  q(c,a).
```

1) tell whether P is stratified;
2) compute the answer sets of P.

Exercise 4 - Solution

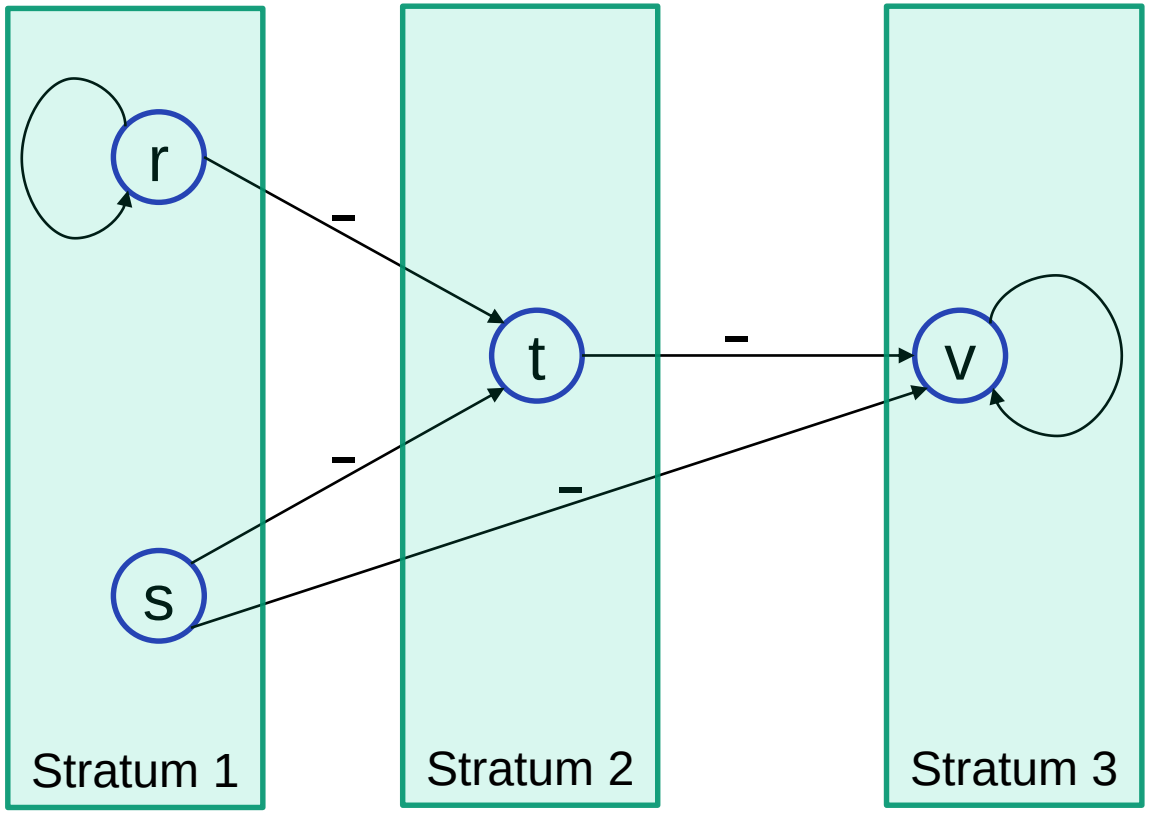
The labeled dependency graph of P is the following:



The above graph does not contain any cycle containing a negated edge.

Consequently, program P is stratified.

Exercise 4 - Solution



Exercise 4 - Solution

$MM_0 = EDB(P) = \{ p(a,b), p(b,c), q(a,b), q(c,a) \}$

Exercise 4 - Solution

Program $P(S_1)$ for Stratum 1:

```
r(X,Y) :- p(X,Y).
r(X,Y) :- p(X,Z), r(Z,Y).
s(X,Y) :- q(X,Y).
t(X,Y) :- r(X,Y), not s(X,Y).
t(X,Y) :- s(X,Y), not r(X,Y).
v(X,Y) :- t(X,Y), not s(Y,X).
v(X,Y) :- v(X,Z), v(Z,Y), not t(Z,X).
p(a,b).  p(b,c).  q(a,b).  q(c,a).
```

Exercise 4 - Solution

MM_1 = Minimal model for $P(S_1)$ U MM_0 =

{ p(a,b), p(b,c), q(a,b), q(c,a),
r(a,b), r(b,c), s(a,b), s(c,a), r(a,c) }

Exercise 4 - Solution

Program $P(S_2)$ for Stratum 2:

```
r(X,Y) :- p(X,Y).
r(X,Y) :- p(X,Z), r(Z,Y).
s(X,Y) :- q(X,Y).
t(X,Y) :- r(X,Y), not s(X,Y).
t(X,Y) :- s(X,Y), not r(X,Y).
v(X,Y) :- t(X,Y), not s(Y,X).
v(X,Y) :- v(X,Z), v(Z,Y), not t(Z,X).
p(a,b).  p(b,c).  q(a,b).  q(c,a).
```

Exercise 4 - Solution

MM_2 = Minimal model for $P(S_2)$ U MM_1 =

{ p(a,b), p(b,c), q(a,b), q(c,a),
r(a,b), r(b,c), s(a,b), s(c,a), r(a,c),
t(b,c), t(a,c), t(c,a) }

Exercise 4 - Solution

Program $P(S_3)$ for Stratum 3:

r(X,Y) :- p(X,Y).
r(X,Y) :- p(X,Z), r(Z,Y).
s(X,Y) :- q(X,Y).
t(X,Y) :- r(X,Y), not s(X,Y).
t(X,Y) :- s(X,Y), not r(X,Y).
v(X,Y) :- t(X,Y), not s(Y,X).
v(X,Y) :- v(X,Z), v(Z,Y), not t(Z,X).
p(a,b). p(b,c). q(a,b). q(c,a).

Exercise 4 - Solution

MM_3 = Minimal model for $P(S_3)$ U MM_2 =

{ p(a,b), p(b,c), q(a,b), q(c,a),
r(a,b), r(b,c), s(a,b), s(c,a), r(a,c),
t(b,c), t(a,c), t(c,a),
v(b,c), v(c,a), v(b,a) }

This is the minimal model for the whole program P.