19/7/2016 EX1

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Exercise 1 Given the following $\mathcal{ALC}$ TBox:

$$\begin{array}{rcl} A & \sqsubseteq & B \\ A & \sqsubseteq & C \\ B & \sqsubseteq & \exists R.\neg D \\ C & \sqsubseteq & \exists R.D \\ E & \sqsubseteq & \forall R.\neg D \end{array}$$

(a) tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;

(b) tell whether the concept A is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where the interpretation of A is non-empty;

(c) given the ABox $\mathcal{A} = \{A(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ entails the assertion $\exists R.D(a)$, explaining your answer;

(d) given the ABox $\mathcal{A} = \{A \sqcap E(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable (consistent), explaining your answer.

a)

Delta^I = {a}
A^I=B^I=C^I=D^I=E^I=r^I= empty set
I is model for Tbox because it satisfies all axioms

b)
Delta^I' ={a,b}

E^I'= empty set
D^I' = {a}
(not D)^I' = {b}
r^I' = {(a,a), (a,b)}
(Exists r. not D)^I' = {a}
(Exists r. D)^I' = {a}
B^I' = C^I' = A^I' = {a}

A is satisfiable

c)

(Not (Exists r.D))(a)
We apply NNF and we obtain
(Forall r. not D)(a)

A0 = {A(a),(Forall r. not D)(a)}
C_GCI = (not A union B) and (not A union C) and (not B union Exists r. not D) and (not C union Exists r. D) and (not E union Forall r. not D)
(C_GCI-rule) A1= A0 union {((not A union B) and (not A union C) and (not B union Exists r. not D) and (not C union Exists r. D) and (not E union Forall r. not D)) (a)}

(and-rule) A2 = A1 union { (not A union B)(a), (not A union C) (a), (not B union Exists r. not D) (a), (not C union Exists r. D) (a), (not E union Forall r. not D)(a)}
(or-rule) A3 = A2 union {not A(a)} - CLASH
A4= A2 union {B(a)}
(or-rule) A5= A4 union {not B(a)} - CLASH
A6= A4 union {Exists r. not D)}
(or-rule) A7 =A6 union {not A(a)} - CLASH
A8 = A6 union{C(a)}
(or-rule) A9 = A8 union{not C(a)} - CLASH
A10 = A9 union {Exists r. D}(a))
(Exist-rule) A11 = A10 union{r(a,x) D(x)}
(C_GCI-rule) A12= A11 union {((not A union B) and (not A union C) and (not B union Exists r. not D) and (not C union Exists r. D) and (not E union Forall r. not D)) (x)}
(and-rule) A13 = A12 union { (not A union B)(x), (not A union C) (x), (not B union Exists r. not D) (x), (not C union Exists r. D) (x), (not E union Forall r. not D)(x)}
(Forall-rule) A14 = A13 union { not D(x)} - CLASH

All Aboxes are closed so tableau return false and so the instance checking problem is true. Exists r. D(a) is entailed by KB

d)

A = {(A and E) (a)}
(and-rule) A0 = {A(a),E(a)}
C_GCI = (not A union B) and (not A union C) and (not B union Exists r. not D) and (not C union Exists r. D) and (not E union Forall r. not D)
(C_GCI-rule) A1= A0 union {((not A union B) and (not A union C) and (not B union Exists r. not D) and (not C union Exists r. D) and (not E union Forall r. not D)) (a)}
(and-rule) A2 = A1 union { (not A union B)(a), (not A union C) (a), (not B union Exists r. not D) (a), (not C union Exists r. D) (a), (not E union Forall r. not D)(a)}
(or-rule) A3 = A2 union {not E(a)} - CLASH
A4 = A2 union {(Forall r. not D)(a)}
(or-rule) A5 = A4 union {not A(a)} - CLASH
A6 = A4 union {B(a)}
(or-rule) A7 = A6 union {not B(a)} - CLASH
A8 = A6 union {(Exists r. not D)(a)}
(or-rule) A9 = A8 union {not A(a)} - CLASH
A10 = A8 union {C(a)}
(or-rule) A11 = A10 union {not C(a)} - CLASH

A12 = A10 union {(Exists r. D)(a)}
(Exist-rule) A13 = A12 union {D(x), r(a,x)}
(C_GCI-rule) A12= A11 union {((not A union B) and (not A union C) and (not B union Exists r. not D) and (not C union Exists r. D) and (not E union Forall r. not D)) (x)}
(and-rule) A13 = A12 union { (not A union B)(x), (not A union C) (x), (not B union Exists r. not D) (x), (not C union Exists r. D) (x), (not E union Forall r. not D)(x)}
(Forall-rule) A14 = A13 union { not D(x)} - CLASH

Tableau return false so the KB is unsatisfiable