21/7/2017 EX2

domenica 3 gennaio 2021 10:20

Exercise 2 Given the following ASP program P:

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r(x,y) :- p1(x), p2(y).
s(x,y) :- r(x,y), not p2(x), not p1(y).
t(x,y) :- s(x,y).
t(y,z) :- s(x,y), t(x,z).
v(x,y) :- r(x,y), not s(x,y).
w(x,y) :- r(x,y), not v(x,y).
p1(a). p1(b). p1(c). p2(b). p2(c). p2(d). p2(e).
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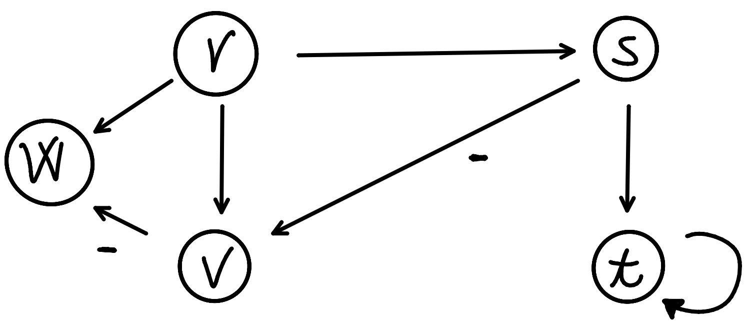
(a) tell whether P is stratified;

(b) compute the answer sets of P.

a) A Datalog program P with Negation is stratified if in the precedence graph there are no cycle with Negated edge

IDB = { r/2 , s/2 , t/2 , v/2 , w/2 }

EDB = { p1/1 , p2/1 }



Precedence graph has no cycle with Negated edge

b) Program is stratified so there is a unique answer set and it is coincide with minimal model of P

To compute MM(P) we use stratification

Stratification of P is a set of strata

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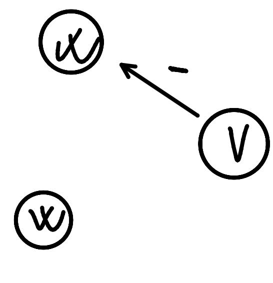
in which we look at precedence graph and
at each iteration we compute a strata.
this strata is compute by looking at vertices in
precedence graph that don't have any negative
dependency. Negative dependency means that
a vertex v has negative dependency if there
exist another vertex v' and there is a path from
v' to v passing through negated edge.
So, we take vertices that have no negative
dependency and then we consider them as a
EDB so we remove from the graph and also
all their edges.
We repeat this operation until the graph
is empty, so at the end we have a
sequence of strata

In our case we have



$S_1 = \{v, s, t\}$

$S_2 = \{v\}$

$S_3 = \{x\}$

Now we are going to compute the MM(P)

Stratification is very important because in this way we can evaluate Negation because in each iteration of the algorithm that we use to compute the minimal model we consider only IDB of that stratum, so we consider only the rules in P that have as head the IDB predicates of stratum S_i. And in those rule the negated predicate that we have in the body is an EDB predicate, we could never have negated IDB predicates because of stratification that we have done. This because in the stratum S_{i-1} we have already computed all positive consequency of the predicates of the lower strata and so we are also able to evaluate Negation now.

But in particular, since, for example I have no derived some facts about a predicate in S_{i-1}, now I KNOW that this facts is negative in the minimal model because if it were positive I would have it in the previous iteration.

We start with

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$MM_0 = EDB(P) = \{ p_1(a), p_1(b), p_1(c), p_2(b), p_2(c), p_2(d), p_2(e) \}$

$P(S_1) = \{$

$r(X,Y) :- p_1(X), p_2(Y).$

$s(X,Y) :- r(X,Y), \text{not } p_2(X), \text{not } p_1(Y).$

$t(X,Y) :- s(X,Y).$

$t(X,Z) :- s(X,Y), t(X,Z).$

$\}$

We apply semi-naive evaluation to compute MM of this strata

$I = MM_0$

$I' = T_P(I) = \{ r(a,b), r(a,c), r(a,d), r(a,e), r(b,b), r(b,c), r(b,d), r(b,e), r(c,b), r(c,c), r(c,d), r(c,e) \}$

$\Delta I' = \{ \Delta' r(a,b), \Delta' r(a,c), \Delta' r(a,d), \Delta' r(a,e), \Delta' r(b,b), \Delta' r(b,c), \Delta' r(b,d), \Delta' r(b,e), \Delta' r(c,b), \Delta' r(c,c), \Delta' r(c,d), \Delta' r(c,e) \}$

Δ - transformation

$\Delta P = \{$

$\Delta' s(X,Y) :- \Delta r(X,Y), \text{not } p_2(X), \text{not } p_1(Y).$

$\Delta' t(x,y) :- \Delta s(x,y)$

$\Delta s(x,y) :- \Delta r(x,y),\ not\ p_2(x),\ not\ p_1(y).$

$\Delta' t(x,y) :- \Delta s(x,y).$

$\Delta' t(x,z) :- \Delta s(x,y),\ t(x,z).$

$\Delta' t(x,z) :- s(x,y),\ \Delta t(x,z).$

}

1-iteration

$I = I \cup \{r(a,b), r(a,c), r(a,d), r(a,e),$
$r(b,b), r(b,c), r(b,d), r(b,e),$
$r(c,b), r(c,c), r(c,d), r(c,e)\}$

$\Delta I = \{\Delta r(a,b), \Delta r(a,c), \Delta r(a,d), \Delta r(a,e),$
$\Delta r(b,b), \Delta r(b,c), \Delta r(b,d), \Delta r(b,e),$
$\Delta r(c,b), \Delta r(c,c), \Delta r(c,d), \Delta r(c,e)\}$

$\Delta' I = \{\Delta' s(a,d), \Delta' s(a,e)\}$

2-iteration

$I = I \cup \{s(a,d), s(a,e)\}$

$\Delta I = \{\Delta s(a,d), \Delta s(a,e)\}$

$\Delta' I = \{\Delta' t(a,d), \Delta' t(a,e)\}$

3-iteration

$I = I \cup \{t(a,d), t(a,e)\}$

$\Delta I = \{\Delta t(a,d), \Delta t(a,e)\}$

$\Delta' I = \{\Delta' t(d,d), \Delta' t(d,e), \Delta' t(e,d), \Delta' t(e,e)\}$

4-iteration

$I = I \cup \{t(d,d), t(d,e), t(e,d), t(e,e)\}$

$\Delta I = \{\Delta t(d,d), \Delta t(d,e), \Delta t(e,d), \Delta t(e,e)\}$

$I = I \cup \{ t(d,d), t(d,e), t(e,d), t(e,e) \}$

$\Delta I = \{ \Delta t(d,d), \Delta t(d,e), \Delta t(e,d), \Delta t(e,e) \}$

$\Delta' I = \{ \}$

STOP

$MM_1 = MM_0 \cup \{ r(a,b), r(a,c), r(a,d), r(a,e),$
$r(b,b), r(b,c), r(b,d), r(b,e),$
$r(c,b), r(c,c), r(c,d), r(c,e),$
$s(a,d), s(a,e),$
$t(a,d), t(a,e), t(d,d), t(d,e),$
$t(e,d), t(e,e) \}$

$P(S_2) = \{$
$v(X,Y) :- r(X,Y), \text{not } s(X,Y).$
$\}$

$MM_2 = MM_1 \cup \{ v(a,b), v(a,c), v(b,b), v(b,c),$
$v(b,c), v(b,d), v(b,e), v(c,b),$
$v(c,c), v(c,d), v(c,e) \}$

$P(S_3) = \{$
$w(X,Y) :- r(X,Y), \text{not } v(X,Y).$
$\}$

$MM_3 = MM_2 \cup \{ w(a,d), w(a,e) \}$

$MM(P) = \{ p_1(a), p_1(b), p_1(c)$
$p_2(b), p_2(c), p_2(d), p_2(e),$
$r(a,b), r(a,c), r(a,d), r(a,e),$

$r(a,b), r(a,c), r(a,d), r(a,e),$
$r(b,b), r(b,c), r(b,d), r(b,e),$
$r(c,b), r(c,c), r(c,d), r(c,e),$
$s(a,d), s(a,e),$
$t(a,d), t(a,e), t(d,d), t(d,e),$
$t(e,d), t(e,e),$
$v(a,b), v(a,c), v(b,b), v(b,c),$
$v(b,c), v(b,d), v(b,e), v(c,b),$
$v(c,c), v(c,d), v(c,e),$
$w(a,d), w(a,e)\}$