

Exercise 2 Given the following ASP program P:

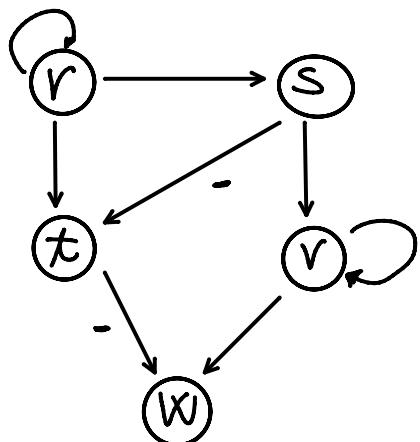
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r(x,y) :- p(x,y).
r(x,y) :- p(x,z), r(z,y).
s(x,y) :- r(x,y), not p(y,x).
t(x,y) :- r(x,y), not s(x,y).
v(x,y) :- s(x,y).
v(x,y) :- s(x,z), v(z,y).
w(x,y) :- v(x,y), not t(x,y).
p(a,b). p(b,c). p(c,d). p(c,e).
```

- (a) tell whether P is stratified;
- (b) compute the answer sets of P.

e) P is stratified IF the precedence graph has no cycles with negative edges

$$IDB = \{r/2, s/2, t/2, v/2, w/2\}$$

$$EDB = \{p/2\}$$



Program is stratified because this precedence graph has cycles with no negative edge

b) P is stratified so there is only 1 answer set that coincides with minimal model of P

Need to compute stratification, return a set of states in which we compute each stratum at iteration.

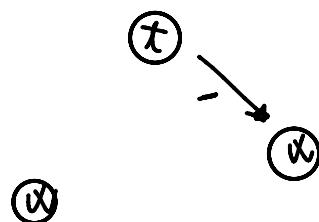
At each iteration we look at precedence graph and see which are the vertices / IDB predicates that have no negative dependencies. Negative dependencies means that a vertex v has a negative dependency if there is another vertex v' and

negative dependency' if there is another vertex v' and there is a path from v' to v that pass through a negated edge. After we take vertices that don't have negative dependency we consider them as a EDB predicates and so we remove them and all edges connected to them from the preconcise graph and continue until there are no more vertex to take.

$$S_0 = \{r, s, v\}$$

$$S_1 = \{t\}$$

$$S_2 = \{w\}$$



Stratification is important because in this way we can evaluate negation because in the algorithm that we use to compute the minimal model, at each iteration, the rules that we consider couldn't have the negated head in the body part, in the body we have existential predicates, predicates that we have already computed in the lower strata,

$$\text{MM}_0 = \text{EDB}(P) = \{p(a, b), p(b, c), p(c, d), p(c, e)\}$$

$$P(S_0) = \{$$

$$r(x, y) : -p(x, y).$$

$$r(x, y) : -p(x, z), r(z, y).$$

$$s(x, y) : -r(x, y), \text{not } p(y, x).$$

$$v(x, y) : -s(x, y).$$

$$v(x, y) : -c, v, \dots, \dots$$

$V(X, Y) := S(X, Y)$
 $V(X, Y) := S(X, Z), V(Z, Y)$.

}

Apply semi Naive-Evaluation algorithm

$$I = \{ p(a, b), p(b, c), p(c, d), p(c, e) \}$$

$$I' = T_p(I) = \{ r(a, b), r(b, c), r(c, d), r(c, e) \}$$

$$\Delta I = \{ \Delta r(a, b), \Delta r(b, c), \Delta r(c, d), \Delta r(c, e) \}$$

$$\Delta P = \{$$

$$\Delta r(x, y) := p(x, z) \Delta r(z, y).$$

$$\Delta s(x, y) := \Delta r(x, y), \text{not } p(y, x).$$

$$\Delta v(x, y) := -\Delta s(x, y).$$

$$\Delta' v(x, y) := -\Delta s(x, z), v(z, y).$$

$$\Delta' v(x, y) := s(x, z), \Delta v(z, y).$$

↳

1-iteration

$$I = I \cup \{ r(a, b), r(b, c), r(c, d), r(c, e) \}$$

$$\Delta I = \{ \Delta r(a, b), \Delta r(b, c), \Delta r(c, d), \Delta r(c, e) \}$$

$$\Delta' I = T_p(I \cup \Delta I) = \{ \Delta' r(a, c), \Delta' r(b, d), \Delta' r(b, e) \}$$

2-iteration

$$I = I \cup \{ r(a, c), r(b, d), r(b, e), r(a, d), r(a, e) \}$$

$$\Delta I = \{ \Delta r(a, c), \Delta r(b, d), \Delta r(b, e), \Delta r(a, d), \Delta r(a, e) \}$$

$$\Delta' I = \{ \Delta' s(a, b), \Delta' s(b, c), \Delta' s(c, d), \Delta' s(c, e), \\ \Delta' s(a, c), \Delta' s(b, d), \Delta' s(b, e), \Delta' s(a, d), \Delta' s(a, e) \}$$

3-iteration

$$I = I \cup \{ s(a,b), s(b,c), s(c,d), s(c,e), \\ s(a,c), s(b,d), s(b,e), s(a,d), s(a,e) \}$$

$$\Delta I = \{ \Delta s(a,b), \Delta s(b,c), \Delta s(c,d), \Delta s(c,e), \\ \Delta s(a,c), \Delta s(b,d), \Delta s(b,e), \Delta s(a,d), \Delta s(a,e) \}$$

$$\Delta' I = \{ \Delta' v(a,b), \Delta' v(b,c), \Delta' v(c,d), \Delta' v(c,e), \Delta' v(a,c), \\ \Delta' v(b,d), \Delta' v(b,e), \Delta' v(a,d), \Delta' v(a,e) \}$$

4-iTeoton

$$I = I \cup \{ v(a,b), v(b,c), v(c,d), v(c,e), v(a,c), v(b,d), \\ v(b,e), v(a,d), v(a,e) \}$$

$$\Delta I = \{ \Delta v(a,b), \Delta v(b,c), \Delta v(c,d), \Delta v(c,e), \Delta v(a,c), \\ \Delta v(b,d), \Delta v(b,e), \Delta v(a,d), \Delta v(a,e) \}$$

$$\Delta' I = \{ \}$$

STOP

$$MM_1 = MM_0 \cup \{ r(a,b), r(b,c), r(c,d), r(c,e), \\ r(a,c), r(b,d), r(b,e), r(a,d), r(a,e) \\ s(a,b), s(b,c), s(c,d), s(c,e), \\ s(a,c), s(b,d), s(b,e), s(a,d), s(a,e) \\ v(a,b), v(b,c), v(c,d), v(c,e), v(a,c), \\ v(b,d), v(b,e), v(a,d), v(a,e) \}$$

P(S₁)

$t(x,y) :- r(x,y), \text{not } s(x,y).$

}

MM₂ = MM₁ ∪ { }

P(S₂) = { }

w(x,y) :- v(x,y), not t(x,y).

1. $\text{WZ} \rightarrow$

$\text{W}(x,y) := V(x,y), \text{not } T(x,y).$

\downarrow

$\text{HM}_3 = \text{HM}_2 \cup \{ W(a,b), W(b,c), W(c,d), W(c,e),$
 $W(a,c), W(b,d), W(b,e), W(a,d),$
 $W(a,e) \}$

$\text{HM}(P) = \{ P(a,b), P(b,c), P(c,d), P(c,e),$
 $V(a,b), V(b,c), V(c,d), V(c,e)$
 $V(a,c), V(b,d), V(b,e), V(a,d), V(a,e)$
 $S(a,b), S(b,c), S(c,d), S(c,e),$
 $S(a,c), S(b,d), S(b,e), S(a,d), S(a,e)$
 $V(a,b), V(b,c), V(c,d), V(c,e), V(a,c),$
 $V(b,d), V(b,e), V(a,d), V(a,e)$
 $W(a,b), W(b,c), W(c,d), W(c,e),$
 $W(a,c), W(b,d), W(b,e), W(a,d),$
 $W(a,e) \}$