17/06/2016 EX1

martedì 5 gennaio 2021 13:50

Exercise 1 Given the following $\mathcal{ALC}$ TBox:

$$\begin{array}{rcl} C & \sqsubseteq & D \\ D \sqcap E & \sqsubseteq & \exists R.C \\ E & \sqsubseteq & F \\ D & \sqsubseteq & \forall R.\neg C \end{array}$$

(a) tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;

(b) tell whether the concept C is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where the interpretation of C is non-empty;

(c) given the ABox $\mathcal{A} = \{C \sqcap E(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable (consistent), explaining your answer;

(d) given the ABox $\mathcal{A} = \{E(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable (consistent), explaining your answer.

a)

Delta^I = {a}
C^I=D^I=E^I=F^I=r^I= empty set

I is model for the T and so, T is satisfiable

b)

Delta^I' = {a}
C^I'={a}
D^I'={a}
E^I'=F^I=empty set
r^I= empty set
(forall r. not C)^I' = {a}

I' is a model for T and C is satisfiable because C^I' is non empty

c)
A = {(C and E)(a)}
(and-rule) A0 = A union {C(a),E(a)}
C_GCI = (not C or D) and (not (D and E) or Exists r. C) and (not E or F) and (not D or Exists r. not C)
NNF (not D(and E)) -> (not D or not E)
(GCI-rule) A1=A0 union {(not C or D) and (not D or not E or Exists r. C) and (not E or F) and (not D or Forall r. not C) (a)}
(and-rule) A2 = A1 union {(not C or D)(a),(not D or not E or Exists r. C)(a),(not E or F) (a)(not D or Forall r. not C)(a)}
(or-rule) A3 = A2 union {not C(a)} - CLASH
A4 = A2 union {D(a)}
(or-rule) A5 = A4 union {not D(a)}-CLASH
A6 = A4 union {not E(a)} - CLASH
A7 = A4 union {(Exists r. C)(a)}
(Exist-rule) A8 = A7 union {r(a,x), C(x)}
(GCI-rule) A9=A8 union {(not C or D) and (not D or not E or Exists r. C) and (not E or F) and (not D or Forall r. not C) (x)}

(and-rule) A10 = A9 union {(not C or D)(x),(not D or not E or Exists r. C)(x),(not E or F) (x)(not D or Forall r. not C)(x)}
(or-rule) A11 = A10 union {not D(a)} - CLASH
A12 = A10 union {(Forall r. not C)(a)}
(Forall-rule) A13 = A12 union {not C(x)} - CLASH

Tableau return false, this means that our KB is inconsistent, so unsatisfiable

d)

A0 = {E(a)}
C_GCI = (not C or D) and (not (D and E) or Exists r. C) and (not E or F) and (not D or Exists r. not C)
NNF (not D(and E)) -> (not D or not E)
(GCI-rule) A1=A0 union {(not C or D) and (not D or not E or Exists r. C) and (not E or F) and (not D or Forall r. not C) (a)}
(and-rule) A2 = A1 union {(not C or D)(a),(not D or not E or Exists r. C)(a),(not E or F) (a)(not D or Forall r. not C)(a)}
(or-rule) A3 = A2 union {not E(a)} - CLASH
A4 = A2 union {not D(a)}
A5 = A2 union {(Exists r. C)(a)}
(or-rule) A6 = A4 union{D(a)} - CLASH
A7 = A4 union {not C(a)}
(or-rule) A8 = A7 union {not E(a)} - CLASH
A9 = A7 union {F(a)} - open and complete

Tableau has a Abox that it is open and complete so our KB is consistent