EX3 DL

giovedì 7 gennaio 2021 09:54

Exercise 3 Given the knowledge base (KB) $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ is the following TBox:

$$\begin{array}{rcl} A & \sqsubseteq & B \sqcup C \\ B & \sqsubseteq & \exists r.D \\ C & \sqsubseteq & \exists r.E \\ A & \sqsubseteq & \forall r.F \\ D \sqcap F & \sqsubseteq & G \end{array}$$

and $\mathcal{A}$ is the following ABox:

$$A(a)$$

1. using the tableau method, tell whether the concept assertion $\exists r.(E \sqcap G)(a)$ is entailed by $\mathcal{K}$;
2. using the tableau method, tell whether the concept assertion $(\exists r.E) \sqcap (\exists r.G)(a)$ is entailed by $\mathcal{K}$.

1)

NNF: not (Exist r. (E and G))(a) - > (Forall r. not(E and G))(a) - > (Forall r. not E or not G)(a)

A0 = {A(a), (Forall r. not E or not G)(a)}
C_GCI = (not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not (D and F) or G)

NNF: not(D and F) -> not D or not F

C_GCI = (not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not D or not F or G)
(C_GCI-rule) A1 = A0 union {((not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not D or not F or G))(a)}
(and-rule) A2 = A1 union {(not A or B or C)(a), (not B or Exist r. D)(a), (not C or Exists r. E) (a), (not A or Forall r. F) (a), (not D or not F or G)(a)}
(or-rule) A3 = A2 union {not A(a)} - CLASH
A4 = A2 union {(Forall r. F)(a)}
(or-rule) A5 = A4 union {not A(a)} - CLASH
A6 = A4 union {B(a)}
A7 = A4 union {C(a)}
(or-rule) A8 = A6 union {not B(a)} - CLASH
A9 = A6 union {(Exists r. D)(a)}
(or-rule) A10 = A9 union {not C(a)}
A11 = A9 union {(Exists r. E)(a)}
(or-rule) A12 = A10 union{not D(a)}
A13 = A10 union {not F(a)}
A14 = A10 union {G(a)}
(Exist-rule) A15 = A12 union {D(x), r(a,x)}
(C_GCI-rule) A16 = A15 union {((not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not D or not F or G))(x)}
(and-rule) A17 = A16 union {(not A or B or C)(x), (not B or Exist r. D)(x), (not C or Exists r. E) (x), (not A or Forall r. F) (x), (not D or not F or G)(x)}
(Forall-rule) A18 = A17 union {F(x)}

(or-rule) A19 = A18 union {not D(x)} - CLASH
A20 = A18 union {not F(x)} - CLASH
A21 = A18 union {G(x)}
(Forall-rule) A22 = A21 union {not G(x)} - CLASH
A23 = A21 union {not E(x)}
(or-rule) A24 = A23 union {not A(x)}
A25 = A23 union {(Forall r. F)(x)
(or-rule) A26 = A24 union {not B(x)}
A27 = A24 union {(Exists r. D)(x)}
(or-rule) A28 = A26 union {not C(x)} - open and complete
A29 = A26 union {(Exists r. E)(x)}

Tableau return true, the instance checking problem is false

2)

NNF: not ((Exists r. E) and (Exists r. G))(a) - > (not (Exists r. E) or not (Exists r. G))(a) ->
(Forall r. not E or Forall r. not G)(a)

A0 = {A(a), (Forall r. not E or Forall r. not G)(a)}
C_GCI = (not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not (D and F) or G)

NNF: not(D and F) -> not D or not F

C_GCI = (not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not D or not F or G)
(C_GCI-rule) A1 = A0 union {((not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not D or not F or G))(a)}
(and-rule) A2 = A1 union {(not A or B or C)(a), (not B or Exist r. D)(a), (not C or Exists r. E) (a), (not A or Forall r. F) (a), (not D or not F or G)(a)}
(or-rule) A3 = A2 union {not A(a)} - CLASH
A4 = A2 union {B(a)}
A5 = A2 union {C(a)}
(or-rule) A6 = A4 union {not B(a)} - CLASH
A7 = A4 union {(Exist r. D)(a)}
(or-rule) A8 = A7 union {not C(a)}
A9 = A7 union {(Exists r. E)(a)}
(or-rule) A10 = A8 union {not A(a)} - CLASH
A11 = A8 union {(Forall r. F)(a)}
(or-rule) A12 = A11 union {not D(a)}
A13 = A11 union {not F(a)}
A14 = A11 union {G(a)}
(Exist -rule) A15 = A12 union {D(x), r(a,x)}

(C_GCI-rule) A16 = A15 union {((not A or B or C) and (not B or Exist r. D) and (not C or Exists r. E) and (not A or Forall r. F) and (not D or not F or G))(x)}
(and-rule) A17 = A16 union {(not A or B or C)(x), (not B or Exist r. D)(x), (not C or Exists r. E) (x), (not A or Forall r. F) (x), (not D or not F or G)(x)}
(Forall -rule) A18 = A17 union {F(x)}
(or-rule) A19 = A18 union {not D(x)} - CLASH
A20 = A18 union {not F(x)} - CLASH
A21 = A18 union {G(x)}
(or-rule) A22 = A18 union {(Forall r. not G)(a)}
A23 = A18 union {(Forall r not E)(a)}
(Forall-rule) A24 = A22 union {not G(x)} -CLASH
A25 = A23 union {not E(x)}
(or-rule) A26 = A25 union {not A(x)}
A27 = A25 union {(Forall r. F)(x)}
(or-rule) A28 = A26 union {not B(x)}
A29 = A26 union {(Exists r. D)(x)}
(or-rule) A30 = A28 union {not C(x)} - open and complete
A31 = A28 union {(Exists r. E)(x)}

Tableau return true, so the instance checking problem is false