Datalog and Answer Set Programming (part 6)

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Adding disjunction to Datalog

Disjunctive rules

A **disjunctive** rule is an expression of the form

$$\alpha_1 \vee \ldots \vee \alpha_k :\!- \beta_1, \ldots, \beta_n, not\ \gamma_1, \ldots, not\ \gamma_m.$$

where:

- n,m,k are non-negative integers
- every α_i, β_i, γ_i is an atom
- (**safeness** condition) every variable symbol occurring in the rule must appear in at least one of the positive atoms of the rule body $\beta_1, \ldots, \beta_n$

The rule is called **positive** if m=0, is called **non-disjunctive** if k=1, and is called **constraint** if k=0.

A **disjunctive program** (also called **Answer Set Program**) is a set of disjunctive rules.

Semantics of disjunctive rules

A ground disjunctive rule $\alpha_1 \vee \ldots \vee \alpha_k$:– $\beta_1, \ldots, \beta_n, not\ \gamma_1, \ldots, not\ \gamma_m$. is **satisfied** in an interpretation I if at least one of the following conditions holds:

- at least one of the atoms $\beta_1, \ldots, \beta_n$ does not belong to I
- at least one of the atoms $\gamma_1, \ldots, \gamma_m$ belongs to I
- at least one of the atoms $\alpha_1, \ldots, \alpha_k$ belongs to I

Example:

the rule $q(a) \vee q(b)$:– $p(b,a)$.

- is satisfied in the interpretations { }, $\{q(a)\}$, $\{q(b)\}$, $\{q(a), q(b)\}$, $\{p(b,a), q(a)\}$, $\{p(b,a), q(b)\}$, $\{p(b,a), q(a), q(b)\}$
- is not satisfied in the interpretation $\{p(b,a)\}$

Answer Set Semantics

(same notion of reduct and answer set as the ones given for non-disjunctive programs)

Let P be a disjunctive program and let I be an interpretation. The **reduct** of P with respect to I is the positive disjunctive ground program obtained as follows:

Delete from ground(P) every rule R such that an atom not β occurs in the body of R and β belongs to I;

Delete from every rule R in ground(P) every atom not β occurring in the body of R such that β does not belong to I.

Let P be a disjunctive program. An interpretation I is an **answer set** of P if I is a minimal model of the positive disjunctive program P/I.

Reduct of a disjunctive program: Example

Example: let P be the program

$r(a) \lor s(a) :\!- \, p(a), not \; q(a).$

$s(a) \lor t(a) :\!- \, p(a), not \; r(a).$

$p(a).$

1) Let I_1 be the interpretation $\{p(a)\}$. Then, P/I_1 is the positive disjunctive program

$r(a) \lor s(a) :\!- \, p(a).$

$s(a) \lor t(a) :\!- \, p(a).$

$p(a).$

It is easy to verify that the first two rules of P/I_1 are not satisfied in I_1, therefore I_1 is not a model (and consequently not a minimal model) for P/I_1, so I_1 is not an answer set of P.

Example (contd.)

Example (contd.): let P be the program

$r(a) \vee s(a) \coloneq p(a), not\ q(a).$

$s(a) \vee t(a) \coloneq p(a), not\ r(a).$

$p(a).$

2) Let I_2 be the interpretation $\{p(a), r(a)\}$. Then, P/I_2 is the positive program

$r(a) \vee s(a) \coloneq p(a).$

$p(a).$

It is easy to verify that I_2 is a minimal model of P/I_2, therefore I_2 is an answer set of P.

Example (contd.)

Example (contd.): let P be the program

$r(a) \lor s(a)$:– $p(a), not\ q(a)$.

$s(a) \lor t(a)$:– $p(a), not\ r(a)$.

$p(a)$.

3) Let I_3 be the interpretation $\{p(a), s(a)\}$. Then, P/I_3 is the positive disjunctive program

$r(a) \lor s(a)$:– $p(a)$.

$s(a) \lor t(a)$:– $p(a)$.

$p(a)$.

It is easy to verify that I_3 is a minimal model of P/I_3, therefore I_3 is an answer set of P.

Example (contd.)

Example (contd.): let P be the program
$r(a) \vee s(a)$:– $p(a), not\ q(a)$.
$s(a) \vee t(a)$:– $p(a), not\ r(a)$.
$p(a)$.

4) Let I_4 be the interpretation $\{p(a), r(a), s(a)\}$. Then, P/I_4 is the positive disjunctive program
$r(a) \vee s(a)$:– $p(a)$.
$p(a)$.

It is easy to verify that I_4 is a model of P/I_4, but is **not** a **minimal** model of P/I_4: the minimal models of P/I_4 are $\{p(a), r(a)\}$ and $\{p(a), s(a)\}$. Therefore, I_4 is not an answer set of P.

References

- Serge Abiteboul, Richard Hull, Victor Vianu: Foundations of Databases. Addison-Wesley 1995, ISBN 0-201-53771-0. http://webdam.inria.fr/Alice/
- Vladimir Lifschitz: Answer Set Programming. Springer 2019, ISBN 978-3-030-24657-0.
- Eiter, T., Gottlob, G. and Mannila, H. (2001): Disjunctive Datalog, ACM Transactions on Database Systems 22(3), July 2001
- Nicola Leone, Gerald Pfeifer, Wolfgang Faber, Thomas Eiter, Georg Gottlob, Simona Perri, Francesco Scarcello: The DLV system for knowledge representation and reasoning. ACM Transactions on Computational Logic 7(3): 499-562 (2006)