05/09/2016 EX1

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Exercise 1 Given the following $\mathcal{ALC}$ TBox:

$$\begin{array}{rcl} A & \sqsubseteq & B \\ A \sqcap C & \sqsubseteq & D \\ \forall R.D & \sqsubseteq & \neg E \\ \exists R.\neg D & \sqsubseteq & \neg E \\ D & \sqsubseteq & \neg B \end{array}$$

(a) tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;

(b) tell whether the concept E is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where the interpretation of E is non-empty;

(c) given the ABox $\mathcal{A} = \{A(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ entails the assertion $\exists\neg E(a)$, explaining your answer;

(d) given the ABox $\mathcal{A} = \{A \sqcap C(a)\}$, tell whether the knowledge base $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable (consistent), explaining your answer.

a)

Delta^I = {a}
A^I=C^I=D^I=B^I = E^I= r^I = empty set

All axioms are satisfied, I is a model for T

b)
It is not possible to have E satisfiable, so an interpretation that it is a model of T and E is non empty because if E is non empty also (forall r. not D) and (Exists r. D) should contain the same elements and this is not possible.

c)
A = {A(a)}
NNF: not(Exists r. not E) -> Forall r. E

A0 = {A(a),(Forall r. E)(a)}
C_GCI = (not A or B) and (not (A and C) or D) and (not (Forall r. D) or not E) and (not (Exist r. not D) or not E) and (not D or not B)

NNF:
not (A and C) -> not A or not C
Not (forall r. D) -> Exist r. not D
Not (Exists r. not D) -> Forall r. D

C_GCI = (not A or B) and (not A or not C or D) and (Exist r. not D or not E) and (Forall r. D or not E) and (not D or not B)

(C_GCI-rule) A1 = A0 union {((not A or B) and (not A or not C or D) and (Exist r. not

D or not E) and (Forall r. D or not E) and (not D or not B))(a)}
(and-rule) A2 = A1 union {(not A or B)(a), (not A or not C or D) (a),(Exist r. not D or not E)(a), (Forall r. D or not E)(a), (not D or not B)(a)}
(or-rule) A3 = A2 union {not A(a)} - CLASH
A4 = A2 union {B(a)}
(or-rule) A5 = A4 union {not B(a)} - CLASH
A6 = A4 union {not D(a)}
(or-rule) A7 = A6 union {D(a)} - CLASH
A8 = A6 union {not A(a)} - CLASH
A9 = A6 union {not C(a)}
(or-rule) A10 = A9 union {not E(a)} - CLASH
A11 = A9 union {(Forall r. D)(a)}
(or-rule) A12 = A11 union {not E(a)}-CLASH
A13 = A12 union {(Exist r. not D)(a)}
(Exist-rule) A14 = A13 union {not D(x), r(a,x)}
(C_GCI-rule) A15 = A14 union {((not A or B) and (not A or not C or D) and (Exist r. not D or not E) and (Forall r. D or not E) and (not D or not B))(x)}
(and-rule) A16 = A15 union {(not A or B)(x), (not A or not C or D) (x),(Exist r. not D or not E)(x), (Forall r. D or not E)(x), (not D or not B)(x)}
(Forall-rule) A17 = A16 union {D(x)} - CLASH

All Aboxes are closed, so tableau return false so the instance checking is true

d)

A = {(A and C)(a)}
(and-rule) A0 = {A(a),C(a)}
C_GCI = (not A or B) and (not (A and C) or D) and (not (Forall r. D) or not E) and (not (Exist r. not D) or not E) and (not D or not B)

NNF:
not (A and C) -> not A or not C
Not (forall r. D) -> Exist r. not D
Not (Exists r. not D) -> Forall r. D

C_GCI = (not A or B) and (not A or not C or D) and (Exist r. not D or not E) and (Forall r. D or not E) and (not D or not B)

(C_GCI-rule) A1 = A0 union {((not A or B) and (not A or not C or D) and (Exist r. not D or not E) and (Forall r. D or not E) and (not D or not B))(a)}
(and-rule) A2 = A1 union {(not A or B)(a), (not A or not C or D) (a),(Exist r. not D or not E)(a), (Forall r. D or not E)(a), (not D or not B)(a)}
(or-rule) A3 = A2 union {not A(a)} - CLASH
A4 = A2 union {B(a)}
(or-rule) A5 = A4 union {not B(a)} - CLASH
A6 = A4 union {not D(a)}
(or-rule) A7 = A6 union {not A(a)} - CLASH
A8 = A6 union {not C(a)} - CLASH
A9 = A6 union {D(a)} - CLASH

Tableau return false, KB is unsatisfiable