

# EXAM TEST 13.12.14

## Question 1

1). This is a binary classification task. The target function is  $f: X \rightarrow \{Y, N\}$  with  $X = \{F_N \times N_R \times N_K\}$  and the dataset is  $D = \{(x_i, y_i)\}_{i=1}^5$ .

2). We want to maximize the information gain:  $IG(S, A) = ent(S) - \sum_{v \in V} \frac{|S_v|}{|S|} ent(S_v)$

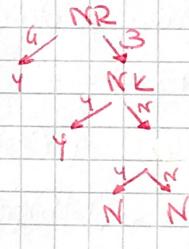
with  $ent(S) = -\sum_{i=1}^n p_i \log_2 p_i$  entropy. The entropy measures the impurity of our information. We define  $V$  like the set of values of  $A$ .

$$3). IG(S, F) = -\frac{3}{5} ent(F_N) - \frac{2}{5} ent(F_Y) + ent(S) = 0.941 - 0.4 - 0.549 = 0.022$$

$$ent(F_N) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.916$$

$$ent(F_Y) = 1$$

$$ent(S) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.941$$



$$IG(S, NR) = ent(S) - \frac{3}{5} ent(NR_3) - \frac{2}{5} ent(NR_4) = 0.941 - 0.549 = 0.422$$

$$ent(NR_4) = 0 \quad ent(NR_3) = 0.916$$

HIGHEST IG!

$$IG(S, NK) = -\frac{4}{5} ent(NK_Y) - \frac{1}{5} ent(NK_N) = 0.941 - 0.8 = 0.141$$

$$ent(NK_N) = 1 \quad ent(NK_Y) = 0$$

$$IG(S, F) = -\frac{2}{3} ent(F_N) - \frac{1}{3} ent(F_Y) + ent(S) = 0.941 - 0.666 = 0.305$$

$$ent(F_N) = 1 \quad ent(F_Y) = 0$$

$$IG(S, NK) = ent(S) - \frac{2}{3} ent(NK_N) - \frac{1}{3} ent(NK_Y) = 0.941$$

$$ent(NK_N) = 0 \quad ent(NK_Y) = 0$$

## Question 2

1). Given a classification problem with a dataset  $\mathcal{D}$  and an hypotheses space  $H$

we are interested in  $P(H|\mathcal{D})$ . We can compute (knowing that  $P(h)$ )  $P(h|\mathcal{D})$ .

$P(h|\mathcal{D}) = \frac{P(\mathcal{D}|h)P(h)}{P(\mathcal{D})}$ . Now we define the Maximum A Posteriori (MAP) hypothesis.

$$h_{MAP} = \operatorname{argmax}_{h \in H} \frac{P(\mathcal{D}|h)P(h)}{P(\mathcal{D})} = \operatorname{argmax}_{h \in H} P(\mathcal{D}|h)P(h)$$

If we assume that  $P(h_i) = P(h_j)$

We can simplify and we obtain the Maximum Likelihood (ML) hypothesis:

$$h_{ML} = \operatorname{argmax}_{h \in H} P(\mathcal{D}|h)$$

2). Is false. Now we do an example:

$$h_1(x) = + \quad h_2(x) = - \quad h_3(x) = -$$

$$P(\mathcal{D}|h_1) = 0.4 \quad P(\mathcal{D}|h_2) = P(\mathcal{D}|h_3) = 0.3$$

$$h_{ML} = \operatorname{argmax}_{h \in H} P(\mathcal{D}|h) = \operatorname{argmax} \{0.4, 0.3, 0.3\} = +$$

Now we introduce the concept of Bayes Optimal Classifier (BOC). BOC is an optimal classifier, in fact returns always the optimal solution.

$$V_{OC} = \operatorname{argmax}_{v \in V} \sum_{h \in H} P(v|x, h)P(h|\mathcal{D})$$

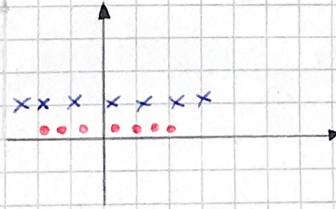
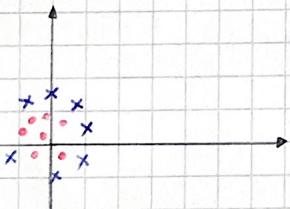
$$P(+|x, h_1) = 1 \quad P(+|x, h_2) = 0 \quad P(+|x, h_3) = 0$$

$$P(-|x, h_1) = 0 \quad P(-|x, h_2) = 1 \quad P(-|x, h_3) = 1$$

$$V_{OC} = \operatorname{argmax}_{v \in V} \{ (1 \cdot 0.4 + 0 + 0), (0 + 1 \cdot 0.3 + 1 \cdot 0.3) \} = -$$

## Question 3

1). In this case i will use a polynomial Kernel function.



2). We introduce some slack variables  $\xi_n \geq 0 \quad \forall n=1\dots N$

$$\text{The modified error function becomes: } E(w) = \operatorname{argmin}_w \frac{1}{2} \|w\|^2 + C \sum_{n=1}^N \xi_n$$

And we modify also:  $y(x_n) \geq 1 - \xi_n \quad \forall n=1\dots N$

### Question 4

1). This is a regression task, a suitable activation function can be the identity activation function  $y = W^T h + b$

2). Using this activation function outputs units don't saturate so the learning step is not affected.

### Question 5

1).  $W_{out} = \frac{W_{in} - W_k + 2p + 1}{S} \rightarrow W_{out} = \frac{3 - W_k + 2p + 1}{S}$

We choose stride = 1, padding = 1 and a 3x3 kernel.

2). tanh, ReLU, Sigmoid

### Question 6

