Medical Robotics

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Stability and Transparency in Bilateral Teleoperation

FROM SLIDE 23

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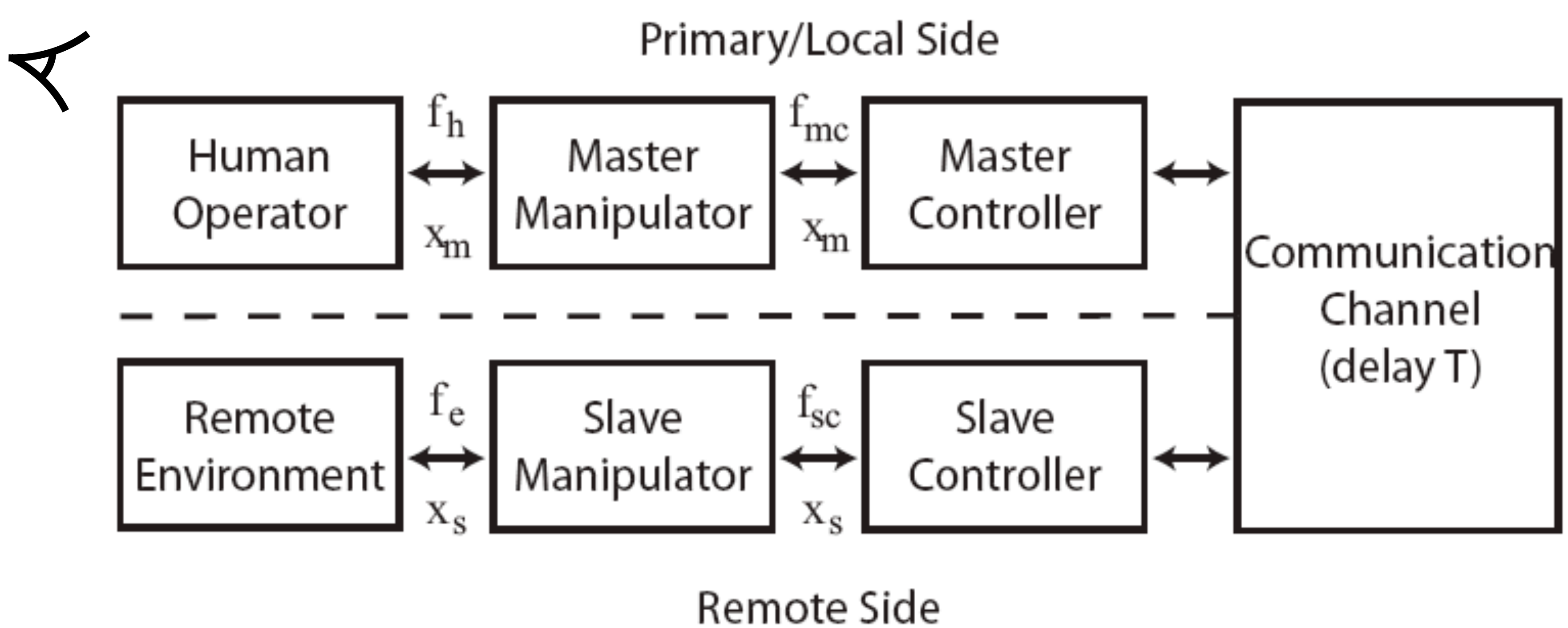
in principle, any control methodology can be applied but a standard approach to the solution does not exist and the definition of a performance criterion by means of which different control schemes can be compared is not clear

the various control schemes differ in

- the computation of the forces applied to the two manipulators: admittance/impedance relationships for master/slave are possible in various combinations although impedance based schemes are more popular because they do not necessarily require force sensors
- the information exchanged between master and slave
- sensors used in the control scheme
- the computational load
- required bandwidth

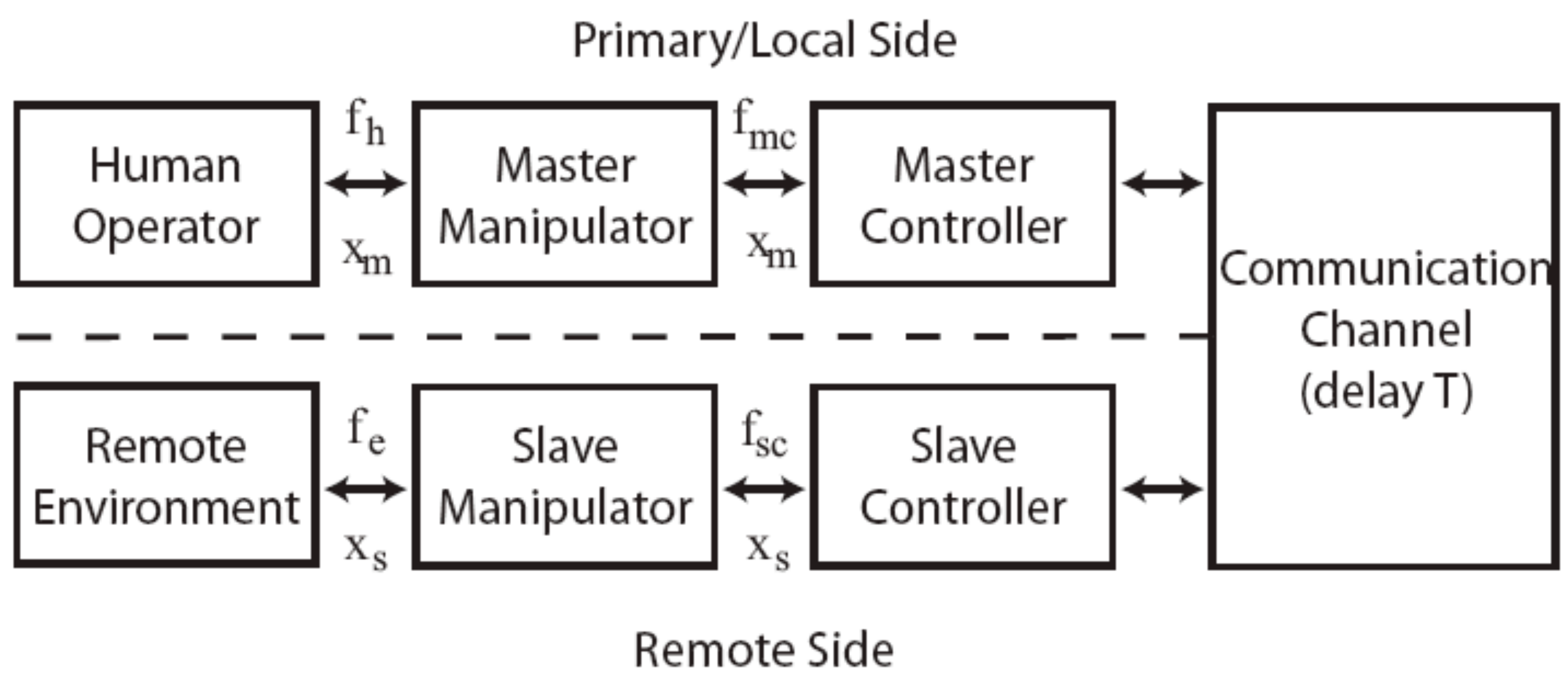


objectives

- **stability**: bounded value of the state (i.e., positions, velocities, internal variables of the local controllers) in response to bounded external inputs (i.e., forces/torques applied by the operator and the environment); delays in communication and controller parameters determine stability properties

- **transparency**: the operator should perceive a direct physical interaction with the remote environment during task execution
  - inertia and damping perceived at the master side by the human operator when no force is exerted on the slave manipulator should be low
  - tracking: at the slave side the master manipulator displacements during movements without interaction should be accurate and “fast”; delays could affect performance
  - stiffness perceived at the master by the operator in case of interaction with a structured environment at the slave should be as the one perceived in the interaction at the slave side
  - drift of position between master and slave in case of interaction at the slave side should be zero

bilateral control components



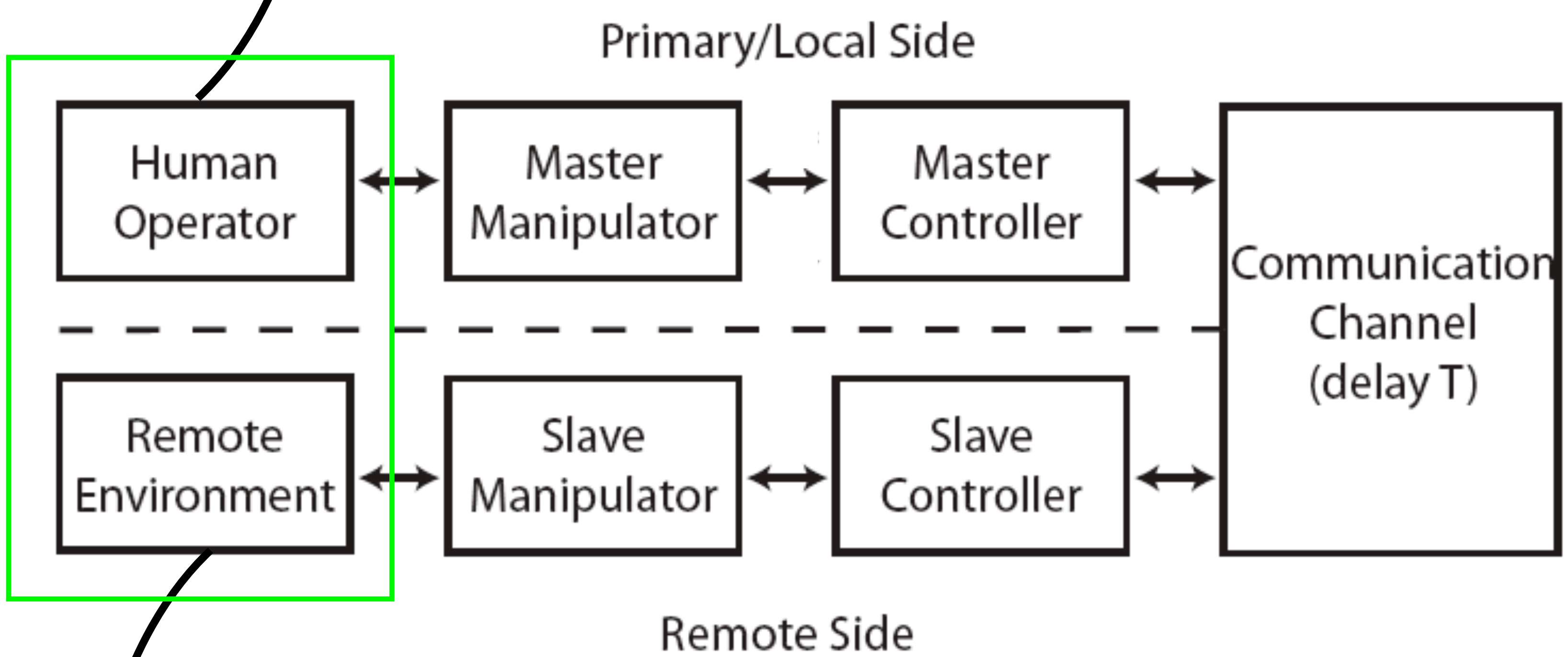
$f_{h/e}$ interaction forces with the human/environment; $f_{mc/sc}$ control force human intentional force

$$F_h = F_h^* - Z_h \dot{x}_h$$

human-master interaction force

human impedance

$$M_h \ddot{x}_h + B_h \dot{x}_h + K_h x_h = F_h^* - F_h \qquad x_h = x_m$$



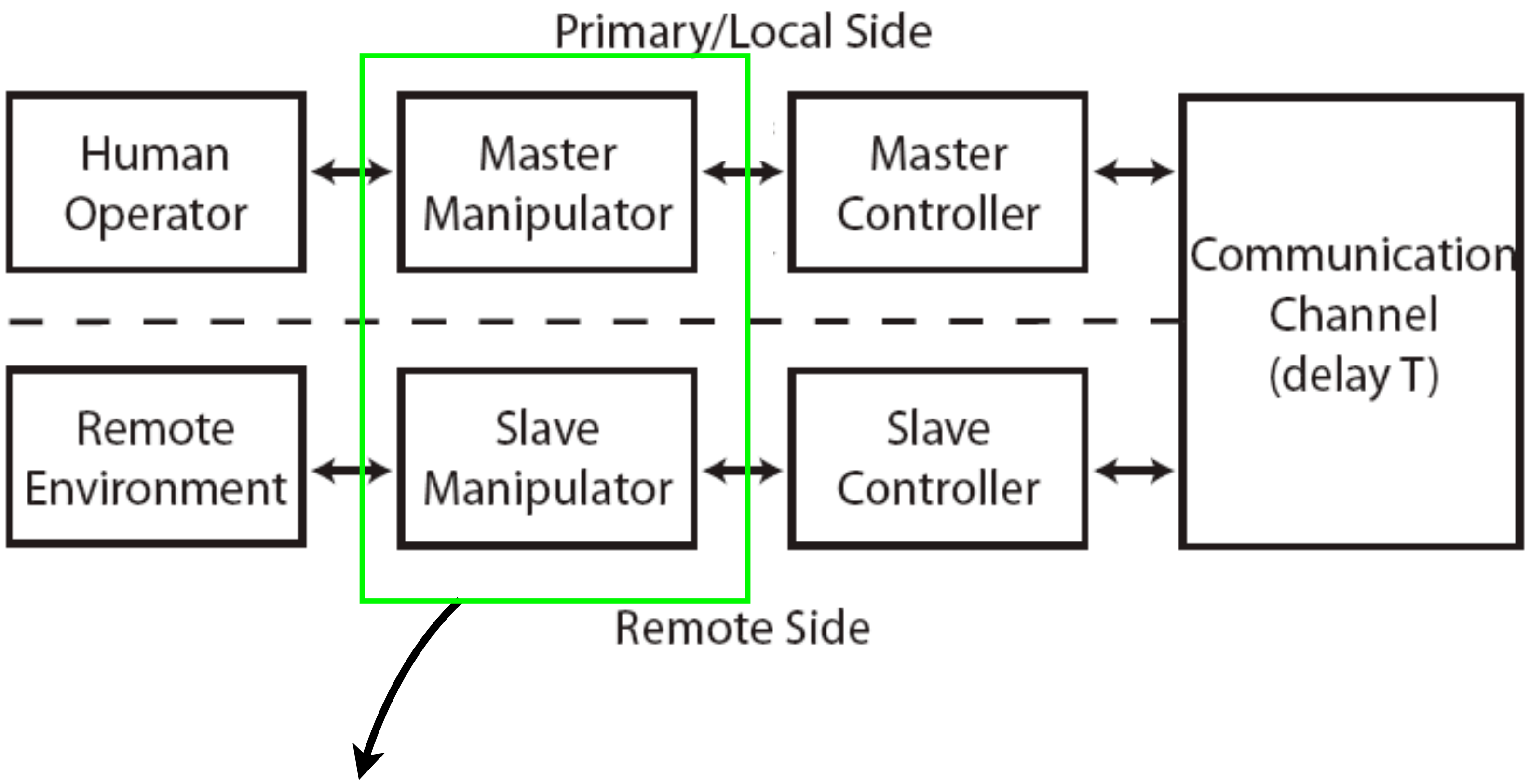
$$F_e = \begin{cases} M_e \ddot{x}_e + B_e \dot{x}_e + K_e (x_e - \bar{x}_e) + F_e^* & \text{contact} \quad x_e = x_s \\ 0 & \text{free motion} \end{cases}$$

environment reaction force

$$\Downarrow$$

$$F_e = Z_e \dot{x}_e + F_e^*$$

exogenous force (usually considered =0)



$$\begin{cases} M(q)\ddot{q} + C(q,\dot{q}) + F(q,\dot{q}) + G(q) = u & \text{free motion} \\ M(q)\ddot{q} + C(q,\dot{q}) + F(q,\dot{q}) + G(q) = u + J^T(q) f & \text{contact} \end{cases}$$

geometric Jacobian

in Cartesian coordinates

(necessary to couple kinematically different master and slave)

$$\Lambda(x)\ddot{x} + \Xi(x,\dot{x})\dot{x} + \Phi(x,\dot{x}) + \gamma(x) = J_a^{-T}(q)u + f_a$$

$$f_a = T_a^{-1}(\phi) f, \quad J_a(q) = T_a(\phi) J(q)$$

analytic Jacobian

feedback linearization in Cartesian space

$$u = J_a^T(q)[\Lambda(x)v + \Xi(x,\dot{x})\dot{x} + \Phi(x,\dot{x}) + \gamma(x)]$$

set a dynamic impedance model

$$v = f_c - M^{-1}(B\dot{x} + Kx) \Rightarrow M\ddot{x} + B\dot{x} + Kx = f_c + f_{ext}$$

⇒ master and slave robot linear single axis approximation

$$M_m\ddot{x}_m + B_m\dot{x}_m + K_m x_m = F_{mc} + F_h$$

$$M_s\ddot{x}_s + B_s\dot{x}_s + K_s x_s = F_{sc} - F_e$$

note: during contact $x_h = x_m$ and $x_e = x_s$

in the frequency domain

IMPEDANCE: ratio between effort (i.e. force) and flow (i.e. velocity)

ADMITTANCE: ratio between flow (i.e. velocity) and effort (i.e. force)

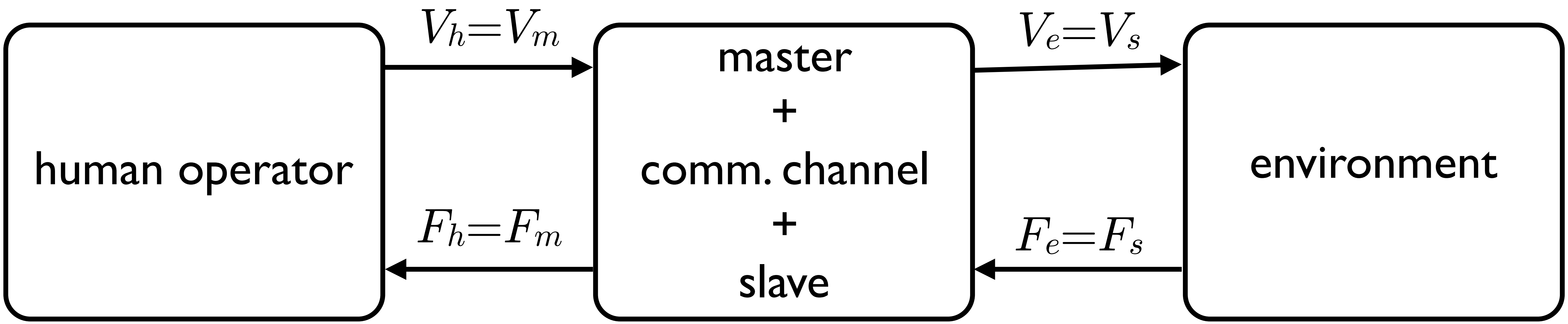
Human $$Z_h(s) = \frac{F_h^\star(s) - F_h(s)}{sX_h(s)} = \frac{m_h s^2 + b_h s + k_h}{s}$$

Master robot $$Z_m^{-1}(s) = \frac{sX_m(s)}{F_{mc}(s) + F_h(s)} = \frac{s}{m_m s^2 + b_m s + k_m}$$

Slave robot $$Z_s^{-1}(s) = \frac{sX_s(s)}{F_{sc}(s) - F_e(s)} = \frac{s}{m_s s^2 + b_s s + k_s}$$

Environment $$Z_e(s) = \frac{F_e(s) - F_e^\star(s)}{sX_e(s)} = \begin{cases} \frac{m_e s^2 + b_e s + k_e}{s}, & \text{contact} \\ 0, & \text{free motion} \end{cases}$$

a two-port model of teleoperator



- **lumped parameter elements**: physical entities who's energy (storage elements) or power (dissipative elements) is defined by a scalar (e.g., mass, resistor)
- **network**: a system described by lumped parameter elements connected in series and parallel (e.g., RLC circuit)
- **port**: a location where energy can move into or out of a network (e.g., contact point between human and haptic interface)

- analysis of energy flow in a network provides key insights

- the derivative of energy is power and can be expressed as the product of two variables: **effort** $\mathcal{E}$ and **flow** $\mathcal{F}$

- effort and flow are linked together by the behavior of systems at their ports through

  - impedance $Z = \mathcal{E}/\mathcal{F}$

  - admittance $A = \mathcal{F}/\mathcal{E} = 1/Z$

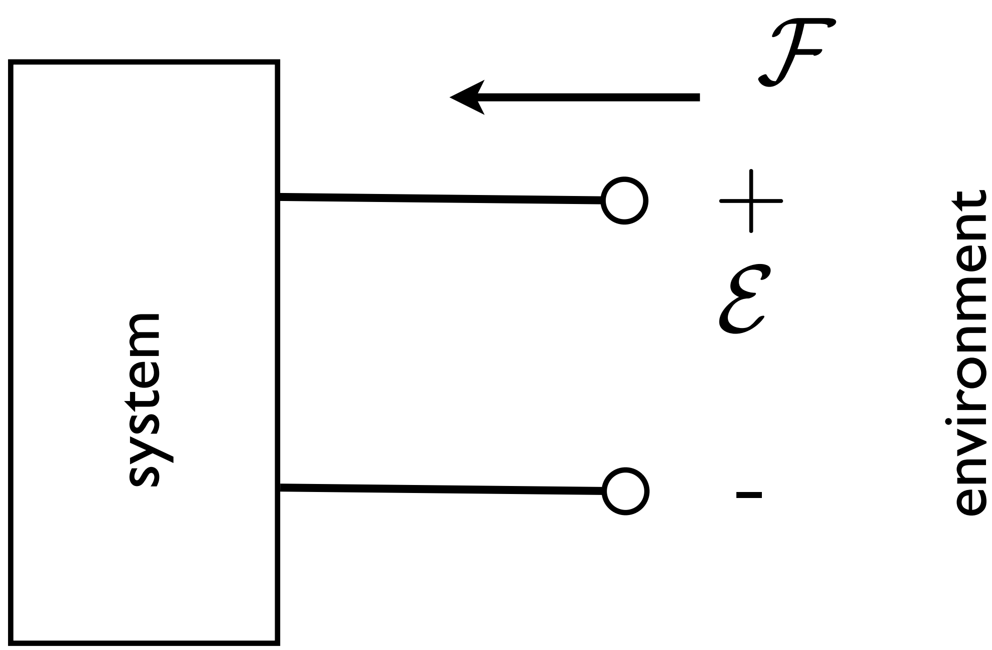
LTI continous systems can be described by the relationships between effort and flow variables

	effort variable	flow variable
mechanical system	force/torque applied to the system	linear/angular velocity of the system
electrical system	voltage across the terminals	current through the network

and similarities between variables

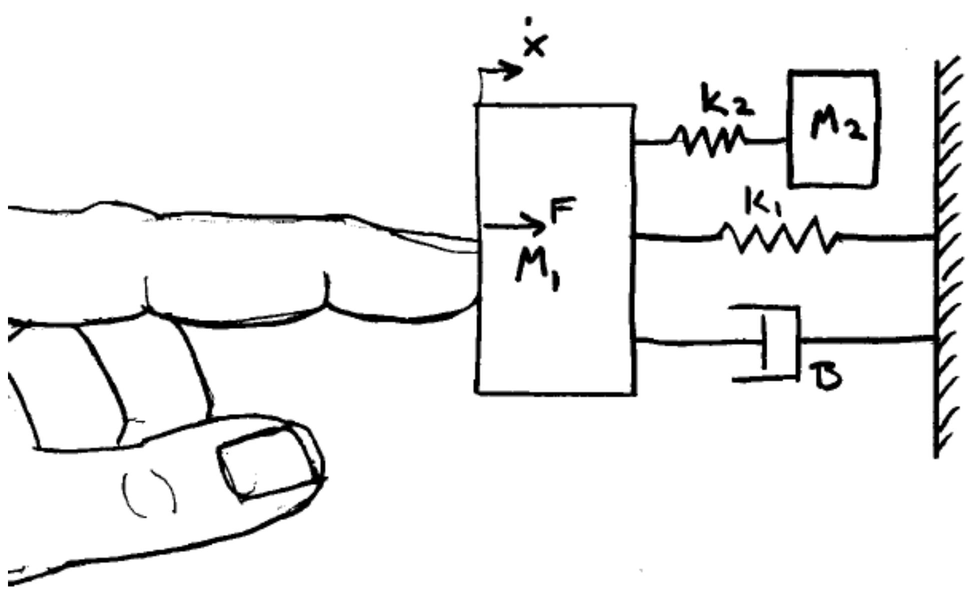
electrical			mechanical	
voltage	$v(t)$		force	$f(t)$
current	$i(t)$		velocity	$V(t)$
resistance	R	$\simeq$	viscous friction	b
inductance	L		inertia	M
capacitance	$1/C$		stiffness	K
one-port impedance	Z		series/parallel of previous elements	$f(s)=Z(s)V(s)$

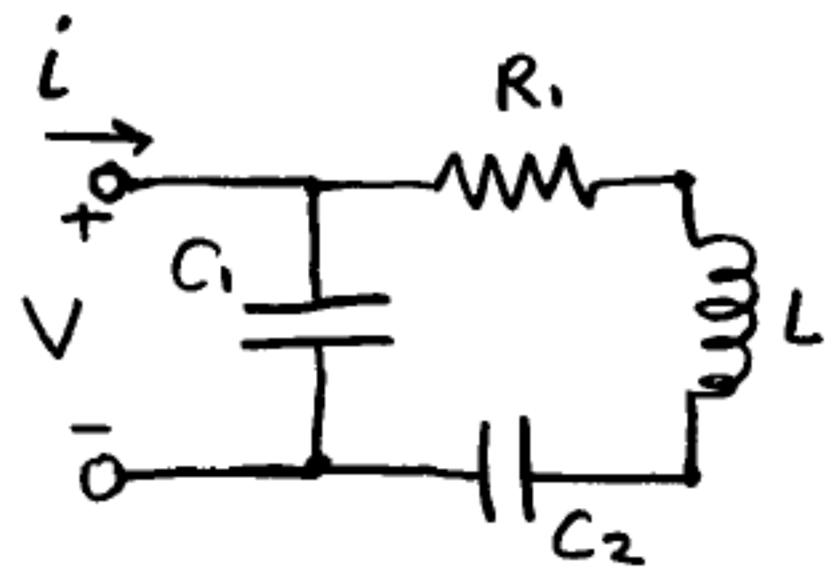
one-port network



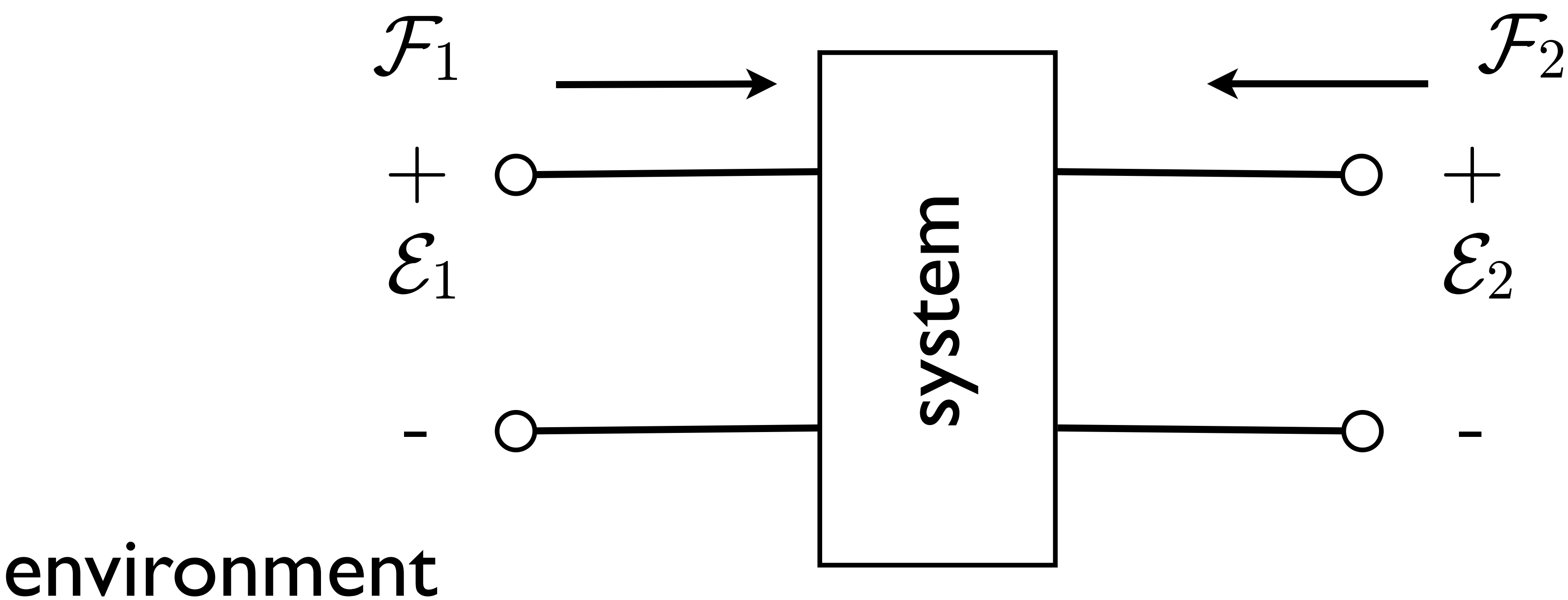
- effort and flow define positive or negative power going into the network at a single location where energy exchange takes place
- signs of effort and flow are usually defined so that power is positive flowing into the port (effort of interest: *on* the port)

- examples of same convention: mechanical (MSD system), electrical (RCL)

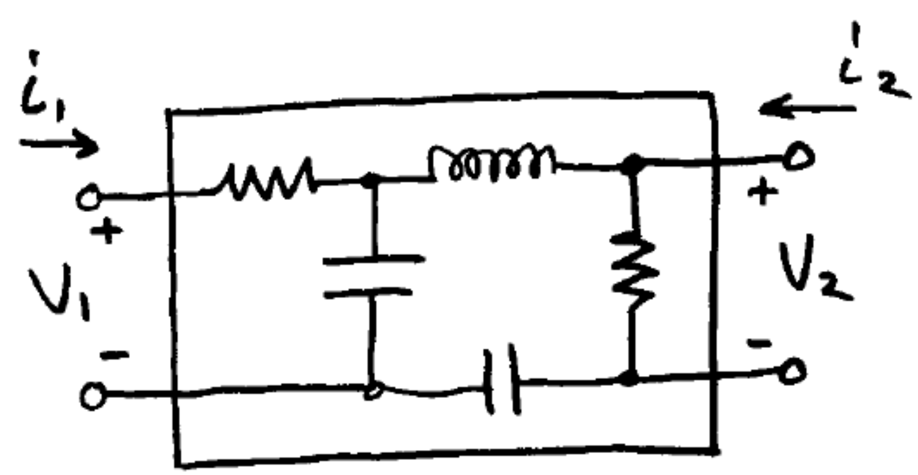




two-port network

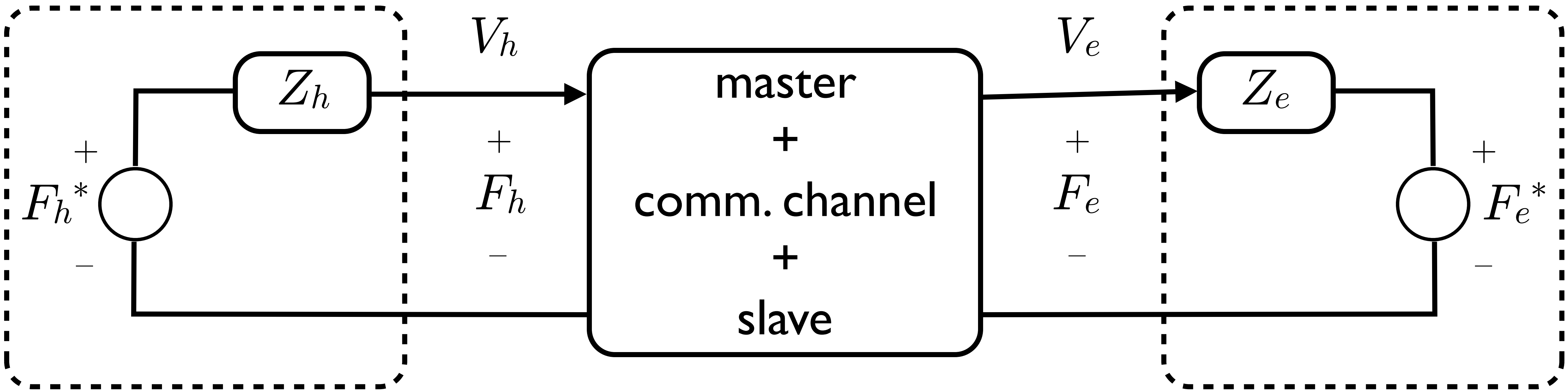


- two-ports have separate effort and flow variables, defined for each port, and a separate coordinate system (sign convention) for each port
- example of same convention: electric network



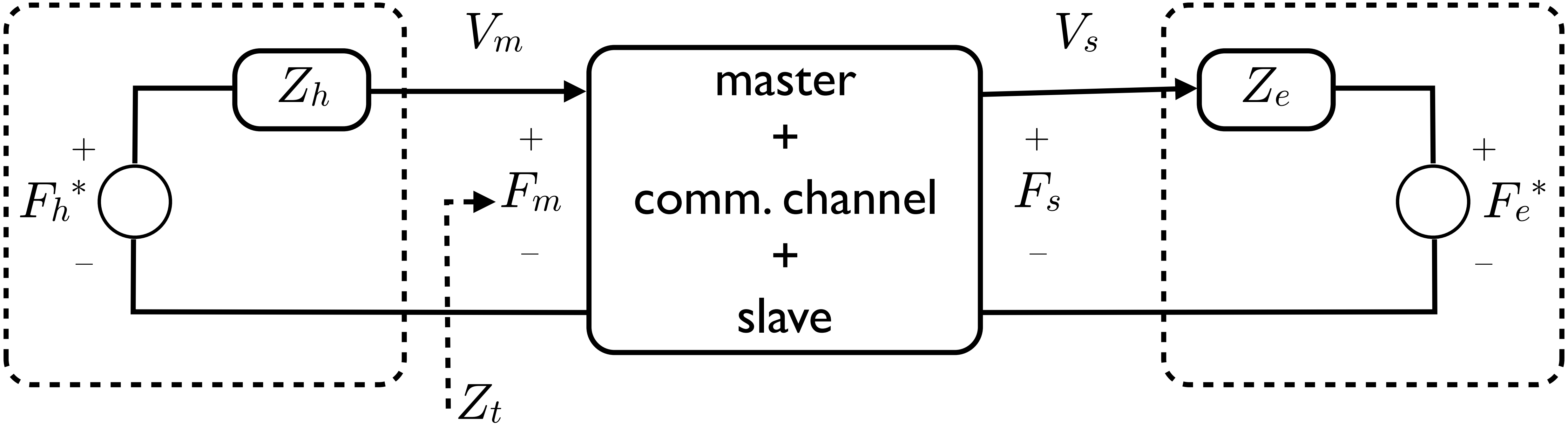
two-port model of teleoperator

- bilateral teleoperation systems can be viewed as a cascade interconnection of two-port (master, communication channel and slave) and one-port (operator and environment) blocks
- using mechanical/electrical analogy and network theory, the teleoperation system is described as interconnection of one and two-port electrical elements



F_h^*, F_e^* exogenous force inputs generated by the operator and the environment respectively; here we assume $F_e^*=0$ (passive environment)

two-port model of teleoperator



Z_t the transmitted impedance (i.e., seen by the human)

$$F_m = Z_t V_m \qquad F_s = Z_e V_s$$

- in the two-port model the behavior of the system is completely characterized by measurements of the forces and velocities at the two ports

$$F_s = Z_e V_s \quad F_m = Z_t V_m$$

- of these four involved variables, two may be chosen as independent and the remaining two dependent
- dependent variables are related to independent ones through the
  - impedance $(F_m, F_s, t) = \boldsymbol{Z}(V_m, V_s, t)$
    - elements of Z can be found through experiments
  - admittance $(V_m, V_s, t) = \boldsymbol{Y}(F_m, F_s, t)$
  - hybrid $(F_m, V_s, t) = \boldsymbol{H}(V_m, F_s, t)$ or $(F_m, V_m, t) = \boldsymbol{H}(V_s, F_s, t)$
  - scattering $(\boldsymbol{F}\text{-}b\boldsymbol{V}, t) = \boldsymbol{S}(\boldsymbol{F}+b\boldsymbol{V}, t)$

operators (transfer matrices for LTI systems)

transparency

for the same forces $F_s = F_m$ we want the same motion $V_s = V_m$

$$\Downarrow$$

transparency condition $Z_t = Z_e$

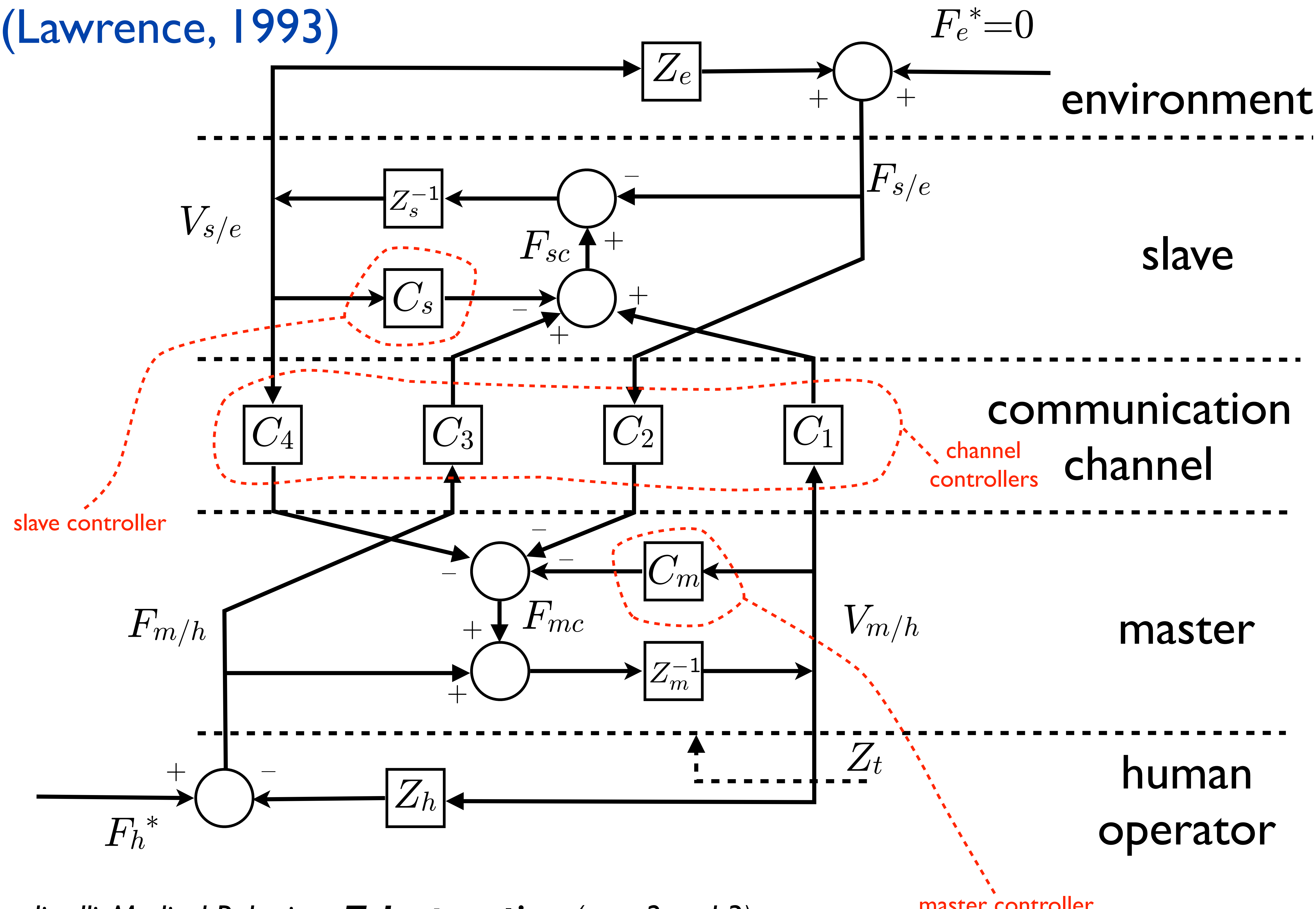
- what degree of transparency is possible?
- what are suitable teleoperator architecture and control laws for achieving necessary or optimal transparency?

for the analysis consider the linearized behavior around contact-operating point using the hybrid matrix formulation (Lawrence)

$$\begin{bmatrix} F_m(s) \\ V_m(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{bmatrix} V_s(s) \\ -F_s(s) \end{bmatrix}$$

$$\boxed{F_m = (H_{11} - H_{12} Z_e)\,(H_{21} - H_{22} Z_e)^{-1} V_m}$$

a general 4-channel architecture
(Lawrence, 1993)

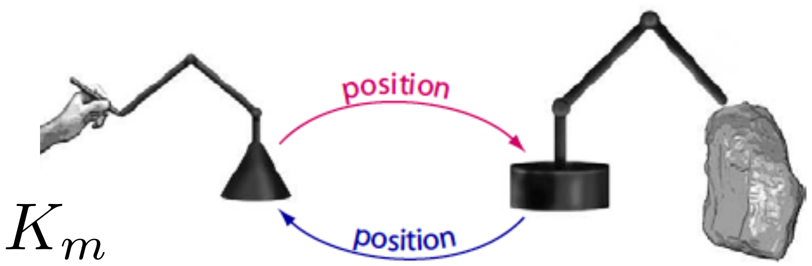


blocks of the 4-channel architecture for position-position and position-force control schemes

block	position-position	position-force
master impedance Z_m	$M_m s$	$M_m s$
master controller C_m	$B_m + K_m/s$	B_m
slave impedance Z_s	$M_s s$	$M_s s$
slave controller C_s	$B_s + K_s/s$	$B_s + K_s/s$
velocity channel C_1	$B_s + K_s/s$	$B_s + K_s/s$
force channel C_2	not used	K_f
force channel C_3	not used	not used
velocity channel C_4	-$(B_m + K_m/s)$	not used
operator impedance Z_h	not a function of	control architecture
task impedance Z_e	not a function of	control architecture

Example: position-position vs the 4-channel architecture

master
- impedance $Z_m = M_m s$
- controller $C_m = B_m + \frac{K_m}{s}$



slave
- impedance $Z_s = M_s s$
- controller $C_s = B_s + \frac{K_s}{s}$

velocity channel $C_1 = B_s + K_s/s$

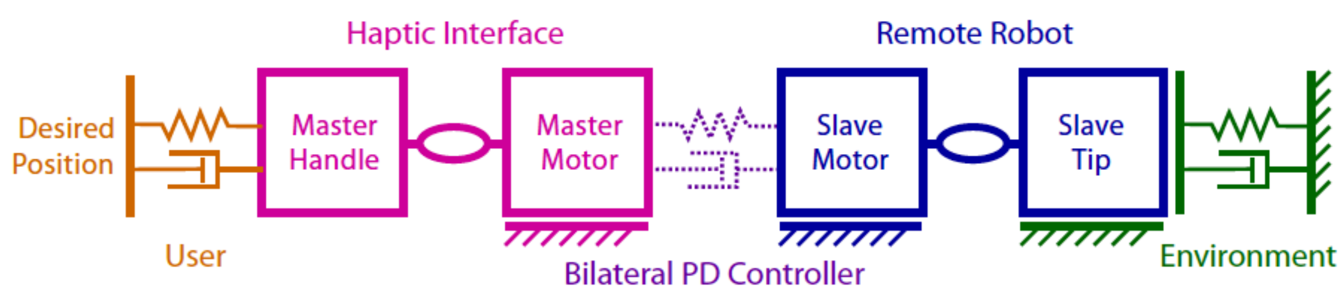
velocity channel $C_4 = -(B_m + K_m/s)$

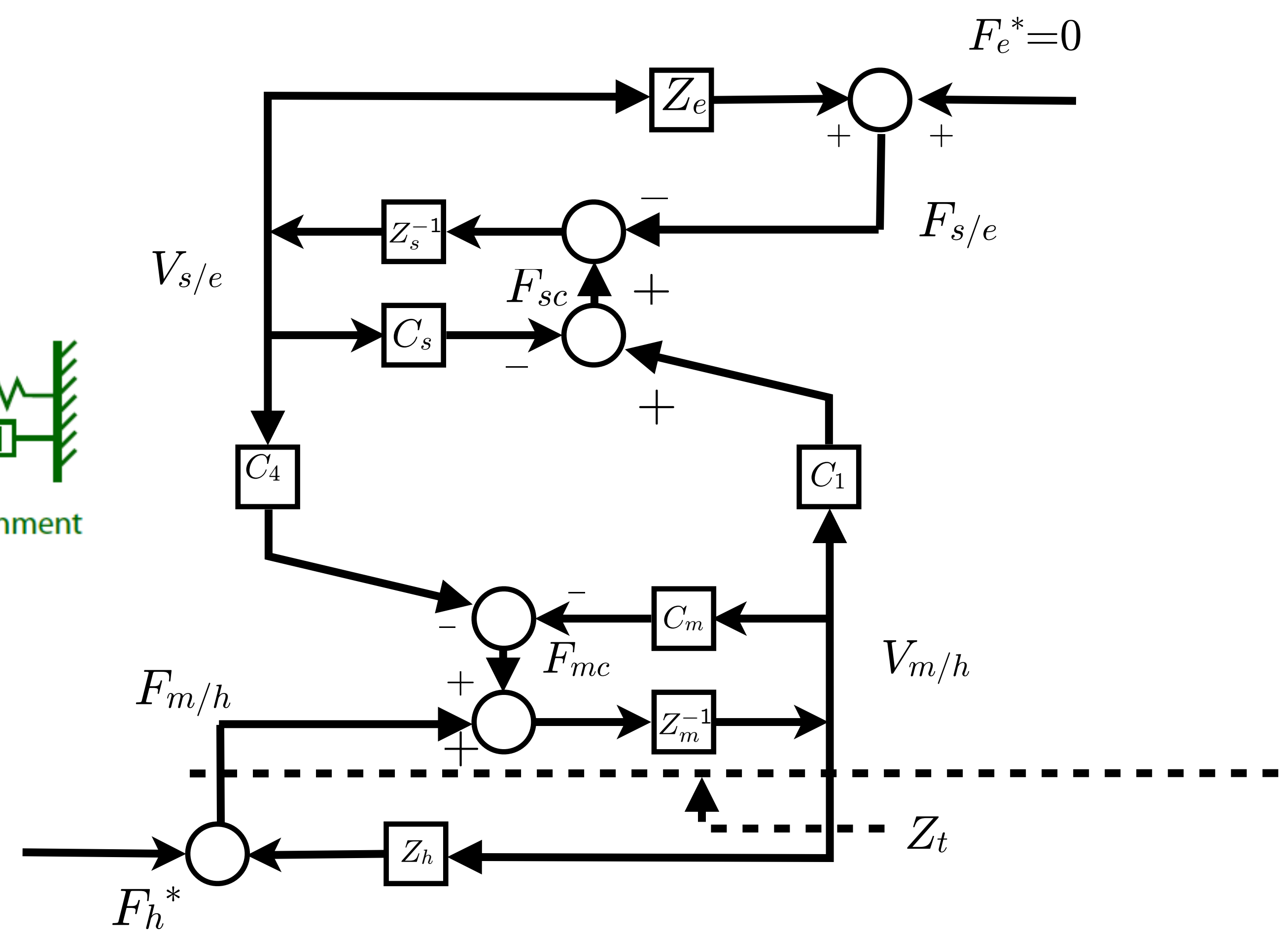
force channels C_2 and C_3 not used

$$\Rightarrow$$

$$F_{mc} = -C_m V_h - C_4 V_e = (B_m + K_m/s)\,(V_e - V_h)$$

$$F_{sc} = -C_s V_e - C_1 V_h = (B_s + K_s/s)\,(V_h - V_e)$$





- solving for the transfer functions between master and slave forces and velocities

$$
\begin{aligned}
H_{11} &= (Z_m + C_m) D (Z_s + C_s - C_3 C_4) + C_4 \\
H_{12} &= -(Z_m + C_m) D (I - C_3 C_2) - C_2 \\
H_{21} &= D (Z_s + C_s - C_3 C_4) - C_2 \\
H_{22} &= -D (I - C_3 C_2) \\
D &= (C_1 + C_3 Z_m + C_3 C_m)^{-1}
\end{aligned}
$$

- these expressions can be used to
    - design suitable control laws $C_i (i = 1, \ldots, 4, m, s)$
    - design suitable master and slave (Z_m, Z_s)
    - compare transparency performance of different teleoperation architectures
    - improve or optimize transparency

optimizing for transparency

- being $Z_t = (H_{11}\text{-}H_{12}Z_e)\ (H_{21}\text{-}H_{22}Z_e)^{-1}$
- perfect transparency ($Z_t{=}Z_e$) can be obtained by choosing C_i $(i{=}1,\ldots,\ 4)$ s.t. $H_{22}{=}0,\ H_{11}{=}0,\ H_{12}{=}I,\ H_{21}{=}\text{-}I$:
- $C_3C_2{=}I, C_4{=}-(Z_m{+}C_m),\ C_1{=}(Z_s{+}C_s),\ C_2{=}I$ ($C_2{\neq}I$ for telefunctioning) ($\Rightarrow$a true 4-channel architecture)
- but $C_4{=}-(Z_m{+}C_m),\ C_1{=}(Z_s{+}C_s)$ require acceleration measurements (see $Z_{m/s}$ and $C_{m/s}$ in slide 19)
- at low frequencies good transparency can be achieved with position and velocity measurements
- in any case, stability is an issue

passivity: a useful tool for stability analysis

- necessary and sufficient conditions for stability are difficult to obtain with a signal-based approach
- an energy-based approach is more suited
- passivity provides sufficient conditions for stability
- transmission delays destroy passivity
- suitable encoding of the transmitted information preserves passivity

definitions

consider the dynamics

the dynamic of the system depends on the input of the system

non linear system
$$\begin{cases} \dot{x} & = & f(x) + g(x)u \\ y & = & h(x) \end{cases}$$

output function

$$x \in R^n, u \in R^m, y \in R^m$$

technical condition chosen to show that the system can be dissipative if there exists a continuous function of the state V(x) which is lower bounded (STORAGE FUNCTION), and a funcion of input-output couples(SUPPLY RATE)

and assume $f(0)=0$ and $h(0)=0$

the system is dissipative if there exist

- a continuous lower bounded function of the state (storage function)

$$V(x) \in \mathcal{C}^1 : R^n \rightarrow R^+$$

- a function of the input/output pair (supply rate)

$$w(u, y) : R^m \times R^m \rightarrow R$$

stored energy

energy supplied to the system

such that (equivalently)
$$\begin{cases} V(x(t)) - V(x(t_0)) & \leq & \int_{t_0}^{t} w(u(s)), y(s))ds \\ \dot{V}(x(t)) & \leq & w(u(t)), y(t)) \end{cases}$$

when the supply rate is the scalar product between the in/out pair

$$w(u, y) =< y, u >= y^T u$$

dissipativity is named passivity and the system is said passive

$V(x(t))$ can be interpreted as the *energy* of the system: physically, a passive system cannot produce energy

passivity condition

$$V(x(t)) \quad \leq \quad V(x(t_0)) + \int_{t_0}^{t} y(s)^T u(s) ds$$

current energy is at most equal to the initial energy + supplied energy from outside

link to Lyapunov stability

- assume $V(0)=0$, i.e., 0 is a (local) minimum of the storage function
- then $V(x)$ is a Lyapunov candidate around 0 and
  - if $u\equiv 0$ then $\dot{V} \leq 0$, i.e., 0 is an unforced (Lyapunov) stable equilibrium (passivity $\Rightarrow$stability of the unforced equilibrium)
  - if $y\equiv 0$ then $\dot{V} \leq 0$, i.e., the zero dynamics of the system is (Lyapunov) stable
  - the system can be stabilized by a static output feedback

$$u = -\phi(y), \qquad y^T \phi(y) < 0 \qquad \forall y \neq 0$$

- for instance $u = -ky, \quad k > 0$

feedback interconnection

- very useful properties to analyze stability of teleoperation systems
- given two passive systems with appropriate in/out dimensions and storage functions $V_1(x_1)$ and $V_2(x_2)$

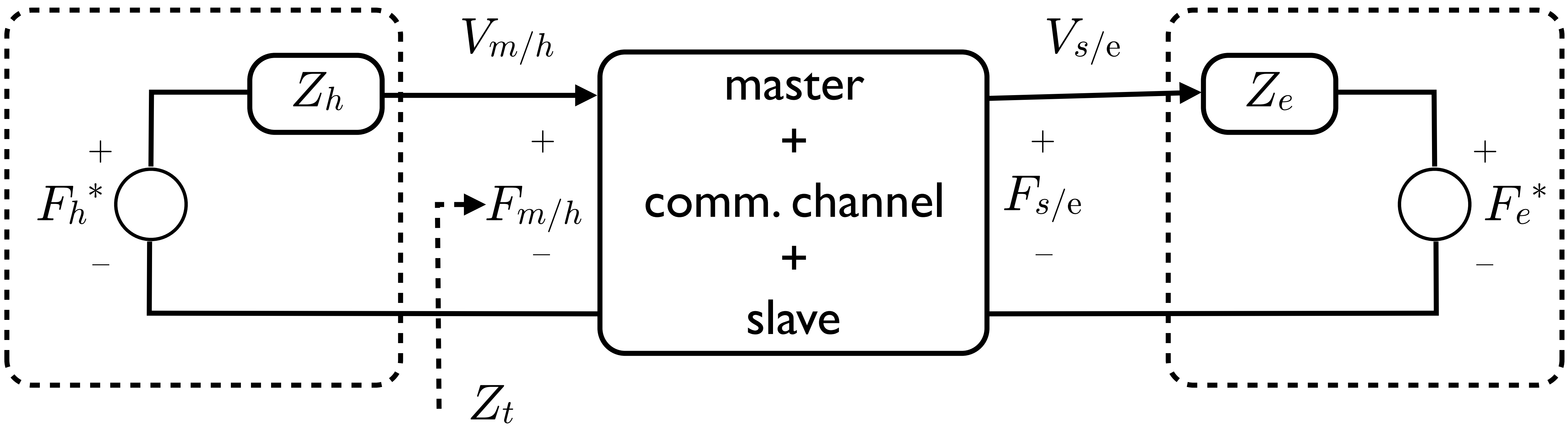
$$\begin{cases} \dot{x}_1 &= f_1(x_1) + g_1(x_1)u_1 \\ y_1 &= h_1(x_1) \end{cases} \qquad \begin{cases} \dot{x}_2 &= f_2(x_2) + g_2(x_2)u_2 \\ y_2 &= h_2(x_2) \end{cases}$$

- their feedback interconnection is passive w.r.t. $V_1(x_1)+V_2(x_2)$ with (new) input $v=(v_1,v_2)$ and output $y=(y_1,y_2)$ pairs

$$\begin{cases} u_1 &= \pm y_2 + v_1 \\ u_2 &= \mp y_1 + v_2 \end{cases}$$

- it easily follows from $\dot{V} = \dot{V}_1 + \dot{V}_2 \leq y_1^T v_1 + y_2^T v_2$

teleoperator passivity



- the two-port teleoperator is passive if (assuming zero energy storage at $t=0$) initial energy storage=0

$$\int_0^t (F_m(\tau)V_m(\tau) - F_s(\tau)V_s(\tau))d\tau \geq 0$$

- passivity of the two-port teleoperator is however a conservative condition
- absolute stability condition **guarantees stability** in a nonconservative way by **ensuring passivity** of the **one-port network** resulting from terminating the two-port teleoperator by **any passive** environment and operator

teleoperator passivity (cont'd)

- for LTI systems **passivity** is equivalent to **positive realness** (PR)
- a linear two-port master-slave network is absolutely or unconditionally stable if any passive (or PR) environment/operator impedance results in a passive (or PR) transmitted impedance to the operator/environment
- assuming a passive environment, we have

impedance of the master manipulator when the contact force at the slave side is equal to zero

Llewellyn's absolute stability criterion

considering the hybrid matrix

complex elements so they have a real part

force transmission ratio

$$\begin{bmatrix} F_m(s) \\ -V_s(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{bmatrix} V_m(s) \\ F_s(s) \end{bmatrix}$$

different from that we consider for transparency

velocity transmission ratio between master and slave

admittance of the slave manipulator

a two-port network is absolutely stable iff

1. $H_{11}(s)$ and $H_{22}(s)$ have no poles in the right half plane, any pole on the imaginary axis is simple with positive positive residues
2. for $s=j\omega$, the following inequalities hold $\forall\omega \geq 0$

$$Re[H_{11}] \geq 0$$

$$\boxed{\eta(\omega) := 2\frac{Re[H_{11}]Re[H_{22}]}{|H_{12}H_{21}|} - \frac{Re[H_{12}H_{21}]}{|H_{12}H_{21}|} \geq 1}$$

teleoperator passivity

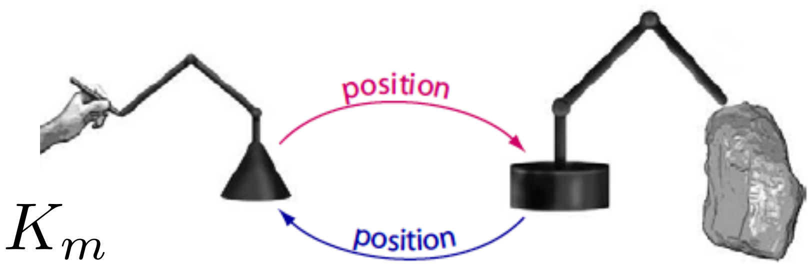
- Llewellyn's criterion can be equivalently applied to any of the possible immittance forms
- satisfaction of the criterion for one immittance form is necessary and sufficient for the satisfaction for the others
- perfect transparency implies a marginally stable teleoperator according to Llewellyn's criterion and master and slave mechanisms with no dynamics (compare with slide 22); hence, a trade-off between transparency and stability is necessary
- absolute stability is a **conservative** analysis method guaranteeing the stability of two-port networks coupled with **any** passive operator and environment impedances
- a system which is not absolutely stable is potentially unstable implying that there exists at least one passive environment which results in a non-passive transmitted impedance

Example: position-position architecture

how to apply the Llewellyn's criterion

master

- impedance $Z_m = M_m s$
- local controller $C_m = B_m + \frac{K_m}{s}$



Ⓒ Kuchenbecker, 2006

slave

- impedance $Z_s = M_s s$
- controller $C_s = B_s + \frac{K_s}{s}$

velocity channel $C_1 = B_s + K_s/s$

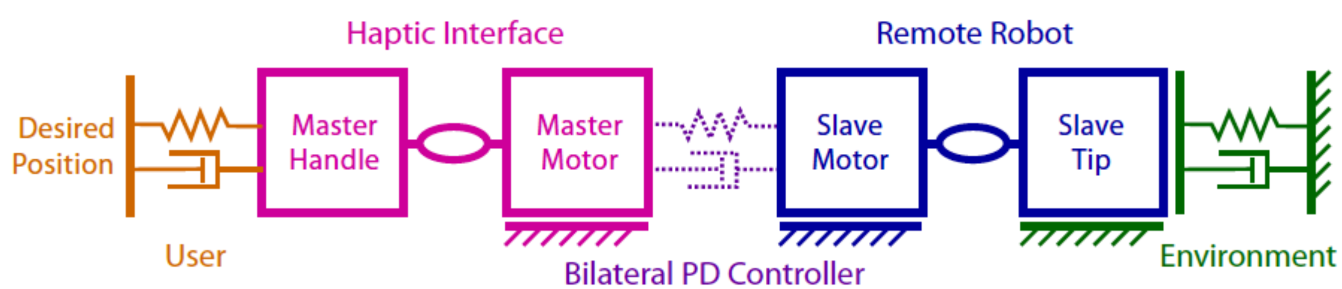
velocity channel $C_4 = -(B_m + K_m/s)$

force channels C_2 and C_3 not used

$\Rightarrow$

$F_{mc} = -C_m V_h - C_4 V_e = (B_m + K_m/s)\ (V_e - V_h)$

$F_{sc} = -C_s V_e - C_1 V_h = (B_s + K_s/s)\ (V_h - V_e)$



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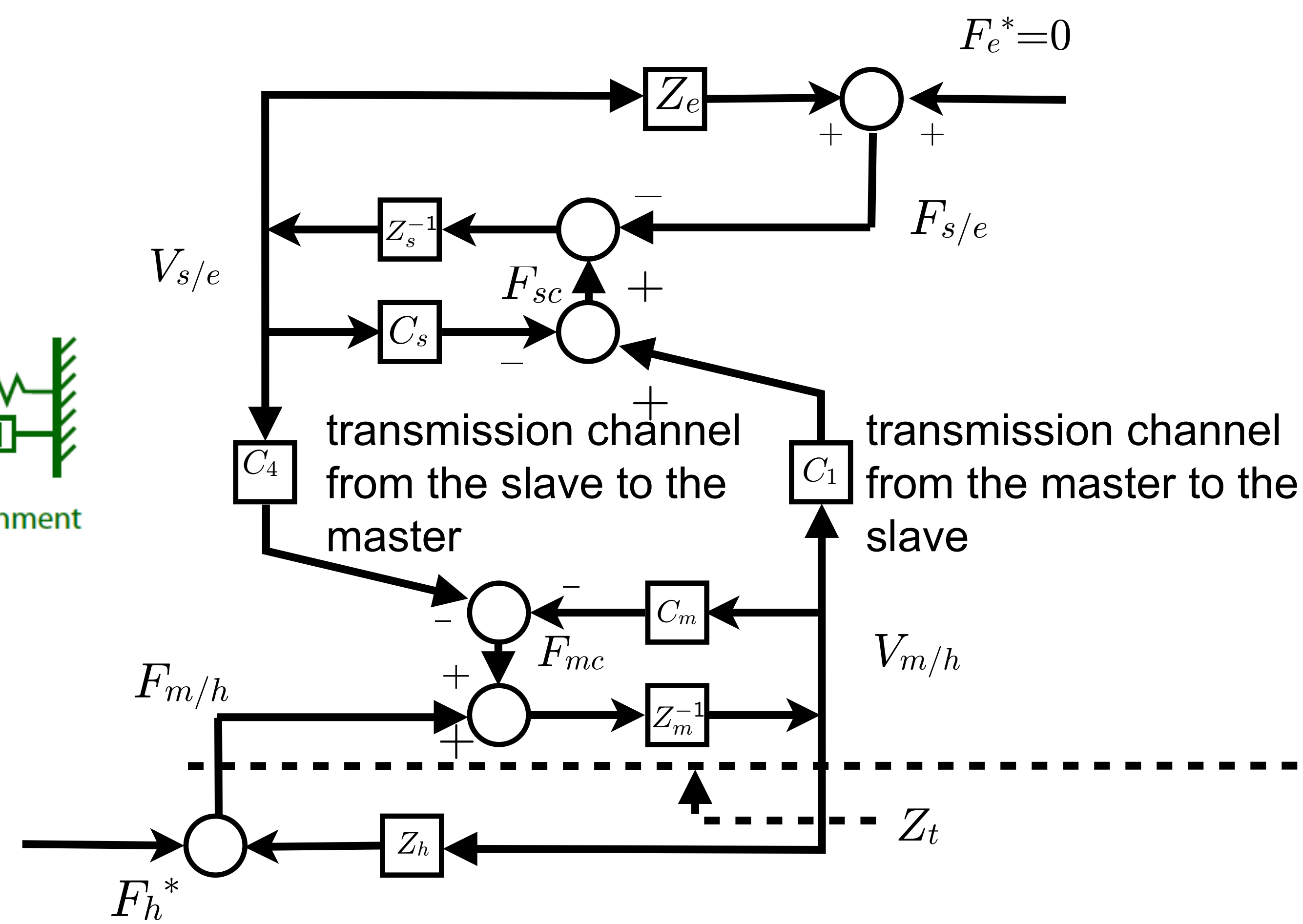

Example: position-position architecture

- determine the hybrid matrix for the position-position control scheme

$$\begin{bmatrix} F_m(s) \\ -V_s(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{bmatrix} V_m(s) \\ F_s(s) \end{bmatrix}$$

- prove that it is possible to conclude that
  - sufficient condition
    the position-position teleoperation scheme is absolutely stable if B_m, B_s , K_m , K_s>0 and C_m/C_s is a positive constant
  - the control scheme is **not** transparent because of non-ideal force tracking and distorted perception of free motion condition

it cannot be transparnt

Example: position-position architecture

- hybrid matrix for the position-position control scheme

$$\boldsymbol{H}(s) = \begin{bmatrix} Z_m + \frac{C_m Z_s}{Z_s + C_s} & \frac{C_m}{Z_s + C_s} \\ -\frac{C_s}{Z_s + C_s} & \frac{1}{Z_s + C_s} \end{bmatrix}$$

with $C_{m/s}$ and $Z_{m/s}$ as in slide 31

- the requested sufficient condition (if) is obtained by first noticing that

we find it by substituting C and Z

  1) the characteristic polynomial of H_{11} and H_{22} is $M_s s^2 + B_s s + K_s$ and it has no roots in the RHP for positive values of the coefficients

- then determine

$$\begin{aligned} \Re[H_{11}] &= \frac{\omega^2 B_m M_s^2 + M_s(K_m B_s - K_s B_m)}{(\omega M_s - K_s/\omega)^2 + B_s^2} \\ \Re[H_{22}] &= \frac{B_s}{(\omega M_s - K_s/\omega)^2 + B_s^2} \end{aligned}$$

- and observe

  1) the real part of H_{11} and H_{22} is non-negative if B_m and B_s are positive and $K_m B_s$ - $K_{ms} B_m = 0$ (implies that C_m/C_s is a positive constant)

  2) this last condition also guarantees that $\eta(\omega) = 1$ if B_m, $K_m > 0$

Homework solution (cont'd)

- the position-position control scheme is **not** transparent because of non-ideal force tracking
    - H_{12} is not equal to 1
- and distorted perception of free motion condition
    - H_{11} is not zero

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