MSc in Artificial Intelligence and Robotics
MSc in Control Engineering
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Neuroengineering

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8- BRAIN NETWORKS II

Learning objectives

1. Understand the two main definitions of causality and their differences
2. Remember the definition of causality in the statistical sense
3. Describe the AR model (and its bivariate version) and its use as a linear predictor
4. Compute the Granger causality index from (BI)AR models
5. Illustrate its values range
6. List its advantages and its limitations

4 Causality

it means that something causes something else

Definitions of causality

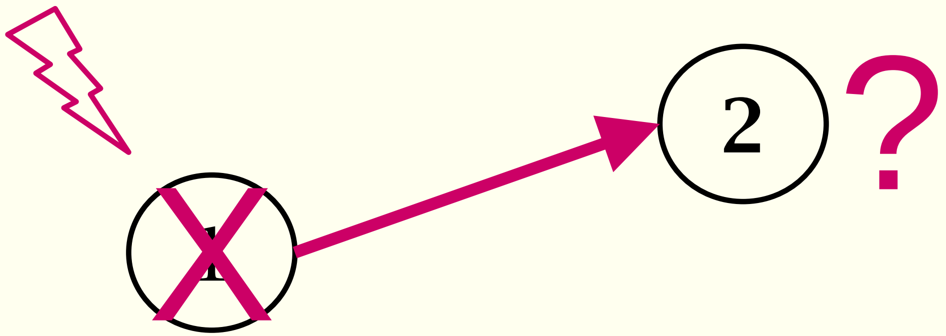
if I change the causes the conseguence change, so there is causality between 2 events A and B if changing A there is, as a conseguence of that, a change in B

1. Physical influence (control) → changing causes changes their consequences

so to verify the presence of causality, we can change something and look at if someother changes as consequence. We need to modify brain activity directly in a controlled way (for ex. through current). But it's not simple for ex in invasive way but also in noninvasive way

- Experimentally controlled interventions, assessment of their distal effect
- Physically acting upon brain activity effectively removes any other physical influence this node receives

Valdes-Sosa et al, Neuroimage, 2011



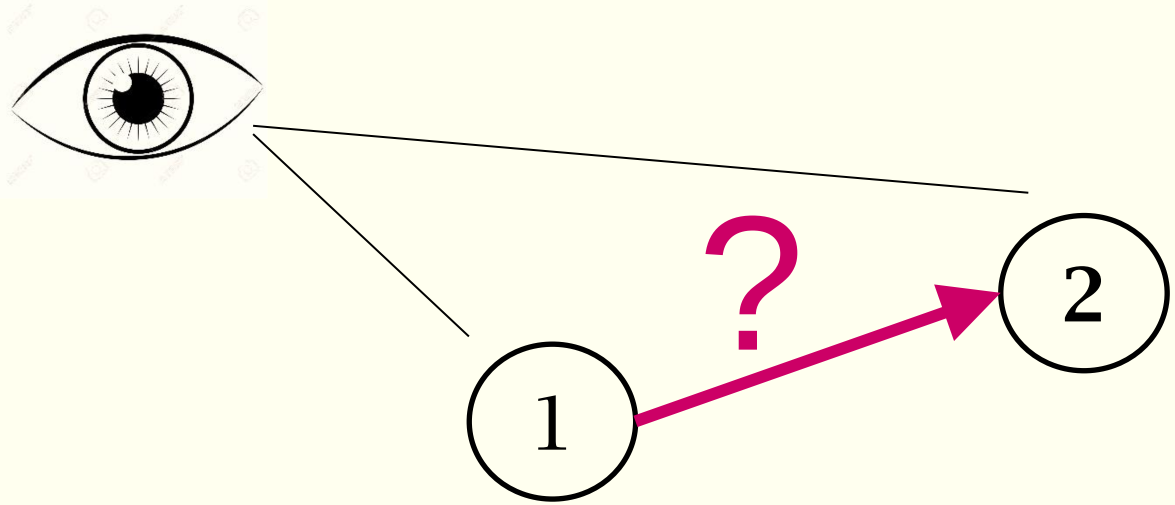
Definitions of causality

phisical intervention on the brain. Noninvasively we have some limitations like accuracy

2. Temporal precedence → causes precede their physically consequences
   - Testing for improvement in predictive capacity between temporally distinct neural events

     non a real measure of the causality but inferred from the data, so we can make assumptions on its existence
   - Observational (non interventional)

Valdes-Sosa et al, Neuroimage, 2011



Definition of causality in the statistical sense

Norbert Wiener (1956). First definition of causality in a statistical framework:

to test for causality in statistical sense, to test for the predictive value of an event with respect to another event, those 2 events can be measured simultaneously

Given two simultaneously measured signals, if one can predict the first signal better by incorporating the past information from the second signal than using only information from the first one, then the second signal can be called causal to the first one (Wiener, 1956).

we can use the simplest predictor we have which is a linear autoregressive predictor

Granger prediction is based on multivariate autoregression rather than a frequency-domain transformation

7 Wiener- Granger causality

(Clive Granger, Nobel Laureate in Economics, 2003)

An observed time series a(n) is said to Granger-cause another series b(n) if knowledge of a(n)'s past significantly improves prediction of b(n) by an autoregressive modelling (Granger, 1969) Granger introduces the predictor into Wiener definition

max length

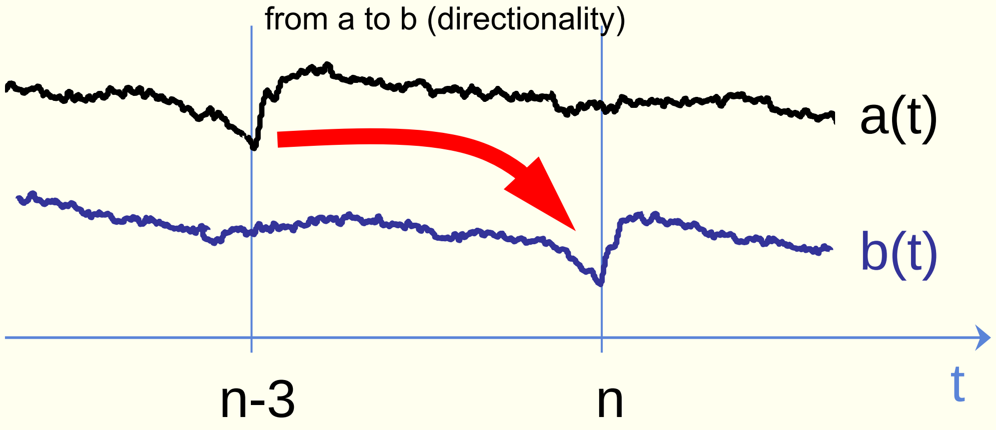
$$\boxed{b(n) = B_1 b(n-1)+\ldots+B_N b(n-N)+e_B(n)}$$

residual which is the difference between the prediction made by the linear combination of the past sample and the actual sample that we will measure (It is the error between the prediction and the real value)

$$\boxed{b(n) = B_1 b(n-1)+\ldots+B_N b(n-N)+A_1 a(n-1)+A_2 a(n-2)+\ldots+A_M a(n-M)+e_{B,A}(n)}$$

both model the same sample b

bivariate autoregressive model



$e_{B,A}(n) < e_B(n) \rightarrow$

the error that includes a and b is < of that that contains only b

It can be a(t)→b(t) without necessarily being b(t)→a(t): DIRECTIONALITY

Linear Autoregressive (AR) Model

a model parameter: the weight we use in the linear combination of the past samples. We have a specific weight for any specific lag

max lag that we consider in the linear regression and it is called the model order. It depends on the properties of data

temporal lag, which is the temporal distance between 2 samples in terms of number of samples

residual

$$x[n] = -\underbrace{\sum_{k=1}^{p} a[k]x[n-k]}_{\text{estimation of the future sample x}} + e[n] =$$

$$= -a[1]x[n-1] - a[2]x[n-2] - \ldots - a[p]x[n-p] + e[n]$$

x[n]= time series

a[k]= autoregressive parameter, lag k

p= model order

e[n]= model residual

Hp:

once we have the model, we need to test, for the residual of the model, that need to be zero mean uncorrelated white noise. If not, it means that some interesting info about the time series was erroneausly put in the model

x[n]: wide-sense stationary

e[n]: zero mean, uncorrelated

white noise

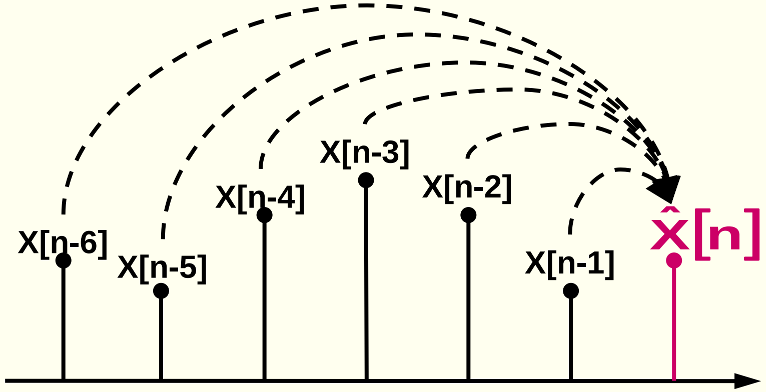
in order to have a good model, the residual should be uncorrelated to the timeseries

Autoregressive linear prediction

- An AR model can be used as a linear predictor:

$$\hat{x}[n] = -\sum_{k=1}^{p} a[k]x[n-k]$$

The residual is here equal to the prediction error:



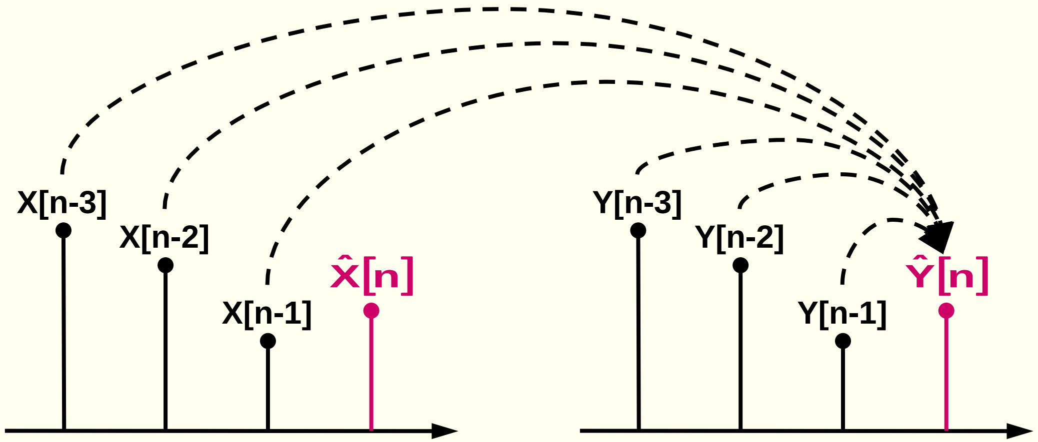
$$e[n] = x[n] - \hat{x}[n]$$

We must determine the coefficients a[k], by minimizing the power of the error e[n]

Bivariate autoregressive modeling

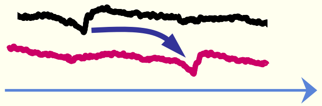
The autoregressive prediction of y is made by including information about the past samples of another signal x:

$$
\left.
\begin{aligned}
\overset{\text{target}}{x}[n] &= \sum_{k=1}^{p} a_{xy}[k]x[n-k] + \sum_{k=1}^{p} b_{xy}[k]\overset{\text{driver}}{y}[n-k] + e_{xy}[n] \\
y[n] &= \sum_{k=1}^{p} a_{y\cancel{x}}^{x}[k]x[n-k] + \sum_{k=1}^{p} b_{yx}[k]y[n-k] + e_{yx}[n]
\end{aligned}
\right\} \longrightarrow \text{bivariate autoregreessive model (2 equations together)}
$$



if there are several models that are fit to the same data, the model with the smallest errors can be said to fit the data better than the other models tested. When there is no influence of Y on X, the coefficients in b are all zero, while if the coefficients in b are nonzero, it means that the bivariate model will fit the data better than the univariate model. Thus, the errors from the bivariate model will be smaller than the errors from the univariate model

Granger Causality Test



- By comparing univariate and bivariate AR:

$$x[n]=\sum_{k=1}^{p}a_x[k]x[n-k]+e_x[n]$$

$$y[n]=\sum_{k=1}^{p}a_y[k]y[n-k]+e_y[n]$$

$a_x[k]$, $a_y[k]$ are the model parameters

p is the model order

e_x and e_y are the uncertainties or the residual noises associated with the model.

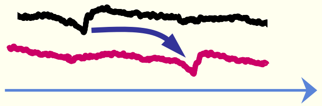
Here, the prediction error depends only on the past values of the own signal.

$$x[n]=\sum_{k=1}^{p}a_{xy}[k]x[n-k]+\sum_{k=1}^{p}b_{xy}[k]y[n-k]+e_{xy}[n]$$

$$y[n]=\sum_{k=1}^{p}a_{y\cancel{x}}^{x}[k]x[n-k]+\sum_{k=1}^{p}b_{yx}[k]y[n-k]+e_{yx}[n]$$

Here, the prediction error for each individual signal depends on the past values of both signals.

Granger Causality Test



The prediction performances for both models can be assessed by the variances of the prediction errors:

$$V_{x|x} = \mathrm{var}(e_x) \qquad V_{y|y} = \mathrm{var}(e_y)$$

For univariate models

$$V_{x|x,y} = var(e_{xy}) \qquad V_{y|y,x} = var(e_{yx})$$

For bivariate models

where X|X and X|X,Y indicate predicting X by its past values alone and by past values of X and Y, respectively.

If $V_{X|X,Y} < V_{X|X}$ then Y causes X in the sense of Granger causality, and a measure of that is given by:

it's not function of frequence or time but a number

$$\boxed{G_{y \to x} = \ln\left(\frac{V_{x|x}}{V_{x|x,y}}\right)}$$

directed from y to x

it's a number

if we have a negative G, it means that we have a worse model, so we made some mistakes. G can be only >= 0

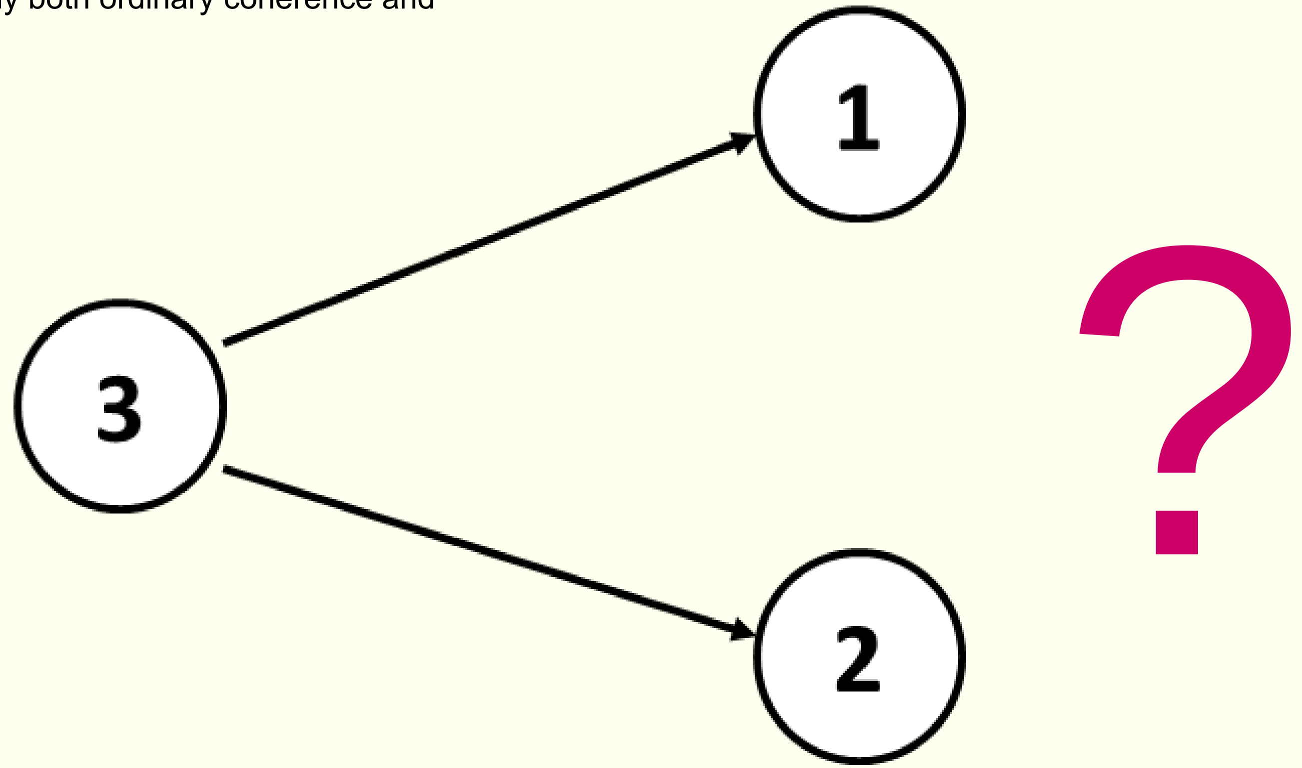
If the past of Y does not improve the prediction of X, then: $V_{x|x,y} \approx V_{x|x} \Rightarrow G \approx 0$

Any improvement in prediction of X by the inclusion of Y: $V_{x|x,y} \downarrow \quad \Rightarrow G \uparrow$

Advantages and limitations

- Advantages:
  1. DIRECTIONALITY: $G_{y \to x} \neq G_{x \to y}$
  2. Statistical meaning
- Limitations:
  1. so we don't have any info about spectral distribution of the casuality in Granger sense
     Defined in the time domain (in the time window we used to identify the model) a condizione provided that the signals are stationary in that window
  2. True causality can only be assessed if the set of two time series contains all possible relevant information and sources of activities for the problem (Granger, 1980).

when we have more regions, for instance 3, we have a problem when we apply both ordinary coherence and granger casuality



References

- Cohen, Chapter 28

Self-evaluation

1. Explain why testing causality as temporal precedence is more practical than testing the physical influence
2. Indicate what's the difference between the Wiener's and Granger's definitions of causality in the statistical sense
3. Given two time series x and y, indicate, for each of the following sentences, if they are true or false:
   a) $G_{x \rightarrow y}$ is always equal to $G_{y \rightarrow x}$
   b) $G_{x \rightarrow y} \in [-\infty, \infty]$
   c) A negative value of $G_{x \rightarrow y}$ means an inverse precedence between the two time series
4. List two advantages and two limitations of the Granger test