MSc in Artificial Intelligence and Robotics
MSc in Control Engineering
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Neuroengineering

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9- BRAIN NETWORKS III

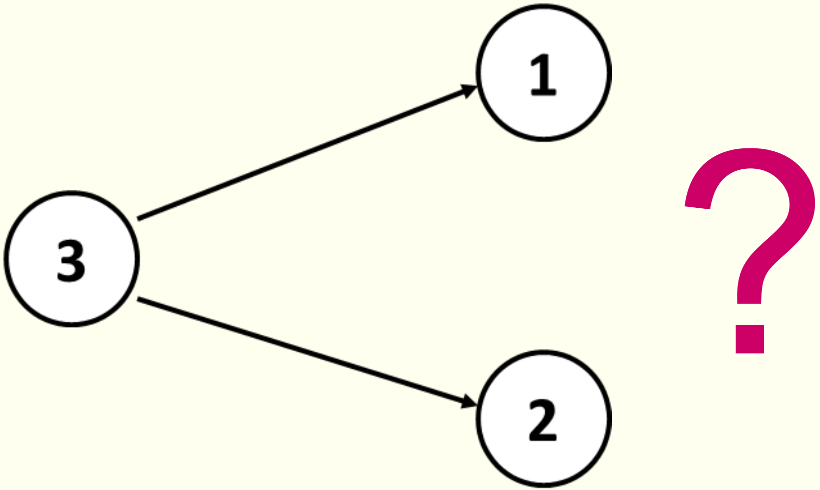
Learning objectives

1. Understand the two main approaches to a multivariate dataset
2. Describe the pairwise and the multivariate approaches
3. Define a method based on a multivariate, spectral AR model
4. Illustrate how it can be used to build brain networks at different frequency ranges
5. Compare the advantages and limitations of pairwise vs multivariate approaches

4 Pairwise and multivariate approaches

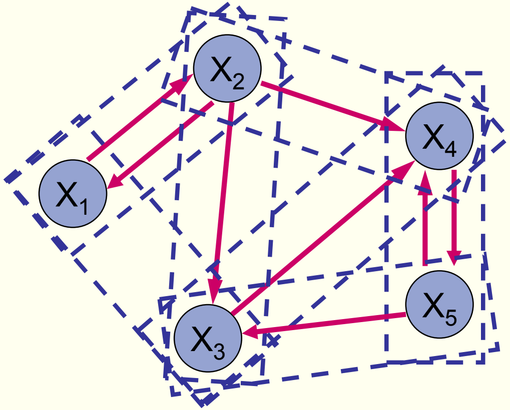
Multivariate dataset

When we have more than 2 signals



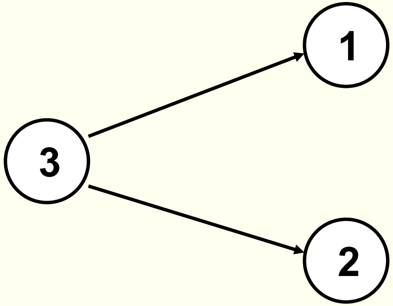
1 - Pairwise approach

All signals in the dataset are taken pairwise and causality is assessed:



But the set of two time series does not contain all possible relevant information → limitation of the Granger definition (Granger, 1980)

Limitations of the pairwise approach



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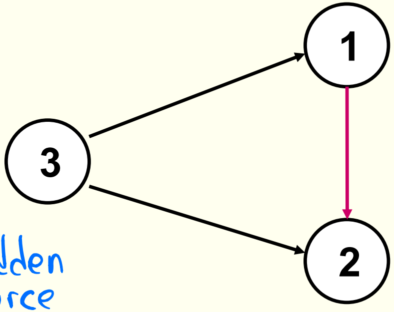
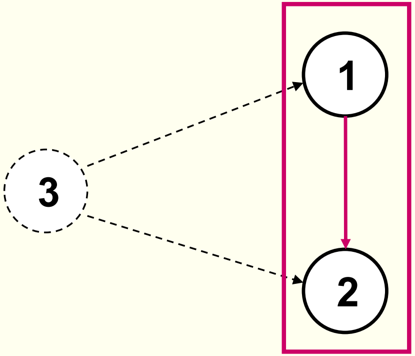
Hidden source problem



Connectivity pattern estimated by a pairwise method → spurious link

false link

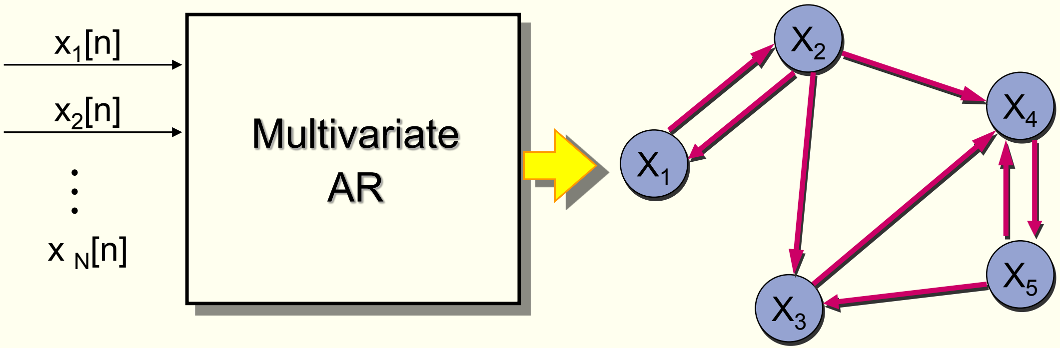
If we perturb 1 it won't influence 2



The bivariate model describing the relation between 1 and 2 does not recognize that the correlation between the two signals is due to a common effect of 3 (which is not included in the model)

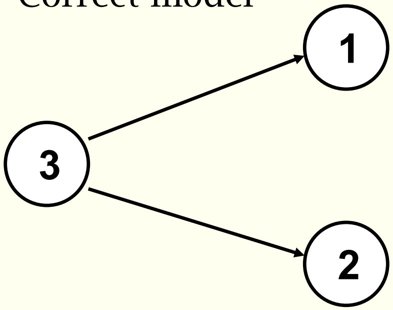
2 - Multivariate approach

The connectivity pattern is obtained by a unique generation model estimated on the entire set of data and takes into account all their interactions:



2 - Multivariate approach

Correct model



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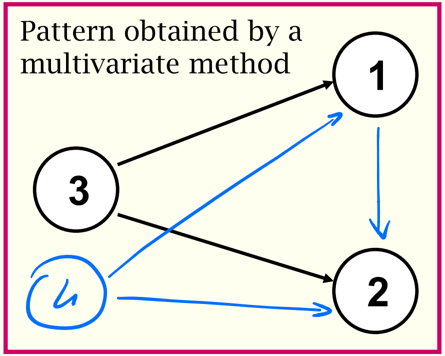
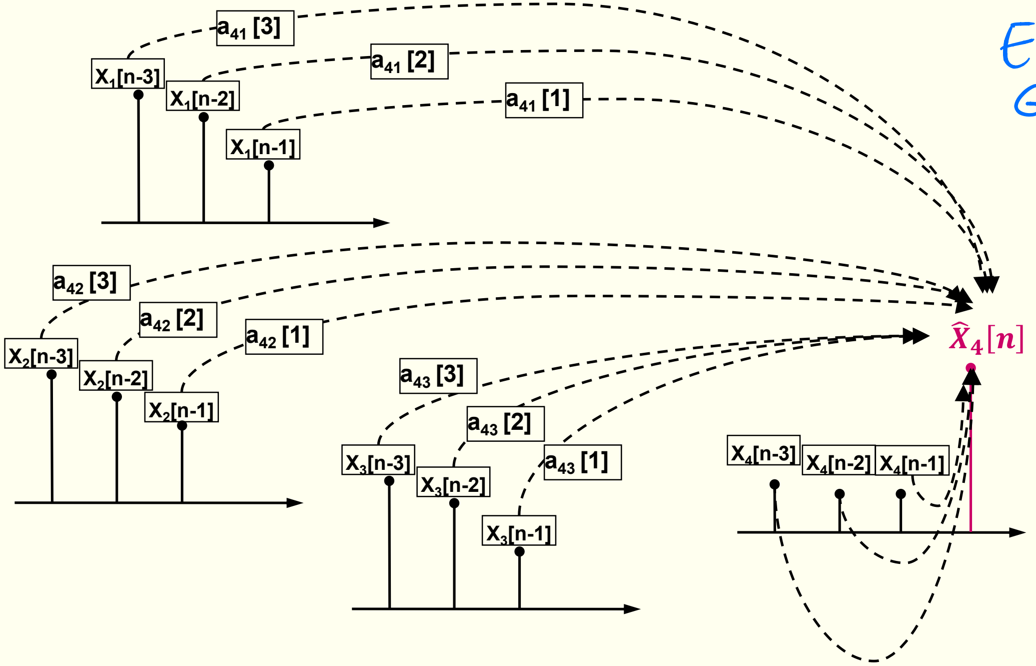


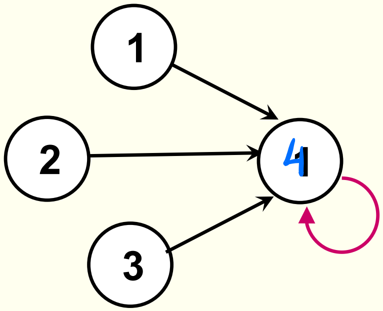
If we have another signal not in our model we could again have spurious links

Multivariate methods, by building a unique model based on all the signals, use all the information at disposal and thus allow a better comprehension of the relationship between the signals

Multivariate Autoregressive Models (MVAR)



Extension of the Granger test



Multivariate Autoregressive Models (MVAR)

- Given a set of N signals: $\bar{X} = \left[x_1[1] \quad x_2[1] \quad \cdots \quad x_N[1] \right]^T$
- A Multivariate Autoregressive Model of dimension N is:

$$\boxed{\begin{aligned}
x_1[n] &= -\sum_{k=1}^{p} a_{11}[k]x_1[n-k] - \sum_{k=1}^{p} a_{12}[k]x_2[n-k] - \quad \cdots \quad -\sum_{k=1}^{p} a_{1N}[k]x_N[n-k] + e_1[n] \\
x_2[n] &= -\sum_{k=1}^{p} a_{21}[k]x_1[n-k] - \sum_{k=1}^{p} a_{22}[k]x_2[n-k] - \quad \cdots \quad -\sum_{k=1}^{p} a_{2N}[k]x_N[n-k] + e_2[n] \\
&\vdots \\
x_N[n] &= -\sum_{k=1}^{p} a_{N1}[k]x_1[n-k] - \sum_{k=1}^{p} a_{N2}[k]x_2[n-k] - \quad \cdots \quad -\sum_{k=1}^{p} a_{NN}[k]x_N[n-k] + e_N[n]
\end{aligned}}$$

Multivariate Autoregressive Models (MVAR)

- The model parameters are N · N · p:

$$\bar{a}[1]=\begin{bmatrix} a_{11}[1] & \cdots & a_{1N}[1] \\ \vdots & \ddots & \vdots \\ a_{N1}[1] & \cdots & a_{NN}[1] \end{bmatrix} \quad \bar{a}[2]=\begin{bmatrix} a_{11}[2] & \cdots & a_{1N}[2] \\ \vdots & \ddots & \vdots \\ a_{N1}[2] & \cdots & a_{NN}[2] \end{bmatrix} \quad \cdots \quad \bar{a}[p]=\begin{bmatrix} a_{11}[p] & \cdots & a_{1N}[p] \\ \vdots & \ddots & \vdots \\ a_{N1}[p] & \cdots & a_{NN}[p] \end{bmatrix}$$

- And the N variances of the residuals:

$$S_E=\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{bmatrix}$$

Total number of parameters to be estimated: N · N · p+N=N(N · p+1)

MVAR in the frequency domain

$$\sum_{k=0}^{p} A[k]X[n-k] = E[n]$$

$$\mathscr{F}$$

$$\overline{A}(f)\overline{X}(f) = \overline{E}(f)$$

Where: $A_{ij}(f) = \sum_{k=0}^{p} a_{ij}[k]e^{-j2\pi fTk}$

$$\overline{A}(f) = \begin{bmatrix} A_{11}(f) & \cdots & A_{1N}(f) \\ \vdots & \ddots & \vdots \\ A_{N1}(f) & \cdots & A_{NN}(f) \end{bmatrix} \qquad \overline{X}(f) = \begin{bmatrix} X_1(f) \\ \vdots \\ X_N(f) \end{bmatrix} \qquad \overline{E}(f) = \begin{bmatrix} E_1(f) \\ \vdots \\ E_N(f) \end{bmatrix}$$

Partial Directed Coherence (PDC)

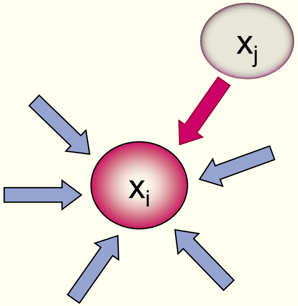
- *PARTIAL DIRECTED COHERENCE (PDC)* from j to i is defined on the basis of matrix A
  (Baccalà and Sameshima, 2001):

$$\boxed{\pi_{ij}(f) = \left|A_{ij}(f)\right|^2}$$

- Different normalization of PDC are provided, for instance (Astolfi et al, 2007):

$$\boxed{\pi_{ij}(f) = \frac{\left|A_{ij}(f)\right|^2}{\sum_{m=1}^{N}\left|A_{im}(f)\right|^2}}$$

Where: $\sum_{n=1}^{N} \pi_{in}(f) = 1$



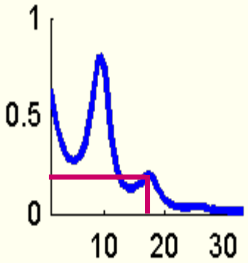
$\sum = 1$

Partial Directed Coherence (PDC)

- Since $A_{ij}(f) \neq A_{ji}(f)$

$$\boxed{\pi_{ij}(f) \neq \pi_{ji}(f)}$$

The value of PDC_{ij} at a certain frequency f_0 represents the existence of a causality link directed from j to i

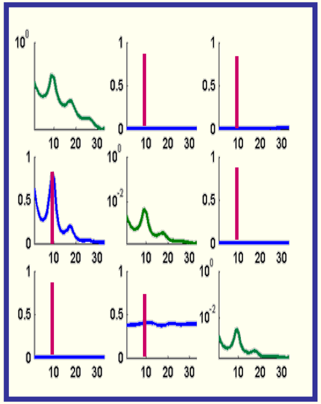


From Spectral Indices to Brain Networks

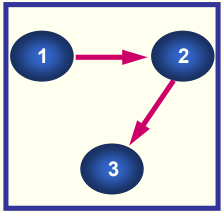
$$\sum_{k=0}^{p} A(k)X(t-k) = E(t) \xrightarrow{\mathcal{F}} A(f)X(f)=E(f)$$

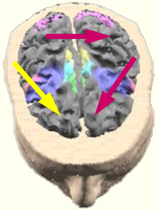
$$\pi_{ij}(f) = \frac{\left|A_{ij}(f)\right|^2}{\sum_{m=1}^{N}\left|A_{im}(f)\right|^2}$$

PDC$_{ij}$ estimates the influence of the region j toward the region I at a given frequency f



f= 10Hz

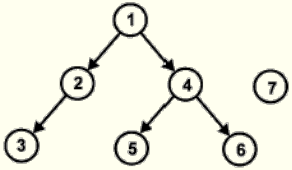




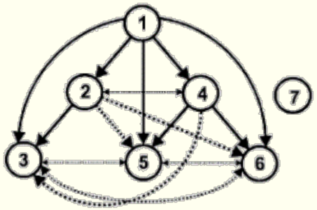
Comparison between different estimators

Kus R, Kaminski M, Blinowska KJ, Determination of EEG activity propagation: pair-wise versus multichannel estimate. *IEEE Trans Biomed Eng*, 2004.

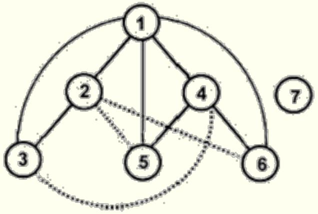
Imposed Pattern



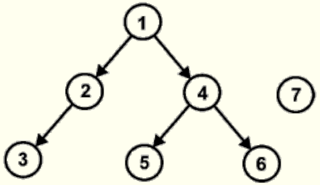
Estimated pattern



Granger Test



Ordinary Coherence



Partial Directed Coherence

Pairwise Vs multivariate estimators

- Bivariate approach:
  - Advantages:
    - No limit to the number of signals
    - To be used when short data segments are available
  - Limitations:
    - Reduced accuracy

Pairwise Vs multivariate estimators

- Multivariate approach:
  - Advantages:
    - Better estimation performances
    - Allows for inserting all data sources in the model
  - Limitations:
    - Limitation in the number of channels/signal that can be modeled → more data required

Self-evaluation

1. Show an example of network for which a pairwise approach is less accurate than a multivariate one
2. Given the PDC estimator, indicate, for each of the following sentences, if they are true or false:
   a) $PDC_{i \to j}$ is always equal to $PDC_{j \to i}$
   b) The normalized PDC $\in [-\infty, \infty]$
   c) PDC can always avoid the problem of the “hidden source”
3. List two advantages and a limitation of the pairwise and of the multivariate approach