

True statements - Cincotti

Electroencephalogram - 1

- An **artifact** is a **potential difference** due to **sources outside the brain**.
- **Artifacts** can have **biological** origin (such as eyes, muscles heart), can be due to **external electromagnetic generators** (power supply, engines, etc) or to **events affecting the recording setup** (electrode movements or loss of contact, saturation of the ADC).
- **Artifacts** can be **partially attenuated or removed** through **signal processing** during data analysis, but it is always **preferable** to make all efforts to **prevent** them when the signal is being **acquired**.
- The **eye** is **more positive in its frontal part** (cornea) **than its posterior part** (retina), and thus its movements can generate **large artifacts** (EOG) on the EEG (of the order of **$500\mu V$**).
- During an **eyeblick**, the **eyelid shortcuts the positive potential of the external surface of the eye**, causing **positive deflections** of the EEG with amplitude of **several hundreds of μV** and duration of a **few hundreds of milliseconds**.
- A sudden **upwards/downwards movement of the eyes** generates a **positive/negative** deflection of EEG potentials (EOG) on frontal EEG channels, respectively.
- A **sudden movement of the eyes** to the **right/left** generates a **positive/negative** deflection of EEG potentials (EOG) on the EEG channel **F8** (right side of the head), respectively.
- The **electrical activity of the muscles** (EMG) has a **spectral content** starting at frequencies of **20 Hz** and up, thus **affecting the beta and gamma bands** of the EEG signal.
- The **amplitude of the electromyogram** (EMG) originated from muscles in the head **can have amplitude ten times higher than the EEG signal** (thus in the order of **$1 mV$**).

- EMG artifact can easily appear on the EEG recording unless the subjects are specifically instructed by the experimenter on how to relax their face and tongue/throat muscles.
- The **heart activity can contaminate an EEG** recording because **an electrocardiographic (ECG/EKG) artifact can directly affect the potentials**, especially if the reference electrode is not placed on the head.
- The heart activity can contaminate an EEG recording because a **ballistocardiographic artifact is indirectly generated by the pulse of a blood vessel causing movements of a nearby electrode**.
- **Sweating can affect the EEG**, causing a slow changing and high amplitude artifact (**below 0.5 Hz, up to a few mV**)
- **Powerline noise is an artifact caused by the capacitive coupling between** (i) the **conductors carrying the alternating** (typically at **50Hz**) **current power supply** and (ii) the **recording setup including the subject**.
- The **powerline noise affects a very narrow frequency band of the recorded signal around 50 Hz** (or 60 Hz, depending on the powerline frequency). **Other frequency bands can be affected at multiple frequencies that at multiple** (typically odd multiples) of 50/60Hz.
- **Powerline noise is accentuated by asymmetries** in the recording electrode pairs, such as impedances and cable path, **because asymmetries prevent the noise to be rejected** by the amplifier's common-mode rejection capabilities.
- **Notch filters effectively remove powerline noise** because they selectively reject the narrow band affected by the artifact, preserving almost entirely the useful signal.
- **Movement of the subject's head may produce slow artifacts** on the EEG recording, **whose waveform is closely related to the time-course of the movement**. Since the potentials originate from the mechanical displacement of the charged double layer at the electrodes interface, **these artifacts are less pronounced when non-polarizable electrodes**.

Electroencephalogram - 2

- As a **preliminary step to EEG data analysis**, the following data can be **discarded**: (i) **one or more channels, if it is extensively contaminated by artifacts**; (ii) **all time intervals (epochs) in which artifacts appear**. Both strategies can be applied on the same dataset.
- The **estimation of event-related or evoked potentials** (ERPs, EPs) **requires the acquisition of numerous repetitions** (typically tens or hundreds) **of the stimulus** or event which evoked or induced the potential.
- When **recording ERPs or EPs** (whose **peak amplitudes are a few microvolts down to a fraction of microvolt**), the **spontaneous EEG** (whose amplitude is of **tens of microvolt**) **is to be considered a noise** that completely masks the EPs or ERPs on the recorded waveform.
- In **ERP analysis**, the **averaging procedure** consists in (i) **segmenting epochs (trials) with fixed duration** from the raw recording, each **aligned** to a repetition of the event; (ii) **performing a synchronized average**, i.e. averaging all corresponding samples sharing the same latency across the set of trials.
- **Synchronized averaging of N trials preserves the amplitude of the ERP and reduces the amplitude of the background spontaneous EEG activity by a factor \sqrt{N}** , under commonly verified hypotheses.
- The **amplitude of ERPs is measured with respect to a baseline epoch** (usually **preceding the stimulus**), in which the **amplitude is assumed to be zero**.
- The **latency of an ERP peak is measured with respect to the relevant event**, usually the **presentation of a stimulus**, rather than with respect to the beginning of the waveform.
- In an **ERP**, **peaks are named with a leading P (N)** if the peak has **positive (negative) polarity**.
- In an **ERP**, **peaks are often named with a trailing number indicating the nominal latency in milliseconds**. In older

conventions, the trailing number represents the order of the peak within the ERP.

- A negative peak in an ERP recorded on a **specific subject with a latency of 108 ms may still be named N100, if it matches the physiological phenomenon of the nominal N100 component.**
- The Stimulus Onset Asynchrony (SOA) measures the time interval between the onset two successive stimuli in a train. If each trial only contains a stimulus, it is equivalent to the Inter-Trial Interval (ITI)
- The Inter-Stimulus Interval (ISI) measures the time interval between the end of a stimulus and the beginning of the following one. It equals the SOA minus the stimulus duration.
- In an ERP, the response to a stimulus has a reduced amplitude when the SOA is too short. Long-latency components of the ERP are especially sensitive to this decrease.

... tradeoff between number of stimuli and SOA ...

... mapping...

- Brain activity in response to a stimulus can be phase-locked to the event, meaning that the whole timecourse (including positive and negative peaks) of the response has the same latency in every repetition. This activity is called **evoked**.
- Brain activity in response to a stimulus can be non-phase-locked, meaning that they show variable latency (jitter) at each repetition. This activity is called **invoked induced**.
- The averaging procedure can reliably uncover components of an ERP corresponding to evoked activity of the brain.
- Induced activity is often examined by analyzing the envelope of the EEG in a relevant frequency band, i.e. by rectifying or squaring the pass-band filtered trials before averaging them.
- In the EEG terminology, **synchronization** (desynchronization) is synonymous of an increase (decrease) of the waveform amplitude (and thus power).
- Event-Related Desynchronization/Synchronization (ERD/S) quantify changes of the power of EEG relative to a baseline period, expressed as percent change. ERD/S is usually evaluated on a whole range of relevant latencies with respect to the event.

- **Computation of ERD/S** from a set of N EEG trials requires the following steps: (i) **band-pass filtering in the relevant frequency band**; (ii) **take the square** of each sample; (iii) **take a synchronized average across trials**; (iv) **smooth by averaging samples within each short time window in a set of adjacent ones covering the whole timecourse**; (iv) **compute the relative percentage change with respect to the average value in a baseline period**.

Clarification: An alternative algorithm to evaluate the ERD/S uses **rectification instead of squaring** in step (ii), thus estimating the amplitude of the signal rather than its power.

- **Time-frequency analysis allows to analyze and visualize changes of the spectral components of a signal over time. Time resolution and frequency resolution cannot be freely chosen - the higher the one, the lower the other.**

Basics of signal processing - 1

- The **Shannon's theorem** (sampling theorem) states that a continuous signal can be properly sampled only if it does not contain frequency components above one-half of the sampling rate.
- In Analog to Digital Conversion (ADC), the **Nyquist frequency equals half of the sampling frequency**.
- The **reconstruction of an analog signal** from its sampled version is equivalent to the sum a set of a set of *sinc()* functions, one for each sample, each centered on the time of the respective sample, whose amplitude equals the sample value.
- **Aliasing occurs** when an analog signal is sampled outside the conditions set by the Shannon's theorem.
- When aliasing occurs in ADC, a sinusoidal component with frequency $f_0 \in (f_{Nyquist}, f_{Sampling})$ is reconstructed as a sinusoidal component at $f_{Aliasing} = f_{Sampling} - f_0 \in (0, f_{Nyquist})$
- Aliasing can be prevented by applying an analog low-pass filter with cutoff frequency lower than $f_{Nyquist}$ to the analog signal (i.e. before it is converted).
- **Quantization** (i.e. approximation of the analog value of a sample to the nearest among the allowed quantization levels) introduces a noise whose amplitude is proportional to the width of the quantization interval ($\delta_{quant} = 1/\sqrt{12} \cdot \text{LSB}$).
- Quantization divides the input range of the ADC into (approximately) 2^{NBITS} intervals, where NBITS is the number of bits of the ADC.
- Given a fixed number of bits NBITS of the ADC, choosing a large input range increases the quantization error, while choosing a small input range increases the chance that the signal is clipped (i.e. the input range is saturated).
- Appropriate application of an analog filter (i.e. before the analog signal is converted) may prevent saturation by removing high amplitude artifacts in specific frequency bands.

Basics of signal processing - 1

- The Average Rectified Value (**ARV**) is a **measure of the amplitude of a signal**, and it is obtained by **summing the absolute values of all samples and dividing the result by the number of samples**.
- $ARV_x = \frac{1}{N} \sum_i |x_i|$ where the sum extends on the N samples of the signal X
- The Root Mean Square (**RMS**) is the **square root of the average of the squared value of the samples of a signal**
- The **variance** of a signal is estimated by **summing the square of all deviations of the N sample values from the sample mean, and then dividing by N - 1**
- $s_x^2 = \frac{1}{N-1} \sum_i (x_i - X_{medio})^2$ where the sum extends over the N samples of the signal X
- The **variance** and the **square of the RMS** of a zero-mean signal **have the same value**. (Consider $N \rightarrow \infty$.)
- The **standard deviation** σ of a signal is the **square root of its variance**.
- In **white noise**, all samples are **uncorrelated**, i.e. when given the value of one sample we have no increased knowledge to predict the value of another sample.
- The **frequency spectrum** of white noise is **flat**, i.e. it has the **same power at any frequency**.
- In a **gaussian noise**, the **probability [density]** that a sample has a given amplitude value follows the **normal (gaussian) distribution with zero mean**.
- Given two ranges of equal width $A = [-0.5, +0.5]$ and $B = [0.5, 1.5]$, it is more likely that samples of a gaussian noise will have amplitude in A rather than B [because the gaussian probability density function is highest around 0]
- Given two ranges of equal width $A = [-0.1, +0.1]$ and $B = [0.8, 1.0]$, it is less likely that samples of a sinewave $x = \sin(t)$ will have amplitude in A rather than B.

- **The amplitude of the samples of a triangular waveform have uniform probability density function**, i.e. samples have the same probability [density] of taking a value between the $-A$ and $+A$ (being A the peak value of the waveform) and zero probability of taking a value outside $[-A, A]$.
- **The Central Limit Theorem (CLT) states that the probability distribution of the average of N independent and identically distributed random variables tends to a normal distribution for N approaching infinity.**
- **The probability distribution of the average of N independent and identically distributed random variables is a normal distribution independently of the value of N .**
- **Given N independent and identically distributed random variables with variance (or standard deviation) equal to σ^2 (or σ), the variance (standard deviation) of their average is σ^2/N (or σ/\sqrt{N})**
- The synchronized average of N trials containing only spontaneous EEG whose $RMS_{trial} = \sigma^2$ is a signal $RMS_{avg} = \sigma^2/N$

Basics of signal processing - 2

- Discrete Fourier Transform (DFT) **associates the** (time-limited and sampled) **time-domain representation of a signal into its** (bandwidth limited and sampled) **representation in the frequency domain.**
- **Given a sampled signal with N samples, its DFT will contain N complex values, of which the first half is enough to completely define the signal.**
- The DFT of a **signal represents the amplitude and initial phases of sinewave components of the signal at frequencies f_i ranging from 0 Hz to the sampling frequency. Only components up to the Nyquist frequency are needed to characterize the signal, since the remaining part of the spectrum is a mirrored copy of the first half.**
- **An arbitrary zero-mean signal of finite duration T can be approximated by the sum of N sinewaves with frequencies $f_i = if_0 = \frac{i}{T}$. In the conditions set by the Shannon's theorem, the approximation error vanishes when N equals half the number of samples.**
- **The spectrum of a sampled signal can be considered periodic with period $f_{sampling}$. Thus, samples of the DFT corresponding to frequencies $\in [f_{Nyquist}, f_{sampling})$ can be as well interpreted as belonging to the interval $[-f_{Nyquist}, 0)$, thus yielding a spectrum defined in $[-f_{Nyquist}, f_{Nyquist})$ and symmetrical around $f = 0$ Hz**
- Fast Fourier Transform (FFT) is an **efficient implementation of the DFT for signals whose number of samples is a power of 2.**
- **The spectrum of a signal is often represented as the pair of Magnitude and Phase plots, in which the absolute value and the angle, respectively, of the complex samples of the DFT are displayed. The phase can be "unwrapped" to eliminate the $\sim 2\pi$ -wide discontinuities.**
- Spectral analysis is well suited at identifying narrowband useful signals in (approximately) white noise, because a peak in the spectrum may still be detected even when the low signal-to-noise ratio (SNR) prevents the signal's waveform to be recognized among the noise samples.

- For a signal sampled with sampling interval $\Delta T = 1/f_s$, the spectrum has a frequency range of width $f_s = 1/T_s$. For a signal of total duration T_{\max} , adjacent samples (frequency bins) of the DTF are $\Delta f = 1/T_{\max}$ apart.
- Zero-padding a signal (i.e. adding trailing samples of zero amplitude) produces a reduction of the distance between adjacent samples of the spectrum, thus visually improving the frequency resolution.

Basics of signal processing - 3

- The **spectral leakage** phenomenon is observed, for instance, when comparing the spectrum of a signal with the spectrum of a short section of the same signal.
- For the purposes of spectral analysis, **extracting a section of duration ΔT** extracted from a longer signal **is equivalent to multiplying the signal by a rectangular windowing function**, whose value is 1 in the section of interest and 0 elsewhere.
- **Spectral leakage spreads each frequency sample of the original spectrum with a pattern defined by the DFT of the windowing function**, and thus is characterized by a **main lobe and several side lobes**.
- The **effects of spectral leakage** can be **controlled by using a tapered windowing function** (such as the **Hamming's** or the **Blackman-Harris'**) instead of a rectangular one. **This yields lower sidelobes but a wider main lobe**.
- A **rectangular windowing function is preferable when the spectral analysis aims at distinguishing two spectral components with similar frequency and power**. A tapered windowing function is preferable when the leakage from a strong component may obscure a much weaker component with a quite different frequency.
- The spectrum of a time-limited sampled signal is formally defined as the square of the absolute value of its DFT.
- The method of the average periodogram to estimate the spectrum of a stochastic signal is applied when the spectral resolution $\Delta f = 1/T_{\max}$ is higher than required [i.e. the distance Δf between adjacent bins is smaller than required],

where T_{\max} is the duration of the signal. In turn, the variability of the estimate of each frequency sample of the spectrum is improved.

- In the **averaged periodogram's method**, a long signal is split into N shorter adjacent epochs, the spectrum of each epoch is computed, and this set of spectra is averaged bin by bin.
 - In the **Welch's method** for spectral estimation the signal is split into N shorter possibly overlapping epochs, the signal in each epoch is multiplied by a windowing function, then the spectrum of each short signal is computed and averaged (bin by bin) with the others.
 - The Short Time Fourier Transform (**STFT**) is a **simple method to estimate a spectrogram**, i.e. the representation of the time-varying spectrum of a non-stationary signal.
 - In a spectrogram, time is usually displayed on the horizontal axis, frequency on the vertical axis, and the magnitude of the spectrum is color-coded. [The third axis of a 3D plot may also be used]
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Basics of signal processing - 4 (filters)

- The purpose a filter is to allow some spectral component of a signal to pass (almost) unaltered, while (almost) blocking other spectral components
- Filters are categorized into four types depending on the basic shape of their frequency response: (i) **low-pass**; (ii) **high-pass**; (iii) **band-pass**; (iv) **band-reject (or band-stop)**
- The **passband** is the interval of frequencies in which the gain of the filter is close to 1. In the **stopband** the gain is close to 0. In the **transition band** the gain has an intermediate value between 0 and 1.
- The **roll-off** of a filter is the **slope of its frequency response in the transition band**. It is high when the transition band is narrow.
- The **cutoff frequency** (or corner frequency) **designates the limit of the passband**. The gain of the filter at cutoff frequency is approximately 0.71 (i.e. $1/\sqrt{2}$, -3 dB [The gain value at the cutoff-frequency might be different for some filter designs, but this concept is beyond the scope of this course])
- The **gain in the passband** can monotonically decrease below 1 when the frequency approaches the cutoff frequency, or it might **ripple** above and below 1.
- The **frequency response of a filter in the stopband should not be evaluated from a graph where the gain axis is in linear scale**, because a gain of 0.001 can hardly be distinguished from a gain of 0.0001. **Rather, a vertical axis in logarithmic scale (i.e. the gain is expressed in dB) should be used.**
- **Good features of a filter include:** (i) **fast roll-off**; (ii) **flat passband (i.e. no ripple)**; (iii) **strong stopband attenuation** (e.g. gain below -40 dB, but the specific value may change depending on applications)
- **Digital filters are categorized into two types depending on their implementation:** (i) **Finite Impulse Response (FIR)**; (ii) **Infinite Impulse Response (IIR)**
- The **output of FIR filters is the linear combination of samples of the input**. The output of IIR filters combines both samples of the input and past samples of the output.

- The **order of a filter** measures the number of recent samples of the input (or the output) are combined to compute the output. [slightly incorrect, but will do for this exam. A more accurate definition that is beyond the scope of the course states that the order of a filter is the maximum delay applied to an input or output sample, whichever is largest]
- The **Butterworth** filter is a design method in the family of **IIR**
- The **corner frequency** of a high-pass filter is called **low cutoff frequency**. The corner frequency of a low-pass filter is called **high cutoff frequency**. Band-pass and band-stop filters have both a low cutoff frequency and a high cutoff frequency.
- A **notch filter** is a type of band-stop filter which removes only attenuates the input in a narrow band around the **notch frequency**. Its most common use is to remove the **powerline artifact** at 50 Hz and its harmonics (60 Hz in some other countries).
- An **IIR filter** is more efficient than a **FIR filter**, meaning that the latter needs to be of a higher order to achieve the same quality specifications.
- A **FIR filter** can be designed to have "linear phase", meaning that it will not introduce time-domain distortions in the waveform of the output signal. IIR filters cannot have linear phase.

BCI

- A **BCI** is a system that measures brain activity and converts it to artificial output that replaces, improves (or extend otherwise) **natural brain output** (naturally subserved by nerves and muscles) and thereby changes the ongoing interactions between the brain and the environment
- The **P300 ERP** generated by attending a target stimulus is exploited to build virtual keyboards based on a BCI
- The **amplitude of sensorimotor rhythms** can be voluntarily modulated through the exercise of motor imagery, to build a cursor control based on a BCI.

