

MSc in Artificial Intelligence and Robotics

MSc in Control Engineering

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# Neuroengineering

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we focus on single neuron

## 5- NEURAL ENCODING

# Learning objectives of the lesson

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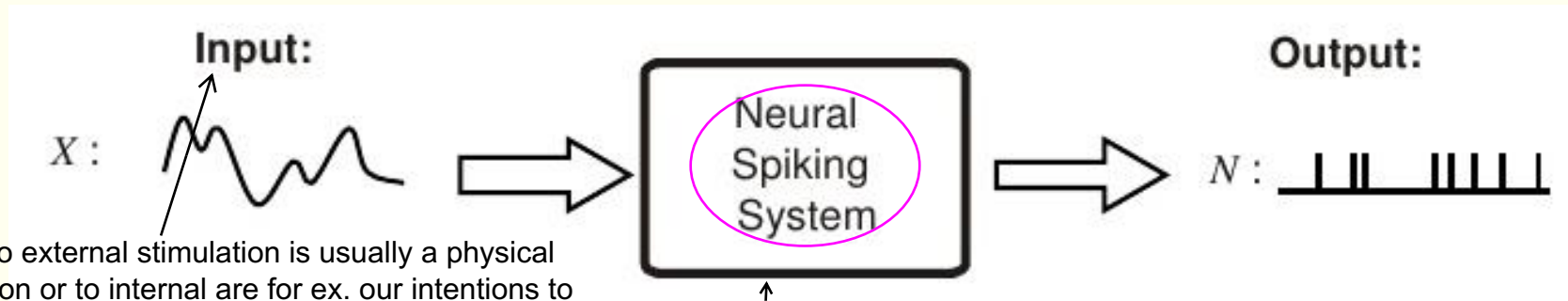
1. **Understand** the definition of neural encoding and decoding
2. **Describe** the rate-coding hypothesis
3. **Define** the neural response function and the spike-count firing rate
4. **Illustrate** the experimental procedure used to build a tuning curve
5. **Interpret** a given a tuning curve, and specifically:
  - a. **Describe** the neuron behavior as a response to stimulus properties
  - b. **Compare** the neural response to specific values of the stimulus  $s$

# Introduction

it's the process in which we are able to characterize a neuron as a function linking inputs and outputs to the neuron

**Neural (En/De)coding:** measuring and characterizing how an external (physical, e.g., light or sound intensity) or internal (e.g. the direction of a planned movement) input received by a neuron is translated into a sequence of action potentials (output):

using a detailed model for each neuron is not feasible because each single model will be very complicated and impossible to use



related to external stimulation is usually a physical stimulation or to internal are for ex. our intentions to move some part of the body or say something

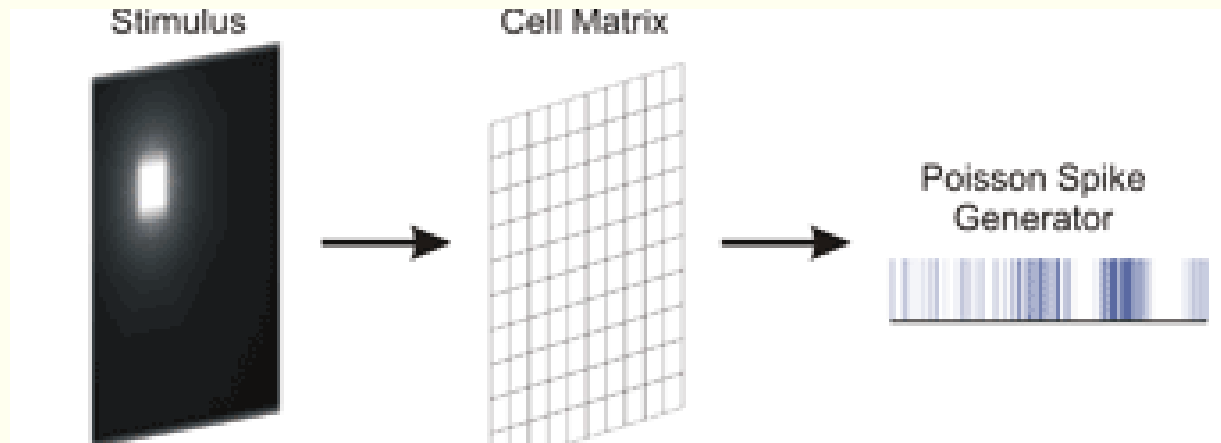
we ignore the neuron feature and we focus on the relation between input and output

# Neural encoding

From stimulus to response

Neural response =  $f(\text{stimulus})$

- *Aim:* describing how neurons react to different stimuli and trying to **predict their response to a new one**

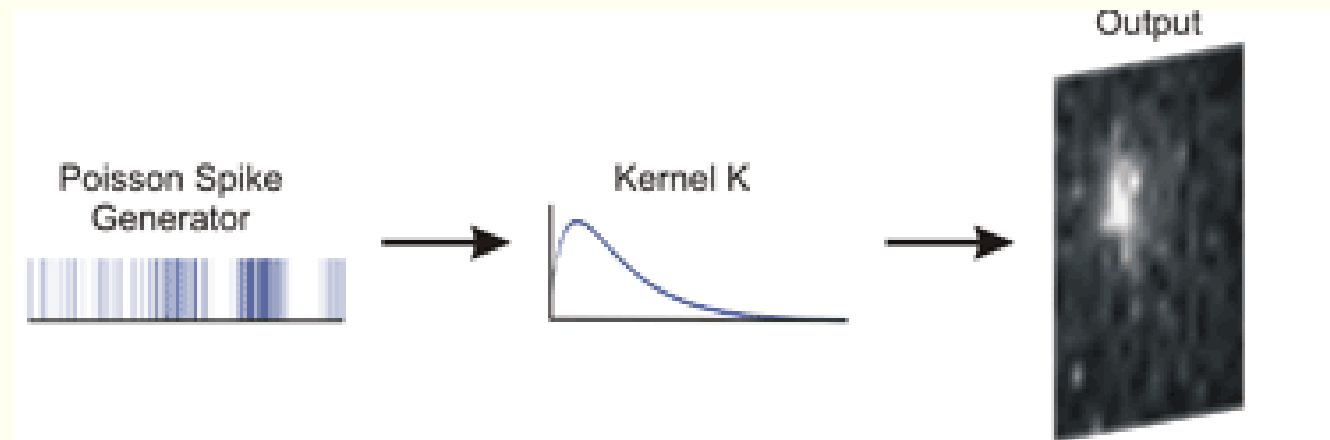


# Neural decoding

From neural response to the stimulus that induced it

**Stimulus =  $f^{-1}(\text{neural response})$**

- *Aim*: **recognize the stimulus** (or its properties) that induced the spike train response

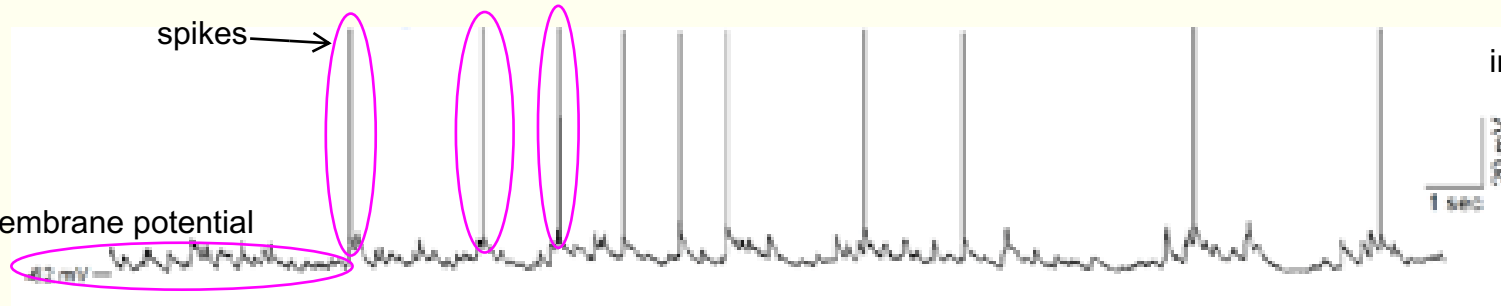


# Rate-coding hypothesis

it's just an hypothesis but supported in several cases

- The rate-coding hypothesis suggests that spike frequency (or rate) is the fundamental mechanism of coding information ↑ how many spike we have in a temporal unit
- e.g.: the number of action potentials from cutaneous nerve fibers in the leg of a cat was proportional to the pressure applied to the footpad (Adrian and Zotterman, 1926)

# Spike trains and firing rate



intracellular recording performed on a single neuron

each action potential is represented by Dirac delta

- Each spike is like a Dirac  $\delta$  function
- For  $n$  spikes,  $t_i$  is the time in which the  $i^{\text{th}}$  spike occurred:

$$i=1,2,\dots,n \text{ and } 0 \leq t_i \leq T \quad (T = \text{trial duration})$$

- The spike train can be thus written as:

neural response →

output of the cell

mathematical description →

$$\rho(t) = \sum_{i=1}^n \delta(t - t_i)$$

we use this to model the sequence

$\rho(t)$  is the **neural response function**



# Spike-count firing rate

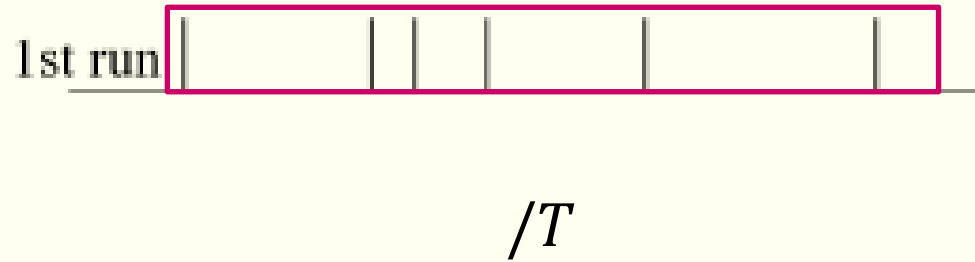
important property because there is a theory according to which the way in which the neurons encode their info is in the frequency with which they produce a train of action potential(spike train).

$$r = \frac{n}{T} = \frac{1}{T} \int_0^T \rho(\tau) d\tau$$

$r(t)$ : firing rate  
 $r$ : spike-count firing rate  
 $\langle r \rangle$ : average firing rate

time window

$$\int_0^T \rho(\tau) d\tau = n$$



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The spike-count firing rate is the time average of the neural response function over the duration of the trial. → number of events/temporal duration of the time window we're recording

$$n/T = \text{Hz}$$

$$\text{e.g.: } T = 1 \text{ s} \quad n = 6$$

$$\text{firing rate} \rightarrow \underline{r} = 6 \text{ Hz}$$

# Stimulus-response link

- Spike sequences reflect both the **intrinsic dynamics of the neuron** and the **temporal characteristics of the stimulus**
- We can use the spike-count firing rate when the stimulus is **stationary** (otherwise we need a **time-resolved** definition for the spike rate)
- Many neurons respond to the same given stimulus → we should examine the **relationships of these firing patterns** to each other across the population of responding cells (neural networks)

# Stimulus-response link

- Neural responses can <sup>number of spikes change</sup> vary across repetitions (trials) even when the same stimulus is presented repeatedly → We cannot describe the timing of each spike deterministically → we use a probabilistic approach (average firing rate  $\langle r \rangle$  across trials)

average number of spikes

$$\langle r \rangle = \frac{\langle n \rangle}{T} = \frac{1}{T} \int_0^T r(t) dt$$

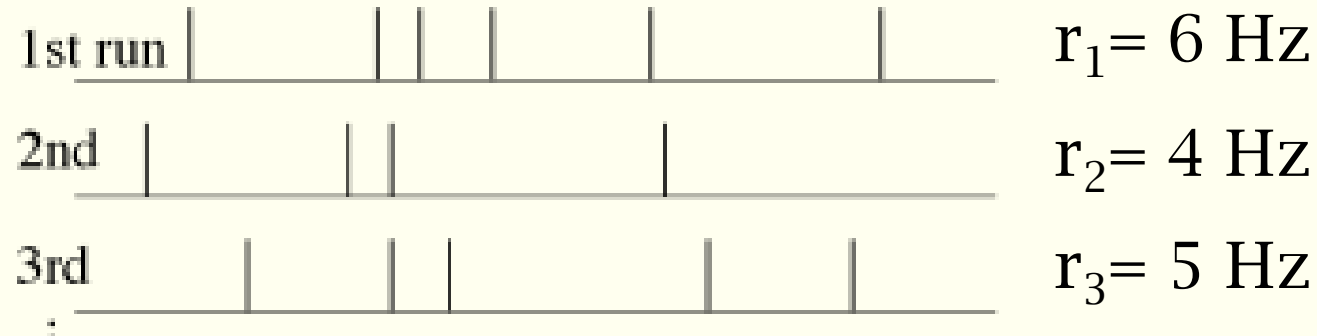
more accurate, robust and meaningful for our

$r_i = n_i/T$

Handwritten:  $r(t) = \frac{1}{\Delta t} \int_t^{t+\Delta t} dt \langle p(t) \rangle$

e.g.:

considering  $T=1s$



$$\langle r \rangle = \langle n \rangle / T = [(6+4+5)/3] / 1 = 5 \text{ Hz}$$

$$\langle r \rangle = 5 \text{ Hz}$$

we can use the firing rate to build the tuning curves. they are the core topic of the neural encoding, so it means building an appropriate tuning curve for that neuron

# Tuning curves

it's a function that links stimulus property  $s$ (input) with firing rate(output). So the output is not the sequence of spikes but the property of firing rate

if we have many trials we use the average firing rate as output

A stimulus is characterized by many properties (e.g.: shape, orientation, contrast, movement, ...)

We focus on a single property  $s$  and we assume it's stationary along each trial (repetition of the stimulation)



$$\langle r \rangle = f(s)$$

Tuning curve of the neuronal response

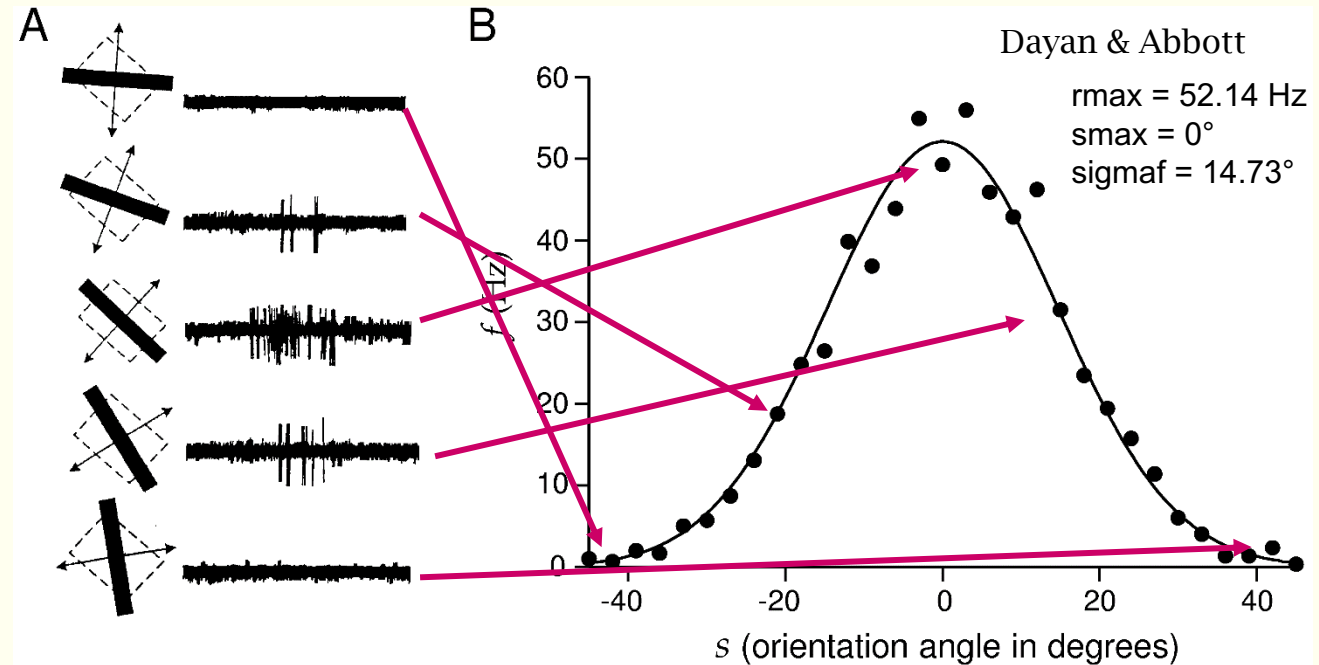
this experiment starts with a actual neuron studied "in vitro" or "in vivo"(this term is better). This was an invasive measurement

you remove the cell from the subject and analyze them out of the brain

means that you measure the cellular activity by keeping it in its natural position(so in the brain)

# Examples of tuning curves - 1

- Primary visual cortex in an animal subject (monkey)
- Visual stimulation (bar with different orientations)
- $s$  = orientation <sup>light bar</sup> angle of the bar (degrees)



$$f(s) = r_{max} e^{-\frac{1}{2} \left( \frac{s - s_{max}}{\sigma_f} \right)^2}$$

in the range  $-90^\circ$  to  $+90^\circ$

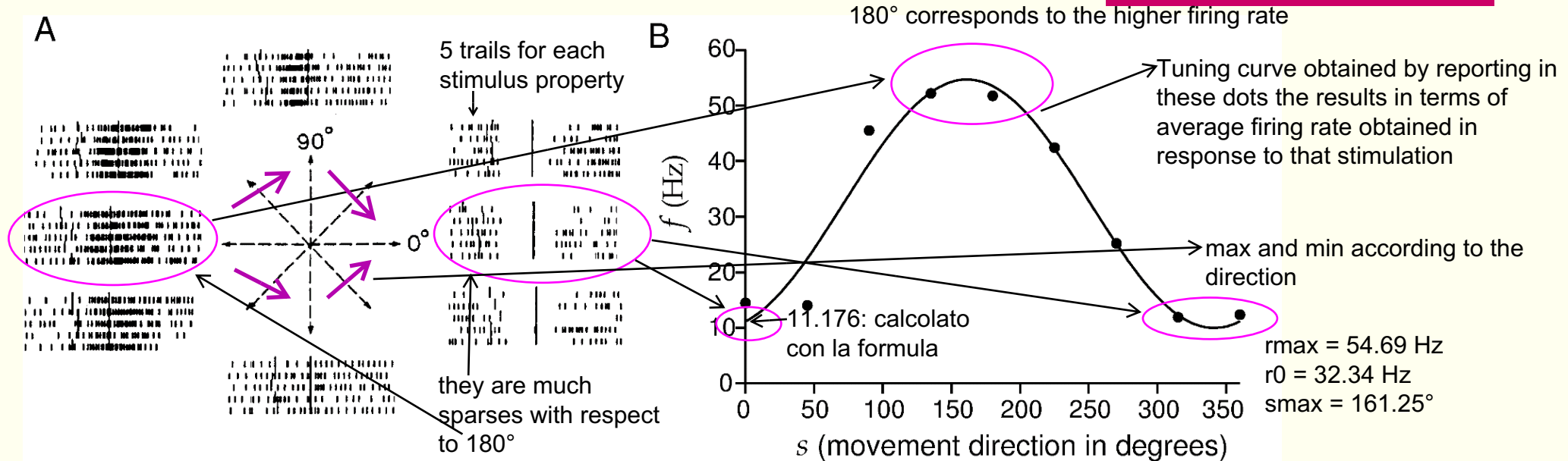
$s_{max}$  = angle evoking the maximum response  $r_{max}$

$\sigma_f$  = amplitude of the tuning curve

neuron in the primary motor cortex. This is organized in somatotopic fashion which means that you have the homunculus on the somatosensory cortex

## Examples of tuning curves - 2

these are the results of experiments performed by making the trained monkey to reach in different directions



- A neuron in the primary motor cortex of a monkey trained to reach in different directions
- The firing rate of the cell is correlated with the direction of arm movement s (in degrees)

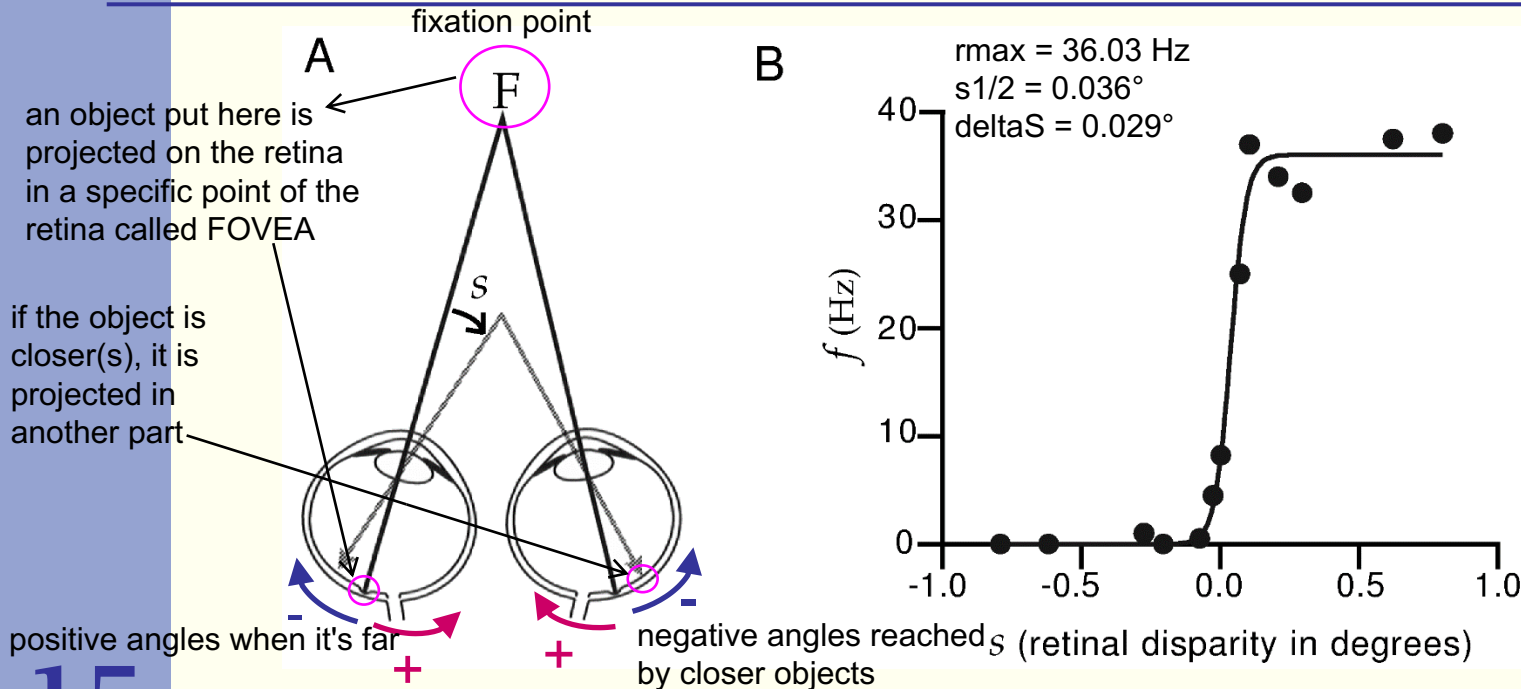
$$f(s) = r_0 + (r_{max} - r_0) \cos(s - s_{max})$$

s = movement direction (degrees)

s<sub>max</sub> = angle evoking the max response r<sub>max</sub>

r<sub>0</sub> = offset (to avoid negative firing rate)  
or background firing rate

# Examples of tuning curves - 3



- Primary visual cortex neuron (cat) reacting to retinal disparity (a difference in the retinal location of an image between the two eyes)
- F: fixation point
- The neuron responds only to positive  $s \rightarrow$  far objects (far-tuned neuron)

$$f(s) = \frac{r_{max}}{1 + \exp[(s_{1/2} - s)/\Delta s]}$$

$s$ =binocular retinal disparity (degrees)

$S_{1/2}$ =disparity inducing a response equal to  $\frac{1}{2}$  of the maximum  $r_{max}$

$\Delta_s$ =controls how quickly the firing rate increases as a function of  $s$

controls how quickly the firing rate moves from 0 to the max. So it's small

If  $\Delta s$  is negative, the firing rate is a monotonically decreasing function of  $s$  rather than a monotonically increasing function

# References

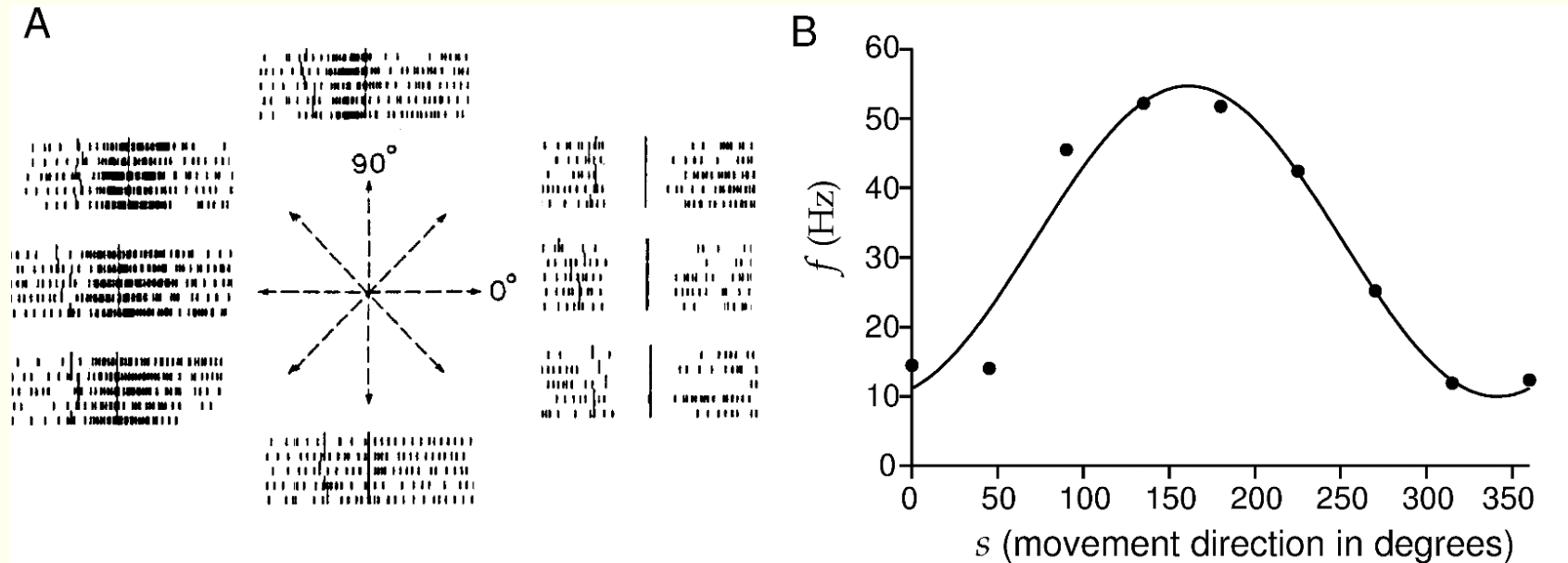
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- Dayan & Abbott:
  - Chapter 1.1 (From Stimulus to Response)
  - Chapter 1.2 (Spike Trains and Firing Rates, Tuning Curves)



# Self-evaluation

- Given the following tuning curve:



- Is the neural response for a movement direction of 90 degrees greater than for 180 degrees?
- Will I build a different tuning curve for each trial?
- Which firing rate can I expect when the movement direction is 250 degrees?
- If the measured firing rate is 55Hz, can I «guess» which was the movement direction that produced that response?