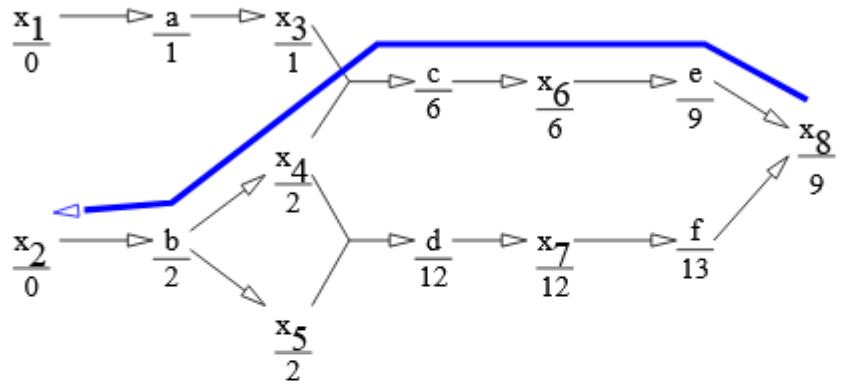
critical path heuristics

another way of viewing the maximal heuristics h^{max}

extend it

maximal heuristics, in retrospect



e and f both make x_8 true
alternative actions: cheapest

x_3 and x_4 both needed to execute c
preconditions: maximal cost only

cheapest way to go the goal
with the additional simplification of a single precondition per action

no way to do it better: *critical path*
minimal absolute possible cost of reaching the goal

critical path: extension

instead of "single precondition per action"

use: "two preconditions per actions"

[note] Switching from one to two creates an additional complication, since a pair of variables may be made true by the same action or by two different ones.

But delete effects can be taken into account somehow.

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maximal of two

base of h^{max}:

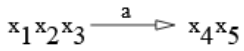
> obtaining all three of x_1, x_2 and x_3
> cannot be easier than obtaining x_1 alone,
> or x_2 alone
> or x_3 alone

safe choice: obtaining three is the maximal cost of obtaining each

also safe is the maximal cost of obtaining each *pair*:

> obtaining all three of x_1, x_2 and x_3
> cannot be easier than obtaining x_1 and x_2,
> cannot be easier than obtaining x_1 and x_3
> and cannot be easier than obtaining x_2 and x_3

preconditions and effects of action



example: action a
preconditions x_1 x_2 x_3
effects x_4 x_5

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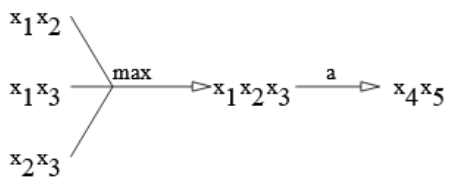
pairs of precondition



before: cost is maximal of obtaining x_1 alone, x_2 alone or x_3 alone

now: maximal of obtaining the pairs

not so easy…

needed pairs

no action has x_5x_6 in a precondition
⇒ do not calculate the cost of this pair, pointless

an action has x_3x_6 in a precondition
⇒ calculate the cost of this pair

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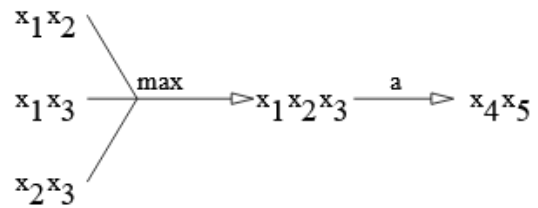
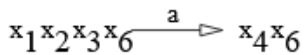
other pairs



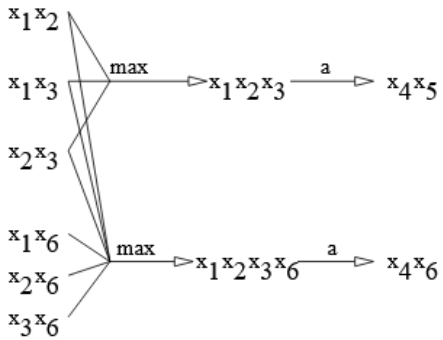


if some other action has x_4x_6 as a precondition
⇒ calculate the cost of this pair
⇒ how is this pair obtained?

x_4x_6 made true by a if x_6 is already true!

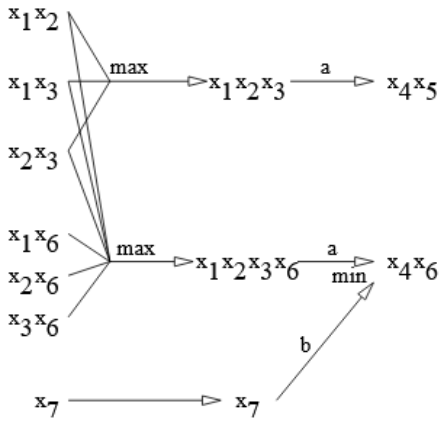
requires: preconditions of a and x_6
so: x_1 x_2 x_3 x_6

pairs, again



to make x_1 x_2 x_3 x_6 true:
maximal cost of making a pair true

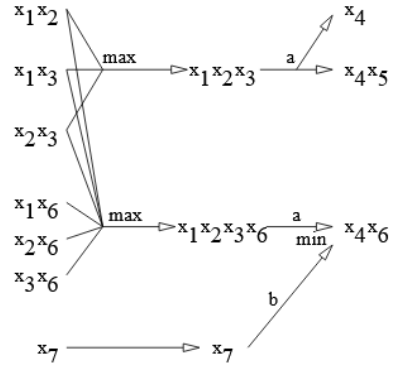
multiple actions



example: action b
precondition x_7
effects x_4x_6

x_4x_6 can be obtained both by a and b
alternative: minimal cost of the two

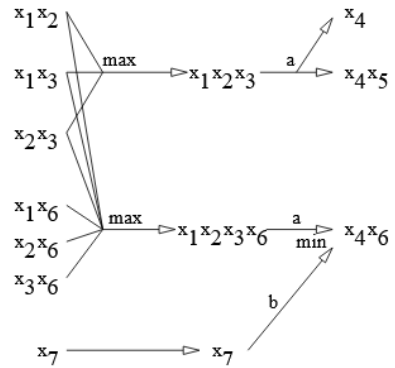
single variables



if some action has one precondition only
⇒ calculate its cost

in the example: a also makes x_4 alone true

delete effects



if a does not delete x_6
⇒ makes x_4x_6 true if x_6 was true before

if a deletes x_6
⇒ does not make x_4x_6 true even if x_6 was true before

[note] Incorporating delete effects this way allows excluding a as a possible way for obtaining x_4x_6.

Since it only applies to *pairs* of variables, it cannot be applied to the maximal heuristics h^{max}, which only consider *single* variables.

triples, etc.

heuristics using subsets of at most m variables: h^m

polynomial for every fixed m

only h^1 (=max heuristics) and h^2 used in practice

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mathematical formalization

example: maximal heuristics

start with $cost(x_i) = 0$ if x_i initially true
otherwise $cost(x_i) = \infty$
$cost(a)$ = cost of executing a alone

keep updating costs until they do not change

$$cost(x_i) = min(cost(x_i), P_i)$$

where P_i is:

$$P_i = min_{a \,.\, x_i \in add(a)} (cost(a) + max_{x_j \in pre(a)} (cost(x_j)))$$

and $pre(a)$ = preconditions of a
and $add(a)$ = positive effects of a

[note] The cost of obtaining a variable x_i is the minimal overall cost of the actions that have x_i as an effect. The overall cost is the cost of the action plus its preconditions. So far, this is an exact calulation.

The approximation enter the scent at this point: instead of computing the cost of the preconditions, their maximum individual cost is considered. In the case of h^2, the maximal cost of pairs of preconditions is used instead.

critical path

max heuristics = generating a variable is the same as generating is hardest-to-obtain precondition

go back from the goal to the initial state

cost of the *critical path* of actions
from initial state to goal

h^m: same, but for pair/triples/quadruples/etc. of variables

all of them: *critical path heuristics*

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relaxation and critical path heuristics

relaxation
    ignore delete effects
critical path
    obtain something = obtain its hardest part

delete effects irrelevant to h^1
also a relaxation heuristics

h^m keeps them into account for $m \geq 2$
not a relaxation heuristics

admissibility

maximal and its generalization h^m:
cost = maximal cost of a subset
indeed: subset needs to be achieved
admissible

sum and FF:
sum the cost of actions
but, some actions may be redundant
in such cases, cost is overestimated
not admissible