Process Automation (MCER), 2016-2017

EXAM

23 Jan. 2017 – 2h30

Exercise 1 (12 pts.)

Consider a process whose step response model is given by the following coefficients:

$g_1 = 0,\ g_2 = 0.15,\ g_3 = 0.2,\ g_4 = 0.21,\ g_5 \approx 0.21,\ g_6 \approx 0.21 \dots .$

Compute the future control action sequence of a MPC algorithm at time $t = 10$, in two cases:

A) reference $r(t)$ unknown to the controller;
B) planned reference dynamics ($r(t)$ known to the controller)

In both cases, consider the following information:

- control horizon $m = 2$;
- reference $r(t) = \begin{cases} 0.1t, & 0 \le t < 12 \\ r(t-1) - 0.1, & t \ge 12 \end{cases}$;
- cost function $J = e^T e + \lambda u^T u$, where e is the vector of future errors between predicted output and reference trajectory and $\lambda = 0.2$;
- control actions and available measured outputs at time $t = 10$:
    - $u(5) = 1, u(6) = 1.2, u(7) = 1.4, u(8) = 1.7, u(9) = 1.9$
    - $y_m(6) = 0.4, y_m(7) = 0.52,\ y_m(8) = 0.64,\ y_m(9) = 0.76,\ y_m(10) = 0.87$

Exercise 2 (12 pts.)

Let the process be described by the transfer function: $P(s) = \frac{s-1}{(s+10)(s+1)} e^{-\theta s}$, with $\theta \in (0.5,11)$, and let the process model be $\tilde{P}(s) = \frac{s-1}{(s+10)(s+1)} e^{-s}$.

Design a controller by using the IMC design such that:

a. the controlled system has 0 steady-state error for step inputs;
b. the controlled system is robustly stable against the uncertainties of the parameter θ.

If possible, write the controller as a Smith Predictor controller.

Questions (6 pt.)

i) Why may a feedback delay cause instability and when is it possible to use a Padé approximation?
(1/2 pg. max, 3pt)

ii) Which are the main advantages of MPC in assuring safety of industrial plants with respect to classic feedback control srategies?
(1/2 pg. max, 3pt)

Solution of exercise 1

Firstly, we note that the coefficient g_1 is 0: this means that the process has an input-output delay $d = 1$. Then, we have to consider a prediction horizon $p \geq m + 1 = 3$. We chose $p = 3$. The samples g_4, g_5 and g_6 have the same value; thus, we select the first 4 samples as the step response model, i.e., $N = 4$.

The dynamic matrix is $G = \begin{pmatrix} g_1 & 0 \\ g_2 & g_1 \\ g_3 & g_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0.15 & 0 \\ 0.2 & 0.15 \end{pmatrix} \in \mathbb{R}^{p \times m}$.

The matrix G is used to compute the solution of the constrained optimization problem: $u = (G^T G)^{-1} G^T (w - f)$, with $(G^T G + \lambda I)^{-1} G^T = \begin{pmatrix} 0 & 0.58 & 0.70 \\ 0 & -0.08 & 0.58 \end{pmatrix}$.

The reference trajectory $r(t)$ is computed from the given formula.

A) If the reference is known to the controller, we consider $w(t) = r(t), \forall t$: at time $t = 10$ we will then consider the following references over the prediction period: $w_A(11) = 1.1, w_A(12) = 1, w_A(13) = 0.9$.

B) If the reference is unknown to the controller, we consider $w(t + k) = r(t), k = 1,2,3,\ \forall t$:
$w_B(10 + k) = r(10) = 1, k = 1,2,3$.

The past control variations are computed as follows:
$\Delta u(6) = u(6) - u(5) = 0.2, \Delta u(7) = u(7) - u(6) = 0.2, \Delta u(8) = u(8) - u(7) = 0.3,$
$\Delta u(9) = u(9) - u(8) = 0.2.$

Then, we compute the free response at time $t = 10$ over the prediction horizon:

$t = 10$

$k = 1$ $\quad f(11) = y_m(10) + \sum_{i=1,\dots,4} (g_{i+1} - g_i) \Delta u(10 - i) =$
$= 0.87 + (g_2 - g_1)\Delta u(9) + (g_3 - g_2)\Delta u(8) + (g_4 - g_3)\Delta u(7) + (g_5 - g_4)\Delta u(6);$
$= 0.87 + 0.15 \cdot 0.2 + 0.05 \cdot 0.3 + 0.01 \cdot 0.2 = 0.92;$

$k = 2$ $\quad f(12) = y_m(10) + \sum_{i=1,\dots,4} (g_{i+2} - g_i) \Delta u(10 - i) =;$
$= 0.87 + (g_3 - g_1)\Delta u(9) + (g_4 - g_2)\Delta u(8) + (g_5 - g_3)\Delta u(7) + (g_6 - g_4)\Delta u(6);$
$= 0.87 + 0.2 \cdot 0.2 + 0.06 \cdot 0.3 + 0.01 \cdot 0.2 = 0.93;$

$k = 3$ $\quad f(13) = y_m(10) + \sum_{i=1,\dots,4} (g_{i+3} - g_i) \Delta u(10 - i) =.$
$= 0.87 + (g_4 - g_1)\Delta u(9) + (g_5 - g_2)\Delta u(8) + (g_6 - g_3)\Delta u(7) + (g_7 - g_4)\Delta u(6);$
$= 0.87 + 0.21 \cdot 0.2 + 0.06 \cdot 0.3 + 0.01 \cdot 0.2 = 0.93;$

The control action sequence is then computed in the two cases:

A) $\begin{pmatrix} \Delta u(10) \\ \Delta u(11) \end{pmatrix} = (G^T G + \lambda I)^{-1} G^T (w_A - f) = \begin{pmatrix} 0 & 0.58 & 0.70 \\ 0 & -0.08 & 0.58 \end{pmatrix} \left(\begin{pmatrix} 1.1 \\ 1 \\ 0.9 \end{pmatrix} - \begin{pmatrix} 0.92 \\ 0.93 \\ 0.93 \end{pmatrix} \right) = \begin{pmatrix} 0.02 \\ -0.02 \end{pmatrix}$

$\Rightarrow u(10) = u(9) + \Delta u(10) = 1.92\ , u(11) = u(10) + \Delta u(11) = 1.9.$

B) $\begin{pmatrix} \Delta u(10) \\ \Delta u(11) \end{pmatrix} = (G^T G + \lambda I)^{-1} G^T (w_A - f) = \begin{pmatrix} 0 & 0.58 & 0.70 \\ 0 & -0.08 & 0.58 \end{pmatrix} \left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0.92 \\ 0.93 \\ 0.93 \end{pmatrix} \right) = = \begin{pmatrix} 0.09 \\ -0.04 \end{pmatrix}$

$\Rightarrow u(10) = u(9) + \Delta u(10) = 1.99\ , u(11) = u(10) + \Delta u(11) = 1.95.$

Solution of exercise 2.

The nominal process $\tilde{P}(s)$ is stable, therefore it is possible to design a stable controller $Q(s)$ to stabilize the closed-loop nominal system.
Moreover, since the time-delay θ of the process is larger then the time constant $\tau = 0.1s$ of the process, we cannot use a Padé approximation to write the delay term as a transfer function.

The IMC design consists in the following steps:

Step 1)

a) Factorize the nominal process in a minimum-phase term and a non-minimum-phase term:

$\tilde{P}(s) = \tilde{P}_+(s)\tilde{P}_-(s)$, with $\tilde{P}_+(s) = (1-s)e^{-s}$ and $\tilde{P}_-(s) = -\frac{1}{10}\frac{1}{(1+s/10)(1+s)}$.

b) Define the controller as follows: $\tilde{Q}(s) = \left(\tilde{P}_-(s)\right)^{-1} = -10 \cdot (1+s/10)(1+s)$.

Step 2)

Design the controller $Q(s) = \tilde{Q}(s)f(s)$, where the IMC filter $f(s)$ must be such that a) the controller $Q(s)$ is proper and b) the overall system is of type 1 (i.e., $\tilde{T}(0) = \tilde{P}(0)Q(0) = 1$).
Thus, we use the well-known filter $f(s) = \frac{1}{(1+\lambda s)^n}$ with $n = 2$. In fact:

a) $Q(s) = \tilde{Q}(s)f(s) = -10\frac{(1+s/10)(1+s)}{(1+\lambda s)^2}$ is proper;

b) $\tilde{T}(0) = \tilde{P}(0)Q(0) = \left[\tilde{P}_+(s)\tilde{P}_-(s)\left(\tilde{P}_-(s)\right)^{-1} f(s)\right]_{s=0} = \left[\frac{1-s}{(1+\lambda s)^2}e^{-s}\right]_{s=0} = 1$.

Step 3)

Determine the value of λ such that the sufficient condition for robust stability holds:

$\left|l_m(j\omega)\tilde{T}(j\omega)\right| < 1, \forall\omega,$

where

$$\left|\tilde{T}(j\omega)\right| = \left|\tilde{P}(j\omega)Q(j\omega)\right| = \left|-\frac{1}{10}\frac{1-j\omega}{\left(1+\frac{j\omega}{10}\right)(1+j\omega)}e^{-j\omega} \cdot \left(-10\frac{\left(1+\frac{j\omega}{10}\right)(1+j\omega)}{(1+\lambda j\omega)^2}\right)\right| = \left|\frac{1-j\omega}{(1+\lambda j\omega)^2}\right|,$$

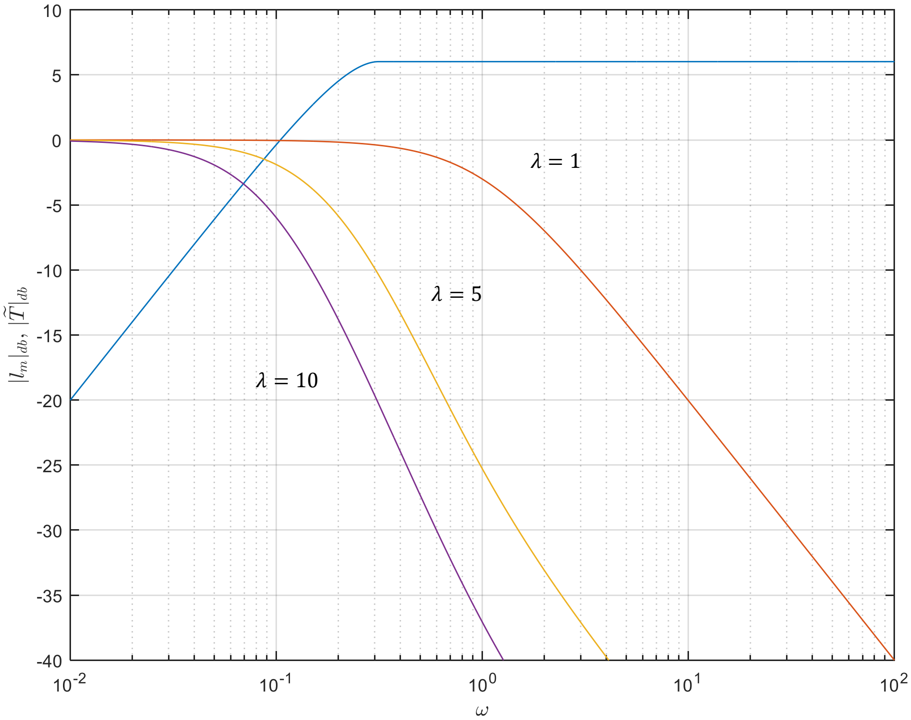
and $l_m(j\omega)$ is an upper-bound of the multiplicative uncertainty $\Delta_m(j\omega)$, i.e., a function such that $|l_m(j\omega)| > |\Delta_m(j\omega)|, \forall\omega$.

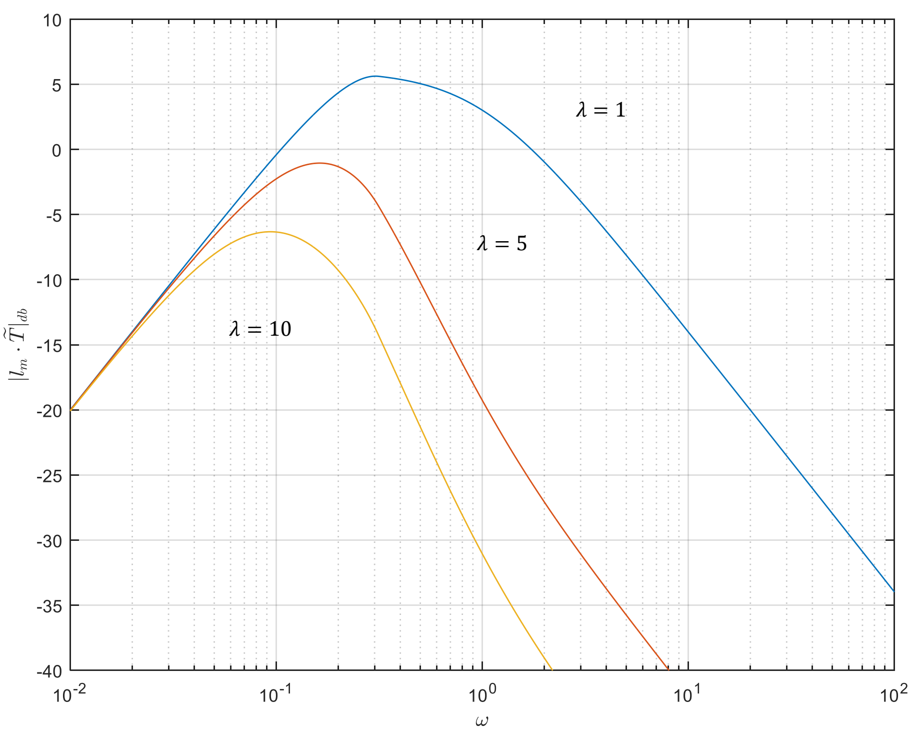
By definition, the multiplicative uncertainty is defined as

$$\Delta_m(j\omega) := \frac{P(j\omega)-\tilde{P}(j\omega)}{\tilde{P}(j\omega)} = \frac{\frac{j\omega-1}{(j\omega+10)(j\omega+1)}e^{-\theta j\omega} - \frac{j\omega-1}{(j\omega+10)(j\omega+1)}e^{-j\omega}}{\frac{j\omega-1}{(j\omega+10)(j\omega+1)}e^{-j\omega}} = e^{-j\omega\delta} - 1, \text{ with } \delta = \theta - 1.$$

From theory, we know that an upper-bound is defined as $l_m(j\omega) = \begin{cases} e^{-j\omega\delta_{max}} - 1, & if\ \omega \le \frac{\pi}{\delta_{max}} \\ 2, & if\ \omega > \frac{\pi}{\delta_{max}} \end{cases}$, with $\delta_{max} = \max\limits_{\theta\in(0.5,3)}|\delta| = \max\limits_{\theta\in(0.5,3)}|\theta - 1| = 10$.

The figures below show that for $\lambda \geq 5$ the condition is met. For instance, we choose $\lambda = 10$.





The resulting IMC controller is then $Q(s) = -10\frac{(1+s/10)(1+s)}{(1+10s)^2}$.

To write the controller in the SP form we recall the SP controller scheme and its IMC form in the following Figure 1. and 2., respectively:

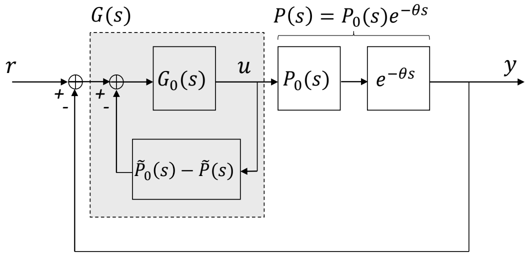


Figure 1.

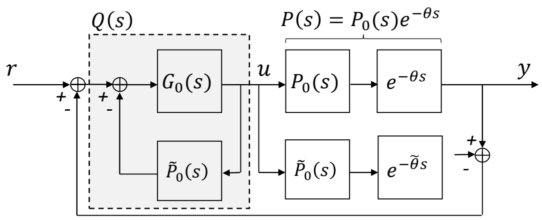


Figure 2.

We then compute $G(s)$ as $G(s) = \frac{G_0(s)}{1+G_0(s)(\tilde{P}_0(s)-\tilde{P}(s))}$ and $G_0(s)$ as $G_0(s) = \frac{Q(s)}{1-Q(s)\tilde{P}_0(s)}$.

The SP primary controller is then $G_0(s) = -\frac{10}{21}\frac{\left(1+\frac{s}{10}\right)(1+s)}{s\left(1+\frac{s}{\frac{100}{21}}\right)}$.