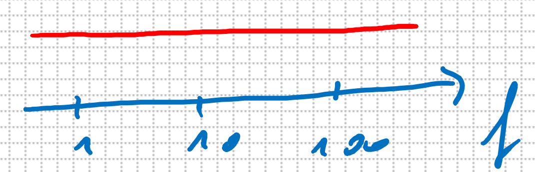
WHITE AND COLOURED NOISE

$w \in \mathbb{R}^{n \times m}$: random vector with components w_i, with 0 mean and $\sigma_i^2 = \sigma^2$

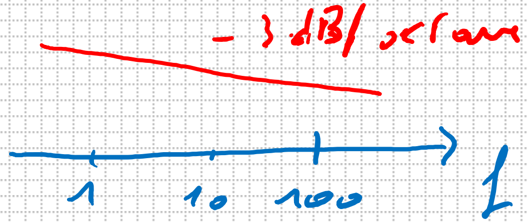
e.g. Gaussian noise $\mathcal{N}(0, \sigma^2)$

Power density per unit of bandwidth $\frac{1}{f^\beta}$

$\beta = 0$ WHITE

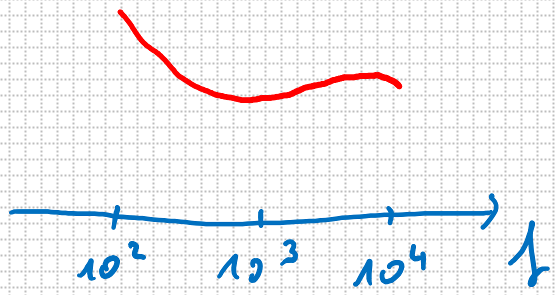


$\beta = 1$ PINK



$\beta = 2$ BROWN

$\beta = -1$ BLUE



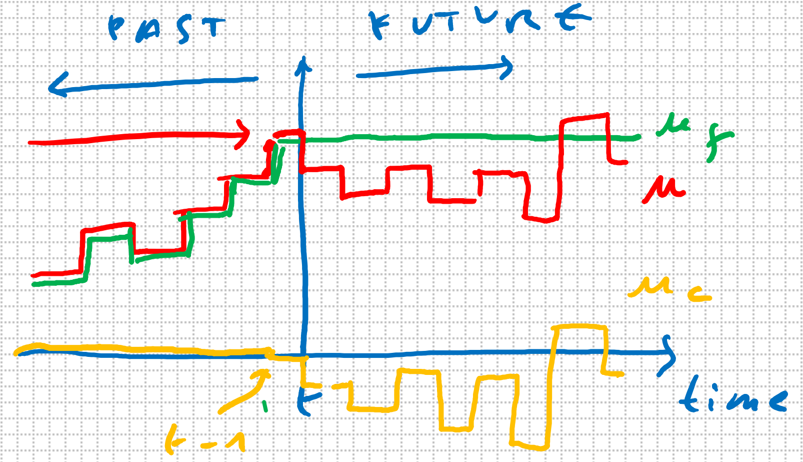
PSYCHOACOUSTIC EQUAL LOUDNESS CURVE

FREE AND FORCED RESPONSES

MPC: $$u(t) = u_f(t) + u_c(t)$$

free   control

$u(t-1)$



$$\begin{cases} u_f(t-j) = u(t-j), & j = 1, 2, \dots \\ u_f(t+j) = u(t-1), & j = 0, 1, 2, \dots \end{cases}$$

$$u_c(t+j) = u(t+j) - u_f(t+j), \quad j = \dots, -1, 0, 1, 2, \dots$$

$$\left(\Rightarrow u_c(t+j) = 0, \quad j = \dots, -2, -1\right)$$

$$\Rightarrow \quad \hat{y}(t+k|t) = \hat{y}_f(t+k|t) + \hat{y}_c(t+k|t)$$

FREE RESPONSE
- Output prediction for $u = u_f$
- Evolution of the process from the present state

FORCED RESPONSE
- Output prediction for $u = u_c$
- Prediction due to future control action variations

OBJECTIVE FUNCTION

- It penalizes the future errors (the difference between the future output $\hat{y}$ and the reference trajectory w)
- It bounds the control effort (u, Δu)

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)\underbrace{\left(\hat{y}(t+j|t) - w(t+j|t)\right)}_{\text{PREDICTED FUTURE ERRORS}}^2 + \sum_{j=1}^{N_u} \lambda(j)\left(\Delta u(t+j)\right)^2$$

it might be $(u(t+j))^2$

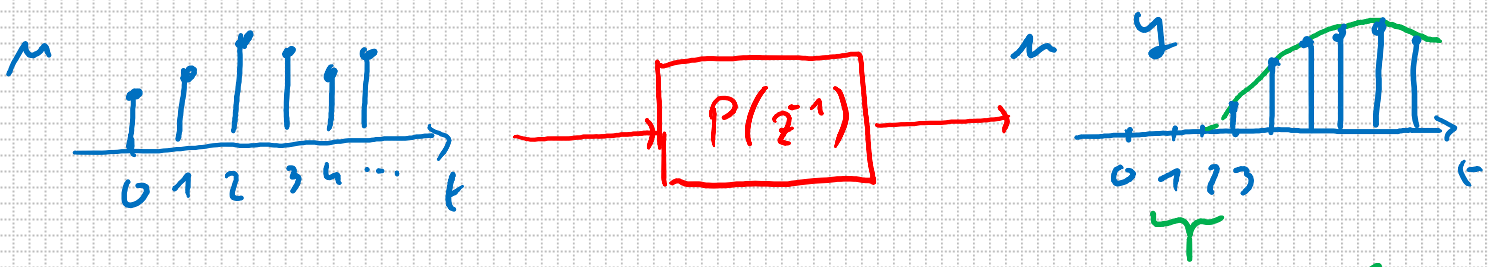
N_2 : PREDICTION HORIZON

$N_1 < N_2$

N_u : CONTROL HORIZON

δ, λ : WEIGHTS FOR THE ERROR AND THE CONTROL ACTION

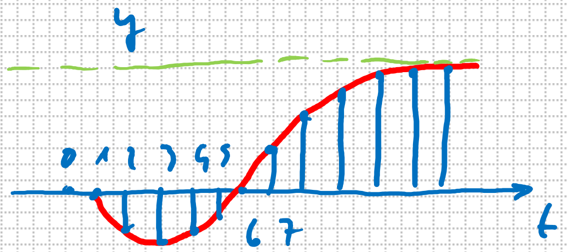
PROCESS WITH TIME-DELAY $d = 2$



$N_1 = 3$ $(N_1 > d)$

$d = 2$ samples will be 0 regardless of u

NON-MINIMUM PHASE PROCESS



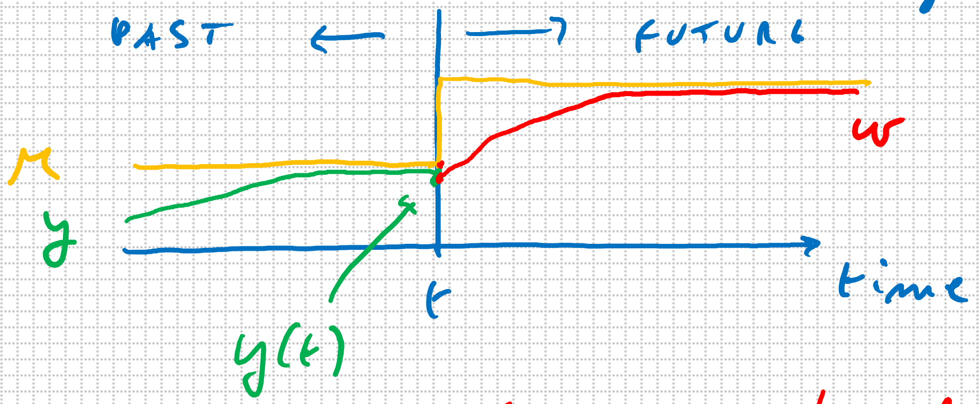
$N_1 = 6$

δ AND λ CAN BE USED TO PENALIZE STEADY-STATE ERRORS MORE THAN INITIAL ONES

REFERENCE TRAJECTORY

- It might be known in the future (trajectory of a robotic arm or of the temperature of a chemical process)
- It might be not known in the future

OFTEN THE REFERENCE TRAJ. w IS DIFFERENT FROM THE REFERENCE SIGNAL r



SET POINT CHANGE AT TIME t

$$w(t+k|t) = \begin{cases} y(t) & \text{if } k=0 \\ \alpha\, w(t+k-1|t) + (1-\alpha)\, r(t+k), & \text{if } k>0 \end{cases}$$

$\alpha \in (0,1)$

OPTIMIZATION PROBLEM

UNCONSTRAINED OPT. PROBLEM

$$\min_{\underline{u}} = J\left(u(t|t), u(t+1|t), \dots, u(t+N_2-1|t)\right)$$

$$\underline{u} = \left[u(t|t), \dots, u(t+N_2-1|t)\right]^t$$

N_2 UNKNOWNS OF THE OPT. PROBLEM

- LINEAR SYSTEM MODEL, QUADRATIC J

$\Rightarrow$ CLOSED-FORM SOLUTION

- NOT TRUE IF WE ADD CONSTRAINTS

CONSTRAINED OPT. PROBLEM

$$\min_{\underline{u}} J(\underline{u})$$

$$\text{s.t.} \quad u_{MIN} \leq u(t+j|t) \leq u_{MAX}, \quad j = 0, \dots, N_2-1$$

$$\Delta_{MIN} \leq u(t+j+1|t) - u(t+j|t) \leq \Delta_{MAX}, \quad j = 0, \dots, N_2-2$$

$$y_{MIN} \leq y(t+j|t) \leq y_{MAX}, \quad j = 0, \dots, N_2-1$$

OBTAINING THE CONTROL LAW

$$J = \sum_{i=N_1}^{N_2} \delta(j)\left(\hat{y}(t+j|t) - w(t+j)\right)^2 + \sum_{i=1}^{N_u} \lambda(j)\left(\Delta u(t+j-1|t)\right)^2$$

$N_u \leq N_2$ CONTROL HORIZON

$$u(t+j-1|t) = u(t+N_u-1|t), \quad N_u+1 \leq j \leq N_2$$

$$\Rightarrow \Delta u(t+j-1|t) = 0, \quad N_u+1 \leq j \leq N_2$$

To avoid undesired high freq. variations of u

$\Rightarrow$ we also reduce the number of unknowns

STRUCTURING THE CONTROL LAW

$$u(t+k|t) = \sum_{i=1}^{n} \underbrace{\mu_i(t)}_{\text{UNKNOWNS}} B_i(k)$$

$, k = 0, 1, \ldots, N_u - 1$

$\{B_1, B_2, \ldots, B_n\}$

SET OF BASE FUNCTIONS TO BE SELECTED BASED ON THE PROCESS, THE REFERENCE, ...

Example : POLYNOMIAL BASE FUNCTIONS $\{1, k, k^2, \ldots, k^n\}$

$B_i = k^{i-1}$

At time t the opt. problem must return n values $\mu_i^*(t)$ (n unknowns vs N_u unknowns)