INTERNAL MODEL CONTROL BASICS

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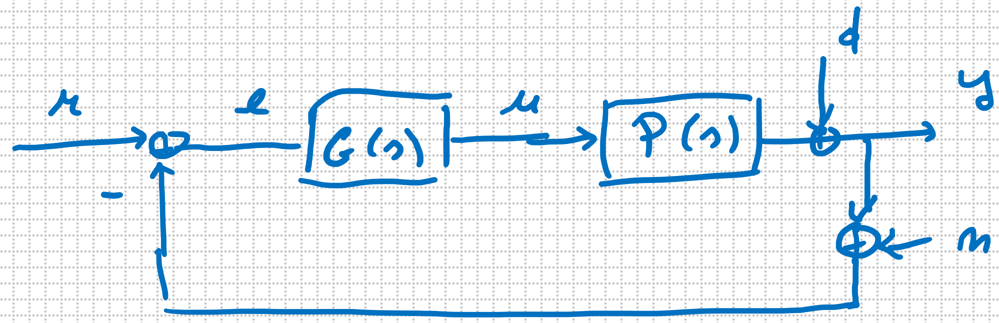
1980's, MORARI and co-workers — ROBUST PROCESS CONTROL, 1989

Idea: CLASSIC CONTROL error: measured output - reference variable

IMC error: measured output - $\underbrace{\text{NOMINAL output}}_{\text{output of the NOMINAL model of the system}}$

- IMC is effective as far as the model is accurate

CLASSIC CONTROL (SISO LTI)



$P(s)$	process
$G(s)$	controller
$y(s)$	output
$r(s)$	reference
$u(s)$	control variable
$d(s)$	disturbance
$n(s)$	measurement noise
$e(s)$	error

$$y(s) = \frac{P(s)G(s)}{1+P(s)G(s)}(r-n) + \frac{1}{1+P(s)G(s)}d =$$

$$= T(s)(r-n) + S(s)\,d$$

$$S(s) = \frac{1}{1+P(s)G(s)}$$ SENSITIVITY FUNCTION

$$T(s) = \frac{P(s)G(s)}{1+P(s)G(s)}$$ COMPLEMENTARY SENSITIVITY FUNCTION

$$(T = 1 - S)$$

$\boxed{S(j\omega)}$ Effectiveness of disturbance rejection

- Physical systems: $\lim_{\omega \to \infty} |P(j\omega)\, G(j\omega)| = 0$

$$\Rightarrow \lim_{\omega \to \infty} |S(j\omega)| = \lim_{\omega \to \infty} \left| \frac{1}{1 + P(j\omega)\, G(j\omega)} \right| = 1$$

$\Rightarrow$ $|S(j\omega)|$ can be "small" only within a limited frequency range

SYSTEM BANDWIDTH ω_s s.t. $|S(j\omega)| > \frac{1}{\sqrt{2}}$

$\boxed{T(j\omega)}$ Accuracy of the regulation

$$\lim_{\omega \to \infty} |T(j\omega)| = \lim_{\omega \to \infty} |1 - S(j\omega)| = 0$$

$T(j\omega)$ can accurate within a freq. range

<u>REQUIREMENTS</u>

- Regulation / Tracking : $T(j\omega) \simeq 1 \quad \forall \omega \in (\omega_1, \omega_2)$

  ω_1, ω_2 : OPERATING FREQ. RANGE of the system

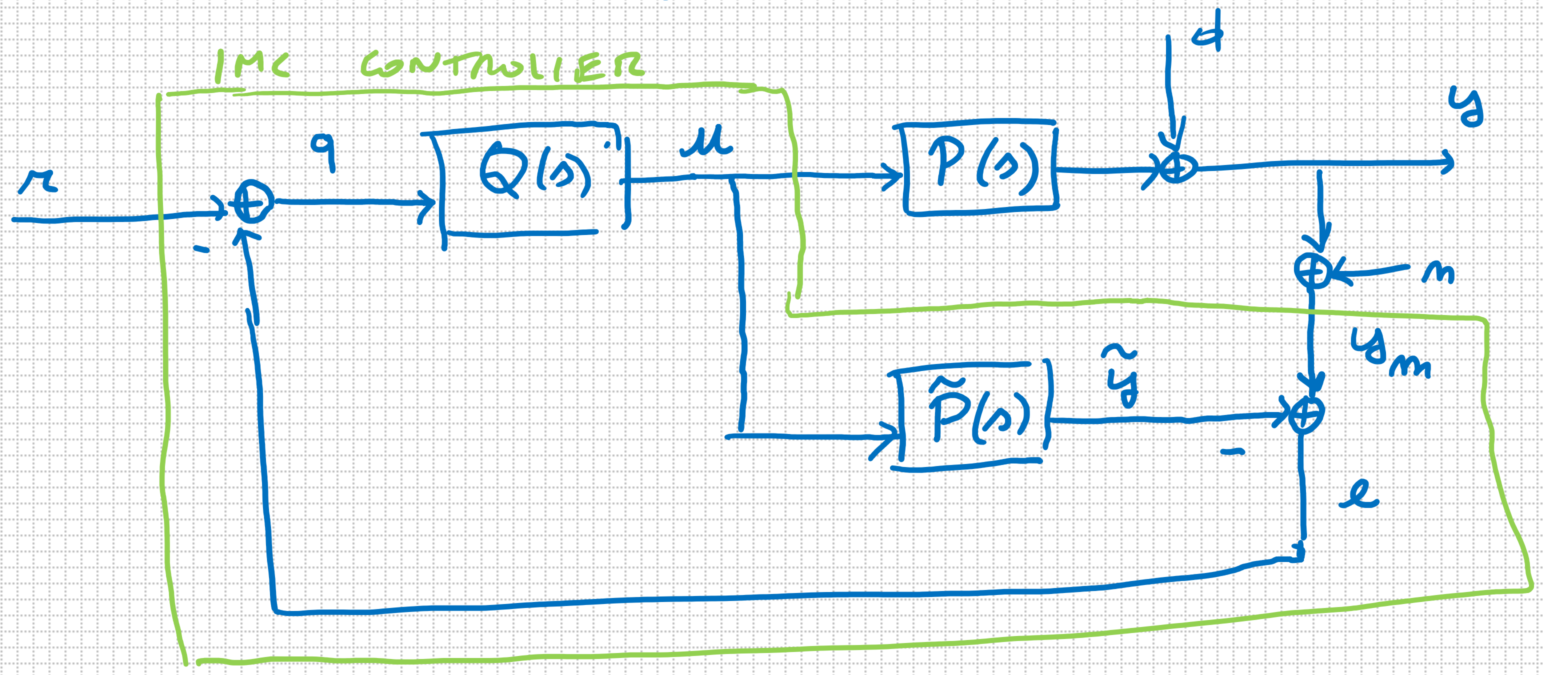
- Disturbance rejection : $S(j\omega) \simeq 0 \quad \forall \omega \in (\omega_1, \omega_2)$

- Measurement noise : $T(j\omega) \simeq 0 \quad \forall \omega > \omega_N$

  $\omega_N > \omega_2$, ω_N : lower freq. of the noise

<u>Complexity</u> : $G(s)$ appears in the DEN of T and S

IMC control scheme



- THE IMC CONTROLLER INCLUDES THE NOMINAL MODEL OF THE PROCESS
- $\tilde{P}(s)$ NOMINAL MODEL

  $\tilde{y}(s)$ NOMINAL OUTPUT

$$e = y_m - \hat{y} = \underbrace{(P - \tilde{P})}_{\text{MODEL UNCERTAINTIES}} u + d + n$$

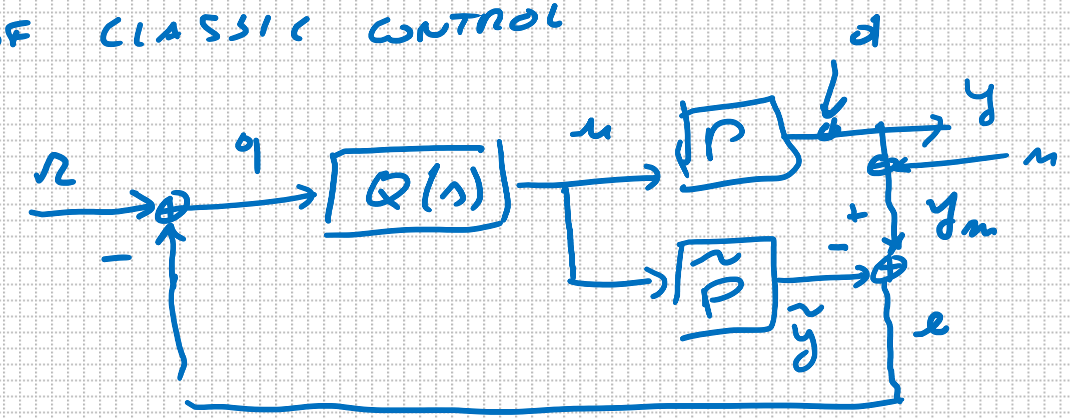
d: disturbance

n: noise

the error $e(s)$ represents all that we don't know about the system

EQUIVALENT CLASSIC CONTROLLER

$e'(s) = r(s) - y_m(s)$ ERROR OF CLASSIC CONTROL



$$
\begin{cases}
u = Q q = Q(r - y_m) + \tilde{P} Q u & (1) \\
q = r - e = (r - y_m) + \tilde{P} u \\
e = y_m - \tilde{y} = y_m - \tilde{P} u \\
\tilde{y} = \tilde{P} u
\end{cases}
$$

$$(1) \Rightarrow (1 - \tilde{P} Q) u = Q(r - y_m)$$

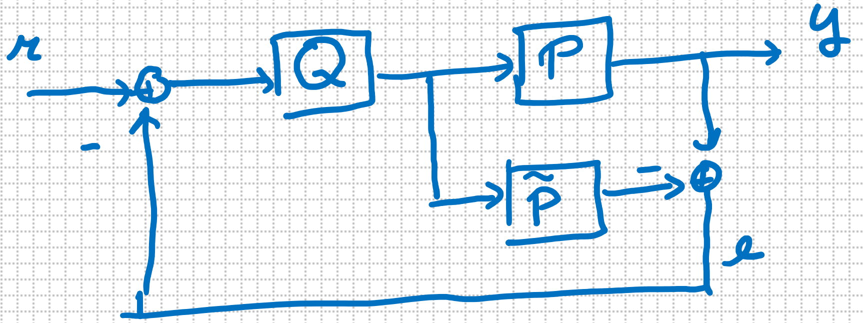
$$\boxed{G(s) = \frac{u}{e'} = \frac{u}{r - y_m} = \frac{Q(s)}{1 - \tilde{P}(s) Q(s)}}$$

EQUIVALENT CLASSICAL FEEDBACK CONTROLLER

$$\left(Q(s) = \frac{G(s)}{1 + \tilde{P}(s) G(s)} \right)$$

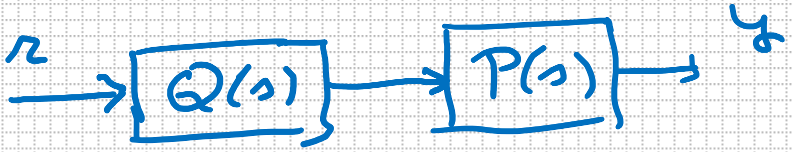
IMC PRINCIPLE

If $\tilde{P}(s) = P(s)$ and $d(s) = n(s) = 0$ (NOMINAL CONDITIONS), the OPEN-LOOP dynamics is the same as the CLOSED-LOOP dynamics



$\equiv$

from the input-output point of view



$y = P(s)\, Q(s)\, r$

(OPEN-LOOP CONTROL)

$\tilde{P} \equiv P$, $d = n = 0$

$e = (P - \tilde{P})u + d + n = 0$

NOMINAL CONDITIONS

For open-loop stable processes the feedback is needed only because of the uncertainties

Sensitivity Functions (SISO LTI)

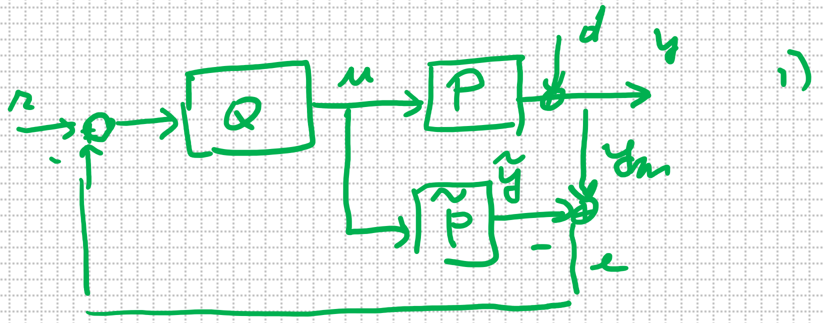
· Nominal Conditions

Performance

$$T(j\omega) = \frac{y(j\omega)}{r(j\omega)} = P(j\omega)\,Q(j\omega)$$

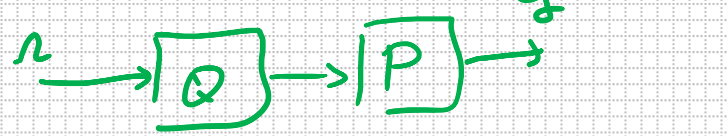
Robustness (against disturbances)

$$S(j\omega) = \left.\frac{y(j\omega)}{d(j\omega)}\right|_{r \equiv 0} \overset{*}{=} 1 - P(j\omega)\,Q(j\omega)$$



$P \equiv \tilde{P}$ $\Downarrow$

$d = 0$



1)
$$\begin{cases} y = Pu + d & (1) \\ u = r - Qe = -Qe = -Qd \quad (r \equiv 0) & (2) \\ e = y - \tilde{y} = \\ \quad = Pu + d - \tilde{P}u = d \quad (\tilde{P} \equiv P) \end{cases}$$

$(2)+(1)$: $y = -PQd + d$

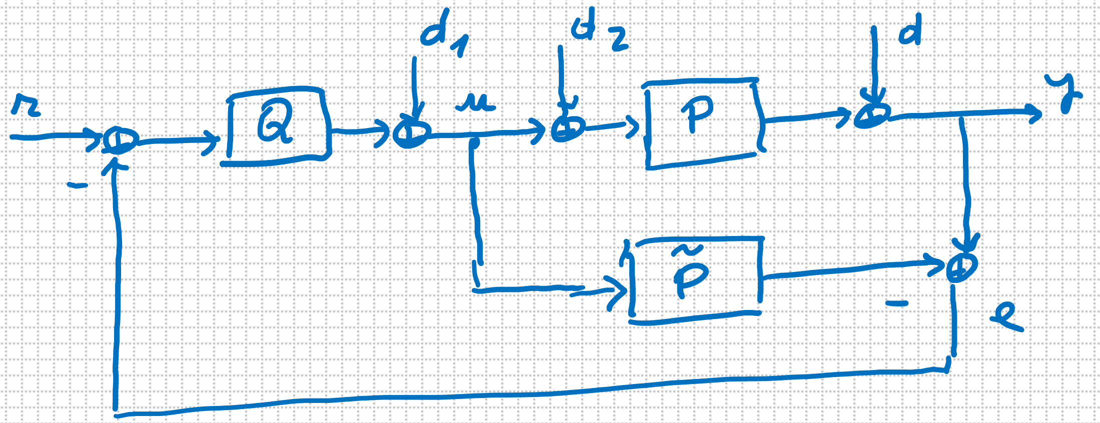
· Design of controller $Q(s)$ appears easier w.r.t. the classic control scheme

NOMINAL STABILITY

ASSUMPTIONS: 1) NOMINAL CONDITIONS

2) $P(s)$ STABLE

$\Rightarrow$ $\boxed{\text{THE CONTROLLED SYSTEM IS STABLE IF } Q(s) \text{ IS STABLE}}$



$$W(s) = \left.\frac{y}{r}\right|_{d=d_1=d_2=0} = P(s)Q(s)$$

$$W_d(s) = \left.\frac{y}{d}\right|_{r=d_1=d_2=0} = 1 - P(s)Q(s)$$

$$W_{d_1}(s) = \left.\frac{y}{d_1}\right|_{r=d=d_2=0} = P(s)$$

$$W_{d_2}(s) = \left.\frac{y}{d_2}\right|_{r=d=d_1=0} = P(s)\left(1 - P(s)Q(s)\right)$$

EQUIVALENT CONTROLLER

$$G(s) = \frac{Q(s)}{1 - \tilde{P}(s)Q(s)} \iff Q(s) = \frac{G(s)}{1 + \tilde{P}(s)G(s)}$$

- NOMINAL CONDITIONS

$\tilde{P}, Q$ STABLE $\iff$ NOMINAL IMC CONTROL SYS. IS STABLE

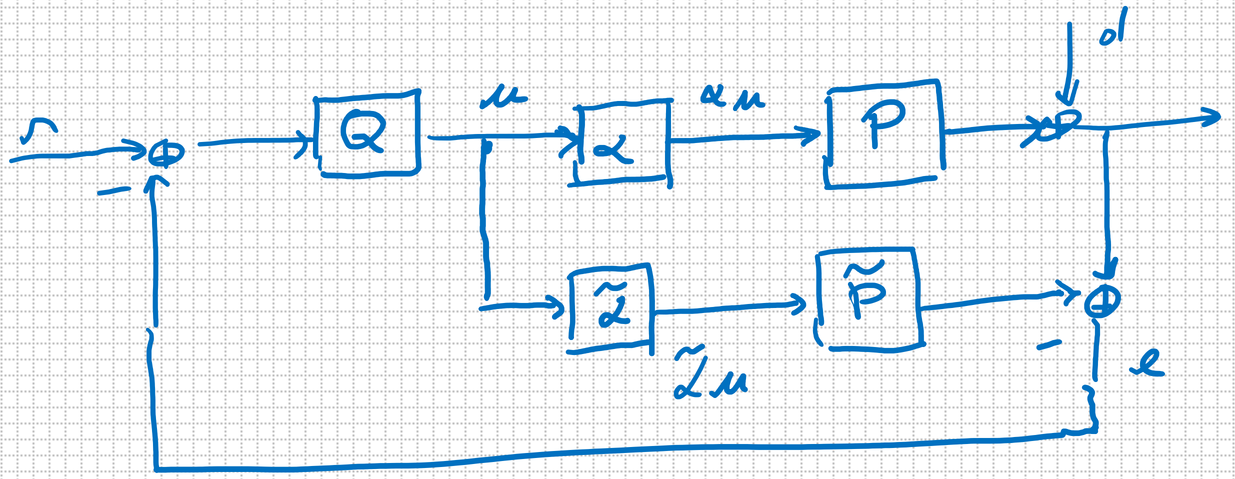
$\Updownarrow$

$D_w(s) = 1 + \hat{P}(s)\,G(s)$ STABLE $\iff$ CLASSIC EQUIVALENT CONTROL SCHEME IS STABLE

IN THE CLASSICAL CONTROL SCHEME

ACTUATOR NONLINEARITIES

Hp: P, Q and α are stable (α is the nonlinear function)



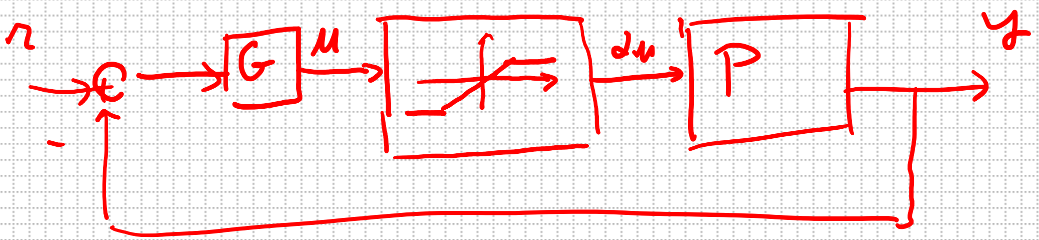
$$e = \underbrace{(\alpha P - \tilde{\alpha}\tilde{P})}u + d + n$$

actuator uncertainties are considered by e

IMC PRINCIPLE STILL HOLDS

$\tilde{P} = P$, $d = 0$, $\tilde{\alpha} = \alpha$ $\Rightarrow$ $e = 0$ (OPEN-LOOP DESIGN)

IMC ANTI - WINDUP

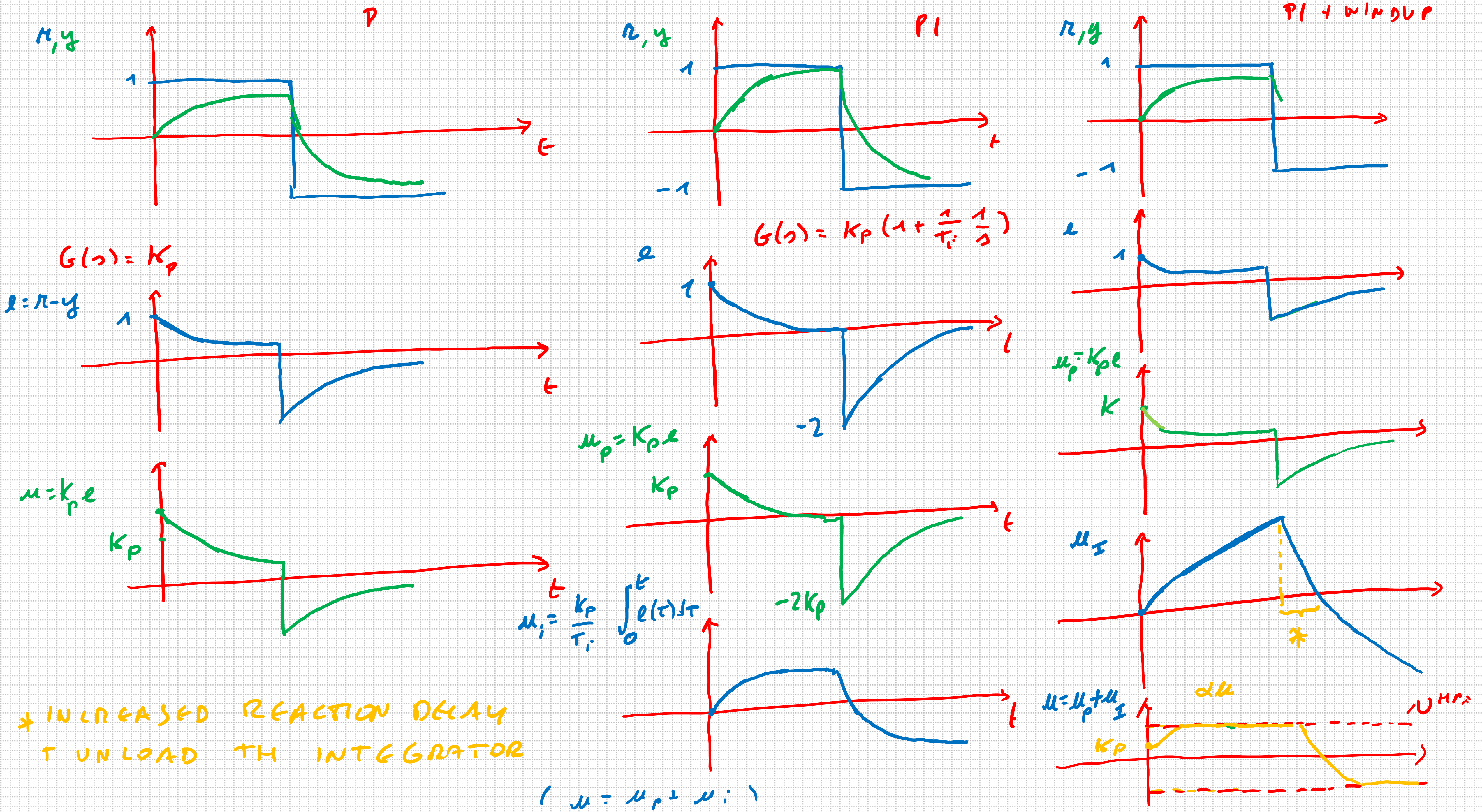


$$\alpha u = \begin{cases} u & u^{MIN} \le u \le u^{MAX} \\ u^{MIN} & u < u^{MIN} \\ u^{MAX} & u > u^{MAX} \end{cases}$$

WINDUP

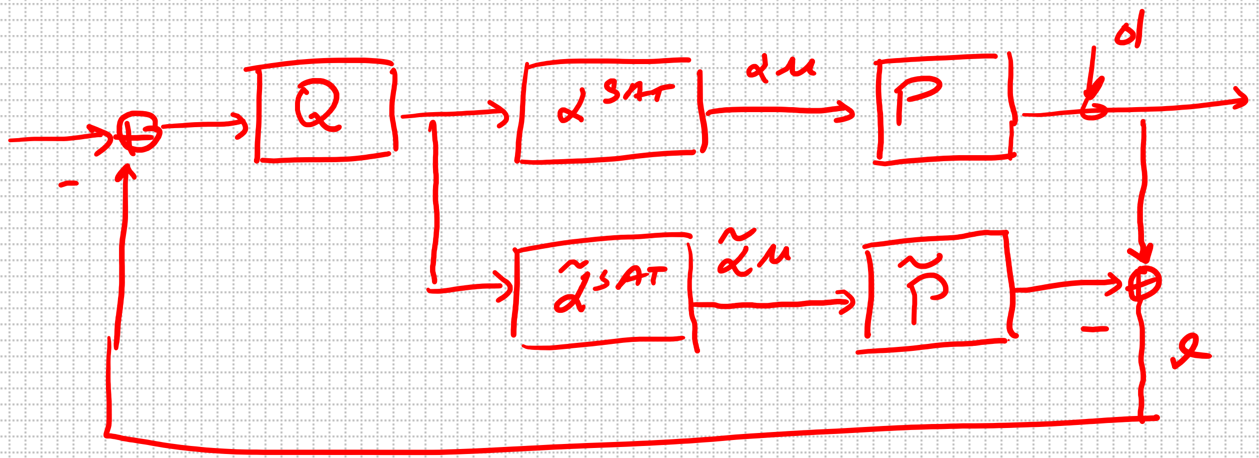
If the controller $G(s)$ has a pole in $s=0$ (integration) and the control variable is saturated, the error accumulates in the integration of $G(s)$, slowing down the control system

(MATLAB)



P
r, y
1
t
$G(s) = K_p$
$e = r - y$
1
t
$u = K_p e$
K_p
t
PI
r, y
1
-1
t
$G(s) = K_p \left(1 + \frac{1}{T_i} \frac{1}{s}\right)$
e
1
-2
t
$u_p = K_p e$
K_p
$-2K_p$
t
$u_i = \frac{K_p}{T_i} \int_0^t e(\tau) d\tau$
t
$(u = u_p + u_i)$
PI + WINDUP
r, y
1
-1
e
1
$u_p = K_p e$
K
u_I
*
$u = u_p + u_I$
Δu
K_p
U^{MAX}
* INCREASED REACTION DELAY
TO UNLOAD THE INTEGRATOR

IMC ANTI-WINDUP SCHEME

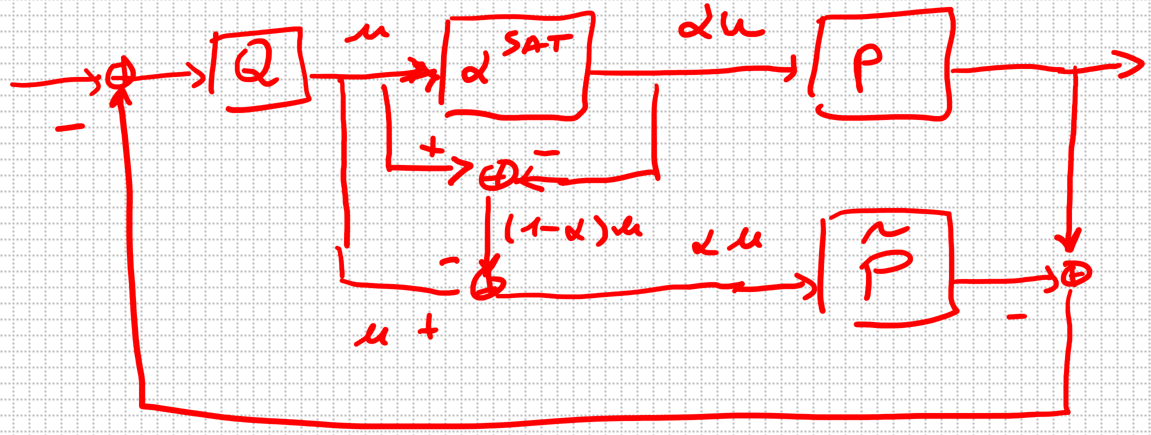
- 

α^{SAT} : ACTUATOR SATURATION

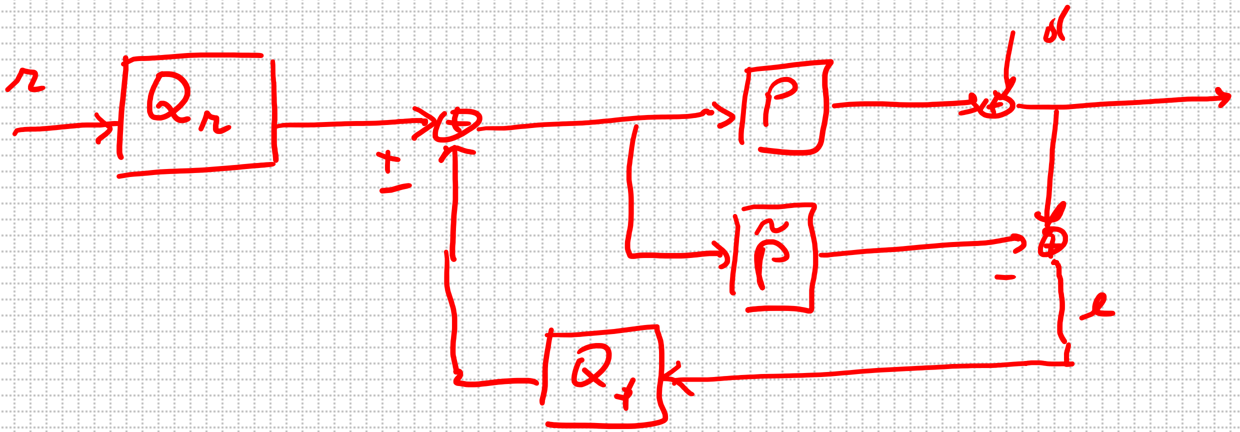
$\tilde{\alpha}^{SAT}$: ACTUATOR SATURATION MODEL

IMC PRINCIPLE : if $P=\tilde{P}$, $\alpha^{SAT}=\tilde{\alpha}^{SAT}$, $d=0$ $\Rightarrow$ $e=0$

- IF αu can be measured:



- 2 DOF IMC DESIGN



$$e = \frac{1-\tilde{P}Q_d}{1+Q_d(\tilde{P}-P)}\,d - \left(1 - \frac{PQ_r}{1+Q_d(\tilde{P}-P)}\right) r$$

1 STEP Design Q_d (ROBUSTNESS)

2 STEP Design Q_r (PERFORMANCE)

IMC PRINCIPLE

$\tilde{P} = P$

$\Rightarrow e = (1-PQ_d)\,d\ +$

$-(1-PQ_r)\,r$

$\Leftarrow$ 2 SEPARATE CONTROLLERS

IMC CONTROL DESIGN

STEP 1.

PERFORMANCE

Design of $Q(s)$ in NOMINAL CONDITIONS $(\tilde{P} = P,\ d = n = 0)$

→ OPEN-LOOP DESIGN

Objective: MINIMIZE A COST FUNCTION

Integral Absolute Error: $IAE(e) = \int_0^\infty |e(t)|\,dt$

Integral Square Error: $ISE(e) = \int_0^\infty e^2(t)\,dt$

Error Variance: $EV(e) = e^{\int_0^\infty |e(t)|\,dt}$

...

⇒ OPTIMAL CONTROLLER WITH RESPECT TO PERFORMANCE

STEP 2.

ROBUSTNESS

DETUNING OF THE CONTROLLER TO ACHIEVE ROBUSTNESS

(the controller will be sub-optimal but robust)