Process Automation (MCER), 2015-2016

FINAL TEST

17 Dec. 2015

Exercise 1 (12 pt.)

Let the process be described by the transfer function: $P(s) = \frac{1-\tau s}{(1+10s)^2} e^{-0.2s}$, with $\tau \in [10,20)$, and let the process model be $\tilde{P}(s) = -0.1 \frac{s-0.1}{(s+0.1)^2} \frac{1-0.1s}{1+0.1s}$.

Design an IMC controller $Q(s)$ such that:

i) the overall system is robustly asymptotically stable;

ii) the overall system has 0 steady-state error for step inputs.

Exercise 2 (12 pt.)

Consider a process whose step response model is given by the following coefficients:

$g_1 = 0,\ g_2 = 0.1,\ g_3 = 0.12,\ g_4 = 0.123,\ g_5 = 0.123,\ \ g_6 = 0.123, \dots,$

and whose state space process is given by the following equations:

$$\begin{cases} x_1(t) = 0.1x_1(t-1) + 0.1x_2(t-1) \\ x_2(t) = 0.1x_2(t-1) + u(t-1) \\ y = x_1(t) \end{cases}$$

Develop the control action $u(2)$ of an MPC controller with the following specifications:

i) Prediction horizon $p = 4$

ii) Step reference signal $r(t) = 1, \forall t \geq 0$

iii) Reference trajectory computed as $w(t) = 0.5w(t-1) + 0.5r(t)$, with $w(0) = 0$

iv) Cost function $J = e^T e$, where e is the vector of predicted errors between predicted output and reference trajectory.

v) The plant-model error is computed as $\hat{n}(t+k|t) = y_m(t) - y(t), \forall k > 0$, with $y_m(0) = y_m(1) = 0.01,\ y_m(2) = 0.1$, .

vi) Initial conditions $x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $u(0) = 0$.

Questions (8 pt.)

i) Briefly discuss the characteristics of feedforward and feedback control (1/3 pg. max, 2pt).

ii) Briefly discuss pros and cons of IMC design vs. classic control design (1/3 pg. max, 2pt).

iii) Briefly discuss pros and cons of the different models used by the MPC algorithms (1/2 pg. max, 4pt).

Solution of exercise 1

The nominal process $\tilde{P}(s)$ is stable, therefore it is possible to design a stable controller $Q(s)$ to stabilize the closed-loop nominal system.
The time-delay θ of the process is much smaller than the time constant $\tau = 10s$ of the process, therefore we can use a Padé approximation to write the delay term of the actual process as a transfer function.

By using the 1/1 Padé approximation $e^{-\theta s} \cong \frac{1-s\frac{\theta}{2}}{1+s\frac{\theta}{2}} = \frac{1-0.1s}{1+0.1s}$, we obtain the following approximated process:

$P^P(s) \cong \frac{1-\tau s}{(1+10s)^2} \cdot \frac{1-0.1s}{1+0.1s}$;

The IMC design procedure to robustly stabilize the approximated process $P^P(s)$ consists in the following 3 steps:

Step 1)

a) Factorize the nominal process in a minimum-phase term and a non-minimum-phase term:
$\tilde{P}(s)(s) = \frac{1-10s}{(1+10s)^2}\frac{1-0.1s}{1+0.1s} = \tilde{P}_+(s)\tilde{P}_-(s)$
with $\tilde{P}(s) = (1-10s)(1-0.1s)$ and $\tilde{P}_-(s) = \frac{1}{(1+10s)^2(1+0.1s)}$

b) Define the controller as follows: $\tilde{Q}(s) = \tilde{P}^{-1}(s) = (1+10s)^2(1+0.1s)$

Step 2)

Design the controller $Q(s) = \tilde{Q}(s)f(s)$, where the IMC filter $f(s)$ must be such that a) the controller $Q(s)$ is proper and b) the overall system is of type 1.
Thus, we use the well-known filter $f(s) = \frac{1}{(1+\lambda s)^n}$ with $n = 3$.
The IMC controller is then $Q(s) = \frac{(1+10s)^2(1+0.1s)}{(1+\lambda s)^3}$

Step 3)

Determine the value of λ such that the sufficient condition for robust stability holds:

$$|l_a(j\omega)Q(j\omega)| < 1, \forall\omega$$

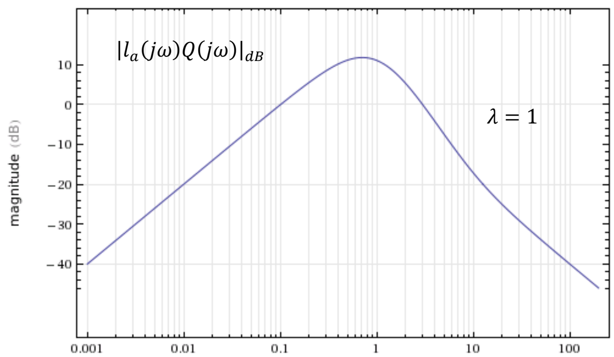
where $l_a(j\omega)$ is an upper-bound of the additive uncertainty $\Delta_a(j\omega)$, i.e., a function such that $|l_a(j\omega)| > |\Delta_a(j\omega)|, \forall\omega$.
By definition, the additive uncertainty is defined as follows

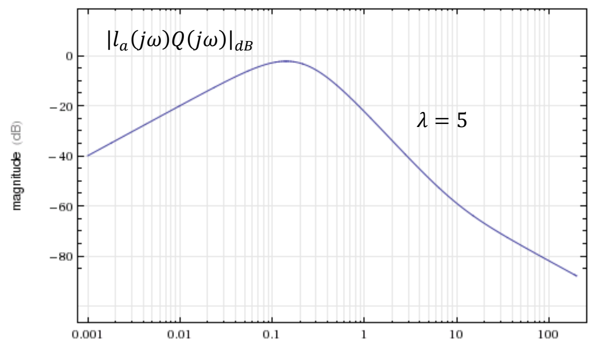
$|\Delta_a(j\omega)| = \left|P^P(j\omega) - \tilde{P}(j\omega)\right| = \left|\frac{1-\tau j\omega}{(1+10j\omega)^2} \cdot \frac{1-0.1j\omega}{1+0.1j\omega} - \frac{1-10j\omega}{(1+10j\omega)^2} \cdot \frac{1-0.1j\omega}{1+0.1j\omega}\right|$
$= \left|(10-\tau)\frac{j\omega}{(1+10j\omega)^2} \cdot \frac{1-0.1j\omega}{1+0.1j\omega}\right| = \left|(10-\tau)\frac{j\omega}{(1+10j\omega)^2}\right|$.

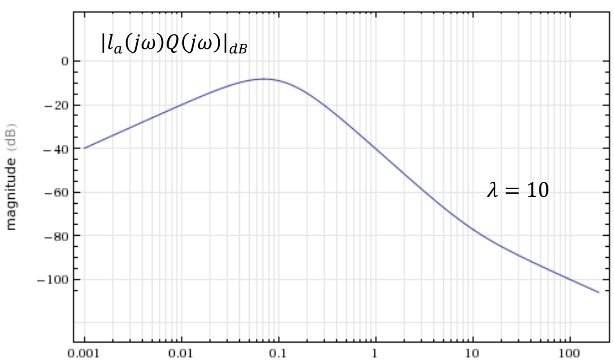
Since $\tau \in [10,20)$, an upper-bound is given by $l_a(j\omega) = -10\frac{j\omega}{(1+10j\omega)^2}$
The sufficient condition for robust stability is then:

$$|l_a(j\omega)Q(j\omega)| = \left|-10\frac{j\omega}{(1+10j\omega)^2} \cdot \frac{(1+10j\omega)^2(1+0.1j\omega)}{(1+\lambda j\omega)^3}\right| = \left|10\frac{j\omega(1+0.1j\omega)}{(1+\lambda j\omega)^3}\right| < 1, \forall\omega$$

Which, according to the Bode diagrams below, is true at least for $\lambda > 5$. To account for the Padé approximation we choose a conservative value $\lambda = 10$.







Solution of exercise 2

Even if both a state-space model and a step response model are available, it is convenient developing a PFC controller with respect to a DMC one due to the (relatively) large prediction horizon.
To develop the PFC controller, we need to select the basis functions and the coincidence points. For the sake of simplicity, and since the reference signal is constant, we chose $n_B = 1$ base function $B_1(k) = k^0 = 1, k = 0,1,2, \dots$ (step function). Considering that the prediction horizon is $p = 4$ and that there is a time-delay $d = 1$, we chose $n_H = 2$, $h_1 = 2$ (since $d = 1$ the input at time t does not affect the output at time $t + 1$) and $h_2 = 4$ (otherwise there is no reason to set $p = 4$).

Firstly, we have to compute the model response to the base function, denoted with y_{B_1}, in the coincidence points, considering null initial conditions $x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Since we choose to use a step function as basis function, we already have the step response of the actual process from the given coefficients g_i. Therefore, we may use the actual step response to compute the free response of the system: $y_{B_1} = (y_{B_1}(h_1) \quad y(h_2)) = (g_2 \quad g_4) = (0.1 \quad 0.123)$; this approach is compliant with the meaning of free response, which, at given time t, accounts for the known information at time t. The matrix $Y_B \in \mathbb{R}^{n_H \times n_B}$ is then $Y_B = (y_{B_1}^T) = \begin{pmatrix} 0.1 \\ 0.123 \end{pmatrix}$. (The other approach is to compute y_{B_1} as the step response of the model; then, the differences between the actual and modelled processes are dealt with by the disturbance, as shown in the following.)

The matrix Y_B is used to compute the solution of the unconstrained optimization problem: $\mu^* = \left({Y_B}^T Y_B\right)^{-1} {Y_B}^T (w - f)$, with $\left({Y_B}^T Y_B\right)^{-1} {Y_B}^T = (4.0 \quad 4.9)$, where μ^* is the vector of the optimal parameters at time t. The control action is the computed as $u(t) = {\mu^*}^T B(0)$, where $B(0)$ is the column vector of base functions $B_i(k), i = 1,2, \dots, n_B$, evaluated for $k = 0$. In our problem, since $n_B = 1$, we need to find a single parameter $\mu^*(t)$.

By considering null initial conditions, we can start computing the free response at time $t = 1$ over the coincidence points. With PFC, the output prediction is

$$\hat{y}(t+k|t) = \sum_{i=1,\dots,n_B} y_{B_i}(k)\mu_i(t) + QM^k x(t) + \hat{n}(t+k|t),$$

where the last two terms constitute the free response:

$$f(t+k|t) = QM^k x(t) + \hat{n}(t+k|t) = QM^k x(t) + y_m(t) - y(t).$$

$\underline{t=1}$ $\quad y_m(1) = 0.01;\ x(0) = \begin{pmatrix}0\\0\end{pmatrix};\ u(0) = 0;$

Model output
$x_1(1) = 0.1x_1(0) + 0.1x_2(0) = 0$
$x_2(1) = 0.1x_2(0) + u(0) = 0$
$y(1) = x_1(1) = 0.$

$\underline{h_1 = 2}$ $\qquad f(3) = QM^{h_1}x(1) + y_m(1) - y(1) = 0.01;$
$\underline{h_2 = 4}$ $\qquad f(5) = QM^{h_2}x(1) + y_m(1) - y(1) = 0.\underline{01}.$

The reference trajectory vector $w = \begin{pmatrix} w(t+h_1) \\ w(t+h_2) \end{pmatrix} = \begin{pmatrix} w(3) \\ w(5) \end{pmatrix}$ is computed as follows

$w(1) = 0.5w(0) + 0.5r(1) = 0.5.$
$w(2) = 0.5w(1) + 0.5r(2) = 0.75.$
$w(3) = 0.5w(2) + 0.5r(3) = 0.875.$
$w(4) = 0.5w(3) + 0.5r(4) = 0.937.$
$w(5) = 0.5w(4) + 0.5r(5) = 0.969.$

$$\mu(1) = \left({Y_B}^T Y_B\right)^{-1} {Y_B}^T (w - f) = (4.0 \quad 4.9)\left(\begin{pmatrix}0.875\\0.969\end{pmatrix} - \begin{pmatrix}0.01\\0.01\end{pmatrix}\right) = 8.25;$$
$u(1) = \mu(1)B_1(0) = 8.25.$

$\underline{t=2}$ $\quad y_m(2) = 0.1;\ x(1) = \begin{pmatrix}0\\0\end{pmatrix};\ u(1) = 8.25;$

Model output
$x_1(2) = 0.1x_1(1) + 0.1x_2(1) = 0$
$x_2(2) = 0.1x_2(1) + u(1) = 8.25$
$y(2) = x_1(2) = 0.$

$\underline{h_1 = 2}$ $\qquad f(4) = QM^{h_1}x(2) + y_m(2) - y(2) = 0.1;$
$\underline{h_2 = 4}$ $\qquad f(6) = QM^{h_2}x(2) + y_m(2) - y(2) = 0.1.$

The reference trajectory vector $w = \begin{pmatrix} w(t+h_1) \\ w(t+h_2) \end{pmatrix} = \begin{pmatrix} w(4) \\ w(6) \end{pmatrix}$ is computed as follows

$w(6) = 0.5w(5) + 0.5r(6) = 0.985.$

$$\mu(2) = \left({Y_B}^T Y_B\right)^{-1} {Y_B}^T (w - f) = (4.0 \quad 4.9)\left(\begin{pmatrix}0.969\\0.985\end{pmatrix} - \begin{pmatrix}0.1\\0.1\end{pmatrix}\right) = 7.81;$$
$u(2) = \mu(2)B_1(0) = 7.81.$