I authorize Prof. Pietrabissa to upload the test results on the mailing list of the Process Automation group.

FAMYLY NAME GIVEN NAME SIGNATURE

Process Automation (MCER), 2019-2020

TEST - B

19 December 2019, 2h00

Exercise 1 (12 pts.)

Let the process be described by the transfer function: $P(s) = \frac{s-1}{(s+1)^2} e^{-\theta s}$, with $\theta \in (5,18)$, and let the process model be $\tilde{P}(s) = \frac{S-1}{(s+1)^2} e^{-\bar{\theta} s}$, with nominal delay $\bar{\theta} = 15$.

A) Design a robust Smith Predictor controller by following the IMC procedure to obtain a type 1 system under the IAE cost.

B) Obtain the parameters of the PID controller corresponding to the designed primary controller.

Exercise 2 (12 pts.)

Consider a process whose transfer function model is:

$$\begin{cases} x(t) = Mx(t-1) + Pu(t-1) \\ y(t) = Qx(t) \end{cases}, \text{ with } M = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, P = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, Q = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

A) Compute the control action at time $t = 5$ by implementing a Predictive Functional Control algorithm, considering:
- prediction and control horizon $p = m = 3$;
- number of base functions $n_B = 1$;
- number of coincident points $n_H = 2$;
- smooth response preferred;
- cost function $J = e^T e$, where e is the vector of future errors between predicted output and reference trajectory;
- known ramp reference signal $r(t) = t$ and reference trajectory $w(t+k|t) = r(t+k)$;
- predicted plant-model errors at time t equal to $\hat{n}(t+k|t) = y_m(t) - y(t)$;
- available data at $t = 5$:
  - control action $u(4) = 1$;
  - state $x(4) = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$;
  - measured output $y_m(5) = 4$.

Questions (6 pt.)

i) Analyze the stability of the controlled system with open-loop transfer function $F(s) = -10 \frac{1+s}{s(1-s)} e^{-10s}$ (approximations are acceptable, 4pt)

ii) Briefly explain why the robust stability condition based on Nyquist theorem arguments (i.e., the condition $\left| l_m(j\omega)\tilde{T}(j\omega) \right| < 1, \forall\omega$) is conservative. (1/2 pg. max, 2pt).

Solution of exercise 1

A)

The nominal process $\tilde{P}(s)$ is stable, therefore it is possible to design a stable controller $Q(s)$ to stabilize the closed-loop nominal system.

Moreover, since the time-delay of the process is larger than the time constant of the process – about $1s$ – we cannot use a Padé approximation to write the delay term as a transfer function and we use a Smith Predictor (SP) controller, depicted in the figure below:

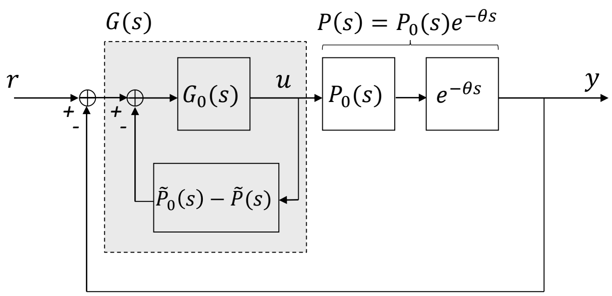


Figure 1)

Thanks to the SP principle, the primary controller $G_0(s)$ can be computed considering the delay-free process $P_0(s) = -\frac{1-s}{(1+s)^2}$.

The IMC design procedure to robustly stabilize the process $P_0(s)$ consists in the following 3 steps:

Step 1)

a) Factorize the nominal process in a minimum-phase term and a non-minimum-phase term (IAE-optimal factorization):

$P_0(s) = P_+(s)P_-(s)$, with $P_+(s) = (1-s)$ and $P_-(s) = -\frac{1}{(1+s)^2}$.

b) Define the preliminary controller as $\tilde{Q}(s) = P_-^{-1}(s) = -(1+s)^2$.

Step 2)

Design the controller $Q(s) = \tilde{Q}(s)f(s)$, where the IMC filter $f(s)$ must be such that a) the controller $Q(s)$ is proper and b) the overall system is of type 1.

Thus, we use the IMC filter $f(s) = \frac{1}{(1+\lambda s)^n}$ with $n = 2$.

The IMC controller is then $Q(s) = -\frac{(1+s)^2}{(1+\lambda s)^2}$.

We check that $T_0(s)|_{s=0} = P_0(0)Q(0) = P_+(0)P_-(0)P_-^{-1}(0)f(0) = P_+(0)f(0) = 1$.

Step 3)

Determine the value of λ such that the sufficient condition for robust stability holds:

$$\left|l_m(j\omega)\tilde{T}(j\omega)\right| < 1, \forall\omega$$

where

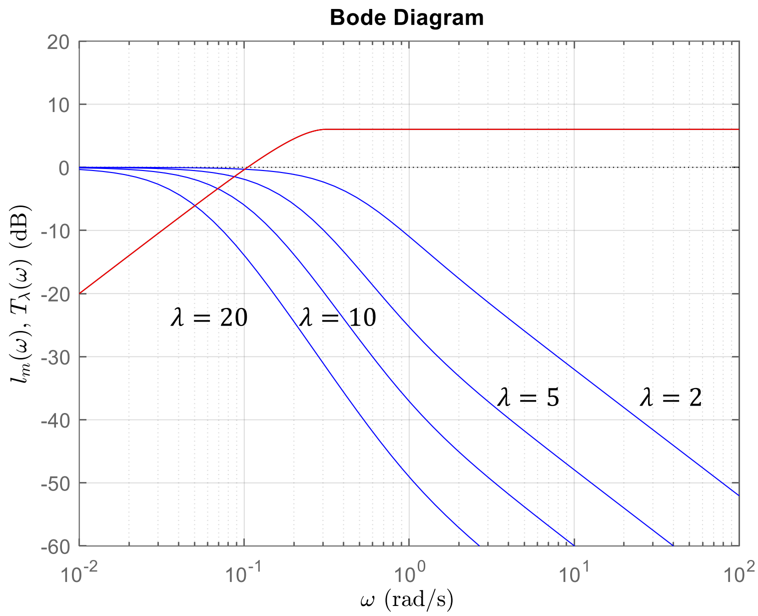
$$|T(j\omega)| = |Q(j\omega)P_0| = |P_-^{-1}(j\omega)f(j\omega)P_+(j\omega)P_-(j\omega)| = \left|\frac{(1-j\omega)}{(1+j\omega\lambda)^2}\right|$$

and $l_m(j\omega)$ is an upper-bound of the multiplicative uncertainty $\Delta_m(j\omega)$, i.e., a function such that $|l_m(j\omega)| > |\Delta_m(j\omega)|, \forall\omega$. By definition, the multiplicative uncertainty is defined as

$$\Delta_m(j\omega) := \frac{P(j\omega)-\tilde{P}(j\omega)}{\tilde{P}(j\omega)} = \frac{-\frac{1-j\omega}{(1+j\omega)^2}e^{-\theta j\omega} + -\frac{1-j\omega}{(1+j\omega)^2}e^{-15j\omega}}{-\frac{1-j\omega}{(1+j\omega)^2}e^{-15j\omega}} = e^{-j\omega\delta} - 1, \text{ with } \delta = \theta - 15.$$

From theory, we know that an upper-bound is defined as $l_m(j\omega) = \begin{cases} e^{-s\delta_{max}} - 1, & if\ \omega \leq \frac{\pi}{\delta_{max}} \\ 2, & if\ \omega > \frac{\pi}{\delta_{max}} \end{cases}$, where, in our example, $\delta_{max} = \max_{\theta\in(10,22)}\left|\theta - \bar{\theta}\right| = \max_{\theta\in(10,22)}|\theta - 15| = 10$.

The figures below show that, at least, for $\lambda \geq 10$ the condition is met (the figure shows that it is met at least for $\lambda \geq 5$). We choose $\lambda = 10$, obtaining $Q(s) = -\frac{(1+s)^2}{(1+10s)^2}$.



B)

From the scheme of Figure 2), it follows that the controller G_0 is computed as

$$G_0(s) = \frac{Q(s)}{1-Q(s)P_0(s)} = \frac{-\frac{(1+s)^2}{(1+10s)^2}}{1-\frac{(1+s)^2}{(1+10s)^2}\frac{1-s}{(1+s)^2}} = -\frac{1+2s+s^2}{100s(s+21)}.$$

The primary controller has 2 zeros, one pole in s=0 and one other negative pole, therefore it is a PID + filter controller:

$$G_0(s) = -\frac{2}{2100}\frac{s+\frac{1}{2}+\frac{1}{2}s^2}{s}\frac{1}{1+\frac{s}{21}} = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{1+\beta_f s}, \text{ with } K_c = -\frac{1}{1050}, T_i = 2, T_d = \frac{1}{2}, \beta_f = \frac{1}{21}.$$

Solution of exercise 2

To develop the PFC controller, we need to select the $n_B = 1$ base function: we select the polynomial (step) function $B_1(k) = k^0 = 1, k = 0,1,2,\ldots$.

Since the smoothness of the controller response is important, we choose $h_1 = 2$ as first coincident point and, since the prediction horizon is $p = 3$, we chose $h_2 = 3$ as second coincident point.

The model responses to the base functions in the coincidence points, considering null initial conditions $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ are computed as follows:

$B_1(k) = 1$

$\underline{t = h_1 = 2}$ $\qquad y_{B_1}(2) = QMPB_1(0) + QPB_1(1) = [0 \quad 1]\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}1 + [0 \quad 1]\begin{bmatrix} 1 \\ 1 \end{bmatrix}1 = 1.5;$

$\underline{t = h_2 = 3}$ $\qquad y_{B_1}(3) = QM^2PB_1(0) + QMPB_1(1) + QPB_1(2) = (QM^2P + QMP + QP)1 = 1.75.$

Thus,
$y_B(h_1) = y_{B_1}(h_2) = [1.5]$;
$y_B(h_2) = y_{B_1}(h_2) = [1.75]$.

The matrix $Y_B \in \mathbb{R}^{n_H \times n_B}$ is then $Y_B = \begin{pmatrix} y_B(h_1) \\ y_B(h_2) \end{pmatrix} = \begin{bmatrix} 1.5 \\ 1.75 \end{bmatrix}$.
The matrix Y_B is used to compute the solution of the unconstrained optimization problem $\mu^* = (Y_B^T Y_B)^{-1} Y_B^T (w - f)$, with $(Y_B^T Y_B)^{-1} Y_B^T = \left([1.5 \;\; 1.75] \begin{bmatrix} 1.5 \\ 1.75 \end{bmatrix}\right)^{-1} [1.5 \;\; 1.75] = [0.29 \;\; 0.33]$,
where μ^* is the vector of the optimal parameters at time t. The control action is the computed as $u(t) = {\mu^*}^T B(0)$, where $B(0)$ is the column vector of base functions $B_i(k), i = 1,2, \dots, n_B$, evaluated for $k = 0$. In our problem, since $n_B = 1$, we need to find one parameter $\mu^*(t)$.

At $t = 5$, considering that $r(t) = t$, the vector of the future reference values evaluated in the coincidence points $t + h_1 = 7$ and $t + h_2 = 8$ is computed as follows:

$w(5|5) = r(5) = 5$;
$w(6|5) = r(6) = 6$;
$w(7|5) = r(7) = 7$.
$w(8|5) = r(8) = 8$.

By considering the given state and measured output values, we can compute the free response at time $t = 5$ over the coincidence points. With PFC, the output prediction is

$$\hat{y}(t + k|t) = y(t + k) + \hat{n}(t + k|t) = \sum_{i=1,\dots,n_B} y_{B_i}(k) \mu_i(t) + Q M^k x(t) + \hat{n}(t + k|t),$$

where the last two terms constitute the free response:

$$f(t + k|t) = Q M^k x(t) + \hat{n}(t + k|t) = Q M^k x(t) + (y_m(t) - y(t)).$$

$\underline{t = 5}$ $\quad y_m(5) = 4$; $x(4) = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$; $u(4) = 1$;

$$\begin{cases} x(5) = M x(4) + P u(4) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 10 \\ 6 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} 1 = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \\ y(5) = Q x(5) = [0 \;\; 1] \begin{bmatrix} 6 \\ 4 \end{bmatrix} = 4 \end{cases};$$

$\underline{h_1 = 2}$ $\qquad f(7) = Q M^{h_1} x(5) + \big(y_m(5) - y(5)\big) = [0 \;\; 1] \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}^2 \begin{bmatrix} 6 \\ 4 \end{bmatrix} + (4 - 4) = 1$;

$\underline{h_2 = 3}$ $\qquad f(8) = Q M^{h_2} x(5) + \big(y_m(5) - y(5)\big) = [0 \;\; 1] \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}^3 \begin{bmatrix} 6 \\ 4 \end{bmatrix} + (4 - 4) = 0.5$.

$d(5) = \begin{bmatrix} \widehat{w}(7) \\ \widehat{w}(8) \end{bmatrix} - \begin{bmatrix} f(7) \\ f(8) \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 6 \\ 7.5 \end{bmatrix}$.
$\mu(5) = [0.29 \;\; 0.33] \begin{bmatrix} 6 \\ 7.5 \end{bmatrix} = 4.2$.

$u(5) = \mu(5) B_1(0) = 4.2$.