# Internal Model Control (IMC)

## 1. Introduction

In the 1980s, IMC has been introduced by Morari and his co-workers. In this notes, the exposition of IMC concepts is mainly based on [2].

The main idea of IMC lies in a different definition of the 'error':

- in classical control theory, the controller computes the control action based on the difference between the measured output and a reference variable;
- in IMC theory, the controller computes the control action based on the difference between the measured output and a *nominal output*, computed as the output of a *nominal model* of the controlled process.

The first consequence is that the IMC effectiveness is tightly linked on the model accuracy.

## 2. IMC Principle

The comparison between the IMC and the classical control structure allows us to underline some desirable features of IMC.

Let us consider the classical control structure shown in Figure 2-1, where *r* is the setpoint, *u* is the control variable, *y* is the controlled variable, *d* is the disturbance (which, in this case, represents also the effect of a load disturbance on the controlled variable *y*), *n* is the measurement disturbance, *P*(*s*) is the plant transfer function and *C*(*s*) is the controller. The controlled variable is given by the following equation:

$$y = \frac{PC}{1+PC}(r-n) + \frac{1}{1+PC}d = T(r-n) + Sd \tag{1}$$

where $S(s) = \frac{1}{1+P(s)C(s)}$ and $T(s) = \frac{P(s)C(s)}{1+P(s)C(s)}$ are the '*sensitivity*' and '*complementary sensitivity*' functions, respectively.

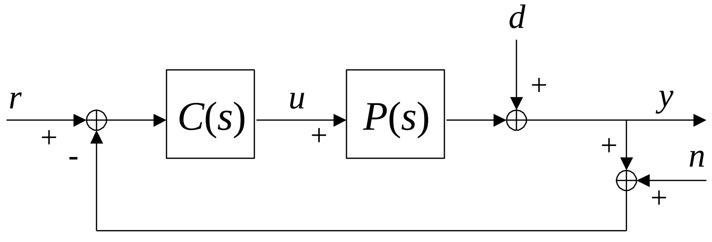


*Figure 2-1: Classical control structure*

The complementary sensitivity function $T(j\omega)$ rules the accuracy of the regulation. From the input-output viewpoint, the *performance* requirement for perfect regulation is $T(j\omega) = 1, \forall\omega$ (or, at least, in a desired frequency range). Note however that $T(j\omega)$ is also the transfer function between the measurement noise and the output: in this regard, $|T(j\omega)|$ should be as small as possible.

The sensitivity function $S(j\omega)$ rules the disturbance rejection, i.e., the *robustness* characteristics of the system. The robustness requirement for perfect disturbance rejection is $S(j\omega) = 0, \forall\omega$ (or, at least, in a desired frequency range). Note however that, since $C(j\omega)$ and $P(j\omega)$ are physical systems, it holds that $\lim_{\omega\to\infty}|P(j\omega)C(j\omega)| = 0$ (physical systems are low-pass filters, otherwise they would have infinite energy). From the sensitivity function definition, it follows that $\lim_{\omega\to\infty}|S(j\omega)| = \lim_{\omega\to\infty}\left|\frac{1}{1+P(j\omega)C(j\omega)}\right| = 1$. Therefore, $|S(j\omega)|$ should be kept as small as possible within the frequency bandwidth of interest. The system bandwidth is often defined in terms of the sensitivity functions as the frequency $\omega_s$ such that $|S(j\omega)| > \frac{1}{\sqrt{2}}, \forall\omega > \omega_s$.

Summarizing, the trade-off among the different requirements of a system can be expressed in terms of sensitivity functions within the frequency range of interest $(\omega_1, \omega_2)$:

- Performance (regulation/tracking): $T(j\omega) \cong 1, \forall\omega \in (\omega_1, \omega_2)$;
- Robustness (disturbance rejection): $S(j\omega) \cong 0, \forall\omega \in (\omega_1, \omega_2)$;
- Measurement noise rejection: $T(j\omega) \cong 0, \forall\omega > \omega_N$, where $\omega_N$ is the lower frequency of the noise signal.

The main drawbacks of the classical control structure when applied to process control are:

1) that the dependencies of the sensitivity function on the controller is not straightforward (the controller is at the denominator of $T(j\omega)$ and $S(j\omega)$);
2) that it cannot easily take into account process and actuator constraints.

Now, let us consider the IMC structure of Figure 2-2, where $y_r$ is the reference output, $e$ is the output error, $Q(s)$ is the IMC controller and $\widetilde{P}(s)$ is the model of the plant transfer function. As shown in Figure 2-2, in the IMC configuration, the controller $C(s)$ includes the plant model.

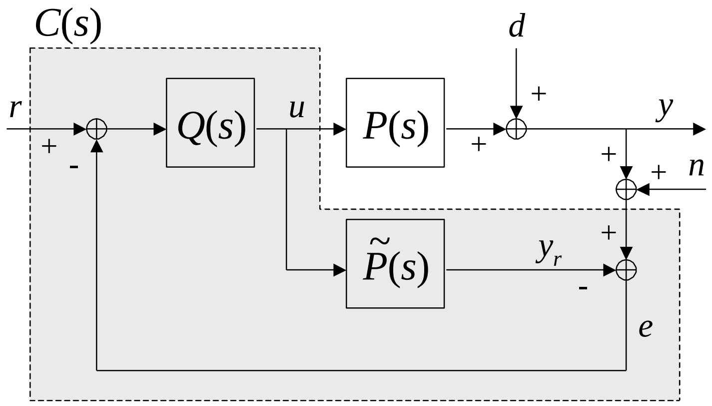


*Figure 2-2: IMC structure*

If the plant model is perfect, i.e., $\tilde{P}(s) \equiv P(s)$, the output variable is given by the following equation:

$$y = PQ(r - n) + (1 - PQ)d \tag{2}$$

Equation (2) shows that if $\tilde{P}(s) \equiv P(s)$ and if $d(s) = n(s) = 0$, the input-output transfer function becomes $y(s) = P(s)Q(s)r(s)$. This is the fundamental IMC principle:

"*if the plant model is perfect, i.e., $\tilde{P} = P$, and if no disturbance is present, i.e., $d \equiv n \equiv 0$, the closed-loop system dynamic is the same as the open-loop system dynamic*".

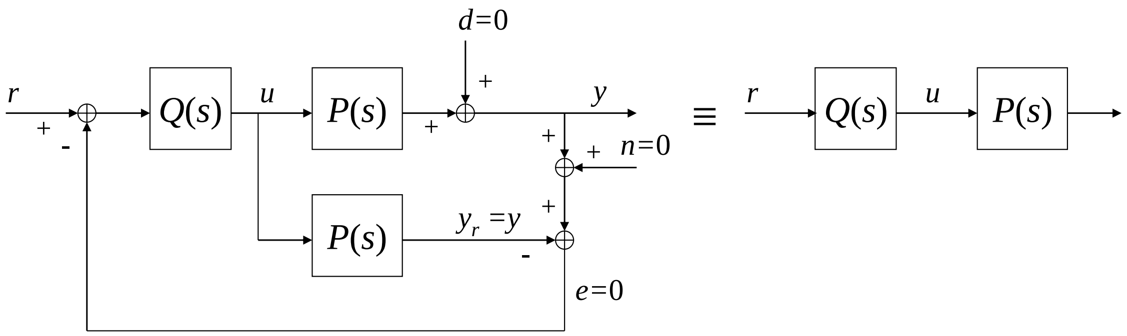


*Figure 2-3: IMC principle*

A consequence of the IMC principle is that, for open-loop stable processes, the feedback is only needed because of the uncertainties.

The output error $e$ is defined as follows (see also Figure 2-2):

$$e = (P - \tilde{P})u + n + d \tag{3}$$

The error expresses all the unknown model uncertainties of the process (with the term $P(s) - \tilde{P}(s)$) and the unknown disturbances, and the magnitude of $e$ is directly related to the magnitude of the unknown characteristics.

Equation (3) shows that if $\tilde{P}(s) \equiv P(s)$ and if $d(s) = n(s) = 0$, the error is null.

From Figure 2-2, it turns out that it is possible to determine an equivalent controller $C(s)$ in terms of the IMC controller :

$$C(s) = \frac{Q(s)}{1 - \tilde{P}(s)Q(s)} \tag{4}$$

Similarly, an IMC controller can be obtained by a classical one:

$$Q(s) = \frac{C(s)}{1 + \tilde{P}(s)C(s)} \tag{5}$$

## 3. IMC Performance

The '*sensitivity*' and '*complementary sensitivity*' functions, $S$ and $T$, respectively, are simply retrieved from  by the equation (2), which can be re-written as follows:

$$y = T(r - n) + Sd \tag{6}$$

where $T(s) = P(s)Q(s)$ and $S(s) = 1 - T(s) = 1 - P(s)Q(s)$.

The simplicity of performance determination with the IMC scheme with respect to the classical control scheme is clear if we compare the sensitivity functions of the IMC scheme with the ones of equation (1).

## 4. Nominal Stability Conditions

It can be shown ([1]) that, for stable processes, it is sufficient to select a stable controller $Q$ in order to stabilize the closed-loop system:

***Theorem 2.1***

*If the plant model is perfect, i.e., $\tilde{P} = P$, the IMC system is internally stable if and only if the process $P$ and the controller $Q$ are stable.*

***Proof:***

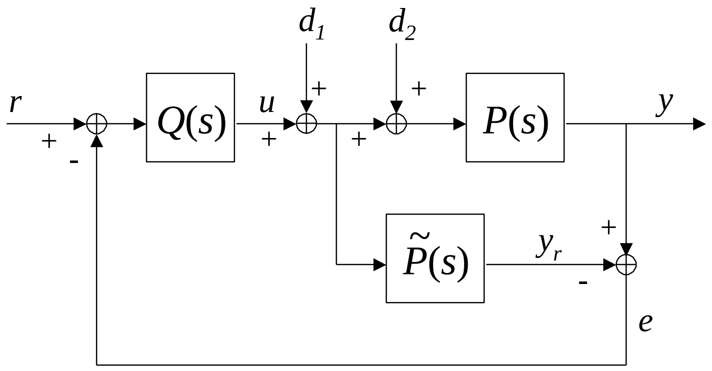


*Figure 4-1: IMC block diagram for internal stability*

The internal stability can be demonstrated by computing all the possible input-output transfer functions (the selected independent inputs and outputs are shown in Figure 4-1):

$$\begin{bmatrix} y \\ u \\ y_r \end{bmatrix} = \begin{bmatrix} PQ & P & P(1-PQ) \\ Q & 0 & -PQ \\ PQ & P & -P^2Q \end{bmatrix} \cdot \begin{bmatrix} r \\ d_1 \\ d_2 \end{bmatrix} \tag{7}$$

Note that the stability Theroem 2.1 requires $Q(s)$ to be stable; from equation (6) it follows that $Q(s)$ is stable if and only if $1 + \tilde{P}(s)C(s)$ is Hurwitz, i.e., if the equivalent classical controller $C(s)$ stabilizes the system.

Finally, another advantage if the IMC structure is that it works even with non-linear plants and non-linear controllers: when no disturbance is present and if the (non-linear) model plant is equal to the actual plant, the error is null and the system is still open-loop. Thus, also in this case, if $P$ and $Q$ are stable, the system is stable.

## 5. IMC in Presence of Saturations

The IMC structure can be used to control systems with actuator and process constraints. For instance, the scheme of Figure 5-1 includes an actuator constraint, which, in this case, is a saturation non-linearity.

By defining the operator $\alpha$ representing the non-linearity, it is sufficient that the same constrained control variable $\alpha u$ feeds the actual plant and the modelled plant; in fact, in this way, $e$ still expresses all the unknown model uncertainties of the process:

$$e = (\tilde{P} - P)\alpha u + n + d \tag{8}$$

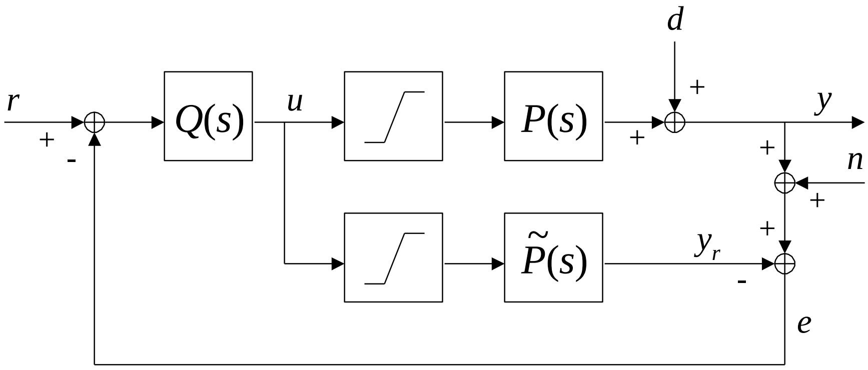


*Figure 5-1: IMC structure with actuator constraints*

In this case, in order to have a stable system, $P$, $Q$ and $\alpha$ must be stable. Furthermore, we explicitly note that, if the model is perfect, the output error only depends on the disturbance; thus, when no disturbances are present, $e$ is equal to 0 and the closed-loop dynamic is equal to the open-loop dynamic, keeping the IMC principle valid.

When the IMC controller $Q$ contains an integrator and the control action $u$ is saturated before feeding the plant $P$, if the non-saturated control action $u$ were used by the plant model $\tilde{P}(s)$, the so-called wind-up problem would arise: the difference between $u$, which feeds the IMC loop, and $\alpha u$, which feeds the system plant, causes an error which is accumulated by the integrator of the controller resulting in a 'deeper' saturation ([3]). To overcome this saturation effect, an IMC-based anti-windup scheme can utilized, which simply requires that the same saturated input feeds both the actual plant and the modelled plant ([3]): in this way, the IMC principle is preserved, since the system still remains open loop if the disturbance is identically null.

## 6. IMC Design Procedure

The objective of setpoint tracking and disturbance rejection is to minimize the error $e = y - y_r$. The IMC design procedure is usually performed in two steps:

1) Neglecting the measurement noise $n$, model uncertainties (i.e., considering $\tilde{P} = P$) and nonlinearities, the nominal IMC controller $\tilde{Q}$ is designed, with the aim of optimal tracking and disturbance rejection.
2) The IMC nominal controller is detuned for robust performance, i.e., to trade-off between performances of the control action and robustness to noise and model errors.

This two-step design procedure is not inherently optimal, but "it constitutes a simple and practical approach for finding controllers which satisfy the typical practical requirements" ([2]).

To develop the optimal nominal controller, $e$ can be minimized with respect to different performance measures. A brief selection of the most common measures is provided in Table 6-1.

| **Performance measure** | **Formula** | **Input** |
|---|---|---|
| Integral Absolute Error (IAE) | $\mathrm{IAE}\{e\} \equiv \int_0^\infty \lvert e(t) \rvert dt$ | Deterministic |
| Integral Square Error (ISE) | $\mathrm{ISE}\{e\} \equiv \int_0^\infty e^2(t) dt$ | Deterministic |
| Error Variance (EV) | $\mathrm{EV}\{e\} \equiv \mathrm{Exp.Value}\left\{\int_0^\infty \lvert e(t) \rvert dt\right\}$ | Stochastic |

*Table 6-1: IMC performance measures*

## 7. Two-degree-of-freedom Internal Model Control

When the dynamics of $r$ and $d$ are significantly different, another IMC structure, named "two-degree-of-freedom" (2DOF), can be used. The 2DOF structure is shown in Figure 7-1, where $Q_r(s)$ and $Q_d(s)$ are the IMC controllers for reference tracking and disturbance rejection, respectively.

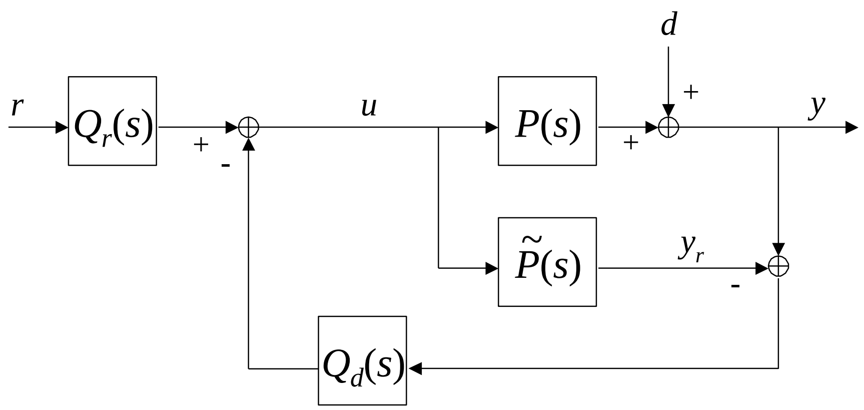


*Figure 7-1: 2DOF IMC structure*

The error $e = y - r$ is given by the following equation:

$$e = \frac{1-\tilde{P}Q_d}{1+Q_d(\tilde{P}-P)}d - \left[1 - \frac{PQ_r}{1+Q_d(\tilde{P}-P)}\right]r \qquad (9)$$

If the plant model is perfect, equation (9) becomes:

$$e = (1-PQ_d)d - (1-PQ_r)r \qquad (10)$$

Equation (9) shows that $Q_r(s)$ and $Q_d(s)$ can be separately designed for reference tracking and disturbance rejection, respectively.

Note, however, that the plant $P$ is still required to be stable: to show this, it is sufficient to note that the transfer function between a load disturbance $d_1$ acting immediately before $P$ (as $d_1$ in Figure 4-1) and the controlled variable $y$ is given by the plant $P$ itself.

## 8. Bibliography


[1] Braatz, R. D. (1995). Internal model control. In *The Control Handbook* (W. S. Levine, ed.) CRC Press, pp. 215-224

[2] Morari, M. & Zafiriou, E. (1989). *Robust Process Control*. Prentice Hall, Englewood Cliffs, New Jersey

[3] Kothare, M. V., Campo, P. J., Morari, M., Nett, C. N. (1994). A Unified Framework for the Study of Anti-Windup Designs. *Automatica*, Vol. 30, N. 12, pp. 1869-1883