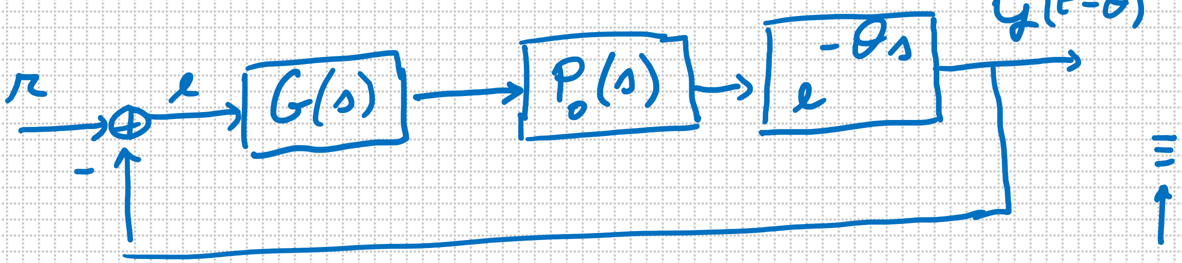
SMITH'S PRINCIPLE (SISO LTI)

- Feedback delay affects the system stability
- " Bring the delay out of the control loop "



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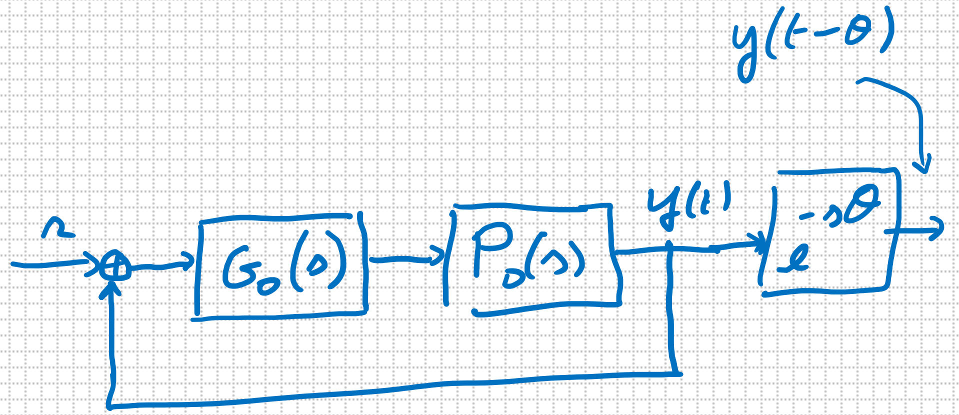
↑

from the input-output viewpoint



$$G(s) \text{ s.t. } W(s) := \frac{y(s)}{r(s)} = \underbrace{\frac{G(s)P_o(s)e^{-\theta s}}{1+G(s)P_o(s)e^{-\theta s}}}_{\text{system with feedback delay}} = \underbrace{\frac{G_o(s)P_o(s)}{1+G_o(s)P_o(s)}}_{\text{delay-free system}} \underbrace{e^{-\theta s}}_{\text{delay}} \quad (1)$$

$$\cancel{G(s)\,P_0(s)\,e^{-\theta s}}\left(1+G_0(s)P_0(s)\right) = \cancel{G_0(s)\,P_0(s)\,e^{-\theta s}}\left(1+G(s)P_0(s)e^{-\theta s}\right)$$

$$G(s)\left(1+G_0(s)P_0(s)\right) = G_0(s) + G(s)\left(G_0(s)P_0(s)e^{-\theta s}\right)$$

$$G(s)\left(1+G_0(s)P_0(s)\left(1-e^{-\theta s}\right)\right) = G_0(s)$$

$$\ast \quad \boxed{G(s) = \frac{G_0(s)}{1+G_0(s)P_0(s)\left(1-e^{-\theta s}\right)}} = \boxed{\frac{G_0(s)}{1+G_0(s)\left(P_0(s)-P(s)\right)}}$$

REALIZABLE

- $G_0(s)$ can be developed considering the delay-free system with STANDARD INSTRUMENTS

$\Rightarrow$ $G_0(s)$ stabilizes $\dfrac{P_0(s)G_0(s)}{1+P_0(s)G_0(s)} = W_0(s)$

SMITH'S PRINCIPLE

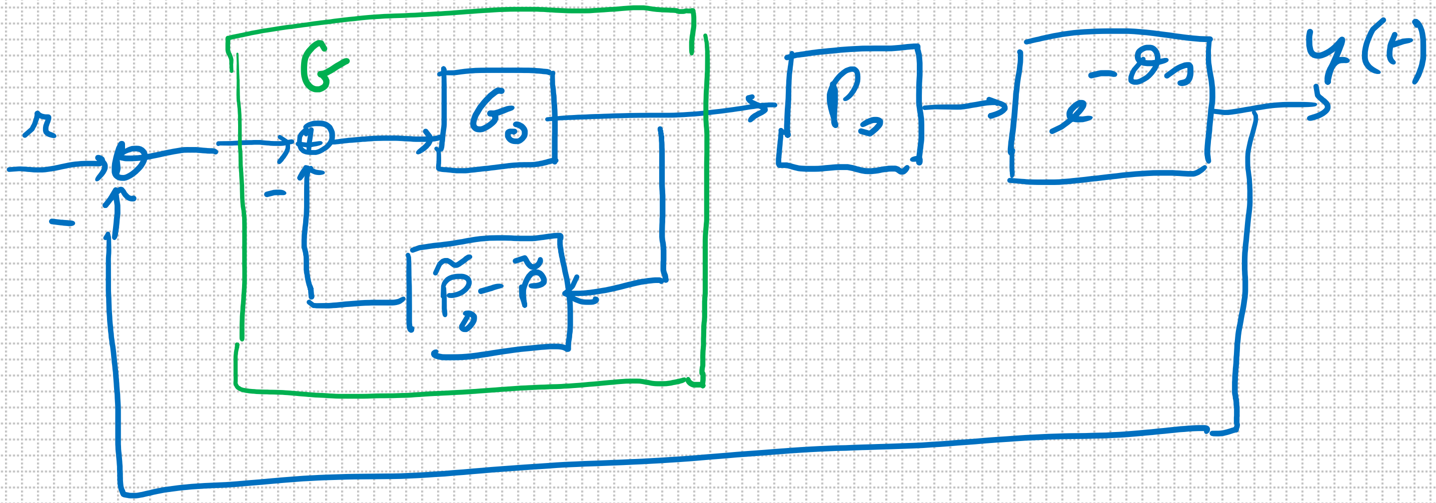
- If $\tilde{P}_0(s)$ and $\tilde{\theta}$ are perfect

$\Rightarrow$ the INPUT-OUTPUT characteristics of the system are the same of that of a delay-free system

$\leftarrow$ NOMINAL CONDITIONS

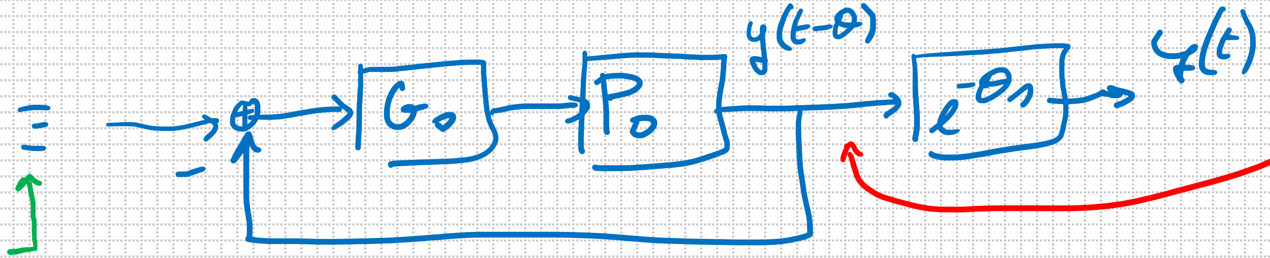
$\tilde{P}_0(s)$ = process model

$\tilde{\theta}$: nominal value of the delay



$\leftarrow$ SP controller

$$G = \frac{G_0}{1+G_0(\tilde{P}_0 - \tilde{P})}$$

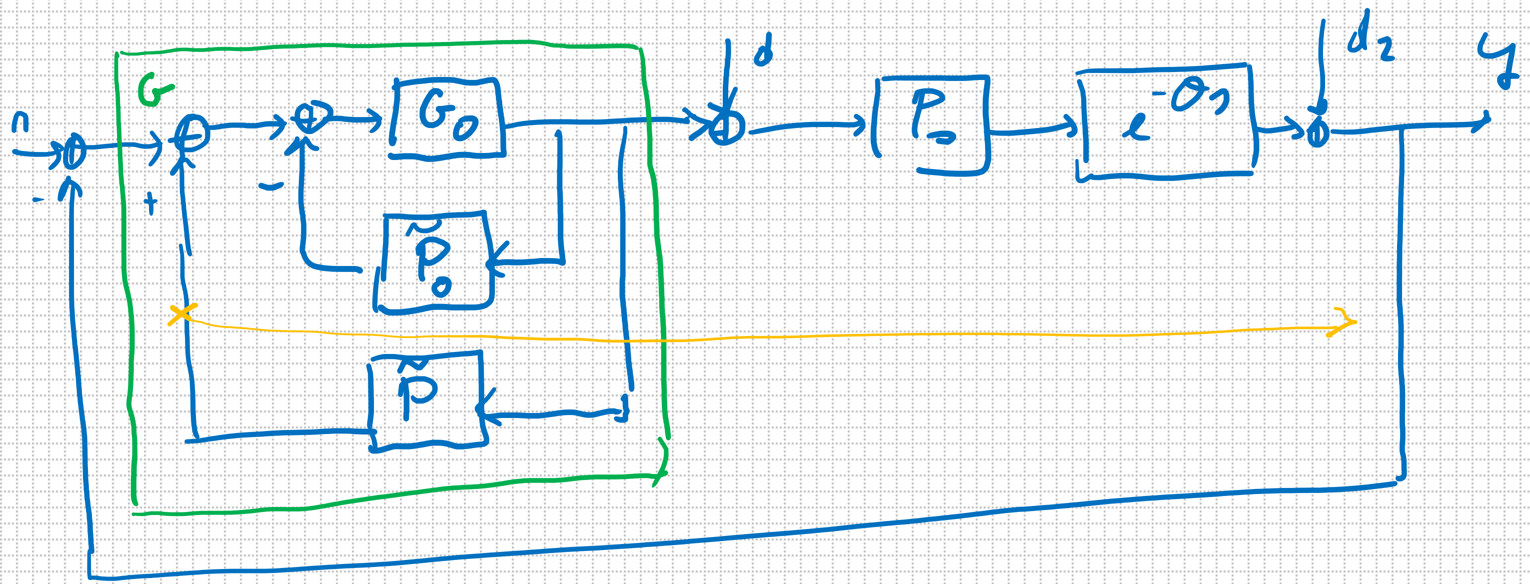


$\tilde{P} = P$

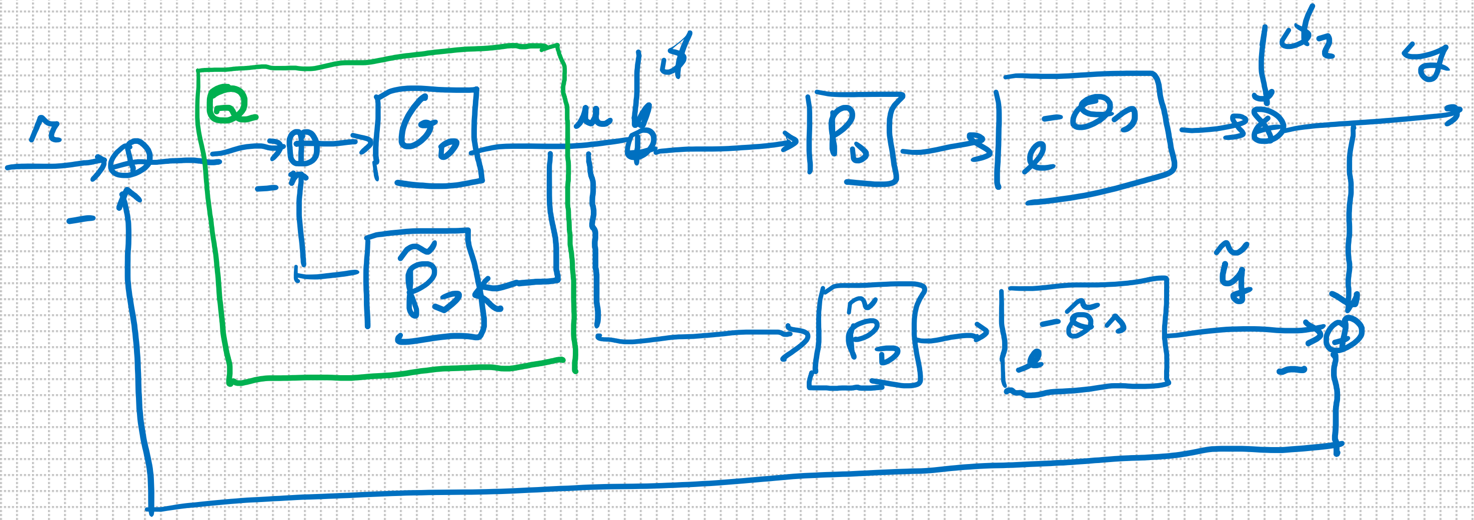
SMITH'S PREDICTOR

It is as the feedback signal was available θ seconds before the actual measure

IMC INTERPRETATION



IMC FORM of the SP



IMC STABILITY THEOREM

- SYS. STABLE $\iff$ P, Q STABLE
  $\uparrow$
  NOMINAL CONDITIONS
- $Q = \dfrac{G_0}{1 + G_0 P_0}$ STABLE

  $\iff$ G_0 STABILIZES P_0

$\tilde{y}$ is the predicted output in the future considering the process model

- $$W_d(s) = \left.\frac{y(s)}{d(s)}\right|_{r=d_2=0} = \left(1 - \frac{G_0 P_0}{1+P_0 G_0} e^{-\theta s}\right) P_0 e^{-\theta s}$$

$$W_{d_2}(s) = \left.\frac{y(s)}{d_2(s)}\right|_{r=d=0} = 1 - \frac{G_0 P_0}{1+P_0 G_0} e^{-\theta s}$$

The delay "disappears" from the denominator of the reference - disturbance transfer functions

- $G(s)$ developed as in delay-free systems!

Example: type 1 system

Hp:
- $P_0(s)$ is stable, without poles in $s=0$
- $G_0(s)$ has a pole in $s=0$ ← $G_0(s) = \frac{1}{s} G_0'(s)$
- $Q(s)$ is stable ($\equiv$ G_0 stabilizes P_0)

Th:
- $\lim_{t\to\infty} y_r(t) = 1$ with $r(t) = 1$
- $\lim_{t\to\infty} y_d(s) = 0$

- The system with time-delay is of Type 1:

$$\lim_{t\to\infty} y_r(t) = \lim_{s\to 0} s\, y_r(s) = \lim_{s\to 0} \cancel{s}\, W(s) \frac{1}{\cancel{s}} =$$

$r(s) = \frac{1}{s}$

$G_0 = \frac{1}{s} G_0'$

$$= \lim_{s\to 0} \underbrace{\frac{P_0 G_0}{1 + P_0 G_0} e^{-\theta s}}_{\text{① Time-delay system controlled by a SP controller}} = \lim_{s\to 0} \frac{P_0 G_0'}{s + P_0 G_0'} e^{-\theta s} = 1$$

$$\lim_{t\to\infty} y_d(t) = \ldots = \lim_{s\to 0} \Big(1 - \underbrace{\frac{G_0' P_0}{s + G_0' P_0} e^{-\theta s}}_{\to 1} \Big) P_0 \underbrace{e^{-\theta s}}_{\to 1} = 0$$

$d(t) = u_{-1}(t)$

DELAY UNCERTAINTIES

$$P(s) = P_0(s)\, e^{-\theta s}$$

$$\tilde{P}(s) : \begin{cases} \tilde{P}_0(s) = P(s) \\ \tilde{\theta} = \theta + \delta \end{cases}$$

δ: DELAY MISMATCH

if $\tilde{\theta} = \theta$

$$W = \frac{G_0 P_0\, e^{-\theta s}}{1 + G_0 P_0}$$

$$W(s) = \frac{G_0(s)\, P_0(s)}{1 + G_0(s) P_0(s) \underbrace{\left(1 - e^{-\tilde{\theta} s} + e^{-\theta s}\right)}} e^{-\theta s}$$

Complex exponential in the denominator in case of delay mismatch (the DELAY COMPENSATION is not perfect) = the stability might be affected

Exercise 5.

$$P(s) = k \, \frac{1+s/10}{(1+2s)^2} \, e^{-\theta s}, \quad \theta = 2, \quad k \in [10, 15], \quad \tilde{k} = 10$$

A) Design an IMC controller $Q(s)$ under the IAE cost:

- ROBUST STABILITY
- TYPE 1

B) Classical controller

- $P(s)$ is stable $\Rightarrow$ IMC design
- $\theta = 2$, $\tau = 2$ $\Rightarrow$ Padé approx of the delay cannot be used
- SP: develop Q_0 for the delay-free process $P_0(s)$ in the IMC design

$$\tilde{P}_0(s) = 10 \, \frac{1+s/10}{(1+2s)^2}$$

STEP 1. $\tilde{P}_+(s) = 1$, $\tilde{P}_- = \tilde{P}_0$

$$\Rightarrow \tilde{Q}_0 = \frac{1}{10} \, \frac{(1+2s)^2}{(1+s/10)}$$

STEP 2. $f(s) = \dfrac{1}{1+\lambda s}$ $\Rightarrow$ $Q_0(s) = \dfrac{1}{10} \, \dfrac{(1+2s)^2}{(1+\frac{s}{10})(1+\lambda s)}$

STEP 3. $\left| l_a(j\omega)\, Q_0(j\omega) \right| < 1, \quad \forall \omega$

$$\Delta_a(j\omega) = P(j\omega) - \tilde{P}(j\omega) = (K-10)\,\frac{1+\frac{j\omega}{10}}{(1+2j\omega)^2}$$

$$l_a(j\omega) = 5\,\frac{1+\frac{j\omega}{10}}{(1+2j\omega)^2}$$

$$|l_a Q| = \left| 5\,\frac{1+\frac{j\omega}{10}}{(1+2j\omega)^2}\;\frac{1}{10}\;\frac{(1+j\omega\lambda)^2}{\left(1+\frac{j\omega}{10}\right)(1+\lambda j\omega)} \right| = \left| \frac{1}{2}\;\frac{1}{1+j\omega\lambda} \right| < 1 \quad \forall \lambda$$

true for all values of λ $\Rightarrow$ we set $\lambda = \frac{1}{10}$

$$\boxed{Q_0(s) = \frac{1}{10}\,\frac{(1+j\omega\lambda)^2}{\left(1+\frac{j\omega}{10}\right)^2}}$$

Primary controller for the delay-free case

$$G_o(s) = \frac{Q_o(s)}{1 - Q_o(s)\tilde{P}_o(s)} = \frac{\frac{1}{10}\frac{(1+2s)^2}{(1+s/10)^2}}{1 - \cancel{10}\,\frac{\cancel{1+s/10}}{\cancel{(1+2s)^2}}\,\frac{1}{\cancel{10}}\,\frac{\cancel{(1+2s)^2}}{(1+s/10)^2}} = \frac{1}{10}\,\frac{(1+2s)^2}{\left(1+\frac{s}{10}\right)^2 - 1} =$$

$$= \frac{1}{10}\,\frac{4s^2+4s+1}{s\left(\frac{s}{100}+\frac{1}{5}\right)} = 2\,\frac{s+\frac{1}{4}+s^2}{s\left(1+\frac{s}{20}\right)} = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{1+\beta_f s}$$

$$\begin{cases} K_c = 2 \\ T_i = 4 \\ T_d = 1 \\ \beta_f = \frac{1}{20} \end{cases}$$

$$\boxed{G(s) = \frac{G_o(s)}{1+G_o(s)\left[\hat{P}_o(s) - \tilde{P}(s)\right]}} \leftarrow \theta = 2$$

ROBUST SP CONTROLLER

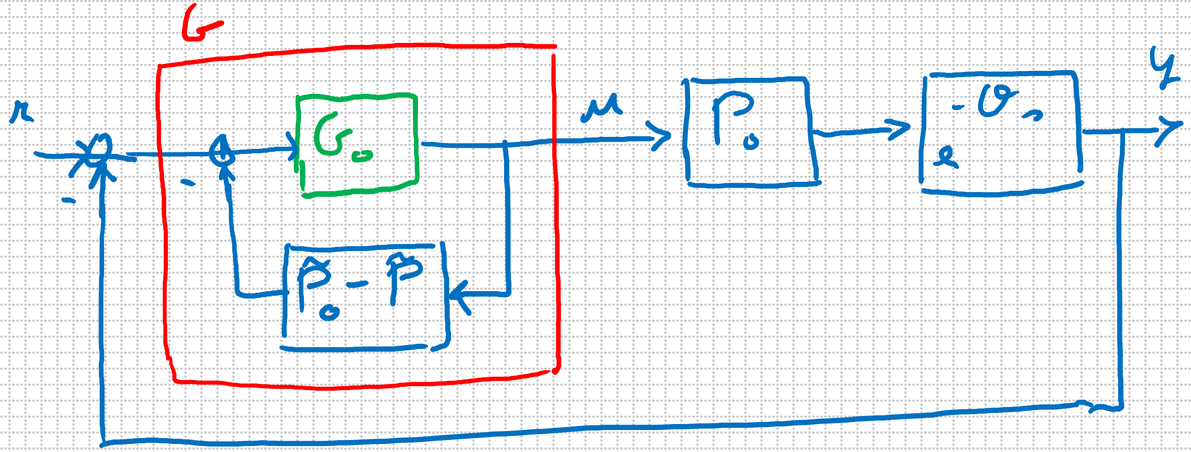
(IMC design for $\hat{P}(s)$ we obtain a $Q(s)$ such that $G = \frac{Q}{1-Q\tilde{P}} \equiv \frac{G_o}{1+G_o(\hat{P}_o - P)}$)

It might be difficult to verify that it is a SP controller

SMITH PREDICTOR

nominal conditions ($\tilde{P}_o = P_o$, $\tilde{\theta} = \theta$)
INPUT-OUTPUT viewpoint

a)



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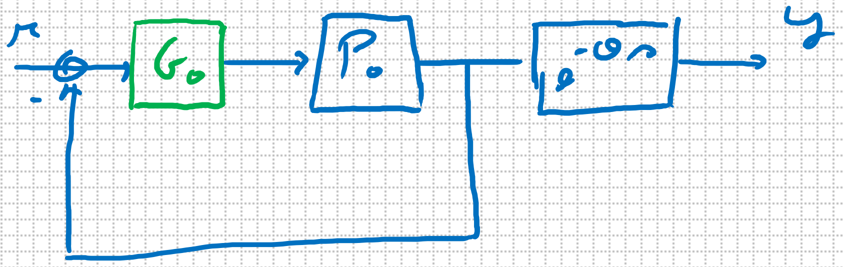
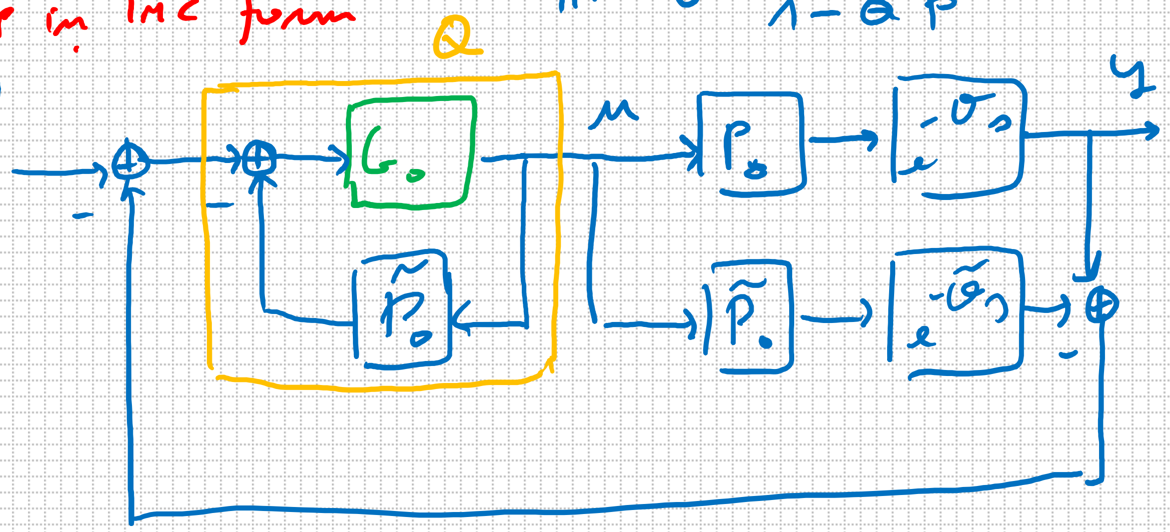
b)



SP in IMC form

$$G = \frac{Q}{1 - Q\tilde{P}}$$

c)



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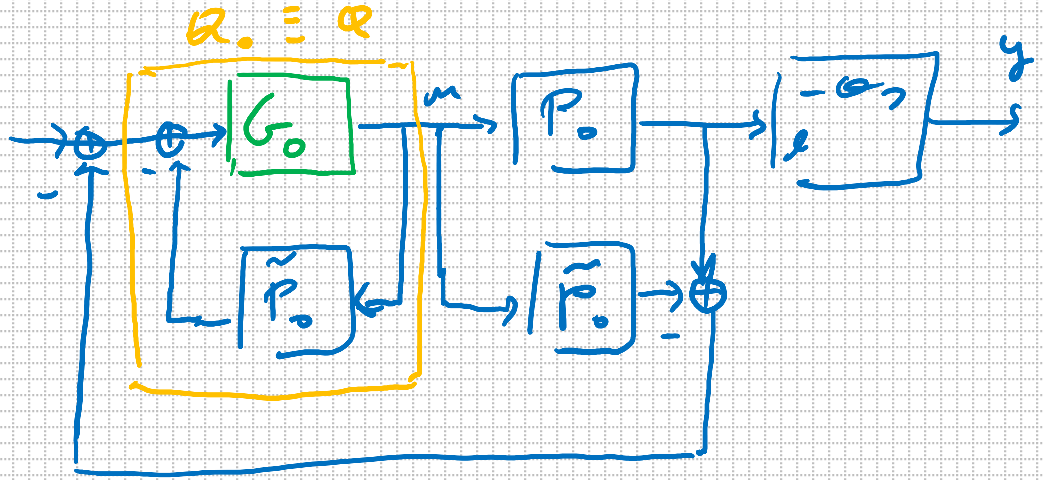
$$Q_o = \frac{G_o}{1 + G_o \tilde{P}_o}$$

d)



1. Find Q_o from d) with robust IMC design

2. Primary controller of the SP in b)
$$G_o = \frac{Q_o}{1 + Q_o \tilde{P}_o}$$

3. SP controller
$$G = \frac{G_o}{1 + G_o(\tilde{P}_o - \tilde{P})}$$