Master in Control Engineering

Process Automation
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Dipartimento di Ingegneria informatica
automatica e gestionale Antonio Ruberti

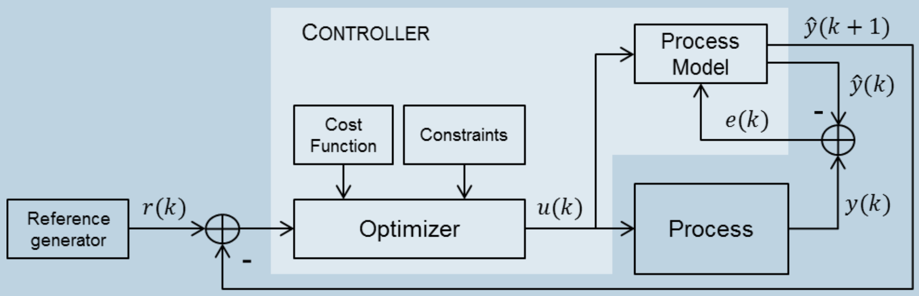
Sapienza
Università di Roma

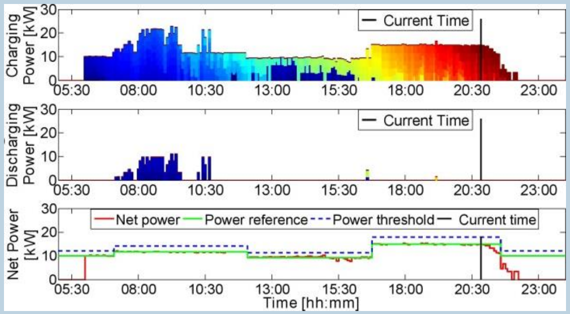
Master in Control Engineering

Process Automation

10. Model Predictive Control principles

Slides based on:
E.F. Camacho, C. Bordons Alba, “Model Predictive Control”, *Advanced Textbooks in Control and Signal Processing*, Springer,-Verlag, XXII, 2nd ed., 2007, 405 p., ISBN 978-0-85729-398-5.





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Outline

- Model Predictive Control (MPC) principles
  - From LQG to MPC
  - Introduction
  - MPC Strategy
  - Historical Perspective
  - Summary

Introduction

- From LQG to MPC
  - LQG (Linear Quadratic Gaussian) controller
    - Kalman, early 1960s
      - Linear discrete-time system

$$x_{k+1} = Ax_k + Bu_k + Gw_k$$
$$y_k = Cx_k + \xi_k$$

        » w_k, ξ_k: Gaussian noises with 0 mean
      - Objective function

$$\Phi = \mathcal{E}\{J\}$$
$$J = \sum_{j=1,\dots,\infty} \left(\|x_{k+j}\|_Q^2 + \|u_{k+j}\|_R^2 \right) = \sum_{j=1,\dots,\infty} \left(x_{k+j}^T Q x_{k+j} + u_{k+j}^T R u_{k+j} \right)$$

        » Q, R: wheight matrices
        » Infinite horizon cost function
      - Otimal input

$$u_k = -K_c \left(y_k - C\hat{x}_{k|k} \right)$$

        » $\hat{x}_{k|k}$: optimal estimate at time k
        » K_c: optimal proportional gain (solution of a Riccati equation)

Introduction

- From LQG to MPC
  - LQG (Linear Quadratic Gaussian) controller
    - Drawbacks in industrial context
      - presence of constraints
      - presence of process nonlinearities;
      - presence of model uncertainty (robustness)
      - unique performance criteria
      - infinite horizon

Introduction

- Model Predictive Control (MPC)
  - Firstly developed in the 70s
  - MPC designates an ample range of control methods which make explicit use of a model of the process to obtain the control signal by minimizing an objective function
    - These design methods lead to controllers which have practically the same structure
- Ideas of predictive control (MPC or receding horizon or receding horizon predictive control (RHPC))
  - Explicit use of a model to predict the process output at future n time instants (prediction horizon)
  - Calculation of a control sequence minimizing an objective function
  - Application of the first control signal of the sequence calculated at each step (receding strategy)
    - At each instant the prediction horizon is displaced one step towards the future
- The various MPC algorithms differ amongst themselves
  - in the model used to represent the process and the noises
  - In the cost function to be minimized

Introduction

- MPC application examples
  - Widely received by the academic world and industry
  - Successful in various application fields
    - Robots
    - Clinical anaesthesia
    - Cement industry
    - Distillation columns
    - PVC plants
    - Steam generators
    - Control of wind turbines
    - …

Introduction

Hierarchy of control system functions in a typical processing plant [QIN03]

1. Plant-wide optimizer

   Determines optimal steady-state settings for each unit in the plant

2. Unit optimizer

   Determine the optimal economic steady state
   Passes this to the dynamic constraint control system for implementation

3. Dynamic constraint control

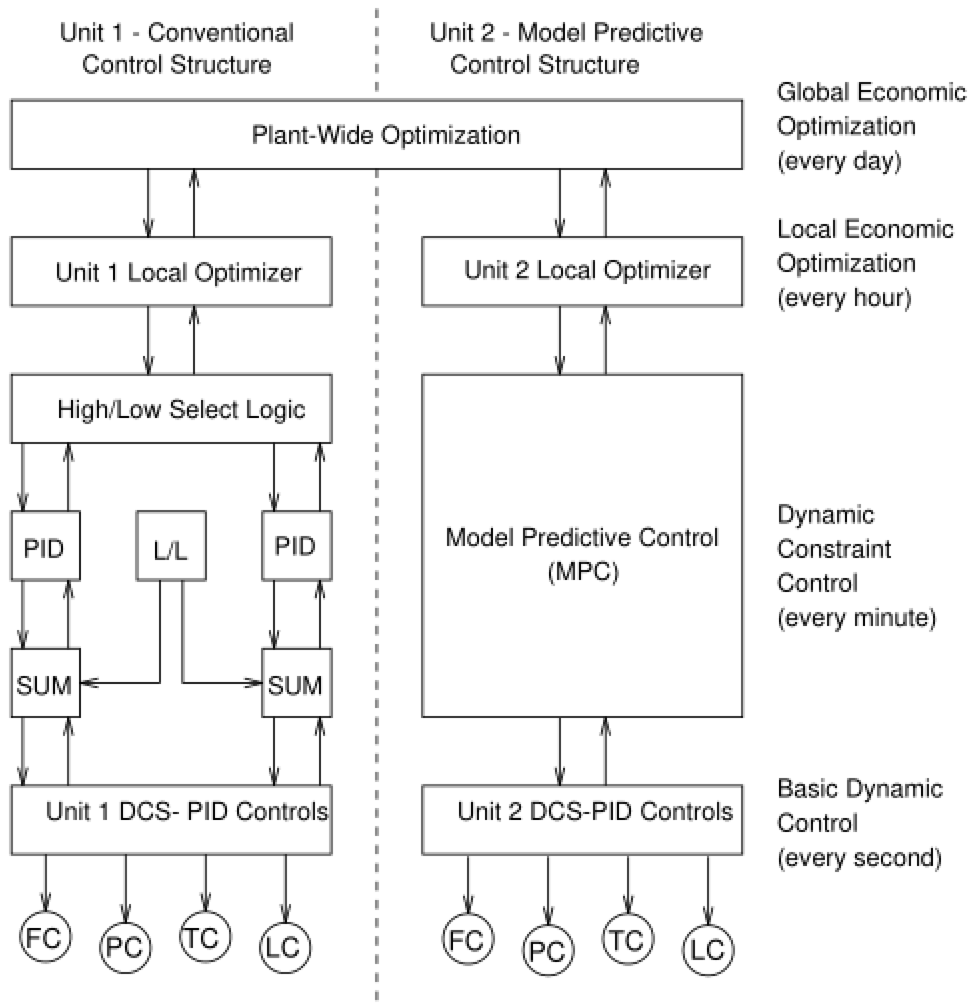
   Must move the plant from one constrained steady state to another while minimizing constraint violations

Left: conventional structure

Dynamic constraint control implemented as a combination of PID algorithms, lead-lag (L/L) blocks and high/low select logic

Right: MPC structure

Dynamic constraint control implemented as a MPC block



[QIN03] Qin et al., A survey of industrial model predictive control technology, Cont. Eng. Pract., 2003

Introduction

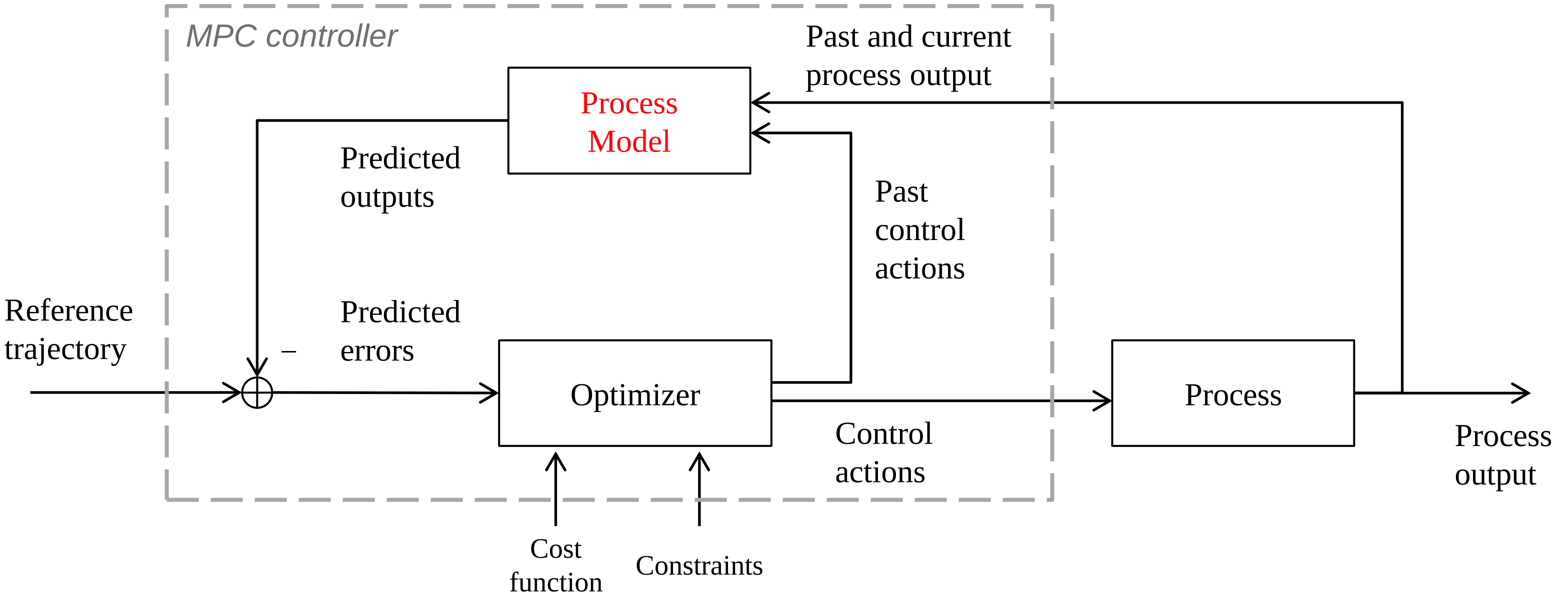
- MPC characteristics
  - Open-loop control
  - The tuning is relatively easy
  - It can be used to control a great variety of processes
    - E.g., systems with long delay times, non-minimum phase systems, unstable systems
  - Multivariable case easily dealt with
  - It intrinsically has compensation for dead times.
  - It introduces feed-forward control in a natural way to compensate for measurable disturbances
  - The resulting controller is an easy-to-implement control law.
  - Its extension to the treatment of constraints is conceptually simple, and these can be systematically included during the design process.
  - It is very useful when future references are known (robotics or batch processes)
  - Open methodology based on certain basic principles which allows for future extensions

Introduction

- MPC drawbacks
  - Although the resulting control law is easy to implement and requires little computation, its derivation is more complex than that of the classical PID controllers
  - If the process dynamics is time-varying all the computation has to be carried out at every sampling time (adaptive control)
  - When constraints are considered, the amount of computation required is even higher
  - Need for an appropriate model of the process to be available
    - The design algorithm is based on prior knowledge of the model and is dependent on it
    - The benefits obtained by this model-based design will be affected by the discrepancies existing between the real process and the model used (*uncertainties*)
- In practice, MPC has proved to be a reasonable strategy for industrial control, in spite of the original lack of theoretical results at some crucial points such as stability and robustness
  - New results on robustness and stability are being achieved by the animate current research on MPC

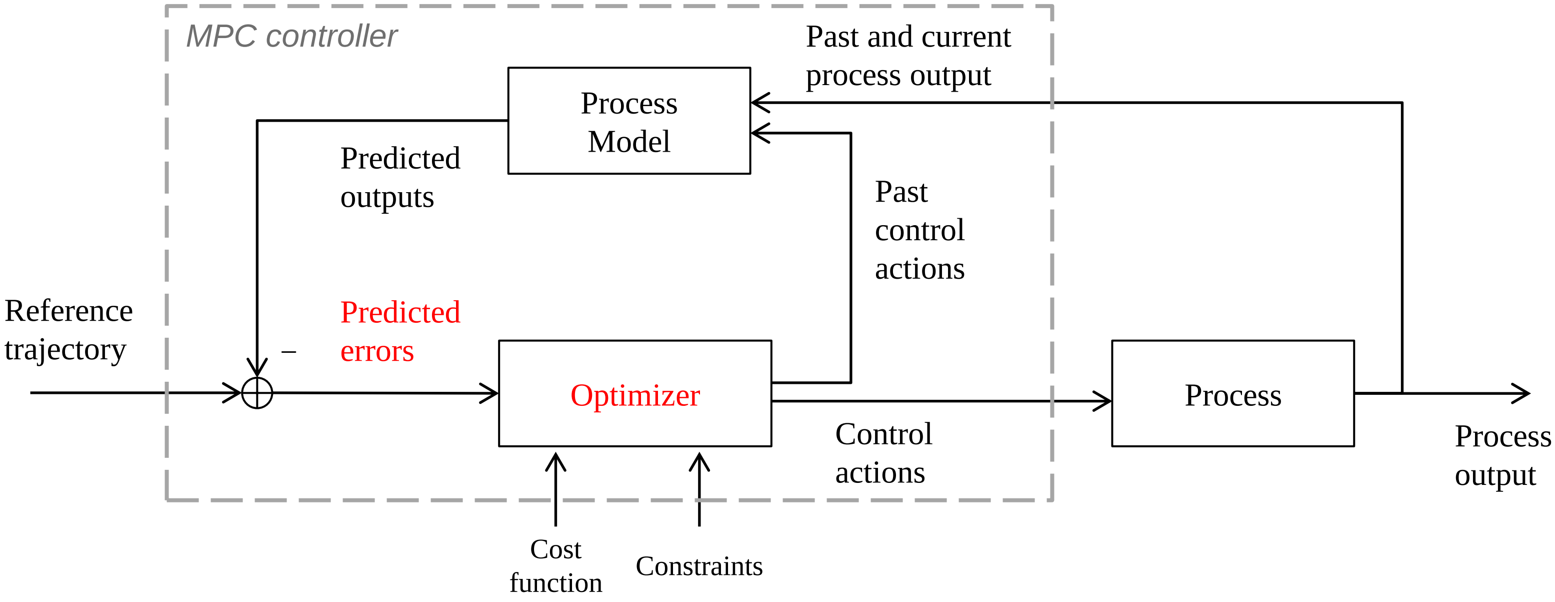
MPC Strategy

- Basic structure of MPC
  - Process model
    - It predicts the *future plant outputs*, based on
      - past and current values of plant outputs
      - past values of the control actions
      - *proposed optimal* future control actions
    - <u>The process model plays a decisive role in the controller</u>
      - It must be able to precisely capture the process dynamics to predict the future outputs



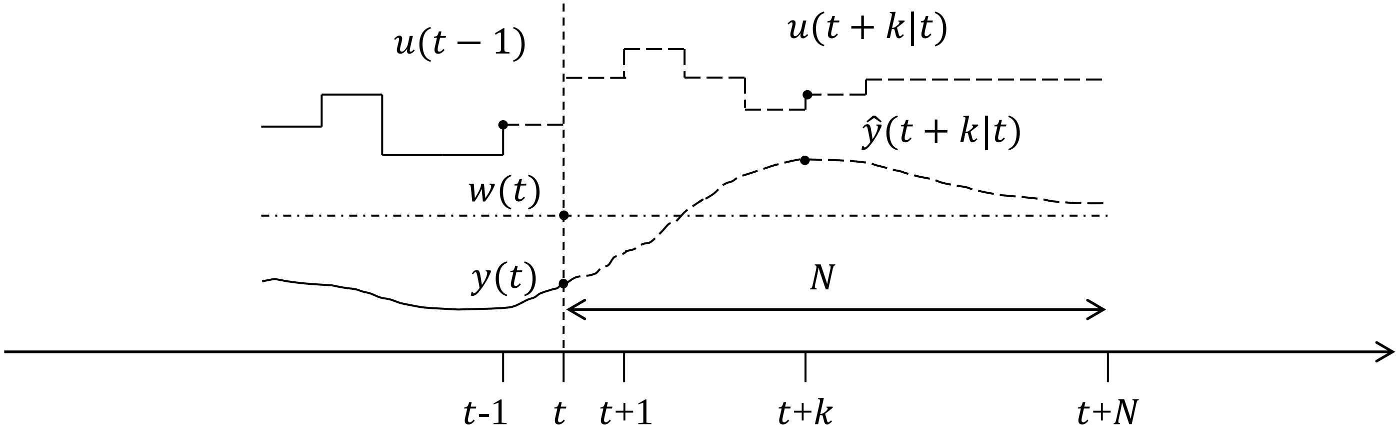
MPC Strategy

- Basic structure of MPC
  - Optimizer
    - It calculates the *optimal control action*, based on
      - a cost function, where the future *tracking error* is considered
      - input and output constraints
  - Comparison to classical feedback control (e.g., PIDs)
    - Classical control: the control actions are taken based on *past errors*
    - MPC: the control actions are taken based on *future predicted errors*



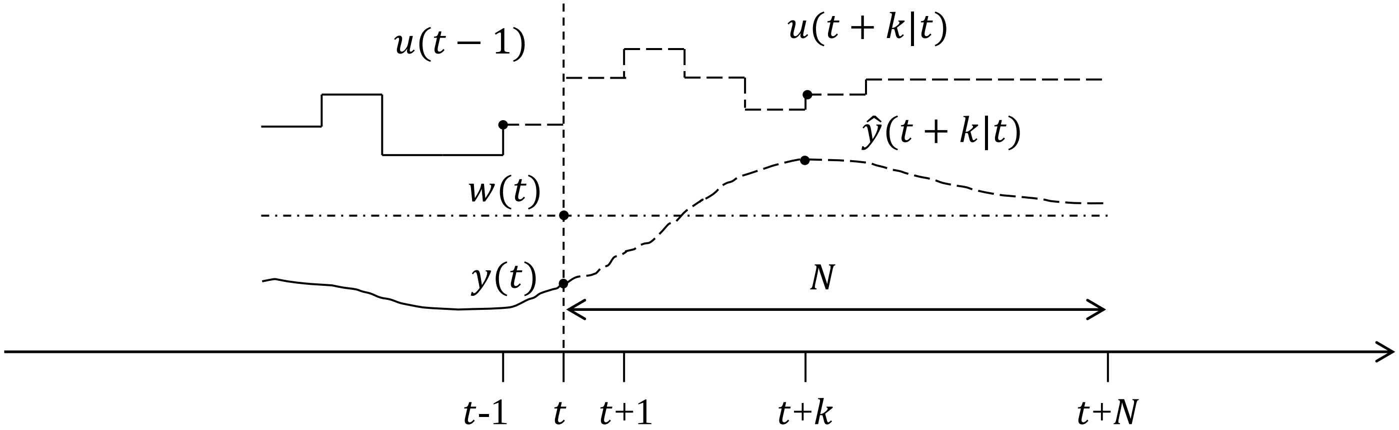
MPC Strategy

- Signals of interest
  - $y(t)$: process output at time instant t
  - $u(t)$: control input at time instant t
  - $w(t)$: reference trajectory at time t
  - $\hat{y}(t+k|t)$: estimated process output at time instant $t+k$, given known inputs and outputs at time t
  - $u(t+k|t)$: control input at time instant $t+k$, computed at time t, given known inputs and outputs at time t
  - N: prediction horizon



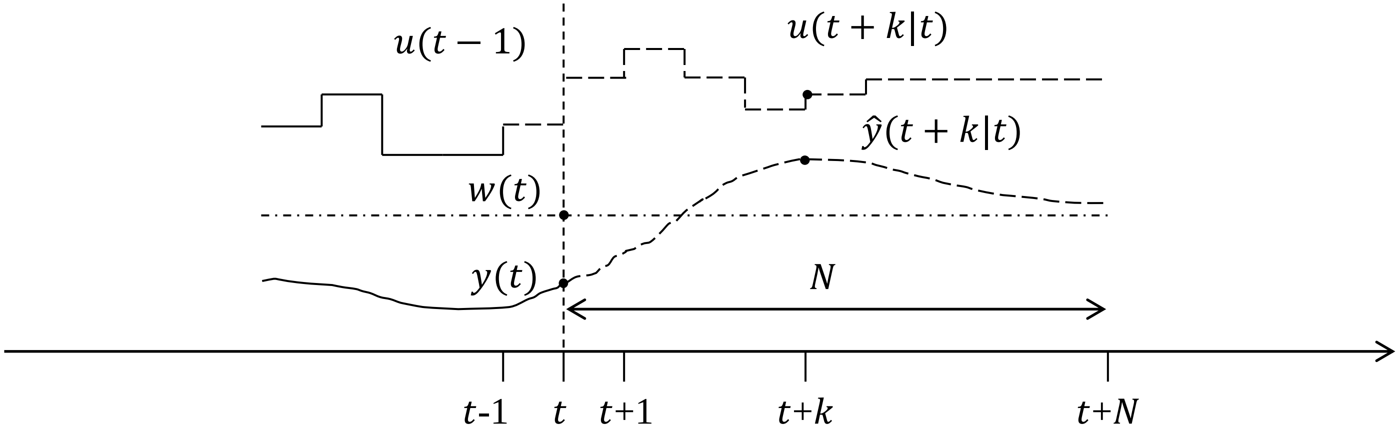
MPC Strategy

– The set of future control signals $u(t+k|t)$ is calculated by optimizing a cost index J to keep the process output as close as possible to the reference trajectory $w(t+k)$
  - e.g., J
    – Quadratic function of the errors between the predicted output signal and the predicted reference trajectory
    – Control effort usually included in the objective function
    – An explicit solution can be obtained if the criterion is quadratic, the model is linear, and there are no constraints, otherwise an iterative optimization method is used



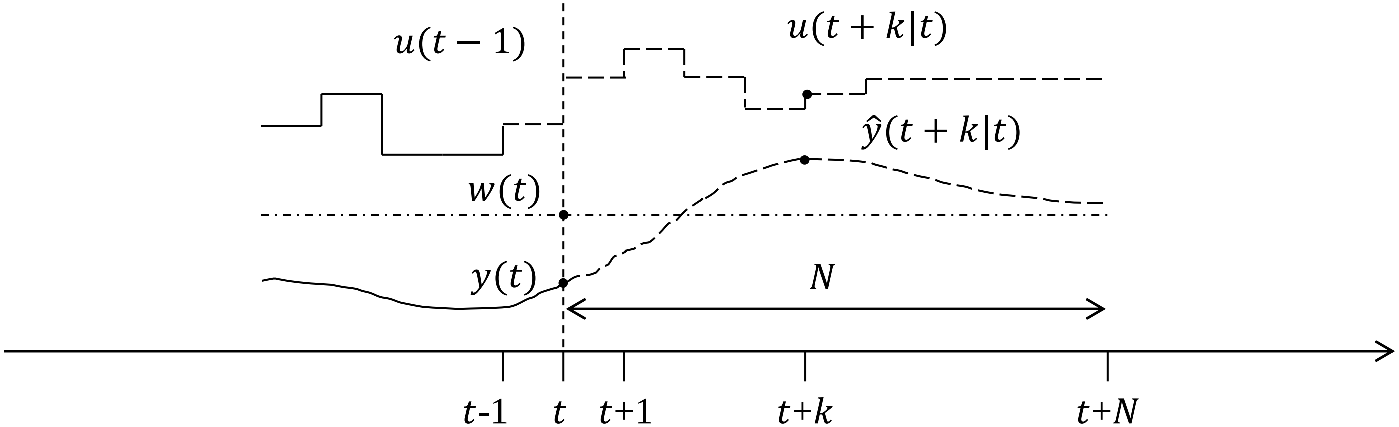
MPC Strategy

- The future outputs $\hat{y}(t+k|t)$ for the prediction horizon N are predicted at <u>each instant</u> t using the process model
- $\hat{y}(t+k|t), k = 1, \dots, N$ is calculated based on
  - $u(t'),\ t' = t-1, t-2, t-3, \dots$: known values of past inputs <u>up to instant</u> t
  - $y(t'),\ t' = t, t-1, t-2, \dots$: known values of past outputs <u>up to instant</u> t
  - $u(t+k|t), k = 0, \dots, N-1$: future control signals (<u>unknown</u>, to be calculated)



MPC Strategy

- *Receding horizon* approach
  - At time instant t we find the optimal control sequence $u^*(t+k|t), k=0,..,N-1$
    - $u^*(t|t)$ is sent to the process
    - $u^*(t+k|t), k=1,\dots,N-1$ are rejected
  - At time instant $t+1$ the control signal $u(t+1|t+1)$ is computed again
    - $u^*(t+1|t+1) \neq u^*(t+1|t)$ (generally) since new information are available: $y(t+1)$, $u(t)$



Historical Perspective

- 1963, receding horizon principle
  - Propoi, ”open-loop optimal feedback”
- Late 1970s, MPC in the industry
  - A dynamic process model is explicitly used in both algorithms to predict the effect of the future control actions
    - Richalet *et al.,* Model Algorithmic Control (MAC)
      - » Model: impulse response
    - Cutler and Ramakter, Dynamic Matrix Control (DMC)
      - » Model: step response
- 80s, MPC quickly became popular in chemical process industries
  - Multivariable systems including constraints
  - Simplicity of the algorithm
  - The use of the impulse or step response model
    - Requiring less *a priori* information for its identification w.r.t. state-space models

Historical Perspective

- Since then, plethora of MCP algorithms
  - » Predictor-Based Self-Tuning Control, Extended Horizon Adaptive Control (EHAC), Extended Prediction Self Adaptive Control (EPSAC), *Generalized Predictive Control (GPC)*, Multistep Multivariable Adaptive Control (MUSMAR), Multipredictor Receding Horizon Adaptive Control (MURHAC), *Predictive Functional Control (PFC)*, Unified Predictive Control (UPC), …
- 90s, state space models
  - Allows for the use of well-known theorems of the state-space theory
  - Stability and robustness
    - The MPC controller = output regulation based on a <u>state observer</u>
      - » Stability, performance and robustness determined by the poles of the observer and the poles of the regulator
  - Generalization to more complex cases
    - Multivariable processes, nonlinear processes, systems with stochastic disturbances and noise in the measured variables
  - State estimation techniques from stochastic optimal control
- 2000s, further results with more complex systems
  - Nonlinear, hybrid, “fast” processes

Summary

- MPC introduced
  - Main MPC controller components
    - Optimizer
      - Cost function, constraints
    - Dynamic process model
  - Main MPC principles
    - Receding horizon
    - Prediction horizon
    - Future predicted errors