Process Automation (MCER), 2017-2018

# Exam - January 23, 2018 (3h00)

## Exercise 1 (11 pt.)

Let the process be described by the transfer function: $P(s) = K\frac{s+0.2}{(s+1)^2}e^{-\theta s}$, with $K \in [1,1.9)$ and $\theta = 0.1s$, and let the nominal gain value be $\widetilde{K} = 1$.

A) By using the Padé approximation, under the IAE cost function, design an IMC controller $Q(s)$ such that:
   i) the overall system is robustly asymptotically stable;
   ii) the overall system has 0 steady-state error for step inputs.

B) Describe (without calculations) how to check that the found controller stabilizes the real process $P^R(s) = 1.9\frac{s+0.2}{(s+1)^2}e^{-0.1}$ – suggestion: consider the equivalent controller $G(s)$ and the classic feedback control scheme.

- $\tilde{P}(s) = \frac{s+0.2}{(s+1)^2}e^{-0.1s}$ STABLE $\Rightarrow$ IMC procedure
- $\tau = 1$, $\theta = 0.1$ $\Rightarrow$ Padé approx $e^{-0.1s} \approx \frac{1-0.05s}{1+0.05s}$ $\leftarrow$ $e^{-\theta s} \approx \frac{1-e^{-\frac{\theta}{2}s}}{1+e^{-\frac{\theta}{2}s}}$

$\tilde{P}^P(s) = 0.2\frac{1+5s}{(1+s)^2}\frac{1-0.05s}{1+0.05s}$, $P^P(s) = 0.2K\frac{s+0.2}{(s+1)^2}\frac{1-0.05s}{1+0.05s}$

STEP 1. factorization $\tilde{P}^p(s) = P_+(s)\,P_-(s)$

IAE $\Rightarrow$ $P_+(s) = 1 - 0.05s \qquad P_-(s) = 0.2\,\dfrac{1+5s}{(1+s)^2(1+0.05s)}$

$$\tilde{Q}(s) = P_-^{-1}(s) = 5\,\frac{(1+s)^2(1+0.05s)}{1+5s}$$

STEP 2. Filter, type 1 : $f(s) = \dfrac{1}{(1+\lambda s)^m}$, $m=2$

$$Q(s) = 5\,\frac{(1+s)^2(1+0.05s)}{(1+5s)(1+\lambda s)^2}$$

$\swarrow$ IMC controller
Proper
$\tilde{F}(0) = 1$

STEP 3. Robustness  Suff. cond. $|l_a(j\omega)\,Q(j\omega)| < 1, \ \forall\omega$

$$l_a(j\omega) = |P^p(j\omega) - \tilde{P}^p(j\omega)| = \left|(0.2K - 0.2)\,\frac{1+5j\omega}{(1+j\omega)^2}\,\frac{1-0.05j\omega}{1+0.05j\omega}\right| =$$

$|\cdot| = 1$

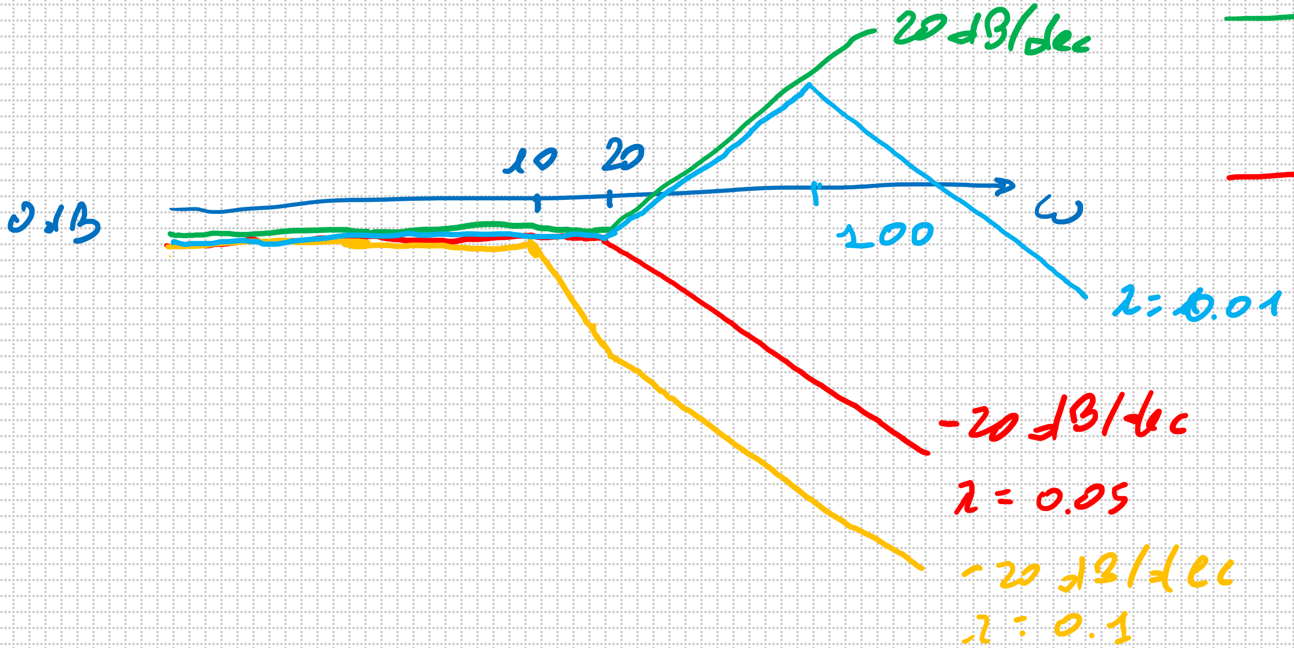
$$= \left|0.2(K-1)\,\frac{1+5j\omega}{(1+j\omega)^2}\right|$$

$\longleftarrow$ maximized for the largest value of $K \in [1, 1.1]$

$l_a(j\omega)$ : $|l_a(j\omega)| > |\Delta_a(j\omega)|$ , $\forall \omega$ $\Rightarrow$ $K = 1.9$

$$l_a(j\omega) = 0.18\,\frac{(1+5j\omega)}{(1+j\omega)^2}$$

$$|l_a(j\omega)\,Q(j\omega)| = \left|\, 0.18\,\frac{\cancel{1+5j\omega}}{\cancel{(1+j\omega)^2}}\; 5\,\frac{\cancel{(1+j\omega)^2}\,(1+0.05j\omega)}{\cancel{(1+5j\omega)}\,(1+\lambda j\omega)^2}\right| = \left|\,0.5\,\frac{1+0.05j\omega}{(1+j\omega\lambda)^2}\right| < 1 ,\quad \forall\omega$$




— $|0.5(1+0.05j\omega)|_{dB}$

— $\left|\,0.5\,\frac{(1+0.05j\omega)}{(1+j\omega\lambda)}\right|$

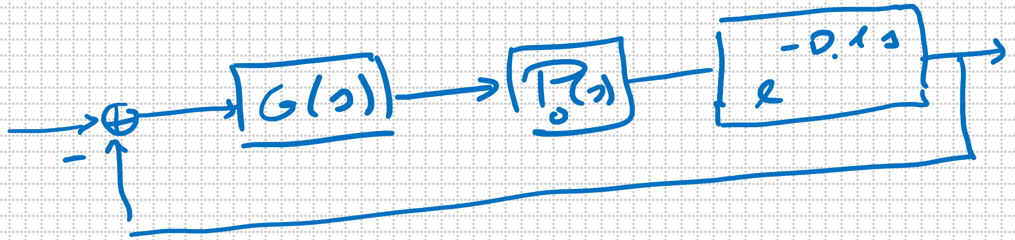
$$\boxed{\lambda \geq 0.05}$$

• It is better to be conservative due to the Padé approx. $\Rightarrow \lambda = 0.1$

$$Q(s) = 5\,\frac{(1+s)^2\,(1+0.05s)}{(1+5s)\,(1+0.1s)^2}$$

**PART B** Does $Q(s)$ stabilize $P(s) = 1.9\,\frac{s+0.2}{(s+1)^2}\,e^{-0.1s}$

1) $G(s) = \dfrac{Q(s)}{1-\tilde{P}(s)\,Q(s)}$ classical controller

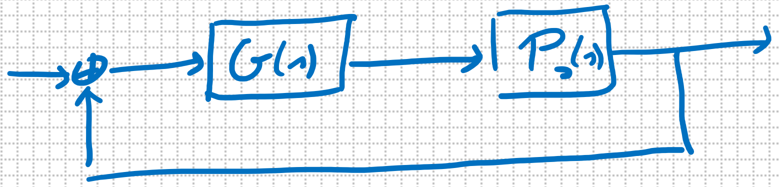



Delay margin

$m_\tau = m_{\tau_0} - \theta$ $m_{\tau_0}$ : maximum delay that the delay-free system can stand (delay margin of the delay-free sys.)

$\theta = 0.1$

2) Delay-free sys.




- Check the stability by means of the Nyquist theorem
- If $P_0(s)$ is stabilized, we can compute $m_{\varphi_0}$

$$\Rightarrow \quad m_{\tau_0} = \frac{m_{\varphi_0}}{\omega_c}$$

3) - If $m_{\tau_0} > 0.1 \Rightarrow G(s)$ stabilizes $P(s)$

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## Exercise 2 (11 pt.)

Consider a process whose state space model is given by the following equations:

$$\begin{cases} x(t) = Mx(t-1) + Nu(t-1) \\ y(t) = Qx(t) \end{cases}, \text{ with } M = \begin{bmatrix} 0.2 & 0.4 \\ 0 & -0.2 \end{bmatrix}, N = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, Q = [0.25 \;\; 0.75].$$

Compute the control action $u(5)$ of a PFC controller with the following specifications:

i) Prediction horizon $p = 4$;

ii) Base functions: $B_1(k) = 1, B_2(k) = k^2, k \geq 0$;

iii) Number of coincidence points $n_H = 2$, chosen to prefer robustness to performance;

iv) Reference signal $r(t) = \begin{cases} t/2, & t = 0, \dots, 6 \\ 3, & t > 6 \end{cases}$;

v) Reference trajectory computed as $w(t+k|t) = r(t)$;

vi) Cost function $J = e^T e$.

vii) The plant-model error is computed as $\hat{n}(t+k|t) = y_m(t) - y(t), \forall k > 0$, with $y_m(0) = 0$, $y_m(1) = 0.2$, $y_m(2) = 0.8$, $y_m(3) = 1.75$, $y_m(4) = 2.5$, $y_m(5) = 3$.

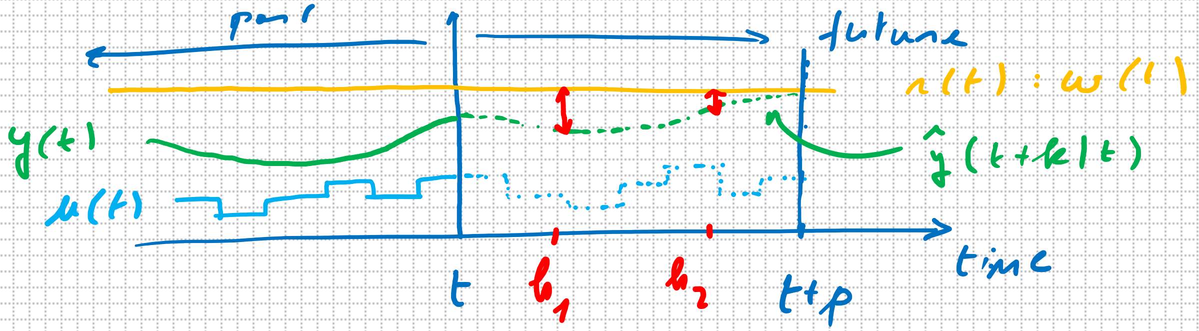
viii) State and control values $x(4) = \begin{pmatrix} 15 \\ 2 \end{pmatrix}$, $u(4) = 5$.

- Coincident points

$h_2 = p = 4$

$h_1 = 3$

ROBUSTNESS vs PERFORMANCE




$n_H = 2$

We are evaluating the errors

$\hat{e}(t+h_1|t)$ , $\hat{e}(t+h_2|t)$

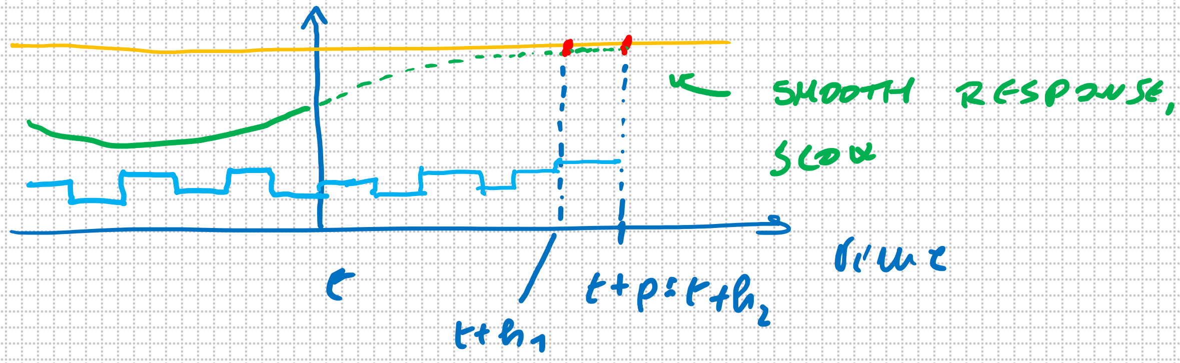
⇒ it makes no sense to pick $p > h_2$ !

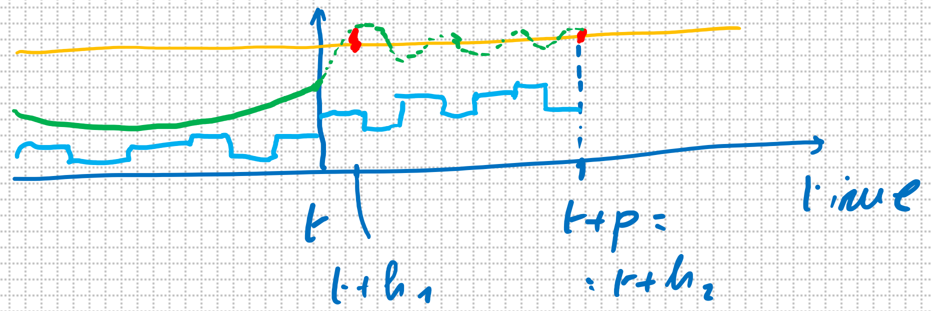
⚠ delay !

$$y(t) = \begin{pmatrix} 0.25 & 0.75 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} ;$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 0.2 & 0.4 \\ 0 & -0.2 \end{pmatrix} \begin{pmatrix} x_1(t-1) \\ x_2(t-1) \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} u(t-1)$$

⇒ $y(t)$ directly depends on $u(t-1)$ ⇒ $d = 0$



- Compute the model response to the base functions at $h_1, h_2$

0 initial conditions

- $B_1(k) = 1$

$h_1 = 3$

$$y_{B_1}(3) = Q M^2 N B_1(0) + Q M N B_1(1) + Q N B_1(2) = \dots = 0.4$$

$h_2 = 4$

$$y_{B_1}(4) = Q M^3 N B_1(0) + Q M^2 N B_1(1) + Q M N B_1(2) + Q N B_1(3) = \dots = 0.4$$

- $B_2(k) = k^2$

$h_1 = 3$

$$y_{B_2}(3) = Q M^2 N B_2(0) + Q M N B_2(1) + Q N B_2(0) = \dots = 2.0$$

$h_2 = 4$

$$y_{B_2}(4) = \dots = 4.1$$

$$\Rightarrow \quad Y_B = \begin{pmatrix} y_{B_1}(h_1) & y_{B_2}(h_1) \\ y_{B_1}(h_2) & y_{B_2}(h_2) \end{pmatrix} = \begin{pmatrix} 0.4 & 2{,}0 \\ 0.4 & 4.1 \end{pmatrix}$$

$$u(5) = \sum_{i=1}^{2} \mu(5)\, B_i(0)$$

$$\mu(5) = Y_B^{-1}(w - f) \quad \longleftarrow$$

$$\mu = (Y_B^T Y_B)^{-1} Y_B^T (w - f)$$

$$Y_B \in \mathbb{R}^{2\times 2}$$

$$\Rightarrow (Y_B^T Y_B)^{-1} Y_B^T = Y_B^{-1}$$

- Free response

$t = 5$

$$f(5+k|5) = QM^k x(5) + \hat{n}(5+k|5) = QM^k x(5) + y_m(5) - y(5)$$

$$\hat{n}(5+k|5) = \hat{n}(5|5) = y_m(5) - y(5)$$

$y_m(5) = 5$, $x(4) = \begin{pmatrix} 15 \\ 2 \end{pmatrix}$, $u(4) = 5$

MODEL

$$\begin{cases} x(5) = M\,x(4) + N\,u(4) = \begin{pmatrix} 0.2 & 0.4 \\ 0 & -0.2 \end{pmatrix}\begin{pmatrix} 15 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} 5 = \begin{pmatrix} -1.2 \\ 4.6 \end{pmatrix} \\ y(5) = Q\,x(5) = (0.25 \;\; 0.75)\begin{pmatrix} -1.2 \\ 4.6 \end{pmatrix} = 3.15 \end{cases}$$

$k = h_1 = 3 \qquad f(8|5) = QM^3 x(5) + y_m(5) - y(5) = \dots = 0.14$

$k = h_2 = 4 \qquad f(9|5) = QM^4 x(5) + y_m(5) - y(5) = \dots = 0.15$

- $$r(t) = \begin{cases} t/2 & , t = 0, \dots, 6 \\ 3 & , t > 6 \end{cases} \quad \leftarrow \quad r(5) = 2.5$$

$$w(t+k|t) = r(t) \Rightarrow w(5+k|5) = r(5) = 2.5 \quad \leftarrow \quad \begin{array}{l} k = h_1 \\ k = h_2 \end{array}$$

- $$\mu(5) = Y_B^{-1}(w - f) = \begin{pmatrix} 0.4 & 2.0 \\ 0.4 & 4.1 \end{pmatrix}^{-1} \left( \begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} 0.14 \\ 0.15 \end{pmatrix} \right) = \dots = \begin{pmatrix} 5.9 \\ -0.01 \end{pmatrix}$$

$$\boxed{u(5) = \begin{pmatrix} \mu_1(5) & \mu_2(5) \end{pmatrix} \begin{pmatrix} B_1(0) \\ B_2(0) \end{pmatrix} = \begin{pmatrix} 5.9 & -0.01 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 5.9}$$