

Smith's Principle

Time-delay systems are important in many applications: chemical processes, thermal systems, communication systems and so on. The main difficulty introduced by time-delay systems is that the feedback is delayed, so that the phase-lag is increased; the consequence is that time-delay systems becomes unstable at relatively low controller gains. Furthermore, the spectrum of the closed-loop transfer function (i.e., the number of poles of the characteristic equation) is infinite. Thus, the control action often becomes ineffective, unless some time-delay compensation technique is introduced.

One of these compensation techniques is the *finite spectrum assignment*. This technique is the equivalent of the spectral assignment of delay-free systems for time-delay systems. The objective is to eliminate the delay from the characteristic equation and to place n poles at arbitrary positions of the complex plane, where n is the dimension of the differential equation describing the system. Differently from the delay-free case, the finite spectrum assignment technique requires an integral feedback action, since a linear one is no more sufficient ([2], [3]). The main drawback of this technique is that the resulting feedback law is very complex even for simple systems and that, to the best of the author's knowledge, no robustness evaluation has been performed. These facts prevented the finite spectrum assignment technique to be applied in practice.

Another technique which leads to the same complexity problem is the *Linear-Quadratic (LQ) optimal approach* ([4]); also in this case, no practical implementation of the resulting optimal controller is available.

Finally, the firstly developed delay compensation technique was the well-known *Smith's principle*, which, for its simplicity and effectiveness, has been successfully implemented in many applications. The following Section provides a brief description of the Smith's principle.

1. Smith's Principle

The Smith's principle has been introduced by O. J. Smith in [5]. The control scheme, known as Smith Predictor (SP), has the capability of improving the control of loops with time-delays, also known as Dead Times (DT). In [5], the SP was developed for SISO systems; however, further studies allowed to apply the Smith's principle also to Multiple-Input Multiple-Output (MIMO) systems with multiple delays ([6], [7]).

Let us consider the LTI SISO system shown in Figure 1-1, where the process $P(s)$ has a delay.

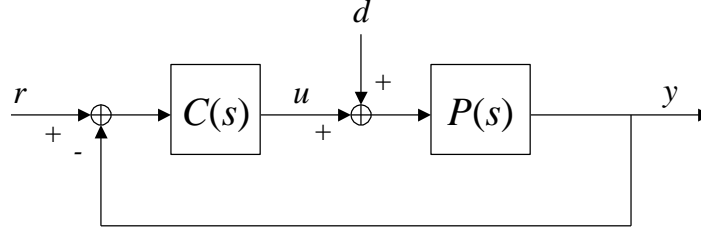


Figure 1-1: SISO control structure

To highlight the presence of the DT, the transfer function $P(s)$ is written as follows:

$$P(s) = P_0(s)e^{-s\theta} \quad (1)$$

where $P_0(s)$ is the delay-free process and θ is the time-delay.

Then, the closed-loop transfer function is the following:

$$W(s) := \left. \frac{y(s)}{r(s)} \right|_{d(s) \equiv 0} = \frac{C(s)P(s)}{1+C(s)P(s)} = \frac{C(s)P_0(s)e^{-\theta s}}{1+C(s)P_0(s)e^{-\theta s}} \quad (2)$$

Equation (2) shows that the time-delay appears in the denominator of the transfer function, which, therefore, becomes a quasi-polynomial. This fact has several consequences: i) the pole number becomes infinite; ii) the delay margin is reduced: a phase lag is introduced and thus, for the sake of stability, controller gains have to be reduced, leading to ineffective performances.

The classical structure of the SP controller, shown in Figure 1-2, consists of a *primary controller*, $C_0(s)$, and a minor feedback loop, in which both the process model, denoted with $\tilde{P}(s)$, and the model of the delay-free process, denoted with $\tilde{P}_0(s)$ appear ([8]). By comparing the schemes of Figure 1-1 and Figure 1-2, it can be inferred that the Smith Predictor controller of the SP scheme plays the role of the controller $C(s)$ of the classical scheme.

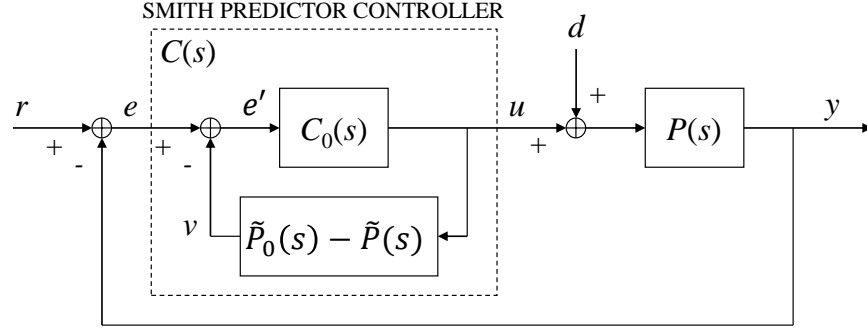


Figure 1-2: SP control structure

As shown by the figure, the SP controller transfer function $C(s)$ contains the process model and the delay-free process model and is given by the following equation:

$$C(s) := \frac{u(s)}{e(s)} = \frac{C_0(s)}{1 + C_0(s)(\tilde{P}_0(s) - \tilde{P}(s))} \quad (3)$$

The signal v in the SP controller, highlighted in Figure 1-2, contains the term $\tilde{P}(s)u(s)$, which is the prediction of the term $P(s)u(s)$ which, in turn, is the part of the output y due to the control action u (this is the reason why the minor feedback loop is called a *predictor*). Consequently, the error e is adjusted by the signal v *before* feeding the primary controller: the adjusted error e' feeding the primary controller $C_0(s)$ does not contain the part of the error which is directly caused by the primary controller itself, thus eliminating the overcorrections which, otherwise, oblige to reduce the controller gain for stability reason ([8]).

Assuming that the plant model is perfect, i.e., that $\tilde{P}(s) = P(s)$, and by substituting the controller $C(s)$ of equation (3) in equation (2), the closed-loop transfer function, denoted with $W(s)$, is the following:

$$W(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} = \frac{\frac{C_0(s)}{1 + C_0(s)(\tilde{P}_0(s) - \tilde{P}(s))}P(s)}{1 + \frac{C_0(s)}{1 + C_0(s)(\tilde{P}_0(s) - \tilde{P}(s))}P(s)} = \frac{C_0(s)P(s)}{1 + C_0(s)P_0(s)} = \frac{C_0(s)P_0(s)}{1 + C_0(s)P_0(s)} e^{-\theta s} \quad (4)$$

Equation (4) shows that the SP controller eliminates the delay in the characteristic equation. Furthermore, equation (4) shows that the input-output transfer function is equal to the one of a delay-free system followed by a pure delay equal to θ . This means that the time-delay is “moved” outside the control loop, and that the output tracks the reference signal with a time delay (equal to the process time-delay).

From the input-output point of view, the resulting equivalent scheme is shown in Figure 1-3: in terms of transfer functions, it is as the output is available to the controller θ seconds before it is actually measured (this is why the scheme is referred to as a prediction scheme).

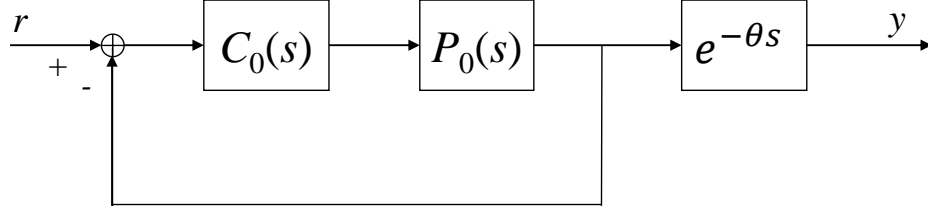


Figure 1-3: Input-output equivalent SP scheme

However, if the process model is not perfect, the delay still appears in the characteristic equation; as a matter of fact, in this case, the closed-loop transfer function is the following:

$$W(s) = \frac{C(s)P(s)}{1+C(s)P(s)} = \frac{\frac{C_0(s)}{1+C_0(s)(\tilde{P}_0(s)-\tilde{P}(s))}P(s)}{1+\frac{C_0(s)}{1+C_0(s)(\tilde{P}_0(s)-\tilde{P}(s))}P(s)} = \frac{C_0(s)P(s)}{1+C_0\tilde{P}_0(s)-C_0\tilde{P}(s)+C_0(s)P(s)} \quad (6)$$

Thus, in this case, the SP cannot completely remove the time-delay from the denominator of the transfer function and can only reduce its effect on the stability, allowing higher gains to be used.

For instance, let us assume that the delay-free process model is perfect but the time-delay of the process model is not, i.e., $\tilde{P}_0(s) = P_0(s)$, $\tilde{P}(s) = P_0(s)e^{-s\tilde{\theta}}$, $\tilde{\theta} \neq \theta$. In this case, the closed-loop transfer function (6) would be the following:

$$W(s) = \frac{C_0(s)P_0(s)e^{-\theta s}}{1+C_0P_0(s)-C_0P_0(s)e^{-s\tilde{\theta}}+C_0(s)P_0(s)e^{-\theta s}} = \frac{C_0(s)P_0(s)e^{-\theta s}}{1+C_0P_0(s)(1-e^{-s\tilde{\theta}}+e^{-\theta s})} \quad (7)$$

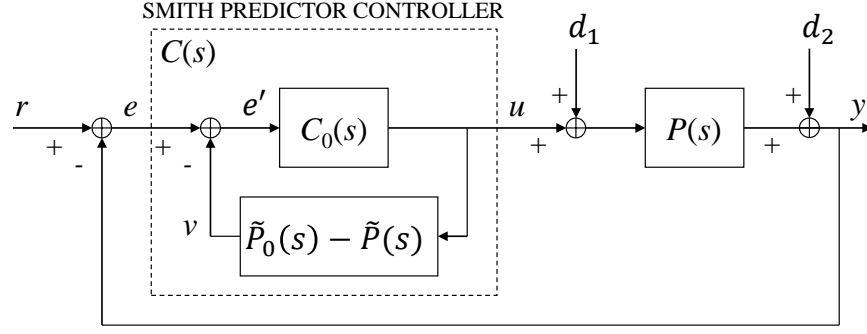


Figure 1-4: SP control structure with disturbances

In the perfect modelling case, i.e., $\tilde{P}(s) = P(s)$, with reference to the system in Figure 1-4, the transfer functions between the disturbances $d_1(s)$, $d_2(s)$ and the output, denoted with $W_{d_1}(s)$ and $W_{d_2}(s)$, respectively, are given by the following equations:

$$W_{d_1}(s) := \left. \frac{y(s)}{d_1(s)} \right|_{r(s) \equiv 0} = \frac{P(s)}{1 + C(s)P(s)} = \frac{1}{1 + \frac{C_0(s)}{1 + C_0(s)(P_0(s) - P(s))}P(s)} P(s) =$$

$$= \frac{1 + C_0(s)P_0(s) - C_0(s)P(s)}{1 + C(s)P_0(s)} P(s) = \left(1 - \frac{C_0(s)P_0(s)}{1 + C(s)P_0(s)} e^{-\theta s} \right) P_0(s) e^{-\theta s} \quad (8)$$

$$W_{d_2}(s) := \left. \frac{y(s)}{d_2(s)} \right|_{r(s) \equiv 0} = \frac{1}{1 + C(s)P(s)} = \frac{1 + C_0(s)(P_0(s) - P(s))}{1 + C(s)P_0(s)} =$$

$$= 1 - \frac{C_0(s)P_0(s)}{1 + C(s)P_0(s)} e^{-\theta s} \quad (9)$$

Again, the time-delay does not appear in the denominator of equations (8) and the output reacts to the disturbance with a time delay. Note that, since the poles of $P(s)$ appear in the poles of equation (8), the SP can be effectively applied to stable plants only.

Finally, it is worth to mention that many different SP-based schemes have been proposed in the literature, which can be applied to unstable plants ([8]), discrete-time systems ([9]). Also the tuning rules for the primary controller have been investigated ([10]), as well as the presence of measurable disturbances – feedforward SP – ([11]). Finally, as described in Section 2, adaptive SP schemes have been considered.

1.1 SP performances

Equation (4) and Figure 1-3 show that the primary controller $C_0(s)$ can be developed by means of standard control theory, considering the delay-free process $P_0(s)$. As a consequence, the gain, phase and delay margins of the process with time-delay $P(s)$

controlled by the SP controller (3) are the same as the margins of the corresponding delay-free process $P_0(s)$ controlled by the primary controller $C_0(s)$: the SP scheme counteracts the phase lag effect of the delay.

Other characteristics of the SP controller can be highlighted. Let us assume that no process model mismatch is present and that:

1. $P_0(s)$ is stable and without zeros in $s = 0$;
2. $C_0(s)$ has a pole in $s = 0$, and thus can be factorized as follows: $C_0(s) = \frac{C'_0(s)}{s}$;
3. $C_0(s)$ stabilizes the delay-free system $W_0 = \frac{C_0(s)P_0(s)}{1+C_0(s)P_0(s)}$.

We can then compute the steady-state system response $y_r(t)$ to a step reference $r(t) = u_{-1}(t)$ and the system responses $y_{d_1}(t)$ and $y_{d_2}(t)$ to a step disturbances $d_1(t) = u_{-1}(t)$ and $d_2(t) = u_{-1}(t)$ by using the final value theorem:

$$\begin{aligned} \lim_{t \rightarrow \infty} y_r(t) &= \lim_{s \rightarrow 0} s y_r(s) = \lim_{s \rightarrow 0} s W(s) r(s) = \lim_{s \rightarrow 0} s \frac{C_0(s)P_0(s)}{1+C_0(s)P_0(s)} e^{-\theta s} \frac{1}{s} = \\ &= \lim_{s \rightarrow 0} \frac{\frac{C'_0(s)}{s} P_0(s)}{1 + \frac{C'_0(s)}{s} P_0(s)} e^{-\theta s} = 1; \end{aligned} \quad (10)$$

$$\begin{aligned} \lim_{t \rightarrow \infty} y_{d_1}(t) &= \lim_{s \rightarrow 0} s y_{d_1}(s) = \lim_{s \rightarrow 0} s W_{d_1}(s) d_1(s) = \lim_{s \rightarrow 0} s \left(1 - \frac{C_0(s)P_0(s)}{1+C_0(s)P_0(s)} e^{-\theta s} \right) P(s) \frac{1}{s} = \\ &= \lim_{s \rightarrow 0} \left(1 - \frac{\frac{C'_0(s)}{s} P_0(s)}{1 + \frac{C'_0(s)}{s} P_0(s)} e^{-\theta s} \right) P_0(s) e^{-\theta s} = 0; \end{aligned} \quad (11)$$

$$\begin{aligned} \lim_{t \rightarrow \infty} y_{d_2}(t) &= \lim_{s \rightarrow 0} s y_{d_2}(s) = \lim_{s \rightarrow 0} s W_{d_2}(s) d_2(s) = \lim_{s \rightarrow 0} s \left(1 - \frac{C_0(s)P_0(s)}{1+C_0(s)P_0(s)} e^{-\theta s} \right) \frac{1}{s} = \\ &= \lim_{s \rightarrow 0} \left(1 - \frac{\frac{C'_0(s)}{s} P_0(s)}{1 + \frac{C'_0(s)}{s} P_0(s)} e^{-\theta s} \right) = 0; \end{aligned} \quad (12)$$

The equations above show that zero steady-state error is achieved in case of step reference and disturbances.

1.2 Internal Model Control (IMC) Interpretation of the SP

By rearranging Figure 1-2, the SP scheme can be designed as shown in Figure 1-5; the represented controller structure is known as the ‘IMC structure of the SP’.


$$Q(s) := \frac{u(s)}{e'(s)} = \frac{c_0(s)}{1+c_0(s)\tilde{p}_0(s)} \quad (13)$$

Since the delay appears in the sensitivity function $S(s) = \frac{1}{1+C(s)P(s)}$, this design procedure is not always the best one. For some processes, an appropriate design method is to design the optimal IMC controller $Q(s)$ considering the plant model with delay, and then to tune it on the basis of the sensitivity and complementary sensitivity functions ([1]). The primary controller of the Smith's predictor is then retrieved from equation (13):

$$C_o(s) = \frac{Q(s)}{1 - \rho(s)\tilde{P}_o(s)} \quad (14)$$

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2.1 Delay mismatch

For the sake of simplicity, we assume that delay-free process model is perfect but the time-delay of the process model is not, i.e., $\tilde{P}_0(s) = P_0(s)$, $\tilde{P}(s) = P_0(s)e^{-s\tilde{\theta}}$, $\tilde{\theta} = \theta + \delta$, with $\delta \in [0, \delta_{max}]$. From equation (16), the multiplicative uncertainty is computed as follows:

$$\Delta_m(s) = \frac{P_0(s)e^{-s\theta} - P_0(s)e^{-s\tilde{\theta}}}{P_0(s)e^{-s\tilde{\theta}}} = \frac{P_0(s)e^{-s\theta} - P_0(s)e^{-s(\theta+\delta)}}{P_0(s)e^{-s(\theta+\delta)}} = e^{-s\delta} - 1 \quad (19)$$

Figure 2-1 shows the module of $\Delta_m(j\omega)$ for different values of δ . Note that $|\Delta_m(j\omega)|$ is periodic with period $\frac{2\pi}{\delta}$ and it is always less than 2.

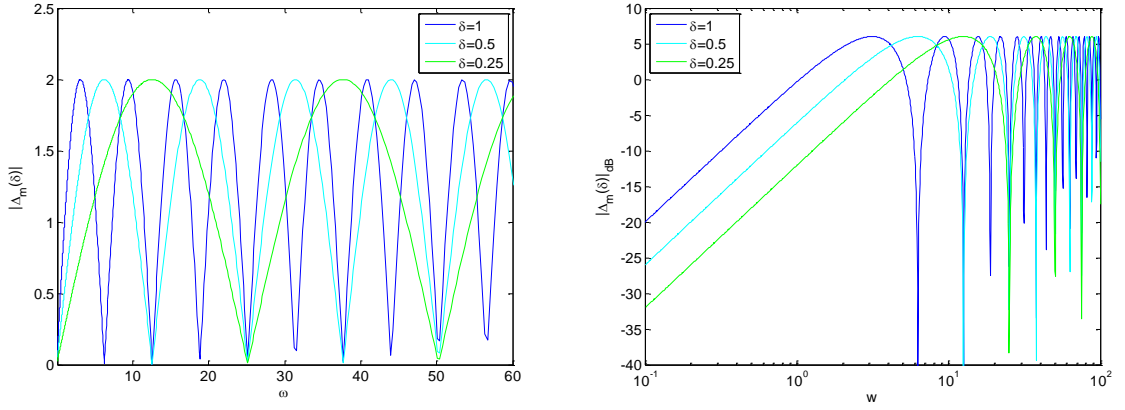


Figure 2-1: Left figure: module of $\Delta_m(j\omega)$ for different values of δ ; right figure: module in dB of $\Delta_m(j\omega)$ for different values of δ .

Based on

Figure 2-1, it can be inferred that an upper-bound for $\Delta_m(j\omega)$ is given by the following function:

$$l_m(j\omega) = \begin{cases} e^{-j\omega\delta_{max}} - 1, & \text{if } \omega \leq \frac{\pi}{\delta_{max}} \\ 2, & \text{if } \omega > \frac{\pi}{\delta_{max}} \end{cases} \quad (20)$$

The upper-bound is shown in Figure 2-2, which highlights that $l_m(j\omega)$ is a high-pass filter.

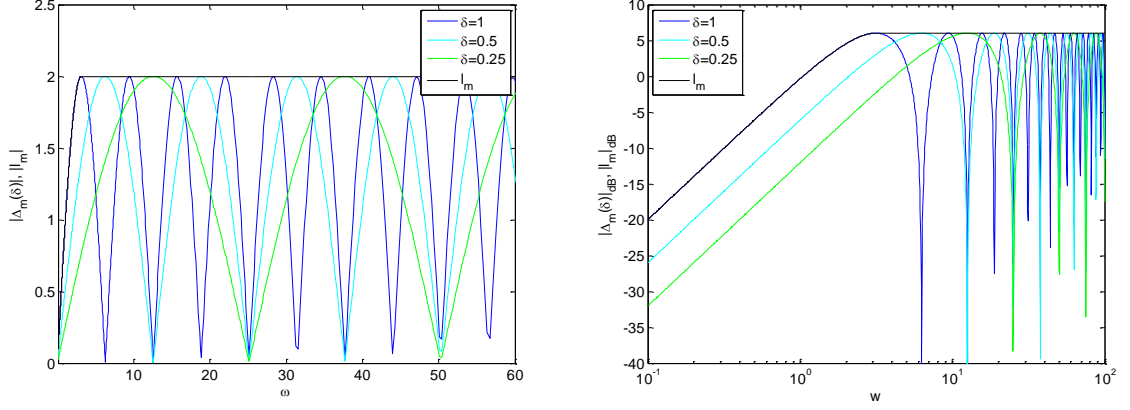


Figure 2-2: Left figure: module of $\Delta_m(j\omega)$ for different δ and of $l_m(j\omega)$, with $\delta_{max} = 1$; right figure: module in dB of $\Delta_m(j\omega)$ for different δ and of $l_m(j\omega)$, with $\delta_{max} = 1$.

In a SP fashion, let us follow the IMC design procedure to control the delay-free process.

Step 1. For the perfect control objective, the IMC controller is defined as $\tilde{Q}(s) = P_0^{-1}(s)$.

Step 2. The IMC controller is rendered proper by means of the IMC filter $f(s)$:

$$Q(s) = \tilde{Q}(s)f(s) = P_0^{-1}(s)f(s) \quad (21)$$

Note that the IMC controller (21) can be put in the form of a SP controller. In fact, the equivalent controller $C_0(s)$ of the SP scheme of Figure 1-5 is obtained by equation (13):

$$C_0(s) = \frac{Q(s)}{1-Q(s)P_0(s)} \quad (22)$$

Considering the primary controller (21) and that the complementary sensitivity function of the system is defined as $\tilde{T}(j\omega) = Q(j\omega)\tilde{P}(j\omega)$, the sufficient condition (18) for robust stability is the following:

$$\begin{aligned} |l_m(j\omega)\tilde{T}(j\omega)| &= |l_m(j\omega)Q(j\omega)\tilde{P}(j\omega)| = |l_m(j\omega)P_0^{-1}(j\omega)f(s)P_0(j\omega)e^{-j\omega\tilde{\theta}}| \\ &= |l_m(j\omega)f(j\omega)| < 1, \forall \omega \end{aligned} \quad (23)$$

Since $l_m(j\omega)$ is a high-pass filter and $f(s)$ is a low-pass filter (we recall that it is used to render the controller proper; a typical filter is $f(s) = \frac{1}{(1+s\lambda)^n}$), it is always possible to find a filter $f(s)$ which satisfies the condition (18). However, a too strong filter would weaken the control effort: then, a trade-off exists between robustness and performances.

2.2 Example

Let us assume that the delay-free process is a first-order process: $P_0(s) = k \frac{1}{1+s\tau}$, $\tau > 0$, and let us assume that $\tilde{\theta} = \theta + \delta$, with $\delta \in [0, \delta_{max}]$ and $\delta_{max}=1s$. The IMC design procedure to control the delay-free process is the following.

Step 1. For the perfect control objective, the IMC controller is $\tilde{Q}(s) = P_0^{-1}(s) = \frac{1}{k}(1+s\tau)$.

Step 2. The IMC controller is rendered proper by means of a first-order filter $f(s) = \frac{1}{1+s\lambda}$:

$$Q(s) = \tilde{Q}(s)f(s) = P_0^{-1}(s)f(s) = \frac{1}{k} \frac{1+s\tau}{1+s\lambda} \quad (24)$$

The IMC controller (24) can be put in the form of a SP controller. By equation (22), the equivalent controller $C_0(s)$ is the following:

$$C_0(s) = \frac{1}{k} \frac{1+s\tau}{s\lambda} \quad (25)$$

By rearranging equation (25), a standard PI controller $C_0(s)$ is obtained:

$$C_0(s) = \frac{1}{k} \frac{1+s\tau}{s\lambda} = \frac{\tau}{k\lambda} \frac{\frac{\lambda}{\tau} + s\lambda}{s\lambda} = \frac{\tau}{k\lambda} \left(1 + \frac{1}{s\lambda\tau}\right) = K_p \left(1 + \frac{1}{sT_i}\right) \quad (26)$$

with proportional constant $K_p = \frac{\tau}{k\lambda}$ and integral constant $T_i = \lambda\tau$.

Considering the IMC controller (24), the sufficient condition (18) for robust stability is the following:

$$|l_m(j\omega)f(s)| = \left| l_m(j\omega) \frac{1}{1+s\lambda} \right| = \begin{cases} \frac{e^{-s\delta_{max}} - 1}{1+s\lambda}, & \text{if } \omega \leq \frac{\pi}{\delta_{max}} \\ \frac{2}{1+s\lambda}, & \text{if } \omega > \frac{\pi}{\delta_{max}} \end{cases} < 1, \forall \omega \quad (27)$$

The function $|l_m(j\omega)f(s)|$ is plot in the following figure. From the figure it can be inferred that the condition (27) is satisfied for $\lambda > \delta_{max} = 1$ (more precise computations lead to the condition $\lambda > 0.67\delta_{max} = 0.67$).

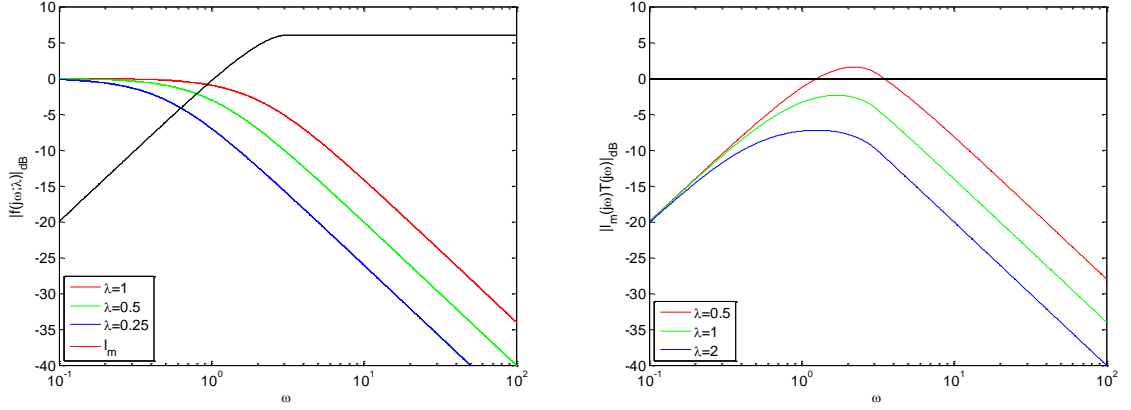


Figure 2-3: Left figure: module of $f(j\omega)$ for different values of λ and of $l_m(j\omega)$; right figure: module in dB of $l_m(j\omega)T(j\omega)$ for different values of λ

3. Adaptive SP Schemes

As already mentioned, the SP is capable of improving the control of time-delay systems, provided that an accurate process model is available. If the parameters of the plant model change in time, an adaptive scheme should be incorporated within the SP scheme, so that the prediction performed by the SP controller loop is still acceptable ([12]).

In [12], an adaptive control scheme for time-delay systems is introduced. The proposed adaptive scheme, which is based on a reference model, i.e., it is essentially a model reference adaptive control (MRAC) scheme, is limited to the special case of no disturbance. The first example given in [12] considers the case of temporal adaptive control, in which the only varying parameter is the time-delay $\theta(t)$. The nominal value of the time-delay, $\theta(0)$, is assumed as known. Thus, the output y also depends on the variation of the time-delay parameter, referred to as $\Delta = \theta(t) - \theta(0)$: $y(t) = y(t, \Delta)$. For small Δ , the process output can be written as follows: $y(t, \Delta) \approx y(t, 0) + \Delta(\partial y / \partial \Delta)$, whereas the reference model returns the output $y(t, 0)$. Thus, the error is defined as follows:

$$e(t, \Delta) = y(t, \Delta) - y(t, 0) = \Delta \frac{\partial y(t, \Delta)}{\partial \Delta} \quad (11)$$

The selected performance criterion is the Integral Square Error (ISE); then, the objective of the adaptive control is to minimize the following cost function:

$$J(t) \equiv \frac{1}{2} \int_0^t e^2(\tau) d\tau \quad (12)$$

The obtained delay variation value which minimizes equation (12), denoted with Δ_{OPT} , is used to update the time-delay estimate $\tilde{\theta}(t)$ of the SP controller. The block diagram of the temporal adaptation scheme is shown in Figure 3-1, where the SP controller is drawn in an MRAC-like fashion.

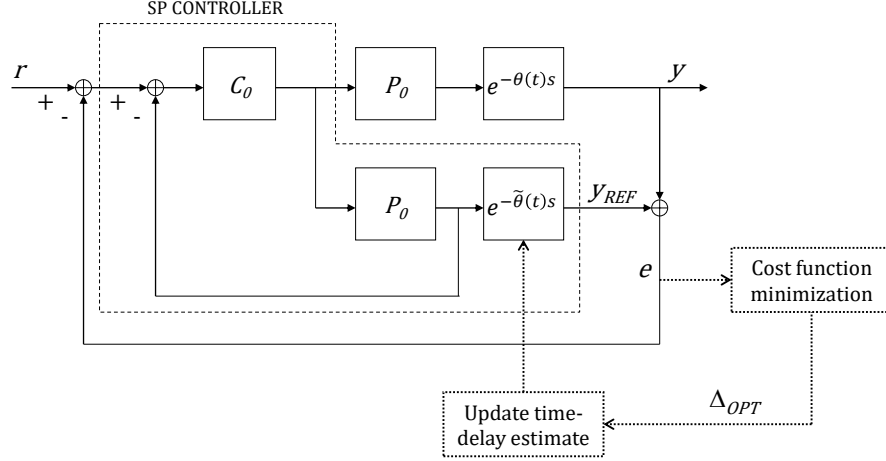


Figure 3-1: Temporal adaptive SP control structure

In [12], the procedure is developed also for plant dynamic errors and for mixed temporal-plant mismatches, but always in case of no disturbances. A similar adaptive SP scheme is described in [13]; also in this case, the adaptation is applicable in case of no disturbance only.

Other adaptive schemes have been proposed ([14], [15]), but, generally, they are still affected by unmeasured load disturbances and/or require complex procedures, not suitable for communication network scenarios.

4. Bibliography

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