ROBUST SP WITH DELAY MISMATCHES

- IMC DESIGN FOR TIME-DELAY SYSTEMS WITH DELAY UNCERTAINTIES ONLY

BOUNDED DELAY UNCERTAINTY

- Process model $\tilde{P}(s) = \dfrac{k}{1+s\tau} e^{-\tilde{\theta} s}$, $\theta = \tilde{\theta} + \delta$, $\tau > 0$

with $|\delta| \leq \bar{\delta}$

Actual process $P(s) = \dfrac{k}{1+s\tau} e^{-\theta s}$

- IMC controller for type 1 system

- Check robustness against the delay uncertainty

$m_F^+ = m_{\tilde{F}}^+$, $F(s) = G(s)P(s)$
$\tilde{F}(s) = G(s)\tilde{P}(s)$
$\Rightarrow$ ROBUST STABILITY DESIGN

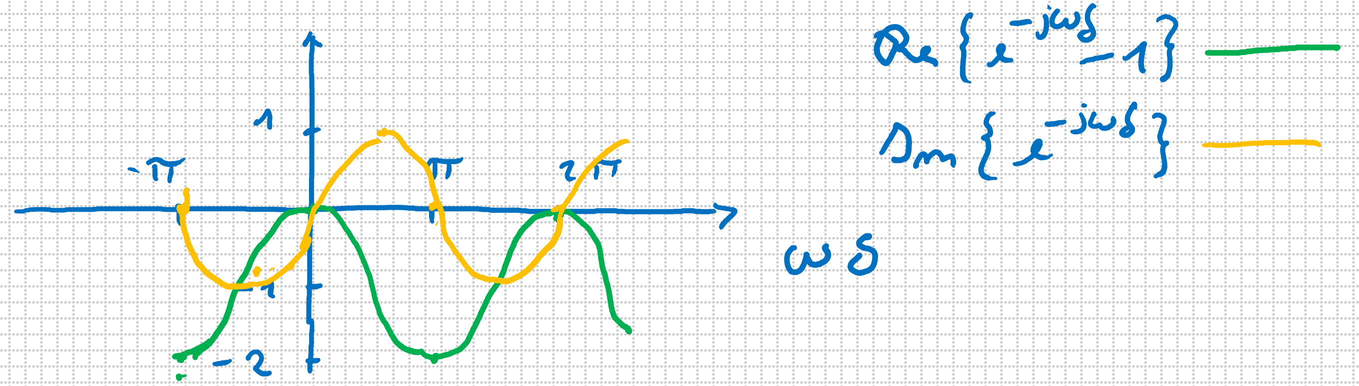
- PARAMETRIC UNCERTAINTY $\longrightarrow$ MULTIPLICATIVE UNCERTAINTY

$$P(s) = \tilde{P}(s)\left(1 + \Delta_m(s)\right) \Rightarrow \Delta_m(s) = \frac{P(s) - \tilde{P}(s)}{\tilde{P}(s)}$$

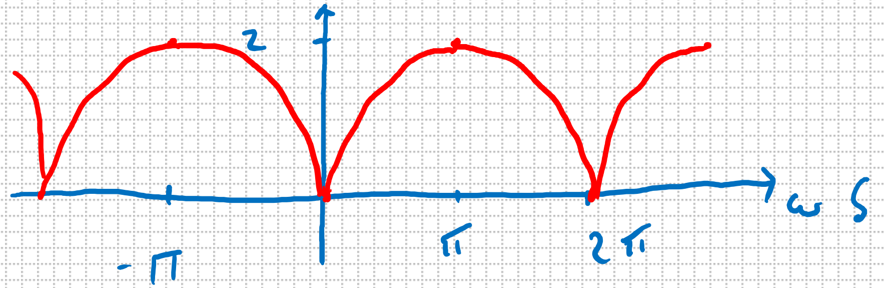
$$\boxed{\Delta_m(s) = \frac{\frac{K}{1+s\tau} e^{-(\tilde{\theta}+\delta)s} - \frac{K}{1+s\tau} e^{-\tilde{\theta}s}}{\frac{K}{1+s\tau} e^{-\tilde{\theta}s}} = e^{-\delta s} - 1}$$

UPPER BOUND $\quad l_m(s) : \quad |l_m(j\omega)| \geq |\Delta_m(j\omega)|, \quad \forall \omega$

$$|\Delta_m(j\omega)| = |e^{-j\omega\delta} - 1|$$

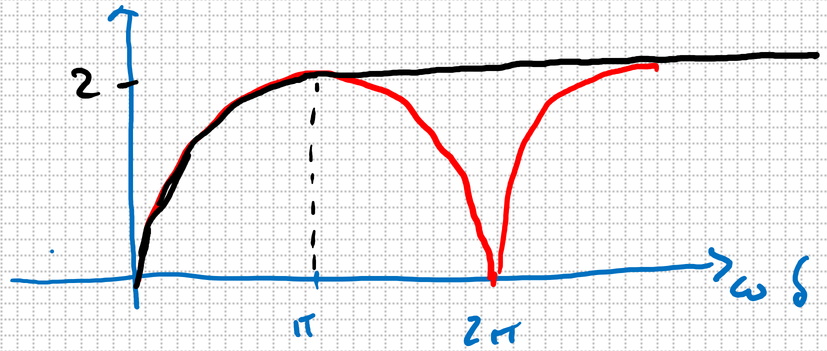


$$|\Delta_m(j\omega)| = \sqrt{Re^2 + Im^2}$$



PERIODIC $\frac{2\pi}{\delta}$

$$|l_m(j\omega)| = \begin{cases} |e^{-j\omega\bar{\delta}} - 1| & \omega \le \frac{\pi}{\bar{\delta}} \\ 2 & \omega > \frac{\pi}{\bar{\delta}} \end{cases}$$



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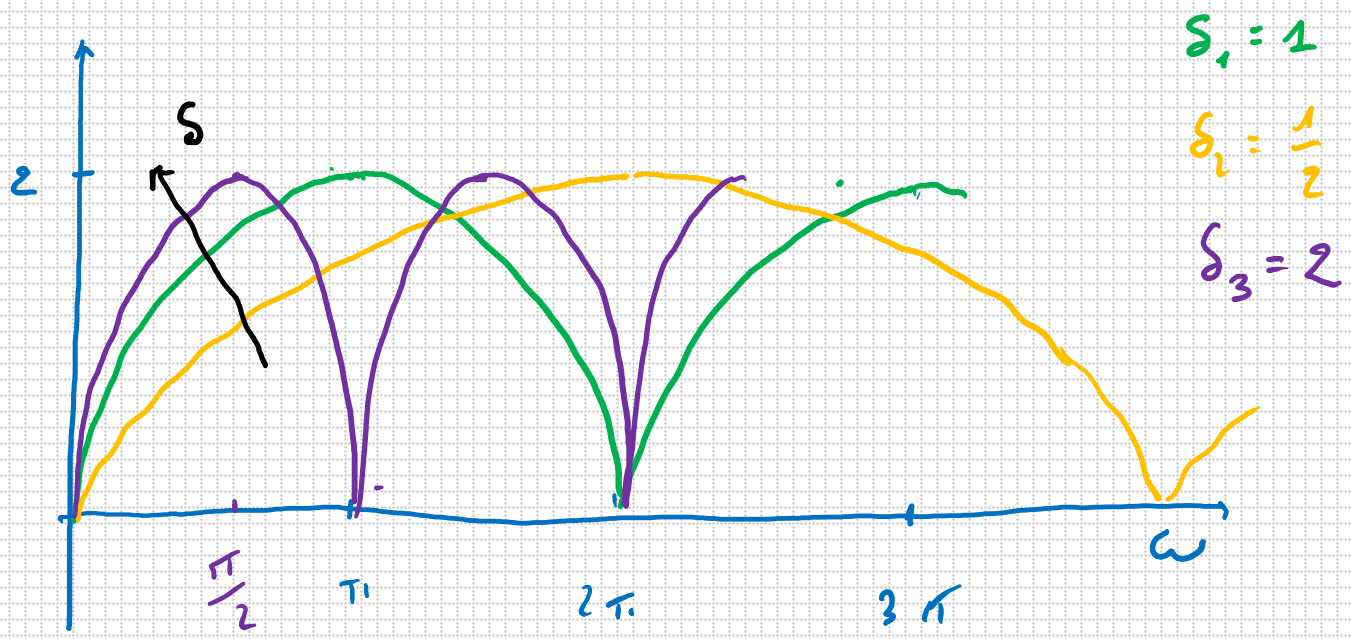
worst-case : $\bar{\delta}$



IMC controller design

STEP 1. $$\tilde{P}(s) = \frac{k}{1+s\tau} e^{-\tilde{\theta} s}$$

(IAE, ISE) $$\tilde{P}_+(s) = e^{-\hat{\theta} s} \quad , \quad \tilde{P}_-(s) = \frac{k}{1+s\tau}$$

$$\tilde{Q}(s) = \tilde{P}_-(s)^{-1} = \frac{1}{k}(1+s\tau)$$

STEP 2. (Type 1) $$f(s) = \frac{1}{(1+\lambda s)^n} \quad , \quad n = 1 \quad \text{PROPERNESS)}$$

$$Q(s) = \tilde{Q}(s) f(s) = \frac{1}{k} \frac{1+\tau s}{1+\lambda s}$$

$$|\tilde{T}(s)|_{s=0} = |Q(s)\tilde{P}(s)|_{s=0} = |\tilde{P}_+(s) f(s)|_{s_0} = \left| e^{-s\theta} \frac{1}{1+\lambda\tau} \right|_{s=0} = 1 \quad \checkmark$$

STEP 3. ROBUSTNESS

Suff. cond. $\quad \left| \ell_m(j\omega)\, \tilde{T}(j\omega) \right| < 1, \quad \forall \omega$

$$\left|\tilde{T}(j\omega)\right| = \left|\tilde{P}_+(j\omega)\, f(j\omega)\right| = \left|f(j\omega)\right| = \left|\frac{1}{1+j\omega\lambda}\right|$$

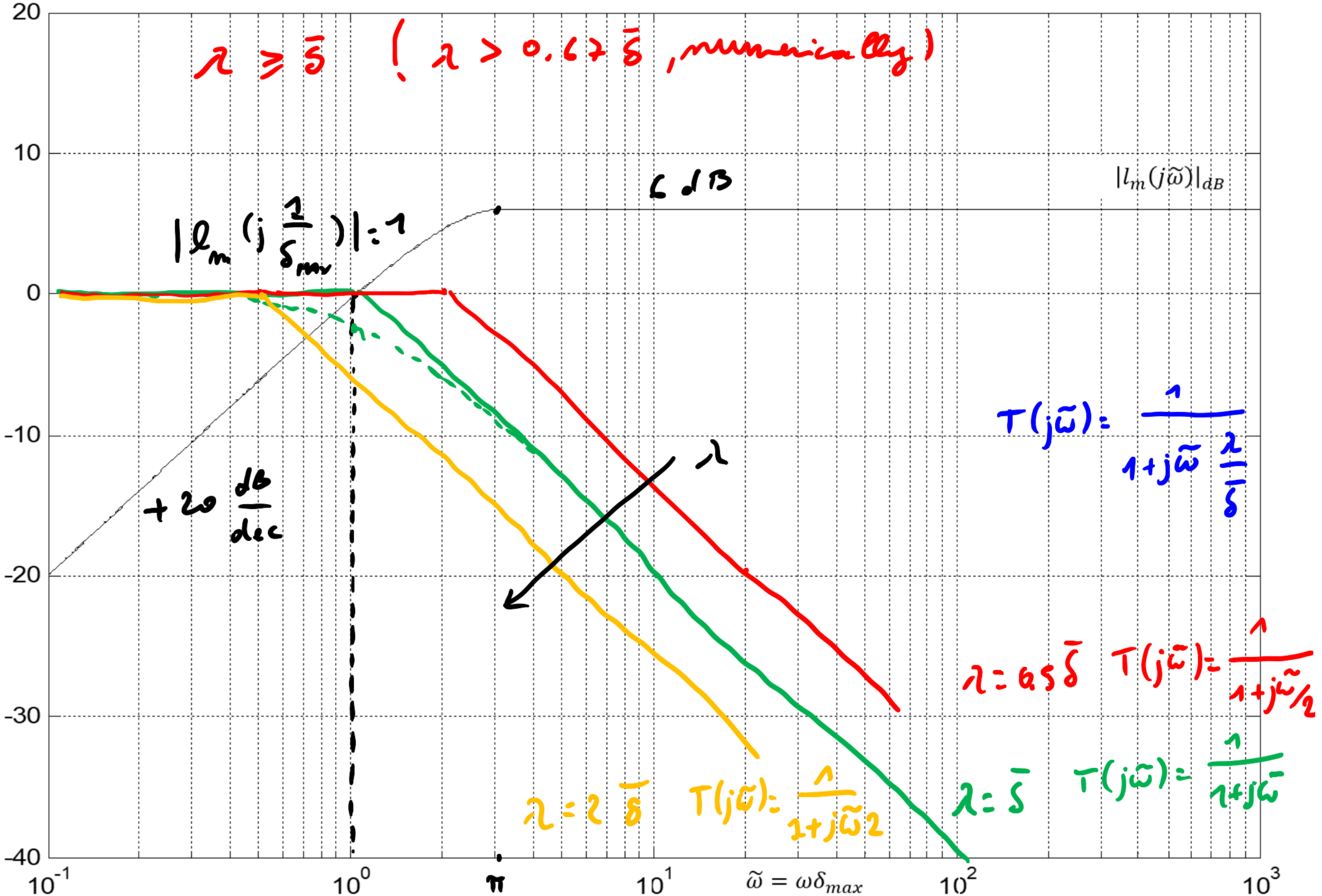
$$\left|\tilde{P}_+(s)\right| = \left|e^{-\theta s}\right| = 1$$

$$\left|\ell_m(j\omega)\right| = \begin{cases} \left|e^{-j\omega\bar{\delta}} - 1\right| & \text{if } \omega \leq \frac{\pi}{\bar{\delta}} \\ 2 & \text{if } \omega > \frac{\pi}{\bar{\delta}} \end{cases} \quad \longleftarrow \; -\bar{\delta} \leq \delta \leq \bar{\delta}$$

$\sim \cdot \sim$

$\boxed{!}$

$$\tilde{T}(j\tilde{\omega}) = \frac{1}{1+j\omega\lambda} = \frac{1}{1+j\underbrace{\omega\bar{\delta}}_{\tilde{\omega}}\frac{\lambda}{\bar{\delta}}} = \frac{1}{1+j\tilde{\omega}\frac{\lambda}{\bar{\delta}}} \qquad \tilde{\omega} = \omega\bar{\delta}$$



$\lambda \geq \bar{\delta}$ ($\lambda > 0.67\,\bar{\delta}$, numerically)
20
10
0
-10
-20
-30
-40
10^{-1}
10^{0}
π
10^{1}
10^{2}
10^{3}
$\tilde{\omega} = \omega\delta_{max}$
6 dB
$|l_m(j\tilde{\omega})|_{dB}$
$|l_m(j\frac{1}{\delta_{max}})| = 1$
$+20\,\frac{dB}{dec}$
λ
$T(j\tilde{\omega}) = \frac{1}{1 + j\tilde{\omega}\frac{\lambda}{\bar{\delta}}}$
$\lambda = 0.5\,\bar{\delta}$ $T(j\tilde{\omega}) = \frac{1}{1 + j\tilde{\omega}/2}$
$\lambda = \bar{\delta}$ $T(j\tilde{\omega}) = \frac{1}{1 + j\tilde{\omega}}$
$\lambda = 2\,\bar{\delta}$ $T(j\tilde{\omega}) = \frac{1}{1 + j\tilde{\omega}2}$

EXERCISE 6 (SP)

$$P(s) = 10\,\frac{1-s}{(1+s)^2}\,e^{-\theta s}, \qquad \theta = 2+\delta, \quad \delta \in [0,2) \quad \leftarrow \bar{\delta} = 2$$

$$\tilde{\theta} = 2$$

- SP robust controller : { robust stability, type 1, IAE cost

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• $\tilde{P}(s)$ stable $\Rightarrow$ IMC design

the controller design is done for the DELAY-FREE CASE

SMITH PRINCIPLE
1. Delay-free case $\quad Q_0(s)$ IMC controller
2. " " " $\quad G_0(s) = \dfrac{Q_0(s)}{1 - Q_0(s)\,\tilde{P}_0(s)}$ classic controller
3. Process with delay $\quad G(s) = \dfrac{G_0(s)}{1 + G_0(s)\left(\tilde{P}_0(s) - \tilde{P}(s)\right)}$, SP controller

IMC

STEP 1. $\tilde{P}_0(s) = 10 \dfrac{1-s}{(1+s)^2}$, $\tilde{P}_+(s) = 1-s$, $\tilde{P}_-(s) = 10 \dfrac{1}{(1+s)^2}$

$$\tilde{Q}_0(s) = \tilde{P}_-(s)^{-1} = \frac{1}{10}(1+s)^2$$

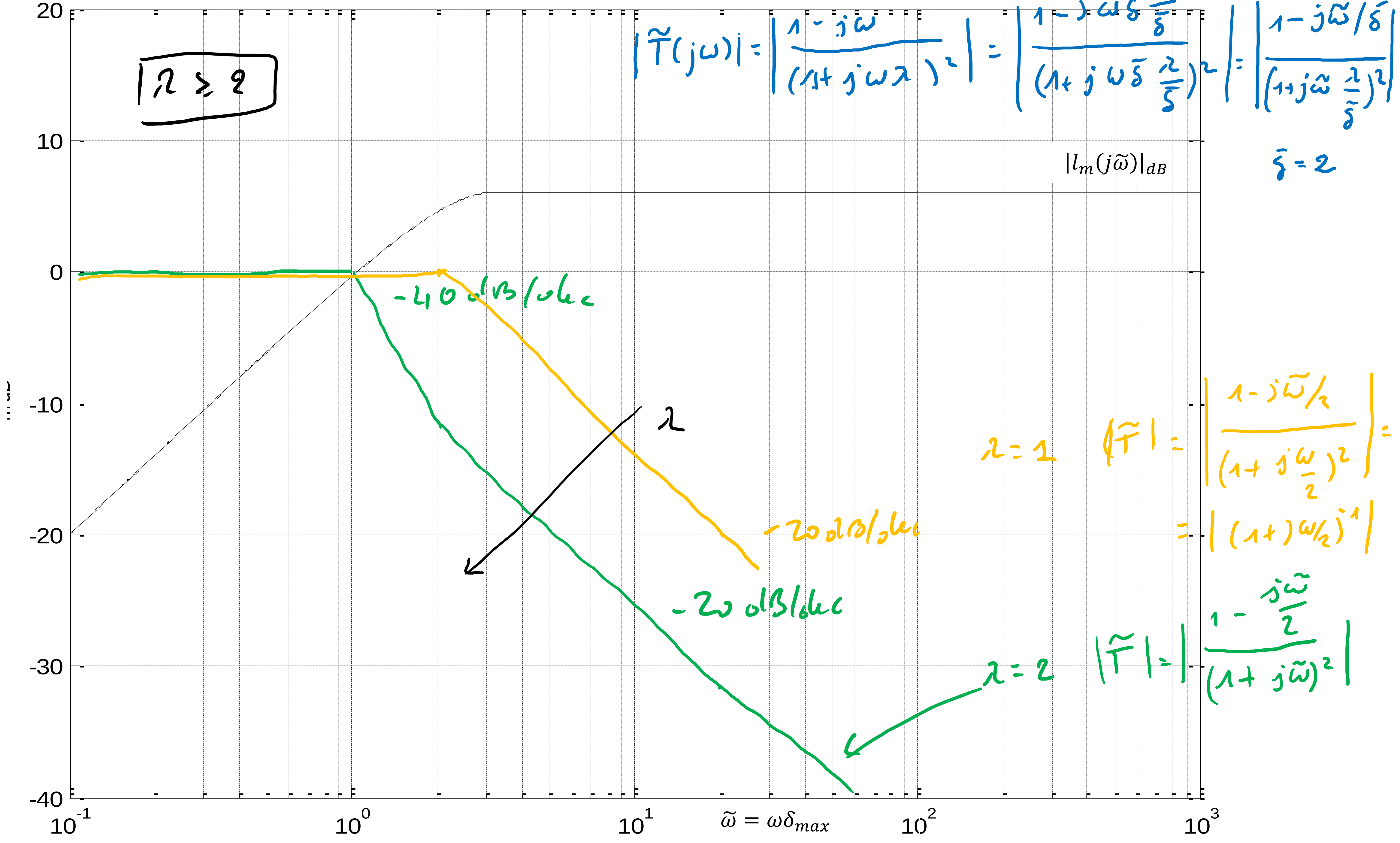
STEP 2. Type 1 $f(s) = \dfrac{1}{(1+\lambda s)^n}$, $n = 2$

$$Q_0(s) = \frac{1}{10} \frac{(1+s)^2}{(1+\lambda s)^2}$$ ← IMC controller for the delay-free process

STEP 3. ROBUSTNESS ! UNCERTAINTY ON THE DELAY ONLY !

$$|l_m(j\omega)\tilde{T}(j\omega)| < 1, \forall \omega$$

$$l_m(j\omega) = \begin{cases} e^{-j\omega\bar{\delta}} - 1 & \text{if } \omega < \frac{\pi}{\bar{\delta}} \\ 2 & \text{otherwise} \end{cases} \quad ; \quad |\tilde{T}(j\omega)| = |Q_0 \tilde{P}_0| = \left| \frac{1-j\omega}{(1+j\omega\lambda)^2} \right|$$



$$|\tilde T(j\omega)| = \left|\frac{1-j\omega}{(1+j\omega\lambda)^2}\right| = \left|\frac{1-j\omega\bar\delta\frac{1}{\bar\delta}}{(1+j\omega\bar\delta\frac{\lambda}{\bar\delta})^2}\right| = \left|\frac{1-j\tilde\omega/\bar\delta}{(1+j\tilde\omega\frac{\lambda}{\bar\delta})^2}\right|$$

$\bar\delta = 2$

$\lambda = 1$ $$|\tilde T| = \left|\frac{1-j\tilde\omega/2}{(1+j\frac{\omega}{2})^2}\right| = \left|(1+j\omega/2)^{-1}\right|$$

$\lambda = 2$ $$|\tilde T| = \left|\frac{1-\frac{j\tilde\omega}{2}}{(1+j\tilde\omega)^2}\right|$$

ROBUST IMC CONTROLLER

$$Q_0(s) = \frac{1}{10}\frac{(1+s)^2}{(1+2s)^2}$$

$$G_0(s) = \frac{Q_0(s)}{1 - Q_0(s)\tilde{P}_0(s)} = \frac{\frac{1}{10}\frac{(1+s)^2}{(1+2s)^2}}{1 - \underbrace{(1-s)}_{\tilde{P}_+}\underbrace{\frac{1}{(1+2s)^2}}_{f}} = \frac{1}{10}\frac{(1+s)^2}{(1+2s)^2 - 1 + s} =$$

PID + filter

$$= \frac{1}{10}\frac{s^2+2s+1}{s(4s+5)} = \frac{2}{10\ 5}\frac{s + \frac{1}{2}s^2 + \frac{1}{2}}{s\left(1+\frac{4}{5}s\right)} = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{1+\beta_f s}$$

$$\begin{cases} K_c = 1/25 \\ T_i = 2 \\ T_d = 0.5 \\ \beta_f = 4/5 \end{cases}$$

ROBUST SP CONTROLLER

$$\boxed{G(s) = \frac{G_0(s)}{1 + G_0(s)\left(\tilde{P}_0(s) - \tilde{P}(s)\right)}}$$

$\leftarrow \hat{\theta} = 2$