Process Automation (MCER), 2017-2018

Exam - February 9, 2018 (3h00)

Exercise 1 (11 pt.)

Let the process be described by the transfer function: $P(s) = 10\frac{s-0.1}{(s+1)^2}e^{-\theta s}$, with $\theta \in [10,20]$s, and let the nominal delay be $\tilde{\theta} = 20$s.

A) Under the ISE cost function, design an IMC controller $Q(s)$ such that:
   i) the overall system is robustly asymptotically stable;
   ii) the overall system has 0 steady-state error for step inputs.

B) Compute the equivalent classic controller in the Smith Predictor form ($G_0(s)$).

Exercise 2 (11 pt.)

Consider a process whose input response model is given by the following samples:

$h_1 = 0, h_2 = 0, h_3 = 0.3, h_4 = 0.5, h_5 = 0.1, h_6 = 0, h_7 = 0, h_8 = 0, \ldots$.

Compute the control action $u(5)$ of a MPC controller with the following specifications:

i) Control horizon $m = 2$;
ii) Reference signal $r(t) = 0.2t, t \geq 0$;
iii) Reference trajectory computed as $w(t+k|t) = \begin{cases} y_m(t), k = 0 \\ 0.5w(t+k-1|t) + 0.5r(t+k), k > 0 \end{cases}$;
iv) Cost function $J = e^T e + 0.1u^T u$.
v) $y_m(0) = 0,\ y_m(1) = 0,\ y_m(2) = 0,\ y_m(3) = 0.2,\ y_m(4) = 0.4,\ y_m(5) = 0.8$;
vi) $u(t) = 0$ for $t \leq 0,\ u(1) = 0.5,\ u(2) = 0.8,\ u(3) = 1.2,\ u(4) = 1.4$.

Questions (8 pt.)

i) Discuss why PFC may improve the scalability of the control algorithm with respect to DMC and MAC. (1/2 pg. max, 3pt).

ii) Considering a time-delay system controlled with a Smith Predictor controller, discuss whether the stability margins of the system are more affected by the absolute value of the delay or by the mismatch between the nominal delay and the actual delay. (1/2 pg. max, 5pt).

Solution of exercise 1.

A) Since the time-delay $\tilde{\theta} = 20s$ of the process is much larger than the time constant $\tau = 1s$ of the process, we cannot use a Padé approximation to write the delay term as a transfer function, and we have to rely on a Smith Predictor controller, depicted in the figure below:

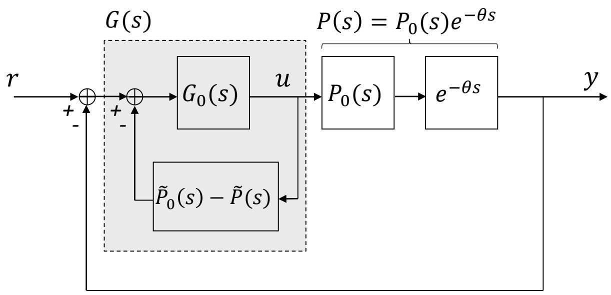


Figure 1)

The nominal process $\tilde{P}(s) = -\frac{1-\frac{s}{0.1}}{(1+s)^2} e^{-20s}$ is stable, therefore it is possible to design a stable controller $Q(s)$ to stabilize the closed-loop nominal system. The IMC form of the SP controller of Figure 1) is shown in the figure below:

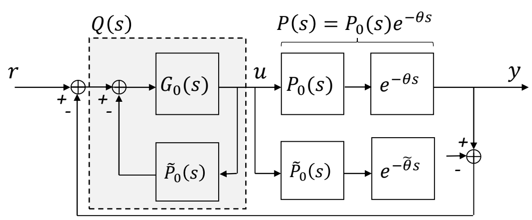


Figure 2)

The IMC design procedure to robustly stabilize the process $P(s)$ consists in the following 3 steps:

Step 1)

a) Factorize the nominal process in a minimum-phase and a non-minimum-phase terms under the ISE cost:

$$\tilde{P}(s) = \tilde{P}_+(s)\tilde{P}_-(s)$$

with $\tilde{P}_+(s) = \frac{1-\frac{s}{0.1}}{1+\frac{s}{0.1}} e^{-20s}$ and $\tilde{P}_-(s) = -\frac{1+\frac{s}{0.1}}{(1+s)^2}$.

b) Define the controller as follows: $\tilde{Q}(s) = \left(\tilde{P}_-(s)\right)^{-1} = -\frac{(1+s)^2}{1+\frac{s}{0.1}}$

Step 2)

Design the controller $Q(s) = \tilde{Q}(s)f(s)$, with the IMC filter $f(s)$ such that a) the controller $Q(s)$ is proper and b) the overall system is of type 1 (i.e., $\tilde{T}(0) = \tilde{P}(0)Q(0) = 1$). We use the filter $f(s) = \frac{1}{(1+\lambda s)^n}$ with $n = 1$. In fact:

a) $Q(s) = \tilde{Q}(s)f(s) = -\frac{(1+s)^2}{\left(1+\frac{s}{0.1}\right)(1+\lambda s)}$ is proper;

b) $\tilde{T}(0) = \tilde{P}(0)Q(0) = \left[\tilde{P}_+(s)\tilde{P}_-(s)\left(\tilde{P}_-(s)\right)^{-1} f(s)\right]_{s=0} = \left[\frac{1-\frac{s}{0.1}}{1+\frac{s}{0.1}}\frac{1}{1+\lambda s}e^{-20s}\right]_{s=0} = 1.$

Step 3)

Determine the value of λ such that the sufficient condition for robust stability holds:

$$\left|l_m(j\omega)\tilde{T}(j\omega)\right| < 1, \forall\omega$$

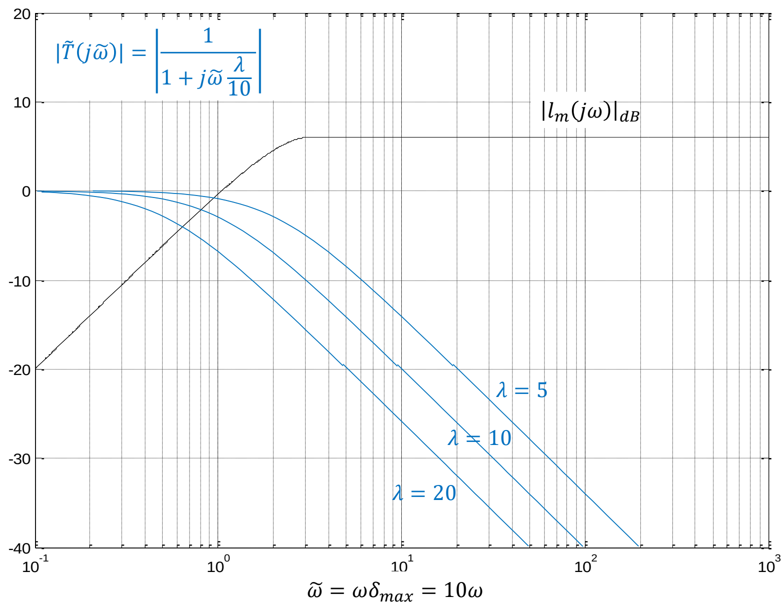
where $|\tilde{T}(j\omega)| = \left|\frac{1-\frac{j\omega}{0.1}}{1+\frac{j\omega}{0.1}}\frac{1}{1+j\omega\lambda}\right| = \left|\frac{1}{1+j\omega\lambda}\right|$ and $l_m(j\omega)$ is an upper-bound of the multiplicative uncertainty $\Delta_m(j\omega)$, i.e., a function such that $|l_m(j\omega)| > |\Delta_m(j\omega)|, \forall\omega$.

By definition, the multiplicative uncertainty is defined as

$$\Delta_m(j\omega) := \frac{P(j\omega)-\tilde{P}(j\omega)}{\tilde{P}(j\omega)} = \frac{-\frac{1-\frac{s}{0.1}}{(1+s)^2}e^{-\theta j\omega}+\frac{1-\frac{s}{0.1}}{(1+s)^2}e^{-20j\omega}}{-\frac{1-\frac{s}{0.1}}{(1+s)^2}e^{-20j\omega}} = e^{-j\omega\delta}-1.$$

From theory, we know that an upper-bound is defined as $l_m(j\omega) = \begin{cases} e^{-s\delta_{max}}-1, & if\ \omega \le \frac{\pi}{\delta_{max}} \\ 2, & if\ \omega > \frac{\pi}{\delta_{max}} \end{cases}$, where, in our example, $\delta_{max} = \max_{\theta\in[10,20]}|\theta-\tilde{\theta}| = \max_{\theta\in[10,20]}|\theta-20| = 10.$

The figure below show that for $\lambda \ge 10$ the condition is met. Since the robust stability condition is conservative, we can choose $\lambda = 10$.



The resulting IMC controller is then: $Q(s) = -\frac{(1+s)^2}{\left(1+\frac{s}{0.1}\right)^2}$.

B)

From the scheme of Figure 2), it follows that the controller G_0 is computed as

$$G_0(s) = \frac{Q(s)}{1-Q(s)\tilde{P}_0(s)} = \frac{-(1+s)^2}{\left(1+\frac{s}{0.1}\right)^2-1-\frac{s}{0.1}} = -0.1\frac{(1+s)^2}{s\left(1+\frac{s}{0.1}\right)}.$$

The primary controller has 2 zeros in $s=-1$, one pole in $s=0$ and one pole in $s=-0.1$, therefore it is a PID + filter controller:

$$G_0(s) = -0.1\frac{1+2s+s^2}{s}\cdot\frac{1}{1+\frac{s}{0.1}} = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{1+\beta_f s},$$ with $K_c=-0.2, T_i=2, T_d=0.5, \beta_f=10$.

Solution of exercise 2.

Firstly, we note that the value of the samples h_1 and h_2 is 0, which indicates that the system has a delay $d = 2$. Therefore we cannot use the MAC algorithm and we have to retrieve the step-response model from the impulse-response one to use the DMC algorithm.

Since the i-th sample of the step-response model is computed as $g_i = \sum_{j=1,\dots,i} h_i$, the obtained step-response samples are:

$g_1 = 0, g_2 = 0, g_3 = 0.3, g_4 = 0.8, g_5 = 0.9, g_6 = 0.9, g_7 = 0.9, g_8 = 0.9, \dots$.

We note that the samples from g_5 on have the same value; therefore, we select the first 5 samples as step-response model, i.e., $N = 5$.
We also note that the first 2 samples are null: this means that the process has an input-output delay $d = 2$.
Then, we need to select the prediction horizon. To avoid that the system dynamic matrix G has a null column (thus generating a singular matrix $G^T G$), we need to set $p \geq m + d = 4$. We select $p = 4$.

The dynamic matrix is then $G = \begin{pmatrix} g_1 & 0 \\ g_2 & g_1 \\ g_3 & g_2 \\ g_4 & g_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.3 & 0 \\ 0.8 & 0.3 \end{pmatrix} \in \mathbb{R}^{p \times m}$.

The matrix G is used to compute the solution of the unconstrained optimization problem: $u = (G^T G + \lambda I)^{-1} G^T (w - f)$, with $\lambda = 0.1$; it holds that

$(G^T G + \lambda I)^{-1} G^T = \begin{pmatrix} 0 & 0 & 0.57 & 0.80 \\ 0 & 0 & -0.72 & 0.57 \end{pmatrix}$.

Since we are interested in the current control increment only, i.e., in the first element of the vector u, we just need the first row of the matrix $(G^T G)^{-1} G^T$. The resulting vector is the control gain

$K = (0 \quad 0 \quad 0.57 \quad 0.80)$

and the control increment is computed as $\Delta u(t) = K(w - f)$.

At time $t = 5$, we need to compute the reference trajectory w up to time $t + p = 9$. From the given equations for the reference signal r (point ii) and for w (point iii), we compute

$w(5) = y_m(5) = 0.8$,
$w(6) = 0.5w(5) + 0.5r(6) = 0.4 + 0.6 = 1$,
$w(7) = 0.5w(6) + 0.5r(7) = 0.5 + 0.7 = 1.2$,
$w(8) = 0.5w(7) + 0.5r(8) = 0.6 + 0.8 = 1.4$,
$w(9) = 0.5w(8) + 0.5r(9) = 0.7 + 0.9 = 1.6$.

From point vi) we compute the control variations:

$\Delta u(0) = 0,\ \Delta u(1) = 0.5,\ \Delta u(2) = 0.3,\ \Delta u(3) = 0.4,\ \Delta u(4) = 0.2$.

Now, we can compute the free response at time $t = 5$ over the prediction horizon, with

$f(t+k) = y_m(t) + \sum_{i=1}^{N} (g_{k+i} - g_i) \Delta u(t-i)\,, k = 1, \dots, p$:

$\underline{t = 5}$

$\underline{k = 1}$ $f(6) = y_m(5) + \sum_{i=1,\dots,5}(g_{i+1} - g_i)\Delta u(5 - i)$
$= y_m(5) + (g_2 - g_1)\Delta u(4) + (g_3 - g_2)\Delta u(3) + (g_4 - g_3)\Delta u(2) + (g_5 - g_4)\Delta u(1) + (g_6 - g_5)\Delta u(0) = 0.8 + 0 \cdot 0.2 + 0.3 \cdot 0.4 + 0.5 \cdot 0.3 + 0.1 \cdot 0.5 + 0 \cdot 0 = 1.12;$

$\underline{k = 2}$ $f(7) = y_m(5) + \sum_{i=1,\dots,5}(g_{i+2} - g_i)\Delta u(5 - i)$
$= y_m(5) + (g_3 - g_1)\Delta u(4) + (g_4 - g_2)\Delta u(3) + (g_5 - g_3)\Delta u(2) + (g_6 - g_4)\Delta u(1) + (g_7 - g_5)\Delta u(0) = 0.8 + 0.3 \cdot 0.2 + 0.8 \cdot 0.4 + 0.6 \cdot 0.3 + 0 \cdot 0.5 = 1.36;$

$\underline{k = 3}$ $f(8) = y_m(5) + \sum_{i=1,\dots,5}(g_{i+3} - g_i)\Delta u(5 - i)$
$= y_m(5) + (g_4 - g_1)\Delta u(4) + (g_5 - g_2)\Delta u(3) + (g_6 - g_3)\Delta u(2) + (g_7 - g_4)\Delta u(1) + (g_8 - g_5)\Delta u(0) = 0.8 + 0.8 \cdot 0.2 + 0.9 \cdot 0.4 + 0.6 \cdot 0.3 + 0.1 \cdot 0.5 + 0 \cdot 0 = 1.55;$

$\underline{k = 4}$ $f(9) = y_m(5) + \sum_{i=1,\dots,5}(g_{i+4} - g_i)\Delta u(5 - i)$
$= y_m(5) + (g_5 - g_1)\Delta u(4) + (g_6 - g_2)\Delta u(3) + (g_7 - g_3)\Delta u(2) + (g_8 - g_4)\Delta u(1) + (g_9 - g_5)\Delta u(0) = 0.8 + 0.9 \cdot 0.2 + 0.9 \cdot 0.4 + 0.6 \cdot 0.3 + 0.1 \cdot 0.5 + 0 \cdot 0 = 1.57.$

$$\Delta u(5) = K(w - f) = (0 \quad 0 \quad 0.57 \quad 0.80)\left(\begin{pmatrix} 1 \\ 1.2 \\ 1.4 \\ 1.6 \end{pmatrix} - \begin{pmatrix} 1.12 \\ 1.36 \\ 1.55 \\ 1.57 \end{pmatrix}\right) = -0.06.$$

$u(5) = u(4) + \Delta u(5) = 1.34.$