IMC - PID TUNING

- PID controllers

$$e \longrightarrow \boxed{\text{PID}} \longrightarrow u$$

$$u(t) = K\Big(\underbrace{e(t)}_{\text{P-term}} + \underbrace{\frac{1}{T_i}\int_0^t e(\tau)\,d\tau}_{\text{I-term}} + \underbrace{T_d\,\frac{de(t)}{dt}}_{\text{D-term}}\Big) \qquad \text{IDEAL}$$

$$\begin{cases} K & \text{PROPORTIONAL GAIN} \\ T_i & \text{INTEGRAL TIME CONSTANT} \\ T_d & \text{DERIVATIVE TIME CONSTANT} \end{cases}$$

I - past

P - present

D - future

P - Proportional controller ($T_i = \infty$, $T_D = 0$) K

- offset which decreases as K increases

I - Integral controller $\frac{K}{T_i}\frac{1}{s}$

- no offset (for typical processes)
- oscillations appear as T_i decreases
  (PI + process have at least 2 poles)

D - Derivative controller $K_d T_d s$

- offset
- improves the transient
- might cause problems with noise

e.g. $e(t) = \sin(t) + n(t) = \sin t + \bar{a} \sin \bar{\omega} t$

$$\frac{d\,e(t)}{dt} = \cos(t) + \bar{\omega}\bar{a} \cos \bar{\omega} t$$

LOW-PASS FILTER

$\Rightarrow$ implemented with a filter $K T_d s \cdot \frac{1}{1 + \beta s}$

D-TERM

EXAMPLE OF FILTER DESIGN

$$f(s) = \frac{1}{1 + \frac{T_d}{N} s} \quad ; \; N: \text{ design parameter}$$

PID

$$\boxed{G(s) = K \left(1 + \frac{1}{T_i s} + \frac{s T_d}{1 + s \frac{T_d}{N}} \right)}$$

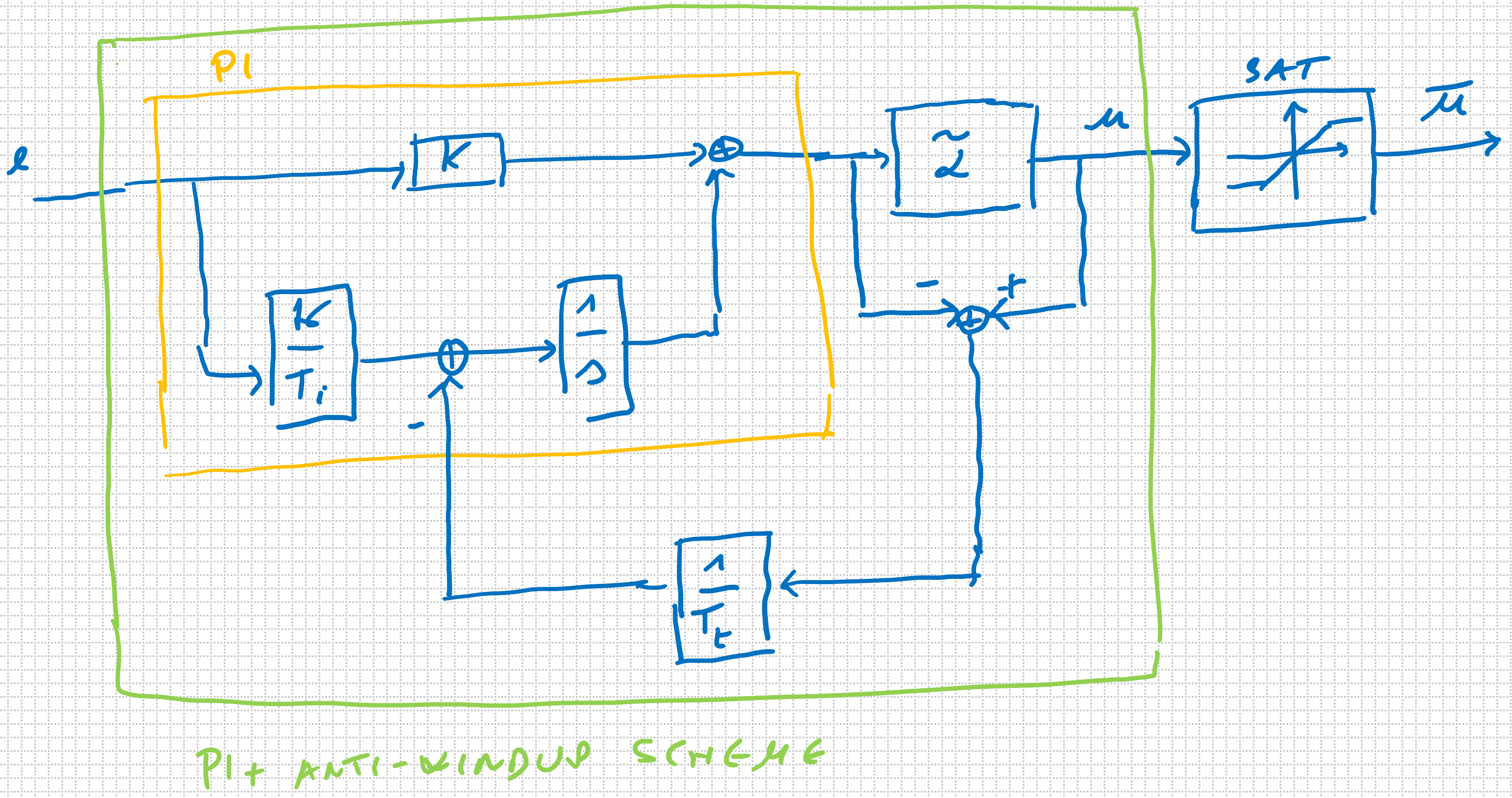
PID with filter (approximation of the derivative Term)

(Still some problems with noise $\lim_{s \to \infty} G(s) = K(1+N)$)

In general, a PID implementation requires a LOW-PASS filter

e.g. 1st order $f(s) = \frac{1}{1 + s \frac{T_d}{N}}$, 2nd order $f(s) = \frac{1}{\left(1 + s \frac{T_d}{N}\right)^2}$

• Example of ANTI-WINDUP scheme (much more complex w.r.t. IMC !)

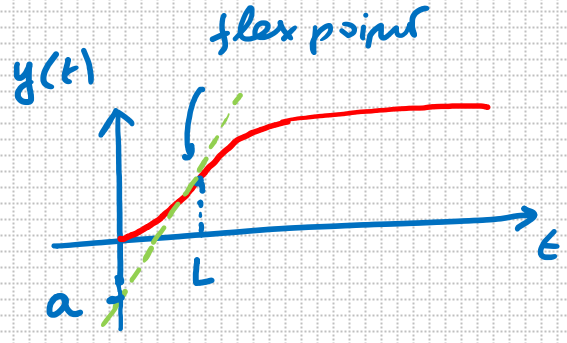


$\tilde{\alpha}$: ACTUATOR MODEL

T_t: time constant to be TUNED

STANDARD PID TUNING

I. ZIEGLER - NICHOLS, step-response method



1) Measure the step response

2) Table

	K	T_i	T_d
P	$1/a$		
PI	$0.9/a$	$3L$	
PID	$1.2/a$	$2L$	$L/2$

II. Frequency response method

1) $T_i \to \infty$, $T_D \to 0$, increase K until you measure perfect oscillation @ K_u

2) Determine the PERIOD of the oscillation T_u

	K	T_i	T_d
P	$0.5 K_u$		
PI	$0.4 K_u$	$0.8 T_u$	
PID	$0.6 K_u$	$0.5 T_u$	$0.125 T_u$

General rules

	RISE TIME	OVERSHOOT	SETTLING TIME	STEADY-STATE ERROR	STABILITY
$K_p \uparrow$	DECREASE	INCREASE	~	DECREASE	DEGRADE
$T_i \downarrow$	SMALL DECREASE	INCREASE	INCREASE	LARGE DECREASE	DEGRADE
$T_D \uparrow$	SMALL DECREAS	DECREASE	DECREASE	~	IMPROVE

REACTION (PRESENT)

TRANSIENT (FUTURE)

STEADY-STAT (PAST)

IMC CONTROLLERS

$$y(s) = \tilde{P}(s)\,Q(s)\,r(s) + \left(1 - \tilde{P}(s)\,Q(s)\right) d(s)$$

NOMINAL CONDITIONS

$\tilde{P} \equiv P$

$$\begin{cases} S(s) = 1 - \tilde{P}(s)\,Q(s) \\ T(s) = \tilde{P}(s)\,Q(s) \end{cases}$$

1. Asymptotic stability : $P(s)$, $Q(s)$ stable

2. Asymptotic behaviour

(CLASSIC CONTROL DESIGN

TYPE 1 SYSTEM : $W_e(0) = 0$

$r(t) = \mu_{-1}(t) \Rightarrow e(\infty) = 0$

TYPE 2 SYSTEM $\begin{cases} W_e(0) = 0 \\ \left.\dfrac{dW_e(s)}{ds}\right|_{s=0} = 0 \end{cases}$

$r(t) = t\,\mu_{-1}(t)$

...)

TYPE 1

$r(s) = \frac{1}{s}$

$$r(t) = \mu_{-1}(t)\,; \quad \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,T(s)\,r(s) = T(0) = 1$$

$$d(t) = \mu_{-1}(t)\,; \quad \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,S(s)\,d(s) = S(0) = 0$$

NO EFFECT TO STEP CHANGES

$$\begin{cases} \boxed{T(0) = 1} \\ S(0) = 1 - T(0) = 0 \end{cases}$$

TYPE 2

$r(t) = t\,\mu_{-1}(t)$

$R(s) = \frac{1}{s^2}$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,T(s)\frac{1}{s^2} = \lim_{s\to} \frac{1}{s}T(s) = T(0)\,t + \left.\frac{dT(s)}{ds}\right|_{s=0}$$

$d(t) = t\,\mu_{-1}(t)$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,(1-T(s))\frac{1}{s^2} = (1-T(0))\,t + \left.\frac{dT(s)}{ds}\right|_{s=0}$$

NO EFFECT TO RAMP CHANGES

$$\boxed{\begin{cases} T(0) = 1 \\ \left.\frac{dT(s)}{ds}\right|_{s=0} = 0 \end{cases}}$$

3. Physical realizability of $Q(s)$

a) Stability (BIBO) $\{\text{poles of } Q(s)\} \subset \mathbb{C}_{<0}$

b) Properness $\#\text{ poles of } Q(s) \geq \#\text{ zeros of } Q(s)$

c) Causal (it must rely on present and past measurements)

DESIGN PROCEDURE FOR STABLE PROCESSES

1) NOMINAL PLANT MODEL $\tilde{P}(s)$

- Perfect control: $y = \tilde{P} Q r = r \iff Q = \tilde{P}^{-1}$

NOT POSSIBLE

- $\tilde{P}$ is STRICTLY PROPER $\Rightarrow$ $\tilde{P}^{-1}$ IMPROPER
- $\tilde{P}$ may have delays $\Rightarrow$ $\tilde{P}^{-1} = \tilde{P}_0^{-1} e^{\theta s}$ $\leftarrow$ anticipation $\Rightarrow Q(s)$ should rely on FUTURE MEASUREMENTS
- $\tilde{P}$ may have RHP zeros $\Rightarrow$ $\tilde{P}^{-1}$ RHP poles

1 **FACTORIZATION OF $\tilde{P}(s)$**

$$\tilde{P}(s) = \tilde{P}_+(s)\,\tilde{P}_-(s)$$

with $|\tilde{P}_+(0)| = 1$

$\tilde{P}_+(s)$ collects the NON-MINIMUM PHASE elements of $\tilde{P}(s)$
- RHP zeros
- Time-delays

$\tilde{P}_-(s)$ collects the MINIMUM-PHASE elements of $\tilde{P}(s)$
- LHP zeros
- Poles (LHP for assumption)

2 $\Rightarrow$ $\boxed{\tilde{Q}(s) = \tilde{P}_-^{-1}(s)}$

Realizability:

1) STABLE — $\tilde{P}_-(s)$ has only LHP zeros $\Rightarrow$ $\tilde{Q}(s)$ - - - - - LHP poles

2) CAUSAL — $\tilde{P}_-(s)$ has no TIME-DELAYS

3) ~~PROPER~~ — $\tilde{P}_-(s)$ STRICTLY PROPER

③ Add a filter $f(s)$ such that $Q(s) = \tilde{Q}(s)\, f(s)$ is PROPER

$$\begin{cases} \tilde{T}(s) = \tilde{P}(s)\, Q(s) = \tilde{P}_+(s)\, P_-(s)\, \tilde{P}_-(s)^{-1} f(s) = \tilde{P}_+(s)\, f(s) \\ \tilde{S}(s) = 1 - \tilde{T}(s) = 1 - \tilde{P}_+(s)\, f(s) \end{cases}$$

NOMINAL SENSITIVITY FUNCTIONS

RULES OF FACTORIZATION

delay (if present)

RHP zeros

IAE - OPTIMAL for step inputs

$$\tilde{P}_+(s) = e^{-\theta s} \prod_i (1 - \beta_i s), \quad \beta_i > 0$$

ISE - OPTIMAL for step inputs

$$\tilde{P}_+(s) = e^{-\theta s} \underbrace{\frac{\prod_i (1 - \beta_i s)}{\prod_i (1 + \beta_i s)}}_{\text{ALL-PASS FILTER}}$$

stable poles

$(\ |P_+(0)| = 1\ )$

...

ASYMPTOTIC BEAVIOUR

TYPE 1 SYSTEM (no offset for step inputs)

$$\iff \quad \tilde{T}(0) = 1$$

$$\tilde{T}(0) = \tilde{P}(0)\, Q(0) = \underbrace{\tilde{P}_+(0)}_{\tilde{P}_+(0)=1} \cancel{\tilde{P}_-(0)}\, \cancel{\tilde{P}_-^{-1}(0)}\, f(0) = 1 \Rightarrow f(0) = 1$$

Example

$$\boxed{f(s) = \frac{1}{(1+\lambda s)^n}}$$

with n large enough to let $Q = \tilde{Q} f$ be proper

$\lambda \uparrow$ ROBUSTNESS

$\lambda \downarrow$ VELOCITY

Type 2 system (no offset to ramp inputs)

$$\begin{cases} \tilde{T}(0) = 1 \\ \left.\dfrac{d\tilde{T}(s)}{ds}\right|_{s=0} = 0 \end{cases} \Longrightarrow \begin{cases} f(0) = 1 \\ \left.\dfrac{d}{ds}\tilde{P}_+(s)\, f(s)\right|_{s=0} = 0 \end{cases}$$

$$\tilde{T} = \tilde{P}_+ f$$

Example

$$\boxed{f(s) = \frac{\left(2\lambda - \left.\dfrac{d\tilde{P}_+(s)}{ds}\right|_{s=0}\right) s + 1}{(\lambda s + 1)^2}}$$

It can be used only if it is sufficient to render Q proper

Example: perfect control is not realizable

RHP zero

delay

$$\tilde{P}(s) = K \frac{(1-\beta s)\, e^{-\theta s}}{(1+\tau_1 s)(1+\tau_2 s)}, \quad \beta, \tau_1, \tau_2 > 0$$

IMPROPER

ANTICIPATION

UNSTABLE

- PERFECT CONTROLLER

$$Q_p(s) = \tilde{P}^{-1}(s) = \frac{1}{K} \frac{(1+\tau_1 s)(1+\tau_2 s)}{(1-\beta s)} e^{\theta s}$$

- IAE-OPTIMAL TYPE 1

$$\tilde{P}(s) = \tilde{P}_+(s)\, \tilde{P}_-(s), \quad \tilde{P}_+(s) = (1-\beta s) e^{-\theta s}, \quad \tilde{P}_-(s) = K \frac{1}{(1+\tau_1 s)(1+\tau_2 s)}$$

$$\Rightarrow \begin{cases} \tilde{Q}(s) = \frac{1}{K}(1+\tau_1 s)(1+\tau_2 s) \\ f(s) = \frac{1}{(1+\lambda s)^2} \end{cases}$$

$$\boxed{Q(s) = \frac{1}{K} \frac{(1+\tau_1 s)(1+\tau_2 s)}{(1+\lambda s)^2}}$$

- ISE-OPTIMAL TYPE 1

$$\tilde{P}_+(s) = \frac{1-\beta s}{1+\beta s} e^{-\theta s}, \quad \tilde{P}_-(s) = K \frac{1+\beta s}{(1+\tau_1 s)(1+\tau_2 s)}$$

$$\Rightarrow \begin{cases} \tilde{Q}(s) = \tilde{P}_-(s)^{-1} = \frac{1}{K} \frac{(1+\tau_1 s)(1+\tau_2 s)}{1+\beta s} \\ f(s) = \frac{1}{1+\lambda s} \end{cases}$$

$$\Rightarrow \boxed{Q = \frac{1}{K} \frac{(1+\tau_1 s)(1+\tau_2 s)}{(1+\beta s)(1+\lambda s)}}$$