PROCESS AUTOMATION

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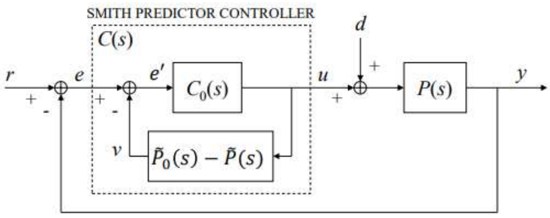
0. EXPLAIN WHY THE STABILITY OF A SYSTEM CONTROLLED BY USING A SMITH'S PREDICTOR IS NOT DEPENDENT ON THE VALUE OF THE NOMINAL DELAY.

To develop a smith predictor, here are the steps to follows:

1. Find the controller Q_0 through the robust IMC design
2. Define the equivalent controller $C_o = \frac{Q_0}{1-Q_0\widetilde{P_0}}$ for the delay free case
3. Consider the smith predictor controller for the process with the delay $C = \frac{C_0}{1+C_0(\widetilde{P_0}-\tilde{P})}$

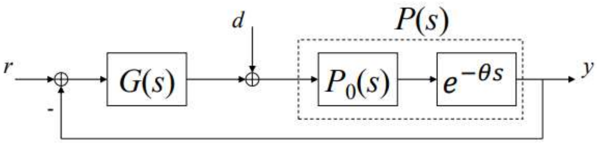
The smith predictor control structure is the following:



Assuming that the plant model is perfect, so $\tilde{P}(s) = P(s)$, the closed loop transfer function is the following:

$$W(s) = \frac{C(s)P(s)}{1+C(s)P(s)} = \frac{\frac{C_0(s)}{1+C_0(s)\left(\bar{P}_0(s)-\tilde{P}(s)\right)}P(s)}{1+\frac{C_0(s)}{1+C_0(s)\left(\bar{P}_0(s)-\tilde{P}(s)\right)}P_0(s)} = \frac{C_0(s)P(s)}{1+C_0(s)P_0(s)} = \frac{C_0(s)P_0(s)}{1+C_0(s)P_0(s)}e^{-\theta s}$$

This equation shows that the SP controller eliminates the delay in the characteristic polynomial. If we consider a classical controller of the form:



Then the characteristic polynomial will be a "quasi polynomial" since it will contain an exponential term. To check the stability, we have to see if the poles of the systems are in the left side part of the complex plane, thus if the roots of the polynomial have negative real part. Due to the presence of the complex exponential, we have infinite roots so infinite poles to take into account. With the smith's predictor controller, we have eliminated this problem since the exponential is no more present in the characteristic polynomial; therefore, it does not affect stability. Moreover, the input output transfer function is the one of a delay free system followed by a pure delay.

1-6-15-8. DISCUSS WHY MPC MAY IMPROVE THE SAFETY OF THE PLANTS

In control system, we always have constraints linked to safety problems; these are hard constraints (in opposite to the soft one that we could also have) therefore, they cannot be violated at any time. In classical control, we have to be sure that the system dynamics is feasible with respect to the constraints and then we can control the dynamic within the feasible state. If we use MPC, we can check the constraints' violation within the control strategy; in fact, in MPC we use the model to predict the dynamic of the system so we are able to predict if a given constraint will be violated. In this way, we can provide early warning of potential problems. Therefore, using MPC we are able to check if the constraints will be violated while we are designing the optimal control strategy.

2. DISCUSS WHY THE LENGTH OF THE PREDICTION HORIZON DEPENDS ON THE ACCURACY OF THE PLANT MODEL (CONSIDER THE TWO CASES: STEP RESPONSE MODEL AND STATE-SPACE MODEL)

3. DISCUSS WHY PFC MAY IMPROVE THE SCALABILITY OF THE CONTROL ALGORITHM WITH RESPECT TO DMC AND MAC

If we use the step response model or the impulse response model, we are limited by the fact that we have to measure the response of the model to a step and to an impulse, and this means that we can use MAC and DMC only for stable process without integrators. On the other hand, PFC can treat every kind of process: unstable process, nonminimum phase terms and also delay. Moreover, it simplifies the computational cost of solving the optimization problem. PFC still uses quadratic cost functions, but the number of unknowns can be diminished because it uses a structured control action. So, in PFC the control actions are expressed as linear combinations of base functions. Moreover, we limit the evaluation of the error in a limited number of coincidence points along the prediction horizon. In this way, the number of unknowns is smaller.

4. CONSIDERING SOME-DELAY SYSTEM CONTROLLED WITH A SMITH PREDICTOR CONTROLLER, DISCUSS WHETHER THE STABILITY MARGINS OF THE SYSTEM ARE MORE AFFECTED BY THE ABSOLUTE VALUE OF THE DELAY OR BY THE MISMATCH BETWEEN THE NOMINAL DELAY AND THE ACTUAL DELAY.

The stability margins of the system controlled with a smith predictor are not influenced by the nominal value of the delay but only by the delay mismatch. In fact, if we have a perfect model, thus $P = \tilde{P}$, then the closed loop transfer function will be the following:

$$W(s) = \frac{C(s)P(s)}{1+C(s)P(s)} = \frac{\frac{C_0(s)}{1+C_0(s)\left(\bar{P}_0(s)-\bar{P}(s)\right)}P(s)}{1+\frac{C_0(s)}{1+C_0(s)\left(\bar{P}_0(s)-\bar{P}(s)\right)}P_0(s)} = \frac{C_0(s)P(s)}{1+C_0(s)P_0(s)} = \frac{C_0(s)P_0(s)}{1+C_0(s)P_0(s)}e^{-\theta s}$$

We have that the exponential is not present in the characteristic polynomial therefore it does not affect the stability of the system. In fact, it is like we are taking out the delay from the closed loop system; so, the feedback signal is available θ seconds before the actual measure.

On the other hand, if the process model is not perfect, the closed loop transfer function, will be:

$$W(s) = \frac{C(s)P(s)}{1+C(s)P(s)} = \frac{\frac{C_0(s)}{1+C_0(s)\left(\tilde{P}_0(s)-\tilde{P}(s)\right)}P(s)}{1+\frac{C_0(s)}{1+C_0(s)\left(\tilde{P}_0(s)-\tilde{P}(s)\right)}P(s)} = \frac{C_0(s)P(s)}{1+C_0\tilde{P}_0(s)-C_0\tilde{P}(s)+C_0(s)P(s)}$$

And if we suppose that the delay free process model is perfect, but the delay is not, thus if:

$$\widetilde{P_0} = P_0, P\tilde{s} = P_0(s)e^{-\tilde{\theta}s},\ \ \tilde{\theta} \neq \theta$$

The closed loop transfer function would be:

$$W(s) = \frac{C_0(s)P_0(s)e^{-\theta s}}{1+C_0P_0(s)-C_0P_0(s)e^{-s\tilde{\theta}}+C_0(s)P_0(s)e^{-\theta s}} = \frac{C_0(s)P_0(s)e^{-\theta s}}{1+C_0P_0(s)\left(1-e^{-s\tilde{\theta}}+e^{-\theta s}\right)}$$

In this case, the SP cannot remove the time delay from the denominator of the transfer function that is still a quasi-polynomial, but the SP can reduce its effect on stability allowing higher gains to be used.

7. ANSWER THE FOLLOWING QUESTIONS:

a. Why it is difficult to extend standard control methods in the Laplace domain and why it is easier within the MPC framework?

The main problem when we extend the standard control methods in Laplace domain is related to stability. In fact, in the Laplace domain we talk about external stability since we can check that only the reachable and observable eigenvalues have real negative parts. The external stability it not asymptotic stability; asymptotic stability implies external stability, but the opposite is not always true; this is true only if the know that the unobservable and unreachable eigenvalues are already stable. In the MPC framework it is easier to consider control methods since it can deal with unstable process, nonminimum phase systems and also with time delays. Moreover, constraints are systematically included in the proceed design.

b. If a given chemical process control problem has a constraint on the maximum temperature of the chemical reactor, a constraint on the maximum value of a control pump which feeds a steam generator, and a constraint on the maximum pressure in a relief tank, which constraint(s) may be enforced by modifying the cost function?

We have a general form for the constraints since we can mix constraints on the input and on the output; we just give some value to the coefficient c_i, c_u, c_j

$$\sum_{i=1}^{N} \left(c_y^{i,j}\hat{y}(t+i|t) + c_u^{i,j}u(t+i-1|t) + c_j\right) \leq 0$$

In particular, we have:

- $c_y^{i,j}$: coefficient of constraint j for the predicted output at time $t+i$
- $c_u^{i,j}$: coefficient of constraint j for the control variable at time $t+i-1$
- c_j: constant term of constraint j

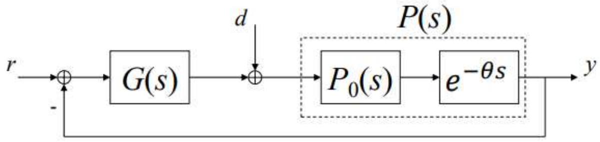
So every kind of constraints can be enforced in this way.

9. WHY THE SENSORS AND ACTUATORS USED FOR CONTROLLING THE PLANT SHOULD NOT BE USED ALSO FOR ENSURING THE PLANT SAFETY

In the different level of process control, we have a first layer devoted to measurement with some sensors and actuators and a second layer related to safety. In this layer, we not only use alarms, but we also need to plan human intervention. It is important to use equipment and instrumentation only devoted to safety indifferently from the one used in the measurement level, and possibly with redundancy. In fact, we need to be aware that sensors and actuators of the measurement level are working properly and in case there is a malfunction, the safety layer can operate. In fact, if there is something wrong, only the measurement level can be affected but the safety level must not.

10 + 14. WHY MAY A FEEDBACK DELAY CAUSE INSTABILITY AND WHEN IT IS POSSIBLE TO USE PADÈ APPROXIMATION?

Let's consider the following scheme:



This is a SISO linear process, whose transfer function is the cascade of a delay free function and a delay.

the open loop transfer function is:

$$F(s) = G(s)P(s) = G(s)P_0e^{-\theta s} = F_0(s)e^{-\theta s}$$

Let's analyse the bode diagram of $e^{-\theta s}$. This is a complex exponential whose module is always equal to 1, thus the magnitude of the diagram is always 0 (the magnitude is computed in db). This is a all-pass filter. The phase of the delay block is linearly proportional to the frequency with constant of proportionality equal to θ. In fact, the phase is $-\omega\theta$.

Therefore, $F_0(jw)$ and $F(jw)$ have the same module since the exponential has module equal to one, but what changes is the phase:

$$\measuredangle\{F(j\omega)\} = \measuredangle\{F_0(j\omega)e^{-\theta j\omega}\} = \measuredangle\{F_0(j\omega)\} - \omega\theta, \forall\omega$$

We have to sum the phase of the exponential to the phase of the open loop transfer function with the delay free.

The phase diminished linearly with the delay. The value $-\omega\theta$ is a phase leg that may drive the system to instabilities. In fact, the system becomes instable if the delay increases to much.

In case the delay is "small", so it is $\theta \leq 0.1\tau$ where τ is the time constant of the system, we can use the padè approximation and we can approximate the delay to a rational function.

11. EXPLAIN WHY THE ROBUST STABILITY CONDITION BASED ON NYQUIST THEOREM ARGUMENTS (I.E., THE CONDITION $|lm(j\omega)\tilde{}(j\omega)|$ < 1, $\forall\omega$) IS CONSERVATIVE

When we deal with robust stability, we fix an upper bound for the uncertainties:

$$|w(j\omega)| > |\Delta_a(j\omega)|$$

$$|l_m(j\omega)| > |\Delta(j\omega)|$$

To check if a process is stabilized, we have to plot the Nyquist diagram and see if the distance between the open loop function and -1 in the Nyquist plot is positive. let's consider the case of the additive uncertainty $P(s) = \tilde{P}(s) + \Delta_a$. We have:

$$F(j\omega) = P(j\omega)G(j\omega) = \left(\tilde{P}(j\omega) + \Delta_a(j\omega)\right)G(j\omega) = P(j\omega)G(j\omega) + \Delta_a(j\omega)G(j\omega) = \tilde{F}(j\omega) + \Delta_aG(j\omega)$$

Then the distance is:

$$d(F(j\omega), 1) = |F(j\omega) - (-1)| = \left|\tilde{F}(j\omega) + \Delta(j\omega)G(j\omega) + 1\right| > 0 \ \ \forall\omega$$

$$\left|\tilde{F}(j\omega) + \Delta(j\omega)G(j\omega) + 1\right| > \left|1 + \tilde{F}(j\omega)\right| - |\Delta(j\omega)G(j\omega)|, \qquad \forall\omega$$

But we know that

$$w(j\omega) > \Delta(j\omega), \forall\omega \rightarrow \quad \left|1 + \tilde{F}(j\omega)\right| - |\Delta(j\omega)G(j\omega)| > \left|1 + \tilde{F}(j\omega)\right| - |w(j\omega)G(j\omega)|$$

So:

$$d(F(j\omega), -1) > \left|1 + \tilde{F}(j\omega)\right| - |w(j\omega)G(j\omega)|$$

We want that this is positive. This is true only if

$$|w(j\omega)G(j\omega)| < \left|1 + \tilde{F}(j\omega)\right|, \quad \forall\omega$$

So we can write that

$$|w(j\omega)G(j\omega)|\left|1 + \tilde{F}(j\omega)\right|^{-1} < 1 \; \forall \; \omega$$

We recall that:

$$\tilde{S}(j\omega) = \frac{1}{1 + \tilde{F}(s)}$$

Therefore:

$$\left|w(j\omega)G(j\omega)\tilde{S}(j\omega)\right| < 1, \forall\omega \rightarrow |w(j\omega)Q(j\omega)| < 1, \forall\omega$$

By this way to compute the distance between F and $\tilde{F}$ we are just considering the module of the processes and not the phase. Moreover, when considering the uncertainty, we are replacing it with the upper bound; therefore, the actual distance between -1 and the actual process is smaller than the one we are considering. These reasons make this a conservative approach.

12. BRIEFLY DISCUSS THE CHARACTERISTICS OF FEEDFORWARD AND FEEDBACK CONTROL

There are two main control strategies: feedback control and feedforward.

Feedback control the controlled variables are the measured ones and the measurements are used to adjust the controlled variables. The disturbances are not measured because the system reacts to the effect of the disturbance and not on the cause. The advantages are that the corrective actions are taken regardless the cause of the disturbance and it is used to reach the set point. The disadvantage is that we may not able to compute the corrective action until the effect of the disturbance is seen on the process.

Feedforward control the measured variables are the disturbance and not the controlled ones. The advantage is that corrective actions are taken before the effect of the disturbance appears (we can also cancel the effect of a disturbance on the controlled variable). the disadvantage is that there may be many disturbances and it is not economical to measure them all; we need to understand which are the one really affecting the process (so we need a process model).

Usually, the control strategy is made of a feedback control and a feedforward control. The feedback control provides the corrective actions for unmeasured disturbances and to reach the set point and the feedforward to eliminate the measured disturbance before the control variables reach the set point.

13 + 16. LIST PROS AND CONS OF THE PREDICTION MODELS OF THE MPC ALGORITHMS DMC, MAC, PFC

The mail difference is in the model that the 3 algorithms use: step response model for DMC, impulse response model for MAC, state-space process model for PFC. If we use the step response model or the impulse response model, we are limited by the fact that we have to measure the response of the model to a step and to an impulse, and this means that we can use MAC and DMC only for stable process without integrators. However, we do not need to know anything about the process, we just measure the output. There are some minor differences between MAC and DMC. We can go from an impulse response model to a step response model and back, so they are equivalent in this sense, but MAC is relatively simpler because in MAC the control horizon is set equal to the prediction horizon. PFC also introduces two innovations to simplify the computational cost of solving the optimization at every step. So, it was born for dealing with fast processes. It uses the state space process model, so we need some a priori knowledge on the process, but then we can use it with integrating processes and also with some attention to unstable processes. PFC still uses quadratic cost functions, but the number of unknowns can be diminished because it uses a structured control action. So, in PFC the control actions are expressed as linear combinations of base functions. The error in the cost function is not computed over all the error samples in the future but only on a selected set.