$$\begin{cases} \rho \dfrac{dV}{dt} = w_1 + w_2 - w & (3) \\ \rho \dfrac{d(Vx)}{dt} = w_1 x_1 + w_2 x_2 - w x & (4) \end{cases}$$

(4) → $$\rho V \frac{dx}{dt} + \rho x \frac{dV}{dt} = w_1 x_1 + w_2 x_2 - w x \quad (6)$$

↓ Eq. 3

$$\rho V \frac{dx}{dt} + x(w_1 + w_2 - w) = w_1 x_1 + w_2 x_2 - w x \quad (7)$$

(3) → $$\frac{dV}{dt} = \frac{1}{\rho}(w_1 + w_2 - w) \quad (8)$$

(7) → $$\frac{dx}{dt} = \frac{w_1}{V\rho}(x_1 - x) + \frac{w_2}{V\rho}(x_2 - x) \quad (9)$$

$$\begin{cases} \rho \dfrac{dV}{dt} = w_1 + w_i - w & (1) \\ \rho V \dfrac{dx}{dt} = w_1 (x_1 - x) + w_2 (x_2 - x) & (2) \end{cases}$$

Assumption: $\begin{cases} V: \text{constant} \\ x_2 = 1 \end{cases}$

NONLINEAR EQ (terms $w_1 x_1$, $w_1 x$, $w_2 x$)

$$w(t) = w_1(t) + w_2(t) \qquad (3)$$

$$\frac{dx(t)}{dt} = \frac{w_1(t)}{\rho V}\left(x_1(t) - x(t)\right) + \frac{w_2(t)}{\rho V}\left(1 - x(t)\right) = f\left(x, x_1, w_1, w_2\right) \qquad (4)$$

LINEARIZATION

1) STEADY-STATE

$\bar{w}_1, \bar{w}_2, \bar{x}_1, \bar{x}_2 = 1$ ← OPERATING CONDITIONS

(3) →
$$\begin{cases} \bar{w} = \bar{w}_1 + \bar{w}_2 & (5) \\ \bar{w}_1 \bar{x}_1 - \bar{w}_1 \bar{x} + \bar{w}_2 - \bar{w}_2 \bar{x} = 0 & (6) \end{cases}$$

(4) → ← $f(\bar{x}, \bar{x}_1, \bar{w}_1, \bar{w}_2) = 0$

$$\Rightarrow \bar{x} = \frac{\bar{w}_1 \bar{x}_1 + \bar{w}_2}{\bar{w}_1 + \bar{w}_2} \qquad (7)$$

DYNAMIC MODEL

$$\frac{dx}{dt} = \frac{w_1}{\rho V}(x_1 - x) - \frac{w_2}{\rho V}(1 - x) = f(\dots) \quad (4)$$

$S: \bar{x}_1, \bar{x}, \bar{w}_1, \bar{w}_2$

$$\left.\frac{dx}{dt}\right|_S = f(\bar{x}_1, \bar{x}, \bar{w}_1, \bar{w}_2) + \left.\frac{df}{dx}\right|_S (x - \bar{x}) + \left.\frac{df}{dx_1}\right|_S (x_1 - \bar{x}_1) + \left.\frac{df}{dw_1}\right|_S (w_1 - \bar{w}_1) +$$

$$+ \left.\frac{df}{dw_2}\right|_S (w_1 - \bar{w}_2)$$

$$\begin{cases} x' = x - \bar{x} \\ x_1' = x_1 - \bar{x} \\ w_1' = w_1 - \bar{w}_1 \\ w_2' = w_2 - \bar{w}_2 \end{cases} ; \quad \left.\frac{df}{dx}\right|_S = -\frac{1}{\rho V}(\bar{w}_1 + \bar{w}_2) = -\frac{1}{\rho V}\bar{w}$$

$$\left.\frac{df}{dx_1}\right|_S = \frac{1}{\rho V}\bar{w}_1 \,, \quad \left.\frac{df}{dw_1}\right|_S = \frac{1}{\rho V}(\bar{x}_1 - \bar{x}) \,, \quad \left.\frac{df}{dw_2}\right|_S = \frac{1}{\rho V}(\bar{x} - 1)$$

$$* \quad \frac{dx(t)}{dt} = -\frac{1}{\rho V}\bar{w}\, x'(t) + \frac{1}{\rho V}\bar{w}_1\, x_1'(t) + \frac{1}{\rho V}(\bar{x}_1 - \bar{x})\, w_1'(t) + \frac{1}{\rho V}(\bar{x} - 1)\, w_2'(t)$$

LINEAR STATE EQ.