Process Automation (MCER), 2015-2016

EXAM

22 Jan. 2016

Exercise 1 (12 pts.)

Let the process be described by the transfer function: $P(s) = \frac{S+1}{(10+s)^2} e^{-\theta s}$, with $\theta = 0.1 + \delta$ and $\delta \in [0,0.1)$, and let the process model be $\tilde{P}(s) = \frac{S+1}{(10+s)^2} e^{-0.1s}$.

Design a robust Smith Predictor controller by following the IMC design.

Exercise 2 (12 pts.)

Consider a process whose impulse response model is given by the following coefficients:

$h_1 = 0,\ h_2 = 0.1,\ h_3 = 0.01,\ h_4 \approx 0,\ h_5 \approx 0,\ h_6 \approx 0 \dots$.

Compute the future control action sequence of a MPC algorithm at time $t = 5$, with:

- prediction horizon $p = 3$;
- reference $r(t) = \begin{cases} 1, & 0 \le t < 3 \\ 0, & t \ge 3 \end{cases}$;
- weighted reference trajectory $w(t) = \alpha w(t-1) + (1-\alpha) r(t), t > 0$, with $w(0) = 0$ and $\alpha = 0.5$.
- cost function $J = e^T e + \lambda u^T u$, where e is the vector of future errors between predicted output and reference trajectory and $\lambda = 0.1$;
- control actions and available measured outputs at time $t = 5$:
    - $u(1) = -0.79, u(2) = -1.44, u(3) = -4.04, u(4) = -9.19$
    - $y_m(1) = 0, y_m(2) = 0,\ y_m(3) = -0.75,\ y_m(4) = -1.44,\ y_m(5) = -4.00$

Questions (6 pt.)

i) Which are the main advantages of MPC in assuring safety of industrial plants? (1/3 pg. max, 2pt)

ii) Why the sensors and actuators used for controlling the plant should not be used also for ensuring the plant safety? (1/3 pg. max, 2pt)

iii) Why may a feedback delay cause instability? (1/3 pg. max, 2pt)

Solution of exercise 1

The nominal process $\tilde{P}(s)$ is stable, therefore it is possible to design a stable controller $Q(s)$ to stabilize the closed-loop nominal system.
Moreover, since the time-delay $\tilde{\theta} = 0.1s$ of the process is comparable to the time constant $\tau = 0.1s$ of the process, we cannot use a Padé approximation to write the delay term as a transfer function.
Then, we use a Smith Predictor controller, depicted in the figure below:

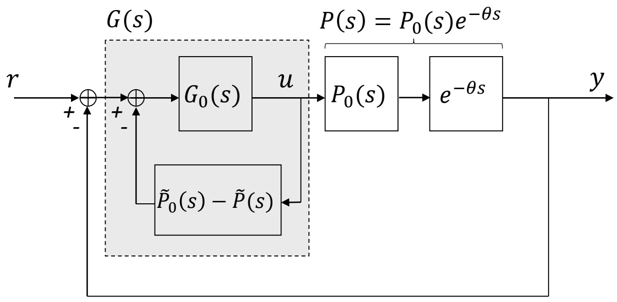


Figure 1)

The IMC form of the SP controller of Figure 1) is shown in the figure below:

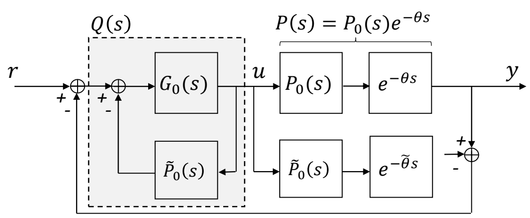


Figure 2)

The IMC design procedure to robustly stabilize the process $P(s)$ consists in the following 3 steps:

Step 1)

a) Factorize the nominal process in a minimum-phase term and a non-minimum-phase term:
   - $\tilde{P}(s) = \tilde{P}_+(s)\tilde{P}_-(s)$
   - with $\tilde{P}_+(s) = e^{-0.1s}$ and $\tilde{P}_-(s) = \frac{1}{100}\frac{1+s}{\left(1+\frac{s}{10}\right)^2}$.

b) Define the controller as follows: $\tilde{Q}(s) = \left(\tilde{P}_-(s)\right)^{-1} = 100\,\frac{\left(1+\frac{s}{10}\right)^2}{1+s}$

Step 2)

Design the controller $Q(s) = \tilde{Q}(s)f(s)$, where the IMC filter $f(s)$ must be such that the controller $Q(s)$ is pro We use the filter $f(s) = \frac{1}{(1+\lambda s)^n}$ with $n = 1$, so that $Q(s) = \tilde{Q}(s)f(s) = 100\,\frac{\left(1+\frac{s}{10}\right)^2}{1+s}\frac{1}{1+\lambda s}$ is proper.

Step 3)

Determine the value of λ such that the sufficient condition for robust stability holds:

$$\left|l_m(j\omega)\tilde{T}(j\omega)\right| < 1, \forall\omega$$

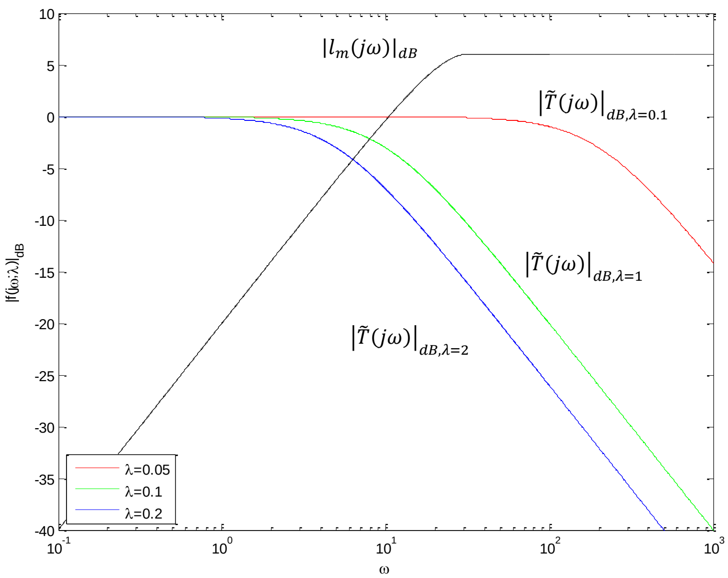
where $|\tilde{T}(j\omega)| = |\tilde{Q}(s)\tilde{P}(s)| = \left|\frac{1}{1+\lambda s}\right|$ and $l_m(j\omega)$ is an upper-bound of the multiplicative uncertainty $\Delta_m(j\omega)$, i.e., a function such that $|l_m(j\omega)| > |\Delta_m(j\omega)|, \forall\omega$.

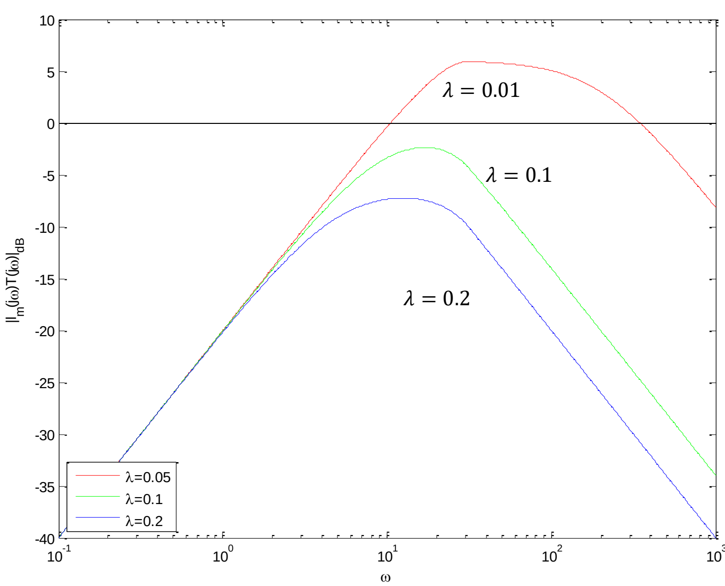
By definition, the multiplicative uncertainty is computed as

$$|\Delta_m(j\omega)| := \left|\frac{P(j\omega)-\tilde{P}(j\omega)}{\tilde{P}(j\omega)}\right| = \left|e^{-j\omega\delta}-1\right|.$$

Since $\delta_{max} = 0.1$, from the theory we know that an upper-bound is defined as $|l_m(j\omega)| = \begin{cases} |e^{-0.1s}-1, & if\ \omega \le 10\pi \\ 2, & if\ \omega > 10\pi \end{cases}$.

The figures below show that for $\lambda > 0.1$ the condition $\left|l_m(j\omega)\tilde{T}(j\omega)\right| < 1, \forall\omega$ is met.





The resulting IMC controller is then $Q(s) = \tilde{Q}(s)f(s) = 100\frac{(1+0.1s)}{(1+s)}$.
From the scheme of Figure 2), it follows that the controller G_0 is computed as
$G_0(s) = \frac{Q(s)}{1-Q(s)\tilde{P}_0(s)} = 10^3\frac{1+0.1s}{s(1+s)}$.

Solution of exercise 2.

Firstly, we note that there is a delay equal to 1 sample since the coefficient h_1 is 0. Then, we have to consider a prediction horizon $p \geq m+1=3$. We chose to use a DMC controller and compute the step response samples from the impulse response samples: $g_i = \sum_{j=1,\dots,i} h_j$.
the samples g_4, g_5 and g_6 have the same value; therefore, we select the first 4 samples as step response model, i.e., $N=4$.
We also note that the first sample g_1 is null: this means that the process has an input-output delay $d=1$.
We select $m=2$.

The dynamic matrix is then $G = \begin{pmatrix} g_1 & 0 \\ g_2 & g_1 \\ g_3 & g_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0 \\ 0.11 & 0.1 \end{pmatrix} \in \mathbb{R}^{p\times m}$.
The matrix G is used to compute the solution of the constrained optimization problem: $u = (G^T G)^{-1} G^T (w-f)$, with $(G^T G + \lambda I)^{-1} G^T = \begin{pmatrix} 0 & 0.9 & 0.09 \\ 0 & -0.09 & 0.9 \end{pmatrix}$. The control gain is then $K = (0 \quad 0.9 \quad 0.09)$.
The reference trajectory is computed from the given formula: $w(1) = 0.5, \dots, w(6) = 0.05, w(7) = 0.02, w(8) = 0.01$.

Then, we compute the free response at time $t=5$ over the prediction horizon:

$\underline{t=5}$

$\underline{k=1}$ $\quad f(6) = y_m(5) + \sum_{i=1,\dots,4}(g_{i+1} - g_i)\Delta u(5-i) = -4.54;$
$\underline{k=2}$ $\quad f(7) = y_m(5) + \sum_{i=1,\dots,4}(g_{i+2} - g_i)\Delta u(5-i) = -4.60;$
$\underline{k=3}$ $\quad f(8) = y_m(5) + \sum_{i=1,\dots,4}(g_{i+3} - g_i)\Delta u(5-i) = -4.62.$

$$\Delta u(5) = K(w-f) = (0 \quad 0.9 \quad 0.09)\left(\begin{pmatrix} 0.05 \\ 0.02 \\ 0.01 \end{pmatrix} - \begin{pmatrix} -4.54 \\ -4.60 \\ -4.62 \end{pmatrix}\right) = 4.6.$$

$u(5) = u(4) + \Delta u(5) = -0.55.$

(Alternatively, the standard MAC algorithm could have been used with $M = m = p = 3$).