Multiplicative Interactions and Where to Find Them[1]

March 26, 2020

[1] "Multiplicative Interactions and Where to Find Them" by Jayakumar et al.

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- They show, that such multiplicative interactions are more *expressive*
  - i.e. can represent broader set of functions with smaller amount of parameters.
- They achieve SotA performance on several tasks simply by replacing concatenation with MI.

General definition of Multiplicative Interaction

Authors define multiplicative interaction as a function between two variables x and z:

$$\mathcal{M}(x, z) = \mathbf{z}^T \mathbb{W} \mathbf{x} + \mathbf{z}^T \mathbf{U} + \mathbf{V}\mathbf{x} + \mathbf{b},$$

where:

- x, z are vectors
- $\mathbb{W}$ is a 3d-tensor
- U, V are 2d-matrices and b is a bias vector.

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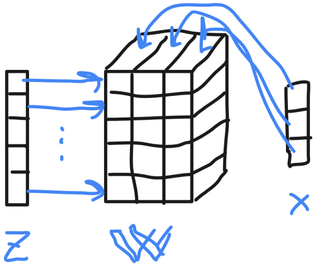
- Imagine that each cell of matrix W is an apple
- Then $z^\top A x$ produces an apple
- Then in case of $\mathbb{W}$ each apple is a 1d-vector



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- **Definition** Hypothesis space $\mathcal{H}_{f(\theta)}$ is a set of those functions that $f(\theta)$ can represent
- **Theorem**: given an MLP and MI, hypotheses class for MI is *strictly* larger than for MLP.
- This means, that MI can represent any function from $\mathcal{H}_{\text{MLP}}$, but there are some functions that MI can represent, but MLP cannot

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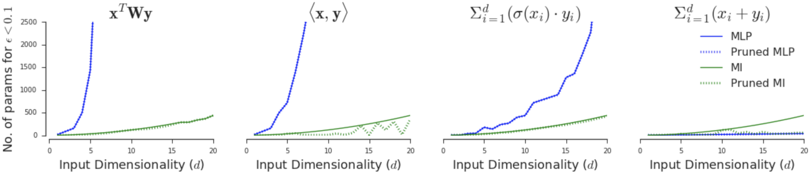
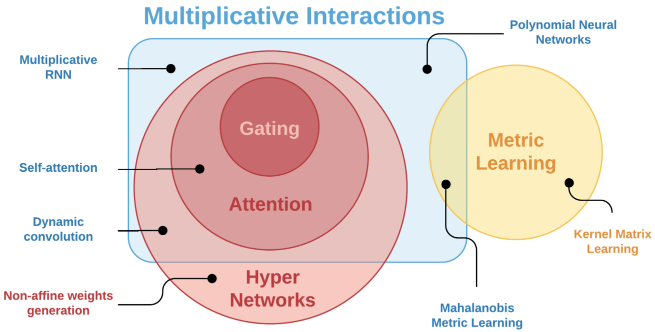


Examples of MIs

There are a lot of examples of MIs in the modern literature:



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  - To have $\mathcal{M}(x, z) = z^\top x$ we just need to set $\mathbb{W}$ to "identity" (zeros everywhere and ones on the main diagonal)
  - More general forms of metric learning as $d_C(x, z) = (\mathbf{x} - \mathbf{z})^T \mathbf{C}^{-1} (\mathbf{x} - \mathbf{z})$ can be also expressed as $\mathcal{M}(x, z)$.
- Hypernetworks, gating, etc

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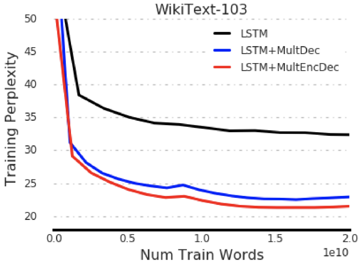
In vanilla LSTM all interactions are "concatentation"-based:

$$
\begin{aligned}
f_t &= \sigma_g(W_f x_t + U_f h_{t-1} + b_f) \\
i_t &= \sigma_g(W_i x_t + U_i h_{t-1} + b_i) \\
o_t &= \sigma_g(W_o x_t + U_o h_{t-1} + b_o) \\
c_t &= f_t \circ c_{t-1} + i_t \circ \sigma_c(W_c x_t + U_c h_{t-1} + b_c) \\
h_t &= o_t \circ \sigma_h(c_t)
\end{aligned}
$$

Authors replaced some of them with multiplicative ones and observed the boost in performance:



Conclusion

- Authors also did some experiments for few-shot learning and multi-task learning
- MI is a powerful tool to integrate different sources of information
- We have countless scenarios to explore them