Whats Hidden in a Randomly Weighted Neural Network? ¹

March 12, 2020

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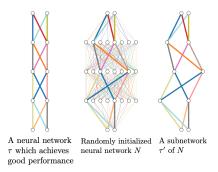
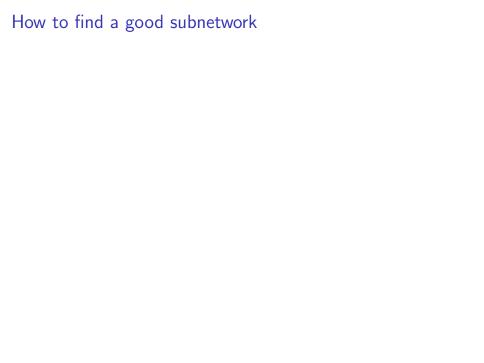


Figure: Since we have combinatorial number of subnetworks and modern models have millions of parameters we are likely to find a good one.



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And we update it via STE:

$$\tilde{\mathbf{s}}_{uv} = \mathbf{s}_{uv} - \alpha \frac{\partial \mathcal{L}}{\partial \mathcal{I}_{v}} \mathbf{w}_{uv} \mathcal{Z}_{u} \tag{3}$$

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The best explanation of STE:

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- \blacktriangleright Signed Constant: set each weight to $\sigma,$ then randomly choose its +/- sign

Method	Model	Initialization	% of Weights	# of Parameters	Accuracy
Learned Dense Weights (SGD)	ResNet-34 [8]	-	-	21.8M	73.3%
	ResNet-50 [8]	-	-	25M	76.1%
	Wide ResNet-50 [28]	-	-	69M	78.1%
edge-popup	ResNet-50	Kaiming Normal	30%	7.6M	61.71%
	ResNet-101	Kaiming Normal	30%	13M	66.15%
	Wide ResNet-50	Kaiming Normal	30%	20.6M	67.95%
edge-popup	ResNet-50	Signed Constant	30%	7.6M	68.6%
	ResNet-101	Signed Constant	30%	13M	72.3%
	Wide ResNet-50	Signed Constant	30%	20.6M	73.3%

Figure: ImageNet results (with k = 30%)

Varying % of remaining weights

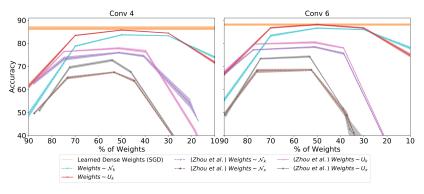


Figure: Varying % of remaining weights for AlexNet for CIFAR-10. Maximum in the middle since $\binom{n}{k}$ is maximized at $k \approx n/2$

Varying width of a specific layer

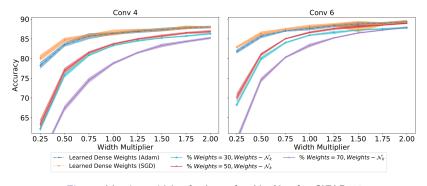


Figure: Varying width of a layer for AlexNet for CIFAR-10

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- ► HAT-algorithm for CL is not "fair"?