Learning deep representations by mutual information estimation and maximization

January 22, 2020

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- If we did a good job in training T_ω then we'll have a good MI estimate between X and Y.

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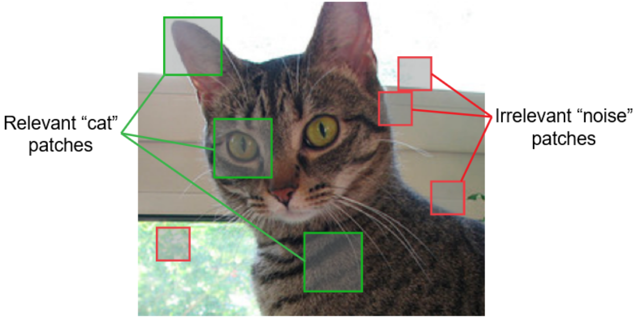

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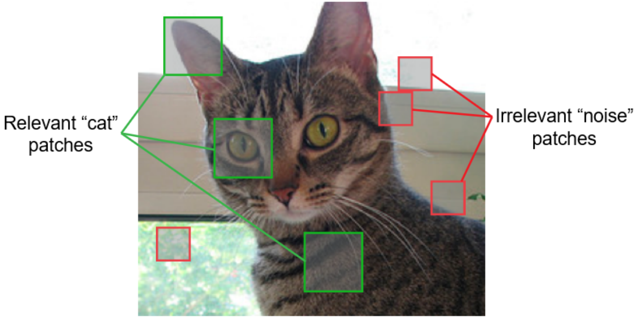

- Let's maximize MI between local patches and the output then!

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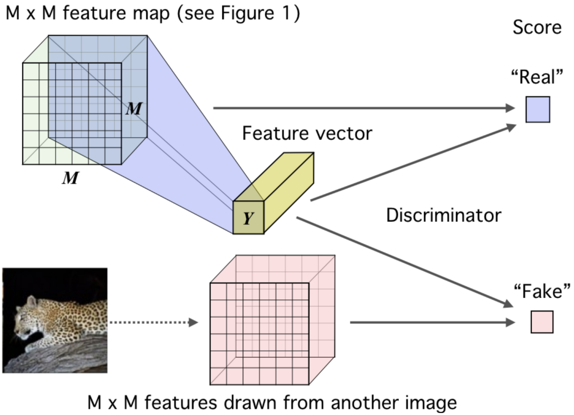

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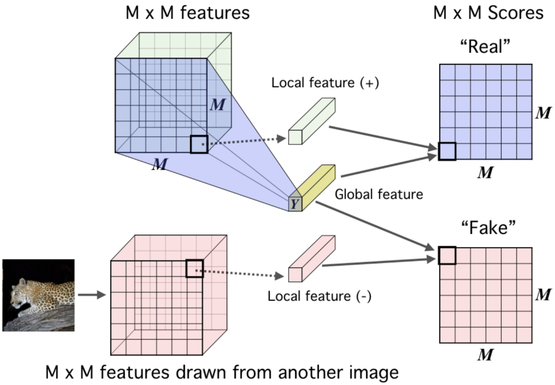

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- The paper is written quite ambigously and a lot of important details are scattered all over the manuscript... (i.e. how we do summarization of c into y, what prior do we use, etc)