Constants

$$\begin{split} c &= 2.99792458 \times 10^{10} \; \mathrm{cm \; s^{-1}} \\ G &= 6.6726 \times 10^{-8} \; \mathrm{cm^3 \; g^{-1} \; s^{-2}} \\ M_\odot &= 1.989 \times 10^{33} \; \mathrm{g} \end{split}$$

Units

Rest mass density ρ , energy density ϵ and pressure p are all measured in $g \, \mathrm{cm}^{-3}$. (One can convert any of them to $\operatorname{erg} \, \mathrm{cm}^{-3} = \operatorname{dyn} \, \mathrm{cm}^{-2}$ by multiplying with c^2). The specific enthalpy h is dimensionless and is defined by $h = 1 + \eta = (\epsilon + p) / \rho$.

Parametrized EOS

In the i-th interval $(\eta_{i-1} \leq \eta < \eta_i)$ the equation of state is parameterized by (astro-ph/0812.2163):

$$\rho(\eta) = \left[\frac{\eta - a_{i}}{K_{i}(n_{i} + 1)}\right]^{n_{i}}; \quad p(\eta) = K_{i}\rho(\eta)^{\Gamma_{i}}; \quad \epsilon(\eta) = \left(1 + \frac{a_{i} + n_{i}\eta}{n_{i} + 1}\right)\rho(\eta) \tag{1.1}$$

where $\Gamma_i = 1 + 1 / n_i$. Parameters for each EOS candidate are given below. Initial candidates (1-9) have only two pieces, but later candidates can have three or four. Please see an example (EOS SLy), appended at the end, for implementing a piecewise polytropic EOS with an arbitrary number of pieces.

EOS Candidate 1 (Candidate HB in gr-qc/0901.3258)

P = 13.45

 $p_{_1}=10^{^P}\,\mathrm{g\,cm^{^{-3}}}$ (pressure at the fiducial density $\rho_{_1}=10^{^{14.7}}\,\mathrm{g\,cm^{^{-3}}})$

Interval	Adiabatic index Γ_i	Dividing density (g cm ⁻³)	K_{i}	a_{i}	Dividing η
$0 \leq \rho < \rho_{\scriptscriptstyle 0}$	$\Gamma_{_0} = 1.35692395$		$K_0 = 3.99873692 \times 10^{-08}$	$a_{0} = 0$	
$\rho_{\scriptscriptstyle 0} \leq \rho$	$\Gamma_{_{1}}=3.0$	$\begin{split} \rho_{_{0}} &= 10^{\frac{P-\Gamma_{_{1}}\times 14.7 - \log_{10}K_{_{0}}}{\Gamma_{_{0}}-\Gamma_{_{1}}}} \\ &= 1.4172898657009434\times 10^{14} \end{split}$	0.00070110070000010=31	$\begin{aligned} a_{_{\! 1}} &= a_{_{\! 0}} + \frac{{_{\! K_{_{\! 0}}}}}{{_{\! \Gamma_{_{\! 0}}-1}}} \rho_{_{\! 0}}^{{_{\! \Gamma_{_{\! 0}}-1}}} - \frac{{_{\! K_{_{\! 1}}}}}{{_{\! \Gamma_{_{\! 1}}-1}}} \rho_{_{\! 0}}^{{_{\! \Gamma_{_{\! 1}}-1}}} \\ &= 0.01035069096964138 \end{aligned}$	$\begin{split} \eta_{_{0}} &= a_{_{\! 1}} + K_{_{\! 1}} \frac{\Gamma_{_{\! 1}}}{\Gamma_{_{\! 1}-1}} \rho_{_{\! 0}}^{\ \Gamma_{_{\! 1}-1}} \\ &= 0.017096105169027462 \end{split}$

With $P, K_0, \Gamma_0, \Gamma_1$ specified, one may algorithmically compute ρ_0, K_1, a_1, η_0 using the formulas above (last row).

EOS Candidate 4 (Candidate B in gr-qc/0901.3258)

$$P = 13.35$$

$$p_{_{\! 1}}=10^{^{P}}\,{\rm g\,cm^{^{-3}}}$$

Interval	Adiabatic index Γ_i	Dividing density (g cm ⁻³)	$K_{_i}$	a_{i}	Dividing η
$0 \leq \rho < \rho_{\scriptscriptstyle 0}$	$\Gamma_{_0}=1.35692395$		$K_0 = 3.99873692 \times 10^{-08}$	$a_{0} = 0$	
$\rho_{\scriptscriptstyle 0} \leq \rho$	$\Gamma_{\scriptscriptstyle 1}=3.0$	$\rho_{\scriptscriptstyle 0} = 1.630497500125504 \times 10^{^{14}}$	$K_1 = 1.778279410038963 \times 10^{-31}$	$a_{_{\! 1}}=0.01088158737430845$	$\eta_{\scriptscriptstyle 0} = 0.017972980036093586$

EOS Candidate 6 (B')

P = 13.35

 $p_{_{1}}=10^{^{P}}\,\mathrm{g\,cm}^{^{-3}}$

Interval	Adiabatic index Γ_i	Dividing density (g cm ⁻³)	K_{i}	a_{i}	Dividing η
$0 \leq \rho < \rho_{_{\! 0}}$	$\Gamma_{_0} = 1.35692395$		$K_0 = 3.99873692 \times 10^{-08}$	$a_0 = 0$	
$\rho_{\scriptscriptstyle 0} \leq \rho$	$\Gamma_{_1}=2.7$	$\rho_{\scriptscriptstyle 0} = 1.268785300163927 \times 10^{^{14}}$	$K_1 = 4.570881896148805 \times 10^{-27}$	$a_{_{\! 1}}=0.00956832301216044$	$\eta_{\scriptscriptstyle 0} = 0.015798256438541058$

EOS Candidate 7 (B'') P = 13.35

 $p_{_{1}}=10^{^{P}}\,\mathrm{g\,cm^{^{-3}}}$

Interval	Adiabatic index Γ_i	Dividing density (g cm ⁻³)	K_{i}	a_{i}	Dividing η
$0 \leq \rho < \rho_{_{0}}$	$\Gamma_{_0} = 1.35692395$		$K_0 = 3.99873692 \times 10^{-08}$	$a_0 = 0$	
$\rho_0 \leq \rho$	$\Gamma_{_1}=2.4$	$\rho_0 = 8.546653314476217 \times 10^{13}$	$K_1 = 1.17489755493955 \times 10^{-22}$	$a_{_{\! 1}}=0.00783658358515553$	$\eta_{\scriptscriptstyle 0} = 0.014272312295948284$

EOS Candidate 8 (HB')

P = 13.45

 $p_{_{1}}=10^{^{P}}\,\mathrm{g\,cm}^{^{-3}}$

Interval	Adiabatic index Γ_i	Dividing density (g cm ⁻³)	$K_{_i}$	$a_{_i}$	Dividing η
$0 \leq \rho < \rho_{\scriptscriptstyle 0}$	$\Gamma_{_0}=1.35692395$		$K_0 = 3.99873692 \times 10^{-08}$	$a_{0} = 0$	
$\rho_0 \leq \rho$	$\Gamma_{_1}=2.7$	$\rho_0 = 1.068887973752668 \times 10^{14}$	$K_1 = 5.754399373371614 \times 10^{-27}$	$a_1 = 0.009000377330119838$	$\eta_{\scriptscriptstyle 0} = 0.015458400028107366$

EOS Candidate 9 (HB'')

P = 13.45

 $p_{_{1}}=10^{^{P}}\,\mathrm{g\,cm}^{^{-3}}$

Interval	Adiabatic index Γ_i	Dividing density $(g\mathrm{cm}^{-3})$	K_{i}	a_{i}	Dividing η
$0 \leq \rho < \rho_{_{0}}$	$\Gamma_{_0} = 1.35692395$		$K_{_{0}} = 3.99873692 \times 10^{-08}$	$a_{_{0}} = 0$	
$\rho_0 \leq \rho$	$\Gamma_{_1}=2.4$	$\rho_0 = 6.853711209810516 \times 10^{13}$	$K_1 = 1.479108388168227 \times 10^{-22}$	$a_1 = 0.007242831283078954$	$\eta_{\scriptscriptstyle 0} = 0.01319094588294563$

Example of realistic EOS with four pieces: EOS SLy

P = 13.430358594144145

$$p_{_{1}}=10^{^{P}}\,\mathrm{g\,cm^{^{-3}}}$$

Interval	Adiabatic index Γ_i	Dividing density (g cm ⁻³)	$K_{_i}$	a_i	Dividing η
$0 \leq \rho < \rho_{_{0}}$	$\Gamma_{_0}=1.35692395$		$K_{_{0}} = 3.99873692 \times 10^{-08}$	$a_{_{0}} = 0$	
$\rho_0 \leq \rho < \rho_1$	$\Gamma_{\scriptscriptstyle 1}=3.005$	$\rho_{\scriptscriptstyle 0} = 10^{\frac{P-\Gamma_{\scriptscriptstyle 1}\times 14.7 - \log_{\scriptscriptstyle 10}K_{\scriptscriptstyle 0}}{\Gamma_{\scriptscriptstyle 0}-\Gamma_{\scriptscriptstyle 1}}}$	$K_{\scriptscriptstyle 1} = K_{\scriptscriptstyle 0} \rho_{\scriptscriptstyle 0}^{\; \Gamma_{\scriptscriptstyle 0} - \Gamma_{\scriptscriptstyle 1}}$	$a_{\!\scriptscriptstyle 1} = a_{\!\scriptscriptstyle 0} + \frac{{\scriptscriptstyle K_0}}{{\scriptscriptstyle \Gamma_0 - 1}} \rho_{\scriptscriptstyle 0}^{ {\scriptscriptstyle \Gamma_0 - 1}} - \frac{{\scriptscriptstyle K_1}}{{\scriptscriptstyle \Gamma_1 - 1}} \rho_{\scriptscriptstyle 0}^{ {\scriptscriptstyle \Gamma_1 - 1}}$	$\eta_{\scriptscriptstyle 0} = a_{\scriptscriptstyle 1} + K_{\scriptscriptstyle 1} \frac{\Gamma_{\scriptscriptstyle 1}}{\Gamma_{\scriptscriptstyle 1}-1} \rho_{\scriptscriptstyle 0}^{\ \Gamma_{\scriptscriptstyle 1}-1}$
$\rho_{\scriptscriptstyle 1} \leq \rho < \rho_{\scriptscriptstyle 2}$	$\Gamma_{_2}=2.988$	$\rho_1 = 10^{14.7} \; \mathrm{g} \; \mathrm{cm}^{-3}$	$K_{\scriptscriptstyle 2} = K_{\scriptscriptstyle 1} \rho_{\scriptscriptstyle 1}^{\;\Gamma_{\scriptscriptstyle 1}-\Gamma_{\scriptscriptstyle 2}}$	$a_{_{\! 2}}=a_{_{\! 1}}+\tfrac{{_{\! K_{_{\! 1}}}}}{{_{\! \Gamma_{_{\! 1}}-1}}}\rho_{_{\! 1}}^{{_{\! \Gamma_{_{\! 1}}-1}}}-\tfrac{{_{\! K_{_{\! 2}}}}}{{_{\! \Gamma_{_{\! 2}}-1}}}\rho_{_{\! 1}}^{{_{\! \Gamma_{_{\! 2}}-1}}}$	$\eta_{_{\! 1}} = a_{_{\! 2}} + K_{_{\! 2}} \frac{_{\Gamma_{_{\! 2}}}}{_{\Gamma_{_{\! 2}}-1}} \rho_{_{\! 1}}^{^{\; \Gamma_{_{\! 2}}-1}}$
$\rho_2 \leq \rho$	$\Gamma_{_3}=2.851$	$\rho_2 = 10^{15.0} \; \mathrm{g \; cm^{-3}}$	$K_{_{3}}=K_{_{2}}\rho_{_{2}}^{^{\;\Gamma_{_{2}}-\Gamma_{_{3}}}}$	$a_{_{\! 3}}=a_{_{\! 2}}+\tfrac{_{K_{_{\! 2}}}}{_{\Gamma_{_{\! 2}}-1}}\rho_{_{\! 2}}^{^{\;\;\Gamma_{_{\! 2}}-1}}-\tfrac{_{K_{_{\! 3}}}}{_{\Gamma_{_{\! 3}}-1}}\rho_{_{\! 2}}^{^{\;\;\Gamma_{_{\! 3}}-1}}$	$\eta_{_{2}}=a_{_{3}}+K_{_{3}}\tfrac{\Gamma_{_{3}}}{\Gamma_{_{3}}-1}\rho_{_{2}}^{\;\Gamma_{_{3}}-1}$

The table above demonstrates how the parameters in a piecewise polytropic EOS subroutine can be obtained:

The parameters $K_{\scriptscriptstyle 0}, \Gamma_{\scriptscriptstyle 0}, \rho_{\scriptscriptstyle 1}$ and $\rho_{\scriptscriptstyle 2}$ are fixed for all EOS;

the free parameters $\,P\,$ (or $\,p_{_{1}}$) and $\,\Gamma_{_{i}}\,$ (i>0) are $\it{specified}$ for each EOS;

then the remaining parameters $\, \rho_{_0} \, , \, \, K_{_i} \, , \, \, a_{_i} \, \, {\rm and} \, \, \, \eta_{_i} \, \, {\rm are} \, \, computed \, {\rm using} \, \, {\rm the} \, \, {\rm above} \, \, {\rm tabulated} \, \, {\rm formulas} \, .$

With all parameters thus obtained and stored, one can then use eq. (1.1) to compute the EOS.

Candidate 1 HB

Candidate 2 2H

Candidate 3 H

Candidate 4

Candidate 5 2B

В

Candidate 6 B'

Candidate 7 B"

Candidate 8 HB'

Candidate 9 HB"